1000+
SOLVED
EXAMPLES
& PRACTICE
QUESTIONS

Pearson SUCCESS COMPANION

MASTER NUMBER SYSTEM for CAT and GMAT®

Also useful for other MBA Entrance Examinations

THIRD EDITION

Pearson

Nishit K. Sinha

About Pearson

Pearson is the world's learning company, with presence across 70 countries worldwide. Our unique insights and world-class expertise comes from a long history of working closely with renowned teachers, authors and thought leaders, as a result of which, we have emerged as the preferred choice for millions of teachers and learners across the world.

We believe learning opens up opportunities, creates fulfilling careers and hence better lives. We hence collaborate with the best of minds to deliver you class-leading products, spread across the Higher Education and K12 spectrum.

Superior learning experience and improved outcomes are at the heart of everything we do. This product is the result of one such effort.

Your feedback plays a critical role in the evolution of our products and you can contact us – reachus@pearson.com. *We look forward to it.*

This page is intentionally left blank

MASTER NUMBER SYSTEM FOR CAT AND GMAT®

THIRD EDITION

Nishit K. Sinha

Pearson

Published by Pearson India Education Services Pvt. Ltd, CIN: U72200TN2005PTC057128, formerly known as TutorVista Global Pvt. Ltd, licensee of Pearson Education in South Asia.

ISBN: 9789332586437
eISBN: 9789332587755

Head Office: 15th Floor, Tower-B, World Trade Tower, Plot No. 1, Block-C, Sector-16, Noida 201 301,Uttar Pradesh, India.
Registered Office: 4th Floor, Software Block, Elnet Software City, TS-140, Block 2 & 9, Rajiv Gandhi Salai, Taramani, Chennai 600 113, Tamil Nadu, India.
Fax: 080-30461003, Phone: 080-30461060
www.pearson.co.in, Email: companysecretary.india@pearson.com

Dedicated to
the Little Angels in my Life
Aarna, Aadya, Samar, Aaradhya and counting....

Contents

This page is intentionally left blank

Preface

'Master Number System for CAT and GMAT®' now in its third edition, provides an array of comprehensive and class-tested material which will help the students to understand the basic concepts of number system as well as how to apply that knowledge to an ever-expanding set of numerical applications/problems/questions. As students progress in this book, they will learn to apply different strategies and will also be developing critical-thinking skill. This new edition offers fully-updated material to capture the recent changes in the CAT and GMAT papers.

It has been observed that every year approximately 20% of QA paper in CAT and other competitive examinations are based on number system questions.

This book is designed keeping in mind the requirement of students who might not have been in touch with their studies or the fundamentals required to understand the questions in the number system. In addition to these, while writing this book, special emphasis has also been laid on:

- How to build an approach to solve a question—so that even if one has never seen a question in his/her life, one should be able to solve that under testing conditions.
- Concepts have been dealt in an easy way to enable application under different contexts—to stimulate your thoughts that 'Why I never thought like this?'.

Every chapter includes variety of solved examples tagged with important concepts and theory. There are three levels (simple to moderate to complex) of practice exercises provided at the end of every chapter, along with previous years' questions. Five sectional tests have been included at the end of the book for self assessment—on understanding basic concepts, critical-thinking skills, and time management techniques for the examination.

New to this edition:

(a) GMAT coverage

(i) A new chapter on Data Sufficiency has been added that deals with 'What' and 'How' of data sufficiency for GMAT including one fully solved practice exercise. Since data sufficiency questions are asked in CAT and other Indian B-School examinations too, this chapter will provide a shot in the arm in students' preparation.

(ii) Data Sufficiency questions with explanations have also been added.

(b) Looking at the relative importance and number of questions asked at different examinations in recent years and extrapolating the same, some of the chapters/concepts have been re-written as per the recent trends for effective preparation.

(c) Solutions and explanations to more than 90% of the questions have been provided now.

Many students reported about how this book has helped them master Number System in a systematic way. Some of the mails also suggested how this book can be made more useful for

GMAT and GRE, as good number of questions in these two exams are asked from Number System. I am overwhelmed by the responses and positive feedback received on the second edition of this book. I accept all the feedback with utmost gratitude and humility. This proved to be a huge stimulus in terms of shaping the current edition of this book.

I am sure this book will help students immensely in learning concepts of number system and cracking the examination. Although I have taken utmost care in preparing the manuscript and checking subsequent proofs, there may be a possibility of some errors creeping in side the book. Constructive comments and suggestions to further improve the book would be acknowledged gratefully.

