A Complete Book of Advanced maths

For All SSC and MBA Entrance Exam

(Strictly according to the latest syllabus and exam oriented prescribed by given syllabus)

By:

Abhinay Sharma

Motivator **Entertainer** **Teacher**

ABHINAY PUBLICATIONS, Delhi - 110009

Ist Edition : 2016

Price : Rs.460

Laser Typed & setting
At Abhinay Publications

For Distribution
07838855815
Buy online: www.mathswithabhinay.com

Marketed by :
Golden Peacock Publications
B-2, 37-39, Ansal Building, Comm. Complex,
Dr. Mukherjee Nagar, Delhi-110009
Ph. : 011-47027303, 9215182295, 9213182295
E-mail : gppublications@ymail.com
Website : gyanworld.com

Printed at :
Shivalik Enterprises

Dedicated to

my beloved father Mr. O.P. Sharma for his love, moral support and encouragement to write this book.

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PREFACE TO THE FIRST EDITION

This book, "Play with advanced maths" has been written for those students preparing for govt. job.

The entire syllabus of advanced maths is divided into six chapters.

A few salient features of the book are :

1. The language used is simple, practical, comprehensible and easily understandable.
2. A "Type approach" has been followed while writing the book becomes it gives a satisfaction after completing a topic.
3. Solved example after each concept are self explanatory and given in a large number .
4. After the theory of each chapter there are large no. of questions in a exercise with detailed and short cut solutions.
5. Questions given in exercise are carefully selected by author according to previous papers.
6. All efforts have been made to get the book printed error free.

I am sure, the book in the present form would be liked by the students as well as teachers.

Finally, I would like to thank all our typing team, printing press and all our well wishers.

Suggestion for further improvement of the book will be thankfully acknowledged.

Abhinay Sharma

abhinay19@ymail.com

www.mathswithabhinay.com

Mob. no. 09015309258(Whatsapp)

Preface of the book

The answers to almost all questions have been checked and every care has been taken to minimize typographical as well as factual errors. However, it is possible that a few errors might have managed to dodge the vigilant eye. The author will be grateful to the readers for bringing these errors to his notice as also for their valuable suggestions. It is earnestly hoped the book will help the students grasp the advanced maths and help them in obtaining a good score in the examination.

It would have been difficult to prepare this book without the aid and support from different people. In my family members, my lovely mother Krishna Sharma, my sweet sister Ekta Sharma, my lovable brother Abhishek Sharma (Special thanks for proof reading) and last cutest member of my family is TukTuk Maharaj.

In my friends special thanks to Mr. Durga Yadav (for arrange typist), Rohit Gour, Shyam Bhai,Jeet Diwedi, Mahi Mam, Sapna Mam, Ranjeeta Mam, Nazia Mam are always available to support me morally at every step.

There is support of my dear students who are close to my heart - Sanchita Choudhary(special thanks to make correction), Monika Yadav, Nisha, Dinesh Yadav, Ankur Sharma, Rohit Rathee, Manjeet and all my dear students.Special thanks to my coaching team Pradeep ji, Naina ji and all staff.

Thanks a lot to all. Love you all.

-Abhinay Sharma

Contents

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1 Algebra

Algebra

Polynomial | **Quadratic Equation** | **Basic Identity**

Expression:– Relation between variable

ex: $y = x + z^2 + w^2$

where *y, x, z,* w are variable and *x, z, w* are independent variable while *y* is dependent variable because value of *y* is depending upon value of *x, z, w*.

POLYNOMIAL

It is an expression having single variable and power of variable must be a positive integer.

Ex. 1 Which one of the following is polynomial?

(a) $x^3 + y^2$ (b) $x^2 + \frac{1}{x^2}$

(c) $x + \sqrt{x}$ (d) $x^3 + x^2 + x$

Ans. (d) $x^3 + x^2 + x$

In options

(i) containing two variables, option

(ii) $\{x^2 + x^{-2}\}$ containing negative power of variable, option

(iii) $\{x + x^{1/2}\}$ containing fraction power of variable.

(iv) while option is in single variable and power of each term of *x* is positive integer.

Degree of Polynomial:–

Maximum power of variable is called degree.

Ex. (i) $x^4 + x^3 + x^2$ (ii) $x^2 + x^3 + 1$

degree = 4 degree = 3

Types of Polynomial :–

1. Linear Polynomial

$\Rightarrow$ degree = 1 (***e.g.*** $x + 1, 3x + 2$)

2. Quadratic Polynomial

$\Rightarrow$ degree = 2 (***e.g.*** $x^2 + x + 1, x^2 + 3$)

3. Cubic Polynomial

$\Rightarrow$ degree = 3 (***e.g.*** $x^3 + x^2 + 1, x^3 + 8$)

Factorization of Polynomial or expression:–

1. $(a + b)^2 = a^2 + b^2 + 2ab$

2. $(a - b)^2 = a^2 + b^2 - 2ab$

3. $(a^2 - b^2) = (a + b)(a - b)$

Ex.2 (i) $xa + ya + xz + yz$

$= a(x + y) + z(x + y)$

$= (x + y)(a + z)$

(ii) $x^2 + 5x + 6$

$= x^2 + 3x + 2x + 6$

$= x(x + 3) + 2(x + 3)$

$= (x + 3)(x + 2)$

(iii) $x^2 + 7xy + 12y^2$

$= x^2 + 3xy + 4xy + 12y^2$

$= x(x + 3y) + 4y(x + 3y)$

$= (x + 3y)(x + 4y)$

Ex. 3 If $\frac{5x^2 - 3y^2}{xy} = \frac{11}{2}$, find the positive value of $\frac{x}{y}$.

Sol^n. $2(5x^2 - 3y^2) = 11xy$

$10x^2 - 6y^2 = 11xy$

$10x^2 - 11xy - 6y^2 = 0$

$10x^2 - 15xy + 4xy - 6y^2 = 0$

$5x(2x - 3y) + 2y(2x - 3y) = 0$

$(2x - 3y)(5x + 2y) = 0$

$2x - 3y = 0$ or $5x + 2y = 0$

$\frac{x}{y} = \frac{3}{2}$ (positive) & $\frac{x}{y} = \frac{-2}{5}$ (negative)

Ex. 4 Which one of the following is not a factor of this polynomial $x^8 + x^4 + 1$?

(a) $(x^2 + 1 + x)$ (b) $(x^2 + 1 - x)$

(c) $(x^4 + 1 - x^2)$ (d) $x^2 - 1 + x$

Sol^n. (d) $x^8 + x^4 + 1 + x^4 - x^4$

$= (x^4 + 1)^2 - (x^2)^2$

$= (x^4 + 1 + x^2)(x^4 + 1 - x^2)$

$= (x^4 + 1 + 2x^2 - x^2)(x^4 + 1 - x^2)$

$= [(x^2 + 1)^2 - x^2][x^4 + 1 - x^2]$

$= (x^2 + 1 + x)(x^2 + 1 - x)(x^4 + 1 - x^2)$

Ex.5 Find $\dfrac{a^4 + b^4}{a^2 - ab\sqrt{2} + b^2}$, if $x = a^2 + b^2$ and $y = ab\sqrt{2}$.

(a) $x + y$ (b) $x - 2y$

(c) $x - y$ (d) $x + 2y$

Sol^n. (a) $a^4 + b^4 = a^4 + b^4 + 2a^2b^2 - 2a^2b^2$

$= (a^2 + b^2)^2 - (ab\sqrt{2})^2$

$= (a^2 + b^2 + ab\sqrt{2})(a^2 + b^2 - ab\sqrt{2})$

$\Rightarrow \dfrac{a^4 + b^4}{a^2 - ab\sqrt{2} + b^2}$

$= \dfrac{(a^2 + b^2 + ab\sqrt{2})(a^2 + b^2 - ab\sqrt{2})}{a^2 - ab\sqrt{2} + b^2}$

$= a^2 + b^2 + ab\sqrt{2} = x + y$

or

$\Rightarrow x = a^2 + b^2$ (squaring both sides)

$x^2 = a^4 + b^4 + 2a^2b^2$ & $y^2 = 2a^2b^2$

$\Rightarrow x^2 - y^2 = a^4 + b^4$

$\Rightarrow \dfrac{a^4 + b^4}{a^2 - ab\sqrt{2} + b^2} = \dfrac{x^2 - y^2}{x - y} = x + y$

Ex. 6 If $a^4 + a^2b^2 + b^4 = 12$, $a^2 + ab + b^2 = 4$, find ab.

Sol^n. $a^4 + 2a^2b^2 + b^4 - a^2b^2 = 12$

$(a^2 + b^2)^2 - (ab)^2 = 12$

$(a^2 + b^2 + ab)(a^2 + b^2 - ab) = 12$

$a^2 + b^2 - ab = \dfrac{12}{4} = 3$

$a^2 + b^2 - ab = 3$ …(i)

$a^2 + b^2 + ab = 4$ …(ii)

Subtracting equation (i) from equation (ii)

$2ab = 1 \Rightarrow ab = \dfrac{1}{2}$

Ex. 7 If $a^4 + a^2b^2 + b^4 = 8$ & $a^2 + b^2 + ab = 4$ then find ab.

Sol^n. We will solve this question without factorisation

$a^2 + b^2 + ab = 4$

$a^2 + b^2 = 4 - ab$

squaring both side

$(a^2 + b^2)^2 = (4 - ab)^2$

$a^4 + b^4 + 2a^2b^2 = 16 + a^2b^2 - 8ab$

$a^4 + b^4 + a^2b^2 = 16 - 8ab$

$8 = 16 - 8ab$ $(\because a^4 + a^2b^2 + b^4 = 8)$

$8ab = 8 \Rightarrow ab = 1$

Ex. 8 If $x^4 + y^4 = 19$ and $x + y = 1$ then find $x^2y^2 - 2xy$.

Sol^n. $x + y = 1$ (squaring both side)

$(x + y)^2 = 1$

$x^2 + y^2 + 2xy = 1$

$x^2 + y^2 = 1 - 2xy$ (again squaring both side)

$(x^2 + y^2)^2 = (1 - 2xy)^2$

$x^4 + y^4 + 2x^2y^2 = 1 + 4x^2y^2 - 4xy$

$19 + 2x^2y^2 = 1 + 4x^2y^2 - 4xy$

$18 = 2x^2y^2 - 4xy \Rightarrow x^2y^2 - 2xy = 9$

Factor Theorem:–

If $(x - a)$ is factor of $f(x)$, then $f(a) = 0$.

For ex. Is $(x - 1)$ factor of $x^2 - 2x + 1$?

Step I : Put factor = 0.

$x - 1 = 0$, then $x = 1$

Step II : Put value of x in function

Put $x = 1$ in $x^2 - 2x + 1$

$(1)^2 - 2 \times 1 + 1 = 1 - 2 + 1 = 0$

hence, $(x - 1)$ is factor of $x^2 - 2x + 1$

Ex. 9 Factor of $x^{29} - x^{26} - x^{23} + 1$.

(a) $(x - 1)$ but not $(x + 1)$

(b) $(x + 1)$ but not $(x - 1)$

(c) both $(x + 1)$ and $(x - 1)$

(d) Neither $(x + 1)$ nor $(x - 1)$

Sol^n. $(x-1)=0$ & $(x+1)=0$

put $x=1$ | put $x=-1$

$1^{29}-1^{26}-1^{23}+1$ | $(-1)^{29}-(-1)^{26}-(-1)^{23}+1$

$=1-1-1+1$ | $=-1-1+1+1$

$=0$ | $=0$

Ans. both $(x+1)$ and $(x-1)$

Ex. 10 If $(x-2)$ is a factor of polynomial x^2+kx+4. Find the value of k.

Sol^n. $x-2=0$

$x=2$

Then, $(2)^2+k(2)+4=0$

$2k=-8 \Rightarrow k=-4$

Ex. 11 If x^3+ax^2+2x+3 is exactly divisible by $(x+1)$. Find the value of a.

Sol^n. $x+1=0$

$\Rightarrow x=-1$

Then, $(-1)^3+a(-1)^2+2(-1)+3=0$

$\Rightarrow -1+a-2+3=0$

$a=0$

Ex. 12 If $(x+1)$ and $(x-1)$ are factor of ax^3+bx^2+3x+5. Find the value of a and b.

Sol^n. If $x+1=0$

$x=-1$

then, $a(-1)^3+b(-1)^2+3(-1)+5=0$

$\Rightarrow -a+b-3+5=0$

$\Rightarrow -a+b=-2$ **…(i)**

If $x-1=0$

$x=1$

then, $a(1)^3+b(1)^2+3(1)+5=0$

$\Rightarrow a+b+3+5=0$

$\Rightarrow a+b=-8$ **…(ii)**

Adding (i) and (ii)

$2b=-8-2=-10$

$b=-5$ put b in equation (i) then $a=-3$

Ex. 13 Find $(x+y+z)^3-(x+y-z)^3-(y+z-x)^3-(z+x-y)^3$

(a) $24xyz$ (b) $27xyz$

(c) $3xyz$ (d) 0

Sol^n. let $\mathbb{F}(x,y,z)=(x+y+z)^3-(x+y-z)^3-(y+z-x)^3-(z+x-y)^3$ .....(i)

If we put $x=0$ in $\mathbb{F}(x,y,z)$, then $\mathbb{F}(x,y,z)$ is 0, so x will be factor of $\mathbb{F}(x,y,z)$.

similarly, we put $y=0$ and $z=0$ in , $\mathbb{F}(x,y,z)$ then $\mathbb{F}(x,y,z)$ is also zero so y, z will also be factor of $\mathbb{F}(x,y,z)$.

$\mathbb{F}(x,y,z)=\text{K}\,xyz$

$(x+y+z)^3-(x+y-z)^3-(y+z-x)^3-(z+x-y)^3=\text{K}xyz$

Now we can put $x=y=z=1$ in above identity to find K

$27-1-1-1=\text{K} \Rightarrow \text{K}=24$

$\Rightarrow \mathbb{F}(x,y,z)=24xyz$

Remainder Theorem :–

If $f(x)$ is divided by $(x-a)$, then

Remainder (R) $=f(a)$

Step 1. divisor $=0$

$x-a=0$

Step 2. put $x=a$ in function then remainder (R) $=f(a)$

Ex.14 $\dfrac{x^2-7x+15}{x-3}$, find remainder.

Sol^n. $x-3=0 \Rightarrow x=3$

$\text{R}=(3)^2-7(3)+15$

$=9-21+15=3$

Ex. 15 If x^2+x+4 is divided by $(x-1)$, find the remainder

Sol^n. $x-1=0 \Rightarrow x=1$

$\text{R}=(1)^2+1+4=6$

Ex. 16 If $x^{11}+3$ is divided by $(x+1)$, find the remainder

Sol^n. $x+1=0 \Rightarrow x=-1$

$\text{R}=(-1)^{11}+3=-1+3=2$

Ex.17 If $x^{51}+51$ is divided by $(x+1)$, find the remainder

Sol^n. $x+1=0 \Rightarrow x=-1$

$\text{R}=(-1)^{51}+51=50$

Ex.18 If $x^{40}+3$ is divided by x^4+1, find the remainder

Sol^n. $x^4+1=0 \Rightarrow x^4=-1$

$\text{R}=(x^4)^{10}+3=(-1)^{10}+3$

$=1+3=4$

Ex. 19 $x^{35} + 3$ is divided by $x^5 + 1$, find remainder

Sol^n. $x^5 + 1 = 0 \Rightarrow x^5 = -1$

$R = (x^5)^7 + 3 = (-1)^7 + 3 = -1 + 3 = 2$

Ex. 20 If $x^2 + bx + 7$ is divided by $(x - 1)$ leaves remainder 12. find b.

Sol^n. $x - 1 = 0 \Rightarrow x = 1$

$R = (1)^2 + b.(1) + 7$

$12 = b + 8 \Rightarrow b = 4 \qquad (\because R = 12)$

Ex.21 If $2x^2 + kx + 8$ is divided by $(x + 2)$ leaves remainder $3k$. find $k = ?$

Sol^n. $x + 2 = 0 \Rightarrow x = -2$

$R = 2(-2)^2 + (-2)k + 8 = 8 - 2k + 8$

$3k = 16 - 2k$

$5k = 16$

$\Rightarrow \quad k = \frac{16}{5}$

Ex.22 $x^2 + 4x + k$ is divided by $(x - 2)$ leaves remainder $2x$. Find k.

Sol^n. $x - 2 = 0 \Rightarrow x = 2$

$R = (2)^2 + 4(2) + k$

$2x = 4 + 8 + k$

$4 = 12 + k \qquad (\because R = 2x = 2 \times 2 = 4)$

$4 - 12 = k$

$k = -8$

LCM & HCF of Polynomial :

HCF:

A polynomial $h(x)$ is called the HCF or GCD of two or more given polynomials, if $h(x)$ is a polynomial of highest degree dividing each one of the given polynomials. The coefficient of highest degree term in HCF is always taken as positive.

to find the HCF of two or more given polynomials step below:

Step I : Express of each polynomial as a product of powers of irreducible factors.

Step II: If there is no common factor, the HCF is 1. If there are common irreducible factors, find the smallest (least) exponents of these irreducible factors in the factorised form of the given polynomials.

Step III: Raise the common irreducible factors to the smallest exponents found in Step II and multiply to get the HCF.

LCM:

A polynomial $p(x)$ is called LCM of two or more given polynomials, if it is a polynomial of smallest degree which is divided by each one of the given polynomials.

To find the LCM of two or more given polynomials follow three-step procedure:

Step I: Express each polynomial as a product of powers of irreducible factors.

Step II: List all the irreducible factors (once only) occurring in the given polynomials. For each of these factors, find the greatest exponent in the factorised form of the given polynomials.

Step III: Raise each irreducible factor to the greatest exponent found in Step II, and multiply to get the LCM.

Ex. 23 Find the HCF of the polynomials $30(x^2 - 3x + 2)$ and $50(x^2 - 2x + 1)$.

Sol^n. Let $f(x) = 30(x^2 - 3x + 2)$,

$g(x) = 50(x^2 - 2x + 1)$

$f(x) = 2 \times 3 \times 5 \times (x - 1) \times (x - 2)$

$g(x) = 2 \times 5^2 \times (x - 1)^2$

$HCF = 2^1 \times 5^1 \times (x - 1)^1 = 10(x - 1)$

Ex. 24 Find the HCF of

$f(x) = 33(2x + 3)^2 (3x - 4)^3 (4x - 5)^4$

and $g(x) = 22\,(x + 1)\,(2x + 3)\,(4x - 5)^2\,(4x^2 - 9)$

Sol^n. $f(x) = 3 \times 11\,(2x + 3)^2(3x - 4)^3\,(4x - 5)^4$

$g(x) = 2 \times 11\,(x+1)(2x+3)(4x-5)^2(2x+3)(2x-3)$

$= 2 \times 11\,(x + 1)\,(2x + 3)^2\,(2x - 3)\,(4x - 5)^2$

$HCF = 11^1 \times (2x + 3)^2 \times (4x - 5)^2$

$= 11(2x + 3)^2\,(4x - 5)^2$

Ex.25 Find the HCF of the polynomials $f(x) = 6(x^3 + 3x^2)\,(x^2 - 16)(x^2 + 9x + 18)$ and $g(x) = 8(x^4 + 4x^3)\,(x^2 + 6x + 9)^2$.

Sol^n. $f(x) = 6\,(x^3 + 3x^2)\,(x^2 - 16)\,(x^2 + 9x + 18)$

$= 2 \times 3\,\{x^2(x+3)\}\{(x-4)(x+4)\}\{(x+6)(x+3)\}$

$= 2 \times 3x^2\,(x + 3)^2\,(x - 4)\,(x + 4)\,(x + 6)$

$g(x) = 8(x^4 + 4x^3)\,(x^2 + 6x + 9)^2$

$= 2^3\,(x^3(x + 4))\{(x + 3)^2\}^2$

$= 2^3\,x^3\,(x + 4)\,(x + 3)^4$

$HCF = 2x^2\,(x + 3)^2\,(x + 4)$.

Ex. 26 Find the LCM of the polynomials
$f(x) = 4(x-1)^2 (x^2+6x+8)$ and
$g(x) = 10(x-1)(x+2)(x^2+7x+10)$.

Sol^n. $f(x) = 2^2 \times (x-1)^2 \times (x+2) \times (x+4)$,
$g(x) = 2 \times 5 \times (x-1) \times (x+2) \times (x+5) \times (x+2)$
$= 2 \times 5 \times (x-1) \times (x+2)^2 \times (x+5)$

LCM $= 2^2 \times 5^1 \times (x-1)^2 \times (x+2)^2 \times (x+4)^1 \times (x+5)^1$
$= 20(x-1)^2 (x+2)^2 (x+4)(x+5)$

Ex. 27 Find the value of b for which the HCF of $x^2 + 2bx + 3b + 3$ and $x^2 + x - 5b$ is $(x+5)$.

Sol^n. $(x+5)$ is the HCF of the given polynomials, it means $(x+5)$ is the factor of the given polynomials this means $f(-5)$ is a zero of both the given polynomials.
$(-5)^2 + 2b(-5) + 3b + 3 = 0$
$28 - 7b = 0 \Rightarrow b = 4$
and $(-5)^2 + (-5) - 5b = 0$
$20 - 5b = 0 \Rightarrow b = 4$
Both the above equations give $b = 4$

Ex. 28 Find the LCM and HCF of the polynomials
$P(x) = (x+1)^2 (x+2)$
$Q(x) = (x+1)(x-2)$

Sol^n. LCM $= (x+1)^2 (x+2)(x-2) = (x+1)^2 (x^2-4)$
HCF $= (x+1)$

Note: The product of the two polynomial is equal to the product of LCM & HCF.

$P(x) \times Q(x) =$ LCM $\times$ HCF

Ex. 29 If HCF and LCM of two polynomial $P(a)$ & $Q(a)$ is $(a+1)$ and $a^3 + a^2 - a - 1$ respectively if $P(a) = (a^2 - 1)$, find $Q(a) = ?$

Sol^n. $P(a) \times Q(a) =$ LCM $\times$ HCF

$$\Rightarrow \quad Q(a) = \frac{(a^3 + a^2 - a - 1) \times (a+1)}{a^2 - 1}$$

$$= \frac{\left[a^2(a+1) - 1(a+1)\right](a+1)}{(a^2-1)}$$

$$= \frac{(a+1)(a^2-1)(a+1)}{(a^2-1)}$$

$Q(a) = (a+1)^2$

Ex. 30 Find HCF of $x^3 + 3x^2y + 2xy^2$ and $x^4 + 6x^3y + 8x^2y^2$

Sol^n. $P(x) = x^3 + 3x^2y + 2xy^2$
$= x[x^2 + 3xy + 2y^2]$
$= x[x^2 + 2xy + xy + 2y^2]$
$= x[x(x+2y) + y(x+2y)]$
$= x(x+2y)(x+y)$

$Q(x) = x^4 + 6x^3y + 8x^2y^2$
$= x^2[x^2 + 6xy + 8y^2]$
$= x^2[x^2 + 4xy + 2xy + 8y^2]$
$= x^2[x(x+4y) + 2y(x+4y)]$
$= x^2(x+2y)(x+4y)$

HCF $= x(x+2y)$

Ex. 31 Find HCF of $10x^3 - 10x^2 - 5x + 9$ & $30x^3 - 61x^2 - 24x + 10$

(a) $31x^2 + 29x + 17$ (b) $31x^2 + 9x + 17$
(c) $31x^2 - 9x - 27$ (d) $30x^2 - 11x + 11$

Sol^n. HCF can never be greater than the difference between the polynomial. HCF may be difference or the factor of difference.
$P(x) = 30x^3 - 30x^2 - 15x + 27$
$Q(x) = 30x^3 - 61x^2 - 24x + 10$
$\quad - \quad + \quad + \quad -$

Difference $= 31x^2 + 9x + 17$

According to the option,
we can see HCF will be $31x^2 + 9x + 17$

QUADRATIC EQUATION

An algebric expression of the form: $ax^2 + bx + c = 0$, where $a \neq 0$, $b, c \in R$ is called a quadratic equation.

