OSWAL

High-Score Series

QUANTITATIVE APTITUDE

for Competitive Examinations

IBPS, SSC, SBI, RBI, AFCAT, CDS, NDA, UPSC, UPPSC, CAT, MAT, XAT, Railways, and Other Competitive Examinations

KEY FEATURES

- An ideal guide-cum-practice book in Quantitative Aptitude for competitive exams
- Explains all relevant topics in a learner-friendly manner
- Plenty of solved examples for easy understanding of the concepts
- Includes over 5000 topic-wise and previous year questions for practice

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Preface

Quantitative aptitude is an essential section of today's competitive exams. Thus, it becomes imperative for all aspirants to have a steady command over the subject to qualify the exams. Oswal's Objective Aptitude is a guidebook that has been prepared keeping in mind the topics and the types of questions asked in the competitive exams nowadays. It follows the 'Learn from Basics' concept which focuses on strengthening the subjective foundation of the learners. Practice questions are segregated on the basis of their difficulty level to increase their speed and accuracy.

The solutions to the questions are provided in a detailed manner to ensure clear understanding in one go. Previous year questions of various competitive exams are also added to help the students gauge the pattern and difficulty level of the exams of current times.

All efforts have been made to make this book error-free and easy to understand. All previous year questions are gathered from genuine sources. Nonetheless, all the readers are welcome to communicate their complaints, queries and suggestions to the publisher. Attempts will be made to inculcate them in the further editions.

Publisher

E-mail Etiquettes

When you are writing an e-mail, pause for a moment and ponder on the purpose of your mail—is it formal or informal? An e-mail is **formal** when you're writing a cover letter for a job or requesting an internship from your future boss and it is **informal** when you could have sent the same message through a text or over a casual conversation. In case you're unsure where your relationship with the recipient stands, it is better to compose a formal e-mail that will help you to present a metaphorical sharp-suited image on the reader.

The **subject line** is as crucial to a mail as the title is to a chapter. It gives a rough idea to the reader what your mail is about. Writing a contextually-relevant subject line will also help the recipient to draw references from the previous trail of mails.

Salutation is another part of a mail that is generally mistaken. It might come as a surprise to you but Hey Tom, style of greeting is technically erroneous; Dear Tom, or Hey, Tom is the correct way to go.

Make sure to proofread your e-mail before you hit the send button. You can now thank Gmail for the 'undo' option that allows you to correct and resend an error-ridden mail. However, you can follow the given tips to write an impeccable mail in the first go.

- Use bullet points wherever possible.
- Change paragraphs when a trail of thoughts ends and another one starts.
- Use **bold** text to emphasize on a point of importance.
- Avoid using informal abbreviations and shorthand spellings.
- Use proper punctuation marks and capitalisation..
- Do not overdo the ellipsis or the three dots that often trail off into Neverland.

It is important to **sign off** your mail in a tone that compliments the style in which it is written. A mail that begins with Respected Ma'am, will certainly not close with XOXO. Ensure that you mention your name and contact details in the signature so that the recipient can contact you further, if need be.

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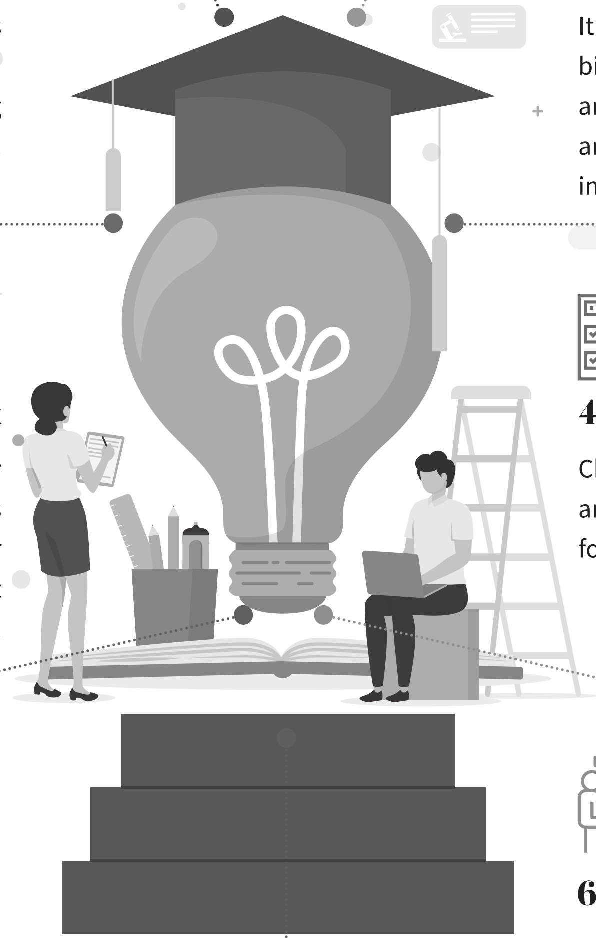
How to choose a GREAT CAREER



1. Know where your heart is

Whatever inspires, motivates and comes naturally to you should be the basis for making a career choice.

2. Explore related courses

It could be science, marine biology, astrophysics or anything, search what courses are there in your area of interest.

3. Reality check

Follow your heart but only after a reality check. It is advisable to explore other fields if your area of interest offers a limited scope.

4. Eligibility criteria

Check the eligibility criteria and know the requirements for the course.

5. Assess yourself

Weigh the pros and cons to see where can you apply.

6. Share and seek advice

Don’t hesitate. Speak to your parents or seek professional advice for the course selected.

7. The final decision

Once you are through with all the steps, you will know where you stand and will be better placed to make the right choice.

Go where your strength is, not where your friends are.

NUMBER SYSTEM

Chapter 1

NUMBERS

A number is a collection of digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

TYPES OF NUMBERS

Natural Number

The set of all positive numbers starting from 1 is called natural numbers. They are denoted by N.

N = {1, 2, 3, 4, 5, 6,..}

Whole Number

Natural numbers including zero are called the whole numbers. They are denoted by W.

W = {0, 1, 2, 3, 4, 5, 6,..}

> **Note :**
>
> *Every natural number is a whole number but every whole number is not a natural number.*

Even Number

A number which is divisible by 2 is called an even number.

Example : 2, 4, 6, 8, 10, 12, …

Odd Number

A number which is not divisible by 2 is called an odd number.

Example : 1, 3, 5, 7, 9, 11, 13, …

Prime Number

A number which is divisible by 1 and the number itself is called a prime number. It is always greater than 1.

Example : 2, 3, 5, 7, 11, 13, 17, 19, …

We know that 2 is an even number. Therefore 2 is called as an even prime number.

Example : How many prime numbers are there in between 1 and 100 ?

Solution :

There are 25 prime numbers in between 1 and 100.

They are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97.

Method to find the prime number greater than 100

Let us consider the given number as x .

Step 1 : Find the nearest whole number (y) which is greater than $\sqrt{x}$.

Step 2 : Check whether the x is divisible by any prime number which is less than y or not.

Step 3 : If yes, x is not a prime number, otherwise x is a prime number.

Ex. 1. Find whether the number 305 is prime or not.

Solution :

Nearest whole number of $\sqrt{305}$ is 18

i.e. $18 > \sqrt{305}$

Prime numbers less than 18 are 17, 13, 11, 7, 5, 3 and 2.

305 is divisible by 5. So, it is not a prime number.

Composite Number

A number which is not a prime number is called a composite number.

Example: 4, 6, 8, 9, 12, 14, 15, …

Composite numbers are always greater than 1.

> **Note :**
>
> *1 is neither prime nor composite.*

Co-Primes

If the HCF of two numbers is 1, then those numbers are called as co-primes.

Example : (3, 11), (5, 7), (2, 9), etc.

H.C.F of 3 and 11 is 1. So they are co-primes.

Similarly, (5, 7) and (2, 9) are also co-primes.

Integers

The set of all positive and negative numbers including 0 is called the integers. They are denoted by Z.

$$Z = \{...,-5,-4,-3,-2,-1,0,1,2,3,4,5,...\}$$

Positive integers: {1, 2, 3, 4, 5, …}

Negative integers: {-1, -2, -3, -4, …}

Non-negative integers: {0, 1, 2, 3, 4, 5, …}

Non-positive integers: {0, -1, -2, -3, -4, …}

Where 0 is neither positive nor negative.

Rational Number

A number in the form of $\frac{p}{q}$ is called a rational number, where p and q are integers and $q \neq 0$. Rational numbers are denoted by Q.

Example : $-\frac{2}{5}, \frac{4}{10}, \frac{2}{1}$......

Irrational Number

A number is said to be irrational if it cannot be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Example : $\sqrt{3}$, $\sqrt{7}$ etc.

Real Number

The set of all rational and irrational numbers is known as real number. It is denoted by a letter R.

$$R = \left\{...., -\frac{1}{2}, 0, \frac{1}{3}, 1, 2, \sqrt{7}, ...\right\}$$

Consecutive number

In a series of numbers, if each number is greater than the preceding number by 1, then the series of numbers is called consecutive numbers.

Example : 5, 6, 7, 8 or 789, 790, 791, 792, 793 or 1001, 1002, 1003.

FACE VALUE AND PLACE VALUE

Every digit of a number has a face value and a place value.

The places can be represented as

Ten Crores	100000000	2
Crores	10000000	1
Ten Lakhs	1000000	7
Lakhs	100000	5
Ten Thousands	10000	1
Thousands	1000	6
Hundreds	100	7
Tens	10	0
Ones	1	1

We can read the number 217516701 as "Twenty-one crores seventy five lakhs sixteen thousand seven hundred and one".

The face value of a digit is the same digit which does not depend on the position of the digit.

The place value of a digit represents the times of digit's position *i.e.* increases in powers of 10, starting from the unit place.

Example : In 2046, the face value of 4 is 4 and the place value of 4 is . $4 \times 10 = 40$

Similarly, the face value of 0 is 0 and the place value of 0 is . $0 \times 100 = 0$

The face value of 6 is 6 and the place value of 6 is. $6 \times 1 = 6$.

The face value of 2 is 2 and the place value of 2 is .

$2 \times 1000 = 2000$

Expanded Form

Any number can be expanded by using the place values.

For example, the expanded form of 2046 is,

$$2 \times 10^3 + 0 \times 10^2 + 4 \times 10^1 + 6 \times 10^0$$

ADDITION

Adding numbers by dividing in terms of place values

Example : 3267 + 531

Step 1 : Divide the numbers in terms of hundreds, tens and ones.

$3267 = 3000 + 200 + 60 + 7$

$531 = 500 + 30 + 1$

Step 2 : Add the numbers

$3000 + 200 + 60 + 7 + 500 + 30 + 1$

$= 3000 + 700 + 90 + 8$

$= 3798$

Therefore, $3267 + 531 = 3798$.

Adding numbers having similar digits in increasing order

Example : 4 + 44 + 444 + 4444 + 44444

Step 1 : Take the common digit and count the number of digits in each number.

$4\,(1 + 11 + 111 + 1111 + 11111)$

Step 2 : Make the digits by counting the digits as a single number

i.e. 1 = 1, 11 = 2, 111 = 3, 1111 = 4, 11111 = 5

Then the number will be 12345

Step 3 : Multiply the number with 4, then

we will get $4 \times 12345 = 49380$.

Therefore, 4 + 44 + 444 + 4444 + 44444 = 49380.

Adding similar digits having decimal points

Example : 0.7 + 0.77 + 0.777 + 0.7777 + 0.77777

Step 1 : Take the common digit from the numbers

$7(0.1 + 0.11 + 0.111 + 0.1111 + 0.11111)$

Step 2 : Solve the addition inside the bracket first.

$0.1 + 0.11 + 0.111 + 0.1111 + 0.11111 = 0.54321$

Step 3 : Multiply the result of step 2 with 7.

$$7 \times 0.54321 = 3.80247$$

Therefore, 0.7 + 0.77 + 0.777 + 0.7777 + 0.77777= 3.80247

Ex. 2. 7.7 + 77.77 + 777.777 + 7777.7777

Solution :

We can take 7 outside as it is common in all the numbers, then we get

7(1.1 + 11.11 + 111. 111 +1111.1111 + 11111.11111)

= 7 ((1 + 11 + 111 + 1111 + 11111) + (0.1 + 0.11 + 0.111 + 0.11111))

= 7 (1234 + 0.4321)

(7 × 1234) + (7 × 0.4321) = 8638 + 3.0247

= 8641.0247

Adding mixed fractional numbers

Example : $2\frac{1}{3}+1\frac{1}{4}+3\frac{1}{2}+5\frac{1}{5}$

Step 1 : Add the whole numbers

2 + 1 + 3 + 5 = 11

Step 2 : Add the fractional values

$$\frac{1}{3}+\frac{1}{4}+\frac{1}{2}+\frac{1}{5}$$

LCM of 3, 4, 2 and 5 is 60

$$\frac{1}{3}+\frac{1}{4}+\frac{1}{2}+\frac{1}{5}=\frac{20+15+30+6}{60}=\frac{71}{60}=1\frac{11}{60}$$

Step 3 : Add the results of both step 1 and step 2.

$$2\frac{1}{3}+1\frac{1}{4}+3\frac{1}{2}+5\frac{1}{5}=11+1\frac{11}{60}=12\frac{11}{60}.$$

SUBTRACTION

Subtracting numbers by dividing in terms of place values

Example: 867 - 531

Step 1 : Divide the number in terms of hundreds and tens.

867 = 800 + 67

531 = 500 + 31

Step 2 : Subtract 500 from 800 and 31 from 67

800 – 500 = 300

67 – 31 = 36

Step 3 : Add the results

300 + 36 = 336.

MULTIPLICATION

Multiplying 2-digit numbers

Example : 42 × 73

Step 1 : Multiply the unit's digits

```
 42
  | ×
 73
----
  6
```

Step 2 : Multiply the last digit of first value and first digit of second value and vice versa. Then add the results.

```
 42        (4×3) + (2×7)
  X ×        12+14=26
 73
----
  6
```

Step 3 : Write the last digit before the step 1's answer and carry forward the first digit to next step.

```
 42        (4×3) + (2×7)
  X ×        12+14=26
 73               ↓
----        Carry Forward
 66
```

Step 4 : Multiply the first digit of the first value with the first digit of the second value.

```
   42        (4×7) = 28
    | ×          +2 (Carry)
   73       -------------
 -----            = 30
 30 66
```

Therefore, $42 \times 73 = 3066$.

Multiplying 3-digit numbers

Example : 142×734

Step 1 :

```
 142        (2×4) = 8
   | ×
 734
 ----
   8
```

Step 2 :

```
 142        (4×4) + (2×3)
  X ×        16 + 6 = 22
 734                 ↓
 ----          Carry Forward
  28
```

Step 3 :

```
 142          (1×4) + (4×3) +(2×7)
  X   ×         4 + 12 + 14 = 30
 734                     + 2 (Carry)
 ---          --------------------
 228                      = 32
                             ↓
                       Carry Forward
```

Step 4 :

```
 142          (1×3) + (4×7)
  X   ×         3 + 28 = 31
 734               + 3 (Carry)
 ----           ---------------
 4228             = 34
                      ↓
                 Carry Forward
```

Step 5 :

```
 142          (1×7) =7
  |   ×            + 3 (Carry)
 734            -----------
 ------             =[10]
 [10]4228
```

Therefore, 142 × 734 = 104228

Multiplying by 9

Put a zero after the last digit of the number and subtract the original number from that number.

Ex. 3. What is the value of 9562417 × 9 ?

Solution :

$$9562417 \times 9 = 95624170 - 9562417$$
$$= 86061753$$

Multiplying by 11

Step 1 : Write 0 in front of the first digit of the multiplicand.

Step 2 : Keep the last digit of the number of the multiplicand in the result as it is.

Step 3 : Add the successive digits of the multiplicand to its preceding digit, then put the last digit of the result before the result of step 2. If carry is there, add it to the successive digit.

Ex. 4. What is the value of 9562417 × 11 ?

Solution :

Step 1 :

09562417 × 11

Step 2 :

09562417 × 11 = _ _ _ _ _ _ _ <u>7</u>

Step 3 :

1 + 7 = 8

09562417 × 11 = _ _ _ _ _ _ <u>87</u>

4 + 1 = 5

09562417 × 11 = _ _ _ _ _ <u>587</u>

2 + 4 = 6

09562417 × 11 = _ _ _ _ <u>6587</u>

6 + 2 = 8

09562417 × 11 = _ _ _ <u>86587</u>

5 + 6 = 11 (1 - Place it before 8, 1 - carry forward)

09562417 × 11 = _ _ <u>186587</u>

9 + 5 + 1 = 15 (5 - Place it before 1, 1 - carry forward)

09562417 × 11 = _ <u>5186587</u>

0 + 9 + 1 = 10

9562417 × 11 = <u>105186587</u>

Therefore, . 9562417 × 11 = 105186587

Multiplying by 12

Step 1 : Write 0 in front of the first digit of the multiplicand

Step 2 : Double the last digit of the multiplicand and place the last digit of the result in answer. If carry is there, then forward it to the next step.

Step 3 : Move left to next digit of the multiplicand, then double it and add to its previous digits. Add the carry from previous step and write down the last digit of the result and forward the carry to next step.

Step 4 : Repeat step 3 for the successive digits.

Ex. 5. What is the value of 9562417 × 12 ?

Solution :

Step 1 :

09562417×12

Step 2 :

7 × 2 = 14

09562417 × 12 = _ _ _ _ _ _ _ 4 [Carry 1]

Step 3 :

(1 × 2) + 7 + 1 = 10

09562417 × 12 = _ _ _ _ _ _ 04 [Carry 1]

Step 4 :

(4 × 2) + 1 + 1 = 10

09562417 × 12 = _ _ _ _ _ 004 [Carry 1]

(2 × 2) + 4 + 1 = 9

09562417 × 12 = _ _ _ _ 9004 [Carry 0]

(6 × 2) + 2 + 0 = 14

09562417 × 12 = _ _ _ 49004 [Carry 1]

(5 × 2) + 6 + 1 = 17

09562417 × 12 = _ _ 749004 [Carry 1]

(9 × 2) + 5 + 1 = 24

09562417 × 12 = _ 4749004 [Carry 2]

(0 × 2) + 9 + 2 = 11

09562417 × 12 = 114749004

Therefore, 09562417 × 12 = 114749004.

Multiplying by 13

Step 1 : Write 0 in front of the first digit of the multiplicand.

Step 2 : Triple the last digit of the multiplicand and place the last digit of the result in answer. If carry is there, then forward it to the next step.

Step 3 : Move left to next digit of the multiplicand, then triple it and add to its previous digits. Add the carry from previous step and write down the last digit of the result and forward the carry to next step.

Step 4 : Repeat step 3 for the successive digits.

Ex. 6. What is the value of 9562417 × 13 ?

Solution:

Step 1 :

09562417×13

Step 2 :

7 × 3 = 21

09562417 × 13 = _ _ _ _ _ _ _ 1 [Carry 2]

Step 3 :

(1 × 3) + 7 + 2 = 12

09562417 × 13 = _ _ _ _ _ _ 21 [Carry 1]

Step 4 :

(4 × 3) + 1 + 1 = 14

09562417 × 13 = _ _ _ _ _ 421 [Carry 1]

(2 × 3) + 4 + 1 = 11

09562417 × 13 = _ _ _ _ 1421 [Carry 1]

(6 × 3) + 2 + 1 = 21

09562417 × 13 = _ _ _ 11421 [Carry 2]

(5 × 3) + 6 + 2 = 23

09562417 × 13 = _ _ 311421 [Carry 2]

(9 × 3) + 5 + 2 = 34

09562417 × 13 = _ 4311421 [Carry 3]

(0 × 3) + 9 + 3 = 12

09562417 × 13 = 124311421

Therefore, 09562417 × 13 = 124311421.

Multiplying a number with 25

Step 1 : Replace 25 with 100/4.

Step 2 : Divide the given number by 4.

Step 3 : Multiply the result with 100 to get the answer.

Ex. 7 What is the value of ?15768 × 25 ?

Solution :

Step 1 : $15768 \times 25 = 15768 \times \frac{100}{4}$

Step 2 : $15768 \times \frac{100}{4} = 3942 \times 100$

Step 3 : 3942 × 100 = 394200

Therefore, 15768 × 25 = 394200.

DIVISION

There are some easy methods to solve 2-digits and 3-digits division.

Division by a 2-digit number

Example : 82964 ÷ 91

Step 1 : Put 9 in the divisor column and 1 on top of the flag and then allocate one place to remainder in the right-end of dividend.

9^1	8	2	9	6	4

Step 2 : As first digit of dividend is lesser than first digit of quotient, take the first 2 digits and then divide it by 9.

i.e. 82 ÷ 9 Quotient = 9 and Remainder 1. Place the remainder before the third digit of the dividend *i.e.* 9.

9^1	82	$_1 9$	6	4
	9			

Step 3 : Now 19 is the dividend. Multiply the last digit of quotient with the flag-digit, then subtract the result from the dividend.

i.e. , 1 × 9 = 9 then 19 – 9 = 10

Then divide 10 by 9.