For any further query and feedback email us at reachus@pearson.com/
nishit.alexander@gmail.com

Nishit K. Sinha
(Alumnus, IIM Lucknow)
Founder Partner, dueNorth India Academics LLP

Examination Pattern and Trends

How to Prepare for Common Admission Test (CAT): Story of Moves

Let me start with the famous story 'Garry Kasparov Vs IBM's Deep Blue computer':

"During 1996–1997, Deep Blue developed by IBM played two series of matches with the then world chess champion Garry Kasparov. While the first series of six matches played in 1996 were won by Garry Kasparov (Wins–3, Draws–2, Loss–1), disproving the IBM's supremacy claim of machine over man, the second series played in May 1997 was won by IBM's machines. After this loss, Kasparov said that he sometimes saw deep intelligence and creativity in the machine's moves, suggesting that during the second game, human chess players, in violation of the rules, intervened. As expected, IBM vehemently denied that it cheated, saying that the only human intervention occurred between games and not during the games. The rules allowed the developers to modify the program between games, an opportunity IBM said it used to shore up weaknesses in the computer's playing prowess that were revealed during the course of the match."

What are the typical responses/reactions we have when we see a question? Some of those most frequently occurring are listed below:

Responses/Reactions
1. Solution is clear end to end
2. Solution is somewhat clear
3. Not at all having any idea of the solution

Response/Reaction 1 I know where to start and how to reach to the final solution. So, it is just a matter of few seconds/minutes that I will have the solution.

Response/Reaction 2 Solution is somewhat clear—knowing how to start, but not absolutely sure if it will lead to the end solution.

Response/Reaction 3 Not at all having any idea of the solution—How to approach or proceed is not clear?

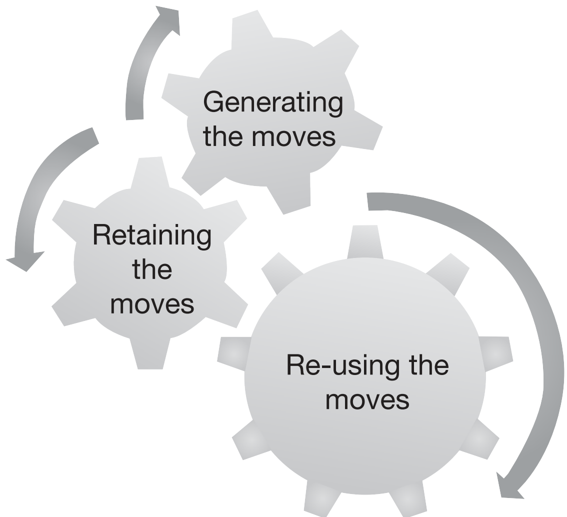
You may wonder why I started with *Kasparov Vs IBM* story and how is it linked to the CAT preparation and the possible reactions. To answer this, I raise another question—Human beings have the thinking capabilities, and so s/he can decide the moves on the basis of the changing situations and so s/he plays chess. How does a computer actually play chess?

If we go through the Kasparov Vs Deep Blue story in flashback, we get to know that there were some chess grandmasters who were the part of IBM Deep Blue manufacturing team and supplying the most crucial information—the database of moves played across thousands of games. So while these moves created a repository of information inside the 'brain' of computer, a complex modelling was done to decide which counter move is best for what move—something on the lines of decision tree analysis through probabilistic calculations. And here was the need for faster calculation. So, for 'X' move placed by Kasparov, computer used to analyse how many times this move has been put and respective countermoves are having what probability of leading to winning position.

This might have answered the most universal query of the students—why are we able to solve a particular question and not been able to solve another question? So its just about the moves–solution to every question involves some moves (loosely speaking, steps are having the same connotation as moves). If we have the moves ready with us from the starting to the end of the solution, we have first response/reaction. And so on, second or third response that how many moves we have.

Summarizing, we can solve a question if either we have the moves already installed inside our brain, or else we would be required to generate it on our own at that very moment inside the examination hall. Probably this is the single most important reason that why a question from 'Function Chapter' seems comparatively easier to a student who has gone through maths in 10+2 with respect to a student who has not gone through maths in his/her 10+2.

So, how can a student who has not gone through maths can come at par with a student who is a maths graduate?



And how moves will be generated (the base of this triad)—By going through the concepts, and solving not just quantity of questions, but quality of questions too.

Looking upon the pattern of CAT QA for the past years, one can be reasonably sure that fetching 40% marks in QA is sufficient to get approximate 97–98 percentile. This translates into approx 10 questions. The idea is—if we can have the moves ready for 6 questions, for an example, then we would be required to generate moves only for 4 questions inside the examination hall, and this saves your considerable time that can be invested in other sections, including QA too.

For example, there was a question in CAT 2008—What is the number of terms in the expansion of (a + b + c)20?

I have put almost similar question in book, What is the number of terms in the expansion of (a + b + c)10?

I am not saying that our objective should be to match the question—that as a student I should have done exactly the same question appearing in CAT of that year before sitting in CAT. Rather, its about thought process—retaining the moves used earlier while solving a question—and the ability to use the same in even slightly different situation. And what is true for QA is mostly true for other sections too.

