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**Arun Sharma**

*Educational Consultant*



**McGraw Hill Education (India) Private Limited**

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## **How to Prepare for Quantitative Aptitude for the CAT, 6e**

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***Dedicated to my Father (Mr MK Sharma), Mother (Mrs Renu Sharma) and Brother (Mr Ravi Sharma)***  
***who have instilled in me the courage to have my convictions***  
***and to stand by them***

# Preface to the Sixth Edition

With the evolution of the CAT in its online avatar, I felt the need to create a comprehensive and updated book that caters to CAT aspirants. Some of its salient features are given below.

**1. Questions from this book:** Over the past decade, it has been noticed that a minimum of 10–20% questions in CAT and other major management entrance examinations have been directly taken from the questions provided in this book. Furthermore, it has been seen that between 2003 (when the book was first released) to 2013, 80–90% of the questions in CAT and other top management entrance tests were covered in this book.

The CAT having gone online saw no change in this trend. Many questions in each of the test papers that the CAT has administered in its online avatar since 2009 have been covered in this book.

In fact, 2009 onwards, the onset of the CAT online pattern has created a significant shift in terms of the CAT preparation process. This is because, 2009 was the first year where there were multiple CAT papers to study, analyse and base our writing and preparation process on. In subsequent years, with the increase of the CAT window, the number of papers every year has gone up to around 30–40 papers. Thus, I am now richer by the experience of around 150 plus test papers when it comes to understanding what I need to provide to my readers for their preparation. It is on the basis of this rather rich insight that I have based the changes in this edition. (Note: Similar changes have been incorporated in my other books *How to prepare for Verbal Ability and Reading Comprehension for the CAT*, *How to Prepare for Data Interpretation for the CAT*, and *How to Prepare for Logical Reasoning for the CAT*).

**2. Not too many changes in the pattern:** Looking rationally into the paper patterns of the 150 plus CAT papers in the past period, there have not been too many changes with regard to the pattern of the examination as compared to previous years. While the QA in the initial years of Online CAT examination (between 2009–2010) was slightly on the easier side, the standard of the questions from 2011 onwards has become pretty much ‘CAT standard’. Hence, the LOD I, LOD II and LOD III scheme of questions followed in this book is even more relevant now than ever before.

In fact, in the near future, the CAT is expected to shift to an adaptive format (like the GMAT) where every test-taker would get a different set of questions. In the adaptive format in the future, question banks of varying difficulty levels would be loaded into the computers and questions would appear for the student one-by-one. The difficulty level of the next question would increase if the previous one has been answered correctly and vice versa.

In order to do this, the examiners would need to build a database of questions which would be parallel to the LOD scheme followed in this book. These factors make this book and the content within all the more relevant for CAT aspirants.

## ***How does Merging Quantitative Aptitude & Data Interpretation change the preparation process?***

As you must be aware, the CAT 2011 introduced a new pattern shift by reducing the number of sections from 3 to 2. Before CAT 2011, there used to be three sections containing 20 questions each in the CAT exam namely:

Section 1: Quantitative Aptitude

Section 2: Verbal Ability & Reading Comprehension

Section 3: Data Interpretation & Logical reasoning.

*In the new pattern the number of sections was reduced to 2 sections containing 30 questions each, namely*

Section 1: Quantitative Aptitude & Data Interpretation

Section 2: Verbal Ability & Logical reasoning.

In many ways, this does not really change your process of preparation. This is because, even though, on the surface the number of sections was changed, the actual number of questions under each category remained the same. In other words, while the three-section CAT used to have 20 questions on Quantitative Aptitude, 20 questions on Verbal Ability and Reading Comprehension, 10-12 questions on Data Interpretation and 8-10 questions on Logical Reasoning, the new two-section examination has 20 questions on Quantitative Aptitude and 10 questions on Data Interpretation merged together under one section, while the 20 questions on Verbal Ability and Reading Comprehension and the 10 questions on Logical reasoning were merged together under the other section.

Thus, the emphasis of your preparation does not really need to change at all. The only change that might occur is in the way you may want to strategise to use the time available to you in each section in order to beat your competition.

**3. The need for greater variety in your preparation:** Prior to the CAT going online, preparing for QA used to be a battle for Blocks 1, 4, 5 and 6. Even out of these, if someone did Blocks I and V well, he stood a strong chance at QA section.

However, as explained in details in the *introductory note* to the *online CAT*, the new avatar of this exam requires the aspirant to be much more balanced in the context of portion coverage.

**4. The tougher level of the CAT exam:** As already stated above, the quality of questions asked in the CAT over the past couple of years has become extremely good—requiring an upgradation of your grasp of concepts and understanding of each particular topic to a level not required before. This shift has necessitated that we do more through this book.

5. I have also come to know that many readers use this book for their preparation of other important management entrance exams (like XAT, IIFT, CMAT, MAT, SNAP, etc). So now, I have also included/modified the contents so that aspirants of the above exams need not look for any resource beyond this book for strengthening their hold on the quantitative aptitude section.

Apart from management entrance examinations, the book also has relevance for aspirants of UPSC and state civil services, Bank PO exams, GATE, Engineering Placement exams, etc. In short the scope of this book has considerably widened to cover the entire subject of quantitative aptitude that finds a resonance for all career aspirants.

The book you now hold in your hand has always been written keeping in mind the avowed objective of developing your quantitative intelligence to a point where you can quickly scale the height of preparation in each chapter of the portion.

### **Key features:**

1. **Comprehensive solutions** (wherever relevant) to questions in all LODs of all chapters.

2. Based on an assessment of any logic I have missed in any chapter, I have introduced extra questions for readers in the form of **additional block-wise practice tests**. The questions in these tests have been carefully selected to ensure that I do not miss out on any probable question type.
3. In some chapters, where I felt that there is some deficiency in the number and variety of questions (due to the increased difficulty level of the CAT) based on the concepts of the chapter, we have **introduced new questions into the LODs** of the chapter.
4. At some places, the need was felt to introduce an entire additional exercise on concepts of a chapter. This too has been accomplished in this revision.
5. **The training ground:** Perhaps the biggest differentiator in this book is the introduction of the ‘Training Ground’ – which is an area through which I teach the readers real time higher-end problem solving. The training ground is a section where I tell you exactly how to think inside the examination hall when faced with questions of varying difficulty levels. Hence, a must read for all management aspirants.

### ***Logic of the Training Ground***

The quality of the questions in the Quantitative Aptitude section (especially in CAT & XAT) is of such a high quality level that even if you know the basics of each chapter within a particular block, it might not be enough to reach a point where you might be able to solve the questions from the chapter/block. In order to have a grip on any chapter/block and be able to handle application-based questions in the actual examination, you would need to raise your level of thinking and ideation in that chapter/block to the point where you are able to tackle any twists and turns that can be thrown up by it.

For this edition, the training ground has been introduced into four of the major blocks of chapters of this book – and you can expect a very extensive training ground section especially for Block V and Block VI. (Block V covers the chapters on Functions, Inequalities, Logs and Quadratic & Other Equations, while Block VI covers the chapters on Permutations and Combinations, Probability and Set Theory).

6. An introductory write up on the QA section of the online CAT to give you a holistic view of how to approach the online CAT from the perspective of the examinee.

The book is now totally in sync with the new trend and pattern of the examination.

Ultimately the endeavour is to provide a one-stop solution for CAT and MBA exam aspirants to tackle the QA section of all major management entrance exams—an endeavour I feel I have managed to do pretty well.

Through this book, I am confident of giving you—the reader—an invaluable resource for enhancing your QA section score drastically. Contained in this book is the very best advisory for each and every question type. Your job is simple—to ensure that you follow the process contained in this advisory.

## **KEY POINTS FOR YOUR PREPARATION**

### **Outline and Strategy**

The first aspect I would like to deal with here is to focus on helping you with the formulation of your strategy with respect to the portion to be covered for the Quantitative Aptitude section of the various management entrance exams including the CAT, XAT, CMAT, IIFT, and other examinations.

Let us start by trying to understand some of the key areas in Quantitative Aptitude (QA).



## Tackling each portion

My experience shows that very often students look at the vast number of chapters and concepts to be studied for QA and get disheartened. This is especially true for students who do not have a strong traditional background in Mathematics. Indeed if you were to look at it with a chapter-wise approach, you can easily define the course to be studied by dividing it into 20+ chapters—preparation for which is such a long-drawn effort that it ends up draining the student's energy enthusiasm and motivation.

It is in this context and for this precise reason that I have divided this book into six manageable blocks—the approach being rationalising the chapters and grouping them according to the amount of shared concepts these chapters have amongst each other.

The outline as defined in the index to this book would divide your work into 6 major areas to prepare for. For your convenience and strategising I have put down the relative importance of each of these six blocks into perspective:

**Block I:** Number Systems and Progressions

*Importance:* **Very High** for CAT, XAT, IIFT, FMS & **High** for MAT, CMAT, SNAP, IRMA, etc.

**Block II:** Averages and Alligations

*Importance:* **Low** for QA in CAT, XAT, IIFT, but **High** for Data Interpretation as a lot of questions in DI are based on the concepts of averages and alligations. Also **High** for MAT, CMAT, IRMA, NMIMS, etc.

**Block III:** Percentages, Ratio, Proportion and Variation, Time and Work, Time, Speed and Distance. (Subsidiary but almost redundant chapters in this block – Interest and Profit & Loss)

*Importance:* **Moderate to High** for QA in CAT, XAT, IIFT, and **Very High** for Data Interpretation (DI) as DI is almost entirely based on the concept of Percentages and Ratio and Proportions. **Very High** for MAT, CMAT, IRMA, NMIMS, etc.

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**Note:** *The chapter on “Time, Speed and Distance” is extremely important for these exams (especially for the CAT as this chapter has been a constant presence in the CAT for almost a decade.)*

**Block IV:** Geometry, Mensuration and Coordinate Geometry

*Importance:* **Very High** for CAT, XAT. **Average** for MAT, CMAT, SNAP, IRMA, IIFT, etc.

**Block V:** Functions, Inequalities, Logs and Quadratic Equations

*Importance:* **Very High** for CAT, XAT. **Low** for MAT, CMAT, IRMA, NMIMS, etc.

**Block VI:** Permutations and Combinations, Probability and Set Theory

*Importance:* **Very High** for CAT, XAT, IIFT etc. **Average** for MAT, CMAT, IRMA etc.

Based on the experience of the online CAT, the strategic preparation imperative for you should be to do at least four blocks and if possible up to 6 blocks “really well”.

What does it mean to prepare a block “really well”? This is something I feel needs emphasis here.

Well what I mean to say is that do not just focus on studying the theory in each of these areas but develop an intuitive knowledge of all problem scenarios which emerge out of each block.

Only then would you be able to reach a situation in the exam—that when the question presents itself to you in the exam—you would have had the logic for the same worked out before hand. This is something that can make a huge difference to your chances in the CAT.

## **Analysing Your Knowledge Level**

The first thing you need to focus on is an analysis of your knowledge level in each of these seven parts. In each of the above areas, first analyse your level of knowledge/ability. In order to do so the typical question you should ask yourself is: For the next 100 questions I face in each of these areas, how many would I be able to handle comfortably?

Think of a number as an answer to this question for each of the six blocks.

Based on your answer, the following analysis would provide you a thumb rule which would tell you how much of a knowledge issue you have:

1. 90+: You know pretty much every question type and variant in the area. You should focus your energies on other aspects rather than knowledge improvement in the area.
2. 80+: Maybe you need to increase your exposure to questions a little bit; around 200–300 more questions in that area would be sufficient.
3. 60–80+: You have a significant knowledge issue in the area. You might need to go back to the basics, but it is less likely to be a theory issue but more of an exposure to questions issue.
4. <60: You need to work on both theory and exposure to questions.

Needless to say, the target and objective for preparations has to be to reach the 90+ range as explained above in any block you intend to do “really well”.