10 ÷ 9 ⇒ Quotient = 1 and Remainder = 1. Place the remainder before 6 of the dividend.

9^1	82	$_1 9$	$_1 6$	4
	9	1		

Step 4 : Now 16 is the dividend. Multiply the last digit of quotient with the flag-digit, then subtract the result from the dividend.

i.e. 1 × 1 = 1, then 16 – 1 = 15

Then divide 15 by 9.

15 ÷ 9 ⇒ Quotient = 1 and Remainder = 6. Place the remainder before 4 of the dividend.

9^1	82	$_1 9$	$_1 6$	$_6 4$
	9	1	1	

Step 5 : Now 64 is the dividend. Multiply the last digit of quotient with the flag-digit, then subtract the result from the dividend.

i.e. 1 × 1 = 1, then 64 – 1 = 63

Then divide 63 by 9.

63 ÷ 9 ⇒ Quotient = 7 and Remainder = 0.

9^{1}	82	$_{1}9$	$_{1}6$	$_{6}4$
	9	1	1	7

Therefore, the quotient is 911.7

Thus, 82964 ÷ 91 = 911.7

Division by a 3-digit number

Example: 1363841 ÷ 642

Step 1 :

6^{42}	13	6	3	8	4	1

Step 2 :

13 ÷ 6 ⇒ Quotient = 2 and Remainder = 1

6^{42}	13	$_{1}6$	3	8	4	1
	2					

Step 3 :

Subtract the dividend from the product of first digit of flag-digit and the last digit of quotient.

16 – (2 × 4) = 16 – 8 = 8

8 ÷ 6 ⇒ Quotient = 1 and Remainder = 2

6^{42}	13	$_{1}6$	$_{2}3$	8	4	1
	2	1				

Step 4 :

Subtract the dividend from the cross multiplication of 2 flag-digits and last 2-quotients to get the actual dividend.

23 – [(4 × 1) + (2 ×2)] = 23 – (4 + 4) = 15

15 ÷ 6 ⇒ Quotient = 2 and Remainder = 3

6^{42}	13	$_{1}6$	$_{2}3$	$_{3}8$	4	1
	2	1	2			

Step 5 :

Subtract the dividend from the cross multiplication of 2 flag-digits and last 2-quotients to get the actual dividend.

36 – [(4 × 2) + (2 × 1)] = 38 – (8 + 2) = 28

28 ÷ 6 ⇒ Quotient = 4 and Remainder = 4

6^{42}	13	$_{1}6$	$_{2}3$	$_{3}8$	$_{4}4$	1
	2	1	2	4		

Step 6 :

Subtract the dividend from the cross multiplication of 2 flag-digits and last 2-quotients to get the actual dividend.

44 – [(4 × 4) + (2 × 2)] = 44 – (16 + 4) = 24

24 ÷ 6 ⇒ Quotient = 4 and Remainder = 0

6^{42}	13	$_{1}6$	$_{2}3$	$_{3}8$	$_{4}4$	$_{0}1$
	2	1	2	4	4	

Therefore, 1363841 ÷ 642 = 2124.4 is the quotient.

TEST OF DIVISIBILITY

"Divisible By" means "when you divide one number by another, the result is a whole number.

Divisibility by 2

Numbers ending with the digits 0, 2, 4, 6, 8 are divisible by 2.

Example: 15086 is divisible by 2 as its last digit is 6.

15085 is not divisible by 2 as its last digit is not ending with 0, 2, 4, 6 and 8.

Divisibility by 3

If the sum of the digits of a number is divisible by 3, then the number is divisible by 3.

Ex. 8. Find whether 15086 is divisible by 3 or not.

Solution :

Sum of the digits = 1 + 5 + 0 + 8 + 6 = 20

20 is not divisible by 3.

So, 15086 is not divisible by 3.

Ex. 9. Find whether 97531245 is divisible by 3 or not.

Solution :

Sum of the digits = 9 + 7 + 5 + 3 + 1 + 2 + 4 + 5 = 36

36 is divisible by 3.

So, 97531245 is divisible by 3.

Divisibility by 4

If the last two digits of a number is divisible by 4, then the number is divisible by 4.

Ex. 10. Find whether 95874314 is divisible by 4 or not.

Solution :

Last two digits of the given number 95874314 is 14.

We know that, 14 is not divisible by 4.

So, 95874314 is also not divisible by 4.

Ex. 11. Find whether 98764392 is divisible by 4 or not.

Solution :

Last two digits of the given number 98764392 is 92.

92 is divisible by 4.

So, 98764392 is also divisible by 4.

Divisibility by 5

A number ending with either 0 or 5, is completely divisible by 5.

Ex. 12. Is the number 856214680 divisible by 5 ?

Solution :

Yes, the number 856214680 is divisible by 5 as its last digit is ending with 0.

Divisibility by 6

Check for the divisibility by 2 and 3.

Ex. 13. Is the number 8572146 divisible by 6 ?

Solution :

To find the divisibility by 6, we need to check whether number is divisible by both 2 and 3 or not.

For divisibility by 2: The last digit of the given number 8572146 is 6 which is divisible by 2.

For divisibility by 3: The sum of the digits in the given number 8572146 is

$8 + 5 + 7 + 2 + 1 + 4 + 6 = 33$

33 is divisible by 3.

Therefore, the number 8572146 is divisible by 6 as it is divisible by both 2 and 3.

Divisibility by 7

Step 1 : Multiply the last digit by 2.

Step 2 : Subtract the result from the remaining digits.

Step 3 : Check whether the answer is divisible by 7 or not. If it is divisible, then the original number will also be divisible.

Ex. 14. Find whether the number 2478 is divisible by 7 or not.

Solution :

To check the divisibility by 7, multiply the last digit by 2 and subtract the result from the remaining digits.

For 2478, $8 \times 2 = 16$

Then $247 - 16 = 231$

231 is divisible by 7.

Therefore, 2479 is divisible by 7.

Divisibility by 8

If the last three digits of a number is divisible by 8, then the number is divisible by 8.

Ex. 15. Find whether the number 5796952 is divisible by 8 or not.

Solution :

We know that, if the last three digits of a number is divisible by 8, then that whole number is also divisible by 8.

The last 3-digits of 5796952 is 952.

952 is divisible by 8.

Therefore, 5796952 is divisible by 8.

Divisibility by 9

If the sum of the digits of a number is divisible by 9, then the number is divisible by 9.

Ex. 16. Find whether 6574123058 is divisible by 9 or not.

Solution :

To find the divisibility by 9, we have to add the digits of the number.

Sum of the digits of 6574123058

$= 6 + 5 + 7 + 4 + 1 + 2 + 3 + 0 + 5 + 8 = 41$

41 is not divisible by 9.

So, 6574123058 is not divisible by 9.

Divisibility by 10

A number ends with 0 is divisible by 10.

Example: 1874620 is divisible by 10.

65247900 is divisible by 10.

1856975 is not divisible by 10.

6784305 is not divisible by 10.

Divisibility by 11

A number is said to be divisible by 11 if the difference between the sum of digits at odd places of a number and the sum of digits at even places of the same number is equal to either '0' or 'a number divisible by 11'.

Ex. 17. Is the number 4984177 divisible by 11 ?

Solution :

Sum of odd digits – Sum of even digits

$= (7+1+8+4) - (7+4+9)$

$= 20 - 20 = 0$

So, 4984177 is divisible by 11.

Divisibility by 12

A number which is divisible by both 4 and 3 can be divided by 12.

Ex. 18. Is the number 96124572 is divisible by 12 ?

Solution :

To check the divisibility by 12, first we have to check the divisibility of 3 and 4.

For divisibility by 3: The sum of the digits in the given number 96124572 is

9 + 6 + 1 + 2 + 4 + 5 + 7 + 2 = 36

36 is divisible by 3.

For divisibility by 4: Last 2-digits of the given number is 72 which is divisible by 4.

Therefore, the number 96124572 is divisible by 12.

Divisibility by 13

Step 1 : Multiply the last digit by 4.

Step 2 : Add the result to the remaining digits.

Step 3 : Check whether the answer is divisible by 13 or not. If it is divisible, then the original number will also be divisible.

Ex. 19. Find whether the number 6877 is divisible by 13 or not.

Solution :

Last digit of the given number is 7.

Then multiply 4 with 7, we get 28

Add 28 to the remaining number 687, then we get

687 + 28 = 715

715 is divisible by 13.

So, 6877 is also divisible by 13.

Divisibility by 14

A number which is divisible by both 2 and 7 can be divided by 14.

Ex. 20. Is the number 85708 is divisible by 14 ?

Solution :

To check the divisibility by 14, first we have to check the divisibility of 2 and 7.

For divisibility by 2: The last digit of the given number 85708 is 8 which is divisible by 2.

For divisibility by 7: Multiply the last digit with 2 and then subtract the result from the remaining digits.

8 × 2 = 16

8570 – 16 = 8554 which is divisible by 7.

Therefore, the number 85708 is divisible by 14 as it is divisible by both 2 and 7.

Divisibility by 15

A number which is divisible by both 3 and 5 can be divided by 15.

Ex. 21. Is the number 591285 is divisible by 15 ?

Solution :

To check the divisibility by 15, first we have to check the divisibility of 3 and 5.

For divisibility by 3: The sum of the digits in the given number 591285 is

5 + 9 + 1 + 2 + 8 + 5 = 30

30 is divisible by 3.

For divisibility by 5: The number 591285 ends with 5, so 591285 is divisible by 5.

Therefore, the number 591285 is divisible by 15 as it is divisible by both 3 and 5.

Divisibility by 17

Step 1 : Multiply the last digit by 5.

Step 2 : Subtract the result from the remaining digits.

Step 3 : Check whether the answer is divisible by 17 or not. If it is divisible, then the original number will also be divisible.

Ex. 22. Is the number 8526 divisible by 17 ?

Solution :

To check the divisibility by 17, multiply the last digit by 5 and subtract the result from the remaining digits.

For 8526, 6 × 5 = 30

Then 852 – 30 = 822

822 is not divisible by 17.

Therefore 8526 is not divisible by 17.

Divisibility by 19

Step 1 : Multiply the last digit by 2.

Step 2 : Add the result to the remaining digits.

Step 3 : Check whether the answer is divisible by 19 or not. If it is divisible, then the original number will also be divisible.

Ex. 23. Is the number 8626 divisible by 19 ?

Solution :

Multiply the last digit by 2

6 × 2 = 12

Then add the result to the remaining digits, we get

862 + 12 = 874 which is divisible by 19.

So, the number 8626 is divisible by 19.

Division Algorithm

Dividend = (Divisor × Quotient) + Remainder

$$\text{Divisor} = \frac{\text{Dividend} - \text{Remainder}}{\text{Quotient}}$$

$(x^n - a^n)$ is divisible by $(x - a)$ for all values of n.

$(x^n - a^n)$is divisible by $(x + a)$ if n is even.

$(x^n + a^n)$ is divisible by $(x + \text{a})$ if n is odd.

Ex. 24. On dividing 42318 by a certain number, the quotient is 61 and the remainder is 45. What is the divisor?

Solution :

We know that,

$$\text{Divisor} = \frac{\text{Dividend} - \text{Remainder}}{\text{Quotient}}$$

$$\text{Divisor} = \frac{42318 - 45}{61} = \frac{42273}{61} = 693$$

Ex. 25. What is the least number that must be added to 6148 to obtain a number exactly divisible by 13 ?

Solution :

$$= \frac{6148}{13}$$

$\Rightarrow$ Quotient = 472, Remainder = 12

$\therefore$ The required number to be added = 13 – 12 = 1.

Verification:

6148 + 1 = 6149

Dividing 6149 by 13, we get quotient = 473 and remainder = 0.

Hence, 1 is the least number that must be added to 6148 to obtain a number exactly divisible by 13.

Ex. 26. What is the least number that must be subtracted from 6148 to get a number exactly divisible by 13 ?

Solution :

$$= \frac{6148}{13}$$

$\Rightarrow$ Quotient = 472, Remainder = 12

$\therefore$ The required number to be subtracted = 12.

i.e. 6148 – 12 = 6136

Dividing 6136 by 13, we get quotient = 472 and remainder = 0.

Hence, 12 is the least number that must be subtracted from 6148 to obtain a number exactly divisible by 13.

UNIT'S DIGIT OF A LARGE EXPONENT

To find the unit's digit of large exponent a^x where a is the base and x is the exponent, follow the steps given below.

Step 1 : Divide the exponent x by 4.

Step 2 : If we get the remainder 0, then

- The last digit of a^x will be 6 for the last digit of base as 2, 4, 6, 8.
- The last digit of a^x will be 1, when the last digit of base is 3, 7, 9.

If we get the remainder non-zero, then

- The last digit of a^x will be the last digit of l^r where is l the last digit of the base and r is the remainder.

Ex. 27. What is the last digit in the expansion 18^{352} ?

Solution :

Let's divide 352 by 4, remainder will be 0.

Since the last digit of base is 8 and 352 is divisible by 4, the unit's digit of 18^{352} is 6.

Ex. 28. Find the unit's digit of .3256^{2147}

Solution :

To find the unit's digit of 3256^{2147}, we have to divide 2147 by 4.

2147 ÷ 4 $\Rightarrow$ Quotient = 536 and Remainder = 3

The last digit of base, l = 6 and remainder, r = 3

Therefore, the unit's digit of 3256^{2147} = The unit's digit of 6^3 = 6.

Ex. 29. When 3^{21} is divided by 5, what will we get as remainder ?

Solution :

$$3^{21} = (3^{10} \times 3^{10}) \times 3 = (3^{10})^2 \times 3$$

$$\text{Unit's digit in } 3^{21} = \text{Unit's digit in } (3^5)^4 \times 3$$

$$= \text{Unit's digit in } (243)^4 \times 3$$

$$= (1 \times 3) = 3$$

remainder = 3

PRACTICE QUESTIONS

LEVEL – EASY

1. If A896 + 107 is completely divisible by 3, the least value of A is

(a) 0 (b) 1

(c) 2 (d) 9

2. What will be the remainder when $29^{100} - 1^{100}$ is divisible by 30 ?

(a) 1 (b) 10

(c) 29 (d) 30

3. The difference between the place and face value of 0 in the number 2017 is

(a) 0 (b) 99

(c) 100 (d) 101

4. The difference between the place values of two 3's in the number 938638.

(a) 9997 (b) 2970

(c) 29997 (d) 29970

5. $8 \times 10^5 + 1 \times 10^3 + 6 \times 10^1 + 9 \times 10^0 = ?$
(a) 801069 (b) 81069
(c) 80169 (d) 8010609

6. Which number is divisible by 13 ?
(a) 1954 (b) 1101
(c) 1872 (d) 1517

7. Multiply the last digit by 2 and add the result to the remaining digits to find the divisibility of which number ?
(a) 19 (b) 13
(c) 17 (d) 14

8. Pick out the prime number.
(a) 802 (b) 803
(c) 809 (d) 807

9. Find the place value of "K" in the number 4K1396. Assume K is the even prime number
(a) 20000 (b) 200
(c) 200000 (d) 2000

10. Tanshika has 265476 flags. 7542 flags are orange colour and rest are violet. How many violet flags does she have ?
(a) 252834 (b) 257934
(c) 247934 (d) 246222

11. The difference between the place value and the face value of 9 in the numeral 429861 is
(a) 8381 (b) 8991
(c) 8101 (d) 7931

12. 8340 – 3215 – 1132 = ?
(a) 3992 (b) 3963
(c) 3923 (d) 3993

13. Which of the following numbers is divisible by 3 ?
(a) 267851 (b) 632817
(c) 547191 (d) 291754

14. Find the least value of * for which 6876*22 is divisible by 3.
(a) 6 (b) 3
(c) 2 (d) 4

15. Which of the following numbers is divisible by 4 ?
(a) 92624782 (b) 47829654
(c) 47820264 (d) 48724838

16. Among the following numbers which is divisible by 8 ?
(a) 5368 (b) 1986
(c) 2897 (d) 2018

17. 8325 is divisible by __________.
(a) 9 (b) 7
(c) 13 (d) 11

18. The sum of all those prime numbers which are not greater than 17 is
(a) 59 (b) 58
(c) 41 (d) 42

19. The number of integers in between 100 and 600, which are divisible by 4 and 6 both, is
(a) 40 (b) 42
(c) 41 (d) 50

20. Find the largest number which when subtracted from 10000, the remainder is divisible by 32, 36, 48 and 50.
(a) 8272 (b) 7408
(c) 9136 (d) 8674

21. 337.62 + 8.591 + 34.4 is equal to
(a) 370.611 (b) 380.511
(c) 380.611 (d) 426.97

22. Find the sum of $\frac{1}{9}+\frac{1}{6}+\frac{1}{12}+\frac{1}{72}$
(a) $\frac{3}{5}$ (b) $\frac{3}{2}$
(c) $\frac{3}{8}$ (d) $\frac{4}{7}$

23. The value of (?) in the equation
365.089 - ? + 89.72 = 302.35 is
(a) 152.456 (b) 152.459
(c) 153.456 (d) 153.459

24. The number of girls in a class is five times the number of boys. Which of the following cannot be the total number of children in the class ?
(a) 24 (b) 30
(c) 35 (d) 54

25. 14.5 × 17.3 × 10 .8 = ?
(a) 2709.18 (b) 2079.08
(c) 2907.28 (d) 2790.80
(e) None of these

26. 333.33 + 33.33 + 3.3 = ?
(a) 369.69 (b) 369.96
(c) 396.96 (d) 396.69
(e) None of these

27. 144 × 83 – 9090 = ?
(a) 2862 (b) 2682
(c) 2286 (d) 2268
(e) None of these

28. 8052 – 9883 + 6048 =? × 25

(a) 168.68 (b) 186.68

(c) 186.86 (d) 168.86

(e) None of these

29. 8536 + 5824 = ? × 40

(a) 336 (b) 359

(c) 363 (d) 395

(e) None of these

30. 566.91 + 551.34 + 114.98 =?

(a) 1233.23 (b) 1222.33

(c) 1223.45 (d) 1235.88

(e) None of these

31. 3463 × 295 – 18611 = ? + 5883

(a) 997091 (b) 887071

(c) 989090 (d) 899060

(e) None of these

32. 4003 × 77 – 21015 = ? × 116

(a) 2477 (b) 2478

(c) 2467 (d) 2476

(e) None of these

33. 18.5 × 21.4 × ? = 6255.22

(a) 15.8 (b) 14.6

(c) 17.4 (d) 17.2

(e) 16.4

34. 302.46 + 395.72 – 123.47 =?

(a) 576.77 (b) 547.17

(c) 547.77 (d) 574.71

(e) 577.71

35. 0.2 × 1.1 + 0.6 × 0.009 = ? – 313.06

(a) 353.2184 (b) 353.2854

(c) 331.54 (d) 313.2854

(e) 331.2854

36. 7589 – ? = 3434

(a) 4242 (b) 4155

(c) 1123 (d) 11023

(e) None of these

37. 3251 + 587 + 369 – ? = 3007

(a) 1250 (b) 1300

(c) 1375 (d) 1200

(e) None of these

38. (4300731) – ? = 2535618

(a) 1865113 (b) 1775123

(c) 1765113 (d) 1675123

(e) None of these

39. $9\frac{3}{4}+7\frac{2}{17}-9\frac{1}{15}=?$

(a) $7\frac{719}{1020}$ (b) $9\frac{817}{1020}$

(c) $9\frac{719}{1020}$ (d) $7\frac{817}{1020}$

(e) None of these

40. – 84 × 29 + 365 = ?

(a) 2436 (b) 2801

(c) –2801 (d) –2071

(e) None of these

41. 4500 × ? = 3375

(a) $\frac{2}{5}$ (b) $\frac{3}{4}$

(c) $\frac{1}{4}$ (d) $\frac{3}{5}$

(e) None of these

42. 8988 ÷ 8 ÷ 4 = ?

(a) 4494 (b) 561.75

(c) 2247 (d) 280.875

(e) None of these

43. 666 ÷ 6 ÷ 3 = ?

(a) 37 (b) 333

(c) 111 (d) 84

(e) None of these

44. 35 + 15 × 1.5 = ?

(a) 75 (b) 51.5

(c) 57.5 (d) 5.25

(e) None of these

45. 5358 × 51 = ?