Root of the Quadratic Equation:

A root of the quadratic equation $ax^2 + bx + c = 0$ is a number α (real or complex) such that $a\alpha^2 + b\alpha + c = 0$ then $(x - \alpha)$ is factor of $ax^2 + bx + c$. The roots of the quadratic equation are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Proof: $\Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$$\Rightarrow x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a} = 0$$

$$\Rightarrow \left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a^2} = 0$$

$$\Rightarrow x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

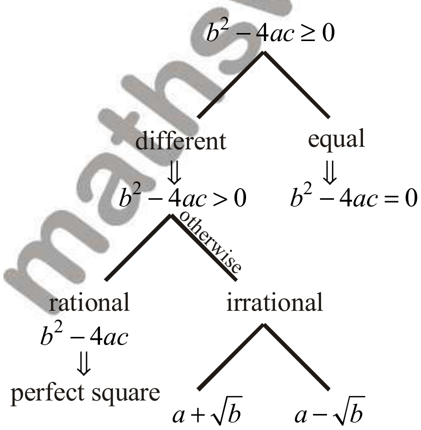
$$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Note: The numbers *a, b, c* are called the coefficients of the quadratic equation and the expression $b^2 - 4ac$ is called its discriminent (denoted by D).

Nature of Roots:

The value of *x* at which value of equation will be zero.

1. **Roots are imaginary:** $b^2 - 4ac \leq 0$
2. **Roots are real:** $b^2 - 4ac \geq 0$



Sum & product of root: let there are two roots named $\alpha \& \beta$, then

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \& \quad \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Sum of root : $\alpha + \beta = \frac{-b}{a}$

Product of root : $\alpha\beta = \frac{c}{a}$

then, $ax^2 + bx + c = 0$ can be written as :

$$\Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$\Rightarrow x^2 - \left(\frac{-b}{a}\right)x + \frac{c}{a} = 0$$

$$\boxed{x^2 - (\text{sum of root})x + \text{product of root} = 0}$$

- If the roots $\alpha \& \beta$ be reciprocal to each other, then $a = c$.
- If the two roots $\alpha \& \beta$ be equal in magnitude and opposite in sign, then $b = 0$
- If *a, b, c* are rational number and $a + \sqrt{b}$ is one root of the quadratic equation, then the other root must be the conjugate $a - \sqrt{b}$ and vice-versa

Ex. 32 Find the Quadratic equation whose one root is $3 + \sqrt{3}$

Sol^n. If one root is $3 + \sqrt{3}$ then second root will be $3 - \sqrt{3}$

Sum of root $= (3 + \sqrt{3}) + (3 - \sqrt{3}) = 6$

Product of root $= (3 + \sqrt{3})(3 - \sqrt{3}) = 6$

using,

$x^2 - (\text{sum of root})x + (\text{product of root}) = 0$

$\Rightarrow x^2 - 6x + 6 = 0$

Ex. 33 Two roots of equation of $2x^2 - 7x + 12 = 0$ are $\alpha \,\&\, \beta$ then, find $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = ?$

Sol^n. $2x^2 - 7x + 12 = 0$

On comparing with standard equation

$ax^2 + bx + c = 0$

$a = 2,\ b = -7\ \&\ c = 12$

$\alpha + \beta = \frac{-b}{a} \Rightarrow \alpha + \beta = \frac{7}{2}$

$\alpha\beta = \frac{c}{a} \Rightarrow \alpha\beta = 6$

$$\Rightarrow \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{\left(\frac{7}{2}\right)^2 - 2\times 6}{6}$$

$$= \frac{\frac{49}{4} - 12}{6} = \frac{49 - 48}{4\times 6} = \frac{1}{24}$$

Ex. 34 Find the product of the root of the equation $x^2 - \sqrt{3} = 0$.

Sol^n. On comparing this equation with $ax^2 + bx + c = 0$

$a = 1,\ b = 0\ \&\ c = -\sqrt{3}$

Product of root $= \alpha\beta = \frac{c}{a} = -\sqrt{3}$

<u>Condition for common Roots:</u>

let two quadratic equations–

$a_1x^2 + b_1x + c_1 = 0 \quad \ldots(i)$

$a_2x^2 + b_2x + c_2 = 0 \quad \ldots(ii)$

(A) If one root is common then,

$$\Rightarrow \left(a_1b_2 - a_2b_1\right)\left(b_1c_2 - b_2c_1\right) = \left(c_1a_2 - c_2a_1\right)^2$$

(B) If two roots are common then,

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Ex. 35 If the equations $x^2 + 2x - 3 = 0$ and $x^2 + 3x - m = 0$ have a common root, then the non- zero value of m.

Sol^n. **using**

$$\left(a_1b_2 - a_2b_1\right)\left(b_1c_2 - b_2c_1\right) = \left(c_1a_2 - c_2a_1\right)^2$$

$\Rightarrow (1.3 - 1.2)[2.(-m) - 3.(-3)] = [(-3).1 - (-m).1]^2$

$\Rightarrow (3 - 2)(-2m + 9) = (-3 + m)^2$

$\Rightarrow -2m + 9 = 9 + m^2 - 6m$

$\Rightarrow m^2 - 4m = 0$

$\Rightarrow m(m - 4) = 0$

$\Rightarrow m = 4 \ (\because m \neq 0)$

Ex. 36 For what value of m equation $4x^2 + 6mx + 9 = 0$ have equal roots.

Sol^n. on comparing this equation with

$ax^2 + bx + c = 0$

$a = 4,\ b = 6m\ \&\ c = 9$

if equation have equal roots: $\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (6m)^2 - 4(4)(9) = 0$

$\Rightarrow 36m^2 = 144$

$\Rightarrow m^2 = 4 \Rightarrow m = \pm\sqrt{4} = \pm 2$

MAX^M & MIN^M VALUE OF QUADRATIC EQUATION:

$$\Rightarrow ax^2 + bx + c = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right)$$

$$= a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a}\right)$$

$$= a\left(\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a^2}\right)$$

$$= a\left(x+\frac{b}{2a}\right)^2+\frac{4ac-b^2}{4a}$$

(A). When $a > 0$ the expression gives minimum value is equal to $\frac{4ac-b^2}{4a}$.

(B). When $a < 0$ the expression gives maximum value is equal to $\frac{4ac-b^2}{4a}$.

Formation of a New Quadratic Equation by changing the roots of a given Qudratic equation:

Ex. 37 If α & β are the roots of equation $ax^2+bx+c = 0$ then find the quadratic equation whose roots are $\alpha+2$ & $\beta+2$.

Sol^n. let $y=\alpha+2$

$\Rightarrow \alpha = y-2$

α is the root of equation $ax^2+bx+c=0$ so put $\alpha = y-2$ in this equation.

$\Rightarrow a(y-2)^2+b(y-2)+c=0$

Replace y by x

$a(x-2)^2+b(x-2)+c=0$

hence, this is the required equation.

Ex. 38 If α & β are the roots of equation $ax^2+bx+c=0$ then find the Quadratic equation whose roots are $\frac{1}{\alpha}$ & $\frac{1}{\beta}$.

Sol^n. let $y=\frac{1}{\alpha}$ [we can take $y=\frac{1}{\beta}$]

$\Rightarrow \alpha = \frac{1}{y}$

α is the root of equation $ax^2+bx+c=0$ so put $\alpha=\frac{1}{y}$ in this equation.

$$\Rightarrow a\left(\frac{1}{y}\right)^2+b\left(\frac{1}{y}\right)+c=0$$

$\Rightarrow a+by+cy^2=0$

Replace y by x

$a+bx+cx^2=0$

hence this is the required equation.

Ex. 39 If α & β are the roots of equation $ax^2+bx+c=0$ then find the quadratic equation whose roots are α^2 & β^2.

Sol^n. let $y=\alpha^2$

$\Rightarrow \alpha=\sqrt{y}$

α is the root of equation $ax^2+bx+c=0$ so put $\alpha=\sqrt{y}$ in this equation.

$$\Rightarrow a\left(\sqrt{y}\right)^2+b\left(\sqrt{y}\right)+c=0$$

$$\Rightarrow b\left(\sqrt{y}\right)=-a\left(\sqrt{y}\right)^2-c=-ay-c$$

Squaring both sides

$b^2y=a^2y^2+c^2+2acy$

$a^2y^2+(2ac-b^2)y+c^2=0$

Replace y by x

$a^2x^2+(2ac-b^2)x+c^2=0$

hence this is the required equation.

Ex. 40 If $\sqrt{3x^2-12x+19}+\sqrt{3x^2-12x-11}=6$ then $\sqrt{3x^2-12x+19}-\sqrt{3x^2-12x-11}=?$

(a) 3 (b) 4

(c) 5 (d) 6

Sol^n. $\sqrt{3x^2-12x+19}+\sqrt{3x^2-12x-11}=6$ ...(1)

$\sqrt{3x^2-12x+19}-\sqrt{3x^2-12x-11}=t$ (Let) ...(2)

multiply both equation (1) and (2)

$\Rightarrow 3x^2-12x+19-3x^2+12x+11=6t$

$\Rightarrow 6t=30 \Rightarrow t=5$

Ex. 41 If $x = 2 - 2^{\frac{1}{3}} + 2^{\frac{2}{3}}$ find the value of $x^3 - 6x^2 + 18x + 18$.

(a) 22 (b) 33

(c) 40 (d) 45

Sol^n.(c) $x - 2 = 2^{\frac{2}{3}} - 2^{\frac{1}{3}}$

cubing both side

$$x^3 - 8 - 3(x)(2).(x-2) = 2^2 - 2^1 - 3.2^{\frac{2}{3}}.2^{\frac{1}{3}}\left(2^{\frac{2}{3}} - 2^{\frac{1}{3}}\right)$$

$$x^3 - 8 - 6x(x-2) = 4 - 2 - 3 \times 2\,(x-2)$$
$$x^3 - 8 - 6x^2 + 12x = 2 - 6x + 12$$
$$x^3 - 8 - 6x^2 + 12x + 6x - 14 = 0$$
$$x^3 - 6x^2 + 18x = 22$$
$$x^3 - 6x^2 + 18x + 18 = 22 + 18$$
$$x^3 - 6x^2 + 18x + 18 = 40$$

Ex. 42 If $x - y = \frac{x+y}{9} = \frac{xy}{6}$ then value of $xy = ?$

(a) $\frac{4}{3}$ (b) $\frac{9}{5}$

(c) $\frac{9}{10}$ (d) $\frac{1}{3}$

Sol^n. (b) $x - y = \frac{x+y}{9} = \frac{xy}{6} = k$ (let)

$x - y = k$

$x + y = 9k$

$2x = 10k$ (adding both equations)

$\Rightarrow x = 5k$

$\Rightarrow y = 4k$

$\therefore \frac{xy}{6} = k \Rightarrow \frac{5k.4k}{6} = k$

$10k = 3 \Rightarrow k = \frac{3}{10}$

$\Rightarrow xy = 6k = \frac{9}{5}$

Ex. 43 If the ratio of roots of equation $lx^2 + nx + n = 0$ is $p : q$ then find the value $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = ?$

(a) 1 (b) 2

(c) 3 (d) 0

Sol^n.(d) $lx^2 + nx + n = 0$ let roots are α & β

$\Rightarrow \alpha + \beta = \frac{-n}{l}$ & $\alpha\beta = \frac{n}{l}$

$\Rightarrow \frac{\alpha}{\beta} = \frac{p}{q}$ (given)

$\Rightarrow \sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = \sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\alpha\beta}$

$$= \frac{\alpha + \beta + \alpha\beta}{\sqrt{\alpha\beta}} = \frac{\frac{-n}{l} + \frac{n}{l}}{\sqrt{\alpha\beta}} = \frac{0}{\sqrt{\alpha\beta}} = 0$$

BASIC IDENTITY

1. $(a + b)^2 = a^2 + 2ab + b^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $a^2 - b^2 = (a + b)(a - b)$
4. $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
5. $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
6. $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$
7. $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$
8. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
9. $(a + b + c)^3 = a^3 + b^3 + c^3 + 3(a + b)(b + c)(c + a)$

Type – 1 (Based on Formulas)

Ex.44 If $a + b + c = 6$,
$a^2 + b^2 + c^2 = 16$, find $ab + bc + ca = ?$

Sol^n. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$36 = 16 + 2(ab + bc + ca)$

$\Rightarrow ab + bc + ca = 10$

Ex. 45 $a + b + c = 3$, $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 2$
$a^2 + b^2 + c^2 = 6$, find $abc = ?$

Solⁿ. $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=2$

$ab + bc + ca = 2abc$

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$9 = 6 + 2\,(2abc)$

$\Rightarrow 3 = 4abc \Rightarrow abc = \frac{3}{4}$

Ex. 46 If $a^3 + b^3 = 0$ find $a + b = ?$

(i) $\sqrt{3ab}$ (ii) $\sqrt{2ab}$

(iii) $3ab$ (iv) $\sqrt{4ab}$

Solⁿ.

$\Rightarrow (a+b)^3 = a^3 + b^3 + 3ab(a+b) = 0 + 3ab(a+b)$

$(a+b)^2 = 3ab$

$a + b = \sqrt{3ab}$

Ex. 47 If $a^4 + b^4 = a^2b^2$ find $a^6 + b^6$.

Solⁿ. $a^6 + b^6 = (a^2)^3 + (b^2)^3$

$= (a^2 + b^2)(a^4 + b^4 - a^2b^2)$

$= (a^2 + b^2)(a^2b^2 - a^2b^2)$

$= 0$

Ex. 48 If $\frac{p}{a}+\frac{q}{b}+\frac{r}{c}=1$ & $\frac{a}{p}+\frac{b}{q}+\frac{c}{r}=0$

find $\frac{p^2}{a^2}+\frac{q^2}{b^2}+\frac{r^2}{c^2}=?$

Solⁿ. let $x=\frac{p}{a}$, $y=\frac{q}{b}$ & $z=\frac{r}{c}$

then, $x + y + z = 1$

& $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0 \Rightarrow xy + yz + xz = 0$

$\Rightarrow$ $(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + xz)$

$1 = x^2 + y^2 + z^2 + 2(0)$

$x^2 + y^2 + z^2 = 1$

hence, $\frac{p^2}{a^2}+\frac{q^2}{b^2}+\frac{r^2}{c^2}=1$

Ex. 49 Given $x + y = 2z$, then

(a) $\frac{x}{x-z}+\frac{y}{y-z}=?$

(i) 0 (ii) 1

(iii) 2 (iv) 3

(b) $\frac{x}{x-z}+\frac{z}{y-z}=?$

(i) 1 (ii) 2

(iii) 3 (iv) 4

Solⁿ. (a) $x + y = z + z$

$x - z = z - y$

$\Rightarrow \frac{x}{z-y}+\frac{y}{y-z}=\frac{x}{z-y}-\frac{y}{z-y}$

$\Rightarrow \frac{x-y}{z-y}=\frac{2z-y-y}{z-y}$

$=\frac{2(z-y)}{(z-y)}=2$

Or

Let $x = 0$ then $y = 2z$

(a) $\frac{x}{x-z}+\frac{y}{y-z}=0+\frac{2z}{2z-z}=2$

(b) $\frac{x}{x-z}+\frac{z}{y-z}=0+\frac{z}{2z-z}=1$

Ex. 50 Given $x+\frac{1}{y}=1$ and $y+\frac{1}{z}=1$ find

(1) $z+\frac{1}{x}=?$ (2) $xyz = ?$

(3) $(x+y+z)+\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)=?$

Solⁿ. $\because$ $x=1-\frac{1}{y}=\frac{y-1}{y} \Rightarrow \frac{1}{x}=\frac{y}{y-1}$

& $\frac{1}{z}=1-y \Rightarrow z=\frac{1}{1-y}$

$\therefore$ **(1)** $z+\dfrac{1}{x}=\dfrac{1}{1-y}+\dfrac{y}{y-1}=\dfrac{1-y}{1-y}=1$

$\therefore$ **(2)** $xyz=\dfrac{(y-1)}{y}\times y\times\dfrac{1}{(1-y)}=-1$

$\therefore$ **(3)** $(x+y+z)+\left(\dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}\right)$

$=\left(x+\dfrac{1}{y}\right)+\left(y+\dfrac{1}{z}\right)+\left(z+\dfrac{1}{x}\right)$

$=1+1+1=3$

Ex. 51 $\left(x+\dfrac{1}{y}\right)=\left(y+\dfrac{1}{z}\right)=\left(z+\dfrac{1}{x}\right)$ and $(x\neq y\neq z)$ find $xyz=?$

(i) 1 (ii) –1
(iii) ±1 (iv) 3

Sol^n. $x-y\neq 0,\ y-z\neq 0$ & $z-x\neq 0$

$$x-y=\frac{1}{z}-\frac{1}{y}=\frac{y-z}{zy} \quad .....(1)$$

$$y-z=\frac{1}{x}-\frac{1}{z}=\frac{z-x}{xz} \quad ....(2)$$

$$z-x=\frac{1}{y}-\frac{1}{x}=\frac{x-y}{xy} \quad ....(3)$$

Multiply equation (1), (2) & (3)

$$(x-y)(y-z)(z-x)=\frac{(y-z)}{yz}\times\frac{(z-x)}{zx}\times\frac{(x-y)}{xy}$$

$$1=\frac{1}{(xyz)^2}$$

$\Rightarrow xyz=\pm 1$

Ex. 52 If $\dfrac{a-b}{c}+\dfrac{b+c}{a}+\dfrac{c-a}{b}=1$ and $(b+c-a\neq 0)$ then which relation will be true?

(i) $\dfrac{1}{a}=\dfrac{1}{b}+\dfrac{1}{c}$ (ii) $\dfrac{1}{b}=\dfrac{1}{a}+\dfrac{1}{c}$

(iii) $\dfrac{1}{c}=\dfrac{1}{a}+\dfrac{1}{b}$ (d) None of these

Sol^n. $\dfrac{a-b}{c}+\dfrac{c-a}{b}=1-\dfrac{b+c}{a}$

$\Rightarrow \dfrac{ab-b^2+c^2-ac}{bc}=\dfrac{a-b-c}{a}$

$\Rightarrow \dfrac{a(b-c)-(b^2-c^2)}{bc}=\dfrac{a-b-c}{a}$

$\Rightarrow \dfrac{a(b-c)-(b-c)(b+c)}{bc}=\dfrac{(a-b-c)}{a}$

$\Rightarrow \dfrac{(b-c)(a-b-c)}{bc}=\dfrac{(a-b-c)}{a}$

$\Rightarrow \dfrac{b-c}{bc}=\dfrac{1}{a}\Rightarrow\dfrac{1}{c}=\dfrac{1}{a}+\dfrac{1}{b}$

Method 2 : Subtract 1 from Ist & IInd term and Add 1 to IIIrd term because we want to make $b+c-a$ in numerator.

$\Rightarrow \dfrac{a-b}{c}-1+\dfrac{b+c}{a}-1+\dfrac{c-a}{b}+1=1-1-1+1$

$\Rightarrow \dfrac{a-b-c}{c}+\dfrac{b+c-a}{a}+\dfrac{c-a+b}{b}=0$

$\Rightarrow -\dfrac{b+c-a}{c}+\dfrac{b+c-a}{a}+\dfrac{b+c-a}{b}=0$

$\Rightarrow (b+c-a)\left(-\dfrac{1}{c}+\dfrac{1}{a}+\dfrac{1}{b}\right)=0$

$\Rightarrow \left(-\dfrac{1}{c}+\dfrac{1}{a}+\dfrac{1}{b}\right)=0 \quad (\because b+c-a\neq 0)$

$\Rightarrow \dfrac{1}{c}=\dfrac{1}{a}+\dfrac{1}{b}$

Ex. 53 Given $pq+qr+rp=0$ find

(1) $\dfrac{1}{p^2-qr}+\dfrac{1}{q^2-rp}+\dfrac{1}{r^2-pq}=?$

(2) $\dfrac{p^2}{p^2-qr}+\dfrac{q^2}{q^2-rp}+\dfrac{r^2}{r^2-pq}=?$

Sol^n. (1) $-qr=pq+rp,\ -rp=pq+qr$ & $-pq=qr+rp$

$\Rightarrow \dfrac{1}{p^2-qr}+\dfrac{1}{q^2-rp}+\dfrac{1}{r^2-pq}$

$$=\frac{1}{p^2+pq+rp}+\frac{1}{q^2+pq+qr}+\frac{1}{r^2+qr+rp}$$

$$=\frac{1}{p(p+q+r)}+\frac{1}{q(p+q+r)}+\frac{1}{r(p+q+r)}$$

$$=\frac{1}{p+q+r}\left[\frac{1}{p}+\frac{1}{q}+\frac{1}{r}\right]$$

$$=\frac{1}{p+q+r}\left[\frac{pq+qr+rp}{pqr}\right]$$

$$=\frac{1}{p+q+r}(0)=0$$

Sol^n.(2) $\frac{p^2}{p^2-qr}+\frac{q^2}{q^2-rp}+\frac{r^2}{r^2-pq}$

$$=\frac{p^2}{p^2+pq+rp}+\frac{q^2}{q^2+pq+qr}+\frac{r^2}{r^2+qr+rp}$$

$$=\frac{p^2}{p(p+q+r)}+\frac{q^2}{q(p+q+r)}+\frac{r^2}{r(p+q+r)}$$

$$=\frac{p+q+r}{p+q+r}=1$$

<u>Method:2</u> $\because pq+qr+rp=0 \Rightarrow \frac{1}{p}+\frac{1}{q}+\frac{1}{r}=0$

we can put $p=q=-2$ & $r=1$

(1). $\frac{1}{p^2-qr}+\frac{1}{q^2-rp}+\frac{1}{r^2-pq}$

$$=\frac{1}{(-2)^2-(-2)(1)}+\frac{1}{(-2)^2-(1)(-2)}+\frac{1}{(1)^2-(-2)(-2)}$$

$$=\frac{1}{6}+\frac{1}{6}+\frac{1}{-3}=0$$

(2). $\frac{p^2}{p^2-qr}+\frac{q^2}{q^2-rp}+\frac{r^2}{r^2-pq}$

$$=\frac{(-2)^2}{(-2)^2-(-2)(1)}+\frac{(-2)^2}{(-2)^2-(1)(-2)}+\frac{(1)^2}{(1)^2-(-2)(-2)}$$

$$=\frac{4}{6}+\frac{4}{6}+\frac{1}{-3}=\frac{2}{3}+\frac{2}{3}-\frac{1}{3}=1$$

Ex. 54 If $(x+y+z)^y=a^x$, $(x+y+z)^z=a^y$
$(x+y+z)^x=a^z$ then,

(a) only $(x+y+z)=a$

(b) $x=y=z=\frac{a}{3}$

(c) $x=y=z=2a$

(d) $x=y=z=a$

Sol^n. $(x+y+z)^y=a^x$ …(i)
$(x+y+z)^z=a^y$ …(ii)
$(x+y+z)^x=a^z$ …(iii)
multiply all three equtions –
$\Rightarrow (x+y+z)^{x+y+z}=a^{x+y+z}$
on comparing power
$\Rightarrow x+y+z=a$ .....(iv)
put $x+y+z=a$ in all three equation
$a^y=a^x \Rightarrow x=y$
$a^z=a^y \Rightarrow y=z$
$a^x=a^z \Rightarrow x=z$
Now, we can say
$x=y=z$ and put in equation (iv)
$\Rightarrow 3x=a$

$\Rightarrow x=\frac{a}{3} \Rightarrow x=y=z=\frac{a}{3}$

Ex. 55 If $x(x+y+z)=4$, $y(x+y+z)=16$ and $z(x+y+z)=29$ and x, y & z are positive numbers. find x, y & z = ?