## **Looking beyond Ability (Quick Reflexes)**

A common frustrating experience for test-takers while taking the test is to not being able to solve a known question/logic and subsequently, not being able to score marks in questions which they knew.

In order to handle this problem, you would need to work on your reactions and reflexes when faced with QA questions. Once you have solved your knowledge/ability issue in a particular block, your next step is to improve your reactions and reflexes while solving a question. Needless to say you would need to do this block wise.

So obviously the main issue is how to improve reflexes and reactions.

- (a) For every block, once you have solved the LODs and the Pre-Assessment/Review tests, the most crucial exercise in this context would be a comprehensive revision and review of each and every question you have solved in that block. Solve every question of every LOD and Pre-Assessment/Review test again and review the logic/process of problem solving used. This need to be done to the point where you almost “recollect” the logic of the question and are able to recognise the same if it is used again in a different context/problem.
- (b) A thorough revision on the theory of the block.

***1. Improve your ability to select what you know and leave what you do not***

In the context of an examination where the required scores for 99 percentile would be 60–70% attempts with 100% accuracy, it is easy to see that perfect knowledge is perhaps not needed in order to crack the CAT. Hence, even if you have around 60–70% knowledge of the questions in an average test, you are perhaps good enough to crack the exam. A good way to test whether you have sufficient knowledge would be to pick up 10–20 test papers and divide your QA section into blocks of 5 questions each. Then test your knowledge by looking at the average number of questions you know. If on an average for every 5 QA test questions that you pick up, if you know more than 3, then the prognosis would be that you have adequate knowledge for cracking the CAT. Thus, while you may want to move towards knowing 5 out of 5 in this context, there are other things that you should focus on—developing your ability to decide on whether you are going to be able to solve a question while reading it for the first time. This would help you stop *fishing* during the test. (*Fishing* can be described as the activity of trying to solve a question without knowing whether you would actually complete the question.)

Your mind should give you a clear indication of whether you would be able to do the last step in a question, before you start doing it. In that sense you should be able to clearly define three types of outcomes when you finish reading a question for the first time:

- (a) **I see a clear flowchart** and the steps are manageable—Obviously you need to go on and solve these questions.
- (b) **I see a clear flowchart** but the steps are too lengthy—In this case you need to see where you stand in your test-time and attempt-wise.
- (c) **I do not see a clear flowchart but I can try as I see a starting point**—This is potentially the most dangerous situation for you in the duration of the test, as once you get sucked into a question, there is a strong tendency to lose track of the time you are using up while trying the question. My advice is that while taking the test you should not even start doing such questions.
- (d) **I see no flowchart and no starting point to the question**—Obviously you should leave such questions and in fact if these are limited to around 20–30% of the paper there is no problem and you need not worry about them.

## ***2. Focus on thorough knowledge of ‘problem scenarios’ rather than theoretical learning***

To illustrate this, I would like to start with a few examples.

Consider the following string of 3 questions. Before I come to my main point here, I would like you to start by solving these questions before looking at the explanations provided:

1. A boy starts adding consecutive natural numbers starting from 1. After some time he reaches a total of 1000 when he realises that he has double counted a number. Find the number double counted.
2. A boy starts adding consecutive natural numbers starting from 1. He reaches a total of 575 when he realises that he has missed a number. What can be said about the number missed?
3. Find the 288<sup>th</sup> term of the series: ABBCCDDDDDEEEEEFFFFFG....

*We can now start to look at each of these 3 questions:*

1. Consider the fact that when you add numbers as stated above ( $1+2+3+4+\dots$ ) the result is known as a triangular number. Hence, numbers like 1,  $1+2=3$ ,  $1+2+3=6$  and so on are triangular numbers. This question asks us to consider the possibility of making the mistake of double counting a number. So instead of  $1+2+3+4$  if you were to do by error  $1+2+3+3+4$  you would realise that the number you would get would be 13 which would be more than 10 (which should have been your

correct addition) and less than 15 (the sum of 1 to 5) which is the next triangular number. And the double counted value could be achieved by spotting 10 as the immediately lower value—and the difference between 10 and 13 would give you the required double counted number.

To carry forward this logic into the given question, we should realise that we are just bothered about finding the last triangular number below 1000—and in trying to work this out is where we really apply our intelligence.

Before one writes about that though, one fully realises that a lot of readers (especially aspirants with an engineering background at this point are thinking about  $n \times (n + 1)/2$ . Knowing that process, one chooses to write about the alternate way to think about in this question.

$$1 + 2 + 3 + 4 \dots + 10 = 55;$$

Hence, we can easily see that  $11+12+13+14+15+\dots+20$  would equal 155 and the sum of 21 to 30 would equal 255 and so on.

Thus, in trying to find the last triangular number below 1000 we can just do:  $55+155+255+355 = 820$  (which is the sum of the first 40 natural numbers) and since we have still not reached close to 1000 we start by adding more numbers as:  $820 + 41 + 42 + 43 + 44 = 990$  and the difference between 990 and 1000 is 10 which is the required answer.

2. For this question we would just need to carry the learning from the previous question forward and realise that when we miss a number, we actually get a total which is lower than the correct total. Hence, if we want to find the number missed all we need to do is to find the first triangular number greater than 575. This can be got simply by  $55+155+255+31+32+33+34 = 595$ , so the number missed has to be 20.

3. In this question all you would need to notice is that in the series ABBCCDDDDDEEEEEE... A ends after the first term; B ends after the third (1+2) term; C ends after the sixth (1+2+3) term and so on.

So we can infer that what we are looking at is how many numbers need to be added before we get to a number just below 288. So  $55 + 155+21+22+23$  gives us 276 which pretty much means that the 24<sup>th</sup> alphabet (i.e. x) would be running in this series when we reach the 288<sup>th</sup> term.

So looking at the three questions above and the solutions, one wants the reader to only answer one specific question:

How much does knowing the first question and developing your thought ability and your intelligence help you in solving the second and the third one? I hope you see the connection. For your information, the three questions presented above were asked in CAT 2001, CAT 2002 and CAT 2003!!!

## CONCLUDING NOTE

You sit in front of your CAT question paper and the first question comes in front of you. If you have identified the logic of the question or seen the question itself earlier, your entire QA preparation is fructified. In fact, every question/logic (that you would face in your test) which you have seen earlier represents a triumph of your preparation process. It is for this very reaction that you prepare for an aptitude exam like the CAT. Any other preparation is quite worthless.

Your battle for CAT would be won if you get a “YES I KNOW THIS PATTERN/LOGIC” reaction to 50–60% of the questions in your test.

Contained in this book is the finest collection questions which you would hope to find anywhere. Remember, each question solved needs to be a learning experience—one that is to be kept in your mind

for future problem solving. Adopt this approach with the problems contained in this book and I am quite confident that you would KNOW over 50% of your actual CAT test paper since you have already solved something like that before!!

**All The BEST !!!!!**

**ARUN SHARMA**

E-pub version of this book is available for downloading from popular online portals.

# Preface to the First Edition

Over the last few years, as a trainer of CAT and other aptitude tests, I have felt the need for a comprehensive book on the subject. Students appearing for the CAT and other aptitude tests usually struggle for appropriate study material to prepare for this vital section of the examination.

This book comes as a humble attempt to fulfil this gap.

## Structure of the book

The book is divided into 19 chapters and five test papers. Each chapter is divided into three broad parts:

- (a) Theory
- (b) Solved examples
- (c) Chapter end exercises (LODs I, II & III), with answer key

The questions in the chapter end exercises have been categorised into three levels of difficulty, viz, Level of Difficulty I, Level of Difficulty II and Level of Difficulty III.

**Level of Difficulty I (LOD I):** These are the basic types of questions pertaining to the chapter. A majority of the MBA entrance tests would test the student with LOD I questions. Tests which ask LOD I questions include MAT, IMT, IRMA, IIFT, NIFT, CET Maharashtra, Bank PO examinations, BBA, BCA, Law, and so on. Besides, there are about 10 questions of LOD I type in the CAT nowadays.

**Level of Difficulty II (LOD II):** These are questions, which are more advanced than the LOD I questions. These questions test all basic as well as applied concepts in the chapter. *LOD II questions are closest to the difficulty levels of the CAT.* Hence, the objective of LOD II questions should be to:

- (a) Clearly understand the concept which underlies the question.
- (b) Create a judgment of time required for different mental processes.
- (c) Identify the time guzzlers.
- (d) Reinforce application of a method in mental processes through the question.
- (e) Learn to flowchart complex questions.

**Level of Difficulty III (LOD III):** LOD III questions build on the previous questions and are a step beyond the LOD II questions. Although they are also normally more difficult than the average CAT question, approximately 5–10 LOD III questions could be asked in the CAT every year. Hence, the learning objectives at LOD III are to:

- (a) Learn applications of the basic concepts at the highest level.
- (b) Sharpen the flowcharting skills learnt at LOD II.
- (c) Use each question as a learning opportunity.

One should not be disheartened if he/she is unable to solve LOD III questions. These questions are extremely tough and uncommon in the CAT and other aptitude tests. Questions in actual tests will appear very simple and elementary if one can solve LOD III questions.

## Approach Taken in Writing This Book

In my experience, the 'math skill' of students appearing for CAT can be classified into three levels:

Level 1: Students who are weak at Mathematics

Level 2: Students who are average at Mathematics

Level 3: Students who are strong at Mathematics

This book has been written keeping in mind all the three kinds of students.

From my experience I have given below my perspective of what one should aim for (based on the category that he/she belongs to). It is important to clearly understand the starting level and accordingly define strategy for the QA section.

**Level 1: Students who are weak at mathematics:** Typically, these are students who were weak at mathematics in school and/or have left mathematics after their 10th or 12th class. They face a mental block in mathematics and have problems in writing equations. They also have severe problems in understanding mathematical language and are unable to convert the mathematical language into mathematical equations. They make mistakes even in interpretation of the most basic statements in mathematics (leave alone the complex statements). Besides, these students also have problems in solving equations. They suffer from the insecurity of knowing that they are unable to solve most problems which they face.

**Level 2: Students who are average at mathematics:** These students lie between the Level 1 and Level 3 students.

**Level 3: Students who are strong at mathematics:** These are the students who have got strong, structured and logical thinking ability. They not only understand the basic repetitive statements in mathematics but also complex statements. They are able to create their own flowcharts to arrive at solutions of these complex mathematical situations.

There are two alternative approaches that a student can take in solving this book.

**Approach 1:** "Start with basic concepts, solved examples then move on to LOD I, then LOD II in the chapter. Do not go into LOD III in the chapter in the first go. Complete all 19 chapters and then re-start with Chapter 1 – review the basic concepts, resolve LOD I and LOD II, then move on to LOD III. This approach is advocated for students who are weak to average in mathematics (i.e. students of Level 1 and Level 2).

After completing the theory and practice exercises of the book for a second time, go to the practice sets 1–5 provided at the end of book. Set a time limit of 40 minutes for each set and take the tests. The questions contained in the sets are questions which have appeared in the CAT over the last 5 years (based on memory)."

**Approach 2:** "Start with the basic concepts, solved examples and then go through the exercises of LOD I, LOD II and LOD III. This is recommended for students who have strong concepts in mathematics (Level 3 students)."

Then go to the 5 practice tests given at the end of the book and take them one by one (time limit of 40 minutes for each test)."

### **An Important Point**

Each of the questions contained in the LOD I, LOD II and LOD III exercises in the chapters have immense learning value. Hence, the approach that one takes while solving the questions should be one of learning.

The reader should try to clearly understand the interpretation of each sentence used in the construction of the questions.

In other words the learning in every chapter should not be restricted to the solved examples or the theory contained in the chapter, but should continue through each of the questions contained in the exercises.

In conclusion, this is a book which is unique in approach and coverage. Any CAT aspirant who goes through the questions contained in this book in the manner advised in this book would get a distinct advantage when he/she faces the CAT.