(a) 273258 (b) 273268

(c) 273348 (d) 273358

46. How many of the following numbers are divisible by 132?

264, 396, 462, 792, 968, 2178, 5184, 6336

(a) 4 (b) 5

(c) 6 (d) 7

47. Which one of the following numbers is exactly divisible by 11 ?

(a) 235641 (b) 245642

(c) 315624 (d) 415624

48. $(800 \div 64) \times (1296 \div 36) = ?$

(a) 420 (b) 460

(c) 500 (d) 540

(e) None of these

49. What is the unit digit in 7^{105} ?

(a) 1 (b) 5

(c) 7 (d) 9

50. The number of prime numbers which are less than 100 is ***[C.D.S., 2017-I]***

(a) 25 (b) 26

(c) 27 (d) 28

51. $7^{10} - 5^{10}$ is divisible by ***[C.D.S., 2016-I]***

(a) 5 (b) 7

(c) 10 (d) 12

52. The seven digit number 876p37q is divisible by 225. The values of p and q can be respectively ***[C.D.S., 2015-II]***

(a) 9, 0 (b) 0, 0

(c) 1, 5 (d) 9, 5

53. The digit in the units place of the product $81 \times 82 \times 83 \times 84 \times ... \times 99$ is ***[C.D.S., 2015-I]***

(a) 0 (b) 4

(c) 6 (d) 8

54. What is the maximum value of m if the number $N = 35 \times 45 \times 55 \times 60 \times 124 \times 75$ is divisible by 5^m ? ***[C.D.S., 2015-I]***

(a) 4 (b) 5

(c) 6 (d) 7

55. The last digit in the expansion of 17^{256} is ***[C.D.S., 2015-I]***

(a) 9 (b) 7

(c) 3 (d) 1

56. What is the remainder when 4^{1012} is divided by 7? ***[C.D.S., 2014-II]***

(a) 1 (b) 2

(c) 3 (d) 4

57. What is the remainder obtained when $1421 \times 1423 \times 1425$ is divided by 12 ? ***[C.D.S., 2015-I]***

(a) 1 (b) 2

(c) 3 (d) 4

58. p, q and r are prime numbers such that $p < q < r < 13$. In how many cases would $(p + q + r)$ also be a prime number ? ***[C.D.S., 2014-II]***

(a) 1 (b) 2

(c) 3 (d) None of these

59. What is the number of divisors of 360 ? ***[C.D.S., 2014-II]***

(a) 12 (b) 18

(c) 24 (d) None of these

60. What is the remainder when 4^{1000} is divided by 7 ? ***[C.D.S., 2014-I]***

(a) 1 (b) 2

(c) 4 (d) None of these

61. If m and n are natural number, then $\sqrt[m]{n}$ is ***[C.D.S., 2014-II]***

(a) Always irrational

(b) Irrational unless n is the m^{th} power of an integer

(c) Irrational unless m is the n^{th} power of an integer

(d) Irrational unless m and n are co-prime

62. If x is positive even integer and y is negative odd integer, then x^y is ***[C.D.S., 2014-II]***

(a) Odd integer (b) Even integer

(c) Rational number (d) None of these

63. The pair of rational number that lies between 1/4 and 3/4 is ***[C.D.S., 2014-I]***

(a) $\frac{262}{1000}, \frac{752}{1000}$ (b) $\frac{24}{100}, \frac{74}{100}$

(c) $\frac{9}{40}, \frac{31}{40}$ (d) $\frac{252}{1000}, \frac{748}{1000}$

64. What is the last digit in $7^{402} + 3^{402}$ ***[C.D.S., 2014-I]***

(a) 0 (b) 4

(c) 8 (d) None of the above

65. $19^5 + 21^5$ is divisible by ***[C.D.S., 2014-I]***

(a) Only 10 (b) Only 20

(c) Both 10 and 20 (d) Neither 10 nor 20

LEVEL – DIFFICULT

66. What least number should be replaced by * in the number 56771*2, so that the number is completely divisible by 8 ?

(a) 2 (b) 3

(c) 4 (d) 5

67. $2^{21} + 2^{22} + 2^{23} + 2^{24}$ is divisible by

(a) 10 (b) 11

(c) 13 (d) 17

68. Which among the following number is exactly divisible by 17 ?

(a) 1071 (b) 2153
(c) 4065 (d) 1849

69. 8937701256 is exactly divisible by which number ?

(a) 19 (b) 11
(c) 17 (d) 13

70. Which among the following numbers is a prime number ?

(a) 621 (b) 317
(c) 539 (d) 493

71. What is the least number that must be added to 39417 to obtain a number exactly divisible by 19 ?

(a) 1 (b) 8
(c) 7 (d) 2

72. What is the least number that must be subtracted from 27537 to get a number exactly divisible by 17 ?

(a) 2 (b) 3
(c) 14 (d) 12

73. If , $\frac{a}{(a+b)} = \frac{17}{23}$ what is $\frac{(a+b)}{(a-b)}$ equal to ?

(a) 13/7 (b) 23/11
(c) 14/5 (d) 25/9

74. Assumption: A number, 'ABCDE' is divisible by 11.

Statement I: E – D + C – B + A is divisible by 11.

Statement II: E – D + C – B + A = 0

Which one of the following is correct ?

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

75. What least number must be added to 4000 to obtain a number exactly divisible by 19 ?

(a) 11 (b) 13
(c) 10 (d) 9

76. A least number of 4 digits when increased by 5 is completely divisible by 12, 15, 20 and 35. The number is

(a) 1275 (b) 1265
(c) 1235 (d) 1255

77. A two digit number becomes $\frac{5}{6}$ of itself when its digits are reversed. The difference of the digits is one. Find the number. *[Delhi Police Constable, 2014]*

(a) 54 (b) 63
(c) 43 (d) 32

78. Four prime numbers are written in ascending order of their magnitudes. The product of the first three is 385 and that of the last three is 1001. The largest given prime number is *[AFCAT, 2017]*

(a) 11 (b) 13
(c) 17 (d) 19

79. In an examination, a student was asked to find $\frac{3}{14}$ of a certain number. By mistake, he found $\frac{3}{4}$ of it. His answer was 150 more than the correct answer. Find the number. *[AFCAT, 2016]*

(a) 180 (b) 280
(c) 380 (d) 480

80. 7 is added to a certain number, the sum is multiplied by 5, the product is divided by 9 and 3 is subtracted from the quotient. The remainder left is 12. What is the number ? *[AFCAT, 2014]*

(a) 20 (b) 30
(c) 40 (d) 5

81. 10 is added to a certain number, the sum is multiplied by 7, the product is divided by 5 and 5 is subtracted from the quotient. The remainder left is half of 88. What is the number ? *[AFCAT, 2014]*

(a) 21 (b) 20
(c) 25 (d) 30

82. If $\frac{a}{a+b} = \frac{15}{21}$ what is $\frac{(a+b)}{(a-b)}$ equal to ?

(a) 13/9 (b) 23/11
(c) 14/5 (d) 21/9

83. If $\frac{a}{b} = \frac{3}{4}$ and 8a + 5b = 22, then the value of a is

(a) 1 (b) 1/2
(c) 3/2 (d) 3/4

84. If (a – b) is 6 more than (c + d) and (a + b) is 3 less than (c – d), then the value of (a – c) is

(a) 0.5 (b) 1.0
(c) 1.5 (d) 2.0

85. A boy was asked to multiply a number by 25. Instead, he multiplied the number by 52 and got the answer 324 more than the correct answer. The number to be multiplied was

(a) 12 (b) 15
(c) 25 (d) 32

86. The sum of two odd numbers is 38 and their product is 325. What is three times of the larger number?

[IBPS Specialist Officer (IT) CWE, 2014]

(a) 42 (b) 39
(c) 75 (d) 72
(e) 78

87. What is the unit digit in the product $(3^{65} \times 6^{59} \times 7^{71})$?

(a) 1 (b) 2

(c) 4 (d) 6

88. Let a two-digit number be k times the sum of its digits. If the number formed by interchanging the digits is m times the sum of the digits, then the value of m is

[C.D.S., 2016-I]

(a) 9 – k (b) 10 – k

(c) 11 – k (d) k – 1

89. Consider the following statements in respect of two different non-zero integers p and q

1. For $(p + q)$ to be less than $(p - q)$, q must be negative.

2. For $(p + q)$ to be greater than $(p - q)$, both p and q must be positive

Which of the above statements is/are correct ?

[C.D.S., 2016-I]

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

90. The digit in the unit's place of the resulting number of the expression $(234)^{100} + (234)^{101}$ is

[C.D.S., 2015-II]

(a) 6 (b) 4

(c) 2 (d) 0

91. A number when divided by 7 leaves a remainder 3 and the resulting quotient when divided by 11 leaves a remainder 6. If the same number when divided by 11 leaves a remainder m and the resulting quotient when divided by 7 leaves a remainder n. What are the values of m and n respectively ?

[C.D.S. 2015-II]

(a) 1 and 4 (b) 4 and 1

(c) 3 and 6 (d) 6 and 3

92. What is the number of possible pairs of (P, Q) if the number 357P25Q is divisible by both 3 and 5 ?

[C.D.S., 2015-I]

(a) 7 (b) 6

(c) 5 (d) None of the above

93. The multiplication of a three-digit number XY5, with digit Z yields X 215. What is X + Y + Z equal to ?

[C.D.S., 2014-II]

(a) 13 (b) 15

(c) 17 (d) 18

94. How many pairs of positive integers m and n satisfy the equation $\frac{1}{m}+\frac{4}{n}+\frac{1}{12,}$ where n is an odd integer less than 60 ?

[C.D.S., 2014-II]

(a) 7 (b) 5

(c) 4 (d) 3

95. How many pairs of X and Y are possible in the number 763X4Y2, if the number is divisible by 9 ?

[C.D.S., 2014-II]

(a) 8 (b) 9

(c) 10 (d) 11

96. Consider the following statements:

I. 7710312401 is divisible by 11.

II. 173 is a prime number.

Which of the statements given above is/are correct ?

(a) Only I (b) Only II

(c) Both I and II (d) Neither I nor II

97. Consider the following statements:

I. To obtain prime numbers less than 121, we have to reject all the multiples of 2, 3, 5 and 7.

II. Every composite number less than 121 is divisible by a prime number less than 11.

Which of the statements given above is/are correct ?

(a) Only I (b) Only II

(c) Both I and II (d) Neither I nor II

98. When a natural number n is divided by 4, the remainder is 3. What is the remainder when $2n$ is divided by 4 ?

(a) 1 (b) 2

(c) 3 (d) 6

99. A three-digit number is divisible by 11 and has its digit in the unit's place equal to 1. The number is 297 more than the number obtained by reversing the digits. What is the number ?

(a) 121 (b) 231

(c) 561 (d) 451

100. What can be said about the expansion of $2^{12n} - 6^{4n}$, where n is positive integer ?

(a) Last digit is 4 (b) Last digit is 8

(c) Last digit is 2 (d) Last two digit are zero

ANSWER KEY

1. (c)	2. (a)	3. (a)	4. (d)	5. (a)	6. (c)	7. (a)	8. (c)	9. (a)	10. (b)
11. (b)	12. (d)	13. (b)	14. (c)	15. (c)	16. (a)	17. (a)	18. (b)	19. (c)	20. (c)
21. (c)	22. (c)	23. (b)	24. (c)	25. (a)	26. (b)	27. (a)	28. (a)	29. (b)	30. (a)
31. (a)	32. (d)	33. (a)	34. (d)	35. (d)	36. (b)	37. (d)	38. (c)	39. (d)	40. (d)
41. (b)	42. (d)	43. (a)	44. (c)	45. (a)	46. (a)	47. (d)	48. (e)	49. (c)	50. (a)
51. (d)	52. (d)	53. (a)	54. (c)	55. (d)	56. (d)	57. (c)	58. (b)	59. (c)	60. (c)

61. (b)	62. (c)	63. (d)	64. (c)	65. (c)	66. (d)	67. (a)	68. (a)	69. (b)	70. (b)
71. (b)	72. (c)	73. (b)	74. (c)	75. (d)	76. (b)	77. (a)	78. (b)	79. (b)	80. (a)
81. (c)	82. (d)	83. (c)	84. (c)	85. (a)	86. (c)	87. (c)	88. (c)	89. (c)	90. (d)
91. (a)	92. (a)	93. (a)	94. (d)	95. (d)	96. (c)	97. (c)	98. (b)	99. (d)	100. (d)

EXPLANATIONS

LEVEL – EASY

1. A8496 + 107 = (1 + A) 003

According to the given data, (1 + A) + 0 + 0 + 3 is completely divisible by 3.

Hence, the least value of A is 2.

Therefore, (1 + 2) + 0 + 0 + 3 = 6 is exactly divisible by 3.

2. For every even natural number n, $(x^n - a^n)$ is exactly divisible by .$(x + a)$

$(29^{100} - 1^{100})$is completely divisible by (29 + 1), *i.e.* 30.

Dividing 29^{100} by 30, we yield 1 as remainder.

3. Face value of 0 is 0.

Place value of 0 is 0 × 100 = 0.

Hence the difference between the place and face value is 0.

4. Difference = 30000 – 30 = 29970

5. $8 \times 10^5 + 1 \times 10^3 + 6 \times 10^1 + 9 \times 10^0$

= 800000 + 1000 + 60 + 9 = 801069

6. Let us take 1872 from the options,

Its last digit is 2.

Multiplying it by 4 ⇒ 2 × 4 = 8

Add to the remaining number ⇒ 187 + 8 = 195

195 is a multiple of 13.

So, the given number 1872 is divisible by 13.

7. Multiply the last digit by 2 and add the result to the remaining digits to find the divisibility of 19.

For 13, multiply the last digit by 4 and add the result to the remaining digits.

For 17, multiply the last digit by 5 and subtract the result from the remaining digits.

For 14, the number should be divisible by both 2 and 7.

8. Option (a), 802 is an even number which is divisible by 2. So, it is not a prime number.

Option (b), the nearest whole number of $\sqrt{803}$ is 29

i.e. $29 > \sqrt{803}$

Prime numbers less than 29 are 23, 19, 17, 13, 11, 7, 5, 3 and 2.

803 is divisible by 11. So, it is not a prime number.

Option (c), the nearest whole number of $\sqrt{809}$ is 29

i.e. $29 > \sqrt{809}$

Prime numbers less than 29 are 23, 19, 17, 13, 11, 7, 5, 3 and 2.

809 is not divisible by any of these prime numbers. So, it is a prime number.

Therefore, option (c) is the correct answer.

9. Given K is the even prime number. So, K = 2

Because we know that 2 is the only even prime number.

So, we have

4K1396 → 421396

To get place value of "K", we have to count the number of digits after 2.

There are four digits after 2.

Hence, the place value of "K" is 20000.

10. Total number of flags = 265476

Number of orange flags = 7542

Number of violet flags = 265476 – 7542 = 257934

Therefore, Tanshika has 257934 violet flags

11. Place value of 9 - face value of 9 = 9000 – 9 = 8991

12. 8340 – 3215 – 1132 = 3993

13. Sum of the digits in 632817 i.e. 6 + 3 + 2 + 8 + 1 + 7 = 27 which is divisible by 3.

14. We know that sum of the digits in a number is divisible by 3, then the number is also divisible by 3.

Replace * by p.

6 + 8 + 7 + 6 + p + 2 + 2 = 31 + p which is divisible by 3.

The least value to be added with 31 to make the number divisible by 3 is 2. i.e. p = 2.

When 2 is placed in the place of * the given number becomes 6876222 and it is divisible by 3.

15. The number formed by the last two digits in the given number is 62, which is not divisible by 4.

Hence, 47820262 is divisible by 4.

16. In 5368, the last three digits are 368, which can be divided by 8.

Hence, 5368 is divisible by 8.

17. Sum of the digits of 8325 = 8 + 3 + 2 + 5 = 18, which is divisible by 9.

Hence, 8325 is divisible by 9.

18. Required sum of all prime numbers (which are not greater than 17)

= 2 + 3 + 5 + 7 + 11 + 13 + 17 = 58

19. LCM of 4 and 6 = 12

∴ Required number of integers

$$= \frac{600 - 100}{12} \cong 41$$

20. LCM of 32, 36, 48, 54 = 864

∴ Required number = 10000 – 864 = 9136

21. 337.62 + 8.591 + 34.4 = 380.611

22. $\frac{1}{9}+\frac{1}{6}+\frac{1}{12}+\frac{1}{72}=\frac{8+12+6+1}{72}$

$$=\frac{27}{72}=\frac{3}{8}$$

23. 365.089 – x + 89.72 = 302.35

454.809 – 302.350 = x

x = 152.459

24. Option (*a*), 6x = 24

Option (*b*), 6x = 30

Option (*c*), But 6x ≠ 35

Therefore 35 cannot be the total number of children in the class.

25. ? = 14.5 × 17.3 × 10.8 = 2709.18

26. ?= 333.33 + 33.33 + 3.3

= 369.96

27. ? = 144 × 83 – 9090

= 11952 – 9090

= 2862

28. 8052 – 9883 + 6048 =? × 25

4217 = ? × 25

∴ ? = $\frac{4217}{25}$ = 168.68

29. 8536 + 5824 = ? × 40

14360 = ? × 40

? = $\frac{14360}{40}$ = 359

30. ? = 566.91 + 551.34 + 114.98

= 1233.23

31. 3463 × 295 – 18611 =? + 5883

1021585 – 18611 =? + 5883

1002974 =? + 5883

? = 1002974 – 5883

= 997091

32. 4003 (70 + 7) – 21015 = ? × 116

280210 + 28021 – 21015 = ? × 116

287216 = ? × 116

? = 287216 ÷ 116 = 2476

33. $? = \frac{6255.22}{18.5 \times 21.4} = 15.8$

34. ? = 302.46 + 395.72 – 123.47

= 698.18 – 123.47

= 574.71

35. 0.22 + 0.0054 = ? – 313.06

0.2254 + 313.06 =?

? = 313.2854

36. Let 7589 – x = 3434

Then, x = 7589 – 3434 = 4155

37. Let 4207 – x – 3007

Then, x = 4207 – 3007 = 1200

38. Let 4300731 – x = 2535618

Then, x = 4300731 – 2535618

= 1765113.

39. Given sum $= 9+\frac{3}{4}+7+\frac{2}{17}-9+\frac{1}{15}$

$$= (9 + 7 - 9) + \frac{3}{4} + \frac{2}{17} - \frac{1}{15}$$

$$= 7 + \frac{765+120-68}{1020} = 7\frac{817}{1020}$$

40. Given Expression = – 84 × (30 – 1) + 365

= –(84 × 30) + 84 + 365

= –2520 + 449 = –2071

41. 4500 × x = 3375

$$x = \frac{3375}{4500}$$

$$= \frac{75}{100} = \frac{3}{4}$$

42. Given Expression $= 8988 \times \frac{1}{8} \times \frac{1}{4}$

$$= \frac{2247}{8} = 280.875$$

43. Given Expression $666 \times \frac{1}{6} \times \frac{1}{3} = 37$

44. Given Expression : $35 + 15 \times \frac{3}{2}$

$$= 35 + \frac{45}{2}$$

= 35 + 22.5 = 57.5

45. 5358 × 51 = 5358 ×(50 + 1)

= 5358 × 50 + 5358 × 1

= 267900 + 5358 = 273258

46. $132 = 11 \times 3 \times 4$

Clearly, 968 is not divisible by 3

None of 462 and 2178 is divisible by 4.

5184 is not divisible by 11.

Each one of the remaining four members is divisible by each one of 4, 3 and 11. So, there are 4 such numbers.

47. $(4 + 5 + 2) - (1 + 6 + 3) = 1$, not divisible by 11.

$(2 + 6 + 4) - (4 + 5 + 2) = 1$, not divisible by 11.

$(4 + 6 + 1) - (2 + 5 + 3) = 1$, not divisible by 11.

$(4 + 6 + 1) - (2 + 5 + 4) = 0$, So, 415624 is divisible by 11.

48. Given Expression $= \frac{800}{64} \times \frac{1296}{36}$

$= 50 \times 9 = 450$

49. Unit digit in 7^{105} = Unit digit in $[(7^4)^{26} \times 7]$

But, unit digit in $(7^4)^{26} = 1$

$\therefore$ Unit digit in $7^{105} = (1 \times 7)$

50. There are 25 prime numbers less than 100. They are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97.

51. The dividend is in the form $a^n - b^n$

Here n is even $\Rightarrow 7^{10} - 5^{10}$ is divisible by both $(7 + 5)$ and $(7 - 5)$

$\Rightarrow 7^{10} - 5^{10}$ is divisible by 12 and 2.

Hence, it is divisible by 12 and 2.

So option (d) is correct

52. 876p37q is divisible by 225 or 25×9.

q has to be 5 and sum of all digits must be divided by 9.

$\therefore \quad p = 0$ or 9.

Hence option (d) is the correct answer.

53. $81 \times 82 \times 83 \times 84 \times ... \times 99$

It can be written as $= 81 \times 82 \times 83 \times ... \times 90 \times ... \times 99$

When we multiply any number by multiple of 10, then resultant number always carry zero at unit place.

54. $N = 35 \times 45 \times 55 \times 60 \times 124 \times 75$

$= 7 \times 5 \times 9 \times 5 \times 11 \times 5 \times 12 \times 5 \times 124 \times 5 \times 5 \times 3$

$= 5^6 \times 7 \times 9 \times 11 \times 12 \times 124 \times 3$

$m = 6$,

i.e. The given number has maximum factor of 5 is 6.

55. $17^{256} \Rightarrow$ Last digit is 7

We know that 7 repeats its unit digit after 4 times. $17^{256} \Rightarrow (17)^{64 \times 4}$

Unit digit of $7 \times 7 \times 7 \times 7 = 2401$.