Sol^n. $x(x+y+z)=4$ …(i)
$y(x+y+z)=16$ …(ii)
$z(x+y+z)=29$ …(iii)
Adding all three equations-
$(x+y+z)(x+y+z)=49$
$(x+y+z)^2=49$
$(x+y+z)=\pm 7$
taking +ve value
$x+y+z=7$

from equation (i), (ii) & (iii)

$7x = 4, \quad 7y = 16, \quad 7z = 29$

$\Rightarrow x = \frac{4}{7} \quad \Rightarrow y = \frac{16}{7} \quad \Rightarrow z = \frac{29}{7}$

Ex. 56 If $(x+y)^2 = 21 + z^2$, $(y+z)^2 = 32 + x^2$ and $(z+x)^2 = 28 + y^2$, find $x + y + z = ?$

Sol^n. $(x+y)^2 - z^2 = 21$

$\Rightarrow (x+y+z)(x+y-z) = 21 \quad ...(1)$

$(y+z)^2 - x^2 = 32$

$\Rightarrow (y+z+x)(y+z-x) = 32 \quad ...(2)$

$(z+x)^2 - y^2 = 28$

$\Rightarrow (z+x+y)(z+x-y) = 28 \quad ...(3)$

Add all three equations –

$(x+y+z)\ [(x+y-z) + (y+z-x) + (z+x-y)] = 81$

$(x+y+z)^2 = 81$

$\Rightarrow x + y + z = \pm 9$

Type – 2

(A). Given: $x + \frac{1}{x} = 2$

Conclusion: $x = 1$

Proof: $\frac{x^2+1}{x} = 2$

$\Rightarrow \quad x^2 + 1 = 2x$

$\Rightarrow \quad x^2 - 2x + 1$

$\Rightarrow \quad (x-1)^2 = 0$

$\Rightarrow \quad x = 1$

Ex. 57 If $x + \frac{1}{x} = 2$, find

(i) $x^{11} + \frac{1}{x^{11}} = ?$

(ii) $x^{112} - \frac{1}{x^{112}} = ?$

Sol^n.

(i) $x^{11} + \frac{1}{x^{11}} = (1)^{11} + \frac{1}{(1)^{11}} = 2$

(ii) $x^{112} - \frac{1}{x^{112}} = (1)^{112} - \frac{1}{(1)^{112}} = 0$

Ex. 58 If $m + \frac{1}{m-2} = 4$, find

(i) $(m-2)^{111} + \frac{1}{(m-2)^{111}} = ?$

(ii) $m^2 + m + 1 = ?$

Sol^n. $(m-2) + \frac{1}{(m-2)} = 2$

then $(m-2) = 1 \Rightarrow m = 3$

(i) $(m-2)^{111} + \frac{1}{(m-2)^{111}} =$

$(1)^{111} + \frac{1}{(1)^{111}} = 2$

(ii) $m^2 + m + 1 = (3)^2 + 3 + 1 = 13$

Ex. 59 If $m + \frac{1}{m+2} = 0$, find

(i) $(m+2)^{112} + \frac{1}{(m+2)^{112}} = ?$

(ii) $m^4 + m^3 + m^2 + m + 1 = ?$

Sol^n. $(m+2) + \frac{1}{(m+2)} = 2$

then $(m+2) = 1 \Rightarrow m = -1$

(i) $(m+2)^{112} + \frac{1}{(m+2)^{112}} = (1)^{112} + \frac{1}{(1)^{112}} = 2$

(ii) $m^4 + m^3 + m^2 + m + 1 = (-1)^4 + (-1)^3 + (-1)^2 - 1 + 1$

$= 1 - 1 + 1 - 1 + 1 = 1$

(B). Given: $x + \frac{1}{x} = -2$

Conclusion: $x = -1$

Proof: $\frac{x^2+1}{x} = -2x$

$\Rightarrow \quad x^2 + 1 = -2x$

$\Rightarrow \quad x^2 + 2x + 1 = 0 \qquad \Rightarrow \quad (x+1)^2 = 0$

$\Rightarrow \quad x = -1$

Ex. 60 If $x+\frac{1}{x}=-2$, find

(i) $x^{11}+\frac{1}{x^{11}}=?$ **(ii)** $x^{112}+\frac{1}{x^{112}}=?$

(iii) $x^{112}-\frac{1}{x^{113}}=?$ **(iv)** $x^{11}+\frac{1}{x^{12}}=?$

Sol^n^. (i) $x^{11}+\frac{1}{x^{11}}=(-1)^{11}+\frac{1}{(-1)^{11}}=-1-1=-2$

(ii) $x^{112}+\frac{1}{x^{112}}=(-1)^{112}+\frac{1}{(-1)^{112}}=1+1=2$

(iii) $x^{112}-\frac{1}{x^{113}}=(-1)^{112}-\frac{1}{(-1)^{113}}=1+1=2$

(iv) $x^{11}+\frac{1}{x^{12}}=(-1)^{11}+\frac{1}{(-1)^{12}}=-1+1=0$

Ex. 61 If $m+\frac{1}{m+2}=-4$, find

(i) $(m+2)^{111}+\frac{1}{(m+2)^{111}}=?$

(ii) $m^2+m+1=?$

Sol^n^. $(m+2)+\frac{1}{(m+2)}=-2 \Rightarrow (m+2)=-1$

$\Rightarrow m=-3$

(i) $(m+2)^{111}+\frac{1}{(m+2)^{111}}=(-1)^{111}+\frac{1}{(-1)^{111}}$

$=-1-1=-2$

(ii) $m^2+m+1=(-3)^2+(-3)+1=9-3+1=7$

Ex. 62 If $m+\frac{1}{m-2}=0$, find

(i) $(m-2)^{12}+\frac{1}{(m-2)^{11}}=?$

(ii) $m^5+m^4+m^3+m^2+m+1=?$

Sol^n^. $(m-2)+\frac{1}{(m-2)}=-2 \Rightarrow m-2=-1$

$\Rightarrow m=1$

(i) $(m-2)^{12}+\frac{1}{(m-2)^{11}}=(-1)^{12}+\frac{1}{(-1)^{11}}$

$=1-1=0$

(ii) $m^5+m^4+m^3+m^2+m+1$

$=(1)^5+(1)^4+(1)^3+(1)^2+1+1$

$=1+1+1+1+1+1$

$=6$

(C). Given: $x+\frac{1}{x}=1$

Conclusion:

(a) $x^3=-1$

(b) if the difference of power of two terms of x is 3 then sum of both terms will be zero.

eg. $x^{12}+x^9=0, x^{10}+x^7=0$

Proof: $\frac{x^2+1}{x}=1$

$\Rightarrow x^2+1=x$ .....(i)

Multiplying both side by x

$\Rightarrow x^3+x=x^2$

$\Rightarrow x^3=x^2-x$ [$\because x^2-x=-1$ from eq^n^(i)]

$\Rightarrow x^3=-1$

$\Rightarrow x^3+1=0$

$\Rightarrow x^3+x^0=0$

(difference between power is 3 and sum is 0)

Ex. 63 If $x+\frac{1}{x}=1$, find

(i) $x^9+\frac{1}{x^9}=?$

$=(x^3)^3+\frac{1}{(x^3)^3}=(-1)^3+\frac{1}{(-1)^3}=-1-1=-2$

(ii) $x^{12}+\frac{1}{x^{12}}=?$

$=(x^3)^4+\frac{1}{(x^3)^4}=(-1)^4+\frac{1}{(-1)^4}=1+1=2$

(iii) $x^{23}+\dfrac{1}{x^{23}}=?$

$=\dfrac{x^{24}}{x}+\dfrac{x}{x^{24}}=\dfrac{(x^3)^8}{x}+\dfrac{x}{(x^3)^8}$

$=\dfrac{(-1)^8}{x}+\dfrac{x}{(-1)^8}=\dfrac{1}{x}+x=1$ (given)

(iv) $x^{10}+\dfrac{1}{x^{10}}=?$

$= x.x^9+\dfrac{1}{x.x^9}=x.(x^3)^3+\dfrac{1}{x.(x^3)^3}$

$=x(-1)^3+\dfrac{1}{x(-1)^3}=-x-\dfrac{1}{x}$

$=-\left(x+\dfrac{1}{x}\right)=-1$ (given)

Ex. 64 If $x+\dfrac{1}{x}=1$, find

(a) $x^{103}+x^{100}+x^{90}+x^{87}+x^{50}+x^{47}+x^9+x^6+x^3+3$

$=\overset{3}{\overbrace{x^{103}+x^{100}}}+\overset{3}{\overbrace{x^{90}+x^{87}}}+\overset{3}{\overbrace{x^{50}+x^{47}}}+\overset{3}{\overbrace{x^9+x^6}}+x^3+3$

$=0+0+0+0+(-1)+3=2$

(b) $x^{82}+x^{68}+x^{55}+x^{35}+x^{30}+x^{27}+x^{21}+x^3+2$

$=x.(x^3)^{27}+\dfrac{(x^3)^{23}}{x}+x.(x^3)^{18}+\dfrac{x^{36}}{x}+0+(x^3)^7+(-1)+2$

$=-x-\dfrac{1}{x}+x+\dfrac{1}{x}-1-1+2=0$

(D). Given: $x+\dfrac{1}{x}=-1$

Conclusion: $x^3=1$

Ex. 65 If $a+\dfrac{1}{a}=-1$ find $a^3+3=?$

Sol^n. $\Rightarrow a^3+3$

$=1+3$

$=4$

Ex. 66 If $x+\dfrac{1}{x}=-1$ find

(1) $x^{12}+\dfrac{1}{x^{12}}=?$ (2) $x^{27}+\dfrac{1}{x^{27}}=?$

(3) $x^{23}+\dfrac{1}{x^{23}}=?$ (4) $x^{10}+\dfrac{1}{x^{10}}=?$

(5) $x^{51}+x^{45}+x^{21}+x^{15}+x^3+2=?$

Sol^n. (1) $x^{12}+\dfrac{1}{x^{12}}=(x^3)^4+\dfrac{1}{(x^3)^4}==1+1=2$

(2) $x^{27}+\dfrac{1}{x^{27}}=(x^3)^9+\dfrac{1}{(x^3)^9}=1+1=2$

(3) $x^{23}+\dfrac{1}{x^{23}}=\dfrac{x^{24}}{x}+\dfrac{x}{x^{24}}=\dfrac{(x^3)^8}{x}+\dfrac{x}{(x^3)^8}$

$=\dfrac{1}{x}+x=-1$

(4) $x^{10}+\dfrac{1}{x^{10}}=x.x^9+\dfrac{1}{x.x^9}=x.(x^3)^3+\dfrac{1}{(x^3)^3}$

$=x+\dfrac{1}{x}=-1$

(5) $x^{51}+x^{45}+x^{21}+x^{15}+x^3+2$

$z=(x^3)^{17}+(x^3)^{15}+(x^3)^7+(x^3)^5+x^3+2$

$=1+1+1+1+1+2=7$

(E) Given: $x+\dfrac{1}{x}=\sqrt{3}$ or $\left(x+\dfrac{1}{x}\right)^2=3$ or $\left(x^2+\dfrac{1}{x^2}\right)=1$

Conclusion: (a) $x^6 = -1$

(b) If the difference of power of two terms of x is 6 then sum of both terms will be zero.

Ex. 67 $x^{18} + x^{12} = 0, x^{20} + x^{14} = 0$

Proof: cubing both sides: $\left(x + \frac{1}{x}\right)^3 = 3\sqrt{3}$

$$\Rightarrow \quad x^3 + \frac{1}{x^3} + 3.x.\frac{1}{x}\left(x + \frac{1}{x}\right) = 3\sqrt{3}$$

$$\Rightarrow \quad x^3 + \frac{1}{x^3} + 3\sqrt{3} = 3\sqrt{3}$$

$$\Rightarrow \quad x^3 + \frac{1}{x^3} = 0$$

$$\Rightarrow \quad x^6 + 1 = 0$$

$$\Rightarrow x^6 = -1$$

$$\Rightarrow \quad x^6 + x^0 = 0$$

Ex. 68 If $x + \frac{1}{x} = \sqrt{3}$ find

(1) $x^{506} + x^{500} + x^{206} + x^{200} + x^{100} + x^{94} + x^{50} + x^{44} + x^{18} + x^{12} + x^6 + 3$

$= 0 + 0 + 0 + 0 + 0 + (-1) + 3 = 2$

(2) $x^{67} + x^{53} + x^{43} + x^{29} + x^{24} + x^{18} + x^6 + 3$

$$= x.x^{66} + \frac{x^{54}}{x} + x.x^{42} + \frac{x^{30}}{x} + 0 + x^6 + 3$$

$$= x.\left(x^6\right)^{11} + \frac{\left(x^6\right)^9}{x} + x.\left(x^6\right)^7 + \frac{\left(x^6\right)^5}{x} - 1 + 3$$

$$= -x - \frac{1}{x} - x - \frac{1}{x} + 2$$

$$= -\left(x + \frac{1}{x}\right) - \left(x + \frac{1}{x}\right) + 2$$

$$= -\sqrt{3} - \sqrt{3} + 2 \; = 2\left(1 - \sqrt{3}\right)$$

(3) $x^{54} + x^{30} + x^{18} + x^6 + 4$

$$= \left(x^6\right)^9 + \left(x^6\right)^5 + \left(x^6\right)^3 - 1 + 4$$

$$= -1 - 1 - 1 - 1 + 4 = 0$$

Ex. 69 If $x + \frac{1}{x} = \sqrt{3}$, find

(1) $x^{102} + \frac{1}{x^{102}} = \left(x^6\right)^{17} + \frac{1}{\left(x^6\right)^{17}} = -1 - 1 = -2$

(2) $x^{48} + \frac{1}{x^{48}} = \left(x^6\right)^8 + \frac{1}{\left(x^6\right)^8} = 1 + 1 = 2$

(3) $x^{17} + \frac{1}{x^{17}} = \frac{x^{18}}{x} + \frac{x}{x^{18}} = \frac{\left(x^6\right)^3}{x} + \frac{x}{\left(x^6\right)^3}$

$$= \frac{-1}{x} - x = -\left(x + \frac{1}{x}\right) = -\sqrt{3}$$

(4) $x^{25} + \frac{1}{x^{25}} = x.x^{24} + \frac{1}{x.x^{24}} = x.\left(x^6\right)^4 + \frac{1}{x.\left(x^6\right)^4}$

$$= x + \frac{1}{x} = \sqrt{3}$$

(5) $x^{26} + \frac{1}{x^{26}} = x.^2 x^{24} + \frac{1}{x^2.x^{24}}$

$$= x^2.\left(x^6\right)^4 + \frac{1}{x^2.\left(x^6\right)^4} = x^2 + \frac{1}{x^2} = 1$$

(6) $x^{117} + \frac{1}{x^{117}} = x^3.x^{114} + \frac{1}{x^3.x^{114}}$

$$= x^3.\left(x^6\right)^{19} + \frac{1}{x^3.\left(x^6\right)^{19}} = -x^3 - \frac{1}{x^3}$$

$$= -\left[\frac{x^6 + 1}{x^3}\right] = -\left[\frac{-1 + 1}{x^3}\right] = 0$$

Or

$$= \frac{x^{234} + 1}{x^{117}} = \frac{\left(x^6\right)^{39} + 1}{x^{117}} = \frac{-1 + 1}{x^{117}} = 0$$

(F). **Given:** $x + \frac{1}{x} = -\sqrt{3}$

Conclusion:

(a) $x^6 = -1$

(b) if the difference of power of two terms of x is 6 then sum of both terms will be zero.

e.g. $x^{18} + x^{12} = 0, x^{20} + x^{14} = 0$

Ex. 70 If $x+\frac{1}{x}=-\sqrt{3}$ find

(1) $x^{17}+\frac{1}{x^{17}}=\frac{x^{18}}{x}+\frac{x}{x^{18}}=\frac{(x^6)^3}{x}+\frac{x}{(x^6)^3}=-\frac{1}{x}-x$

$=-\left(x+\frac{1}{x}\right)=\sqrt{3}$

(2) $x^{25}+\frac{1}{x^{25}}=x.x^{24}+\frac{1}{x.x^{24}}=x.(x^6)^4+\frac{1}{x.(x^6)^4}$

$=x+\frac{1}{x}=-\sqrt{3}$

(G) Given: $x+\frac{1}{x}=a$

(then small power of x will be asked.)

Ex. 71 If $x+\frac{1}{x}=3$, then

(1) $x^2+\frac{1}{x^2}=?$

$\Rightarrow x+\frac{1}{x}=3$, squaring both side

$\Rightarrow \left(x+\frac{1}{x}\right)^2=(3)^2$

$\Rightarrow x^2+\frac{1}{x^2}+2.x.\frac{1}{x}=9$

$\Rightarrow x^2+\frac{1}{x^2}=9-2$

$\Rightarrow x^2+\frac{1}{x^2}=7$

(2) $x^3+\frac{1}{x^3}=?$

$\Rightarrow x+\frac{1}{x}=3$, cubing both side

$\Rightarrow \left(x+\frac{1}{x}\right)^3=(3)^3$

$\Rightarrow x^3+\frac{1}{x^3}+3.x.\frac{1}{x}\left(x+\frac{1}{x}\right)=27$

$\Rightarrow x^3+\frac{1}{x^3}+3(3)=27$

$\Rightarrow x^3+\frac{1}{x^3}=27-9$

$\Rightarrow x^3+\frac{1}{x^3}=18$

(3) $x^4+\frac{1}{x^4}=?$

$\Rightarrow x^2+\frac{1}{x^2}=7$, squaring both side

$\Rightarrow x^4+\frac{1}{x^4}=(7)^2-2 \Rightarrow x^4+\frac{1}{x^4}=47$

(4) $x^5+\frac{1}{x^5}=?$

$\Rightarrow \left(x^2+\frac{1}{x^2}\right)\left(x^3+\frac{1}{x^3}\right)=7\times18$

$\Rightarrow x^5+\frac{1}{x}+x+\frac{1}{x^5}=126$

$\Rightarrow x^5+\frac{1}{x^5}=126-\left(\frac{1}{x}+x\right)=126-3$

$\Rightarrow x^5+\frac{1}{x^5}=123$

(5) $x^6+\frac{1}{x^6}=?$

$\Rightarrow x^3+\frac{1}{x^3}=18$, squaring both side

$\Rightarrow x^6+\frac{1}{x^6}=(18)^2-2=324-2$

$\Rightarrow x^6+\frac{1}{x^6}=322$

Ex. 72 If $\frac{x^2-1}{x}=\sqrt{5}$ and x is positive number find

$$\left(x^2+\frac{1}{x^2}\right)\left(x+\frac{1}{x}\right)=?$$

Sol^n^. $x-\frac{1}{x}=\sqrt{5}$, squaring both side

$$\Rightarrow x^2+\frac{1}{x^2}-2=5$$

$$\Rightarrow x^2+\frac{1}{x^2}=7$$

Now adding 2 both side, $x^2+\frac{1}{x^2}+2=9$

$$\Rightarrow \left(x+\frac{1}{x}\right)^2=9 \Rightarrow x+\frac{1}{x}=3$$

$$\Rightarrow \left(x^2+\frac{1}{x^2}\right)\left(x+\frac{1}{x}\right)=7\times3=21$$

Ex.73 If $x^4+\frac{1}{x^4}=322$ find $x^3-\frac{1}{x^3}=?$

Sol^n^. $x^4+\frac{1}{x^4}+2=324$

$$\Rightarrow \left(x^2+\frac{1}{x^2}\right)^2=324 \Rightarrow x^2+\frac{1}{x^2}=18$$

$\Rightarrow$ Now, subtracting 2 from both side

$$x^2+\frac{1}{x^2}-2=16$$

$$\Rightarrow \left(x-\frac{1}{x}\right)^2=16 \Rightarrow x-\frac{1}{x}=4$$

Now, cubing both side $\left(x-\frac{1}{x}\right)^3=64$

$$\Rightarrow x^3-\frac{1}{x^3}-3x.\frac{1}{x}\left(x-\frac{1}{x}\right)=64$$

$$\Rightarrow x^3-\frac{1}{x^3}-3(4)=64$$

$$\Rightarrow x^3-\frac{1}{x^3}=64+12=76$$

Ex. 74(A) If $(x-a)(x-b)=1$ & $a-b+5=0$ find

$$(x-a)^3-\frac{1}{(x-a)^3}=?$$

(a) 125 (b) –125
(c) 0 (d) 140

Sol^n^.(d) $(x-a)(x-b)=1 \Rightarrow (x-b)=\frac{1}{(x-a)}$

$$\Rightarrow (x-a-5)=\frac{1}{(x-a)} \quad (\because a+5=b)$$

$$\Rightarrow (x-a)-\frac{1}{(x-a)}=5 \quad \text{(cubing both sides)}$$

$$\Rightarrow (x-a)^3-\frac{1}{(x-a)^3}-3(5)=125$$

$$\Rightarrow (x-a)^3-\frac{1}{(x-a)^3}=140$$

Method:-2 Questions and Answer is independent of 'b' so we can put $b=0$. Now question will be $(x-a)(x)=1$ & $a+5=0$ find

$$(x-a)^3-\frac{1}{(x-a)^3}=?$$

Sol^n^. $(x-a)(x)=1 \Rightarrow (x-a)=\frac{1}{x} \Rightarrow 5=\frac{1}{x}-x$

$$\Rightarrow (x-a)^3-\frac{1}{(x-a)^3}$$

$$=\frac{1}{x^3}-x^3=\left(\frac{1}{x}-x\right)^3+3.x.\frac{1}{x}\left(\frac{1}{x}-x\right)$$

$$=(5)^3+3(5)$$

$$=140$$

Ex. 74(B) If $x^2 + x = 5$ then, find the value of

$$(x+3)^3 + \frac{1}{(x+3)^3} = ?$$

Sol^n. Adding $(2x + 1)$ both side

$$x^2 + x + 2x + 1 = 5 + 2x + 1$$

$$x^2 + 3x + 1 = 6 + 2x$$

$$x(x+3) + 1 = 2(x+3)$$

divide by $(x + 3)$ both side

$$x + \frac{1}{x+3} = 2 \text{ (Adding 3 to both sides)}$$

$$(x+3) + \frac{1}{(x+3)} = 2 + 3 = 5$$

Cubing both sides

$$(x+3)^3 + \frac{1}{(x+3)^3} + 3.(5) = 5^3$$

$$(x+3)^3 + \frac{1}{(x+3)^3} = 125 - 15 = 110$$

Type – 3

Think that if $x + y = 0$ then either x or y should be negative but if $x^2 + y^2 = 0$ then both x & y should be zero because neither x nor y can be negative.