For any feedback and suggestion, please contact us at reachus@pearson.com/ nishit.alexander@gmail.com

Nishit K. Sinha

This page is intentionally left blank

ALL ABOUT NUMBERS

1

Learning Objectives

After completion of this chapter, you should have a thorough understanding of the following:

- Types of numbers and their properties
- Methods of counting—digits and numbers
- VBODMAS rule—precedence of the operations
- Types of questions asked from this chapter in CAT and GMAT®

Number is 'a numeral or group of numerals'. In other words, it is a word or symbol, or a combination of words or symbols, used in counting several things.

TYPES OF NUMBER

Broadly, numbers can be defined to be of two types: Real number and imaginary number.

Real Number Real number is a number that can be expressed on the number line. In other words, all the real numbers can be felt or experienced in the real world.

Imaginary Number In contrast to real numbers, imaginary numbers cannot be plotted on the number line, or, they cannot be felt or experienced in the real world.

REAL NUMBER

Types of Real Number

Rational Number

A rational number is a number that can be expressed as a fraction $\left(\frac{p}{q}\right)$, where p and q are integers and $q \neq 0$. The term 'rational' comes from another word ratio, because the rational numbers can be written in the format of a ratio.

For example: $\frac{3}{5}, \frac{4}{8}, \frac{22}{7}$, etc., are rational numbers. Rational numbers can be positive, as well as negative.

In a rational number $\frac{p}{q}$, p is known as numerator and q is known as denominator.

We also understand that,

If numerator is more than denominator, then ratio is known as improper ratio.

Examples $\frac{5}{3}, \frac{7}{4}$.

If the numerator is less than the denominator, then the ratio is known as proper ratio.

Examples $\frac{3}{7}, \frac{7}{18}$.

So what kind of numbers are rational numbers?

1. **All the integers—positive or negative or zero—are rational numbers.**
   For example: (–2), (–100), 10, etc., are rational numbers.
2. **Is 0.5555555…(5 is repeating till infinity) a rational number?**
   Yes, it is, because 0.5555…(5 is repeating till infinity) $= \frac{5}{9}$.

   All the repeating decimal numbers with a fixed period are rational.

   For example: 0.543543543543… (543 is repeating till infinity) $= \frac{543}{999}$.
3. **All finite decimals, like 0.48 are rational. In other words, all the terminating decimals are rational numbers.**
   For example: 0.5678345678 is a rational number because it can be written in the form of $\frac{p}{q}$.

Methods to convert decimals into fractions:

1. If a number is finite decimal like 0.89.

$$0.89 = \frac{89}{100}$$

$$1.27 = \frac{127}{100}$$

$$0.7 = \frac{7}{10} = \frac{70}{100}$$

2. If the number is repeating decimal number like 0.454545…

$0.454545\ldots = \frac{45}{99}$

$0.444444\ldots = \frac{4}{9}$

$0.678678678678\ldots = \frac{678}{999}$

$1.23232323\ldots = 1\frac{23}{99} = \frac{122}{99}$

So dividing by 9 repeats the digits once, dividing by 99 repeats the digits twice, dividing by 999 repeats the digits thrice and so on. In other words, number of 9's in the denominator will be equal to the number of repeating digits in the number.

$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{4}{9}$	$\frac{5}{9}$	$\left(\frac{8}{9}\right)$	$\frac{25}{99}$	$\frac{214}{99}$
0.1111…	0.2222…	0.3333…	0.444…	0.5555…	0.888…	0.252525…	2.161616…

Solved Example 1

Convert 3.1232323… into rational form.

Solution Digits are not repeating after the decimal, rather digits are repeating after one digit from decimal.

So, we can write $3.1232323\ldots = 3.1 + 0.0232323\ldots$

We just have to convert $0.0232323\ldots$ into rational form. Remaining part = 3.1 can be converted into rational part very easily.

$0.02323232\ldots = 0.1 \times 0.232323\ldots = \frac{1}{10} \times \frac{23}{99} = \frac{23}{990}$.

So, $3.1232323\ldots = \frac{31}{10} + \frac{23}{990} = \frac{3092}{990}$.

Alternatively, we can also do this in the following way

Let $P = 3.1232323\ldots \Rightarrow 10P = 31.232323\ldots = 31\frac{23}{99} = \frac{3092}{99}$.

Hence, $P = \frac{3092}{990}$.

Solved Example 2

Convert 3.15474747… into rational form.

Solution Assume $P = 3.15474747\ldots$

We can see that digits are repeating two digits after decimal.

$$100P = 315.474747\ldots = 315\frac{47}{99} = \frac{31232}{99}.$$

Hence, $P = \frac{31232}{9900}$.

Irrational Numbers

Any real number that is not rational is irrational. An irrational number is a number that cannot be expressed as a fraction $\frac{p}{q}$ for any integer p and q. Irrational numbers have decimal expansions that neither terminate nor become periodic.

The most famous irrational number is √2, also known as Pythagoras's constant.

Real numbers are also defined as the set of rational and irrational numbers.

Solved Example 3

Identify which of the following are rational numbers?