Now, 256 is completely divided by 4, so last digit of $17^{256} = 1$

56. $\frac{4^{1012}}{7} = \frac{4^{3\times337+1}}{7} = \frac{4^{3\times337} \times 4}{7}$

$= \frac{(64)^{337} \times 4}{7} = \frac{(9\times7+1)^{337} \times 4}{7}$

We know that $= \frac{(ax+1)^n}{a}$, then its Remainder is 1

$= 1 \times 4 = 4$.

57. When we divide 1421, 1423 and 1425 by 12 then 5, 7 and 9 are the remainders respectively.

$\Rightarrow \frac{5\times7\times9}{12} = \frac{315}{12} = 3$

58. The prime numbers less than 13 are 2, 3, 5, 7, 11.

Also, using the condition, $p < q < r < 13$ and $p + q + r$ is a prime number

Hence, only two possible pairs exist i.e. (3, 5, 11) and (5, 7, 11).

59. $360 = 2^3 \times 3^2 \times 5$

$\therefore$ Number of divisors $= (3 + 1)(2 + 1)(1 + 1)$

$= 4 \times 3 \times 2 = 24$

60. Remainder of $\frac{4^{1000}}{7}$

$= \frac{4^{(333\times3+1)}}{7} = \frac{(64)^{333} \times 4}{7}$

$= 1 \times 4 = 4$

61. If m and n are natural numbers, then $m\sqrt{n}$ is irrational unless n is mth power of an integer.

62. If x is a positive even integer and y is negative odd integer, then x^y is a rational number.

63. $\frac{1}{4} = 0.25$ and $\frac{3}{4} = 0.75$.

Only option (d) with $\frac{252}{1000} = 0.252$ and $\frac{748}{1000} = 0.748$ lie between 0.25 and 0.75.

64. Here 7 and 3 both are repeated its unit digit after four times.

$= 7^{(4\times100+2)} + 3^{(4\times100+2)} = 7^{4\times100} \times 7^2 + 3^{4\times100} \times 3^2$

$= 9 + 9 = 18$. So, its unit digit = 8

65. We know that $a^n + b^n$ where n is odd numbers then it is divisible by $a + b$.

So, $19^5 + 21^5 \Rightarrow 19 + 21 = 40$. Now, 40 is divided by both 10 and 20.

So the number $19^5 + 21^5$ is divisible by 10 and 20.

LEVEL – DIFFICULT

66. Divisibility rule for 8: The last three digits should be divisible by 8.

Replace * by 5, since 152 is completely divisible by 8.

67. $2^{21} + 2^{22} + 2^{23} + 2^{24} = 2^{21}(1 + 2 + 2^2 + 2^3)$

$= 2^{21}(15) = 2^{19}(2^2 \times 15)$

$= 2^{19}(60)$ which is divisible by 10

68. Let us check option (a) 1071, its last digit is 1.

Multiply the last digit by 5 $\Rightarrow 1 \times 5 = 5$

Subtract from the remaining number $= 107 - 5 = 102$.

102 is a multiple of 17.

So, the number 1071 is divisible by 17.

69. To find the divisibility by 11:

Sum of odd digits – Sum of even digits = 0 or multiple of 11.

Sum of odd digits – Sum of even digits

= (8+3+7+1+5) - (9+7+0+2+6)

= 24 – 24=0

Therefore, the given number 8937701256 is divisible by 11.

70. Option (a), the nearest whole number of $\sqrt{621}$ is 25

i.e. $25 > \sqrt{621}$

Prime numbers less than 25 are 23, 19, 17, 13, 11, 7, 5, 3 and 2.

621 is divisible by 23. So, it is not a prime number.

Option (b), the nearest whole number of $\sqrt{317}$ is 18

i.e. $18 > \sqrt{317}$

Prime numbers less than 18 are 17, 13, 11, 7, 5, 3 and 2.

317 is not divisible by any of these prime numbers. So, it is a prime number.

Option (c), the nearest whole number of $\sqrt{539}$ is 24

i.e. $24 > \sqrt{539}$

Prime numbers less than 24 are 23, 19, 17, 13, 11, 7, 5, 3 and 2.

539 is divisible by 11. So, it is not a prime number.

Option (d), the nearest whole number of $\sqrt{493}$ is 23

i.e. $23 > \sqrt{493}$

Prime numbers less than 23 are 19, 17, 13, 11, 7, 5, 3 and 2.

493 is divisible by 17. So, it is not a prime number.

Therefore, option (b) is the correct answer.

71. $\frac{39417}{19} \Rightarrow$ Quotient = 2074, Remainder = 11

$\therefore$ The required number to be added = 19 – 11 = 8.

72. $\frac{27537}{17} \Rightarrow$ Quotient = 1619, Remainder = 14

$\therefore$ The required number to be subtracted = 14.

i.e. 27537 – 14 = 27523

Dividing 27523 by 17, we get quotient = 1619 and remainder = 0.

Hence, 14 is the least number that must be subtracted from 27537 to obtain a number exactly divisible by 17.

73. Given $\frac{a}{a+b} = \frac{17}{23}$

$$1 - \frac{a}{a+b} = 1 - \frac{17}{23}$$

$$\frac{a+b-a}{a+b} = \frac{6}{23}$$

$$\frac{b}{a+b} = \frac{6}{23}$$

$$\frac{a}{a+b} - \frac{b}{a+b} = \frac{17}{23} - \frac{6}{23}$$

$$\frac{a-b}{a+b} = \frac{11}{23}$$

Therefore, $\frac{a-b}{a+b} = \frac{23}{11}$

74. We know that, if the difference of the sum of odd digits and sum of even digits is either 0 or multiple of 11, then the number is divisible by 11.

Given number is $ABCDE$.

Here, A + C + E – (B + D) = 0 or divisible by 11.

Hence, both statements are true.

75. On dividing 4000 by 19, we get 10 as remainder.

Required number to be added = 19 – 10 = 9

76. A least number of 4 digits = 1000

LCM of 12, 15, 20, 35 = 420

The required number is $420 \times n + 5$

(For least)

$= 420 \times 3 + 5$ = 1260 + 5 = 1265

77. Let the unit and tens digits of the number be x and y respectively.

Then, the number is $10y + x$

According to the question

$$10x + y = \frac{5}{6}(10y + x)$$

$$6(10x + y) = 5(10y + x)$$

$$55x - 44y = 0$$

$$5x - 4y = 0 \text{ and } y - x = 1$$

Solving both the equations, $x = 4$, $y = 5$

Therefore, the number is $10 \times 5 + 4 = 54$

78.

385) 1001 (2
  770
  231) 385 (1
    231
    154) 231 (1
      154
      77) 154 (2
        154
        ×

$\therefore$ Product of the middle two numbers = 77

$\therefore$ The largest prime number

$$= 1001 \div 77 = 13$$

79. Let the required number = x

According to the question

$$\frac{3x}{4} - \frac{3x}{14} = 150$$

$$\frac{21x - 6x}{25} = 150$$

$$15x = 150 \times 28$$

$$15x = 4200$$

$$x = \frac{4200}{15} = 280$$

80. Suppose number = x

Then, $\frac{(x+7)\times 5}{9} - 3 = 12$

$$5x + 35 - 27 = 108$$

$$5x + 8 = 108$$

$$5x = 100$$

$$x = \frac{100}{5} = 20$$

81. Suppose number

Then, $\frac{(x+10)\times 7}{5} - 5 = 88 \div 2$

$$7x + 70 - 25 = 44 \times 5$$

$$7x + 45 = 220$$

$$7x = 220 - 45 = 175$$

$$x = \frac{175}{7} = 25$$

82. $\frac{a}{a+b} = \frac{15}{21}$

$$21a = 15a + 15b$$

$$6a = 15b$$

$$a = \frac{15}{6}b$$

$$\frac{a+b}{a-b} = \frac{\frac{15b}{6} + b}{\frac{15b}{6} - b} = \frac{15b + 6b}{15b - 6b}$$

$$= \frac{21b}{9b} = \frac{21}{9}$$

83. $\frac{a}{b} = \frac{3}{4}$

$$a = \frac{3b}{4}$$

$$8a + 5b = 22$$

$$8\left(\frac{3b}{4}\right) + 5b = 22$$

$$11b = 22$$

$$b = 2$$

$$\therefore \quad a = \frac{3b}{4} = \frac{3\times 2}{4} = \frac{3}{2}$$

84. $a - b = c + d + 6$ ...(i)

$a + b = c - d - 3$ ...(ii)

Adding eqn. (i) and (ii)

$$2a = 2c + 3$$

$$2a - 2c = 3$$

$$2(a - c) = 3$$

$$a - c = \frac{3}{2} = 1.5$$

85. Let the number = x

$$52 \times x - 25x = 324$$

$$27x = 324$$

$$x = 12$$

86. First number = x

Second number $= 38 - x$

$\therefore$ $x(38 - x) = 325 = 25(38 - 25)$

$x = 25$

$\therefore$ Required answer $= 3 \times 25 = 75$

87. Unit digit of $3^4 = 1$

Unit digit of $(3^4)^{16} = 1$

$\therefore$ Unit digit of 3^{65} = Unit digit of $[(3^4)^{16} \times 3]$ $= (1 \times 3) = 3$

Unit digit of $6^{59} = 6$

Unit digit of $7^4 = 1$, Unit digit of $(7^4)^{17}$ is 1 .

Unit digit of 7^{71} = Unit digit of $[(7^4)^{17} \times 7^3]$
$= (1 \times 3) = 3$

$\therefore$ Required digit = Unit digit in $(3 \times 6 \times 3) = 4$

88. Let two digit number $= 10y + x$

According to the question,

1st condition, $10y + x = k(x + y)$ ...(1)

2nd condition, $10x + y = m(x + y)$ ...(2)

Adding (1) and (2) we get

$$11x + 11y = (k + m)(x + y)$$
$$11(x + y) = (k + m)(x + y)$$
$$k + m = 11$$
$$\Rightarrow \quad m = 11 - k$$

$\therefore$ Option (c) is correct.

89. 1. If $p + q < p - q$

$\Rightarrow q + q < 0 \Rightarrow q > -q$

$\Rightarrow$ 1 is true

$\Rightarrow q < 0$

$\Rightarrow 2q > 0$

2. If $p + q > p - q$

$\Rightarrow q > 0 \Rightarrow q$ must be positive $\Rightarrow$ p is also
$\Rightarrow 2q > 0$ positive $\Rightarrow$ (2) is true

$\therefore$ Both 1 and 2 are correct.

90. $(234)^{100} + (234)^{101}$

See the pattern:

$$4^1 = 4; 4^2 = 16; 4^3 = 64; 4^4 = 256$$

So, at odd power of 4 we get unit digit as '4' and at even power of 4 we get unit digit as '6'.

$\Rightarrow (234)^{100}$ unit digit is 6

$\Rightarrow (234)^{101}$ unit digit is 4

$\Rightarrow (234)^{100} + (234)^{101}$ unit digit will be $(6 + 4 = 10)$ zero.

91. This is an example of successive division. Let the number be N. The number and successive quotients, the successive divisors and the corresponding remainders are tabulated below:

Quotients	N	q_1	q_2
Divisors	7	11	
Remainder	3	6	

One value of N is $6(7) + 3 = 45$.

In general, $N = 77K + 45$

$\therefore$ $N = 11(7k + 4) + 1$ i.e. $m = 1$

and $q_1 = 7k + 4$, $q_2 = k$ and $n = 4$

$\therefore$ $(m,n) = (1,4)$

92. Given number is 357P25Q.

If it is divided by 3, then sum of the digit must be divided by 3 and if it is divided by 5 then its unit digit must be 0 or 5.

$= 3 + 5 + 7 + P + 2 + 5 + Q$

1^{st} Case Q = 0

$\Rightarrow 3 + 5 + 7 + P + 2 + 5 + 0 = 22 + P$

Possible values of P = 2, 5, 8

2^{nd} Case when Q = 5

$\Rightarrow 3 + 5 + 7 + P + 2 + 5 + 5 = 27 + P$

Possible values of P = 0, 3, 6, 9

Possible pairs of (P, Q) = (2, 0), (5, 0), (8, 0) (0, 5), (3, 5) (6, 5) (9, 5)

Total no. of pairs = 7.

93. Here, three–digits number = $XY5$

```
  XY5
× XZ
-----
 X215
```

So, Z can take value 1, 3, 5, 7 and 9. But only 9 satisfies it, then $X = 1$, $Y = 3$ and $Z = 9$

```
  135
×   9
-----
 1215
```

$\therefore$ Value of X + Y + Z = 1 + 3 + 9 = 13

94. Given equation, $\frac{1}{m} + \frac{4}{n} = \frac{1}{12}$

$$\Rightarrow \quad 12(n + 4m) = mn$$
$$\Rightarrow \quad m(48 - n) = -12n$$
$$\Rightarrow \quad m(n - 48) = 12n$$
$$\therefore \quad m = \frac{12n}{n - 48} \quad \text{...(i)}$$

Here, as m and n are positive integers, therefore $n > 48$. But n is an odd integer less than 60, therefore possible values of n = 49, 51, 53, 55, 57 and 59.

But on putting n = 53, 55, and 59 in Eq. (i), we get the non-integer values of m

On putting n = 49, 51 and 57, we get the value of m = 588, 204 and 76, respectively.

Hence, there are three possible pairs of m and n that satisfy the equation.

95. If sum of all the digits is divisible by 9 then the number is divisible by 9

Given number is 763 × 4Y 2.

Given number is divisible by 9.

So, $7 + 6 + 3 + X + 4 + Y + 2 = 9k$

$\Rightarrow 22 + X + Y = 9k$

It is clear that LHS is divisible by 9, if $x + y = 5, 14$

Now sum of X and Y in 5, then possible pairs are (1, 4), (4, 1), (2, 3) (3, 2), (0, 5) and (5, 0).

When sum of X and Y is 14, then possible pairs are (5, 9), (9, 5), (6, 8), (8, 6) and (7, 7).

Total possible pairs are 11.

96. Statement I :

Divisibility rule of 11: The difference between the sum of even places and the sum of odd places is 0 or that number is divisible by 11.

Difference = Sum of odd place – Sum of even place

= (1 + 4 + 1 + 0 + 7) – (0 + 2 + 3 + 1 + 7)

= 13 – 13 = 0, it is divisible by 11.

Statement II :

173 is square of approximately 13. So, below 13, prime numbers are 2, 3, 5, 7, 11

Now, 173 is not divisible by 2, 3, 5, 7 and 11. So, it is a prime number.

So, both the statements I and II are correct.

97. Both the statements given are correct. As 121 is the square of 11. So, to obtain prime numbers less than 121, we reject all the multiples of prime numbers less than 11 i.e., 2, 3, 5 and 7.

Similarly, every composite number less than 121 is divisible by a prime number less than 11 i.e., 2, 3, 5 and 7.

98. When a number n is divided by 4 then remainder is 3. Now, the number is double then remainder is also double.

So, remainder = 6. But remainder never greater than its divisor. So, remainder = 6 – 4 = 2

99. On taking option (d), the reverse digit of 451 is 154. Now, 154 + 297 = 451 is equal to the original number.

100. $2^{12n} - 6^{4n} = (2^{12})^n - (6^4)^n$

$= (4096)^n - (1296)^n$

$= (4096 - 1296)\,[(4096)^{n-1} + (4096)^{n-2}(1296) + \ldots (1296)^n]$
$= 2800(k)$

Hence, last two digits are always be zero.

❐❐

Chapter

2

NUMBER SERIES

DEFINITION

The number series often denotes the sequence of numbers which is of a particular pattern or in a certain sequence. Before solving these kinds of questions, we need to observe the pattern first to find out the missing term or the wrong one out from the series.

DIFFERENT TYPES OF NUMBER SERIES

The following are the different types of Number Series :

Prime Number Series

A number is said to be prime if it is divisible by 1 and itself. A series which contains only prime numbers are called as prime number series.

Ex. 1. Find out the missing term in the series :
19, 23, 29, 31

Solution :

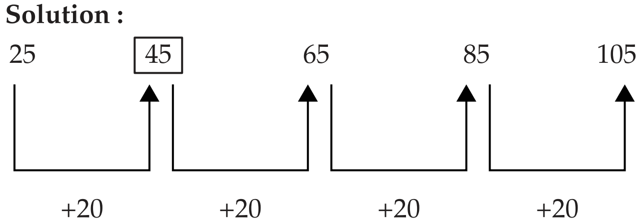
The next prime number in the given prime number series is 37.

Addition Series

This series is formed by the addition of a number to the previous term in order to make a new one.

Ex. 2. Find out the missing term in the series :
25, ?, 65, 85, 105

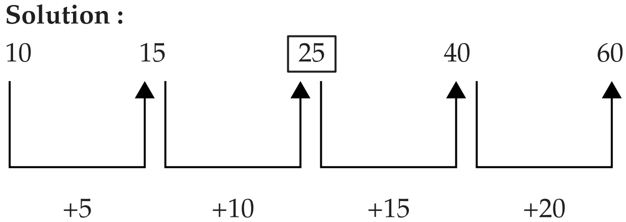
Solution :



The missing term in this series is 45, since each term gets added by the number 20.

Ex. 3. Find out the missing term in the series :
10, 15, ?, 40, 60

Solution :



The missing term in this series is 25 since the value that is being added to the previous number gets increased by 5 each time.

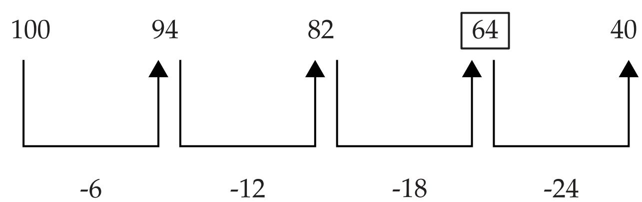
Note :
The number which is added to the terms can be a fixed one or a variable.

Difference Series

In a difference series, a new number is formed when each number is subtracted by a specific number from the previous number.

Ex. 4. Find out the missing term in the series :
100, 94, 82, ?, 40

Solution :



The missing term here is 64 since the value that is being subtracted from the previous number gets increased by -6 each time.

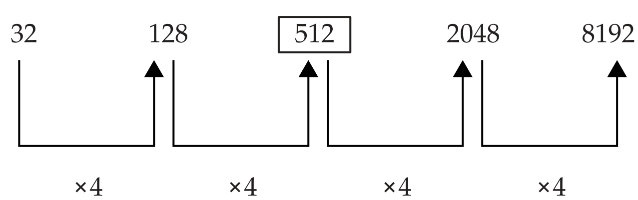
Note :
The number which is subtracted to the terms can be a fixed one or a variable.

Multiplication Series

When the numbers in a series is obtained by multiplying a number with the previous term, the series is said to be multiplication series.

Ex. 5. Find out the missing term in the series :
32, 128, ?, 2048, 8192

Solution :



Missing term= $128 \times 4 = 512$

Ex. 6. Find out the missing term in the series :
3, 15, ?, 450, 2700

Solution :

3 → 15 → [90] → 450 → 2700

×5, ×6, ×5, ×6

Missing term = $15 \times 6 = 90$

In this series, the multiplying term keeps on interchanging every time. Hence, the missing term here is 90.

> **Note :**
> *The number which is multiplied to the terms can be a fixed one or a variable.*

Division series

This series is formed when the previous number is divided by a number to form the next number in the series.

> **Ex. 7.** Find out the missing term in the series :
> 900, 150, ?, 4.17, 0.69

Solution :

900 → 150 → [25] → 4.17 → 0.69

$\frac{900}{6} = 150$ $\frac{150}{6} = 25$ $\frac{25}{6} = 4.17$ $\frac{4.17}{6} = 0.69$

Missing term = $\frac{150}{6} = 25$

In this series, each term gets divided by 6 to form a new term.

> **Note :**
> *The number which is used to divide the terms can be a fixed one or a variable.*

Mixed Series

When a series is formed by more than one series is known as mixed series. For example, addition combined with multiplication, division followed by subtraction etc.

> **Ex. 8.** Find out the missing term in the series :
> 40, 39, ?, 34, 50

Solution :

$40 - 1^2 = 39$

$39 + 2^2 = 43$

$43 - 3^2 = 34$

$34 + 4^2 = 50$

Here, the missing term is 43. It is a mixed series since each term is formed by the interchanging addition and subtraction of the square of the numbers starting from 1.

Geometric Series

This series is basically the multiplication series or the division series but it involves the multiplication or division by a static number throughout the series either in an ascending or descending manner.

> **Ex. 9.** Find out the missing term in the series :
> 6875, 1375, 275, ?, 11

Solution :

6875 → 1375 → 275 → [55] → 11

$\frac{6875}{5} = 1375$ $\frac{1375}{5} = 275$ $\frac{275}{5} = 55$ $\frac{55}{5} = 11$

Here, the missing term is 55 and it is a geometric series since each number is divided by 5 and in descending order.

Two stage Type series

A two stage arithmetic series is one in which the formation of arithmetic series are obtained from the differences of continuous numbers themselves.

> **Ex. 10.** Find out the missing term in the series :
> 1, 7, 14, ?, 33, 45

Solution :

1 → 7 → 14 → [23] → 33 → 45

+6, +7, +9, +10, +12

+1, +2, +1, +2

This is called the two stage series as it involves two stages to find out the pattern of how the series is formed.