So, if **Given** : $x^2 + y^2 + z^2 = 0$ then

Conclusion : $x = y = z = 0$

Or

Given : $(x - a)^2 + (y - b)^2 + (z - c)^2 = 0$ then

Conclusion : $x = a$, $y = b$ & $z = c$

Ex. 75 If $(x - 1)^2 + (y - 2)^2 = 0$ then $x + y = ?$

Sol^n. $(x - 1)^2 + (y - 2)^2 = 0$

$\Downarrow \quad \Downarrow$

$0 \quad 0$

$x = 1 \quad y = 2$

$\Rightarrow \quad x + y = 1 + 2 = 3$

Ex. 76 If $(a - 2)^2 + (b - 3)^2 + (c - 11)^2 = 0$ find $\sqrt{a+b+c} = ?$

(i) 4 (ii) –4

(iii) ±4 (iv) 16

Sol^n. $(a - 2)^2 + (b - 3)^2 + (c - 11)^2 = 0$

$\Downarrow \quad \Downarrow \quad \Downarrow$

$0 \quad 0 \quad 0$

$a = 2 \quad b = 3 \quad c = 11$

$\Rightarrow \quad \sqrt{a+b+c} = \sqrt{2+3+11} = \sqrt{16} = 4$

Ex. 77 If $a^2 + b^2 + c^2 = 2(a - b + c) - 3$ then find $a - b + c = ?$

Sol^n. $a^2 + b^2 + c^2 - 2a + 2b - 2c + 3 = 0$

$(a^2 - 2a + 1) + (b^2 + 2b + 1) + (c^2 - 2c + 1) = 0$

$(a - 1)^2 + (b + 1)^2 + (c - 1)^2 = 0$

$\Downarrow \quad \Downarrow \quad \Downarrow$

$0 \quad 0 \quad 0$

$a = 1 \quad b = -1 \quad c = 1$

$\Rightarrow \quad a - b + c = 1 + 1 + 1 = 3$

Ex. 78 If $a^2 + b^2 + c^2 = 2(a + 2b - 2c) - 9$ then find $a + b + c = ?$

Sol^n. $a^2 + b^2 + c^2 - 2a - 4b + 4c + 9 = 0$

$(a^2 - 2a + 1) + (b^2 - 4b + 4) + (c^2 + 4c + 4) = 0$

$(a - 1)^2 + (b - 2)^2 + (c + 2)^2 = 0$

$\Downarrow \quad \Downarrow \quad \Downarrow$

$0 \quad 0 \quad 0$

$a = 1 \quad b = 2 \quad c = -2$

$\Rightarrow \quad a + b + c = 1 + 2 - 2 = 1$

Ex. 79 If $5x^2 + 4xy + y^2 + 2x + 1 = 0$ then find the value of x, y.

Sol^n. $(x^2 + 2x + 1) + (4x^2 + 4xy + y^2) = 0$

$(x + 1)^2 + (2x + y)^2 = 0$

$\Downarrow \quad \Downarrow$

$0 \quad 0$

$x = -1 \quad 2x = -y$ or $y = -2x$

$y = -2(-1) = 2$

$\Rightarrow \quad x = -1$ & $y = 2$

Ex. 80 If $x^2 + y^2 + z^2 + 2x + 4y + 5 = 0$, find $x^{12} + y + z^{30} = ?$

Sol^n. $(x^2 + 2x + 1) + (y^2 + 4y + 4) + z^2 = 0$

$\Downarrow \quad \Downarrow \quad \Downarrow$

$(x + 1)^2 \quad + \quad (y + 2)^2 \quad + \quad z^2$

$\Downarrow \quad \Downarrow \quad \Downarrow$

$0 \quad 0 \quad 0$

$x = -1 \quad y = -2 \quad z = 0$

$\Rightarrow \quad x^{12} + y + z^{30} = (-1)^{12} - 2 + 0 = 1 - 2$

$= -1$

Ex. 81 If $(x+y-z-1)^2+(z+x-y-2)^2+(z+y-x-4)^2 = 0$, then find $x+y+z=?$

Solⁿ. $(x+y-z-1)^2+(z+x-y-2)^2+(z+y-x-4)^2=0$

$\Downarrow \quad \Downarrow \quad \Downarrow$

$0 \quad 0 \quad 0$

$x+y-z=1 \quad z+x-y=2 \quad z+y-x=4$

Adding all three equations

$x+y-z+z+x-y+z+y-x=1+2+4$

$\Rightarrow x+y+z=7$

Type – 4

(A). **Formula:** $a^2+b^2+c^2-ab-bc-ca$

$$=\frac{1}{2}[(a-b)^2+(b-c)^2+(c-a)^2]$$

Ex. 82 If $a=297, b=298, c=299$ find $a^2+b^2+c^2-ab-bc-ca=?$

Solⁿ. $a^2+b^2+c^2-ab-bc-ca=\frac{1}{2}[(a-b)^2+(b-c)^2+(c-a)^2]$

$=\frac{1}{2}[(-1)^2+(-1)^2+(2)^2]=\frac{1}{2}\times 6=3$

(B). **Given** : $a^2+b^2+c^2=ab+bc+ca$

Conclusion: $a=b=c$

Proof : $a^2+b^2+c^2-ab-bc-ca=0$

$\Rightarrow \frac{1}{2}\left(2a^2+2b^2+2c^2-2ab-2bc-2ca\right)=0$

$\Rightarrow \frac{1}{2}\left[\left(a^2+b^2-2ab\right)+\left(b^2+c^2-2bc\right)+\left(c^2+a^2-2ca\right)\right]=0$

$\Downarrow \quad \Downarrow \quad \Downarrow$

$\Rightarrow \frac{1}{2}[(a-b)^2+(b-c)^2+(c-a)^2]=0$

$\Rightarrow [(a-b)^2+(b-c)^2+(c-a)^2]$

$\Downarrow \quad \Downarrow \quad \Downarrow$

$0 \quad 0 \quad 0$

$a=b \quad b=c \quad c=a$

$\Rightarrow a=b=c$

Ex. 83 If $a^2+b^2+c^2=ab+bc+ca$, then find $\frac{a+c}{b}=?$

Solⁿ. If $a^2+b^2+c^2=ab+bc+ca$

$a=b=c$

$\Rightarrow \frac{a+c}{b}=\frac{a+a}{a}=\frac{2a}{a}=2$

Ex. 84 If $a^2+b^2+c^2=ab+bc+ca$ then

(1) $\frac{a+b}{c}+\frac{b+c}{a}+\frac{c+a}{b}=?$

Solⁿ. $a=b=c$

$\Rightarrow \frac{a+b}{c}+\frac{b+c}{a}+\frac{c+a}{b}=2+2+2=6$

(2) $\frac{a}{b+c}+\frac{b}{a+c}+\frac{c}{a+b}=?$

Solⁿ. $=\frac{a}{a+a}+\frac{a}{a+a}+\frac{a}{a+a}$

$=\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=\frac{3}{2}$

(3)

$\left(\frac{a+b}{c}+\frac{b+c}{a}+\frac{c+a}{b}\right)\left(\frac{a}{b+c}+\frac{b}{a+c}+\frac{c}{a+b}\right)=?$

Solⁿ. $=6\times\frac{3}{2}=9$

(C). **Given:** $a+b+c=0$

Conclusion: $a+b=-c, b+c=-a \,\&\, c+a=-b$

Ex. 85 If $a+b+c=0$, then $\frac{a+b}{c}=?$

Solⁿ. $\because a+b=-c$

$\frac{a+b}{c}=\frac{-c}{c}=-1$

Ex. 86 If $a+b+c=0$, then

(a) $\frac{a+b}{c}+\frac{b+c}{a}+\frac{c+a}{b}=?$

Solⁿ. $\frac{-c}{c}+\frac{-a}{a}+\frac{-b}{b}$

$=-1-1-1=-3$

(b) $\dfrac{a}{b+c}+\dfrac{b}{a+c}+\dfrac{c}{a+b}=?$

Sol^n. $\dfrac{a}{-a}+\dfrac{b}{-b}+\dfrac{c}{-c}$

$=-1-1-1=-3$

(c). $\left(\dfrac{a+b}{c}+\dfrac{b+c}{a}+\dfrac{c+a}{b}\right)\left(\dfrac{c}{a+b}+\dfrac{b}{a+c}+\dfrac{a}{b+c}\right)=?$

Sol. $-3\times-3=9$

Type – 5

(A). Formula: $a^3+b^3+c^3-3abc$

$=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$=\dfrac{1}{2}(a+b+c)\,[(a-b)^2+(b-c)^2+(c-a)^2]$

Ex. 87 If $a=b=333,\ c=334$ find $a^3+b^3+c^3-3abc$.

Sol^n. $a^3+b^3+c^3-3abc=\dfrac{1}{2}(a+b+c)[(a-b)^2+(b-c)^2+(c-a)^2]$

$=\dfrac{1}{2}(333+333+334)\,[(0)^2+(-1)^2+(1)^2]$

$=\dfrac{1}{2}\times1000\times2=1000$

Ex. 88 If $a=20, b=25, c=15$ find

$\dfrac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2-ab-bc-ca}=?$

Sol^n. $\dfrac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2-ab-bc-ca}$

$=\dfrac{(a+b+c)(a^2+b^2+c^2-ab-bc-ca)}{a^2+b^2+c^2-ab-bc-ca}$

$=(a+b+c)=20+25+15=60$

Ex. 89 If $a=25, b=15, c=-10$, then

$\dfrac{a^3+b^3+c^3-3abc}{(a-b)^2+(b-c)^2+(c-a)^2}$

Sol^n. $\dfrac{a^3+b^3+c^3-3abc}{(a-b)^2+(b-c)^2+(c-a)^2}$

$=\dfrac{\frac{1}{2}(a+b+c)\left[(a-b)^2+(b-c)^2+(c-a)^2\right]}{(a-b)^2+(b-c)^2+(c-a)^2}$

$=\dfrac{1}{2}(a+b+c)=\dfrac{1}{2}(25+15-10)$

$=15$

(B).

(1). If $a+b+c=0$ then, $a^3+b^3+c^3-3abc=0$ or $a^3+b^3+c^3=3abc$

(2). If $a^3+b^3+c^3=3abc$ then two cases are possible

(a) $a+b+c=0$

or (b) $a^2+b^2+c^2-ab-bc-ca=0 \Rightarrow a=b=c$

Ex. 90 If $a^3+b^3+c^3=3abc$ and a, b, c are positive numbers.

Which option is correct?

(a) $a+b+c=0$

(b) $a=b=c$

Sol^n. $a=b=c$ ($\because$ Sum of positive numbers can't be zero)

Ex. 91 If $a^3+b^3+c^3=3abc$ and a, b, c are distinct numbers.

Which option is correct?

(a) $a+b+c=0$

(b) $a=b=c$

Sol^n. $a+b+c=0$ ($\because$ Distinct numbers can't be equal)

Ex. 92 If $a+b+c=0$ find $\dfrac{a^2}{bc}+\dfrac{b^2}{ca}+\dfrac{c^2}{ab}=?$

Sol^n. $\dfrac{a^2}{bc}+\dfrac{b^2}{ca}+\dfrac{c^2}{ab}=\dfrac{a^3+b^3+c^3}{abc}=\dfrac{3abc}{abc}=3$

Or

Let value such that $a+b+c=0$

$a=-1, b=-1, c=2$

$\dfrac{a^2}{bc}+\dfrac{b^2}{ca}+\dfrac{c^2}{ab}=\dfrac{1}{-2}+\dfrac{1}{-2}+\dfrac{4}{1}=3$

Ex. 93 If $a+b+c=0$ find $a^3+b^3+c^3+3abc$.

Sol^n. If $a+b+c=0$ then $a^3+b^3+c^3=3abc$

Adding $3abc$ both sides

$a^3+b^3+c^3+3abc=3abc+3abc$

$=6abc$

Ex. 94 Find $(a-b)^3+(b-c)^3+(c-a)^3=?$

Sol^n. $\because$ $(a-b)+(b-c)+(c-a)=0$

$\therefore (a-b)^3+(b-c)^3+(c-a)^3$
$=3(a-b)(b-c)(c-a)$

Ex. 95 Find $\dfrac{(x^2-y^2)^3+(y^2-z^2)^3+(z^2-x^2)^3}{(x-y)^3+(y-z)^3+(z-x)^3}$

(a) $3(x+y)(y+z)(z+x)$
(b) $3(x+y)(y+z)(z-x)$
(c) $(x+y)(y+z)(z+x)$
(d) $9(x+y)(y+z)(z+x)$

Sol^n. $\because (x^2-y^2)+(y^2-z^2)+(z^2-x^2)=0$

$\therefore (x^2-y^2)^3+(y^2-z^2)^3+(z^2-x^2)^3$
$=3(x^2-y^2)(y^2-z^2)(z^2-x^2)$

$\because$ $(x-y)+(y-z)+(z-x)=0$

$\therefore$ $(x-y)^3+(y-z)^3+(z-x)^3$
$=3(x-y)(y-z)(z-x)$

$$\Rightarrow \frac{(x^2-y^2)^3+(y^2-z^2)^3+(z^2-x^2)^3}{(x-y)^3+(y-z)^3+(z-x)^3}$$

$$=\frac{3(x^2-y^2)(y^2-z^2)(z^2-x^2)}{3(x-y)(y-z)(z-x)}$$

$$=\frac{3(x+y)(x-y)(y+z)(y-z)(z+x)(z-x)}{3(x-y)(y-z)(z-x)}$$

$=(x+y)(y+z)(z+x)$

Ex. 96. What will be factors of $(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3$

(a) $(a+b)(a-b)$ (b) $(a+b)(a+b)$
(c) $(a-c)(a-c)$ (d) $(b-c)(b-c)$

Sol^n. $\because$ $(a^2-b^2)+(b^2-c^2)+(c^2-a^2)=0$

$\therefore (a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3=3(a^2-b^2)(b^2-c^2)(c^2-a^2)$
$=3(a-b)(a+b)(b+c)(b-c)(c-a)(c+a)$

$\therefore$ $(a+b)(a-b)$ is factor.

Ex. 97 If $x+y+z=2s$ find
$(s-x)^3+(s-y)^3+3(s-x)(s-y)z=?$

(a) z^3 (b) $-z^3$
(c) 0 (d) $2z^3$

Sol^n. $0=(s-x)+(s-y)+(-z)$

[$\therefore$ $a+b+c=0$ then $a^3+b^3+c^3-3abc=0$]

$(s-x)^3+(s-y)^3+(-z)^3-3(s-x)(s-y)(-z)=0$
$(s-x)^3+(s-y)^3-z^3+3(s-x)(s-y)z=0$
$(s-x)^3+(s-y)^3+3(s-x)(s-y)z=z^3$

Method-2 Answer is independent of 's' so we can take $s=0$ now question will be if $x+y+z=0$ then $(-x)^3+(-y)^3$
$+3(-x)(-y)z=-x^3-y^3+3xyz=?$

$\because x+y+z=0 \Rightarrow$ $x^3+y^3+z^3-3xyz=0$

$\Rightarrow z^3=-x^3-y^3+3xyz$

Ex. 98 If $a=1.21$, $b=2.23$ & $c=3.44$ find $a^3+b^3-c^3+3abc=?$

(a) $6ab$ (b) $6abc$
(c) 1 (d) 0

Sol^n. $\because a+b+(-c)=0$ then $a^3+b^3+(-c)^3-3ab(-c)=0$

$\Rightarrow a^3+b^3-c^3+3abc=0$

Ex. 99 If $a=5.431$, $b=2.121$ & $c=-7.552$ find $a^3+b^3+c^3+3abc=?$

Sol^n. $\because$ $a+b+c=0$ then $a^3+b^3+c^3=3abc$

Adding $3abc$ to both sides

$\Rightarrow a^3+b^3+c^3+3abc=3abc+3abc$
$=6\,abc$

Ex. 100 If $a^{\frac{1}{3}}+b^{\frac{1}{3}}=c^{\frac{1}{3}}$ then which option is correct?

(a) $a^3+b^3+c^3=3abc$
(b) $a+b+c=3abc$
(c) $(a+b-c)^3+27abc=0$
(d) $(a+b+c)^3+27abc=0$

Sol^n. $\Rightarrow a^{\frac{1}{3}}+b^{\frac{1}{3}}-c^{\frac{1}{3}}=0$

$$\Rightarrow \left(a^{\frac{1}{3}}\right)^3+\left(b^{\frac{1}{3}}\right)^3+\left(-c^{\frac{1}{3}}\right)^3=3\left(a^{\frac{1}{3}}\right)\left(b^{\frac{1}{3}}\right)\left(-c^{\frac{1}{3}}\right)$$

$a+b-c=3a^{\frac{1}{3}}b^{\frac{1}{3}}(-c)^{\frac{1}{3}}$ (cubing both sides)

$\Rightarrow$ $(a+b-c)^3=-27abc$

$\Rightarrow$ $(a+b-c)^3+27abc=0$

Ex. 101 If $a^2+b^2=c^2$ then find $\dfrac{a^6+b^6-c^6}{a^2b^2c^2}=?$

Sol^n. We know that $a^3+b^3+c^3=3abc$ if $a+b+c=0$

$\because$ $a^2+b^2+(-c^2)=0$

$\therefore (a^2)^3+(b^2)^3+(-c^2)^3=$ $3a^2b^2(-c^2)$

$a^6+b^6-c^6=-3a^2b^2c^2$

$$\frac{a^6+b^6-c^6}{a^2b^2c^2}=\frac{-3a^2b^2c^2}{a^2b^2c^2}=-3$$

Method : 2 put $a=b=1$ & $c=\sqrt{2}$

$\left[\therefore 1^2+1^2=\left(\sqrt{2}\right)^2\right]$

$$\Rightarrow \frac{a^6+b^6-c^6}{a^2b^2c^2}=\frac{(1)^6+(1)^6-(\sqrt{2})^6}{(1)^2(1)^2(\sqrt{2})^2}=\frac{1+1-8}{2}=-3$$

Type – 6 (Add 1 or Subtract 1)

Ex. 102 If $\frac{1}{1+a}+\frac{1}{1+b}+\frac{1}{1+c}=1$

find $\frac{a}{1+a}+\frac{b}{1+b}+\frac{c}{1+c}=?$

Sol^n. Subtract 1 from each term

$$\Rightarrow \frac{1}{1+a}-1+\frac{1}{1+b}-1+\frac{1}{1+c}-1=1-1-1-1$$

$$\Rightarrow \frac{-a}{1+a}+\frac{-b}{1+b}+\frac{-c}{1+c}=-2$$

$$\Rightarrow \frac{a}{a+1}+\frac{b}{b+1}+\frac{c}{c+1}=2$$

Method-2:

$$\frac{1}{1+a}+\frac{1}{1+b}+\frac{1}{1+c}=1 \qquad \ldots(i)$$

$$\frac{a}{1+a}+\frac{b}{1+b}+\frac{c}{1+c}=t\ (let) \qquad \ldots(ii)$$

Adding equ^n (i) & (ii)

$$\Rightarrow \frac{1+a}{1+a}+\frac{1+b}{1+b}+\frac{1+c}{1+c}=1+t$$

$$1+1+1=1+t$$

$$t=2$$

Ex. 103 If $\frac{1}{x+1}+\frac{2}{y+2}+\frac{1009}{z+1009}=1$

find $\frac{x}{x+1}+\frac{y}{y+2}+\frac{z}{z+1009}=?$

Sol^n. Subtract 1 from each term

$$\Rightarrow \frac{1}{x+1}-1+\frac{2}{y+2}-1+\frac{1009}{z+1009}-1=1-1-1-1$$

$$\Rightarrow \frac{-x}{x+1}+\frac{-y}{y+2}+\frac{-z}{z+1009}=-2$$

$$\Rightarrow \frac{x}{x+1}+\frac{y}{y+2}+\frac{z}{z+1009}=2$$

Ex. 104 If $\frac{a}{x-a}+\frac{b}{y-b}+\frac{c}{z-c}=2$

find $\frac{x}{x-a}+\frac{y}{y-b}+\frac{z}{z-c}=?$

Sol^n. Add 1 to each term

$$\Rightarrow \frac{a}{x-a}+1+\frac{b}{y-b}+1+\frac{c}{z-c}+1=2+1+1+1$$

$$\Rightarrow \frac{x}{x-a}+\frac{y}{y-b}+\frac{z}{z-c}=5$$

Ex. 105 If $\frac{a}{x-1}+\frac{4b}{y-2b}+\frac{9c}{z-3c}=6-a$

find $\frac{ax}{x-1}+\frac{2y}{y-2b}+\frac{3z}{z-3c}=?$

Sol^n. $\frac{a}{x-1}+a+\frac{4b}{y-2b}+2+\frac{9c}{z-3c}+3=6-a+a+2+3$

$$\Rightarrow \frac{ax}{x-a}+\frac{2y}{y-2b}+\frac{3z}{z-3c}=11$$

Ex. 106 If $\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}=1$

find $\frac{a^2}{b+c}+\frac{b^2}{c+a}+\frac{c^2}{a+b}=?$

Sol^n. Multiply by $(a+b+c)$ both sides

$$\Rightarrow \frac{a(a+b+c)}{b+c}+\frac{b(a+b+c)}{c+a}+\frac{c(a+b+c)}{a+b}=a+b+c$$

$$\Rightarrow \frac{a^2+a(b+c)}{b+c}+\frac{b^2+b(c+a)}{c+a}+\frac{c^2+c(a+b)}{a+b}=a+b+c$$

$$\Rightarrow \frac{a^2}{b+c}+a+\frac{b^2}{c+a}+b+\frac{c^2}{a+b}+c=a+b+c$$

$$\Rightarrow \frac{a^2}{b+c}+\frac{b^2}{c+a}+\frac{c^2}{a+b}=0$$

Method:-2

we can put $\frac{a}{b+c}=\frac{b}{c+a}=\frac{c}{a+b}=\frac{1}{3}$ then

$3a=b+c,\ 3b=c+a\ \&\ 3c=a+b$

adding all these relation $\Rightarrow a+b+c=0$

it means $b+c=-a,\ c+a=-b\ \&\ a+b=-c$

$$\Rightarrow \frac{a^2}{b+c}+\frac{b^2}{c+a}+\frac{c^2}{a+b}=a\left(\frac{a}{b+c}\right)+b\left(\frac{b}{c+a}\right)+c\left(\frac{c}{a+b}\right)$$

$$=-a-b-c=0$$

Ex. 107 If $\frac{a^2-bc}{a^2+bc}+\frac{b^2-ac}{b^2+ac}+\frac{c^2-ab}{c^2+ab}=1$ then find

$$\frac{a^2}{a^2+bc}+\frac{b^2}{b^2+ac}+\frac{c^2}{c^2+ab}=?$$

Sol. Adding 1 to each term

$$\left(\frac{a^2-bc}{a^2+bc}+1\right)+\left(\frac{b^2-ac}{b^2+ac}+1\right)+\left(\frac{c^2-ab}{c^2+ab}+1\right)=1+1+1+1$$

$$\left(\frac{2a^2}{a^2+bc}\right)+\left(\frac{2b^2}{b^2+ac}\right)+\left(\frac{2c^2}{c^2+ab}\right)=4$$

$$\Rightarrow \frac{a^2}{a^2+bc}+\frac{b^2}{b^2+ac}+\frac{c^2}{c^2+ab}=2$$

<u>Type – 7</u> (Symmetry in max^m case Ans= 1)

Ex. 108 Given, $a^2 = b + c$, $b^2 = c + a$ & $c^2 = a + b$

Or $\frac{a^2}{b+c}=\frac{b^2}{c+a}=\frac{c^2}{a+b}=1$

find **(1).** $\frac{1}{1+a}+\frac{1}{1+b}+\frac{1}{1+c}=?$

Sol. $a^2 = b + c$ .....(i)

$b^2 = c + a$ .....(ii)

$c^2 = a + b$ ......(iii)

Adding *a, b* & *c* to all three equations respectively

$a + a^2 = (b + c) + a = k$ (let)

$b + b^2 = (c + a) + b = k$ (let)