**ARUN SHARMA**



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I would also like to express my heartfelt thanks to my wonderful students due to whom I have had a wonderful opportunity to live a highly fulfilled life. This book would not have been possible in the absence of this opportunity. This work would not have been possible but for the support and encouragement from McGraw-Hill Education, India. My sincere thanks and words of appreciation to them. I would like to make specific mention of Tanmoy Roychowdhury, K.N. Prakash, Niju Sreedharan, Bhavna Malhotra, Anubha Srivastava and Medha Arora.

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No amount of thanks can ever repay the great debt that I owe my wife Meenakshi (along with my son Shaurya) who have provided me with the constant inspiration over the past few years. In fact, this book would not have been possible but for the direct and indirect support, inspiration and guidance from Meenakshi who has been a constant partner in all my efforts over the past years.

**ARUN SHARMA**

# Online CAT: From the QA Perspective

Welcome to the world of online CAT!

The advent of the online version of the Common Admission Test (CAT) in 2009 and beyond brought with it a whole lot of opinions and views about what has changed in the examination and what should be the ideal preparation pattern. Therefore, one objective in this revised edition of this widely read book is to look at the issues that an aspirant needs to consider while preparing for the online CAT. I would like to discuss this issue in the following parts:

1. *What has changed?* A comprehensive analysis of what are the critical dimensions of the changes that have taken place in the CAT in its online avatar and what it means for the aspirant, both in respect of positive and negative factors, taking into account the following:
  - (a) Changes in the Test-Taking Experience
  - (b) Changes in the Exam pattern
  - (c) Changes in the Marking process
2. *What does all this mean for the Preparation Process?* How it should change in the context of an online examination and how has it remained constant- whether online or paper-and-pen?

While doing so I have taken the help of a varied experiential sample of test-takers from across India and also my own personal experience of taking (and may I add dominating) the CAT. Given below are some of the implications of the online version of the CAT in the context of the Quantitative Aptitude section (which this book is all about).

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**Note:** In this book, whenever I refer to preparation for the Quantitative Aptitude Section, I am referring to the preparation for the approximately 20 questions on Quantitative Aptitude, that are being asked under the Quantitative Aptitude and Data Interpretation sections of the CAT.

## I. WHAT HAS CHANGED?

### The ‘experience’ of taking the test

**1. Cleaner & More Efficient:** Compared to the paper-and-pen based CAT, the online version is much cleaner as the clarity of questions, their visibility, as well as the overall feel of the question solving experience is much better. Consequently, the efficiency (of the thought processes) is much higher, leading to a much superior test-solving experience.

**2. Space Management on the Table:** In the paper-and-pen version, the aspirant had to typically manage the test paper, admit card, watch, pencils (at least 2), eraser, sharpener as well as the answer sheet on the table. To add to their woes, the paper-and-pen versions of the exam were mostly conducted in schools. Very often the aspirants had to contend with the additional challenge of managing all this paraphernalia on a school boy’s small table. In addition, if luck did not run your way and you were made to sit in a classroom meant for juniors, (between classes 3 to 6) you really had a challenge.

Most of these problems have disappeared in the new version. The fact that computer terminals at most colleges and universities are of standard shape and size eliminates the imbalance created due to non-uniformity. Besides, while writing the online version of the CAT, all you need to manage on the table are the mouse, the key board, a pencil and a sheet of paper for rough work; no watches, erasers, sharpeners and most importantly, no test paper and no answer sheet.

**3. Moving Questions in the Test:** Unlike the paper-and-pen version, where test-takers could scan the whole question paper in one look, in the online CAT, aspirants had to move one question at a time. This had both its advantages and disadvantages in terms of the overall test experience. The obvious disadvantage that most aspirants faced was the fact that since you could not really see the whole paper in one look, you could not make a judgment about the balance, the difficulty level or the portion wise question distribution in the paper. (Although I am referring to the quantitative aptitude section here, this was also true for all the sections in the exam)

Ironically, the biggest advantage for the examinee in terms of the online CAT was exactly the same i.e. since you could not see the paper entirely at one go, the only option while taking the test was to look at the questions one by one.

*This turned out to be a huge advantage because of two main reasons mentioned below:*

**Higher Focus while Solving an Individual Question:** Not knowing the exact number of questions from various areas and not being able to estimate the difficulty level of the paper, left individuals with no choice but to focus on the one question that was visible to them on the screen. The result was that achieving the all important ‘tunnel vision’ while solving a question was much easier. The immediate result of this was that the focus on the ‘problem at hand’ was infinitely more in the online version than in the conventional paper-and-pen format. Thus, ironically, not knowing the pattern of the paper resulted in giving examinees their best chance to solve a question.

The main reason for this was that while solving the question in front of the computer screen the experience of the previous question was totally blanked out. In the paper-and-pen version, students who had a negative experience while solving a question or two carried that negativity to the next question.

Thus the specific advantage of the online version was that “forgetting” a bad experience was relatively easier. The moment you navigated away from the question in front of you, it went away from your mind as well. So much so, that remembering a question that was just two questions back was close to impossible. Naturally the ‘carry over’ emotions from a previous negative experience were significantly reduced.

**The Imperative for Faster Navigation (less time wasted on unsolvable questions):** Since the examinees had not seen the full question paper right at the beginning, the imperative to move to the next question was extremely strong. This resulted in students seeing a higher percentage of the questions in the online test than in the paper-and-pen version.

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**Author’s Note:** One of the problems I had noticed in the paper-and-pen version was that most examinees were not able to ‘see’ the entire paper. i.e. the fraction of the quantitative aptitude section that they were able to process was a fraction of the entire test paper. As a result they used to miss out on a large number of sitters! On an average, out of a 5-page question paper in quantitative aptitude, students were able to process at most upto 2–3 pages. So they would naturally miss out on all easy ones on the pages they did not process. A lot of time would get wasted in questions that they tried and were unable to solve or even if they solved, they were unable to get them correct.

Part of this time mismanagement also occurred due to the fact that they did not have the clock ticking on the screen in front of them. Therefore, they naturally lost track of how much time they had spent in attempting to solve a question. A good percentage of the time the aspirants used to spend in the QA section was spent in trying to solve a question which they were eventually unable to solve.

All this changed for the better in the online version. There was a greater imperative to move to the next question due to the twin facts that you had not seen the entire paper as you were moving from one question to the other, and that the ticking clock was omnipresent in front of your eyes on the screen. As a result, you were aware of the exact amount of time you had spent on a particular question. The net result was that after trying a question for maybe 60 to 90 seconds, in case you did not have a clue about what to do in the same, you moved to the next question. Thus time management improved drastically for the examinee.

I believe this is one of the main reasons why a lot of students who were trying to compare the two versions of the CAT said that the online version was easier. Since the amount of time spent in questions which they were eventually not able to solve, reduced drastically, they got a feeling that they were solving questions all the time as opposed to the paper-and-pen version where aspirants used to have an overall negative experience of the test (as they would end up spending a lot of time in attempting “unsolvable” questions).

**4. Mark/Unmark Button & the Review Button:** A very important feature in the online version was the introduction of the REVIEW button. In the paper-and-pen version, it was extremely difficult to track the number of your attempts and especially so in the context of questions that you were unsure about and/or questions which you wanted to come back to. There was simply no way in which you could keep a track of those and as a result there was effectively ‘no second chance’ at a question.

This too changed in the online CAT. For every question, apart from the facility to answer it, you also had a MARK button, which would give you easy access to the question at the end of the paper. When you have completed the paper (reached the last question in the paper), you also got access to a review screen that in one visual showed you all the questions you had solved as well as all the questions you had marked with the MARK button. So going back to a specific question in the paper was just the click of the mouse away.

To sum up, the net effect of the online CAT was a superior test-taking experience — something that gives you a chance to be more in control of your test— and thus aim for a higher score assuming that the same set of questions would have been asked in the paper-and-pen version.

## **What has changed in terms of the exam pattern?**

Having seen the specific changes that have occurred in terms of the test-taking experience, let us now examine another crucial aspect.

**Changes in Exam Pattern:** Obviously for the purpose of this book, the analysis will pertain to the QA portion only. In order to read a similar analysis with respect to the other sections namely, *Verbal Ability & Logical Reasoning* you can refer to my book on these subjects, also published by McGraw Hill. The major changes in the pattern of the Quantitative Aptitude paper can be summarised through the following points:

1. More balanced portion coverage
2. Reduction in number of questions

3. Lack of uniformity
4. Higher percentage cutoffs

**1. More Balanced Portion Coverage:** As per the scheme followed in this book, the QA portion can be divided into 6 major parts (or blocks as I call them in this book).

The underlying constant that used to exist in the paper-and-pen version (through the entire decade prior to the first online CAT) was the prominence of Block I and Block V. (Block I comprising Number Systems and Progressions and Block V comprising the chapters on Functions, Inequalities, Quadratic and other Equations and Logarithms.)

In each of the years from 1999 to 2008, the QA section required you to get a net score of approximately 30 – 40% of the total marks in order to score a high 90 percentile in this section.

In the light of this fact, the importance of Block I and Block V can be gauged from the table below:

<b>Block</b>	<b>Weightage (as a % of total marks)</b>
Block I	30 – 50%
Block V	15 – 50%
Combined weightage of Blocks I & V	60 – 80%

Add to this, the chapter on Time, Speed and Distance with a minimum weightage of 5–10% and you pretty much had the QA section well covered. In a nutshell, QA for CAT preparation had become “do 10 chapters well”.

However, this scenario has changed in the context of the online version of the exam.

The balance of weightage of questions shifted and each of the six blocks have become important. The aspirant of CAT online version saw a weightage distribution of the kind illustrated below.

<b>Block</b>	<b>Total Out of 20 Questions</b>
Block I	3 – 4 questions
Blocks II & III	3 – 5 questions
Block IV	3 – 6 questions
Block V	4 – 5 questions
Block VI	1 – 3 questions

**2. Reduction in number of Questions:** The second major change in the QA section is the reduction of questions to 20. From 55 questions in the late nineties to 50 between 2000 to 2003 to 30 & then 25 in the last years of the paper-and-pen version, the number of questions has further gone down to 20 in the online version. Naturally, this reduced the amount of choice the aspirant had for leaving out a question.

For instance in CAT 2003 out of 50 questions, you needed to solve 15 to get to the cut off. This meant that at 100% accuracy you could afford to leave 35 questions. This scenario has now changed drastically as in evident from the following table.

<b>Year</b>	<b>No. of Questions in QA</b>	<b>Number of Marks</b>	<b>Cut off at (approx number of marks)</b>	<b>No. of Questions you could leave @ 100% accuracy</b>	<b>No. of Questions you could leave @ 90% accuracy</b>
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CAT 1999	55	55	16-18	37+	32+
CAT 2000–04	50	50	12-14	36+	32+
CAT 2005	30	50	12-14	20+	16+
2006–08	25	100	28–32	17+	14+
Online CAT 2009 & 2010	20	80	40–48	8+	5+
Online CAT 2011–2013	20 (QA) + 10 (DI)	80 (QA) + 40 (DI)	68–72	12+	8+
QA & DI Section					

- As you can see, there is very little elbowroom available in the online version now to leave out questions and expect a good percentile score.
- The expectation in the future is that students taking the CAT would have to really use their mathematical intelligence and attempt as many questions as possible in order to get a top percentile in the test.

**3. Lack of uniformity:** The third major factor in terms of paper pattern was the lack of uniformity of the test paper. Different students got tests with differing difficulty levels. The papers on the first few days of CAT 2009 were quite simple, but after the third day most papers had a pretty good difficulty level.

An issue that is being discussed widely on the Net is fairness. A lot of voices rose against the CAT committee and the online version of the exam questioning the fairness of the testing process.

*The key criticism was:* In the context of multiple papers with varying difficulty levels, how would the IIMs judge fairly between students who solved a high percentage of the questions in an easy test versus students who were able to solve a lower number of questions in a more difficult paper? The answer to this is really simple. Since the population size of each paper was significantly large, the IIMs could easily define individual percentiles in each test and ensure fairness to all.