Perfect Square Series

A number which has been multiplied once by the same number itself is called as the perfect square number. The series which contains only square numbers are called as perfect square series. This series is also called as the n^2 series.

> **Ex. 11.** Find out the missing term in the series :
> 121, 144, 169, 196, ?

Solution :

$11^2 = 121$, $12^2 = 144$, $13^2 = 169$, $14^2 = 196$, $15^2 = 225$

The above-given series is a perfect square series so the missing term will be 225 (*i.e.*, square of 15)

Perfect Cube Series

A number is said to be a cube number when it has been multiplied twice by the same number itself. A series of numbers which contains only cube numbers are called as perfect cube series. This series is also called as the n^3 series.

Ex. 12. Find out the missing term in the series :
729, 1000, 1331, 1728, ?

Solution :

$9^3 = 729, 10^3 = 1000, 11^3 = 1331, 12^3 = 1728, 13^3 = 2197$

The missing term in this perfect cube series is 2197.

(n^x + 1)series

In this series, each term is formed by the addition of n to the power of x and the number 1. The value of x may be an integer or a decimal number.

Ex. 13. Find out the missing term in the series :
26, 37, 50, 65, ?, 101.

Solution :

$5^2 + 1 = 26, 6^2 + 1 = 37, 7^2 + 1 = 50, 8^2 + 1 = 65, 9^2 + 1 = 82,$

$10^2 + 1 = 101$

Here, the missing term in this series is 82 and the value of x in this series is 2.

Ex. 14. Find out the missing term in the series :
126, 217, 344, ?, 730, 1001.

Solution :

$5^3 + 1 = 126, 6^3 + 1 = 217, 7^3 + 1 = 344, 8^3 + 1 = 513,$

$9^3 + 1 = 730, 10^3 + 1 = 1001$

Here, the missing term in this series is 513 and the value of x in this series is 3.

(n^x – 1) series

In this series, each term is formed by the subtraction of the number 1 from the x^{th} power of n. The value of **x** may be an integer or a decimal number.

Ex. 15. Find out the missing term in the series :
24, 35, 48, ?, 80, 99.

Solution :

$5^2 - 1 = 24, 6^2 - 1 = 35, 7^2 - 1 = 48,$

$8^2 - 1 = 63, 9^2 - 1 = 80, 10^2 - 1 = 99$

Here, the missing term in this series is 63 and the value of x in this series is 2.

Ex. 16. Find out the missing term in the series :
124, 215, 342, 511, ?, 999.

Solution :

$5^3 - 1 = 124, 6^3 - 1 = 215, 7^3 - 1 = 342, 8^3 - 1 = 511, 9^3 - 1 = 728, 10^3 - 1 = 999$

Here, the missing term in this series is 728 and the value of x in this series is 3.

(n^x + n) series

In this series, each term is formed by the addition of n to the power of x and the number n itself. The value of x may be of an integer or a decimal number.

Ex. 17. Find out the missing term in the series :
6, 12, 20, ?, 42, 56.

Solution :

$2^2 + 2 = 6, 3^2 + 3 = 12, 4^2 + 4 = 20,$

$5^2 + 5 = 30, 6^2 + 6 = 42, 7^2 + 7 = 56$

Here, the missing term in this series is 30 and the value of x in this series is 2.

Ex. 18. Find out the missing term in the series :
10, 30, 68, ?, 222, 350.

Solution :

$2^3 + 2 = 10, 3^3 + 3 = 30, 4^3 + 4 = 68, 5^3 + 5 = 130,$

$6^3 + 6 = 222, 7^3 + 7 = 350$

Here, the missing term in this series is 130 and the value of x in this series is 3.

(n^x + n) series

In this series, each term is formed by the subtraction of the number n itself from the x^{th} power of n. The value of x may be an integer or a decimal number.

Ex. 19. Find out the missing term in the series :
2, 6, 12, ?, 30, 42.

Solution :

$2^2 - 2 = 2, 3^2 - 3 = 6, 4^2 - 4 = 12,$

$5^2 - 5 = 20, 6^2 - 6 = 30, 7^2 - 7 = 42$

Here, the missing term in this series is 20 and the value of x in this series is 2.

Ex. 20. Find out the missing term in the series :
6, 24, 60, 120, ?, 336.

Solution :

$2^3 - 2 = 6, 3^3 - 3 = 24, 4^3 - 4 = 60, 5^3 - 5 = 120,$

$6^3 - 6 = 210, 7^3 - 7 = 336$

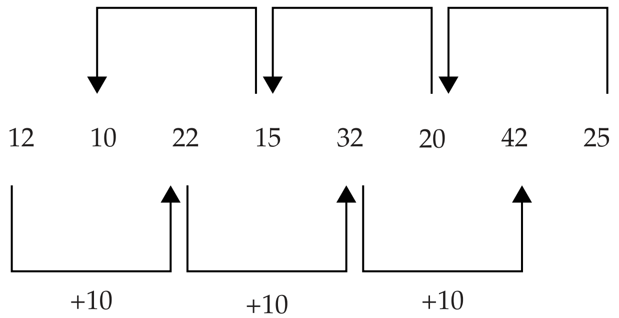
Here, the missing term in this series is 210 and the value of x in this series is 3.

Alternating Series

In this series, the alternative numbers in the series follows the same pattern of series formation. It is most commonly known as the combination of two different series as a single series.

Ex. 21. Find the next term in the given series :
12, 10, 22, 15, 32, 20, 42, …………..

Solution :



Hence, the next number in the given alternating series is 25.

ARITHMETIC PROGRESSION (AP)

A sequence is said to be an arithmetic progression, or AP, if the difference between any two consecutive terms of the sequence is a constant. The pattern of the arithmetic progression is

a, a +d, a + 2d, a + 3d, ...

Where, the n^{th} term is $t_n = a + (n - 1) d$

Sum of an Arithmetic Progression

The sum of the terms of an arithmetic progression gives an arithmetic series. Let the starting value is a and the common difference is d, then the sum of the first n terms is calculated by the formula,

$$S_n = \frac{1}{2} n \ \{2a + (n - 1)d\}$$

If the last term l is known, the sum of the series is given by

$$S_n = \frac{1}{2} n(a + l)$$

Arithmetic Mean (AM)

If a, b, c are in AP, b is called the Arithmetic Mean (AM) of a and c.

In this case, $b = \frac{1}{2}(a + c)$

a, a_1, a_2 ... a_n, b are in AP we can say that a_1, a_2 ... a_n are the n Arithmetic Means between a and b.

Ex. 22. Find the sum of all odd natural numbers less than 75.

Solution :

Required sum = 1 + 3 + 5 + 7 +… + 73.

This is an Arithmetic Progression in which

First term a = 1,

Difference, $d = (3 - 1) = 2$

Last term, l = 73

$$t_n = a + (n-1)d$$
$$73 = 1 + (n-1)\,2$$
$$73 = -1 + 2n$$
$$74 = 2n$$
$$37 = n$$

Sum of the series, $S_n = \frac{1}{2} n(a + l)$

$$S_n = \frac{1}{2} \times 37 \times (1 + 73)$$
$$= \frac{1}{2} \times 37 \times 74 = 1369$$

Ex. 23. How many two digit natural numbers are divisible by 9 ?

Solution :

The first two digit number divisible by 9 is 18.

The last two digit number divisible by 9 is 99.

Therefore the sequence is 18, 27, 36, … ,99.

This is an Arithmetic progression in which

First term $a = 18$,

Difference, $d = (27 - 18) = 9$

Last term, $l = 99$

Let the number of these terms be n.

Then, $t_n = a + (n-1)d$

$$99 = 18 + (n-1)9$$
$$99 = 9 + 9n$$
$$90 = 9n$$
$$\Rightarrow 10 = n$$

Therefore, there are 10 natural two digit numbers are divisible by 9.

GEOMETRIC PROGRESSION (GP)

A sequence in which the ratio of any two consecutive number is a constant, then it is called as geometric sequence. The first term is denoted by a and the common ratio is denoted by r. The basic pattern of the geometric progression is

a, ar, ar^2, ar^3...

Where, the n^{th} term is $t_n = ar^{n-1}$

Sum of Geometric Progression

The sum of the terms of a geometric progression gives a geometric series. Let the first term be a and the common ratio be r, then the sum of the first n terms is

$$S_n = \frac{a(1-r^n)}{(1-r)}, \quad r \neq 1$$

Note :

The sum to infinity of a geometric progression with starting value a and common ratio r is given by

$$S_n = \frac{a}{(1-r)}, -1 < r < 1$$

Geometric Mean (GM)

If non-zero numbers a, b, c are in GP, b is said to be the Geometric Mean (GM) of a and c.

In this case, $b = \sqrt{ac}$

Note :

If a and c are of opposite sign, then their GM cannot be defined.

Ex. 24. Find the number of terms in the G.P. 2, 4, 8, 12, 16, …, 128 ?

Solution :

The given G.P series is 2, 4, 8, 12, 16, …, 128

First term $a = 2$

Common ratio, $r = \frac{4}{2} = 2$

Let the number of terms be n.

$$t_n = 128$$
$$\Rightarrow \quad ar^{n-1} = 128$$
$$2 \times (2)^{n-1} = 128$$
$$(2)^{n-1} = \frac{128}{2}$$
$$(2)^{n-1} = 2^6$$
$$n = 6 + 1 = 7$$

Ex. 25. Find the sum of the geometric series

$$\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots$$

Solution :

Note that the given geometric series is infinite.

First term, $a = \frac{1}{2}$

Common ratio, $r = \frac{1}{2}$

So, Sum of the geometric series, $S_\infty = \frac{a}{(1-r)}$

$$S_\infty = \frac{\frac{1}{2}}{\left(1 - \frac{1}{2}\right)} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$

Ex. 26. Find out the 9^{th} term in the Geometric Progression 1, 9, 18, …

Solution :

From the given series 1, 9, 18, …

$$a_1 = 1,\ a_2 = 9,\ a_3 = 18$$

Common ratio $r = \frac{a_2}{a_1} = \frac{9}{1} = 9$

$$n^{th}\ term,\ a_n = a_1 r^{n-1}$$
$$9^{th}\ term,\ a_9 = a_1 r^{9-1}$$
$$= 1 \times 9^{9-1} = 1 \times 9^8 = 43046721$$

The required number is 43046721.

HARMONIC PROGRESSION (HP)

Non-zero numbers $a_1, a_2, a_3, \cdots a_n$ are said to be in Harmonic Progression (HP) $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \ldots \frac{1}{a_n}$ if are in AP. Harmonic Progression is also known as harmonic sequence.

If $\frac{1}{a}, \frac{1}{(a+d)}, \frac{1}{(a+2d)}, \ldots$ are in HP, n^{th} term of the HP,

$$t_n = \frac{1}{(a + (n-1)d)}$$

Harmonic Mean (HM)

If a, b, c are in HP, b is said to be Harmonic Mean (HM) of a and c

In this case, $b = \frac{2ac}{(a+c)}$

If a, a_1, a_2 ... a_n, b are in HP, we can say that a_1, a_2 ... a_n are the n Harmonic Means between a and b.

If a, b and c are in HP, $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

Ex. 27. Find the 5th and 10th term of the series 6, 4, 3, ...

Solution :

Consider $\frac{1}{6}, \frac{1}{4}, \frac{1}{3}, \ldots, \infty$

Here $T_2 - T_1 = T_3 - T_2 = \frac{1}{12}$

$\Rightarrow \frac{1}{6}, \frac{1}{4}, \frac{1}{3}$ is an A.P

5th term of this A.P $= \frac{1}{6} + 4 \times \frac{1}{12}$

$= \frac{1}{6} + \frac{1}{3} = \frac{1}{2}$

And the 10th term $= \frac{1}{6} + 9 \times \frac{1}{12} = \frac{11}{12}$

Relationship between Arithmetic Mean, Harmonic Mean, and Geometric Mean of Two Numbers

If AM, GM and HM are the Arithmetic Mean, Geometric Mean and Harmonic Mean of two positive numbers respectively, then

$GM^2 = AM \times HM$

SPECIAL SERIES

Sum of first n natural numbers

$$\sum_{i=1}^{n} \mathbf{1 + 2 + 3 \ldots\ldots + n = \frac{n(n+1)}{2}}$$

$$1 + 2 + 3 + 4 + \ldots + n = \frac{n(n+1)}{2}$$

Ex. 28. What is the sum of first 22 natural numbers ?

Solution :

We know that, the sum of first n natural numbers,

$$1 + 2 + 3 + 4 + \ldots + n = \frac{n(n+1)}{2}$$

Here, n = 22

$$\Rightarrow \frac{n(n+1)}{2} = \frac{22(22+1)}{2} = 253$$

Therefore, the sum of first 22 natural numbers = 253.

Sum of first n odd numbers

$$\sum_{i=1}^{n} \mathbf{1 + 3 + 5 + \ldots + (2n-1) = n^2}$$

$$1 + 3 + 5 + 7 + \ldots + (2n-1) = n^2$$

Ex. 29. What is the sum of first 34 odd numbers ?

Solution :

We know that, the sum of first n odd numbers,

$$1+3+5+7+\ldots+(2n-1)=n^2$$

Here, $n = 34$

$\Rightarrow$ $n^2 = 34^2 = 1156$

Therefore, the sum of first 34 odd numbers = 1156.

Sum of first n even numbers

$$\sum_{i=1}^{n} 2+4+6+8+\ldots+2n = n(n+1)$$

$$2+4+6+8+\ldots+2n = n(n+1)$$

Ex. 30. Find the value of $2+4+6+8+\ldots+32$.

Solution :

The given numbers are even numbers.

The sum of first n even numbers is calculated by the formula

$$2+4+6+8+\ldots+2n = n(n+1)$$

Here, $2n = 32$

$\Rightarrow$ $n = \frac{32}{2} = 16$

So, $n(n+1) = 16(16+1) = 272$

Therefore, $2+4+6+8+\ldots+32 = 272$.

Sum of first n square numbers

$$\sum_{i=1}^{h} 1^2+2^2+3^2+\ldots\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^2+2^2+3^2+4^2+\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

Ex. 31. What is the value of

$1^2+2^2+3^2+4^2+\ldots+12^2$ **?**

Solution :

Formula to find the sum of first n square numbers is

$$1^2+2^2+3^2+4^2+\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

Here, $n = 12$

So, $\frac{n(n+1)(2n+1)}{6} = \frac{12(12+1)(24+1)}{6} = 650$

Therefore, $1^2+2^2+3^2+4^2+\ldots+12^2 = 650$.

Ex. 32. $(11^2+12^2+13^2+\ldots+20^2) = ?$

Solution :

$$(11^2+12^2+13^2+\ldots+20^2)$$

$$=(1^2+2^2+3^2..+20^2)-(1^2+2^2+3^2\ldots+10^2)$$

$$=\left\{\frac{20\times21\times41}{6}-\frac{10\times11\times21}{6}\right\}$$

$$\left[\because (1^2+2^2+3^2..+n^2) = \tfrac{1}{6}n(n+1)(2n+1)\right]$$

$$= 2870 - 385 = 2485$$

Sum of first n cube numbers

$$\sum_{i=1}^{n} 1^3+2^3+3^3+\ldots\ldots+n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

$$1^3+2^3+3^3+4^3+\ldots+n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Ex. 33. What is the sum of first 11 cube numbers ?

Solution :

We know that,

Sum of first n cube numbers

$$1^3+2^3+3^3+4^3+\ldots+n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Here, n = 11

So, $1^3+2^3+3^3+4^3+\ldots+11^3$ is

$$\left[\frac{n(n+1)}{2}\right]^2 = \left[\frac{11(11+1)}{2}\right]^2$$

$$= 66^2 = 4356$$

Therefore, the sum of first 11 cube numbers = 4356.

Sum of first n odd square numbers

$$\sum_{i=1}^{n} 1^2+3^2+..+(2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$$

$$1^2+3^2+5^2\ldots+(2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$$

Ex. 34. What is the sum of squares of first 9 add numbers ?

Solution :

The formula to find the sum of first n odd square numbers

$$1^2+3^2+5^2+..+(2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$$

Here, n = 9,

$$\Rightarrow \frac{n(2n-1)(2n+1)}{3} = \frac{9(18-1)(18+1)}{3}$$

$$= \frac{2907}{3} = 969$$

Therefore, the sum of squares of first 9 odd numbers is 969.

Sum of first n even square numbers

$$\sum_{i=1}^{n} 2^2+4^2+6^2\ldots\ldots(2n)^2 = \frac{2n(n+1)(2n+1)}{3}$$

$$2^2+4^2+6^2+\ldots(2n)^2 = \frac{2n(n+1)(2n+1)}{3}$$

Ex. 35. Find the sum of squares of first 7 even numbers.

Solution :

We know that,

$$2^2+4^2+6^2+\ldots(2n)^2 = \frac{2n(n+1)(2n+1)}{3}$$

Here, $n = 7$

So, $\frac{2n(n+1)(2n+1)}{3} = \frac{14(7+1)(14+1)}{3}$

$= \frac{1680}{3} = 560$

Therefore, the sum of the squares of first 7 even numbers is 560.

PRACTICE QUESTIONS

LEVEL-EASY

1. Find the value of $1+3+5+\ldots$ up to 30 terms
(a) 30 (b) 100
(c) 300 (d) 900

2. $3+3^2+3^3+\ldots+3^{10}$
(a) 88500 (b) 88570
(c) 88572 (d) 88574

3. $\left(1^2+2^2+3^2+\ldots+12^2\right)=$
(a) 651 (b) 653
(c) 655 (d) 650

Directions (Q- 4-17) :
What should come in place of question mark (?) in the following series :

4. 14, 14, ?, 84, 336, 1680
(a) 24 (b) 28
(c) 32 (d) 34

5. 5, 5, 45, ?, 55125
(a) 1122 (b) 1125
(c) 1120 (d) 1122

6. 9, 31, 73, 141, (?)
(a) 164 (b) 280
(c) 239 (d) 241
(e) None of these

7. 35, 256, 451, 620, 763, (?)
(a) 680 (b) 893
(c) 633 (d) 880
(e) None of these

8. 130, 139, 155, 180, 216, (?)
(a) 260 (b) 290
(c) 265 (d) 996
(e) None of these

9. 2890, (?), 1162, 874, 730, 658,
(a) 1684 (b) 1738
(c) 1784 (d) 1672
(e) None of these

10. 16, 8, 12, 30, ?,
(a) 75 (b) 105
(c) 95 (d) 115
(e) None of these

11. 5, 6, 14, 45, ?
(a) 138 (b) 154
(c) 118 (d) 184
(e) None of these

12. 11, 23, 47, 95, ?
(a) 189 (b) 193
(c) 181 (d) 195
(e) None of these

13. 9, 17, 33, 65, ?
(a) 113 (b) 131
(c) 129 (d) 118
(e) None of these

14. 960, 839, 758, 709, ?, 675
(a) 698 (b) 694
(c) 684 (d) 648
(e) 680

15. 3, ?, 14, 55, 274, 1643
(a) 5 (b) 6
(c) 7 (d) 8
(e) None of these

16. 37, ?, 103, 169, 257, 367
(a) 49 (b) 46
(c) 56 (d) 59
(e) 69

17. 4, 6, 12, ?, 90, 315
(a) 25 (b) 27
(c) 30 (d) 45
(e) None of these

18. "The wrong number in the series
2, 9, 28 , 65, 126, 216, 344 is"
(a) 9 (b) 65
(c) 216 (d) none of these

19. The odd one out from the sequence of numbers
19, 23, 29, 37, 43, 46, 47 is
(a) 37 (b) 19
(c) 23 (d) 46

20. The next number of the sequence
1/2, 3/4, 5/8, 7/16, ….is
(a) 9/24 (b) 9/32
(c) 10/24 (d) 11/32

21. Find out the wrong number in the sequence :
169, 144, 121, 100, 82, 64, 49
(a) 144 (b) 49
(c) 64 (d) 82

22. The sum of a natural number and its square equals the product of the first three prime numbers. The number is
(a) 2 (b) 3
(c) 5 (d) 6

Directions (Q. 23 - 35) :
What should come in place of the question mark (?) in the following number series :

23. 800, 400, 200, 100, 50, ?
(a) 20 (b) 30
(c) 25 (d) 35
(e) None of these

24. 2, 13, 35, 68, 112, ?
(a) 173 (b) 178
(c) 163 (d) 167
(e) None of these

25. 650, 601, 565, 540, 524, ?
(a) 512 (b) 514
(c) 511 (d) 515
(e) None of these

26. 2, 4, 16, 96, 768, ?, 92160
(a) 7680 (b) 7580
(c) 7608 (d) 7090
(e) 7860

27. 14, 36, ?, 300, 894, 2676, 8022
(a) 101 (b) 102
(c) 103 (d) 104
(e) None of these