$c + c^2 = (a + b) + c = k$ (let)

$$\frac{1}{1+a}+\frac{1}{1+b}+\frac{1}{1+c}=\frac{a}{a(1+a)}+\frac{b}{b(1+b)}+\frac{c}{c(1+c)}$$

$$=\frac{a}{a+a^2}+\frac{b}{b+b^2}+\frac{c}{c+c^2}$$

$$=\frac{a}{k}+\frac{b}{k}+\frac{c}{k}=\frac{a+b+c}{k}=\frac{k}{k}=1$$

Method:-2

Check value of $a = b = c = 2$ in $a^2 = b + c$, $b^2 = c + a$ & $c^2 = a + b$ and these value is satisfying above equation so we can put in Question.

$$\Rightarrow \frac{1}{1+a}+\frac{1}{1+b}+\frac{1}{1+c}=\frac{1}{1+2}+\frac{1}{1+2}+\frac{1}{1+2}$$

$$=\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=1$$

(2). find $\frac{a}{1+a}+\frac{b}{1+b}+\frac{c}{1+c}=?$

Sol. put value of $a = b = c = 2$

$$\Rightarrow \frac{a}{1+a}+\frac{b}{1+b}+\frac{c}{1+c}=\frac{2}{1+2}+\frac{2}{1+2}+\frac{2}{1+2}$$

$$=\frac{2}{3}+\frac{2}{3}+\frac{2}{3}=\frac{6}{3}=2$$

Ex. 109 $\frac{x^2}{by+cz}=\frac{y^2}{ax+cz}=\frac{z^2}{ax+by}=1$ find

(1). $\frac{a}{x+a}+\frac{b}{y+b}+\frac{c}{z+c}=?$

(2). $\frac{x}{x+a}+\frac{y}{y+b}+\frac{z}{z+c}=?$

Sol. **(1).** $x^2 = by + cz$ …(i)

$y^2 = ax + cz$ …(ii)

$z^2 = ax + by$ …(iii)

Adding *ax, by* & *cz* to all three equations respectively

$ax + x^2 = (by + cz) + ax = k$ (let)

$by + y^2 = (ax + cz) + by = k$ (let)

$cz + z^2 = (ax + by) + cz = k$ (let)

$$\Rightarrow \frac{a}{x+a}+\frac{b}{y+b}+\frac{c}{z+c}$$

$$=\frac{ax}{x(x+a)}+\frac{by}{y(y+b)}+\frac{cz}{z(z+c)}$$

$$=\frac{ax}{ax+x^2}+\frac{by}{by+y^2}+\frac{cz}{cz+z^2}$$

$$=\frac{ax}{k}+\frac{by}{k}+\frac{cz}{k}$$

$$=\frac{ax+by+cz}{k}=\frac{k}{k}=1$$

Method:-2

Check value of $x = y = z = 2$ & $a = b = c = 1$ in $x^2 = by + cz$, $y^2 = ax + cz$ & $z^2 = ax + by$ and these value is satisfying above equation so we can put in Question.

put value of $x = y = z = 2$ & $a = b = c = 1$

$$\Rightarrow \frac{a}{x+a}+\frac{b}{y+b}+\frac{c}{z+c}$$

$$=\frac{1}{2+1}+\frac{1}{2+1}+\frac{1}{2+1}=\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=1$$

(2). $\Rightarrow \frac{x}{x+a}+\frac{y}{y+b}+\frac{z}{z+c}$

$$=\frac{2}{2+1}+\frac{2}{2+1}+\frac{2}{2+1}=\frac{2}{3}+\frac{2}{3}+\frac{2}{3}=2$$

Ex. 110 If $\frac{x^2}{by+cz}=\frac{y^2}{ax+cz}=\frac{z^2}{ax+by}=2$

(1). $\frac{a}{x+2a}+\frac{b}{y+2b}+\frac{c}{z+2c}=?$

(2). $\frac{x}{x+2a}+\frac{y}{y+2b}+\frac{z}{z+2c}=?$

Sol. $x^2 = 2by + 2cz$ …(i)

$y^2 = 2ax + 2cz$ …(ii)

$z^2 = 2ax + 2by$ …(iii)

Adding $2ax$, $2by$ & $2cz$ to all three equations respectively

$2ax + x^2 = (2by + 2cz) + 2ax = k$ (let)

$2by + y^2 = (2ax + 2cz) + 2by = k$ (let)

$2cz + z^2 = (2ax + 2by) + 2cz = k$ (let)

$$\Rightarrow \frac{a}{x+2a}+\frac{b}{y+2b}+\frac{c}{z+2c}$$

$$=\frac{ax}{x(x+2a)}+\frac{by}{y(y+2b)}+\frac{cz}{z(z+2c)}$$

$$=\frac{ax}{2ax+x^2}+\frac{by}{2by+y^2}+\frac{cz}{2cz+z^2}$$

$$=\frac{ax}{k}+\frac{by}{k}+\frac{cz}{k} \quad (\because 2ax + 2by + 2cz = k \quad \therefore ax + by + cz = k/2)$$

$$=\frac{ax+by+cz}{k}=\frac{k}{2k}=\frac{1}{2}$$

Method:-2

Check value of $x = y = z = 4$ & $a = b = c = 1$ in $x^2 = 2by + 2cz$, $y^2 = 2ax + 2cz$ & $z^2 = 2ax + 2by$ and these value is satisfying above equation so we can put in Question.

put value of $x = y = z = 4$ & $a = b = c = 1$

$$\Rightarrow \frac{a}{x+2a}+\frac{b}{y+2b}+\frac{c}{z+2c}$$

$$=\frac{1}{4+2(1)}+\frac{1}{4+2(1)}+\frac{1}{4+2(1)}$$

$$=\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{3}{6}=\frac{1}{2}$$

(2). $\frac{x}{x+2a}+\frac{y}{y+2b}+\frac{z}{z+2c}$

$$=\frac{4}{4+2(1)}+\frac{4}{4+2(1)}+\frac{4}{4+2(1)}$$

$$=\frac{4}{6}+\frac{4}{6}+\frac{4}{6}=\frac{12}{6}=2$$

Ex.111. If $xy + yz + zx = 1$

find $\frac{x+y}{1-xy}+\frac{y+z}{1-yz}+\frac{z+x}{1-zx}=?$

(a) $x + y + z$ (b) xyz

(c) 1 (d) $\frac{1}{xyz}$

Sol. $\frac{x+y}{1-xy}+\frac{y+z}{1-yz}+\frac{z+x}{1-zx}$

$$=\frac{x+y}{yz+zx}+\frac{y+z}{xy+zx}+\frac{z+x}{xy+yz}$$

$$=\frac{(x+y)}{z(x+y)}+\frac{(y+z)}{x(y+z)}+\frac{(z+x)}{y(x+z)}$$

$$= \frac{1}{z}+\frac{1}{x}+\frac{1}{y} = \frac{xy+yz+zx}{xyz} = \frac{1}{xyz}$$

Type – 8 (in this type of questions Ans = 2)

Ex. 112 $x = \frac{4\sqrt{15}}{\sqrt{5}+\sqrt{3}}$, find $\frac{x+\sqrt{20}}{x-\sqrt{20}} + \frac{x+\sqrt{12}}{x-\sqrt{12}} = ?$

Sol. $x = \frac{\sqrt{240}}{\sqrt{5}+\sqrt{3}} = \frac{\sqrt{20}\sqrt{12}}{\sqrt{5}+\sqrt{3}}$

$$\frac{x}{\sqrt{20}} = \frac{\sqrt{12}}{\sqrt{5}+\sqrt{3}}$$

$$\frac{x}{\sqrt{20}} = \frac{2\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

Applying Component & Dividend (C & D) rule

$$\frac{x+\sqrt{20}}{x-\sqrt{20}} = \frac{2\sqrt{3}+\sqrt{5}+\sqrt{3}}{2\sqrt{3}-\sqrt{5}-\sqrt{3}}$$

$$\frac{x+\sqrt{20}}{x-\sqrt{20}} = \frac{3\sqrt{3}+\sqrt{5}}{\sqrt{3}-\sqrt{5}} \quad \text{...(i)}$$

$$\frac{x}{\sqrt{12}} = \frac{\sqrt{20}}{\sqrt{5}+\sqrt{3}}$$

Applying Component & Dividend (C & D) rule

$$\frac{x+\sqrt{12}}{x-\sqrt{12}} = \frac{2\sqrt{5}+\sqrt{5}+\sqrt{3}}{2\sqrt{5}-\sqrt{5}-\sqrt{3}}$$

$$\frac{x+\sqrt{12}}{x-\sqrt{12}} = \frac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \quad \text{...(ii)}$$

Adding equation (i) & (ii)

$$\frac{x+\sqrt{20}}{x-\sqrt{20}} + \frac{x+\sqrt{12}}{x-\sqrt{12}} = \frac{3\sqrt{3}+\sqrt{5}}{\sqrt{3}-\sqrt{5}} + \frac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$$

$$= \frac{3\sqrt{3}+\sqrt{5}-3\sqrt{5}-\sqrt{3}}{\sqrt{3}-\sqrt{5}}$$

$$= \frac{2\sqrt{3}-2\sqrt{5}}{\sqrt{3}-\sqrt{5}} = \frac{2\left(\sqrt{3}-\sqrt{5}\right)}{\left(\sqrt{3}-\sqrt{5}\right)} = 2$$

Type – 9

If $xy = 1$ then $\frac{1}{1+x}+\frac{1}{1+y}$ will be 1.

Proof: $\frac{1}{1+x}+\frac{1}{1+y} = \frac{1+y+1+x}{(1+x)(1+y)}$

$$= \frac{x+y+2}{x+y+xy+1} = \frac{x+y+2}{x+y+2} = 1$$

Ex. 113 If $x = \frac{1}{\sqrt{3}+\sqrt{2}}, y = \frac{1}{\sqrt{3}-\sqrt{2}}$ then find $\frac{1}{x+1}+\frac{1}{y+1}$

(i) 0 (ii) 1
(iii) 2 (iv) –1

Sol. here $x.y = 1$ then $\frac{1}{x+1}+\frac{1}{y+1} = 1$

$$\left(\because xy = \frac{1}{\sqrt{3}+\sqrt{2}}.\frac{1}{\sqrt{3}-\sqrt{2}} = \frac{1}{3-2} = 1\right)$$

Ex. 114. If $x = \frac{\sqrt{87}-\sqrt{71}}{\sqrt{55}+\sqrt{39}}$ & $y = \frac{\sqrt{87}+\sqrt{71}}{\sqrt{55}-\sqrt{39}}$ then find $\frac{1}{x+1}+\frac{1}{y+1} = ?$

Sol. here $x.y = 1$ then $\frac{1}{x+1}+\frac{1}{y+1} = 1$

$$\left(\because x.y = \frac{\sqrt{87}-\sqrt{71}}{\sqrt{55}+\sqrt{39}}.\frac{\sqrt{87}+\sqrt{71}}{\sqrt{55}-\sqrt{39}} = \frac{87-71}{55-39} = 1\right)$$

MAX[M] & MIN[M] VALUE OF ALGEBRIC FUNCTION

Function	maximum	minimum
x	∞	$-\infty$
x^2	∞	0
$-x$	∞	$-\infty$
$-x^2$	0	$-\infty$

⇒ When the coefficient of x^2 is positive only min^m value will be asked (max^m value will be infinite)

⇒ When the coefficient of x^2 is negative only max^m value will be asked (min^m value will be –Ve infinite)

Note-1 If expression is in the form of $10 + X^2$ then min^m value will be 10 when X =0.

Note-2 If expression is in the form of $10 - X^2$ then max^m value will be 10 when X =0.

Ex. 115 What will be the minimum value of $12+(x-2)^2$\

Sol. minimum value = 12, when $x-2=0 \Rightarrow x=2$

Ex. 116 What will be the maximum value of $15-(x-3)^2$\

Sol. maximum value = 15, when $x-3=0 \Rightarrow x=3$

Ex. 117 If $5-(3a-2b)^2$ will be max^m, when $\frac{a}{b}=?$

Sol. Expression $5-(3a-2b)^2$ will be max^m when

$\Rightarrow 3a-2b=0$

$\Rightarrow 3a=2b \Rightarrow \frac{a}{b}=\frac{2}{3}$

Note: Question are asked in this form $x^2-6x+19$

Sol. $x^2-6x+19$

$= 10+x^2+9-6x = 10+(x-3)^2$

min^m value = 10 when $x-3=0 \Rightarrow x=3$

But in these type Questions we have to face difficulties to make perfect square so we can use the method of differentiation but don't think it is very easy only three things are to be remembered.

Step 1- Take Differentiation of given equation.

Differentiation of (i) x^2 is $2x$

(ii) x is 1

(iii) constant is 0

Step 2-Put Differentiation = 0 and find the value of x.

Step 3- Put the value of x in original equation to find maximum and minimum value.

Note: In quadratic equation you don't have to think value is maximum or minimum if the cofficient of x is positive minmum value will be asked or cofficient of x is negative maximum value will be asked and in method of diifferentiation required value would be obtained.

Using this method in previous question

$x^2-6x+19$ ...(i)

⇒ Differentiation = 0 $(x^2 \to 2x$

⇒ $2x-6+0=0$ $x \to 1$

⇒ $2x=6$ const. $\to 0)$

⇒ $x=3$

put $x = 3$ in equation (i)

min^m value $= (3)^2-6(3)+19$

$= 9-18+19 = 10$

⇒ **If you are unable to understand this method then learn two formula given in quadratic equation to find maximum and minimum value.**

Ex. 118 Find the minimum value of $3x^2-6x+11$.

$\Rightarrow$ Differentiation = 0

$\Rightarrow 6x-6+0=0$

$\Rightarrow 6x=6$

$\Rightarrow x=1$

put $x = 1$ in $3x^2-6x+11$

min^m value $= 3(1)^2-6(1)+11$

$= 3-6+11 = 8$

Ex. 119 Find the maximum value of $13-4x-x^2$.

$\Rightarrow$ Differentiation = 0

$\Rightarrow 0-4-2x=0$

$\Rightarrow 2x=-4$

$\Rightarrow x=-2$

put $x=-2$ in $13-4x-x^2$

max^m value $= 13-4(-2)-(-2)^2$

$= 13+8-4 = 17$

Ex. 120 Expression $15-7x-2x^2$ will be maximum when x =?

Sol. $\Rightarrow$ Differentiation = 0

$\Rightarrow 0-7-4x=0$

$\Rightarrow 4x=-7$

$\Rightarrow x=-\frac{7}{4}$

Ex. 121 Expression $4x^2-16x+17$ will be minimum when x = ?

Sol. $\Rightarrow$ Differentiation $= 0$

$\Rightarrow 8x - 16 + 0 = 0$

$\Rightarrow 8x = 16$

$\Rightarrow x = 2$

Ex. 122 Find minimum value of $(9 - x)(2 - x)$.

Sol. $\Rightarrow (9 - x)(2 - x) = 18 - 9x - 2x + x^2$

$= x^2 - 11x + 18$

Differentiation $= 0$

$\Rightarrow 2x - 11 + 0 = 0$

$\Rightarrow 2x = 11$

$\Rightarrow x = \frac{11}{2}$

put $x = \frac{11}{2}$ in $(9 - x)(2 - x)$

$\min^m$ value $= \left(9 - \frac{11}{2}\right)\left(2 - \frac{11}{2}\right)$

$= \left(\frac{7}{2}\right)\cdot\left(\frac{-7}{2}\right) = -\frac{49}{4}$

Ex. 123 Find maximum value of $(6 - x)(x + 4)$.

Sol. $\Rightarrow (6 - x)(x + 4) = 6x + 24 - x^2 - 4x$

$= -x^2 + 2x + 24$

Differentiation $= 0$

$\Rightarrow -2x + 2 + 0 = 0$

$\Rightarrow -2x + 2 = 0$

$\Rightarrow 2x = 2 \quad \Rightarrow x = 1$

put $x = 1$ in $(6 - x)(x + 4)$

$\max^m$ value $= (6 - 1)(1 + 4) = 5 \times 5 = 25$

(B). How to find minimum value if the expression in the form of $x^2 + \frac{1}{x^2}, x \in R$ or $x + \frac{1}{x}$ if x is +ve.

$\Rightarrow x^2 + \frac{1}{x^2} = x^2 + \frac{1}{x^2} - 2 + 2 = \left(x - \frac{1}{x}\right)^2 + 2$

$\min^m$ value $= 2$ when, $\left(x - \frac{1}{x}\right) = 0 \Rightarrow x^2 = 1$

Ex. 124 Find the minimum value of $x^2 + \frac{1}{x^2} - 3$.

Sol. $\min^m$ value of $x^2 + \frac{1}{x^2} - 3 = 2 - 3 = -1$

Ex. 125 Find the minimum value of $x^2 + \frac{1}{x^2 + 1} - 3$.

Sol. $x^2 + \frac{1}{x^2 + 1} - 3 = \left(x^2 + 1\right) + \frac{1}{\left(x^2 + 1\right)} - 3 - 1$

$= \left(x^2 + 1\right) + \frac{1}{\left(x^2 + 1\right)} - 4$

$\min^m$ value $= 2 - 4 = -2$

Ex.126 If $\sqrt{x^2 - x + 1} + \frac{1}{\sqrt{x^2 - x + 1}} = 2 - x^2$, how many value possible for x?

(a) 0 (b) 1

(c) 2 (d) 3

Sol. let $X = \sqrt{x^2 - x + 1}$

$\Rightarrow X + \frac{1}{X} = 2 - x^2$

$\min^m$ value of $X + \frac{1}{X} = 2$

$\Rightarrow X + \frac{1}{X} \geq 2$

$\Rightarrow 2 - x^2 \geq 2$

$\Rightarrow 0 \geq x^2$ or $x^2 \leq 0$

it is only possible when $x = 0$

One value is possible for x.

(C). If $x + y$ will be given then xy will be maximum when $x = y$

e.g. $x + y = k$ (given) then maximum value of $xy = k^2/4$ because $x = y = k/2$

Proof: We can write $\Rightarrow xy = \left(\frac{x + y}{2}\right)^2 - \left(\frac{x - y}{2}\right)^2$

xy will be maximum when

$\left(\frac{x - y}{2}\right)^2 = 0 \Rightarrow x = y$

Ex. 127 If $x+y=6$ then max^m value of xy.

Sol. $x+y=6 \Rightarrow x=y=3$

max^m value of $xy = 3 \times 3 = 9$

Ex. 128 If $a+b+c+d=1$ then maximum value of $ab + bc + cd + da$.

Sol. $a+b+c+d=1 \Rightarrow a=b=c=d=\frac{1}{4}$

$$\begin{pmatrix} ab & + & bc & + & cd & + & da \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ a=b & & b=c & & c=d & & d=a \end{pmatrix}$$

maximum value of $ab+bc+cd+da$

$$=\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}=\frac{1}{4}$$

Ex. 129 If $x+y+z=21$ then find the maximum value of $(x-2)(y-1)(z+9)$.

Sol. We have to find maximum value of $(x-2)(y-1)(z+9)$ so $(x-2)=(y-1)=(z+9)=k$ (let)

then $x=2+k$, $y=1+k$ & $z=k-9$, put these values in $x=2+k$, $y=1+k$ & $z=k-9$ in given equation

$x+y+z=21$

$2+k+1+k+k-9=21 \Rightarrow 3k=21+9-2-1$

$3k=27 \Rightarrow k=9$

So maximum value of $(x-2)(y-1)(z+9)=k.k.k=k^3$

$=(9)^3=729$

Method 2

$\because x+y+z=21$

$(x-2)+(y-1)+(z+9)=21-2-1+9=27$

$$\begin{matrix} \Downarrow & \Downarrow & \Downarrow \\ 9 & 9 & 9 \end{matrix}$$

maximum value of $(x-2)(y-1)(z+9)=9\times9\times9=729$

(D). **If xy will be given then $x+y$ will be minimum when $x=y$ (x & y are positive numbers)**

e.g. $xy=k$ (given) then minimum value of $x+y=2\sqrt{k}$ because $x=y=\sqrt{k}$

Proof :

$$xy=k \Rightarrow y=\frac{k}{x}$$

$$x+y = x+\frac{k}{x}=x+\frac{k}{x}-2\sqrt{k}+2\sqrt{k}$$

$$=\left(x-\frac{k}{x}\right)^2+2\sqrt{k}$$

Then minimum value $=2\sqrt{k}$

when $\left(x-\frac{k}{x}\right)=0 \Rightarrow x=\frac{k}{x} \Rightarrow x^2=k$

$\Rightarrow x=\sqrt{k}$

Ex. 130 If $xy=16$, find minimum value of $x+y$.

Sol. $x=y=k$ (let)

$xy=16 \Rightarrow k.k=16 \Rightarrow k^2=16 \Rightarrow k=4$

min^m value of $x+y=4+4=8$

Ex. 131 If $(x-7)(y-10)(z-12)=1000$, then find the minimum value of $(x+y+z)$.