The key point to be noted here is that there are infinite statistical ways through which processes like this can be made fair to everyone. As a future CAT aspirant, however, what you need to worry about is preparing diligently and facing the exam with a positive attitude.

**4. Higher percentage cutoffs:** In the online version, aspirants wasted less time in questions which they thought was unsolvable and moved on to those they could solve. The result— most students were able to raise their scores in this section significantly.

Consequently sectional cut offs which used to be in the range of 30% of the net marks rose to around 40 – 45% of the marks.

## Changes in the Marking process

The key change that an analysis of CAT 2009 results showed was that there was an increased emphasis on accuracy. Mistakes were heavily penalised. This was evident from the fact that two students solving the

same test paper (December 3 evening slot) scored:

- (a) 14 attempts 1 incorrect – score 98.23 percentile
- (b) 19 attempts 3 incorrect – score 92.6 percentile

There were innumerable such examples where students solving more questions with higher errors scored significantly lower than students who attempted much less but got most correct.

Hence, the key learning for you while preparing is to focus on improving your accuracy as well as the belief in your process of solving. This is especially true while preparing for the QA section. While solving a QA question, you should be able to know that if your process is correct then so would your answer would also be correct. The need to check the answer to a QA question is something that is only required for minds weak in Quantitative Aptitude. This is where an under-prepared aspirant loses out to the best—in the knowledge of whether what they are solving is correct or not.

Unfortunately, most students I see are more interested in seeing the answer to the question as soon as they solve the same. This is a habit I would strongly discourage you from. The ideal preparation process for you should be:

- (a) solve the question,
- (b) review your process and tell yourself, “if your process is correct, so is your answer”, and
- (c) only check your answer after you have reviewed your process.

This is important, because when you are solving a QA question inside the CAT, you would not have the cushion to ‘look’ at the answer. The only thing you have is the question and the process you use in solving the same. Your mind should be able to tell you whether the answer you have got is correct or not. This is a key difference in solving questions in practice and solving them under exam pressure.

Hence, developing more confidence in your QA problem-solving processes becomes a key ingredient and objective of your preparation process for this section.

## **II. WHAT DOES ALL THIS MEAN FOR THE PREPARATION PROCESS? HOW HAS IT CHANGED AND HOW HAS IT REMAINED CONSTANT?**

Let us look at this aspect in two broad parts:

1. What are the changes that need to happen in the preparation processes for the online CAT vis-a-vis the preparation process for the traditional paper-and-pen version?
  2. What are the things and issues that remain constant in the preparation process?
1. For the first question, the specific things that come to my mind are:

**(a) More Balanced portion coverage needed:** As explained above, in the paper-and-pen version the best approach for Quantitative Aptitude preparation was “do 10 chapters well — really well”. In fact, even 4 chapters done well were mostly sufficient to crack this section. However, in the new online version, since the weightage of distribution of questions is much more even, this approach is no longer going to work. Hence, the need to cover all aspects of the portion well and not ignore any particular portion is perhaps the first and the biggest change that needs to be done in the preparation process.

**(b) Need to cover the basics well, namely, speedily solving LOD I questions and the ability to think through LOD II and LOD III questions:** In the early years (1980s and upto the late

1990s), the CAT used to be essentially a speed test (including the QA section). There were times when the paper used to consist of upto 225+ questions to be solved in 120 minutes. Questions used to be one-liners and could be solved in 1–2 steps. The key differentiation used to be the speed at which the aspirants could solve questions. However, from late 1990s onwards the QA section of the CAT had become a real test of quantitative intelligence. Questions ceased to be one-liners unless you had a very high degree of mathematical understanding and intelligence. The online CAT in its first year tended to be a mix of both these extremes. Papers consisted of between 4–6 one-liners topped up by LOD II and LOD III questions. So while most aspirants found 4–6 very easy questions in each paper, they also had to really use their mathematical strengths to cross 10–12 attempts. In the future, as the IIMs improve the quality of the database of quantitative questions, one can expect the quality of the questions to improve drastically and hence the LOD II and LOD III questions contained in this book would be an extremely important resource to solve for maximising your score in this section in the exam. [In fact after the first 3–4 days of the exam in 2009, examinees taking the test on the subsequent days found the paper to be of really good quality].

For the future CAT aspirants and the readers of my books, my advice is short and simple. Cover both the flanks—the short cuts and quicker methods to solve the easier LOD I questions *and* improving your mathematical and logical intelligence to cover the higher end questions of LOD II and LOD III level.

- (c) **The need to take computer based tests in order to be able to think on the computer:** Thinking and solving questions from the computer screen is a slightly different experience than solving from a physical book. Thus students and aspirants are advised to experience this change by going for online solving experience. It is in this context that we have tied up with [www.mindworkzz.in](http://www.mindworkzz.in) to give our readers a feel of the online problem solving experience. However, in spite of these seemingly big external changes, my personal opinion is that the changes are mostly external in nature.

2. The essence of preparation of the Quantitative Aptitude section remains the same in a lot many fundamental ways. Some of these that come readily to mind are:

- (a) **The need to develop mental structures for the CAT:** QA preparation has always been associated with the development of the mathematical thinking processes and thought structures for specific mathematical situations. The smart CAT aspirant is able to create mathematical thoughts in his mind to situations that he would encounter in the exam.

The whole battle for QA preparation in the CAT can be essentially summarised in terms of the quality of pre formed logic that you have to the specific questions that you are going to face in the exam. In other words, your battle in QA section is won if you have during your preparation process, encountered the logic to the question which is in front of you. Hence, the focus of your QA preparation process has to be on creating the logics for as many questions and mathematical situations as possible. You would have well and truly won your battle at the CAT in case you encounter 15 ‘known’ mathematical situations out of the 20 questions in your set. Hence, the **imperative to form “thought algorithms” for standard and non-standard mathematical situations related to various chapters and concepts in the portion remains as strong as ever.** In fact, if anything this imperative is expanded to the entire portion base due to the wider and more balanced portion coverage in the exam.

- (b) **The need for thoroughness in your preparation:** This is again something that does not



change.

The key point you need to remember is that the CAT still remains a test of your intelligence and an aspirant should focus on this aspect. This book provides plenty of mathematical thinking situations and alternatives related to each and every part of this section that help you hone your skills in the QA section of the examination.

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# DEVELOPING YOUR CALCULATIONS

This special section contains the best available approaches for all kind of calculations that you are likely to face in the CAT or any other aptitude examination...

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*Developing your ability to calculate well should be one of your major thrust areas for your preparation strategy for Quantitative Aptitude. In fact, most of the times (in most coaching programs and books) this area is totally bypassed leaving the student broadly to develop his/her own methods to calculate faster. Needless to say, your work in trying to develop your ability to calculate would always be greatly superior if you are guided properly with approaches that have been tested and have stood the test of time. The following advisory contained in this special section of the book not just aims to give you the best advice for each and every type of calculation, but also gives you a comprehensive plan to develop your calculation speed—for every conceivable type of calculation.*

*My focus throughout this special note on calculations is to help you to develop the relevant calculations only, viz., calculations that you are likely to encounter inside the CAT based on the experiences of the past CAT examinations.*

*For this purpose this section has been divided into the following chapters:*

**Chapter 1: ADDITIONS and SUBTRACTIONS** Ideas for developing your ability to add & subtract well;

**Chapter 2: MULTIPLICATIONS** Ideas for developing your ability to multiply well;

**Chapter 3: DIVISIONS, PERCENTAGE CALCULATIONS and RATIO COMPARISONS** Ideas for developing your ability to divide well as well as to compare ratios more efficiently;

**Chapter 4: SQUARES AND CUBES OF NUMBERS**

# 1 Chapter

## Additions and Subtractions (As an Extension of Additions)

### 📡 IDEAS FOR ADDING AND SUBTRACTING WELL

Addition is perhaps the most critical skill when it comes to developing your calculations. As you would see through the discussions in the remaining chapters of this section of the book, if you have the ability to add well you would be able to handle all the other kinds of calculations with consummate ease.

**Skill 1 for addition:** The ability to react with the addition of two numbers when you see them.

The first and foremost skill in the development of your addition abilities is the ability to react to 2 two digit numbers when you come across them. You simply have to develop the ability to react with their totals whenever you come across 2 two digit numbers.

For instance, suppose I were to give you two numbers at random—5, 7 and ask you to **STOP!! STOP YOUR MIND BEFORE IT GIVES YOU THE SUM OF THESE TWO NUMBERS!!** What happened? Were you able to stop your mind from saying 12? No! of course not you would say.

TRY AGAIN:  $12 + 7$  STOP YOUR MIND!! You could not do it again!!

TRY AGAIN:  $15+12$  STOP!! Could not?

TRY AGAIN:  $88+ 73 = ??$  STOP!! If you belong to the normal category of what I call “addition disabled aspirants” you did not even start, did you?

TRY AGAIN:  $57 + 95 =??$

TRY AGAIN:  $78+88 =??$

What went wrong? You are not used to such big numbers, you would say. Well, if you are serious about your ability to crack aptitude exams, you better make this start to happen in your mind. You would know what I mean if you just try to look at a 5 year old child who has just learnt to add, struggle with a calculation like  $12 + 7$  on his fingers or his abacus.

His struggle with something like  $12 + 7$  or even  $15 + 12$  would be akin to the average aspirant’s ability to react to  $88+ 78$ . However, just as you know  $15 + 12$  is not a special skill so also  $88+78$  is not a special skill. It is just a function of how much you practice your calculations especially in the domain of 2 digit additions.

So what am I trying to tell you here?

All I am trying to communicate to you is to tell you to work on developing your ability to react to 2 two

digit numbers with their addition as soon as these numbers hit your mind. What I am trying to tell you that the moment you make your mind adept at saying something like  $74 + 87 = 161$  just the way you would do  $9 + 6 = 15$  you would have made a significant movement in your mind's ability to crack aptitude exams.

Why do I say that—you might be justified in asking me at this point of time? In order to answer your question I would like to present the following argument to you:

In numerical questions, a normal student/aspirant would be roughly calculating for approximately 50% of the time that he/she takes to solve a question. This means that half the total time that you would spend in solving questions of basic numeracy or data interpretation would essentially go into calculations. Thus, if the test paper consists of say 30-40% questions on basic numeracy and data interpretation you would be expected to spend somewhere between 36 to 48 minutes on these questions—which would in turn translate to approximately 18 to 24 minutes in calculations inside the test paper.

So the contention is this: If you can improve your calculation speed to 5x, the time you would require to do the same calculations would come down to  $1/5^{\text{th}}$  of your original time. In other words, 18 minutes would come down to 3.6 minutes—a saving of 14.4 minutes; 24 minutes would come down to 4.8 minutes—a saving of 19.2 minutes just by improving your calculation speed!!

In an exam like the CAT where you would always run out of time (rather than running out of solvable questions) 19 extra minutes could easily mean anywhere between 15–20 marks (even if you able to solve an additional 6–7 questions in this time).

15–20 extra marks in the prelims exam could very well make the difference between getting a chance to write the mains examination in the same year versus going back to the drawing board and preparing for the prelims for another year!!