28. 5, 8, 13, 20, ?, 44, 61
(a) 29 (b) 30
(c) 31 (d) 32
(e) 37

29. 11, 16, 31, 56, 91, 136, ?
(a) 171 (b) 181
(c) 185 (d) 191
(e) 197

30. 3, 4, 12, 45, 196 ?
(a) 985 (b) 990
(c) 995 (d) 1000
(e) 1005

31. 2, 26, 286, ?, 18018, 90090, 270270
(a) 3088 (b) 2667
(c) 3862 (d) 2574
(e) None of these

32. 358, 356, 352, 344, 328, 296, ?
(a) 232 (b) 247
(c) 225 (d) 255
(e) None of these

33. 8, ?, 30, 105, 472.5, 2598.75, 16891.875
(a) 24 (b) 10
(c) 12 (d) 16
(e) None of these

34. 3, 4, ?, 21, 85, 110, 326
(a) 7 (b) 10
(c) 12 (d) 14
(e) None of these

35. 50000, 10000, 2500, 500, 125, ?, 6.25
(a) 75 (b) 25
(c) 50 (d) 31.5
(e) None of these

36. $1 + 2 + 3 + ….. + 100 = x$, then x is
(a) 5050 (b) 5000
(c) 10100 (d) 10000

Directions (Q. 37-38) :
In each of the following questions, a series is given, with one/two term(s) missing. Choose the correct alternative from the given ones that will complete the series :

37. 56, 90, 132, 184, 248, _ ?__
(a) 368 (b) 316
(c) 362 (d) 326

38. 0, 4, 8, 24, 64, 176, ?
(a) 180 (b) 480
(c) 280 (d) 300

39. "Find the wrong number in the series: 6, 12, 21, 32, 45, 60"
(a) 6 (b) 12
(c) 21 (d) 32

40. In each of the following questions, find the missing number/letters/figure from the given responses.
1, 1, 6, 6, 11, 11, 16, ?,? ***[IBPS PO Pre, 2018-Memory based]***
(a) 13, 11 (b) 16, 21
(c) 17, 21 (d) 21, 16

Directions (Q.41-44) :

What will come in place of the question mark (?) in each of the following number series :

41. 9, 5, 6, 10.5, 23, ? *[IBPS PO, 2015]*

(a) 85 (b) 60
(c) 78 (d) 49
(e) 97

42. 1, 2, 6, 21, 88, ? *[IBPS PO, 2015]*

(a) 539 (b) 398
(c) 216 (d) 445
(e) 615

43. 9, 10.8, 14.4, 21.6, ?, 64.8 *[IBPS PO, 2015]*

(a) 36 (b) 44
(c) 34 (d) 41.8
(e) 37.6

44. 6, 5, 8, 21, 80, ? *[IBPS PO, 2015]*

(a) 268 (b) 192
(c) 255 (d) 364
(e) 395

45. Find two natural numbers whose sum is 85 and the least common multiple is 102. *[AFCAT, 2014]*

(a) 30, 55 (b) 17, 68
(c) 35, 55 (d) 51, 34

LEVEL- DIFFICULT

46. Sum of the following series

$1+1^3+2+2^3+3+3^3+\ldots+n+n^3$

(a) $\frac{n^2+2n}{2}$ (b) $\frac{n(n+1)(2n+1)}{3}$

(c) $\frac{n^4+\ 2n^3+\ 3n^2+2n}{4}$ (d) $1+\left[\frac{n(n+1)}{2}\right]^2$

47. Find the sum of $3^3+6^3+9^3+\ldots+81^3$

(a) 3857868 (b) 3812578
(c) 5812768 (d) 5769854

48. $\left(21^2+22^2+23^2+\ldots+30^2\right)=?$

(a) 6550 (b) 6580
(c) 6585 (d) 6555

49. Find the missing term in the series : 3, 5, 12, ?, 154, 772

(a) 38 (b) 49
(c) 39 (d) 33

50. How many terms are there in the G.P. 4, 8, 16, 32, ……., 512

(a) 10 (b) 8
(c) 7 (d) 15

Directions (Q. 51 - 53)

What will come in place of the question mark (?) in the following number series :

51. 155, 151, 144, 132, 113, ? *[IBPS PO Pre, 2018-Memory based]*

(a) 89 (b) 71
(c) 85 (d) 92
(e) 60

52. 18, 18, 24, 48, 108, ? *[SSC CGL, 2018-Memory based]*

(a) 254 (b) 228
(c) 212 (d) 176
(e) 194

53. 13, 6, 5, 6, 10, ? *[SSC CGL, 2018-Memory based]*

(a) 19 (b) 25
(c) 17.5 (d) 28
(e) 22.5

54. The first term of an arithmetic progression is 22 and the last term is -11. If the sum is 66, the number of terms in the sequence are *[SSC CGL, 2018-Memory based]*

(a) 10 (b) 12
(c) 9 (d) 8

55. Find the nth term of the following sequence.
$5 + 55 + 555 + \ldots\ldots + T_n$ *[IBPS PO Pre, 2018-Memory based]*

(a) $5(10^n - 1)$ (b) $5^n(10^n - 1)$
(c) $5/9\ (10^n - 1)$ (d) $(5/9)^n(10^n - 1)$

Directions (Q. 56 - 60)

What will come in place of the question mark (?) in the following number series :

56. 14, 1004, 1202, 1251.5, 1268, (?) *[IBPS PO Pre, 2018-Memory based]*

(a) 1267.5 (b) 1276.25
(c) 1324.5 (d) 1367.25
(e) None of these

57. 7, 12, 32, 105, ? *[IBPS PO Pre, 2018-Memory based]*

(a) 428 (b) 214
(c) 218 (d) 416
(e) None of these

58. 59, 66, 80, 108, ?, 276 *[IBPS PO, 2015]*

(a) 150 (b) 125
(c) 164 (d) 132
(e) 178

59. 47, 23, 11, 5, 2, ? *[IBPS PO, 2015]*

(a) 0.2 (b) 1
(c) 0.4 (d) 2
(e) 0.5

60. 300, 298, 307, 279, 344, ? *[IBPS PO, 2015]*

(a) 265 (b) 218
(c) 253 (d) 289
(e) 298

ANSWER KEY

1. (d)	2. (c)	3. (d)	4. (b)	5. (b)	6. (d)	7. (d)	8. (c)	9. (b)	10. (b)
11. (d)	12. (e)	13. (c)	14. (c)	15. (a)	16. (d)	17. (c)	18. (c)	19. (d)	20. (b)
21. (d)	22. (c)	23. (c)	24. (d)	25. (d)	26. (a)	27. (b)	28. (c)	29. (d)	30. (e)
31. (d)	32. (a)	33. (c)	34. (c)	35. (b)	36. (a)	37. (d)	38. (b)	39. (a)	40. (b)
41. (b)	42. (d)	43. (a)	44. (e)	45. (d)	46. (c)	47. (a)	48. (c)	49. (a)	50. (b)
51. (c)	52. (b)	53. (e)	54. (b)	55. (c)	56. (b)	57. (a)	58. (c)	59. (e)	60. (b)

EXPLANATIONS

LEVEL - EASY

1. $1+3+5+\ldots+\ n\ terms = n^2$

$1+3+5+\ldots+upto\,30\ terms\ = 30^2 = 900$

2. This is a GP,

Here $a=3,\ r=\frac{3^2}{3}=3\ \ and\ \ n=10$

$$S_n=\frac{a(r^n-1)}{r-1}=\frac{3(3^{10}-1)}{3-1}=88572$$

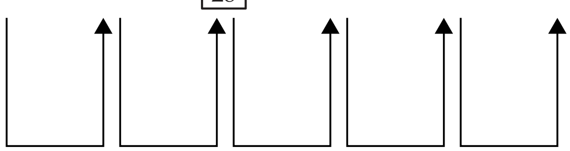
3. $\left(1^2+2^2+3^2+\ldots+n^2\right)=\frac{1}{6}n(n+1)(2n+1)$

Here $n=12$

(12 + 22 + 32) + ... + 122 = $\frac{1}{6}12(12+1)\{2(12)+1\}=$

$=\frac{1}{6}(156)(25)=\frac{3900}{6}=650$

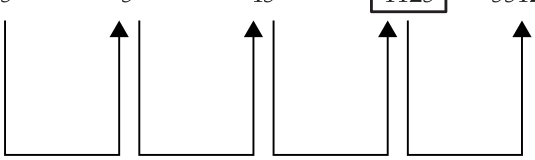
4. 14 14 [28] 84 336 1680



×1 ×2 ×3 ×4 ×5

Here, every next term is getting multiplied by numbers starting from 1. Hence the missing term is 28.

5. 5 5 45 [1125] 55125



$\times 1^2$ $\times 3^2$ $\times 5^2$ $\times 7^2$

Here, each term is being multiplied by the square of the odd numbers. Hence the missing term here is 1125.

6. The pattern is :

$2^3+1^2=9$

$3^3+2^2=31$

$4^3+3^2=73$

$5^3+4^2=141$

$6^3+5^2=241$

7. The pattern is :

$35+221=256$

256 + 195 (221 – 26) = 451

$451+169(195-26)=620$

$620+143(169-26)=763$

763 + 117 (143 – 26) = 880

8. The pattern is :

$130+3^2=139$

$139+4^2=155$

$155+5^2=180$

$180+6^2=216$

$216+7^2=265$

9. The pattern is :

$658+72=730$

$730+144=874$

$874+288=1162$

$1162+576=1738$

10. The pattern is :

16 × 0.5 = 8

$8\times1.5=12$

$12\times2.5=30$

$30\times3.5=105$

11. The pattern is :

5× 1+1= 6

6× 2 + 2 = 14

14× 3 + 3 = 45

45× 4 + 4= 184

12. The pattern is :

$11\times2+1=23$

$23\times2+1=47$

$47\times2+1=95$

$95\times2+1=191$

13. The pattern is :

$9\times2-1=17$

$17\times2-1=33$

$33\times2-1=65$

$65\times2-1=129$

14. The pattern is :

$960-839=121=11^2$

$839-758=81=9^2$

$758-709=49=7^2$

$\Rightarrow\ ?=709-25$

$=684$

15. The pattern is :

$3\times2-1=5$

$5\times3-1=14$

$14\times4-1=55$

$55\times5-1=274$

$274\times6-1=1643$

16. The pattern is :

$37+22=59$

$59+44=103$

$103+66=169$

$169+88=257$

$257+110=367$

17. The pattern is :

$4\times1.5=6$

$6\times2=12$

$12\times2.5=30$

$30\times3=90$

$90\times3.5=315$

18. Series as follows

$1^3+1=2$

$2^3+1=9$

$3^3+1=28$

$4^3+1=65$

$5^3+1=126$

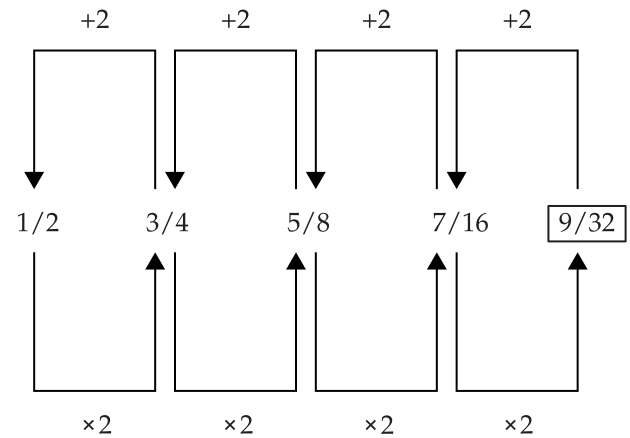
$6^3+1=217$

$7^3+1=344$

Therefore 216 is the wrong number

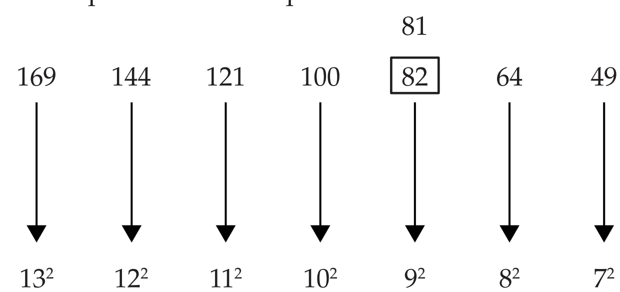
19. Numbers in sequence, 19, 23, 29, 37, 43, 46, 47. In the given sequence, there are all the prime numbers except 46.

20.



Hence, the number of the sequence = 9/32

21. The pattern of the sequence is



22. $5+(5)^2=5+25=30$

$\because 30=2\times3\times5$

Hence, the number is 5.

23. The pattern of the number series is :

$\frac{800}{2}=400$

$\frac{400}{2}=200$

$\frac{200}{2}=100$

$\frac{100}{2}=50$

$\frac{50}{2}=25$

24. The pattern of the number series is :

$2+1\times11=2+11=13$

$13+2\times11=13+22=35$

$35+3\times11=35+33=68$

$68+4\times11=68+44=112$

$112+5\times11=112+55=167$

25. The pattern of the number series :

$650-7^2=650-49=601$

$601-6^2=601-36=565$

$565-5^2=565-25=540$

$540 - 4^2 = 540 - 16 = 524$

$524 - 3^2 = 524 - 9 = 515$

26. The pattern is :

$2 \times 2 = 4$

$4 \times 4 = 16$

$16 \times 6 = 96$

$96 \times 8 = 768$

$768 \times 10 = 7680$

$7680 \times 12 = 92160$

27. The pattern is :

$14 \times 3 - 6 = 42 - 6 = 36$

$36 \times 3 - 6 = 108 - 6 = 102$

$102 \times 3 - 6 = 306 - 6 = 300$

$300 \times 3 - 6 = 900 - 6 = 894$

$894 \times 3 - 6 = 2682 - 6 = 2676$

$2676 \times 3 - 6 = 8028 - 6 = 8022$

28. The pattern is :

$5 + 3 = 8$

$8 + 5 = 13$

$13 + 7 = 20$

$20 + 11 = 31$

$31 + 13 = 44$

$44 + 17 = 61$

Note :

Consecutive prime numbers have been added.

29. The pattern is :

$11 + 5 = 16$

$16 + 15 = 31$

$31 + 25 = 56$

$56 + 35 = 91$

$91 + 45 = 136$

$136 + 55 = 191$

30. The pattern is :

$3 \times 1 + 1^2 = 3 + 1 = 4$

$4 \times 2 + 2^2 = 8 + 4 = 12$

$12 \times 3 + 3^2 = 36 + 9 = 45$

$45 \times 4 + 4^2 = 180 + 16 = 196$

$196 \times 5 + 5^2 = 980 + 25 = 1005$

31. The pattern of the number series is :

$2 \times 13 = 26$

$26 \times 11 = 286$

$286 \times 9 = 2574$

$2574 \times 7 = 18018$

$18018 \times 5 = 90090$

32. The pattern of the number series is :

$358 - 2 = 356$

$356 - 4 = 352$

$352 - 8 = 344$

$344 - 16 = 328$

$328 - 32 = 296$

$296 - 64 = 232$

33. The pattern of the number series is :

$8 \times 1.5 = 12$

$12 \times 2.5 = 30$

$30 \times 3.5 = 105$

$105 \times 4.5 = 472.5$

34. The pattern of the number series is :

$3 + 1^2 = 4$

$4 + 2^3 = 12$

$12 + 3^2 = 21$

$21 + 4^3 = 85$

$85 + 5^2 = 110$

$110 + 6^3 = 326$

35. The pattern of the number series is :

50000/5 =10000

10000/4=2500

2500/5=500

500/4=125

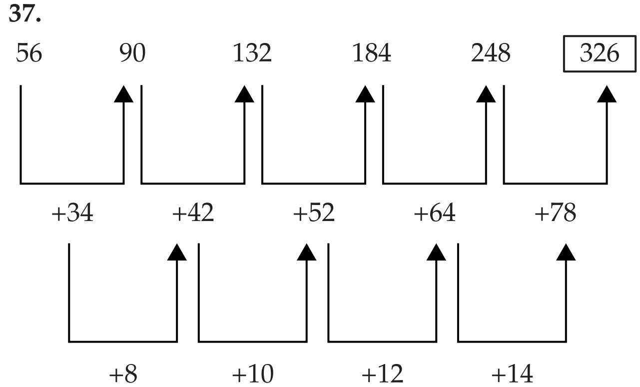
125/5=25

36. $1 + 2 + 3 + 4 + \ldots + 100 = \frac{n(n+1)}{2}$

$$\frac{100(100+1)}{2} = x$$

$x = 50 \times 101 = 5050$

37.



38. $(0 + 4) \times 2 = 8$

$(4 + 8) \times 2 = 24$

$(8 + 24) \times 2 = 64$

$(24 + 64) \times 2 = 176$

$(64 + 176) \times 2 = 480$

39. 6 is wrong. Because the numbers are increased by odd numbers. i.e., the first number should be 5.

5 +7 = 12.

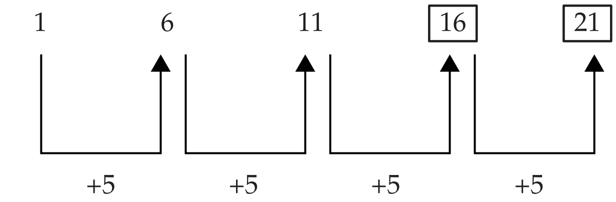
12+9 =21

21+11 =32

32+13=45

45+15 =60

40.



41. The pattern is :

$9\times0.5+0.5=4.5+0.5=5$

$5\times1+1=5+1=6$

$6\times1.5+1.5=9+1.5=10.5$

$10.5\times2+2=21+2=23$

$23\times2.5+2.5=57.5+2.5\ =60$

42. The pattern is :

$1\times1+1=1+1=2$

$2\times2+2=4+2=6$

$6\times3+3=18+3=21$

$21\times4+4=84+4=88$

$88\times5+5=440+5=445$

43. The pattern is :

$9+1.8=10.8$

$10.8 + 2 \times 1.8 = 10.8 + 3.6 = 14.4$

$14.4+2\times3.6=14.4+7.2=21.6$

$21.6+2\times7.2=21.6+14.4=36$

$36+2\times14.4=36+28.8=64.8$

44. The pattern is :

$6\times1-1=6-1=5$

$5\times2-2=10-2=8$

$8\times3-3=24-3=21$

$21\times4-4=84-4=80$

$80\times5-5=400-5=395$

45. $51 + 34 = 85$ and $51\times2=102$

$34\times3=102$

LEVEL - DIFFICULT

46. $1+1^3+2+2^3+3+3^3+\ldots+n+n^3$

$=(1+2+3+\ldots+n)+\left(1^3+2^3+3^3+..+n^3\right)$

$=\dfrac{n(n+1)}{2}+\left[\dfrac{n(n+1)}{2}\right]^2$

$=\dfrac{n^4+\ 2n^3+\ 3n^2+2n}{4}$

47. $3^3+6^3+9^3+\ldots+81^3=\ 3^3\left(1^3+2^3+3^3+\ldots+27^3\right)$

$=27\left[\dfrac{27(27+1)}{2}\right]^2=27\left[27(14)\right]^2$

$=27(142884)=3857868$

48.

$$\sum_{i=1}^{n}1^2+2^2+3^2+....+n^2=\frac{n(n+1)(2n+1)}{6}$$

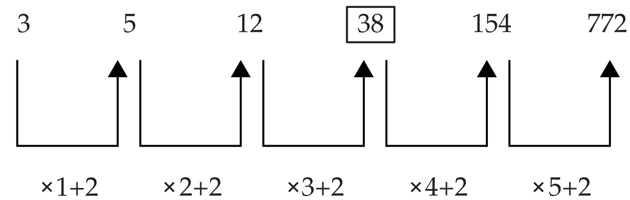
$\left(21^2+22^2+23^2+\ldots+30^2\right)$

$= (1^2 + 2^2 + 3^2 + ... + 30^2) - (1^2 + 2^2 + 3^2 + ... + 20^2)$

$=\dfrac{30\times31\times61}{6}-\dfrac{20\times21\times41}{6}$

$=\dfrac{56730}{6}-\dfrac{17220}{6}=9455-2870=6585$

49.



Here each term is multiplied by the increasing numbers and gets added by the number two. Hence the missing term is 38.