(a) $\frac{5}{9}$

(b) 0.33333 (3 is repeating till infinity)

(c) $\sqrt{2}$

(d) $\frac{22}{7}$

(e) π

(f) 0.573573573 (573 is repeating till infinity)

(g) 5.63796246067726496778346521 (till infinity)

(h) $(\sqrt{5})^{100}$

(i) $(\sqrt{5})^{100} + \frac{1}{(\sqrt{5})^{100}}$

Solution

(a) Yes

(b) Yes. 0.3333333333333 (3 is repeating till infinity) $= \frac{3}{9}$ = Rational number.

(c) √2 cannot be written in the form of $\frac{p}{q}$, where p and q are integers ($q \neq 0$). Hence, irrational number.

(d) Yes. $\frac{22}{7}$ is in the form of $\frac{p}{q}$. Hence, rational number.

(e) Irrational number $\pi\left(\frac{22}{7}\right)$ is only an approximate value, and not the exact value. Hence, No.

(f) 0.573573753 (573 is repeating till infinity) $= \dfrac{573}{999}$ = Rational number.

(g) Irrational number. Despite 5.63796246067726496778346521 ... goes till infinity, but it does not show any pattern (unlike previous question), hence, cannot be presented as a ratio $= \dfrac{p}{q}$.

(h) $(\sqrt{5})^{100} = 5^{50}$ = rational number.

(i) $(\sqrt{5})^{100} + \dfrac{1}{(\sqrt{5})^{100}}$ rational number + rational number = rational number.

Alternatively, $(\sqrt{5})^{100} + \dfrac{1}{(\sqrt{5})^{100}} = 5^{50} + \dfrac{1}{5^{50}} = \dfrac{5^{100}+1}{5^{50}} = \dfrac{p}{q}$ = Rational number.

A dilemma—Is 0.9999999 ... equal to 1?

Obviously, when we write this down mathematically, $0.9999999\ldots = 0.\overline{9} = \dfrac{9}{9} = 1$.

Although a more logical question arises that any number of times we write 9 after the decimal as in 0.99999 ..., this should not be equal to 1.

An explanation to this is given in the following manner:

Do not look at 0.99999 ..., as if we are approaching towards 1, see this number as if we are moving away from 1 and look at the distance between 1 and this number.

So, distance between 1 and 0.9 = 0.1

Distance between 1 and 0.99 = 0.01

Distance between 1 and 0.999 = 0.001

Distance between 1 and 0.9999 = 0.0001

We can see that the distance is very slowly getting smaller, and is tending towards zero. So after writing a number of 9's as in 0.999999 ..., distance will become equal to zero, and number = 1.

Integers

Integers can be either positive or negative or zero. Integers are also classified as odd or even integers.

Even Integers Any integer that can be written in the format of $2N$ is an even integer, where N is an integer. In other words, any number that is divisible by 2 is an even integer. For example, 2, 4, 100, etc.

Is (–10) an even integer → Yes (–10) is an even integer.
Is zero an even integer → Yes, 0 is an even integer.

Odd Integers Any integer that can be written in the format of $2N + 1$ is an odd integer, where N is an integer. For example, 1, 5, 101, –23 are odd integers.

Odd ± Odd = Even

Even ± Even = Even

Odd ± Even = Odd

Even ± Odd = Odd

$(\text{Odd})^{\text{Even}} = \text{Odd}$

$(\text{Even})^{\text{Odd}} = \text{Even}$

Odd × Even = Even

Odd × Odd = Odd

Even × Even = Even

Well, the idea is not asking you to mug it up. Rather, develop an understanding and whenever required, you should be able to verify it on your own.

Solved Example 4

Let x, y and z be distinct integers, x and y are odd and positive, and z is even and positive. Which one of the following statements cannot be true? **{CAT 2001}**

(a) $y(x - z)^2$ is even
(b) $y^2(x - z)$ is odd
(c) $y(x - z)$ is odd
(d) $z(x - y)^2$ is even

Solution Answer can be verified by assuming the values $x = 1$, $y = 3$, $z = 2$. Let us verify options:

(a) $(x - z)$ = Odd. So $y(x - z)^2$ is also odd. Hence option a is not true.
(b) Since y and $(x - z)$ both are odd, hence, $y^2(x - z)$ is odd. So option b is true.
(c) Since y and $(x - z)$ both are odd, hence, $y(x - z)$ is odd. So option c is true.
(d) Since z and $(x - y)$ both are even, hence, $z(x - y)^2$ is even. So option d is true.

Hence, answer = option a.

Solved Example 5

1st 100 natural numbers are written on a black board. Two persons A and B are playing a game of putting '+' and '–' sign one by one between any two consecutive integers out of these 100 natural numbers. Both A and B are free to put any sign (+ or –) anywhere, provided there is no sign already placed between the two natural numbers. At the end, when the signs were put between all such two consecutive natural numbers, result is calculated. If the result is even, then A wins and if the result is odd, then B wins. Who will win?

Solution It can be seen that there are 50 odd and 50 even numbers.