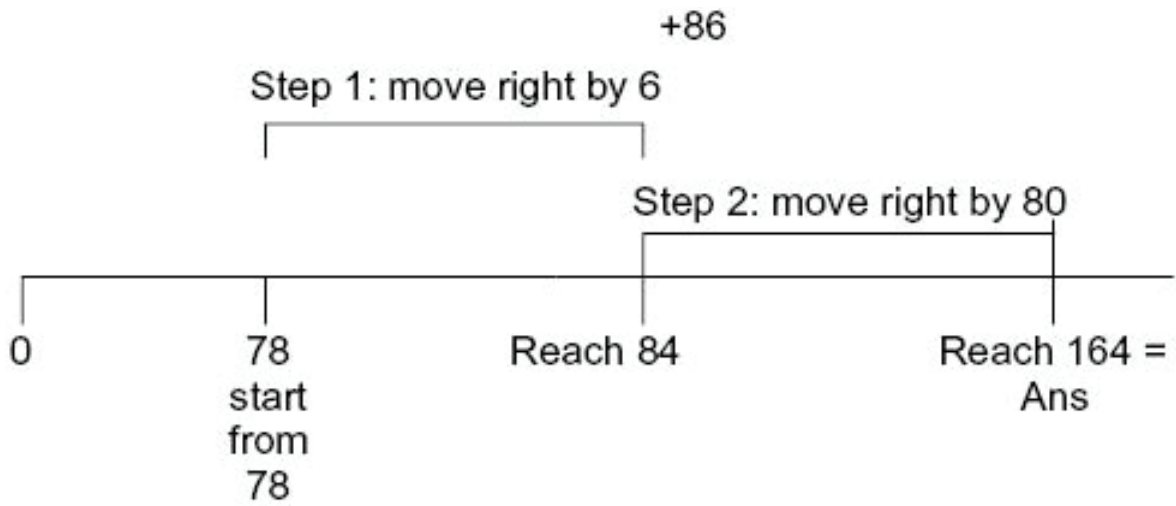
Addition being the mother of all calculations has the potential of giving you the extra edge you require to dominate this all important examination.

Over the next few chapters in this section of the book, all I am going to show you is how knowing additions well would have an impact on each and every calculation type that you might encounter in this exam and indeed for all aptitude tests. However, before we go that far you need to develop your ability to add well.

Let us look at the simple calculation of  $78 + 86$ . For eternity you have been constrained to doing this as follows using the carry over method:

$$\begin{array}{r} 1 \\ 78 \\ + 86 \\ \hline 164 \end{array}$$

The problem with this thought is that no matter how many times you practice this process you would still be required to write it down. The other option of doing this same addition is to think on the number line as this:



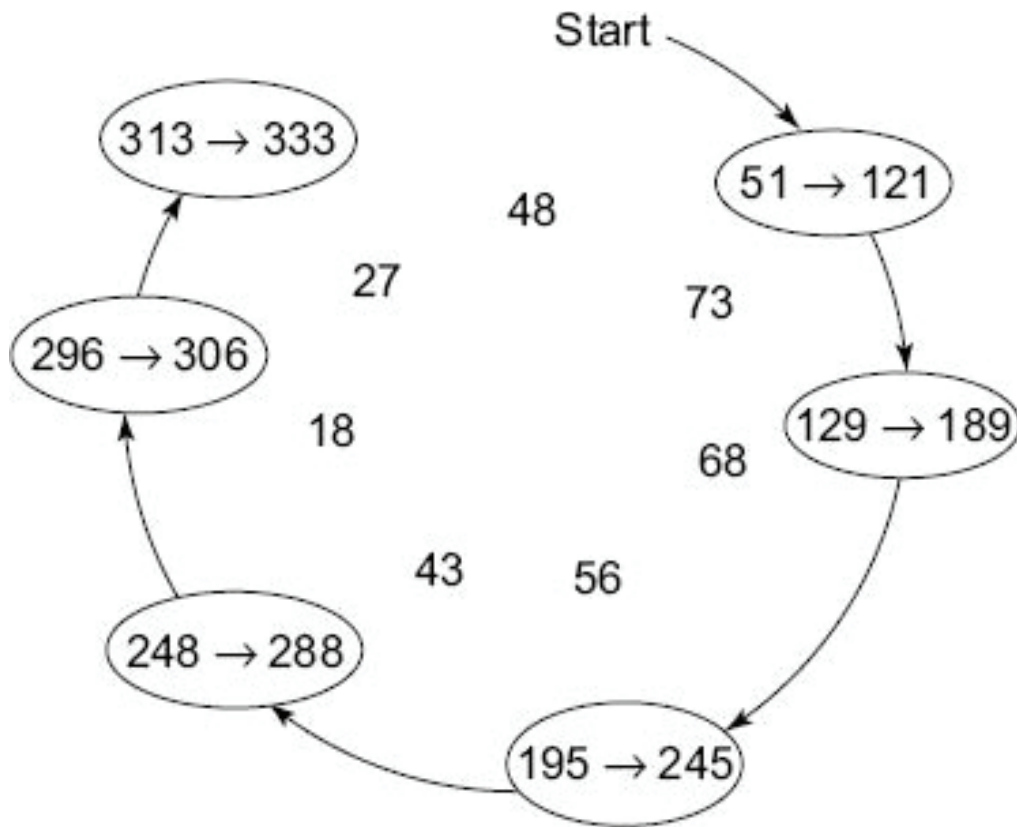
78 + 86 – how to think of this addition problem

As you can see, the above thinking in an addition situation requires no carry over and after some practice would require no writing at all. It is just an extension of how you are able to naturally react to 5+11 so also you can train your mind to react to 58+63 and react with a two step thought (as 61 Æ 121—with practice this can be done inside a fraction of a second. It is just a matter of how much you are willing to push your mind for this). Once you can do that your next target is to be able to add multiple 2 digit numbers written randomly on a single page:

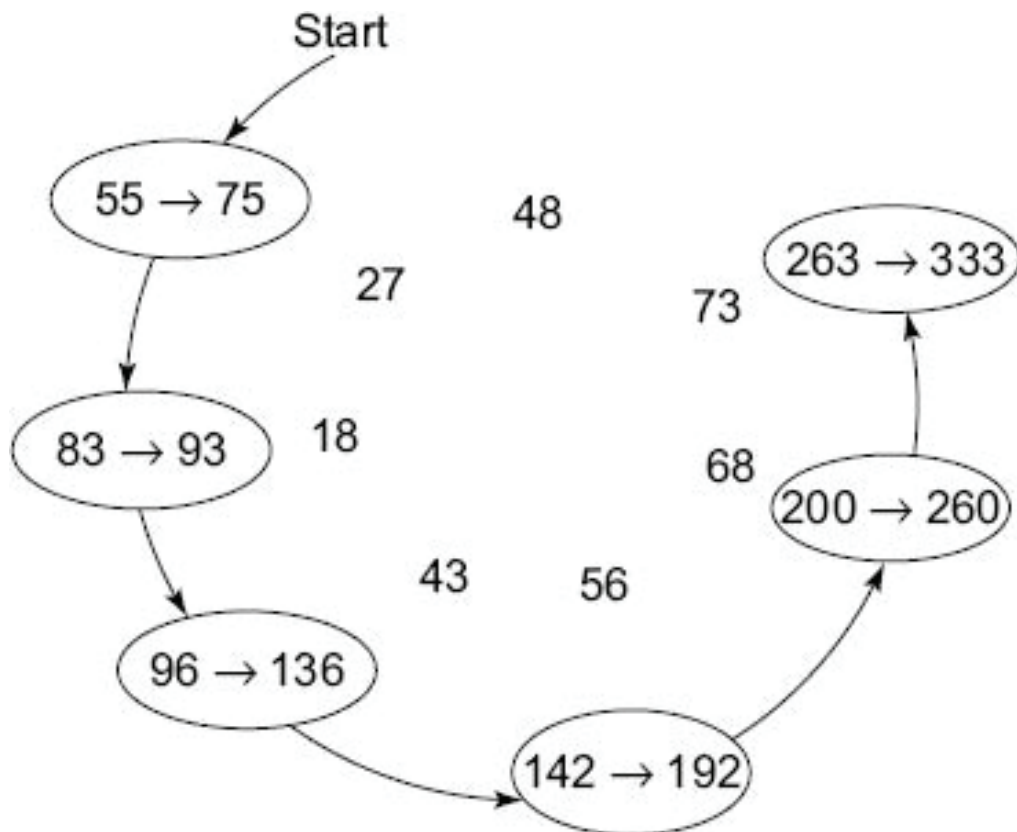
**Try this: Add the following**

- 48
- 27
73
- 18
68
- 43
56

In order to do this addition your thinking should go like this:



Alternately you may also do this the other way. The result would be quite the same:



While you are trying to work on this addition you would realize the following about your abilities to add (if you belong to the normal category of aspirants')

1. Something like  $121 + 68$  would be easier than  $189 + 56$  because the latter requires you to shift hundreds— something that the former does not require you to do.
2. Something like  $48 + 27$  would be easier for you to do initially than  $136 + 56$ ; and  $136 + 56$  would



be easier than  $543 + 48$  because your mind would be more comfortable with smaller numbers than you would be with larger numbers.

However as you start practicing your additions, these additions would become automatic for your mind—as they would then fall into the range where your mind can react with the answers. That is the point to which we would want you to target your skill levels for additions.

To put it in other terms, you would need to work on your additions in such a way that 10 numbers written around a circle (as shown above) should be done in around 10-12 seconds in your mind.

Till the time your addition skill levels reach that point, I would want you to work aggressively on your addition ability.

The following  $10 \times 10$  table done at least once daily might be a good way to work on your additions:

	59	68	77	96	84	32	17	69	81	38	TOTALS
48											
54				= 96 + 54 = 150							
67											
89											
56											
73											105
88											
24											
47											
96											
TOTALS											

Inside the table you would broadly do two things:

- For each cell you would add the values in the corresponding row and the corresponding column in order to get the value inside the cell. Thus, the second row and 4<sup>th</sup> column intersection would give you  $54+96=150$ , the sixth row and the sixth column would add to  $73+32 = 105$  as shown in the table.
- Add the total of the 10 numbers seen in each row after you finish doing the values inside the cells in the total. This would give you the final total of the row. Repeat the same process for the addition of the 10 numbers in the columns.

By this time, I guess you would have realized that we are targeting two broad addition skills—

- Your ability to react with the total when you see two 2 digit numbers (like  $57+78=135$ )
- Your ability to add multiple 2 digit numbers if they are given to you consecutively (like  $57+78+43+65+91+38+44+18+64+72=570$  in 8–10 seconds)

You might require around 1–2 months of regular practice to get proficient at this. However once you acquire this skill, every conceivable calculation that any aptitude exam can throw at you (or indeed has thrown at you over the past 20 years) would be very much within your zone.

How do you do larger additions?

One you have the skills to handle two digit additions as specified above handling bigger additions should be a cakewalk.

Suppose you were adding:

$57436 + 64123 + 44586 + 78304 + 84653 + 5836$ . In order to do this, first add the thousands.  $57 + 64 (=121) + 44 (=165) + 78 (=243) + 84 (=327) + 5 (=332)$ . Thus, you have an interim answer of 332 thousands. At this stage you know that your answer would be  $332000 +$  a maximum of 6000 ( as there are 6 numbers whose last 3 digits you have neglected). If a range of 332000 to 338000 suffices for you in the addition based on the closeness of the options, you would be through with your calculation at this point. In the event that you need to get to a closer answer than this, the next step would involve taking the 100s digit into account.

Thus for the above calculation:  $57436 + 64123 + 44586 + 78304 + 84653 + 5836$  when you add the hundreds, you get  $4+1+5+3+6+8 = 27$  hundreds. Your answer gets refined to 334700 and at this point you also know that the upper limit of the addition has to be a maximum of 600 more than 334700 i.e. the answer lies between 334700 to 335300. In case this accuracy level is still not sufficient you may then look at the last 2 digits of the numbers. Our experience tells us that normally that would not be required.

However, in case you still need to add these digits-it would amount to 2 digit additions again. So you would need to add  $57436 + 64123 + 44586 + 78304 + 84653 + 5836 \approx 36 + 23 (=59) + 86 (=145) + 4 (=149) + 53 (=202) + 36 (=238)$ .

**Thus, the correct total would be  $334700 + 238 = 334938$  and while doing this entire calculation we have not gone above 2 digit additions anywhere.**

Apart from that, the biggest advantage of the process explained above is that in this process, you could stop the moment you had an answer that was sufficient in the context of the provided options.