50. Here, $a = 4$, $r = \dfrac{8}{4} = 2$, $t_n = 512$

Let n be the number of terms

$ar^{n-1}=512$

$4\times2^{n-1}=512\Rightarrow2^{n-1}=128$

$2^{n-1}=2^7\Rightarrow n-1=7$

$n=8$

$\therefore$ Number of terms = 8

51. The pattern is :

$155-4=151$

$151-7(=4+3)=144$

$144-12(=7+5)=132$

$132 - 19\ (== 12 + 7) = 113$

$113 - 28\ (= 19 + 9) = 85$

52. The pattern is :

$18+\left(1^3-1\right)=18+0=18$

$18+\left(2^3-2\right)=18+6=24$

$24+\left(3^3-3\right)=24+24=48$

$48 + (4^3 - 4) = 48 + 60 = 108$

$108+\left(5^3-5\right)=108+120=228$

53. The pattern is :

$13\times0.5-0.5=6.5-0.5=6$

$6\times1-1=6-1=5$

$5\times1.5-1.5=7.5-1.5=6$

$6\times2-2=12-2=10$

$10\times2.5-2.5=25-2.5=22.5$

54. Let n be the number of terms in the sequence of AP and common difference be d.

According to the question,

$22+(n-1)d=-11$ $\qquad [\because l = a_n = a + (n-1)]$

$(n-1)d=-11-22=-33\ldots.(1)$

$\frac{n}{2}\left[2\times22+(n-1)d\right]=66$

$\left[\because S_n=\frac{n}{2}\left(2a+(n-1)d\right)\right]\ -\frac{5}{9}$

$\frac{n}{2}\left[44+(-33)\right]=66$

$\frac{n}{2}(11)=66$

$\frac{n}{2}=\frac{66}{11}$

$\frac{n}{2}=\frac{66\times2}{11}=6\times2=12$

55. Given sequence,

$5+55+555+\ldots+T_n$

$=5\left[1+11+111+\ldots+n \text{ terms}\right]$

$=\frac{5}{9}\left[(10-1)+(100-1)+(1000-1)+\ldots+n \text{ terms}\right]$

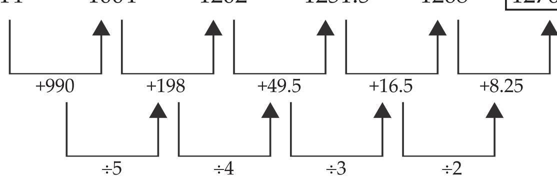
$=\frac{5}{9}\left[10+10^2+10^3+\ldots+n \text{ terms}\right]$

$=\left[1+1+1+\ldots+n \text{ terms}\right]$

$=\frac{5}{9}\left[10\times10^{n-1}-1\times1^{n-1}\right]$

$=\frac{5}{9}\left[10^n-1\right]$

56. 14 1004 1202 1251.5 1268 **[1276.25]**



57. The pattern is :

$7\times1+1\times5=12$

$12\times2+2\times4=32$

$32\times3+3\times3=105$

$105\times4+4\times2=428$

58. The pattern is :

$59+1\times7=59+7=66$

$66+2\times7=66+14=80$

$80+2\times14=80+28=108$

$108+2\times28=108+56$

$=164$

59. The pattern is :

$\frac{47-1}{2}=\frac{46}{2}=23$

$\frac{23-1}{2}=\frac{22}{2}=11$

$\frac{11-1}{2}=\frac{10}{2}=5$

$\frac{5-1}{2}=\frac{4}{2}=2$

$\frac{2-1}{2}=\frac{1}{2}=0.5$

60. The pattern is :

$300-2=\left(1+1^3\right)=298$

$298+9=\left(1+2^3\right)=307$

$307-28=\left(1+3^3\right)=279$

$279+65=\left(1+4^3\right)=344$

$344-126=\left(1+5^3\right)=218$

❒❒

Chapter 3

H.C.F AND L.C.M OF NUMBERS

METHODS OF FINDING H.C.F.

Highest Common Factor (H.C.F.)

H.C.F. stands for Highest Common Factor or also known as Greatest Common Divisor (GCD) or Greatest Common Measure (GCM), the greatest number which exactly divides the given numbers.

There are two methods to find H.C.F. which are :

- **Prime Factorization method**
- **Division Method**

Prime Factorization method

Express each one of the given numbers as the product of prime factors. The product of least powers of common prime factors gives H.C.F.

Finding H.C.F. of two numbers by division method

Step 1 : Divide the larger number by the smaller one.

Step 2 : Now, divide the divisor by the remainder.

Step 3 : Repeat the same process till you get zero as the remainder.

Step 4 : The last divisor is said to be the H.C.F. of given two numbers.

Ex. 1. Find the H.C.F. of

$2^2 \times 3^3 \times 5 \times 7^2, 2^2 \times 3^5 \times 7^3, \times 2^3 \times 5^3 \times 7$

Solution :

Least powers of common prime factors are 2^2 and 7.

H.C.F. = $2^2 \times 7$ = 28.

Ex. 2. Find the H.C.F. of 216, 96 and 480.

Solution :

$216 = 2^3 \times 3^3, 96 = 2^5 \times 3$

and, $360 = 2^5 \times 3 \times 5.$

H.C.F. $= 2^3 \times 3 = 8 \times 3 = 24$

Ex. 3. Find the H.C.F of 875, 625 and 125

Solution :

```
625 ) 875 ( 1
      625
      -----
      250 ) 625 ( 2
            500
            -----
            125 ) 500 ( 4
                  500
                  -----
                   ×
```

∴ H.C.F of 875, 625 is 125

So, required H.C.F = H.C.F of 125 and 125

```
125 ) 125 ( 1
      125
      -----
       ×
```

∴ H.C.F of given numbers = 125

Ex. 4. Reduce $\frac{225}{300}$ to lowest terms.

Solution :

H.C.F of 225 and 300 is 75

On dividing the numerator and denominator by 75,

$$\frac{225}{300} = \frac{225 \div 75}{300 \div 75} = \frac{3}{4}$$

METHODS OF FINDING L.C.M.

Least Common Multiple (L.C.M.)

L.C.M. is the Least Common Multiple, the smallest number which is exactly divisible by all the given numbers.

There are two methods to find L.C.M. of given numbers. They are :

- **Prime Factorization Method**
- **Division Method.**

Finding L.C.M. using Prime Factorization Method

Resolve each one of the given numbers into a product of prime factors.

Then, L.C.M. is the product of highest powers of all the factors.

Finding L.C.M. using Division Method

Step 1 : Write the given numbers in a row in any order. Use comma to separate them.

Step 2 : Divide these numbers by a suitable prime number.

Step 3 : Make sure that the prime number divides at least two of the given numbers.

Step 4 : Now, carry forward the non-divisible numbers.

Step 5 : Repeat the above process till no two numbers are divisible by the same prime number.

Step 6 : The product of the prime numbers and the undivided numbers is the required L.C.M. of the given numbers.

Product of two numbers = Product of their H.C.F. and L.C.M.

Ex. 5. Two numbers are in the ratio 13:12. If their H.C.F is 15, then find the numbers

Solution :

Let the required numbers be $13x$ and $12x$

Then, their H.C.F is x so $x = 15$

$\therefore$ The numbers are (13×15 and 12×15) i.e., 195 and 180

H.C.F of given numbers always divides their L.C.M

Ex. 6. The H.C.F of two numbers is 13 and their L.C.M is 105. If one of the numbers is 39, then find the other?

Solution :

Other number = $\left(\frac{13 \times 105}{39}\right) = 35$

Note :

Two numbers are said to be co-prime if their H.C.F. is 1.

Ex. 7. Find the least square number which is exactly divisible by 12, 15 and 18.

Solution :

L.C.M. of 12, 15, 18 = 180.

Now, $180 = 2 \times 2 \times 3 \times 3 \times 5 = 2^2 \times 3^2 \times 5$.

To make it a perfect square, it must be multiplied by 5.

Required number = . $(2^2 \times 3^2 \times 5) = 900$.

Ex. 8. Find the L.C.M of

$2^3 \times 3^2 \times 5 \times 7^2$, $2^2 \times 3^2 \times 5^2 \times 7^2$, $2^4 \times 3^3 \times 5^2 \times 7^2$

Solution :

L.C.M = Product of highest powers of 2 ,3 ,5, and $7 = 2^4 \times 3^3 \times 5^2 \times 7^2$

Ex. 9. Find the L.C.M of 54 , 48 , 24

Solution :

2	54, 48, 24
2	27, 24, 12
2	27, 12, 6
3	27, 6, 3
3	9, 2, 1
3	3, 2, 1
2	1, 2, 1

$\therefore$ L.C.M = $2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 2 = 432$

H.C.F and L.C.M of Fractions :

- **H.C.F =** $\dfrac{\textbf{H.C.F of Numerators}}{\textbf{L.C.M of Denominatos}}$
- **L.C.M =** $\dfrac{\textbf{L.C.M of Numerators}}{\textbf{H.C.F of Denominatos}}$

Ex. 10. Find the H.C.F and L.C.M of

$\frac{3}{5}, \frac{9}{5}, \frac{12}{10}$ and $\frac{18}{30}$

Solution :

$$\text{H.C.F} = \frac{\text{H.C.F of Numerators}}{\text{L.C.M of Denominators}}$$

$$\text{L.C.M} = \frac{\text{L.C.M of Numerators}}{\text{H.C.F of Denominators}}$$

$$\text{H.C.F} = \frac{\text{H.C.F of } 3, 9, 12, 18}{\text{L.C.M of } 5, 5, 10, 15} = \frac{3}{30} = \frac{1}{10}$$

$$\text{L.C.M} = \frac{\text{L.C.M of Numerators}}{\text{H.C.F of Denominators}} = \frac{36}{5}$$

How to find H.C.F and L.C.M for Decimal Fractions :

Step 1 : Write the given numbers and make the same number of decimal places by annexing zeros in some numbers if necessary.

Step 2 : Consider these numbers without decimal point.

Step 3 : Find the H.C.F and L.C.M as per the given question.

Step 4 : Now, in the result, mark off as many decimal places as there in each of the given numbers.

Ex. 11. Find the H.C.F and L.C.M of

1.15, 1.61 and 1.84

Solution :

Given numbers are 1.15, 1.61 and 1.84

Without decimal places these numbers are 115, 161 and 184.

H.C.F of 115, 161 and 184 is 23.

$\therefore$ H.C.F of 1.15, 1.61 and 18.4 is 0.23.

L.C.M of 115, 161 and 184 is 6440

$\therefore$ L.C.M of 1.15, 1.61 and 1.84 is 64.40 .

Comparison of fractions :

Step 1 : For the given fractions find the L.C.M of the denominators.

Step 2 : Convert each of the fractions into an equivalent fractions with L.C.M.

Step 3 : By multiplying both the numerator and denominator by the same number. The resultant fraction with the greatest numerator is the greatest.

Ex. 12. Arrange the fractions $\frac{1}{9}, \frac{3}{6}, \frac{7}{3}, \frac{5}{12}$ in ascending order.

Solution :

L.C.M of 9, 6, 3 and 12 = 72

$$\frac{1\times 8}{9\times 8} = \frac{8}{72}$$

$$\frac{3\times 12}{6\times 12} = \frac{36}{72}$$

$$\frac{7\times 24}{3\times 24} = \frac{188}{72}$$

$$\frac{5\times 6}{12\times 6} = \frac{30}{72}$$

Since, $8 < 30 < 36 < 188$,

So, $\frac{8}{72} < \frac{30}{72} < \frac{36}{72} < \frac{188}{72}$

Hence, $\frac{1}{9} < \frac{5}{12} < \frac{3}{6} < \frac{7}{3}$

PRACTICE QUESTIONS

LEVEL-EASY

1. The H.C.F of two numbers is 11, and their L.C.M is 616. If one of the numbers is 88. Find the other.
(a) 77 (b) 87
(c) 97 (d) None of these

2. What is the greatest possible rate at which a man can walk 51 km and 85 km in an exact number of minutes ?
(a) 11 km/min (b) 13 km/min
(c) 17 km/min (d) None of these

3. The H.C.F. and L.C.M. of two numbers are 12 and 144 respectively. If one of the numbers is 36, the other number is
(a) 4 (b) 48
(c) 72 (d) 432

4. Find out the smallest square which is exactly divisible by 2, 3, 4, -9, 6, 18, 30 and 60 is :
(a) 900 (b) 1600
(c) 3600 (d) None of these

5. The L.C.M. of two numbers is 135. If their H.C.F. is 5 and one of the numbers is 75, the other is :
(a) 8 (b) 7
(c) 9 (d) 10

6. Find the H.C.F. of $(3^{125} - 1)$ *and* $(3^{35} - 1)$
(a) $(3^4 - 1)$ (b) $(3^5 - 1)$
(c) $(3^6 - 1)$ (d) $(3^7 - 1)$

7. Find the L.C.M. of 2.5, 0.5 and 0.0175
(a) 2.5 (b) 5
(c) 7.5 (d) 17.5

8. The L.C.M. of two numbers is 1890 and their H.C.F. is 30. If one of the number is 270, then the other will be
(a) 210 (b) 220
(c) 310 (d) 320

9. 4 Bells toll together at 9.00 AM. They toll after 7, 8, 11 and 12 seconds respectively. How many times will they toll together again in the next 3 hours ?
(a) 3 (b) 4
(c) 5 (d) 6

10. The H.C.F. of 2472, 1284 and a third number 'N' is 12. If their L.C.M. is $2^3 \times 3^2 \times 5^1 \times 103 \times 107$, then the number 'N' is
(a) $2^2 \times 3^2 \times 7^1$ (b) $2^2 \times 3^3 \times 103$
(c) $2^2 \times 3^2 \times 5^1$ (d) None of these

11. Find the L.C.M. of $2^2 \times 3^3 \times 5 \times 7^2$, $2^3 \times 3^2\ \ 5^2 \times 7^4$, $2 \times 3 \times 5^3 \times 7 \times 11$
(a) $2^3 \times 3^3 \times 5^3 \times 7^4 \times 11$ (b) $2^3 \times 3^3 \times 5^3 \times 7^3 \times 11$
(c) $2^3 \times 3^2 \times 5^3 \times 7^4 \times 11$ (d) $2^2 \times 3^3 \times 5^3 \times 7^4 \times 11$

12. Reduce $\frac{391}{667}$ to lowest terms
(a) $\frac{17}{29}$ (b) $\frac{29}{17}$
(c) $\frac{15}{29}$ (d) $\frac{29}{15}$

13. Find the least numbers which when divided by 20, 25, 35 and 40 leaves remainders 14, 19, 29 and 34 respectively.
(a) 1337 (b) 1352
(c) 1330 (d) 1394

14. Three lights changes colour after every 48 sec, 72 sec and 108 sec. If they all change simultaneously at 8:20:00 hours, then at what time will they again change simultaneously ?
(a) 8: 27: 12 hours (b) 8: 25: 12 hours
(c) 8: 22: 12 hours (d) 8: 29: 12 hours

15. Find the H.C.F. and L.C.M. of 0.63, 1.05 and 2.1
(a) 0.21, 6.3 (b) 6.3, 0.21
(c) 0.25, 6.5 (d) 6.5, 0.25

16. The H.C.F. of two numbers is 98 and their L.C.M. is 2352. The sum of the number may be :
(a) 1372 (b) 1398
(c) 1426 (d) 1484

17. What is the LCM of and ?$x^3 + 8$, $x^2 + 5x + 6$ and $x^3 + 4x^2 + 4x$? ***[CDS, 2017 I]***
(a) $x(x + 2)^2 (x + 3) (x^2 – 2x + 4)$
(b) $x(x + 2)^2 (x – 3) (x^2 + 2x + 4)$
(c) $(x + 2)^2 (x +3) (x^2 – 2x + 4)$
(d) $(x – 2)^2 (x – 3) (x^2 – 2x + 4)$

18. Consider the following statements
I. If a = bc with HCF (b, c) = 1, then HCF (c, bd) = (c, d)
II. If a = bc with HCF (b, c) = 1, then LCM (a, d) = LCM (c, bd)
Which of the above statements is/are correct ? ***[CDS, 2017 I]***
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

19. Consider the following in respect of natural numbers a, b and c : ***[CDS, 2016-I]***
1. L.C. M. (ab, ac) = a L.C.M. (b, c)
2. H.C.F. (ab, ac) = a H.C.F. (b, c)
3. H.C.F. (a, b) < L.C.M. (a, b)
4. H.C.F. $(a$, b) divides L.C.M. (a, b)
Which of the above are correct ?
(a) 1 and 2 only (b) 3 and 4 only
(c) 1, 2 and 4 only (d) 1, 2, 3 and 4

20. Two pipes of length 1.5 m and 1.2 m are to be cut into equal pieces without leaving any extra length of pipes. The greatest length of the pipe pieces of same size which can be cut from these two lengths will be ***[SSC CGL MAINS, 2016]***
(a) 0.13 m (b) 26 m
(c) 0.3 m (d) 0.41 m

21. Three bells ring at interval of 36 seconds, 40 seconds and 48 seconds respectively. They start ringing together at a particular time. They will ring together after every ***[SSC CGL MAINS, 2016]***
(a) 6 minutes (b) 12 minutes
(c) 18 minutes (d) 24 minutes

22. If the product of three consecutive numbers is 210. Then the sum of the smaller numbers is :
[SSC CPO (Re), 2016]
(a) 3 (b) 4
(c) 5 (d) 11

23. The LCM of four consecutive numbers is 60. The sum of the first two numbers is equal to the fourth number. What is the sum of the four numbers ?
[SSC CPO (Re), 2016]
(a) 17 (b) 14
(c) 21 (d) 24

24. Let x be the smallest number which when added to 2000 makes the resulting number divisible by 12, 16, 18 and 21. The sum of the digits of x is
[CGL MAINS, 2015]
(a) 6 (b) 5
(c) 7 (d) 15

25. A number when divided by 361 gives remainder 47. When the same number is divided by 19 then find the remainder ? ***[CGL MAINS, 2015]***
(a) 9 (b) 1
(c) 8 (d) 3

26. The HCF and LCM of two numbers are 21 and 84 respectively. If the ratio of the two numbers is 1 : 4, then the larger of the two numbers is
[CGL MAINS, 2015]
(a) 48 (b) 12
(c) 84 (d) 108

27. The H.C.F. of two natural numbers m and n is 24 and their product is 552. How many sets of values of m and n are possible ? ***[CDS, 2014-I]***
(a) 1 (b) 2
(c) 4
(d) No set of m and n is possible satisfying the given conditions

28. The L.C.M. of two numbers is 90 times their H.C.F.. The sum of L.C.M. and H.C.F. is 1456. If one of the numbers is 160, then what is the other number ?
[CDS, 2014-I]
(a) 120 (b) 136
(c) 144 (d) 184

LEVEL- DIFFICULT

29. What is the H.C.F. of the polynomials $x^3 + 3x^2y + 2xy^2$ and $x^4 + 6x^3y + 8x^2y^2$?
(a) $x (x + 2y)$ (b) $x (x +3y)$
(c) $x + 2y$ (d) None of these

30. If $(x + k)$ is the H.C.F. of $(x^2 + ax + b)$ and $(x^2 + cx + d)$, then what is the value of k ?
(a) $\frac{b+d}{a+c}$ (b) $\frac{b+d}{c+d}$
(c) $\frac{a-b}{c-b}$ (d) $\frac{b-d}{a-c}$

31. L.C.M. of two numbers is 16 times their H.C.F.. The sum of L.C.M. and H.C.F. is 850. If one number is 50, then what is the other number ?

(a) 800 (b) 1200

(c) 1600 (d) 2400

32. 5 bells start tolling together and toll at intervals of 2, 4, 6, 8 and 10 s, respectively. How many times do the five bells toll together in 20 min ?

(a) 10 (b) 11

(c) 12 (d) 15

33. If H.C.F. of m and n is 1, then what are the H.C.F. of m + n, m and H.C.F. of m − n, n respectively ? (m > n)

(a) 1 and 2 (b) 2 and 1

(c) 1 and 1

(d) Cannot be determined

34. Let p, q and r be natural numbers. If m is their L.C.M. and n is their H.C.F., consider the following

I. mn = pqr if each p, q and r is prime.

II. mn = pqr if p, q and r are relatively prime in pairs.

Which of the above statement is/are correct ?

(a) Only I (b) Only II

(c) Both I and II (d) Neither I nor II

35. The L.C.M. of three different numbers is 150. Which of the following cannot be their H.C.F.?

(a) 15 (b) 25

(c) 50 (d) 55

36. What is the H.C.F. of $(x^2 + bx - x - b)$ and,

$[x^2 + x\,(a - 1) - a]$?

(a) $x + b$ (b) $x + a$

(c) $x + 1$ (d) $x - b$

37. If the H.C.F. of $(x^2 + x - 12)$ and $(2x^2 - kx - 9)$ is $(x - k)$, then what is the value of k ?

(a) − 3 (b) 3

(c) − 4 (d) 4

38. What is the L.C.M. of $(6x^3 + 60x^2 + 150x)$ and $(3 + 12 - 15)$?

(a) $6x^2 (x + 5)^2 (x - 1)$ (b) $3x^2 (x + 5)^2 (x - 1)$

(c) $6x^2 (x + 5)^2 (x - 1)^2$ (d) $3x^2 (x + 5)^2 (x - 1)^2$

39. What is the L.C.M. of $(x + 2)^2 (x - 2)$ and $x^2 - 4x - 12$?

(a) $(x + 2)(x - 2)$ (b) $(x+2)^3 (x-2)(x-6)$

(c) $(x + 2)(x - 2)^2$ (d) $(x + 2)^2 (x - 2)(x - 6)$

40. What is the value of k for which the H.C.F. of $2x^2 + kx - 12$ and $x^2 + x - 2k - 2$ is $(x + 4)$?

(a) 5 (b) 7

(c) 10 (d) − 4

41. 21 mango trees, 42 apple trees and 56 orange trees have to be planted in rows such that each row contains the same number of trees of one variety only. What is the minimum number of rows in which the above trees may be planted ?