So, 100 consecutive natural numbers = set of 50 odd and 50 even numbers.

Whatever sign we put between two odd numbers, resultant of 50 odd numbers = Even, and similarly, whatever sign we put between two even numbers, resultant of 50 even numbers = Even.

Hence the net result is Even + Even or Even – Even. In any case, net result = Even.

Hence A will win.

We can also see here that '+' sign or ' – ' sign does not matter here. Irrespective of the signs put by A or B, A is always going to win.

Solved Example 6

When 98 is added to a perfect square, another perfect square is obtained. How many such pairs of perfect squares exist?

Solution

$A^2 = B^2 + 98$

$\Rightarrow A^2 - B^2 = 98$

$\Rightarrow (A - B)(A + B) = 98$

Different possibilities for A and B are:

A	B
Even	Even
Odd	Odd
Even	Odd
Odd	Even

Consequently, different possibilities for A^2 and B^2 are:

A^2	B^2	$A^2 - B^2$	Remarks
Even	Even	Even	May be possible
Odd	Odd	Even	May be possible
Even	Odd	Odd	Not possible (as the difference = 98)
Odd	Even	Odd	Not possible (as the difference = 98)

If both A and B are even, then both A^2 and B^2 are going to be multiples of 4 (even^2). Hence, $A^2 - B^2 = 4K \neq 98 \Rightarrow$ So, A = Even and B = Even are not possible.

Last possibility for this calculation to be true is, A and B both are odd. Let us verify that:

If A and B both are odd, then $(A - B)$ and $(A + B)$ both will be even. Hence, product of $(A - B)(A + B)$ = Even × Even = $4K \neq 98 \Rightarrow$ So, A = Odd and B = Odd are not possible.

Hence, we can conclude now that no such set exists.

Whole Numbers

When 0 is added to the set of natural numbers, we obtain whole numbers. So, whole numbers are 0, 1, 2, 3, 4, 5,… up to infinity. Whole numbers are also known as non-negative integers.

Lowest whole number = 0.

Natural Numbers

Natural numbers are counting numbers: 1, 2, 3, 4, 5,… up to infinity. Natural numbers are also known as positive integers.

Lowest natural number = 1 = 1st natural number. Since the difference between any two consecutive natural number = 1, we say that 10th natural number from starting = 10, or 22nd natural number from starting = 22 and so on.

Equation format of natural number → $a_n = a_{n-1} + 1$, where, $a_1 = 1$ and a_1 is the lowest natural number.

Function format of natural number → $f(n) = f(n - 1) + 1$, where $f(1) = 1$ and $f(1)$ is the lowest natural number.

Natural numbers, further, can be categorized as either prime number or composite number except 1. [1 is neither prime number nor composite number]. We will see this on next pages.

Formula/generalization related to natural numbers:

1. Sum of 1st N natural numbers $= \dfrac{N(N+1)}{2}$. Understand that this formula is applicable only if we are adding the numbers from 1, then 2, and so on.
2. Sum of the natural numbers from 1–10 = 55
   Sum of the natural numbers from 11–20 = 155
   Sum of the natural numbers from 21–30 = 255, and so on.

Solved Example 7

Find the sum of 1st 100 consecutive natural numbers.

Solution $N = 100$. Using the formula given above, sum of 1st 100 natural numbers $= 100 \times \dfrac{101}{2} = 5050$.

Solved Example 8

A child was asked to add first few natural numbers (i.e., 1 + 2 + 3 + ⋯) as long as his patience permitted. As he stopped, he gave the sum as 575. When the teacher declared the result wrong, the child discovered that he had missed one number in the sequence during addition. The number he missed was **{CAT 2002}**

(a) less than 10 (b) 10 (c) 15 (d) more than 15

Solution Let us first try to see that summation till what number gives us a value close to 575.

Sum of the natural numbers from 1–10 = 55
Sum of the natural numbers from 11–20 = 155
Sum of the natural numbers from 21–30 = 255

Net summation from 1–30 = 55 + 155 + 255 = 465, so we are required to add some more numbers.

Next number = 31, summation from 1–31 = 496

Next number = 32, summation from 1–31 = 528

Next number = 33, summation from 1–31 = 561

Next number = 34, summation from 1–31 = 595.

Hence, the child has missed out 595 – 575 = 20.

(Look at the end of this chapter under the section 'CAT questions' to see a different method of solving the same question.)

Solved Example 9

A child tore off one leaf from a book, having page number from 1 to N. Summation of the remaining number of pages = 1010. Which page numbers are missing from this book?

Solution We are required to find out the 1st number larger than 1010 in the summation 1 to *N*.

Going through hit and trial, summation from 1 to 40 = 820. So we are required to add more numbers to it.

820 + 41 = 861. 861 + 42 = 903. 903 + 43 = 946. 946 + 44 = 990. 990 + 45 = 1035.