Sol. $\Rightarrow (x-7)=(y-10)=(z-12)=k$ (let)

$\Rightarrow k.k.k=1000 \Rightarrow k^3=1000 \Rightarrow k=10$

$\Rightarrow (x-7)=10 \Rightarrow x=17$

$\Rightarrow (y-10)=10 \Rightarrow y=20$

$\Rightarrow (z-12)=10 \Rightarrow z=22$

minimum value of $(x+y+z)=17+20+22=59$

☆☆☆☆

Exercise - (Polynomial & Quadratic Equation)

1. If $(x-2)$ is a factor of $(x^2+3qx-2q)$, then the value of q is:

(a) 2 (b) –2
(c) –1 (d) 1

2. If x^3+6x^2+4x+k is exactly divisible by $(x+2)$, then the value of k is:

(a) –6 (b) –7
(c) –8 (d) –10

3. Let $f(x)=x^3-6x^2+11x-6$. Then, which one of the following is not a factor of $f(x)$:

(a) $x-1$ (b) $x-2$
(c) $x+3$ (d) $x-3$

4. If $(x+t)$ is a factor of both (x^2+px+q) and (x^2+lx+m), then the value of $'t'$ is:

(a) $m-q$ (b) $l-p$
(c) $\frac{l-p}{m-q}$ (d) $\frac{m-q}{l-p}$

5. $(x^{29}-x^{25}+x^{13}-1)$ is divisible by:

(a) both $(x-1)$ & $(x+1)$
(b) $(x-1)$ but not by $(x+1)$
(c) $(x+1)$ but not by $(x-1)$
(d) neither $(x-1)$ nor $(x+1)$

6. Value of k for which $(x-1)$ is a factor of (x^3-k) is:

(a) –1 (b) 1
(c) 8 (d) –8

7. If $x^{100}+2x^{99}+k$ is divisible by $(x+1)$, then the value of k is:

(a) 1 (b) –3
(c) 2 (d) –2

8. If (x^3-5x^2+4p) is divisible by $(x+2)$, then the value of p is:

(a) 7 (b) –2
(c) 3 (d) –7

9. If $(x-a)$ is a factor of $(x^3-3x^2a+2a^2x+b)$, then the value of b is:

(a) 0 (b) 2
(c) 1 (d) 3

10. If x^3+3x^2+4x+k contains $(x+6)$ as a factor, the value of k is:

(a) 66 (b) 33
(c) 132 (d) 36

11. If $(x+2)$ and $(x-1)$ are the factors of (x^3+10x^2+mx+n), the values of m and n are:

(a) $m=5, n=-3$ (b) $m=17, n=-8$
(c) $m=7, n=-18$
(d) $m=23, n=-19$

12. On dividing (x^3-6x+7) by $(x+1)$, then remainder is:

(a) 2 (b) 12
(c) 0 (d) 7

13. If $(x^5-9x^2+12x-14)$ is divided by $(x-3)$, the remainder is:

(a) 184 (b) 56
(c) 2 (d) 1

14. When $(x^4-3x^3+2x^2-5x+7)$ is divided by $(x-2)$, then remainder is:

(a) 3 (b) –3
(c) 2 (d) 0

15. If $5x^3+5x^2-6x+9$ is divided by $(x+3)$, then remainder is:

(a) 135 (b) –135
(c) 63 (d) –63

16. If $(x^{11}+1)$ is divided by $(x+1)$, then remainder is:

(a) 2 (b) 0
(c) 11 (d) 12

17. If $2x^3+5x^2-4x-6$ is divided by $2x+1$, then remainder is:

(a) $-\frac{13}{2}$ (b) 3
(c) –3 (d) 6

18. If x^3+5x^2+10k leaves remainder $-2x$ when divided by x^2+2, then the value of k is:

(a) –2 (b) –1
(c) 1 (d) 2

19. The factors of $(x^2-1-2a-a^2)$ are:

(a) $(x-a+1)(x-a-1)$
(b) $(x+a-1)(x-a+1)$
(c) $(x+a+1)(x-a-1)$
(d) None of these

20. The factors of $(x^2-8x-20)$ are:

(a) $(x+10)(x-2)$ (b) $(x-10)(x+2)$
(c) $(x-5)(x+4)$ (d) $(x+5)(x-4)$

21. The factors of $(x^2-xy-72y^2)$ are:

(a) $(x-8y)(x+9y)$
(b) $(x-9y)(x+8y)$
(c) $(x-y)(x+72y)$
(d) $(x-6y)(x+12y)$

22. The factors of $(x^2-11xy-60y^2)$ are:

(a) $(x+15y)(x-4y)$
(b) $(x-15y)(x+4y)$
(c) $(15x+y)(4x-y)$
(d) None of these

23. (x^n-a^n) is divisible by $(x-a)$:

(a) for all values of n
(b) only for even values of n
(c) only for odd values of n
(d) only for prime values of n

24. (x^n-a^n) is divisible by $(x+a)$:

(a) for all values of n
(b) only for even values of n
(c) only for odd values of n
(d) only for prime values of n

25. (x^n+a^n) is divisible by $(x+a)$:

(a) for all values of n
(b) only for even values of n
(c) only for odd values of n
(d) only for prime values of n

26. Find k for which the system $6x-2y=3$ and $kx-y=2$ has a unique solution.

(a) $k \neq 2$ (b) $k \neq 3$
(c) $k \neq 5$ (d) $k \neq 4$

27. Find k for which the system $5x+2y=3$ and $x+ky=-7$ is inconsistent.

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$
(c) $\frac{5}{2}$ (d) $\frac{3}{2}$

28. Find k for which the system $2x+3y-5=0$ and $4x+ky-10=0$ has an infinite number of solutions.

(a) $k=5$ (b) $k=6$
(c) $k=7$ (d) $k=8$

29. The solution of the system of simultaneous linear equations $4x-3y=7$ and $7x+5y=2$ is

(a) $x=1, y=1$ (b) $x=-1, y=1$

(c) $x=-1, y=-1$ (d) $x=1, y=-1$

30. The solution of the equations $x-y=0.9$ and $\frac{11}{2(x+y)}=1$ is:

(a) $x=3.2, y=2.3$ (b) $x=1, y=0.1$

(c) $x=2, y=1.1$ (d) None of these

31. The solution of the equations –

$\frac{3x-y+1}{3}=\frac{2x+y+2}{5}=\frac{3x+2y+1}{6}$ is given by:

(a) $x=2, y=1$ (b) $x=1, y=1$

(c) $x=-1, y=-1$ (d) $x=1, y=2$

32. The value of $x+y$ is the solution of the equations $\frac{x}{4}+\frac{y}{3}=\frac{5}{12}$ and $\frac{x}{2}+y=1$ is

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$

(c) 2 (d) $\frac{5}{2}$

33. If $\frac{2}{x}+\frac{3}{y}=\frac{9}{xy}$ and $\frac{4}{x}+\frac{9}{y}=\frac{21}{xy}$, where $x\neq 0, y\neq 0$, then the values of x and y are respectively:

(a) 0 and 1 (b) 1 and 2

(c) 2 and 3 (d) 1 and 3

34. The number of solutions of the equations $x+\frac{1}{y}=2$ and $2xy-3y=-2$ is:

(a) 0 (b) 1

(c) 2 (d) None of these

35. The total cost of 6 books and 4 pencils is ₹ 34 and that of 5 books and 5 pencils is ₹ 30. The cost of each book and pencil (in ₹) respectively is:

(a) 1 and 5 (b) 5 and 1

(c) 6 and 1 (d) 1 and 6

36. If 3 chairs and 2 tubes cost ₹ 1200 and 5 chairs and 3 tubes cost ₹ 1900, then the cost of 2 chairs and 2 tubes (in ₹) is:

(a) 700 (b) 900

(c) 1000 (d) 1100

37. If $x+2y\leq 3, x>0$ and $y>0$, then one of the solutions is:

(a) $x=-1, y=2$ (b) $x=2, y=1$

(c) $x=1, y=1$ (d) $x=0, y=0$

38. What are the values of x in the equation $4^x - 3.2^{x+2}+32=0$?

(a) 1, 2 (b) 3, 4

(c) 2, 3 (d) 1, 3

39. What are the roots of the equation $(a+b+x)^{-1}=a^{-1}+b^{-1}+x^{-1}$?

(a) *a, b* (b) *–a, b*

(c) *a, –b* (d) *–a, –b*

40. If the ratio of the roots of the equation $lx^2+nx+n=0$ is $p:q$, then the value of $\sqrt{\frac{p}{q}}+\sqrt{\frac{q}{p}}+\sqrt{\frac{n}{l}}$ is

(a) 1 (b) 2

(c) 3 (d) 0

41. If $x=11$ find the value of $x^5-12x^4+12x^3-12x^2+12x-1$ is

(a) 10 (b) 11

(c) 12 (d) 0

42. If $\frac{5x-7y+15}{1}=\frac{3x-2y+1}{8}=\frac{11x-6y+10}{9}$, then the value of $(x+y)$ is equal to :

(a) 1 (b) 2

(c) 3 (d) –3

43. If $\frac{2x-13y+1}{2}=\frac{x+4y+8}{3}=\frac{4x-7y+2}{5}$ then the value of $(x+2y)$ is equal to :

(a) 3 (b) 2

(c) 7 (d) –2

44. If $x = 2t$ and $y = \frac{2t-1}{3}$, then the value of t, when $x = y$

(a) $\frac{1}{3}$ (b) $-\frac{1}{4}$

(c) $\frac{1}{4}$ (d) $-\frac{1}{2}$

45. If $(8x^2 - 15y^2) : xy = 14 : 1$, then the positive value of $\frac{x}{y}$ is:

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$

(c) $\frac{5}{3}$ (d) $\frac{7}{2}$

46. If a, b are rational numbers and $(a-1)\left(\sqrt{2}+3\right) = (b\sqrt{2} + a)$, then the value of $(a + b)$ is

(a) 2 (b) 5

(c) 5 (d) 3

47. Find the value of x for which $\sqrt{4x-9}+\sqrt{4x+9} = 5+\sqrt{7}$.

(a) $\frac{3}{\sqrt{7}}$ (b) 4

(c) $\sqrt{5}$ (d) $2\sqrt{3}$

48. If $\sqrt{7-4\sqrt{3}} = \sqrt{3}a - b$ (a, b are rational numbers), then value of $(a - b)$ is:

(a) 1 (b) –1

(c) –2 (d) 2

49. If $\frac{x+\sqrt{x^2-1}}{x-\sqrt{x^2-1}} + \frac{x-\sqrt{x^2-1}}{x+\sqrt{x^2-1}} = 34$, then the value of x is $(x < 0)$

(a) –1 (b) –2

(c) –3 (d) –4

50. If one factor of equation $x^2 - x(a + b) + (a - 1)(b + 1) = 0$ is $(x - a + 1)$ then other factor is :

(a) $x - b$ (b) $x - a - b$

(c) $x - b + 1$ (d) $x - b - 1$

51. What are the factors of $x^2 + 4y^2 + 4y - 4xy - 2x - 8$?

(a) $(x - 2y - 4)$ and $(x - 2y + 2)$

(b) $(x - y + 2)$ and $(x - 4y + 4)$

(c) $(x - y + 2)$ and $(x - 4y - 4)$

(d) $(x + 2y - 4)$ and $(x + 2y + 2)$

52. The HCF of two polynomials $p(x)$ and $q(x)$ is $2x(x+2)$ and LCM is $24x(x+2)^2 (x-2)$. If $p(x) = 8x^3 + 32x^2 + 32x$, then what is $q(x)$ equal to ?

(a) $x^3 - 16x$ (b) $6x^3 - 24x$

(c) $12x^3 + 24x$ (d) $12x^3 - 24x$

53. If $2\sqrt{3}$ is a root of equation $x^2 + px - 6 = 0$ and roots of equation $x^2 + px + q = 0$ are equal. Find the value of q.

(a) 3/4 (b) 2/3

(c) 4/3 (d) 3/2

54. Find the value of b if $12^{2b+4} = 3^{3b}.4^{b+8}$

(a) +4 (b) 2

(c) Data insufficient (d) None of these

55. If $x^2 - 1$ is a factor of polynomial $f(x) = 2x^3 + Ax^2 + Bx + 3$ then $A^2 - B^2 =$

(a) 5 (b) 6

(c) –6 (d) 0

56. The sum of the roots of the equation $\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$ is zero. What is the product of the roots of equation ?

(a) $-\frac{(a+b)}{2}$ (b) $\frac{(a+b)}{2}$

(c) $-\frac{(a^2+b^2)}{2}$ (d) $\frac{(a^2+b^2)}{2}$

57. If $3x^3 - 2x^2y - 13xy^2 + 10y^3$ is divided by $x - 2y$, then what is the remainder ?

(a) 0 (b) x

(c) $y + 5$ (d) $x - 3$

58. If HCF of $x^3 - 27$ and $x^3 + 4x^2 + 12x + k$ is a quadratic polynomial. Find the value of k.
(a) 27 (b) 9
(c) 3 (d) 4

59. If $x^{40} + 2$ is divided by $x^4 + 1$, then the remainder?
(a) 1 (b) 2
(c) 4 (d) 3

60. Find the least integer value of k if the root of equation $x^2 - 2(k-1)x + (2k+1) = 0$ are equal–
(a) 1 (b) $-\frac{1}{2}$
(c) 4 (d) 0

61. The number of real roots of $3^{2x^2-7x+7} = 9$ is
(a) 3 (b) 1
(c) 2 (d) 4

62. Which one of the following is not the factor of $x^{16} + x^8 + 1$
(a) $x^2 + x + 1$ (b) $x^2 + 1 - x$
(c) $x^2 + 1$ (d) $x^4 - x^2 + 1$

63. If $ax^2 + bx + c = a(x - p)^2$, then the relation among a, b, c should be :
(a) $b^2 = 4ac$ (b) $2b = a + c$
(c) $abc = 1$ (d) $b^2 = ac$

64. The solution to the inequality $11x - 61 \leq 5$ is
(a) $-6 \leq x \leq 6$ (b) $-6 \leq x \leq 0$
(c) $x \leq 6$ (d) $0 \leq x \leq 6$

65. Find the coefficient of x^2 in in the expansion of $(x^2 + x + 1)(x^2 - x + 1)$.
(a) 2 (b) –2
(c) 1 (d) –1

66. What is the value of x satisfying the equation

$$16\left(\frac{a-x}{a+x}\right)^3 = \frac{a+x}{a-x} ?$$

(a) $a/2$ (b) $a/3$
(c) $3a$ (d) Both (b) and (c)

67. If HCF of $ax^2 + bx + c$ and $bx^2 + ax + c$ is $(x + 1)$, then value of c :
(a) –1 (b) 1
(c) 0
(d) Can't be determined

68. If the equation $2x^2 - 7x + 12 = 0$ has two roots α and β, then the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$
(a) $\frac{97}{24}$ (b) $\frac{7}{2}$
(c) $\frac{1}{24}$ (d) $\frac{7}{24}$

69. If $3x^3 - kx^2 + 4x + 16$ is exactly divisible by $\left(x - \frac{k}{2}\right)$, then $k = ?$
(a) 4 (b) – 4
(c) 2 (d) 0

70. If $\frac{2}{x} + \frac{3}{y} = \frac{9}{xy}$ and $\frac{4}{x} + \frac{9}{y} = \frac{21}{xy}$ where $x \neq 0$ and $y \neq 0$, then $x + y = ?$
(a) –1 (b) 4
(c) 0 (d) 1

71. If $4^x + 2^{2x-1} = 3^{x+\frac{1}{2}} + 3^{x-\frac{1}{2}}$, then $x = ?$
(a) 1/2 (b) 3/2
(c) 5/2 (d) 1

72. Find the rational root of polynomial $f(x) = 3x^4 - 15x^3 + 17x^2 + 5x - 6$ if one roots is $\frac{1}{\sqrt{3}}$
(a) –3, 2 (b) 3, 2
(c) 3, –2 (d) –3, –2

73. If $x^3 + 3x^2 - kx + 4$ is divided by $(x - 2)$, then remainder is k. The value of k is equal to :
(a) 8 (b) 2
(c) 4 (d) 6

74. What is the solution of the equation

$$\sqrt{\frac{x}{x+3}} - \sqrt{\frac{x+3}{x}} = -\frac{3}{2} :$$

(a) 1 (b) 2
(c) 4 (d) –2

75. Find the number of solution of equation

$$\sqrt{2x^2-3x+1}+\frac{1}{\sqrt{2x^2-3x+1}}=2-x^2$$

(a) 0 (b) 1
(c) 2 (d) 4

76. If one factor of expression $3a(3a+2c)-4b(b+c)$ is $(3a-2b)$ then other factor is :

(a) $2(a+b+c)$ (b) $(3a+2b+2c)$
(c) $(3a-2b+2c)$ (d) $(3a+2b-2c)$

77. If $x=3$ is a solution of the equation $3x^2+(k-1)x+9=0$, find k.

(a) 13 (b) –13
(c) 11 (d) –11

78. The common roots of the equations $x^2-7x+10=0$ and $x^2-10x+16=0$ is :

(a) –2 (b) 3
(c) 5 (d) 2

79. If α, β are the roots of the equation $x^2-5x+6=0$, then find the quadratic equation whose roots are $\frac{1}{\alpha},\frac{1}{\beta}$.

(a) $6x^2-5x+1=0$ (b) $6x^2+5x+1=0$
(c) $6x^2-5x-1=0$ (d) $6x^2+5x-1=0$

80. The value of k for which the roots α, β of the equation $x^2-6x+k=0$ satisfy the relation $3\alpha+2\beta=20$

(a) 8 (b) – 8
(c) 16 (d) – 16

81. If the equations $x^2+2x-3=0$ and $x^2+3x-k=0$ have a common roots then the non zero value of k is :

(a) 1 (b) 2
(c) 3 (d) 4

82. The positive value of m for which the roots of equation $12x^2+mx+5=0$ are in ratio 3 : 2 is:

(a) $5\sqrt{10}$ (b) $-\frac{5\sqrt{10}}{2}$
(c) $\frac{5}{12}$ (d) $\frac{12}{5}$

83. If α and β are the roots of the equation $x^2-3kx+k^2=0$ find k if $\alpha^2+\beta^2=\frac{7}{4}$

(a) $\pm\frac{1}{2}$ (b) $\frac{1}{2}$
(c) $-\frac{1}{2}$ (d) None of these

84. If α, β are the roots of equation $x^2+kx+12=0$ such that $\alpha-\beta=1$ the value of k is :

(a) 0 (b) ±5
(c) ±1 (d) ±7

85. Find the value of k so that the sum of the roots of equation $3x^2+(2x+1)x-k-5=0$ is equal to the product of the roots :

(a) – 4 (b) 6
(c) 2 (d) 8

86. If $2^{2y+3}=65(2^y-1)+57$ then y =?

(a) ±3 (b) ±1
(c) ±2 (d) ±4

87. If x is real number then value of expression $\frac{x^2+2x+1}{x^2+2x+7}$ will be between—

(a) 1 and 2 (b) –1 and 1
(c) 0 and 1 (d) 1/2 and 1

88. If α and β are the roots of the equation $6x^2+x-2=0$, find the value of $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=?$

(a) $-\frac{25}{12}$ (b) $-\frac{12}{75}$
(c) $-\frac{4}{9}$ (d) None of these

89. If $(5x^2 + 14x + 2)^2 - (4x^2 - 5x + 7)^2$ is divided by $x^2 + x + 1$, then what is the remainder ?

(a) –1 (b) 0

(c) 1 (d) 2

90. A factor of $a^4 - 11a^2b^2 + b^4$ is

(a) $(a^2 - b^2 - 3ab)$ (b) $(a^2 + b^2 - 3ab)$

(c) $(a^2 + b^2 + 3ab)$ (d) $(a^2 - b^2 + 4ab)$

91. Find the value of k, so that the roots of $7x^2 - 14x + k = 0$ are reciprocal of each other.

(a) 4 (b) 7

(c) 6 (d) 3

92. Find the value of k if one root of the equation $x^2 - 9x + k = 0$ is twice the other root.

(a) 18 (b) 16

(c) 12 (d) 9

93. If $\sqrt{2x-5} + \sqrt{3x+4} = 8$ then $x = ?$

(a) 8 (b) 6

(c) 7 (d) Either (b) or (c)

94. $4x^3 + ax^2 - bx + 3$ divided by $(x - 2)$ leaves remainder 2, divided by $(x + 3)$ leaves remainder 3. Find remainder when it is divided by $(x + 2)$.

(a) 26.8 (b) 29.2

(c) 32.2 (d) 35.2

95. If two factors of $a^4 - 2a^3 - 9a^2 + 2a + 8$ are $(a + 1)$ and $(a - 1)$, then what are the other two factors ?

(a) $(a - 2)$ and $(a + 4)$

(b) $(a + 2)$ and $(a + 4)$

(c) $(a + 2)$ and $(a - 4)$

(d) $(a - 2)$ and $(a - 4)$

96. If $\frac{a}{b} - \frac{b}{a} = \frac{x}{y}$ and $\frac{a}{b} + \frac{b}{a} = x - y$, then what is x equal to ?

(a) $\frac{a+b}{a}$ (b) $\frac{a+b}{b}$

(c) $\frac{a-b}{a}$ (d) None of these

97. If one root of the equation $\frac{x^2}{a} + \frac{x}{b} + \frac{1}{c} = 0$ reciprocal of the other, then which one of the following is correct ?

(a) $a = b$ (b) $b = c$

(c) $ac = 1$ (d) $a = c$

98. What are the roots of the quadratic equation $a^2b^2x^2 - (a^2 + b^2)x + 1 = 0$?

(a) $\frac{1}{a^2}, \frac{1}{b^2}$ (b) $-\frac{1}{a^2}, -\frac{1}{b^2}$

(c) $\frac{1}{a^2}, -\frac{1}{b^2}$ (d) $-\frac{1}{a^2}, \frac{1}{b^2}$

99. What should be added to the $x(x + a)(x + 2a)(x + 3a)$, so that the sum be a perfect square?

(a) $9a^2$ (b) $4a^2$

(c) a^4 (d) None of these

100. The quantity which must be added to $(1-x)(1+x^2)$ to obtain x^3 is

(a) $2x^3 + 3x^2 + x + 1$

(b) $2x^3 - x^2 + x - 1$

(c) $a + b + c = abc$

(d) $-x^2 + x - 1$

101. The factors of $5px - 10qy + 2rpx - 4qry$ is/are

(a) only $(5 + 2r)$

(b) only $(px - 2qy)$

(c) both $(5 + 2r)$ and $(px - 2qy)$

(d) neither $(5 + 2r)$ nor $(px - 2qy)$

102. $x^4 + xy^3 + x^3y + xz^3 + y^4 + yz^3$ is divisible by

(a) only $(x + y)$

(b) only $(x^3 + y^3 + z^3)$

(c) both $(x + y)$ and $(x^3 + y^3 + z^3)$

(d) None of the above

103. If the expression px^3+3x^2-3 and $2x^3-5x+p$ when divided by $(x-4)$ leave the same remainder, then what is value of p?

(a) 1 (b) -1

(c) -2 (d) 2

104. If the roots of the equation–

$(a^2-bc)x^2+2(b^2-ac)x+(c^2-ab)=0$ are equal, where $b\neq 0$, which one of the following is correct?

(a) $a+b+c=abc$

(b) $a^2+b^2+c^2=0$

(c) $a^3+b^3+c^3=0$

(d) $a^3+b^3+c^3=3abc$

105. If α and β are the roots of the equation $x^2-x-1=0$, then what is $\frac{\alpha^2+\beta^2}{(\alpha^2-\beta^2)(\alpha-\beta)}$ equal to?

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$

(c) $\frac{4}{5}$ (d) None of these

106. If K_1 and K_2 are the remainders when the polynomials $x^3+2x^2-5bx-7$ and $x^3+bx^2-12x+6$ are divided by $x+1$ and $x-2$ respectively. If $2K_1+K_2=6$. Find the value of b.

(a) 2 (b) –1

(c) 4 (d) –2

107. If $x^4-2x^3+3x^2-ax+b$ is divided by $x-1$ and $x+1$ the the remainders are 5 and 19 respectively. Find the remainder when this polynomial is divided by $x-2$.

(a) 10 (b) 12

(c) –1 (d) 8

108. If $x-2$ and $x-\frac{1}{2}$ are factors of ax^2+5x+b then under value of $\frac{a}{b}$

(a) 1 (b) 2

(c) –1 (d) 3

109. If α and β are the roots of the equation $3x^2-6x+2=0$, find the value of $\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta=$?

(a) 6 (b) 8

(c) 5 (d) 12

110. If α and β are the roots of the equation $\sqrt{\frac{x}{1-x}}+\sqrt{\frac{1-x}{x}}=2\frac{1}{6}$, find the value of $\alpha+\beta$.

(a) 1 (b) 8

(c) 5 (d) 6

****

Answer

1. (c)	**2.** (c)	**3.** (c)	**4.** (d)	**5.** (b)	**6.** (b)	**7.** (a)	**8.** (a)	**9.** (a)
10. (c)	**11.** (c)	**12.** (b)	**13.** (a)	**14.** (b)	**15.** (d)	**16.** (b)	**17.** (c)	**18.** (c)
19. (c)	**20.** (b)	**21.** (b)	**22.** (b)	**23.** (a)	**24.** (b)	**25.** (c)	**26.** (b)	**27.** (a)
28. (b)	**29.** (d)	**30.** (a)	**31.** (b)	**32.** (b)	**33.** (d)	**34.** (a)	**35.** (b)	**36.** (c)
37. (c)	**38.** (c)	**39.** (d)	**40.** (d)	**41.** (a)	**42.** (b)	**43.** (c)	**44.** (b)	**45.** (b)
46. (a)	**47.** (b)	**48.** (a)	**49.** (c)	**50.** (d)	**51.** (a)	**52.** (b)	**53.** (a)	**54.** (a)
55. (a)	**56.** (c)	**57.** (a)	**58.** (b)	**59.** (d)	**60.** (c)	**61.** (c)	**62.** (c)	**63.** (a)
64. (c)	**65.** (c)	**66.** (b)	**67.** (d)	**68.** (c)	**69.** (d)	**70.** (b)	**71.** (b)	**72.** (b)
73. (a)	**74.** (a)	**75.** (b)	**76.** (b)	**77.** (d)	**78.** (d)	**79.** (a)	**80.** (d)	**81.** (d)
82. (a)	**83.** (a)	**84.** (d)	**85.** (a)	**86.** (a)	**87.** (c)	**88.** (a)	**89.** (b)	**90.** (a)
91. (b)	**92.** (a)	**93.** (c)	**94.** (d)	**95.** (c)	**96.** (d)	**97.** (d)	**98.** (a)	**99.** (c)
100. (b)	**101.** (c)	**102.** (c)	**103.** (a)	**104.** (d)	**105.** (b)	**106.** (a)	**107.** (a)	**108.** (a)
109. (d)	**110.** (a)							

Solution & Hints

Sol. 1 $(x-2)$ is a factor of $x^2 + 3qx - 2q$

so, $x - 2 = 0 \Rightarrow x = 2$

Put , $x = 2$ in $x^2 + 3qx - 2q$ and equate to zero.