## SUBTRACTIONS—JUST AN EXTENSION OF ADDITIONS

The better your additions are, the better you would be able to implement the process explained for subtractions. So, a piece of advice from me—make sure that you have worked on your additions seriously for at least 15 days before you attempt to internalize the process for subtractions that is explained in this chapter.

Throughout school you have always used the conventional carry over method of subtracting. But, I am here to show you that you have an option—something that would be much faster and much more superior to the current process you are using. What is it you would ask me? Well what would you do in case you are trying to subtract 38 from 72?

The conventional process tells us to do this as:

		<i>Carry over 1</i>
7		2
– 3		8
3		4

Well, the alternative and much faster way of thinking about subtractions is shown on the number line below:

Difference between any 2 numbers is equal to the distance between the numbers on the number line



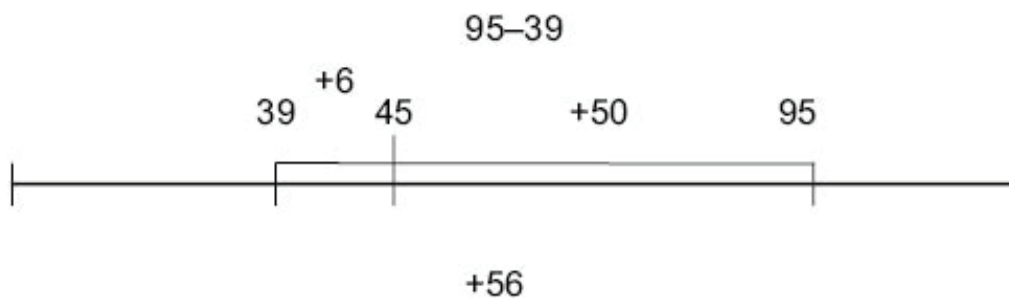
The principle used for doing subtractions this way is that the difference between any two numbers can be seen as the distance between them on the number line.

Thus, imagine you are standing on the number 38 on the number line and you are looking towards 72. To make your calculation easy, your first target has to be to reach a number ending with 2. When you start to move to the right from 38, the first number you see that ends in 2 is the number 42. To move from 38 to 42 you need to cover a distance of +4 (as shown in the figure). Once you are at 42, your next target is to move from 42 to 72. The distance between 42 to 72 is 30.

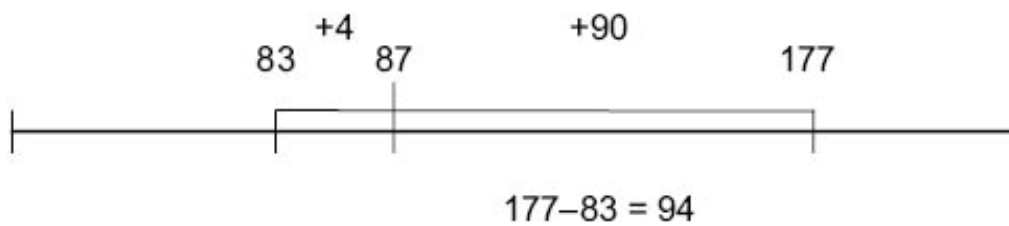
Thus, the subtraction's value for the numbers  $72 - 38$  would be 34.

Consider, the following examples:

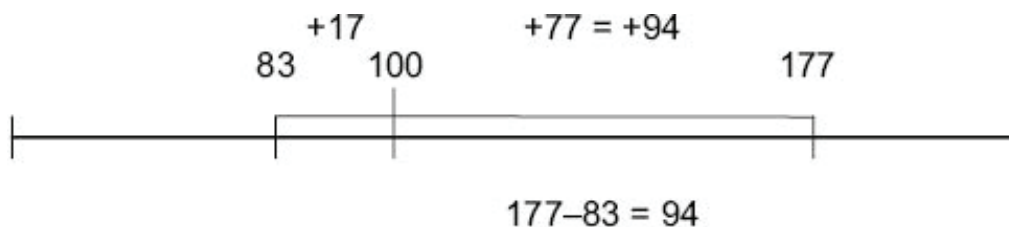
**Illustration 1**  $95 - 39$



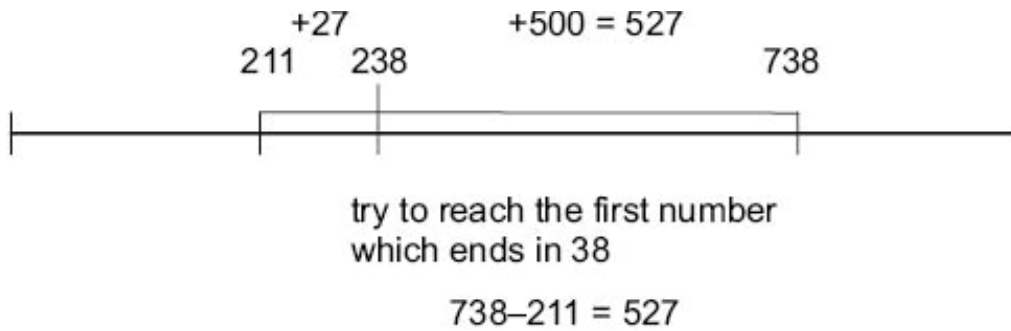
**Illustration 2**  $177 - 83$



Alternately:

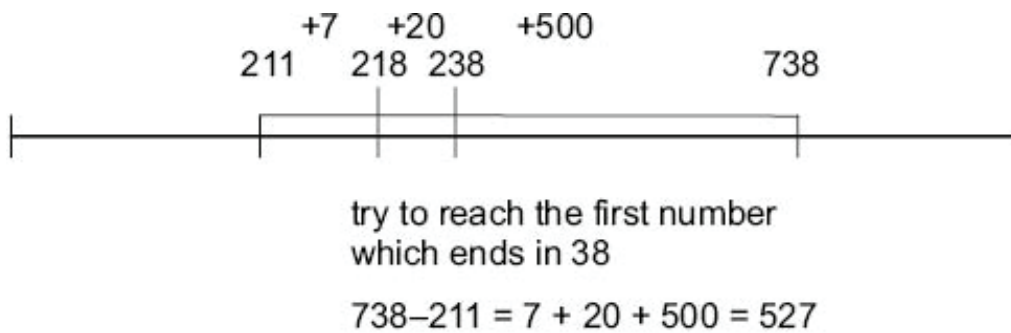


**Illustration 3**  $738 - 211$

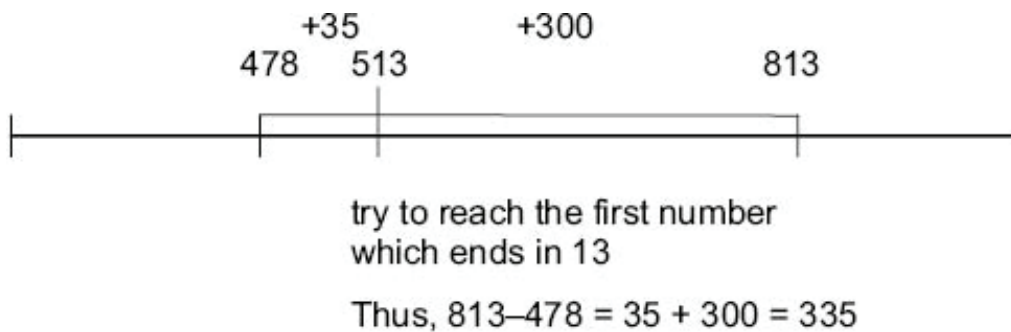


In this case the first objective is to reach the first number ending in 38 as you start moving to the right of 211. The first such number to the right of 211 being 238, first reach 238 (by adding 27 to 211) and then move from 238 to 738 (adding 500 to 238 to reach 738)

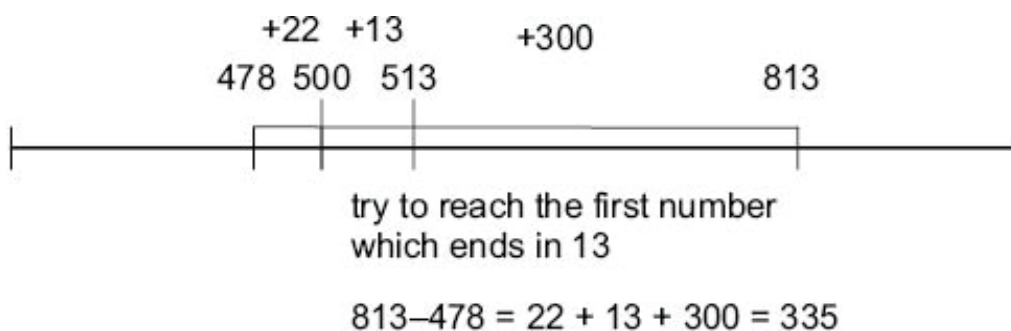
In case you need an intermediate number before reaching 238 you can also think of doing the following:



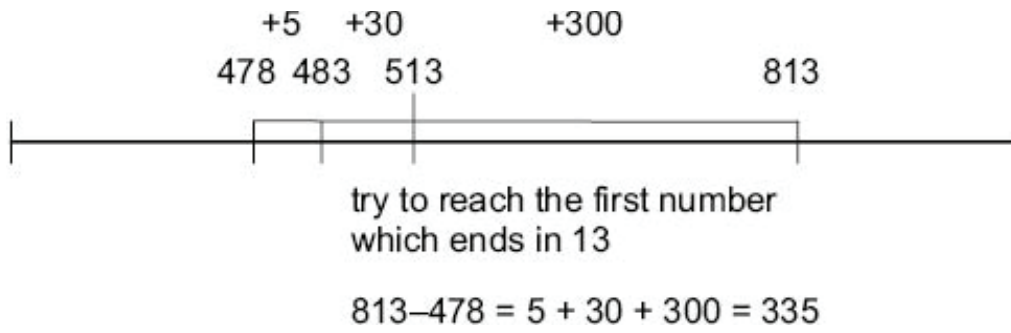
#### Illustration 4 $813 - 478$



Alternately, this thought can also be done as:



Also, you could have done it as follows:



Even if we were to get 4 digit numbers, you would still be able to use this process quite easily.

# 2

Chapter

## Multiplications

Multiplications are the next calculation which we need to look at—these are obviously crucial because most questions in Mathematics do involve multiplications.

The fundamental view of multiplication is essentially that when we need to add a certain number consecutively—say we want to add the number 17 seven times:

i.e.  $17 + 17 + 17 + 17 + 17 + 17 + 17$  it can also be more conveniently done by using  $17 \times 7$ .

Normally in aptitude exams like the CAT, multiplications would be restricted to 2 digits multiplied by 2 digits, 2 digits multiplied by 3 digits and 3 digits multiplied by 3 digits.

So what are the short cuts that are available in Multiplications? Let us take a look at the various options you have in order to multiply.

### **1. The straight line method of multiplying two numbers (From Vedic Mathematics and also from the Trachtenberg System of Speed Mathematics)**

Let us take an example to explain this process:

Suppose you were multiplying two 2 digit numbers like  $43 \times 78$ .

The multiplication would be done in the following manner:

$$\begin{array}{r} 43 \\ \times 78 \\ \hline \end{array}$$

#### ***Step 1: Finding the Unit's digit***

The first objective would be to get the unit's digit. In order to do this we just need to multiply the units' digit of both the numbers. Thus,  $3 \times 8$  would give us 24. Hence, we would write 4 in the units' digit of the answer and carry over the digit 2 to the tens place as follows:

$$\begin{array}{r} 43 \\ \times 78 \\ \hline 4 \end{array}$$

2 carry over to the tens place

At this point we know that the units' digit is 4 and also that there is a carry over of 2 to the tens place of

the answer.