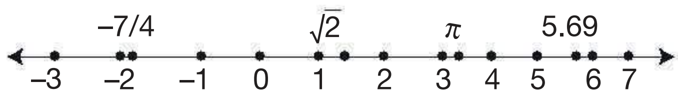
Ideally the summation should have been = 1035, but it is 1010. So its 25 less than the actual summation that it should have been.

So the page numbers missing from the book = page numbers 12 and page numbers 13 (it will be on one leaf only).

Real Number Line

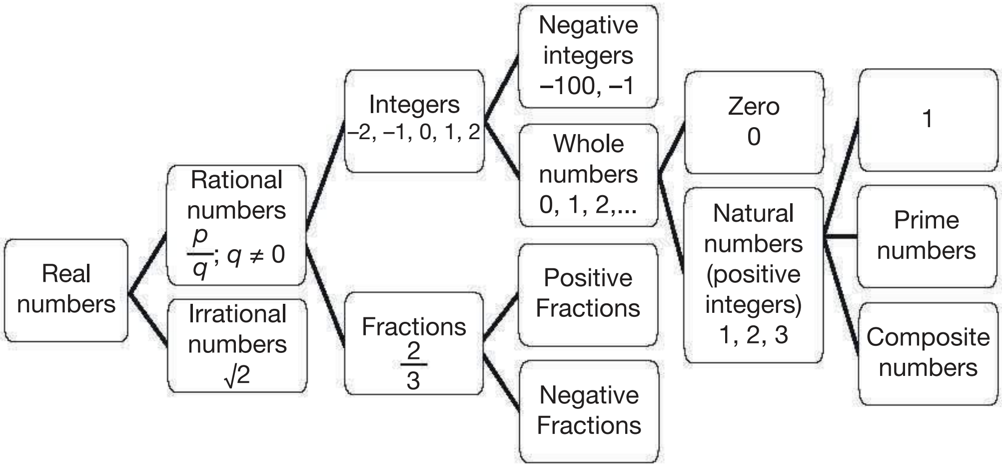
Real number line (in short also known as number line) consists of the union of the rational and irrational numbers.

Every real number can be associated with a single point on the **real number line.**

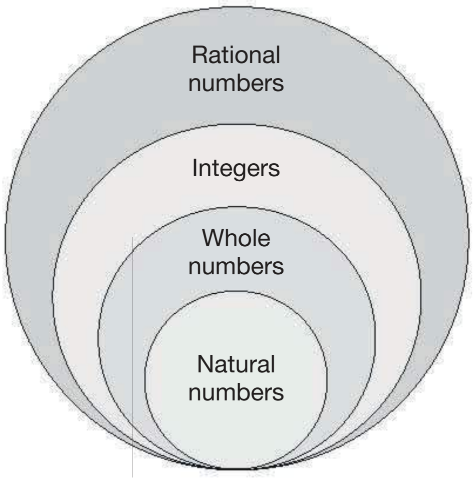


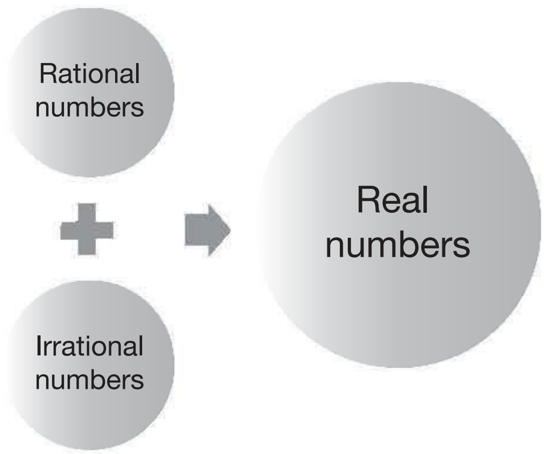
- It extends from infinity (–) on the left hand side to infinity (+) on the right hand side.
- Numbers on the right hand side are always larger than the numbers on the left hand side.
- When we do addition operation, we move on to the right hand side of numbers line. *For example*: when we add 4 units to 3 (= 3 + 4), we are required to move 4 units right hand side to 3, and landing up at 7.
- When we do subtraction operation, we move on to the left hand side of numbers line. *For example*: when we subtract 3 units from 4 (= 4 – 3), we are required to move 3 units left hand side to 4, and landing up at 1.

SUMMARY OF THE WHOLE DISCUSSION



We can present the above graphics by using Set theory too:





Solved Example 10

The sum of four consecutive two-digit odd numbers, when divided by 10, becomes a perfect square. Which of the following can be one of these four numbers? **{CAT 2006}**

(a) 21 (b) 25 (c) 41 (d) 67 (e) 73

Solution Maximum sum of any four consecutive two-digit odd numbers = 389. Since question is asking for a number that is divisible by 10, sum can be 360, 250, 160 or 90 and hence, the number will be an odd number close to 90, 62, 40 or 22.

Now we will go through the options.