$\because (2)^2 + 3.q.2 - 2q = 0$

$\Rightarrow 4 + 6q - 2q = 0$

$\Rightarrow 4 + 4q = 0$

$4q = -4 \Rightarrow q = -1$

Sol. 2 Factor $\Rightarrow x + 2 = 0 \Rightarrow x = -2$ (put)

$\because (-2)^3 + 6(-2)^2 + 4(-2) + k = 0$

$\Rightarrow -8 + 24 - 8 + k = 0$

$\Rightarrow k = -8$

Sol. 3 Hint : Check by option

Sol. 4 $(x + t)$ is a factor of both equations, put $x = -t$ in both equations

$\Rightarrow (-t)^2 + p(-t) + q = 0$

$t^2 - pt + q = 0$ ...(i)

$\Rightarrow (-t)^2 + l(-t) + m = 0$

$t^2 - lt + m = 0$ ...(ii)

From equation (i) and (ii)

$t^2 - pt + q = t^2 - lt + m$

$lt - pt = m - q$

$\Rightarrow t(l-p) = m - q \Rightarrow t = \dfrac{m-q}{l-p}$

Sol. 5 Hint : Check by option

Sol. 6 $x - 1 = 0 \Rightarrow x = 1$ (Put)

$\Rightarrow 1^3 - k = 0$

$\Rightarrow 1 - k = 0 \Rightarrow k = 1$

Sol. 7 $x + 1 = 0 \Rightarrow x = -1$ (Put)

$\Rightarrow x^{100} + 2x^{99} + k = 0$

$\Rightarrow (-1)^{100} + 2(-1)^{99} + k = 0$

$\Rightarrow 1 - 2 + k = 0 \Rightarrow -1 + k = 0$

$\Rightarrow k = 1$

Sol. 8 $x + 2 = 0 \Rightarrow x = -2$ (Put)

$\Rightarrow x^3 - 5x^2 + 4p = 0$

$\Rightarrow (-2)^3 - 5(-2)^2 + 4p = 0$

$\Rightarrow -8 - 20 + 4p = 0$

$\Rightarrow -28 + 4p = 0 \Rightarrow 4p = 28$

$\Rightarrow p = 7$

Sol. 9 $x - a = 0 \Rightarrow x = a$ (Put)

$\Rightarrow x^3 - 3x^2a + 2a^2x + b = 0$

$\Rightarrow a^3 - 3a^2a + 2a^2a + b = 0$

$\Rightarrow a^3 - 3a^3 + 2a^3 + b = 0$

$\Rightarrow b = 0$

Sol. 10 Hint: Same as Q.9

Sol. 11 $(x + 2)$ & $(x - 1)$ are the factor of $x^3 + 10x^2 + mx + n$, so put $x = -2$ and $x = 1$ respectively

$\Rightarrow (-2)^3 + 10(-2)^2 + m(-2) + n = 0$

$\Rightarrow -8 + 40 - 2m + n = 0$

$\Rightarrow -2m + n + 32 = 0$

$\Rightarrow -2m + n = -32$ ...(i)

put $x = 1$

$\Rightarrow (1)^3 + 10(1)^2 + m.1 + n = 0$

$\Rightarrow 1 + 10 + m + n = 0$

$\Rightarrow m + n = -11$ ...(ii)

Subtract equation (ii) from (i)

$-2m + n = -32$

$m + n = -11$

$- \quad - \quad +$

$-3m = -21$

$\Rightarrow m = 7$

$7 + n = -11$

$\Rightarrow n = -18$

Sol. 12 $x + 1 = 0 \Rightarrow x = -1$

put the value of $x = -1$ in equation

$R = x^3 - 6x + 7 = (-1)^3 - 6(-1) + 7$

$= -1 + 6 + 7$

Remainder $= 12$

Sol. 13 $x - 3 = 0 \Rightarrow x = 3$

put of value of $x = 3$ in $x^5 - 9x^2 + 12x - 14$

$R = (3)^5 - 9(3)^2 + 12(3) - 14$

$R = 243 - 81 + 36 - 14$

$R = 279 - 95$

$R = 184$

Sol. 14 $x-2=0 \Rightarrow x=2$

put the value of $x=2$ in $x^4-3x^3+2x^2-5x+7$

R $= (2)^4-3(2)^3+2(2)^2-5(2)+7$

R $= 16-24+8-10+7$

R $= 31-34=-3$

Sol. 15 $x+3=0 \Rightarrow x=-3$

put the value of $x=-3$ in $5x^3+5x^2-6x+9$

R $= 5(-3)^3+5(-3)^2-6(-3)+9$

R $= -135+45+18+9$

R $= -135+72=-63$

Sol. 16 $x+1=0 \Rightarrow x=-1$

put the value of $x=-1$ in $x^{11}+1$

R $=(-1)^{11}+1=-1+1=0$

Sol. 17 $2x+1=0 \Rightarrow x=-\frac{1}{2}$

put the value of $x=-\frac{1}{2}$ in $2x^3+5x^2-4x-6$

R $= 2\left(-\frac{1}{2}\right)^3+5\left(-\frac{1}{2}\right)^2-4\left(-\frac{1}{2}\right)-6$

R $= -2\times\frac{1}{8}+\frac{5}{4}+\frac{4}{2}-6$

R $= -\frac{1}{4}+\frac{5}{4}+2-6$

R $= \frac{4}{4}-4=-3$

Sol. 18 $x^2+2=0 \Rightarrow x^2=-2$

put the value of $x^2=-2$ in x^3+5x^2+10k

R $= x^2.x+5x^2+10k=-2x$ (given)

$\Rightarrow (-2).x+5.(-2)+10k=-2x$

$\Rightarrow -10+10k=0 \Rightarrow k=1$

Sol. 19 $x^2-1-2a-a^2=[x^2-(a^2+1+2a)]$

$=[x^2-(a+1)^2]=(x+a+1)(x-a-1)$

Sol. 20 $x^2-8x-20=x^2-10x+2x-20$

$= x(x-10)+2(x-10)$

$=(x-10)(x+2)$

Sol. 21 $x^2-9xy+8xy-72y^2$

$= x(x-9y)+8y(x-9y)$

$=(x+8y)(x-9y)$

Sol. 22 $x^2-11xy-60y^2$

$= x^2-15xy+4xy-60y^2$

$= x(x-15y)+4y(x-15y)$

$=(x+4y)(x-15y)$

Sol. 23 x^n-a^n is divisible by $x-a$ for all values of n

(We can check by putting $x=a$ in x^n-a^n)

Sol. 24 Hint : Same as 23.

Sol. 25 Hint : Same as 23.

Sol. 26 $6x-2y=3$

$kx-y=2$

For unique solution, $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$

$\therefore \frac{6}{k}\neq\frac{-2}{-1} \Rightarrow \boxed{k\neq 3}$

Sol. 27 System of linear equation is inconsistent, it means it has no solution.

For no solution, $\frac{a_1}{a_2}=\frac{b_1}{b_2}$

$\therefore \frac{5}{1}=\frac{2}{k} \Rightarrow k=\frac{2}{5}$

Sol. 28 $2x+3y-5=0,\ 4x+ky-10=0$

For infinite solution,

$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

$\therefore \frac{2}{4}=\frac{3}{k} \Rightarrow \boxed{k=6}$

Sol. 29 $4x-3y=7$ …(i)

$7x+5y=2$ …(ii)

Multiply equation (i) by 5 and equation (ii) by 3

$20x-15y=35$ …(iii)

$21x+15y=6$ …(iv)

Adding (iii) and (iv)

$20x-15y=35$

$\underline{21x+15y=6}$

$41x=41 \Rightarrow x=1$

put the value of x in Eq^n (i)

$\Rightarrow 4(1)-3y=7 \Rightarrow -3y=3$

$\Rightarrow y=-1$

Hence, $x=1, y=-1$

Sol. 30 $x - y = 0.9 \Rightarrow x - y = \frac{9}{10}$ …(i)

$\frac{11}{2(x+y)} = 1 \Rightarrow x + y = \frac{11}{2}$ …(ii)

Adding equation (i) and (ii)

$x - y = \frac{9}{10}$

$x + y = \frac{11}{2}$

$2x = \frac{9}{10} + \frac{11}{2} = \frac{9+55}{10}$

$\Rightarrow 2x = \frac{64}{10}$

$\Rightarrow x = 3.2$

Put the value of x in equation (i)

$\Rightarrow 3.2 - y = 0.9$

$\Rightarrow y = 3.2 - 0.9 = 2.3$

Sol. 31 $\frac{3x - y + 1}{3} = \frac{2x + y + 2}{5}$

$\Rightarrow 15x - 5y + 5 = 6x + 3y + 6$

$\Rightarrow 9x - 8y = 1$ …(i)

Again , $\frac{3x - y + 1}{3} = \frac{3x + 2y + 1}{6}$

$\Rightarrow 18x - 6y + 6 = 9x + 6y + 3$

$\Rightarrow 9x - 12y = -3$ …(ii)

Subtract equation (i) from (ii)

$9x - 12y = -3$

$9x - 8y = 1$

$- \quad + \quad -$

$-4y = -4 \Rightarrow y = 1$

Put the value of y in equation (i)

$9x - 8(1) = 1$

$9x = 9$

$\boxed{x = 1, \ y = 1}$

Sol. 32 $\frac{x}{4} + \frac{y}{3} = \frac{5}{12}$

$\frac{3x + 4y}{12} = \frac{5}{12} \Rightarrow 3x + 4y = 5$ …(i)

$\frac{x}{2} + y = 1 \Rightarrow x + 2y = 2$ …(ii)

Multiply equation (ii) by 2

$2x + 4y = 4$ …(iii)

Subtract equation (iii) from (i)

$3x + 4y = 5$

$2x + 4y = 4$

$- \quad - \quad -$

$x = 1$

Put the value in equation (ii)

$1 + 2y = 2$

$2y = 1 \Rightarrow y = \frac{1}{2}$

$x + y = 1 + \frac{1}{2} = \frac{3}{2}$

Sol. 33 $\frac{2}{x} + \frac{3}{y} = \frac{9}{xy}$

$\Rightarrow 2y + 3x = 9$ ......(i)

$\frac{4}{x} + \frac{9}{y} = \frac{21}{xy}$

$\Rightarrow 4y + 9x = 21$ ......(ii)

Multiply equation (i) by 2

$4y + 6x = 18$

$4y + 9x = 21$

$- \quad - \quad -$

$-3x = -3 \Rightarrow x = 1$

Put the value of x in equation (i)

$2y + 3(1) = 9$

$\Rightarrow 2y = 6 \quad \Rightarrow y = 3$

Sol. 34 $x + \frac{1}{y} = 2 \Rightarrow xy + 1 = 2y \Rightarrow xy = 2y - 1$

put value of xy in $2xy - 3y = -2$

$2(2y - 1) - 3y = -2 \Rightarrow y = 0$

but y can't be zero from equation (i)

hence, both equations have no solutions or zero solutions.

Sol. 35 6 books + 4 pencil = 34 …(i)

5 books + 5 pencil = 30

$\Rightarrow$ books + pencil = 6 …(ii)

Multiply equation (ii) by 4

4 book + 4 pencil = 24 …(iii)

Subtract equation (iii) from (i)

$6b + 4p = 34$

$4b + 4p = 24$

$-\quad -\quad -$

$2b = 10$

book = 5 Rs.

Put the value in euqation (ii)

5 + pencil = 6

Pencil = 1 Rs.

Sol. 36 3 chair + 2 tube = 1200 …(i)

5 chair + 3 tubes = 1900 …(ii)

Multiply equation (i) by 3 & (ii) by 2 and subtract

9 chair + 6 tubes = 3600

10 chair + 6 tubes = 3800

$-\quad -\quad -$

– chair = –200

Chair = 200 Rs.

Put the value in (i)

3 × 200 + 2 tubes = 1200

2 tubes = 1200 – 600 = 600

tubes = 300 Rs.

$\because$ 2 chair + 2 tubes = 2 × 200 + 2 × 300

= 400 + 600 = 1000 Rs.

Sol. 37 $x + 2y \le 3, x > 0$ and $y > 0$

It is clear that one of the solution is

$x = 1$ and $y = 1$

$1 + 2(1) \le 3 \Rightarrow 3 \le 3$

Sol. 38 Hint : Put option to satisfy given Eq^n

Sol. 39 Hint : Put option to satisfy given Eq^n

Sol. 40 $lx^2 + nx + n = 0$

Let the root α, β of this equation

$$\frac{\alpha}{\beta} = \frac{p}{q}, \ \alpha + \beta = \frac{-n}{l} \ \& \ \alpha\beta = \frac{n}{l}$$

$$\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = \sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\alpha\beta}$$

$$= \frac{\sqrt{\alpha}}{\sqrt{\beta}} + \frac{\sqrt{\beta}}{\sqrt{\alpha}} + \sqrt{\alpha\beta} = \frac{\alpha + \beta + \alpha\beta}{\sqrt{\alpha\beta}}$$

$$= \frac{\frac{-n}{l} + \frac{n}{l}}{\sqrt{\frac{n}{l}}} = \frac{0}{\sqrt{\frac{n}{l}}} = 0$$

Sol. 41. $x^5 - 12x^4 + 12x^3 - 12x^2 + 12x - 1$

$= x^5 - 11x^4 - x^4 + 11x^3 + x^3 - 11x^2 - x^2 + 11x + x - 1$

$= x^4(x-11) - x^3(x-11) + x^2(x-11) - x(x-11) + 11 - 1$

$\because x = 11 \Rightarrow x - 11 = 0$

$= 11 - 1 = 10$

Sol. 42. $\dfrac{5x - 7y + 15}{1} = \dfrac{3x - 2y + 1}{8} = \dfrac{11x - 6y + 10}{9} = k$

$5x - 7y + 15 = k$ ...(i)

$3x - 2y + 1 = 8k$ ...(ii)

$11x - 6y + 10 = 9k$ ...(iii)

Adding equation (i) & (ii) and then subtract from equation (iii)

$3x + 3y - 6 = 0 \Rightarrow x + y = 2$

Sol. 43. Hint: Same as Q.42

Sol. 44. $x = 2t$ and $y = \dfrac{2t - 1}{3}$

When $x = y$, then

$2t = \dfrac{2t - 1}{3}$

$6t = 2t - 1$

$4t = -1$

$t = -\dfrac{1}{4}$

Sol. 45 $\frac{(8x^2 - 15y^2)}{xy} = 14$

$\Rightarrow 8x^2 - 15y^2 = 14xy$

$\Rightarrow 8x^2 - 14xy - 15y^2 = 0$

$\Rightarrow 8x^2 - 20xy + 6xy - 15y^2 = 0$

$\Rightarrow 4x(2x - 5y) + 3y(2x - 5y) = 0$

$\Rightarrow (4x + 3y)(2x - 5y) = 0$

$\because 2x - 5y = 0 \Rightarrow 2x = 5y \Rightarrow \frac{x}{y} = \frac{5}{2}$

Sol. 46 $(a-1)(\sqrt{2} + 3) = b\sqrt{2} + a$

$(a-1)\sqrt{2} + 3a - 3 = b\sqrt{2} + a$

(comparing both side)

$a - 1 = b \Rightarrow a - b = 1$ ....(1)

$3a - 3 = a \Rightarrow 2a = 3 \Rightarrow a = \frac{3}{2}$

$a = \frac{3}{2}$ put in equation (1)

then, $b = \frac{1}{2}$

now, $a + b = \frac{3}{2} + \frac{1}{2} = 2$

Sol. 47 $\sqrt{4x-9} + \sqrt{4x+9} = 5 + \sqrt{7} = \sqrt{25} + \sqrt{7}$

On comparing,

$4x + 9 = 25 \Rightarrow x = 4$

or

$4x - 9 = 7 \Rightarrow x = 4$

Sol. 48

$\sqrt{7 - 4\sqrt{3}} = \sqrt{(\sqrt{4})^2 + (\sqrt{3})^2 - 2 \times \sqrt{4}\sqrt{3}}$

$= \sqrt{(\sqrt{4} - \sqrt{3})^2}$

$= \sqrt{4} - \sqrt{3}$

Now,= $2 - \sqrt{3}$

$\because 2 - \sqrt{3} = \sqrt{3}a - b$

By comparing

$a = -1$ and $b = -2$

$\because a - b = -1 + 2 = 1$

Sol. 49 $\frac{x + \sqrt{x^2-1}}{x - \sqrt{x^2-1}} + \frac{x - \sqrt{x^2-1}}{x + \sqrt{x^2-1}} = 34$

$\Rightarrow \frac{\left(x + \sqrt{x^2-1}\right)^2 + \left(x - \sqrt{x^2-1}\right)^2}{\left(x - \sqrt{x^2-1}\right)\left(x + \sqrt{x^2-1}\right)} = 34$

$\Rightarrow \frac{x^2 + x^2 - 1 + x^2 + x^2 - 1}{x^2 - (x^2 - 1)} = 34$

$\Rightarrow 4x^2 - 2 = 34 \Rightarrow 4x^2 = 36$

$\Rightarrow x^2 = 9 \Rightarrow x = \pm 3$

hence, $x = -3$ $(\because x < 0)$

Sol. 50 $x^2 - x(a + b) + (a - 1)(b + 1) = 0$

one root $\Rightarrow (x - a + 1) = 0$

$x = a - 1$

Sum of roots = $\alpha + \beta = -\frac{B}{A} = (a + b)$

$a - 1 + \beta = a + b \Rightarrow \beta = b + 1$

then other factor will be $(x - b - 1)$

Sol. 51 Hint: Solve through options.

Sol. 52 Hint: See Example 25

Sol. 53 Hint: See Example 36

Sol. 54 Hint: Compare powers

Sol. 55 Hint: See Q.9

Sol. 56 $\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$

Let roots are α & $-\alpha$

then put $x = \alpha$ & $-\alpha$ in given equation.

$\frac{1}{\alpha + a} + \frac{1}{\alpha + b} = \frac{1}{c}$ & $\frac{1}{-\alpha + a} + \frac{1}{-\alpha + b} = \frac{1}{c}$

Equate above both equation

$\frac{1}{\alpha + a} + \frac{1}{\alpha + b} = \frac{1}{-\alpha + a} + \frac{1}{-\alpha + b}$

$\frac{1}{\alpha + a} - \frac{1}{-\alpha + a} = \frac{-1}{\alpha + b} + \frac{1}{-\alpha + b}$

on solving,

product of roots = $-\alpha^2 = -\frac{a^2 + b^2}{2}$

Sol. 57 $x-2y=0 \Rightarrow x=2y$

$\because 3x^3-2x^2y-13xy^2+10y^3$

$3(2y)^3-2(2y)^2y-13(2y)y^2+10y^3$

$24y^3-8y^3-26y^3+10y^3$

$34y^3-34y^3=0$

Remainder = 0

Sol. 58 Hint: See Example 27

Sol. 59 $x^4+1=0 \Rightarrow x^4=-1$

$\because x^{40}+2=(x^4)^{10}+2=(-1)^{10}+2$

$= 1+2=3$

Sol. 60 $x^2-2(k-1)x+(2k+1)=0$

When roots of equation are equal then

$b^2-4ac=0$

$[-2(k-1)]^2-4(1)(2k+1)=0$

$\Rightarrow 4(k^2+1-2k)-4(2k+1)=0$

$\Rightarrow 4k^2+4-8k-8k-4=0$

$\Rightarrow 4k^2-16k=0$

$\Rightarrow 4k-16=0$

$\Rightarrow 4k=16 \Rightarrow k=4$

Sol. 61 $3^{2x^2-7x+7}=9=3^2$

Hint: Compare power and solve or we can put option in given equation.

Sol. 62 $x^{16}+x^8+1=(x^8)^2+2.x^8+1-x^8$

$=[(x^8+1)^2-(x^4)^2]$

$=[(x^8+x^4+1)(x^8-x^4+1)]$

$=[(x^4)^2+2x^4+1-x^4][x^8-x^4+1]$

$=[(x^4+1)^2-(x^2)^2](x^8-x^4+1)$

$=(x^4+x^2+1)(x^4-x^2+1)(x^8-x^4+1)$

$=[(x^2)^2+2x^2+1-x^2](x^4-x^2+1)(x^8-x^4+1)$

$=[(x^2+1)^2-x^2](x^4-x^2+1)(x^8-x^4+1)$

$=(x^2+x+1)(x^2-x+1)(x^4-x^2+1)(x^8-x^4+1)$

$=(x^2+2x+1-x)(x^2-x+1)(x^4-x^2+1)(x^8-x^4+1)$

$=[(x+1)^2-\left(\sqrt{x}\right)^2](x^2-x+1)(x^4-x^2+1)(x^8-x^4+1)$

$=(x+1+\sqrt{x})(x+1-\sqrt{x})(x^2-x+1)(x^4-x^2+1)(x^8-x^4+1)$

$\therefore x^2+1$ is not a factor.

Sol. 63 $ax^2+bx+c=a(x-p)^2$

$=a(x^2+p^2-2xp)$

$\therefore ax^2+bx+c=ax^2+ap^2-2axp$

Comparing both sides

$a=a,\ b=-2ap \Rightarrow p=\dfrac{b}{-2a}$ and $c=ap^2$

Put the value of p

$c=a\times\left(\dfrac{b}{-2a}\right)^2=a\times\dfrac{b^2}{4a^2}$

$\boxed{b^2=4ac}$

Sol. 64 $11x-61\le 5$

$\Rightarrow 11x\le 66 \Rightarrow x\le 6$

Equation of inequality

Sol. 65 $(x^2+x+1)(x^2-x+1)=x^4+x^2+1$

coefficient of $x^2=1$

Sol. 66 $16\left(\dfrac{a-x}{a+x}\right)^3=\dfrac{a+x}{a-x}$

$\Rightarrow 16(a-x)^4=(a+x)^4$

Taking square root, we get

$4(a-x)^2=(a+x)^2$

Again taking square root

$2(a-x)=a+x$

$\Rightarrow 2a-2x=a+x$

$a=3x$

$\Rightarrow \boxed{x=\dfrac{a}{3}}$

Sol. 67 If $(x+1)$ is H.C.F. of ax^2+bx+c and bx^2+ax+c, then $(x+1)$ is also factor of the equation

$\therefore x+1=0 \Rightarrow x=-1$

$ax^2+bx+c=a(-1)^2+b(-1)+c=0$

$\Rightarrow a-b+c=0$ ...(i)

and $bx^2+ax+c=0$

$\Rightarrow b(-1)^2-a+c=0$

$\Rightarrow b-a+c=0$ ...(ii)

Comparing (i) and (ii)

$a-b+c=b-a+c$

hence, can't be determined

Sol. 68 $2x^2 - 7x + 12 = 0$

$$\alpha + \beta = \left(-\frac{7}{2}\right) = \frac{7}{2} \quad \ldots(i)$$

and $\alpha\beta = \frac{12}{2} = 6 \quad \ldots(ii)$

Now,

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$$

Put the value from equation (i) and (ii)

$$\therefore \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{\left(\frac{7}{2}\right)^2 - 2(6)}{6} = \frac{\frac{49}{4} - 12}{6}$$

$$= \frac{49-48}{4\times 6} = \frac{1}{24}$$

Sol. 69 $x - \frac{k}{2} = 0 \Rightarrow x = \frac{k}{2}$

$\therefore \quad 3x^3 - kx^2 + 4x + 16 = 0$

$\Rightarrow \quad 3\left(\frac{k}{2}\right)^3 - k\left(\frac{k}{2}\right)^2 + 4\left(\frac{k}{2}\right) + 16 = 0$

$\Rightarrow \quad \frac{3}{8}k^3 - \frac{k^3}{4} + 2k + 16 = 0$

$\Rightarrow \quad \frac{3k^3 - 2k^3}{8} + 2k + 16 = 0$

$\Rightarrow \quad \frac{k^3}{8} + 2k + 16 = 0$

$\Rightarrow \quad k^3 + 16k + 128 = 0$

From option, k = – 4, satisfying this equation.