### **Step 2: Finding the tens' place digit**

$$\begin{array}{r} 43 \\ \times 78 \\ \hline 54 \end{array}$$

5 carry over to  
hundreds place

#### **Thought Process:**

$$4 \times 8 + 3 \times 7 = 32 + 21 = 53$$

$$53 + 2 \text{ (from carry over)} = 55$$

Thus we write 5 in the tens place and carry over 5 to the hundreds place

In the above case, we have multiplied the units digit of the second number with the tens digit of the first number and added the multiplication of the units digit of the first number with the tens digit of the second number. Thus we would get:

$$\begin{aligned} &8 \text{ (units digit of the second number)} \times 4 \text{ (tens digit of the first number)} + 7 \text{ (tens digit of the second} \\ &\text{number)} \times 3 \text{ (units digit of the first number} + 2 \text{ (carry over from the units' digit calculation)} \\ &= 32 + 21 + 2 = 55. \end{aligned}$$

We write down 5 in the tens' digit of the answer and carry over 5 to the hundreds digit of the answer.

### **Step 3: Finding the hundreds' place digit**

$$\begin{array}{r} 43 \\ \times 78 \\ \hline 3354 \end{array}$$

#### **Thought Process:**

$$4 \times 7 = 28$$

$$28 + 5 = 33$$

Since, 4 and 7 are the last digits on the left in both the numbers this is the last calculation in this problem and hence we can write 33 for the remaining 2 digits in the answer

Thus, the answer to the question is 3354.

With a little bit of practice you can do these kinds of calculations mentally without having to write the intermediate steps.

Let us consider another example where the number of digits is larger:

Suppose you were trying to find the product of  $43578 \times 6921$

### **Step 1: Finding the units digit**

$$\text{Units digit: } 1 \times 8 = 8$$

$$\begin{array}{r} 4\ 3\ 5\ 7\ 8 \\ \times 6\ 9\ 2\ 1 \\ \hline \end{array}$$

### Step 2: Finding the tens digit

$$\begin{array}{r} 4\ 3\ 5\ 7\ 8 \\ \times 6\ 9\ 2\ 1 \\ \hline \end{array}$$

3 8

Carry over 2

#### Thought Process:

Tens digit would come by multiplying tens with units and units with tens

$$7 \times 1 + 2 \times 8 = 7 + 16 = 23$$

In order to think about this, we can think of the first pair - by thinking about which number would multiply 1 (units digit of the second number) to make it into tens.

Once, you have spotted the first pair the next pair would get spotted by moving right on the upper number (43578) and moving left on the lower number (6921)

### Step 3: Finding the hundreds digit

Let us look at the broken down thought process for this step:

$$\begin{array}{r} 4\ 3\ 5\ 7\ 8 \\ \times 6\ 9\ 2\ 1 \\ \hline \end{array}$$

3 8

Carry over 2

#### Thought Process:

Locate the first pair that would give you your hundreds digit.

For this first think of what you need to multiply the digit in the units place of the second number (digit 1) with to get the hundreds digit of the answer.

Since:

$$\text{Units} \times \text{Hundreds} = \text{Hundreds}$$

We need to pair 1 with 5 in the upper number as shown in the figure.

Once you have identified  $5 \times 1$  as the first pair of digits, to identify the next pair, move 1 to the right of the upper number and move 1 to the left of the lower number.

Thus, you should be able to get  $7 \times 2$  as your next pair.

$$\begin{array}{r} 4\ 3\ 5\ 7\ 8 \\ \times 6\ 9\ 2\ 1 \\ \hline \end{array}$$

3 8

Carry over 2

For the last pair, you can again repeat the above thought- move to the right in the upper number and move



to the left in the lower number.

Thus, the final thought for this situation would look like:

$$\begin{array}{r} 4\ 3\ 5\ 7\ 8 \\ \times 6\ 9\ 2\ 1 \\ \hline 3\ 3\ 8 \end{array}$$

9 carry over to thousands place

**Thought Process:**

First pair:  $5 \times 1$

Second pair:  $7 \times 2$  (move right on upper number and move left on the lower number)

Third pair:  $8 \times 9$  (move right on upper number and move left on the lower number)

Thus,  $5 \times 1 + 7 \times 2 + 8 \times 9 = 91$

$91 + 2$  (carry over) = 93

Hundreds place digit would be 3 and carry over to the thousands place would be 9

**Step 4: Finding the thousands digit**

We would follow the same process as above. For doing the same first identify the first pair as  $3 \times 1$  (thousands from the first number multiplied by units from the second number) and then start moving right and moving left on both the numbers to find the other pairs.

$$\begin{array}{r} 4\ 3\ 5\ 7\ 8 \\ \times 6\ 9\ 2\ 1 \\ \hline 3\ 3\ 1\ 8 \end{array}$$

13 carry over to  
ten thousands place

**Thought Process:**

$3 \times 1 + 5 \times 2 + 7 \times 9 + 8 \times 6 = 124$

thousands  $\times$  units + hundreds  $\times$  tens + tens  $\times$  hundreds + units  $\times$  thousands

$124 + 9$  (from the carry over) = 133

put down 3 as the thousands place digit and carry over 13 to the ten thousands place

**Step 5: Finding the ten thousands digit**

$4 \times 1$  would be the first pair here followed by  $3 \times 2$ ;  $5 \times 9$  and  $7 \times 6$  as shown below:

$$\begin{array}{r} 4\ 3\ 5\ 7\ 8 \\ \times 6\ 9\ 2\ 1 \\ \hline 0\ 3\ 3\ 3\ 8 \end{array}$$

11 carry over to  
the lacs place

**Thought Process:**

$4 \times 1 + 3 \times 2 + 5 \times 9 + 7 \times 6 = 97$

$97 + 13$  (from the carry over of the previous step) = 110.

Hence, 0 becomes the digit which would come into the answer and 11 would be carried over

### **Step 6: Finding the digit in the lakhs' place**

$$\begin{array}{r} 4\ 3\ 5\ 7\ 8 \\ \times 6\ 9\ 2\ 1 \\ \hline 6\ 0\ 3\ 3\ 3\ 8 \end{array}$$

7 carry over to  
the ten lakhs place

#### **Thought Process:**

Since we have already used up  $4 \times 1$  (the left most digit in the first number multiplied with the right most digit in the second number, it is no longer possible to use the digit 1 (units digit of the second number) to form a pair.

Use this as a signal to fix the digit 4 from the first number and start to write down the pairs by pairing this digit 4 (ten thousands' digit of the first number) with the corresponding digit of the second number to form the lacs digit.

It is evident that ten thousands  $\times$  tens = lacs.

Thus, the first pair is  $4 \times 2$ .

Note that you could also think of the first pair as  $4 \times 2$  by realising that since we have used  $4 \times 1$  as the first pair for the previous digit we can use the number to the left of 1 to form  $4 \times 2$ .

Subsequent pairs would be  $3 \times 9$  and  $5 \times 6$ .

Thus,  $4 \times 2 + 3 \times 9 + 5 \times 6 = 8 + 27 + 30 = 65$

$65 + 11$  (carry over) = 76.

Thus, 6 becomes the lacs digit and we get a carry over of 7.

### **Step 7: Finding the ten lakh's digit**

$$\begin{array}{r} 4\ 3\ 5\ 7\ 8 \\ \times 6\ 9\ 2\ 1 \\ \hline 1\ 6\ 0\ 3\ 3\ 3\ 8 \end{array}$$

6 carry over to the next place

#### **Thought Process:**

First pair:  $4 \times 9$

Next pair:  $3 \times 6$

Thus,  $4 \times 9 + 3 \times 6 = 54$

$54 + 7$  (from the carry over) = 61

1 becomes the next digit in the answer and we carry over 6.

### **Step 8: Finding the next digit**

$$\begin{array}{r}
 43578 \\
 \times 6921 \\
 \hline
 30160338
 \end{array}$$

**Thought Process:**

$$4 \times 6 = 24$$

$$24 + 6 \text{ (from the carry over)} = 30$$

This is the last step because we have multiplied the left most two digits in the number.

The above process of multiplication- although it looks extremely attractive and magical – especially for larger numbers, it’s actual usage in the examination context might actually be quite low. This is because there are better ways of doing multiplication of 2 to 3 digits and larger multiplications might not be required to be executed in an exam like the CAT.

However, in order to solve questions where you might be asked to find the hundreds or even the thousands’ digit of a big multiplication like the one showed above, this might be your only option.

Let us look at a few more alternative approaches in order to calculate multiplication problems.

**2. Using squares to multiply two numbers**

In this approach the usage of the mathematical result  $a^2 - b^2 = (a - b)(a + b)$  helps us to find the result of a multiplication.

For instance  $18 \times 22$  can be done using  $20^2 - 2^2 = 400 - 4 = 396$ , taking  $a = 20$  and  $b = 2$

Similarly,  $22 \times 28 = 25^2 - 3^2 = 625 - 9 = 616$

$35 \times 47 = 41^2 - 6^2 = 1681 - 36 = 1645$

In case the difference between the two numbers is not even, we can still use this process by modifying it thus:

$$\begin{aligned}
 24 \times 33 &= 24 \times 32 + 24 = 28^2 - 4^2 + 24 \\
 &= 784 - 16 + 24 = 792
 \end{aligned}$$

However, obviously this process might be ineffective in the following cases:

- (i) If the values of the squares required to calculate an multiplication are difficult to ascertain (For two digit numbers, we can bypass this by knowing the short cut to calculate the squares of 2 digit numbers- **You may want to look at the methods given in Chapter 4 of this part to find out the squares of 2 digit numbers in order to be more effective with these kinds of calculations.**)
- (ii) When one is trying to multiply two numbers which are very far from each other, there might be other processes for multiplying them that might be better than this process. For instance, if you are trying to multiply  $24 \times 92$  trying to do it as  $58^2 - 34^2$  obviously would not be a very convenient process.
- (iii) Also, in case one moves into trying to multiply larger numbers, obviously this process would fail. For instance  $283 \times 305$  would definitely not be a convenient calculation if we use this process.

**3. Multiplying numbers close to 100 and 1000**

A specific method exists for multiplying two numbers which are both close to 100 or 1000 or 10000.

For us, the most important would be to multiply 2 numbers which are close to 100.

The following example will detail this process for you:

Let us say you are trying to multiply  $94 \times 96$ .

**Step 1:** Calculate the difference from 100 for both numbers and write them down (or visualize them) as follows:

$$\begin{array}{r} \text{Difference} \\ \text{from 100} \\ 94 - 6 \\ \times 96 - 4 \\ \hline \end{array}$$

**Step 2:** The answer would be calculated in two steps-

(a) The last two digits of the answer would be calculated by multiplying  $-6 \times -4$  to get 24.

$$\begin{array}{r} 94 - 6 \\ \times 96 - 4 \\ \hline \end{array}$$

$\frac{\text{initial digits of the multiplication}}$	$\frac{\text{last 2 digits would be got by multiplying } -6 \times -4}$
------------------------------------------------------	-------------------------------------------------------------------------

Note here that we divide the answer into two parts:

### ***Last 2 digits and Initial digits***

(In case the numbers were close to 1000 we would divide the calculation into the last three digits and the initial digits)

When we multiply  $-6 \times -4$  we get 24 and hence we would write that as our last 2 digits in the answer.

We would then reach the following stage of the multiplication:

$$\begin{array}{r} 94 - 6 \\ \times 96 - 4 \\ \hline \text{initial digits} \quad | \quad 24 \\ \text{of the} \\ \text{multiplication} \end{array}$$

The next task is to find the initial digits of the answer:

This can be done by cross adding  $94 + (-4)$  or  $96 + (-6)$  to get the digits as 90 as shown in the figure below:

$$\begin{array}{r} \text{Difference from 100} \\ 94 - 6 \\ \times 96 - 4 \\ \hline 90 \quad | \quad 24 \end{array}$$

**Thought Process:**

Add along the diagonal connecting line shown in the figure to get the value of the initial digits of the answer:

$$\text{Thus we would get } 94 + (-4) = 90$$

or

$$96 + (-6) = 90$$

Thus, our initial digits of the answer are 90.