As the numbers are consecutive, for any of the options, sum should be approximately four times of the value given in the option. Going by this logic, 5th option can be easily eliminated as 4×73 is not close to any of the probable numbers. Doing a bit of hit and trial will ravel that $(43 + 41 + 39 + 37) = 160$, and hence 41 is the right answer. Look at the end of this chapter under CAT questions for 2nd method of solving this question.

Now we will see the types of natural numbers:

All the natural numbers greater than 1 has at least two factors—and on the basis of number of factors we can categorize natural numbers greater than 1 either as prime number or composite number.

Prime Numbers

Any number that has exactly two distinct factors is known as prime number. In other words, any number which is divisible by 1 and itself only is a prime number. For example, 2 is a prime number, because 2 is divisible by 1 and 2 only (two distinct factors only). Similarly, 103 is a prime number because it is divisible by 1 and 103 only. 2 is the only even prime number, and all the other prime numbers are odd.

In that way prime numbers are building blocks of numbers, i.e., prime numbers are used to build the other numbers—known as composite numbers.

Solved Example 11

A, B, C, D and E are five prime numbers, not necessarily consecutive. Sum of these five prime numbers = 264. It is also given that A < B < C < D < E. What is the value of A^5?

Solution You should not try to solve this question by trying to find out the actual values of prime numbers, rather think of the logic behind the question.

Summation of five prime numbers = 264 = Even number.

We know that except 2, all the other prime numbers are odd. Since summation of five prime numbers is even, hence, four of these prime numbers are odd and one is even.

Since A is the lowest prime number, hence A = Even prime number = 2.

So, $A^5 = 2^5 = 32$.

Properties of Prime Numbers

1. There are infinite prime numbers.
2. There are infinite set of prime numbers such that, difference between them = 2 (represented as N and $N + 2$, where N is a prime number). For example, 3 and 5, 5 and 7, 11 and 13, etc. Set of these two prime numbers are known as 'Twin Primes'.
3. There is exactly one set of prime number triplet, i.e., N, $N + 2$, $N + 4$, where N is a prime number (3, 5, 7).
4. All the natural numbers greater than 1 has at least one prime factor.
5. If a number P has a prime factor $N < P$, then it has another prime divisor $M \leq \sqrt{P}$.
6. All the prime numbers greater than 3 are of the format $6N \pm 1$. It means that all the prime numbers will give either a remainder of +1 or –1 when divided by 6. Understand that this is only a sufficient condition and not necessary one. In other words, if a number is a prime number, it will be of the format $6N \pm 1$, but it does not mean that if any number is of the format $6N \pm 1$, then its going to be a prime number. For example, $25 = 6N + 1$, but 25 is not a prime number.

Test of Primality Using conditions 5 and 6, we will now learn the methods to check if a number N is a prime number:

Step 1 1st divide the number by 6, if number does not give remainder = either +1 or –1, then its not a prime number. If in Step 1, number gives either +1 or –1 as the remainder, then we will go through the next steps.

Step 2 Find out the approximate value of the square root of N.

Step 3 Starting from 2 to $\sqrt{N}$, check one by one, if any of these numbers divide N.

Step 4 If N gets divided by any one of 2 to $\sqrt{N}$ (other than 1), then N is not prime, otherwise prime.

Solved Example 12

Check whether N = 142 is a prime number or not?

Solution 2 is the only even prime number. Hence, 142 is not a prime number.

Solved Example 13

Check whether N = 143 is a prime number or not?

Step 1 Remainder obtained when 143 is divided by 6 = –1. Since remainder obtained = –1, hence, we will now go to next steps.

Step 2 $\sqrt{143} \cong 12$

Step 3 Starting from 1 to 12, we will try dividing 143 by all the numbers.

Outcome when 143 is divided by numbers from 2 to 12:

2	Not divisible
3	Not divisible
4	Not divisible
5	Not divisible
6	Not divisible
7	Not divisible
8	Not divisible
9	Not divisible
10	Not divisible
11	**Divisible**
12	Not divisible

Since, 143 is divisible by 11, hence, 143 is not prime.

Note Wilson theorem also gives the test for the primality of a natural number. See 'Remainder' chapter to know more about this.

Solved Example 14

What is the number of natural numbers n in the range of $2 < n < 20$ such that $(n – 1)!$ is not divisible by n?

Solution Take some initial values of n and try to find a pattern.

$N = 3 \Rightarrow (n - 1)! = 2!$ Which is not divisible by 3.

$N = 4 \Rightarrow (n - 1)! = 3!$ Which is not divisible by 4.

$N = 5 \Rightarrow (n - 1)! = 4!$ Which is not divisible by 5.

$N = 6 \Rightarrow (n - 1)! = 5!$ Which is divisible by 6.

$N = 7 \Rightarrow (n - 1)! = 6!$ Which is not divisible by 7.

Now we can generalize that for N = Prime numbers, $(n - 1)!$ is not divisible by n.

Hence, values of n (greater than 6) for which $(n - 1)!$ is not divisible by n = Prime values of n = 7, 11, 13, 17, 19.

Total values = 3, 4, 5, 7, 11, 13, 17, 19 = 8 numbers.