(a) 3 (b) 15

(c) 17 (d) 20

42. What is the greatest number which divides 392, 486 and 627 so as to leave the same remainder in each case ?

(a) 47 (b) 43

(c) 37 (d) 34

43. Find the greatest possible length which can be used to measure exactly the lengths 4 m 95 cm, 9 m and 16 m 65 cm.

(a) 41 cm (b) 42 cm

(c) 44 cm (d) 45 cm

44. The LCM of $(x^3 - x^2 - 2x)$ and $(x^3 + x^2)$ is

(a) $x^3 - x^2 - 2x$ (b) $x^2 + x$

(c) $x^4 - x^3 - 2x^2$ (d) $x - 2$

45. The H.C.F. of $(x^4 - y^4)$ and $(x^6 - y^6)$ is

(a) $x^2 - y^2$ (b) $x - y$

(c) $x^3 - y^3$ (d) $x^4 - y^4$

46 The product of two numbers is 2160 and their HCF is 12. Number of such possible pairs are

(a) 1 (b) 2

(c) 3 (d) 4

47. The ratio of HCF and LCM of two numbers a and b is 1 : 30 and the difference between the HCF and LCM is 493. Find the possible number of pairs of a and b. ***[SSC CPO (Re), 2016]***

(a) One (b) Two

(c) Four (d) Five

48. What is the sum of digits of the least multiple of 13, which when divided by 6, 8 and 12 leaves 5, 7 and 11 respectively as the remainders ? ***[CDS, 2015-I]***

(a) 5 (b) 6

(c) 7 (d) 8

49. A tin of oil was $\frac{4}{5}$ full. When 6 bottles of oil were taken out from this tin and 4 bottles of oil were poured into it, it was $\frac{3}{4}$ full. Oil of how many bottles can the tin contains ? (All bottles are of equal volume) ***[CDS, 2015-I]***

(a) 35 (b) 40

(c) 45 (d) 50

50. The HCF and LCM of two polynomials are (x + y) and $(3x^5 + 5x^4y + 2x^3y^2 - 3x^2y^3 - 5xy^4 - 2y^5)$ respectively. If one of the polynomials is $(x^2 - y^5)$, then the other polynomial is ***[CDS, 2015-I]***

(a) $3x^4 - 8x^3y + 10x^2y^2 + 7xy^3 - 2y^4$

(b) $3x^4 - 8x^3y - 10x^2y^2 + 7xy^3 + 2y^4$

(c) $3x^4 + 8x^4y + 10x^2y^2 + 7xy^3 + 2y^4$

(d) $3x^4 + 8x^3y - 10x^2y^2 + 7xy^3 + 2y^4$

51. Let x be the least number which when divided by 5, 6, 7 and 8 leaves a remainder 3 in each case but when divided by 9 leaves remainder 0, the sum of digits of x is ***[CGL MAINS, 2015]***

(a) 24 (b) 21

(c) 22 (d) 18

52. For any integers '*a*' and '*b*' with H.C.F. (*a*, *b*) = 1, what is H.C.F. (*a* + *b*, *a* – *b*) equal to ? ***[CDS, 2014-I]***

(a) It is always 1 (b) It is always 2

(c) Either 1 or 2 (d) None of these

ANSWER KEY

1. (a)	2. (c)	3. (b)	4. (a)	5. (c)	6. (b)	7. (d)	8. (a)	9. (c)	10. (c)
11. (a)	12. (a)	13. (d)	14. (a)	15. (a)	16. (a)	17. (a)	18. (c)	19. (c)	20. (c)
21. (b)	22. (d)	23. (b)	24. (c)	25. (a)	26. (c)	27. (d)	28. (c)	29. (a)	30. (d)
31. (a)	32. (a)	33. (c)	34. (c)	35. (d)	36. (d)	37. (b)	38. (a)	39. (d)	40. (a)
41. (c)	42. (a)	43. (d)	44. (c)	45. (a)	46. (b)	47. (c)	48. (d)	49. (b)	50. (c)
51. (d)	52. (c)								

EXPLANATIONS

LEVEL-EASY

1. Using the property,
HCF × LCM = product of the numbers
We get,

$$616 \times 11 = 88 \times N_2$$
$$N_2 = 77$$

2. H.C.F. of 51 and 85 is given by 17.
Hence, the greatest possible rate at which a man walk is 17 km/min.

3. By property,
HCF × LCM = product of the numbers
WKT,

$$12 \times 144 = 36 \times N_2$$
$$N_2 = 48$$

4. Taking L.C.M., We get 180.
Hence, all the multiples of 180 will be divisible by all these given numbers.
In the series 180, 360, 540, 720, 900
900 seems to be the perfect square.

5. Using property,
HCF × LCM = product of the numbers
We get,

$$135 \times 5 = 75 \times N_2$$
$$N_2 = 9$$

6. Based on the rule,
HCF of $(a^m - 1)$ and $(a^n - 1)$ is $(a^{\text{HCF of } m,\, n} - 1)$
5 is the H.C.F. of 35 and 125
∴ The answer is $(3^5 - 1)$

7. Let the numbers be $\frac{5}{2}, \frac{1}{2}, \frac{175}{1000} = \frac{7}{40}$

LCM of three fractions is

$$\frac{\text{LCM of numerators}}{\text{HCF of denominators}} = \frac{\text{LCM of 5, 1 and 7}}{\text{HCF of 2and 40}}$$

$$\frac{35}{2} = 17.5$$

8. Using the formula,
HCF × LCM = product of the numbers
WKT,

$$1890 \times 30 = 270 \times N_2$$
$$N_2 = 210$$

9. L.C.M. of 7, 8, 11 and 12 is 1848.
The answer will be the quotient of the ratio

$$\frac{3 \times 60 \times 60}{1848} = \frac{10800}{1848} = 5$$

Hence they toll again 5 times in the next 3 hours.

10. Here,
$2472 = 2^3 \times 103 \times 3$; $1284 = 2^2 \times 107 \times 3$
Since the H.C.F. is 12, the number must have a component of $2^2 \times 3^1$ at the very least in it.

L.C.M. is $2^3 \times 3^2 \times 5^1 \times 103 \times 107$

WKT, the required number has to be $3^2 \times 5^1$

Combining these two requirements, we get the number should have $2^2 \times 3^2 \times 5^1$ at the minimum and the power of 2 could also be 2^3 while we could also have either 1 of 103^1 and/or 107^1 as a part of the required number.

Hence, $2^2 \times 3^2 \times 5^1$ gives the possible value of the number and it is the correct answer.

11. L.C.M.= Product of highest powers of 2, 3, 5, 7 and 11

$$= 2^3 \times 3^3 \times 5^3 \times 7^4 \times 11$$

12. H.C.F. of 391 and 667 is 23.

On dividing the numerator and denominator by 23, we get

$$\frac{391}{667} = \frac{391 \div 23}{667 \div 23} = \frac{17}{29}$$

13. Here, (20-14) = 6, (25-19) = 6,

(35-29) = 6 and (40-34) = 6

Required number = (L.C.M. of 20, 25, 35, 40)-6=1394

14. Changing interval= (L.C.M. of 48, 72, 108) sec= 432 sec

So, the three lights will again change simultaneously after every 432 seconds. *i.e.,* 7 min. 12 sec.

Hence, next simultaneous change will take place at 8:27:12 hours.

15. Without decimal places, these numbers are 63, 105, 210

Now, H.C.F. of 63, 105, 210 is 21

H.C.F. of 0.63, 1.05, and 2.1 is 0.21

L.C.M. of 63, 105, and 210 is 630

L.C.M. of 0.63, 1.05, and 2.1 is 6.3

Hence the H.C.F. and L.C.M. would be 0.21 and 6.3

16. H.C.F. of two numbers is 98. It means that 98 is common in both the numbers. Therefore, the sum of these two numbers also be multiple of 98. So, 1372 is divided by 98.

17. We have,

$$\begin{aligned} x^3 + 8 &= (x)^3 + 2^3 \\ &= (x+2)(x^2 - (x)(2) + 2^2) \\ &= x(x+2) + 3(x+2) \\ &= (x+2)(x+3) \\ &= (x+2)(x^2 - 2x + 4) \\ x^2 + 5x + 6 &= x^2 + 2x + 3x + 6 \\ &= x(x+2) + 3(x+2) \\ &= (x+2)(x+3) \end{aligned}$$

and

$$\begin{aligned} x^3 + 4x^2 + 4x &= x(x^2 + 4x + 4) \\ &= x(x+2)^2 \end{aligned}$$

LCM = $x(x+2)^2(x+3)(x^2 - 2x + 4)$

18. I. If a = bc with HCF (b, c) = 1

$\Rightarrow$ b and c are co-prime numbers.

$\therefore$ HCF (c,bd) = HCF (c,d) , which is correct

II. If a = bc with HCF (b, c) = 1

$\Rightarrow$ b and c are co-prime numbers.

$\therefore$ LCM (b,c) = bc

Now, LCM (a, d) = LCM (bc, d)

$\therefore$ LCM (a,d) = LCM (c,bd) , which is correct

19. Option c is correct. *i.e.* the statements 1, 2 and 4 are correct.

20. The greatest length of pipes of same size will be HCF of 1.5 m and 1.2 m.

So HCF of 15 and 12 is 3, then HCF of 1.5 and 1.2 is 0.3 m.

21. Time interval = 36, 40, 48

$$\text{LCM of (36, 40, 48) sec} = 720 \text{ sec} = \frac{720}{60} = 12 \text{ min}$$

So, they will ring together after 12 min.

22. 210 = 21 × 10 = 7 × 3 × 2 × 5

Take 2 and 3 together then the obtained number is 5, 6, and 7 which is the consecutive number.

So, $1^{st} + 2^{nd} = 5 + 6 = 11$

23. $x, x+1, x+2, x+3$

$$1^{st} \text{ term} + 2^{nd} \text{ term} = 4^{th} \text{ term}$$

$$x + x + 1 = x + 3$$

$$\Rightarrow \quad x = 2$$

$x, x+1, x+2, x+3$

$\Rightarrow$ 2,3,4,5

$\Rightarrow \quad 2 + 3 + 4 + 5 = 14$

24. LCM of 12, 16, 18, 21 = 1008

Next number = 1008 × 2 = 2016

Divisible by all

$\therefore$ 16 is added

Sum of the digits = 1 + 6 = 7

25. $\dfrac{\text{Remainder of the number}}{19} = \dfrac{47}{19}$

= [Remainder] = 9

26. We know,

LCM × HCF = First number Second number

Let First number be K and the second number be 4K.

$$K \times 4K = 21 \times 84$$

$$K = 21$$

Then, Number 21, 84

So, the larger number = 84.

27. H.C.F. of two natural numbers m and n = 24

Product of m and n = 552

L.C.M. of two natural numbers

$$= \frac{\text{Product of } m \text{ and } n}{\text{H.C.F. of } m \text{ and } n} = \frac{552}{24} = 23$$

Therefore, no set of m and n is possible satisfying the given condition.

28. Let the H.C.F. of two number = x

The L.C.M. of two numbers = 90 x

According to question,

L.C.M. + H.C.F. = 1456

$$90x + x = 1456$$

$$x = 16$$

H.C.F. of two numbers = 16,

L.C.M. of two number = 1440

L.C.M. × H.C.F. = 1^{st} numbers × 2^{nd} number

$\Rightarrow$ 2^{nd} Number = $\frac{1440 \times 16}{160} = 144$

LEVEL - DIFFICULT

29. Let $f_1(x) = x^3 + 3x^2y + 2xy^2$

$= x(x^2 + 3yx + 2y^2)$

$= x(x^2 + 2xy + xy + 2y^2)$

$= x[x(x + 2y) + y(x + 2y)]$

$= x(x + y)(x + 2y)$

And $f_2(x) = x^4 + 6x^3y + 8x^2y^2$

$= x^2(x^2 + 6xy + 8y^2)$

$= x^2(x^2 + 2xy + 4xy + 8y^2)$

$= x^2[x(x + 2y) + 4y(x + 2y)]$

$= x^2(x + 2y)(x + 4y)$

H.C.F. of $f_1(x)$ and $f_2(x) = x(x + 2y)$

30. $(x + k)$ is the H.C.F. of $(x^2 + ax + b)$ and $(x^2 + cx + d)$.

$(-k)^2 + a(-k) + b = 0$

$= (-k)^2 + c(-k) + d = 0$

$(a - c)k = (b - d)$

$k = \frac{b - d}{a - c}$

31. Let first number = x, second number = y

L.C.M. × H.C.F. = Product of numbers = $x \times y$

Given, L.C.M. = 16 × H.C.F

L.C.M. + H.C.F. = 850 and $x = 50$

16 × H.C.F. + H.C.F. = 850

17 × H.C.F. = 850

H.C.F. = 50

Now, L.C.M. = 16 × 50 = 800

L.C.M. × H.C.F. = Product of numbers = $x \times y$

800 × 50 = 50 × y

$y = 800$

The other number,

32. L.C.M. of 2, 4, 6, 8 and 10 is 120s.

i.e., 2 min after tolling together.

Total in 20 min = $\frac{\text{Total time}}{\text{L.C.M. intervals}}$

In 20 min tolling = $\frac{20 \text{ min}}{2 \text{ min}} = 10$ times

33. H.C.F. of m and n is 1.

H.C.F. (m + n, m) = 1

And H.C.F. (m – n, n) = 1

34. Let $p = 10$, $q = 11$, $r = 13$ (Co-prime numbers)

L.C.M. of (10, 11, 13) = 1430

H.C.F. (10, 11, 13) = 1

$mn = 1430 \times 1 = 1430$

Also, $pqr = 10 \times 11 \times 13 = 1430$

So, $mn = pqr$

35. We know that L.C.M. is the multiple of H.C.F.. So that 55 cannot be H.C.F. because it is not a divisor of 150.

36. Let $f_1(x) = x^2 + bx - x - b$

$= x(x + b) - 1(x + b)$

$= (x - 1)(x + b)$

And $f(x) = x^2 + xa - x - a$

$= x(x + a) - 1(x + a)$

$= (x + a)(x - 1)$

H.C.F. of $f_1(x)$ and $f(x) = (x - 1)$

37. H.C.F. of $x^2 + x - 12$ and $2x^2 - kx - 9$ is $(x - k)$,

Then $x = k$ will be the factor of $2x^2 - kx - 9$

$2k^2 - k^2 - 9 = 0$

$\Rightarrow$ $k^2 - 9 = 0$

$k = \pm 3$

And factor of $x^2 + x - 12$ are $(x + 4)(x - 3)$.

Hence, the value of k is 3.

38. Let $f_1(x) = 6x^3 + 60x^2 + 150x$

$= 6x(x^2 + 10x + 25)$

$= 3x^2 \times 2 \times (x + 5)^2$ and

$f_1(x) = 3x^4 + 12x^3 - 15x^2$

$= 3x^2(x^2 + 4x - 5)$

$= 3x^2(x^2 + 5x - x - 5)$

$= 3x^2(x + 5)(x - 1)$

L.C.M. of $f_1(x)$ and $x^2(x)$

$= 3 \times 2 \times x^2 \times (x + 5)^2 (x - 1)$

$= 6x^2(x + 5)^2(x - 1)$

39. Let $f_1(x) = (x + 2)^2(x - 2)^2$

And $f_2(x) = x^2 - 4x - 12 = (x - 6)(x + 2)$

L.C.M. of $f_1(x)$, $f_2(x) = (x + 2)^2(x - 2)(x - 6)$

40. $(x + 4)$ is H.C.F., so it will be common in both expressions

$x = -4$ will make each one zero.

$2(-4)^2 + k(-4) - 12 = 0$

and $(-4)^2 + (-4) - 2k - 2 = 0$

$32 - 12 = 4k$ and $16 - 6 = 2k$

$k = 5$.

41. Minimum number of rows = $\frac{21}{7} + \frac{42}{7} + \frac{56}{7} = 17$

42. Given numbers are 392, 486 and 627.

For same remainder

486 – 392 = 94

627 – 486 = 141

627 – 392 = 235

H.C.F. of (94, 141, 235) = 47

43. Required length= H.C.F. of 495 cm, 900 cm and 1665 cm.

$495 = 3^2 \times 5 \times 11$, $900 = 2^2 \times 3^2 \times 5^2$

$1665 = 3^2 \times 5 \times 37$

HCF $= 3^2 \times 5 = 45$

Hence, required length = 45 cm

44. Let $f_1(x) = x^3 - x^2 - 2x$

$= x\,(x^2 - x - 2)$

$= x\,\{x^2 - 2x + x - 2\}$

$= x\,\{x\,(x - 2) + 1\,(x - 2)\}$

$= x\,(x + 1)\,(x - 2)$

And $f_2(x) = x^3 + x^2 = x^2\,(x + 1) = x.\,x\,(x + 1)$

$\therefore$ LCM of $[f_1(x), f_2(x)] = x\,(x + 1).\,x\,(x - 2)$

$= x^2(x + 1)\,(x - 2) = x^2\,(x^2 - x - 2)$

$= x^4 - x^3 - 2x^2$

45. Let $f_1(x) = (x^4 - y^4) = [((x^2)^2 - (y^2)^2]$

$= (x^2 - y^2)\,(x^2 + y^2)$

$= (x - y)\,(x + y)\,(x^2 + y^2)$

And $f_2(x) = (x^6 - y^6)$

$= (x^3)^2 - (y^3)^2 = (x^3 + y^3)\,(x^3 - y^3)$

$= (x + y)\,(x^2 - xy + y^2)\,(x - y)\,(x^2 + xy + y^2)$

$= (x - y)\,(x + y)\,(x^2 - xy + y^2)\,(x^2 + xy + y^2)$

H.C.F. of $[f_1(x), f_2(x)] = (x - y)\,(x + y) = x^2 - y^2$

46. HCF = 12

$\therefore$ Let numbers are $12x$ and $12y$

$\therefore$ Product of two numbers $= 12x \times 12y = 144xy$

$\Rightarrow \quad 144xy = 2160$

$\Rightarrow \quad xy = 15$

Possible pairs are (1, 15), (3, 5), factors should be co-prime. Two pairs are possible.

47. $\dfrac{\text{HCF}}{\text{LCM}} = \dfrac{1}{30}$

HCF $= x$ (Let), LCM $= 30x$

LCM – HCF = 493

$30x - x = 493$

$29x = 493$

$x = 17$

HCF = 17, LCM = 510

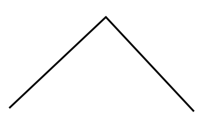
So, Number $= 17a \times 17b$

LCM × HCF = First term × Second term

510 × 17 = 17a × 17 b

$ab = 30$

30



1	×	30
2	×	15
3	×	10
5	×	6

Possible number of pairs = 4 pairs.

48. LCM of 6, 8, 12 is 24

Number is when divided by 6, 8 and 12 leaves 5, 7 and 11 as remainders, as 6 – 5 = 1, 8 – 7 = 1, 12 – 11 = 1 So (24 k – 1) will be divisible by 13.

$\Rightarrow \dfrac{24k - 1}{13}$, is divisible for $k_{min} = 6$

At $k = 6$, Number is $= 24\,k - 1$

$= 24 \times 6 - 1 = 143$

$\Rightarrow$ Sum of its digit = 1 + 4 + 3 = 8

49. Let the number of bottles in the tin be 20 n.

[LCM of (5, 4) = 20]. Initially it had 16 n bottles. 6 bottles were removed and 4 were poured into the tin. Then it was ¾ full.

$16n - 6 + 4 = 15n$

$\Rightarrow \quad n = 2$

$\therefore \quad 20n = 20 \times 2 = 40$

50. HCF × LCM = 1st polynomial × 2nd polynomial

$\Rightarrow 2^{nd}$ polynomial $= \dfrac{\text{HCF LCM}}{1^{st}\text{ Polynomial}}$

$$= \frac{(x+y)\times(3x^5 + 5x^4y^2 + 2x^3y^2 - 3x^2y^3 - 5xy^4 - 2y^5}{(x^2 - y^2)}$$

$3x^4 + 8x^3y + 10\,x^2\,y^2 + 7xy^3 + 2y^4$

51. LCM of 5, 6, 7 & 8 = 840

$\dfrac{840n + 3}{9} \Rightarrow \dfrac{3n + 3}{9}$

Take $n = 2$

$3(2) + 3 = 9$

$\Rightarrow \quad \dfrac{9}{9}$ = Remainder = 0

Number is $840n + 3$

$(n = 2)$

$\Rightarrow \quad 840\,(2) + 3$

$\Rightarrow \quad 1683$

sum of the digits = 18

52. Given that H.C.F. $(a, b) = 1$ means that a and b are co-prime numbers.

So, H.C.F. $(a + b, a - b)$

Let $a = 4, b = 3$

H.C.F. (4, 3) = 1

Now, H.C.F. (3 + 4, 4 – 3)

= H.C.F. (7, 1)

H.C.F. is equal = 1

Let $a = 23$ and $b = 17$

H.C.F. (23, 17) = 1

H.C.F. (23 + 17, 23 – 17) = H.C.F. (40, 6) = 2

So, H.C.F. $(a + b, a - b)$ = either 1 or 2

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