Let us take another example to illustrate a few more points which might arise in such a calculation:

Let us say, you were doing  $102 \times 103$ .

$$\begin{array}{r|l} 102 & +2 \\ 103 & +3 \\ \hline 105 & 06 \\ \text{Initial digits} & \text{Last 2 digits} \end{array}$$

**Thought Process:**

In this case, the second part of the answer (the last two digits) turns out to be  $2 \times 3 = 6$ . In such a case, since we know that the second part of the answer has to be compulsorily in 2 digits, we would naturally need to take it as 06.

The initial digits of the answer would be got by cross addition:

$$102 + 3 = 103 + 2 = 105$$

Consider:  $84 \times 88$

$$\begin{array}{r|l} 84 & -16 \\ 88 & -12 \\ \hline 73 & 92 \\ \text{Initial} & \text{Last 2} \\ \text{digits} & \text{digits} \end{array}$$

7392 is the answer

**Thought Process:**

In this case, the difference from 100 for the two numbers are  $-12$  and  $-16$  respectively.

We multiply them to get the last 2 digits of the number. However,  $-12 \times -16 = 192$  which is a 3 digit number.

Hence, retain 92 as the last 2 digits and carry over 1 to the initial digits.

Then while finding the initial digits you would need to add this carry over when you are writing the answer.

Thus, initial digits:

$$84 + (-12) + 1 \text{ (from the carry over)} = 72 + 1 = 73$$

Alternately:

$$88 + (-16) + 1 \text{ (from the carry over)} = 72 + 1 = 73$$

Now consider the situation where one number is above 100 and the other below 100.

For instance:

$$92 \times 104$$

The following figure would show you what to do in this case:

$$\begin{array}{r|l}
 104 & +4 \\
 \times 92 & -8 \\
 \hline
 96 & 00 \\
 - & 32 \\
 \hline
 95 & 68 \\
 \hline
 \text{Initial} & \text{Last 2} \\
 \text{digits} & \text{digits}
 \end{array}$$

**Thought Process:**

The problem in this calculation is that  $+4 \times (-8) = -32$  and hence cannot be directly written as the last two digits of the answer.

In this case, first leave the last 2 digits as 00 and find the initial digits of the answer.

Initial digits:

$$104 + (-8) = 96 = 92 + 4$$

When you write 96 having kept the last 2 digits of the number as 00, the meaning of the number's value would be 9600.

Now, from this subtract  $= 4 \times (-8) = -32$  to get the answer as  $9600 - 32 = 9568$ .

For a multiplication like  $994 \times 996$  the only adjustment you would need to do would be to look at the second part of the answer as a 3- digit number:

The following would make the process clear to you for such cases:

$$\begin{array}{r|l}
 994 & -6 \\
 \times 996 & -4 \\
 \hline
 990 & 024 \\
 \hline
 \text{Initial} & \text{Last 3} \\
 \text{digits} & \text{digits}
 \end{array}$$

**Thought Process:**

Last 3 digits =  $-6 \times -4 = 24 \text{ } \text{\AA}$  Hence, we write it as 024

Initial digits: Cross addition of  $994 + (-4) = 990$

Alternately,  $996 + (-6) = 990$

Thus, the answer would be 990024.

**Note:**

- (i) The above process for multiplication is extremely good in cases when the two numbers are close to any power of 10 (like 100,1000,10000 etc)

However, when the numbers are far away from a power of 10, the process becomes infeasible.

Thus, this process would not be effective at all in the case of  $62 \times 34$ .

- (ii) For finding squares of numbers between 80 to 120, this process is extremely good and hence you

should use this whenever you are faced with the task of finding the square of a number in this range.

For instance, if you are multiplying  $91 \times 91$  you can easily see the answer as 8281.

#### 4. Using additions to multiply

Consider the following view of an option for multiplying

Let us say we are trying to multiply  $83 \times 32$

**This can be converted most conveniently into  $80 \times 30 + 3 \times 30 + 2 \times 83 = 2400 + 90 + 166 = 2656$**

**This could also have been done as:  $83 \times 30 + 2 \times 83 = 2490 + 166 = 2656$**

**However in the case of  $77 \times 48$  the second conversion shown above might not be so easy to execute- while the first one would be much easier:**

**$70 \times 40 + 7 \times 40 + 8 \times 77 = 70 \times 40 + 7 \times 40 + 8 \times 70 + 8 \times 7 = 2800 + 280 + 560 + 56 = 3696.$**

The advantage of this type of conversion is that at no point of time in the above calculation are you doing anything more than single digit  $\times$  single digit multiplication.

#### 5. Use of percentages to multiply

Another option that you have can be explained as below:

Let us say you are trying to find  $43 \times 78$ .

In order to calculate  $43 \times 78$  first calculate 43% of 78 as follows:

$43\% \text{ of } 78 = 10\% \text{ of } 78 + 10\% \text{ of } 78 + 10\% \text{ of } 78 + 10\% \text{ of } 78 + 1\% \text{ of } 78 + 1\% \text{ of } 78 + 1\% \text{ of } 78 = 7.8 + 7.8 + 7.8 + 7.8 + 0.78 + 0.78 + 0.78 = 33.34$

This can be done using:  $7 \times 4 = 28$  as the integer part.

For adding the decimals, consider all the decimals as two digit numbers. In the addition if your total is a 2 digit number, write that down in the decimals place of the answer. If the number is a 3 digit number, carry over the hundreds' digit to the integer part of the answer.

Thus, in this case you would get:

$80 + 80 + 80 + 80 + 78 + 78 + 78 = 554$ . This 554 actually means 5.54 in the context that we have written down 0.80 as 80.

Thus, the total is 33.54.

We have found that 43% of 78 is 33.54 and our entire addition has been done in single and 2 digits. We of course realize that 43% of 78 being the same as  $0.43 \times 78$  the digits for  $43 \times 78$  would be the same as the digits for what we have calculated.

Now, the only thing that remains is to put the decimals back where they belong.

There are many ways to think about this- perhaps the easiest being that  $43 \times 78$  should have 4 as it's units digit and hence the correct answer is 3354.

Of course, this could also have been done by calculating 78% of 43 as  $21.5 + 10.75 + 0.43 + 0.43 + 0.43 = 31 + 2.54 = 33.54$  Hence, the answer is 3354.

You can even handle 2 digit  $\times$  3 digit multiplication through the same process:

Suppose you were multiplying  $324 \times 82$ , instead of doing the multiplication as given, find 324 % of 82.

The question converts to:  $3.24 \times 82 = 82 \times 3 + 8.2 + 8.2 + 0.82 + 0.82 + 0.82 + 0.82 = 246 + 16 + 3.68 = 265.68$ .

Hence, the answer is 26568.

We would encourage you to try to multiply 2 digits  $\times$  2 digits and 2 digits  $\times$  3 digits and 3 digits  $\times$  3 digits by the methods you find most suitable amongst those given above.

**In my view, the use of percentages to multiply is the most powerful tool for carrying out the kinds of multiplications you would come across in Aptitude exams.** Once you can master how to think about the decimals digits in these calculations, it has the potential to give you a significant time saving in your examination.

Obviously, when you convert a multiplication into an addition using any of the two processes given above, the speed and efficiency of your calculation would depend largely on your ability to add well. If your 2 digit additions are good (or if you have made your additions of 2 digit numbers good by using the process given in the chapter on additions) you would find the addition processes given here the best.

The simple reason is because this process has the advantage of being the most versatile- in the sense that it is not dependant on particular types of numbers.

Besides, after enough practice you would be able to do 2 digits  $\times$  2 digits and 2 digits  $\times$  3 digits and 3 digits  $\times$  3 digits orally.



# 3 Chapter

## Divisions, Percentage Calculations and Ratio Comparisons

### ● CALCULATING DECIMAL VALUES FOR DIVISION QUESTIONS USING PERCENTAGE CALCULATIONS

I have chosen to club these two together because they are actually parallel to each other—in the sense that for any ratio we can calculate two values—the percentage value and the decimal value. The digits in the decimal value and the percentage value of any ratio would always be the same. Hence, calculating the percentage value of a ratio and the decimal value of the ratio would be the same thing.

How do you calculate the percentage value of a ratio?

### ● PERCENTAGE RULE FOR CALCULATING PERCENTAGE VALUES THROUGH ADDITIONS

Illustrated below is a powerful method of calculating percentages. In my opinion, the ability to calculate percentage through this method depends on your ability to handle 2 digit additions. Unless you develop the skill to add 2 digit additions in your mind, you are always likely to face problems in calculating percentage through the method illustrated below. In fact, trying this method without being strong at 2-digit additions/subtractions (including 2 digits after decimal point) would prove to be a disadvantage in your attempt at calculating percentages fast.

This process, essentially being a commonsense process, is best illustrated through a few examples:

**Example** What is the percentage value of the ratio:  $53/81$ ?

**Solution** The process involves removing all the 100%, 50%, 10%, 1%, 0.1% and so forth of the denominator from the numerator.

Thus,  $53/81$  can be rewritten as:  $(40.5 + 12.5)/81 = 40.5/81 + 12.5/81 = 50\% + 12.5/81$   
 $= 50\% + (8.1 + 4.4)/81 = 50\% + 10\% + 4.4/81$   
 $= 60\% + 4.4/81$

At this stage you know that the answer to the question lies between 60 – 70% (Since 4.4 is less than 10% of 81)

At this stage, you know that the answer to the calculation will be in the form:  $6a.bcde \dots$

All you need to do is find out the value of the missing digits.

In order to do this, calculate the percentage value of  $4.4/81$  through the normal process of multiplying the numerator by 100.

$$\text{Thus the \% value of } \frac{4.4}{81} = \frac{4.4 \times 100}{81} = \frac{440}{81}$$

**[Note:** Use the multiplication by 100, once you have the 10% range. This step reduces the decimal calculations.]

$$\text{Thus } \frac{440}{81} = 5\% \text{ with a remainder of } 35$$

Our answer is now refined to  $65.bcde$ . (1% Range)

Next, in order to find the next digit (first one after the decimal) add a zero to the remainder;

Hence, the value of 'b' will be the quotient of

$$b \text{ } \text{Æ} \text{ } 350/81 = 4 \text{ Remainder } 26$$

Answer:  $65.4cde$  (0.1% Range)

$$c \text{ } \text{Æ} \text{ } 260/81 = 3 \text{ Remainder } 17$$

Answer:  $65.43$  (0.01% Range)

and so forth.

The advantages of this process are two fold:

(1) You only calculate as long as you need to in order to eliminate the options. Thus, in case there was only a single option between 60–70% in the above question, you could have stopped your calculations right there.

(2) This process allows you to go through with the calculations as long as you need to.

However, remember what I had advised you right at the start: Strong Addition skills are a primary requirement for using this method properly.

**To illustrate another example:**

What is the percentage value of the ratio  $\frac{223}{72}$  ?

$$223/72 \text{ } \text{Æ} \text{ } 300 - 310\% \text{ Remainder } 7$$

$$700/72 \text{ } \text{Æ} \text{ } 9. \text{ Hence } 309 - 310\%, \text{ Remainder } 52$$

$$520/72 \text{ } \text{Æ} \text{ } 7. \text{ Hence, } 309.7, \text{ Remainder } 16$$

$$160/72 \text{ } \text{Æ} \text{ } 2. \text{ Hence, } 309.72 \text{ Remainder } 16$$

Hence,  $309.7222$  (2 recurs since we enter an infinite loop of  $160/72$  calculations).

In my view, percentage rule (as I call it) is one of the best ways to calculate percentages since it gives you the flexibility to calculate the percentage value up to as many digits after decimals as you are required to and at the same time allows you to stop the moment you attain the required accuracy range.

Of course I hope you realize that when you get  $53/81 = 65.43\%$  the decimal value of the same would be  $0.6543$  and for  $223/72$ , the decimal value would be  $3.097222$ .

The kind of exam that the CAT is, I do not think you would not need to calculate ratios beyond 2 digits