Composite Numbers

A composite number is a positive integer > 1, which is not prime number. In other words, composite numbers have factors other than 1 and itself, and hence, composite numbers will have at least 3 factors or divisors.

Lowest composite number = 4.

List of 1st few composite numbers:

Number	4	6	8	9	10	12	14	15
Prime factorization	2^2	$2^1 \times 3^1$	2^3	3^2	$2^1 \times 5^1$	$2^2 \times 3^1$	$2^1 \times 7^1$	$3^1 \times 5^1$
Number of factors	3	4	4	3	4	6	4	4

Properties of Composite Numbers

1. A composite number has at least three factors.
2. All the natural numbers other than prime numbers and 1 are composite numbers.

Euler wrote in 1770—Mathematicians have tried in vain to discover some order in the sequence of prime numbers but we have every reason to believe that there are some mysteries which the human mind will never penetrate.

Methods of Counting

Generally, we encounter two types of counting problems—1st one related to digits counting and 2nd one related to numbers counting. Let us see this one by one with the help of questions.

Digit Counting In these types of questions, some numbers will be given and occurrence of a particular digit will be asked.

Solved Example 15

How many times the digits of a computer keyboard will be required to be pressed in typing 1st 100 natural numbers?

Solution Following observations are required to be made:

1. Numbers are going to be of 1-digit, 2-digits, 3-digits.
2. Number of numbers of 1-digit/2-digits/3-digits are going to be different.

Number of natural numbers of 1-digit = 9 ⇒ Number of digits = $9 \times 1 = 9$

Number of natural numbers of 2-digits = 90 ⇒ Number of digits = $90 \times 2 = 180$

Number of natural numbers of 3-digits = 1 ⇒ Number of digits = $1 \times 3 = 3$

Hence, total number of digits from 1–100 = 9 + 180 + 3 = 192.

Observation 1 There are 9 1-digit natural number (1–9)

Observation 2 There are 90 2-digits natural number (10–99)

Observation 3 There are 900 3-digits natural number (100–999)

It can be generalized now that, the number of four digit natural numbers = 9000 and so on.

Solved Example 16

How many times does the digit 6 appear when writing from 6 to 400?

Solution One method of solving this is obviously through the actual counting method—6, 16, 26, 36, 46, 56, 60, 61,… and so on.

Best method is to understand that how actually numbers occur or numbers are made in succession;

Every digit from 1 to 9 appears ten times at units place, and ten times at tens place.

For example, let us count number of times digit '4' appears in 1st 100 natural numbers.

At the units place—04, 14, 24, 34, 44, 54, 64, 74, 84, 94—10 times (Given in bold)

At the tens place—40, 41, 42, 43, 44, 45, 46, 47, 48, 49—10 times (Given in bold)

It can also be seen that digits have actually interchanged their positions.

Observation 1 All the digits from 1 to 9 appear 10 times at units place and ten times at tens place. In other words, when we count any 100 consecutive natural numbers, i.e., it may not be from 1 to 100, even if it is from 23–122, number of times any digit from 1 to 9 will appear a total of 20 times—10 times at the unit place and 10 times at the tens place. We are required to count the occurrence at the 100's place separately.

Observation 2 All the digits from 1–9 appear to be a total of 300 times from 1 to 999 (including both the limits):

	Units place	Tens place	
1–100	10	10	
101–200	10	10	
201–300	10	10	
301–400	10	10	
401–500	10	10	
501–600	10	10	
601–700	10	10	
701–800	10	10	
801–900	10	10	
901–999	10	10	
Sum	100	100	200

Number of times any of the digits (1–9) occurs at units place = 100
Number of times any of the digits (1–9) occurs at tens place = 100
Number of times any of the digits (1–9) occurs at hundreds place = 100
Number of times any of the digits (1–9) occurs from 1 to 999 = 100 + 100 + 100 = 300

Number Counting

Out of 1st *N* consecutive natural numbers,

1. Every 2nd number is divisible by 2.

   So if we take 1st 100 natural numbers, 50 are divisible by 2 and remaining 50 are not divisible by 2.

   Out of 1st 99 natural numbers, since 1st number is not divisible by 2, 49 numbers are divisible by 2 and 50 numbers are not divisible by 2.

2. Every 3rd number is divisible by 3.

   So, if we take 1st 100 natural numbers, $\frac{100}{3} = 33$ are divisible by 3 and remaining 67 are not divisible by 3.

   However, if we have to find out number of numbers divisible by 3 in any 100 natural numbers, we cannot find out exact number which are divisible by 3. Key is to know if the 1st number or the last number is divisible or not?

Solved Example 17

Difference between two natural numbers A and B = 101. How many natural number between A and B (excluding both A and B) will be divisible by 3?

Solution Let us take the values:

$A = 102$, $B = 1$. Excluding both 1 and 102, numbers which are divisible by 3 = 3, 6, 9, 12, …, 99 = 33 numbers.