Sol. 70 $\frac{2}{x} + \frac{3}{y} = \frac{9}{xy} \Rightarrow \frac{2y+3x}{xy} = \frac{9}{xy}$

$3x + 2y = 9 \quad \ldots(i)$

$\frac{4}{x} + \frac{9}{y} = \frac{21}{xy} \Rightarrow \frac{4y+9x}{xy} = \frac{21}{xy}$

$\Rightarrow \quad 9x + 4y = 21 \quad \ldots(ii)$

Multiple equation (i) by 2 and subtract from equation (ii)

$$\begin{aligned} 9x + 4y &= 21 \\ 6x + 4y &= 18 \\ - \quad - \quad &\quad - \\ \hline 3x \quad\quad &= 3 \end{aligned}$$

$\Rightarrow x = 1$

Put the value of x in (i)

$3 + 2y = 9 \Rightarrow 2y = 6$

$y = 3$

$\therefore x + y = 1 + 3 = 4$

Sol. 71 Hint: Put option in given equations.

Sol. 72 $f(x) = 3x^4 - 15x^3 + 17x^2 + 5x - 6$

if one roots is $\alpha = \frac{1}{\sqrt{3}}$ then other is $\beta = -\frac{1}{\sqrt{3}}$.

Sum of roots $= \alpha + \beta + \gamma + \delta = -\frac{b}{a} = -\frac{-15}{3} = 5$

$\gamma + \delta = 5 \quad \ldots(i)$

Product of roots $= \alpha\beta\gamma\delta = -\frac{e}{a} = -\frac{-6}{3} = 2$

$\gamma\delta = 6 \quad \ldots(ii)$

On solving, $\gamma = 3, \delta = 2$

Sol. 73 $x - 2 = 0 \Rightarrow x = 2$

Now,

$x^3 + 3x^2 - kx + 4 = k$

$\Rightarrow \quad (2)^3 + 3(2)^2 - k(2) + 4 = k$

$\Rightarrow \quad 8 + 12 - 2k + 4 = k$

$\Rightarrow \quad 24 = 3k$

$\Rightarrow \quad \boxed{k = 8}$

Sol. 74 $\sqrt{\frac{x}{x+3}}-\sqrt{\frac{x+3}{x}}=-\frac{3}{2}$

Take $x=1$

$\because$ L.H.S.

$\sqrt{\frac{x}{x+3}}-\sqrt{\frac{x+3}{x}}=\sqrt{\frac{1}{4}}-\sqrt{\frac{4}{1}}$

$=\frac{1}{2}-2=-\frac{3}{2}=$ R.H.S.

Sol. 75 Hint: See example 126

Sol. 76 $3a(3a+2c)-4b(b+c)=9a^2+6ac-4b^2-4bc$

$=9a^2-4b^2+6ac-4bc$

$=(3a-2b)(3a+2b)+2c(3a-2b)$

$=(3a-2b)(3a+2b+2c)$

then, other factor is $(3a+2b+2c)$

Sol. 77 Put $x=3$ in $3x^2+(k-1)x+9=0$

$\Rightarrow$ $3(3)^2+(k-1)3+9=0$

$\Rightarrow$ $27+3k-3+9=0$

$\Rightarrow$ $3k+33=0$

$\Rightarrow$ $3k=-33$

$\boxed{k=-11}$

Sol. 78 Hint: See example 35

Sol. 79 $x^2-5x+6=0$

let $y=\frac{1}{\alpha}$

$\alpha=\frac{1}{y}$

α is the root of equation $x^2-5x+6=0$

So, Put $\alpha=\frac{1}{y}$ in this equation.

$\left(\frac{1}{y}\right)^2-5\left(\frac{1}{y}\right)+6=0$

$1-5y+6y^2=0$

Replace y by x

$\Rightarrow$ $6x^2-5x+1=0$

Sol. 80 $x^2-6x+k=0\Rightarrow\alpha+\beta=6\ \&\ \alpha\beta=k$

on solving $\alpha+\beta=6$ & $3\alpha+2\beta=20$

$\Rightarrow\alpha=8,\ \beta=-2$ (put these values in $\alpha\beta=k$)

$8\times(-2)=k\Rightarrow k=-16$

Sol. 81 Hint : See example 35

Sol. 82 $12x^2+mx+5=0\Rightarrow\alpha+\beta=-\frac{m}{12}\ \&\ \alpha\beta=\frac{5}{12}$

$\frac{\alpha}{\beta}=\frac{3}{2}$ (let $\alpha=3k\ \&\ \beta=2k$)

$(3k)(2k)=\frac{5}{12}\Rightarrow k^2=\frac{5}{72}\Rightarrow k=\pm\frac{\sqrt{5}}{6\sqrt{2}}$

$(3k+2k)=-\frac{m}{12}\Rightarrow 5k=-\frac{m}{12}$

$m=-60k=-60\left(-\frac{\sqrt{5}}{6\sqrt{2}}\right)=5\sqrt{10}$

Sol. 83 $x^2-3kx+k^2=0$

$\alpha+\beta=3k$ …(i)

$\alpha\beta=k^2$ …(ii)

$\alpha^2+\beta^2=\frac{7}{4}$

$\Rightarrow$ $(\alpha+\beta)^2-2\alpha\beta=\frac{7}{4}$

From equation (i) and (ii)

$(-3k)^2-2k^2=\frac{7}{4}$

$9k^2-2k^2=\frac{7}{4}$

$$7k^2 = \frac{7}{4}$$

$$k = \pm\frac{1}{2}$$

Sol. 84 $x^2 + kx + 12 = 0$

$\alpha + \beta = -k$ …(i)

$\alpha\beta = 12$ …(ii)

Also $\alpha - \beta = 1$ …(iii)

Adding equation (i) and (iii)

$$\alpha + \beta = -k$$
$$\alpha - \beta = 1$$
$$2\alpha = 1 - k$$

$$\Rightarrow \alpha = \frac{1-k}{2}$$

Put the value of α in equation (iii)

$$\frac{1-k}{2} - \beta = 1 \qquad \Rightarrow \beta = \frac{1-k}{2} - 1$$

$$\Rightarrow \beta = \frac{1-k-2}{2} = \frac{-1-k}{2}$$

Put the value of $\alpha + \beta$ in equation (ii)

$$\alpha\beta = 12$$

$$\frac{1-k}{2} \times \frac{-(1+k)}{2} = 12$$

$$(k-1)(k+1) = 48$$

$$k^2 = 49 \Rightarrow k = \pm 7$$

Sol. 85 $\alpha + \beta = \alpha\beta$ $(5x^2 + x - k - 5 = 0)$

$$-\frac{1}{5} = -\frac{k+5}{5} \Rightarrow k + 5 = 1 \Rightarrow k = -4$$

Sol. 86 Hint: Put option in given Equation.

Sol. 87 let $\frac{x^2+2x+1}{x^2+2x+7} = y$

$$x^2(y-1) + x(2y-2) + 7y - 1 = 0$$

x is real , then $b^2 - 4ac \geq 0$

$$(4y^2 + 4 - 8y) - 4(y-1)(7y-1) \geq 0$$

$$(y^2 + 1 - 2y) - (7y^2 - 8y + 1) \geq 0$$

$$-6y^2 + 6y \geq 0$$

$$y(1-y) \geq 0 \Rightarrow y(y-1) \leq 0$$

hence, $y \in [0,1]$

Sol. 88 $\alpha + \beta = -\frac{1}{6}$, $\alpha.\beta = -\frac{1}{3}$

$$\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$= \left(\frac{\left(-\frac{1}{6}\right)^2 - 2\left(-\frac{1}{3}\right)}{-\frac{1}{3}}\right) = -\frac{25}{12}$$

Sol. 89 Using formula $A^2 - B^2 = (A+B)(A-B)$

$(5x^2 + 14x + 2)^2 - (4x^2 - 5x + 7)^2$

$= (9x^2 + 9x + 9)(x^2 + 19x - 5)$

$= 9(x^2 + x + 1)(x^2 + 19x - 5)$

If this is divided by $(x^2 + x + 1)$ then remainder will be zero.

Sol. 90 Hint: See example 4.

Sol. 91 let roots are $\alpha \,\&\, \frac{1}{\alpha}$

$$\Rightarrow \text{product of roots} = \alpha.\frac{1}{\alpha} = 1 = \frac{c}{a}$$

$$\frac{k}{7} = 1 \Rightarrow k = 7$$

Sol. 92 $x^2 - 9x + k = 0$

lets roots are $\alpha \,\&\, 2\alpha$

$$\alpha + 2\alpha = 9 \Rightarrow \alpha = 3$$

$$\alpha.2\alpha = k \Rightarrow k = 2\alpha^2 = 18$$

Sol. 93 Hint: put option in given equations.

Sol. 94 Hint: See remainder theorem concept (find the value of a & b firstly)

Sol. 95 Hint: Check by option

Sol. 96 $\frac{a}{b} - \frac{b}{a} = \frac{x}{y} \Rightarrow y = \frac{x}{\frac{a}{b} - \frac{b}{a}} = \frac{xab}{a^2 - b^2}$

$$\because \frac{a}{b} + \frac{b}{a} = x - y$$

$$\therefore \frac{a}{b} + \frac{b}{a} = x - \frac{xab}{a^2 - b^2}$$

$$\Rightarrow x\left(\frac{a^2-b^2-ab}{a^2-b^2}\right)=\frac{a^2+b^2}{ab}$$

$$x=\frac{a^4-b^4}{ab(a^2-b^2-ab)}$$

Sol. 97 When roots are reciprocal to each other then

$$\alpha\beta=\frac{C}{A}=1$$

$$\alpha\beta=\frac{C}{A}=1\Rightarrow\frac{\frac{1}{c}}{\frac{1}{a}}=1\Rightarrow\frac{1}{c}=\frac{1}{a}$$

hence, $c=a$

Sol. 98 $a^2b^2x^2-(a^2+b^2)x+1=0$

$$\alpha+\beta=\frac{a^2+b^2}{a^2b^2}=\frac{1}{a^2}+\frac{1}{b^2}$$

$$\alpha\beta=\frac{1}{a^2b^2}$$

hence, $\alpha=\frac{1}{a^2},\beta=\frac{1}{b^2}$

Sol. 99 put anything in place of x
let $x=a$
$x(x+a)(x+2a)(x+3a)=a(a+a)(a+2a)(a+3a)=24a^4$
hence, a^4 is required to be added to make perfect square.

Sol. 100 let t is added to $(1-x)(1+x^2)$ to obtain x^3

$t+(1-x)(1+x^2)=x^3$
$t+1+x^2-x-x^3=x^3$
$t=2x^3-x^2+x-1$

Sol. 101 $5px-10qy+2rpx-4qry$
$=5px+2rpx-10qy-4qry$
$=px(5+2r)-2qy(5+2r)$
$=(5+2r)(px-2qy)$

Sol. 102 $x^4+xy^3+x^3y+xz^3+y^4+yz^3$

$=x(x^3+y^3+z^3)+y(x^3+y^3+z^3)$

$=(x^3+y^3+z^3)(x+y)$

Sol. 103 Hint: Use Remainder theorem Concept

Sol. 104 Hint: If roots are equal then $B^2-4AC=0$

Sol. 105 $x^2-x-1=0\Rightarrow\alpha+\beta=1$ & $\alpha\beta=-1$

$$\frac{\alpha^2+\beta^2}{(\alpha^2-\beta^2)(\alpha-\beta)}=\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha+\beta)(\alpha-\beta)^2}$$

$$=\frac{1+2}{(1)(\alpha^2+\beta^2-2\alpha\beta)}=\frac{3}{(3+2)}=\frac{3}{5}$$

Sol. 106 Hint: Use Remainder theorem Concept

Sol. 107 Hint: Use Remainder theorem Concept

Sol. 108 Hint: Use Factor theorem Concept

Sol. 109 $\alpha+\beta=-\frac{b}{a}=-\frac{(-6)}{3}=2$ & $\alpha\beta=\frac{c}{a}=\frac{2}{3}$

$$\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta$$

$$=\frac{\alpha^2+\beta^2}{\alpha\beta}+2\left(\frac{\alpha+\beta}{\alpha\beta}\right)+3\alpha\beta$$

$$=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}+2\left(\frac{\alpha+\beta}{\alpha\beta}\right)+3\alpha\beta$$

$$=\frac{(2)^2-2\times\frac{2}{3}}{\frac{2}{3}}+2\left(\frac{2}{2/3}\right)+3\times\frac{2}{3}=12$$

Sol. 110 $\sqrt{\frac{x}{1-x}}+\sqrt{\frac{1-x}{x}}=2\frac{1}{6}$

$$\frac{x+1-x}{\sqrt{x(1-x)}}=\frac{13}{6}$$

Squaring both sides,

$$\frac{1}{x-x^2}=\frac{169}{36}$$

$$x-x^2=\frac{36}{169}$$

$$x^2-x+\frac{36}{169}=0$$

$$\alpha+\beta=\frac{-b}{a}=\frac{-(-1)}{1}$$

$$\alpha+\beta=1$$

Exercise - (Basic Identity)

1. If $x+\frac{1}{x}=2$, then find the value of $x^9+\frac{1}{x^9}=?$

(a) 0 (b) 4
(c) 2 (d) 6

2. If $x+\frac{1}{x}=2$, then find the value of $x^5+x^4+x^3+x^2+x+1=?$

(a) 0 (b) 6
(c) 2 (d) 4

3. If $x+\frac{1}{x}=3$, then find the value of $x^4+\frac{1}{x^4}=?$

(a) 49 (b) 42
(c) 38 (d) 47

4. If $x+\frac{1}{x}=5$, then find the value of $x^5+\frac{1}{x^5}$.

(a) 2525 (b) 2550
(c) 2500 (d) 2700

5. If $x^2-2x+1=0$, find the value of $\frac{x^5+x^4+x^3+x^2}{x}$.

(a) 0 (b) 2
(c) 3 (d) 4

6. If $x+\frac{1}{x}=2$, then find the value of $\frac{10x}{3x^2-4x+3}$.

(a) 0 (b) 5
(c) 3 (d) 4

7. If $x+\frac{1}{x}=5$, then find the value of $\frac{x^6+x^4+x^2+1}{x^3}$.

(a) 115 (b) 110
(c) 140 (d) 125

8. If $x^2-7x+1=0$, then find the value of

$$\frac{20x}{5x^2-15x+5}\Rightarrow\frac{20x}{5(x^2-3x+1)}=\frac{4x}{4x}=1$$

(a) 1 (b) 2
(c) 3 (d) 4

9. If $2x+\frac{1}{3x}=5$, then find the value of $16x^2+\frac{4}{9x^2}$.

(a) $\frac{284}{3}$ (b) $\frac{84}{3}$
(c) $\frac{184}{3}$ (d) 3

10. If $x+\frac{1}{x}=-2$, then find $x^{2n+1}+\frac{1}{x^{2n+1}}$, where n is an positive integer.

(a) 0 (b) 2
(c) -2 (d) -5

11. If $5a+\frac{1}{3a}=5$, then find the value of $9a^2+\frac{1}{25a^2}$.

(a) 0 (b) 5
(c) 7.8 (d) 4

12. If $x-\frac{1}{x-3}=0$, then find the value of $(x-3)^2+\frac{1}{(x-3)^2}$.

(a) 9 (b) 7
(c) 11 (d) 14

13. If $x^2+\frac{1}{x^2+1}=6$, then find the value of $(x^2+1)^2+\frac{1}{(x^2+1)^2}$.

(a) 0 (b) 5
(c) 47 (d) 4

14. If $x^2+y^2+\frac{1}{x^2}+\frac{1}{y^2}=4$, then the value of x^2+y^2 is

(a) 2 (b) 4
(c) 8 (d) 16

15. If $x^2+y^2+\frac{1}{x^2}+\frac{1}{y^2}=0$, then the value of x^2+y^2 is

(a) 0 (b) 4
(c) 8 (d) 16

16. If $\frac{2p}{p^2-2p+1}=\frac{1}{4}, p\neq 0$, then the value of $p+\frac{1}{p}$ is

(a) 4 (b) 5
(c) 10 (d) 12

17. If $\left(x+\frac{1}{x}\right)^2=3$, then the value of $x^{206}+x^{200}+x^{90}+x^{84}+x^{18}+x^{12}+x^6+1$ is

(a) 0 (b) 1
(c) 84 (d) 206

18. If $x+\frac{1}{x}=5$, then the value of $\frac{x^4+3x^3+5x^2+3x+1}{x^4+1}$ is

(a) $\frac{43}{23}$ (b) $\frac{47}{21}$

(c) $\frac{41}{23}$ (d) $\frac{45}{21}$

19. If $a+b+c=0$, then the value of $\frac{3(a+b)(b+c)(c+a)}{abc}$.

(a) 3 (b) -1
(c) 1 (d) -3

20. If $x^2+y^2+2x+1=0$, then the value of $x^{31}+y^{35}$.

(a) -1 (b) 0
(c) 1 (d) 2

21. If $a^2+b^2+2b+4a+5=0$, then the value of $\frac{a-b}{a+b}$.

(a) 3 (b) -3

(c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

22. If $x^2+y^2-4x-4y+8=0$, then the value of $x-y$.

(a) 4 (b) -4
(c) 0 (d) 8

23. If $x+\frac{1}{x}=\sqrt{3}$, then the value of $x^{17}+\frac{1}{x^{17}}$ is:

(a) $\sqrt{3}$ (b) $-\sqrt{3}$
(c) 1 (d) 0

24. If $x+\frac{1}{x}=\sqrt{3}$, then the value of $x^6-\frac{1}{x^6}+2$ is:

(a) $\sqrt{3}$ (b) 2
(c) 1 (d) 0

25. If $x^2+x+1=0$, then the value of x^3+1 is:

(a) 0 (b) 1
(c) 2 (d) -1

26. If $x+\frac{1}{x}=1$, then the value of $x^{12}+x^9+x^6+x^3+1$.

(a) -1 (b) -2
(c) 1 (d) 2

27. If $\left(x+\frac{1}{x}\right)^2=3$, then the value of

$x^{206}+x^{200}+x^{90}+x^{84}+x^{18}+x^{12}+1$

(a) 84 (b) 206
(c) 0 (d) 1

28. If $\left(x+\frac{1}{x}\right)=\sqrt{3}$, then the value of

$x^{506}+x^{500}+x^{206}+x^{200}+x^{90}+x^{84}+x^{18}+x^{12}$ is

(a) 84 (b) 206
(c) 0 (d) 1

29. If $x=(a+b-c), y=(b+c-a)$ & $z=(c+a-b)$,

then $(x-a)^3+(y-b)^3+(z-c)^3=?$

(a) $3(x-a)(y-b)(z-c)$ (b) $3xyz$
(c) $(x-a)(y-b)(z-c)$ (d) $3abc$

30. If $a+b+c=0$, then find $\frac{a+b}{c}-\frac{2b}{c+a}+\frac{b+c}{a}$.

(a) -1 (b) 0
(c) 1 (d) 2

31. If $a+b+c=0$, then the value of

$$\frac{1}{(a+b)(b+c)}+\frac{1}{(a+c)(b+c)}+\frac{1}{(b+a)(c+a)}$$

(a) 1 (b) 0
(c) -1 (d) -2

32. $\frac{(a+b)^2-(a-b)^2}{a^2b-ab^2}=?$

(a) $\frac{1}{a-b}$ (b) $\frac{2}{a-b}$
(c) $\frac{4}{a-b}$ (d) $\frac{1}{ab}$

33. If $4x=8y$, then the value of $\left(\frac{x}{y}-1\right)$ is :

(a) 1 (b) 2
(c) 3 (d) 4

34. If $x+\frac{1}{x}=a$, then what is the value of

$x^3+x^2+\frac{1}{x^3}+\frac{1}{x^2}$.

(a) a^3+a^2 (b) a^3+a^2-5a
(c) a^3+a^2-3a-2
(d) a^3+a^2-4a-2

35. If $x^4+\frac{1}{x^4}=119$ and $x>1$, then the value of

$x^3-\frac{1}{x^3}$ is

(a) 54 (b) 18
(c) 72 (d) 36

36. If $\frac{x^2-1}{x}=2\sqrt{3}$, then $\left(x^2+\frac{1}{x^2}\right)\left(x+\frac{1}{x}\right)=?$

(a) 21 (b) 15
(c) 56 (d) 12

37. If $x+y=a$ and $xy=b^2$, then find the value of $x^3-x^2y-xy^2+y^3$ in terms of a and b.

(a) $(a^2+4b^2)a$ (b) a^3-3b^2
(c) a^3-4b^2a (d) a^3+3b^2

38. If $a=298, b=297, c=296$, then find the value of $a^2+b^2+c^2-ab-bc-ca$.

(a) 4 (b) 5
(c) 3 (d) -3

39. If $a=874, b=875, c=877$, then find the value of $a^2+b^2+c^2-ab-bc-ca$.

(a) 4 (b) 5
(c) 3 (d) 7

40. If $a=9, b=10, c=11$, then find the value of $a^3+b^3+c^3-3abc$.

(a) 70 (b) 50
(c) 30 (d) 90

41. If $a = 115, b = 116, c = 117$, then find the value of $a^3 + b^3 + c^3 - 3abc$.
(a) 1404 (b) 2044
(c) 1044 (d) 2444

42. If $(a-1)^2 + (b-2)^2 = 0$, then find the value of $a + b$.
(a) 5 (b) -3
(c) -2 (d) 3

43. If $(a-1)^2 + (b-2)^2 + (c-3)^2 = 0$, then find the value of $a + b + c$.
(a) -6 (b) -3
(c) -2 (d) 6

44. If $(a+1)^2 + (b+2)^2 + (c+3)^2 + (d+4)^2 = 0$, then find the value of $a + b + c + d$.
(a) 10 (b) -30
(c) 24 (d) -10

45. If $a^2 + b^2 + 4a + 6b + 13 = 0$, then find the value of $a + b$.
(a) -5 (b) -3
(c) 5 (d) 6

46. If $a^2 + b^2 + c^2 - 4a - 6b - 8c + 29 = 0$, then find the value of $a + b + c$.
(a) -9 (b) -3
(c) 5 (d) 9

47. If $a^2 + b^2 + c^2 + 2(a+b+c) + 3 = 0$, then find the value of $(a + b + c)$.
(a) 9 (b) 3
(c) -3 (d) -9

48. Solve $\dfrac{(a-b)^3 + (b-c)^3 + (c-a)^3}{(a-b)(b-c)(c-a)}$.
(a) 9 (b) -3
(c) 3 (d) -9

49. If $x^a x^b x^c = 1$, then find $a^3 + b^3 + c^3$.
(a) $a + b + c$ (b) $3abc$
(c) 9 (d) $-3abc$

50. If $x = a(b-c), y = b(c-a)$ & $z = c(a-b)$, then the value of $\dfrac{x^3}{a^3} + \dfrac{y^3}{b^3} + \dfrac{z^3}{c^3}$ is
(a) $\dfrac{3abc}{xyz}$ (b) $\dfrac{abc}{xyz}$
(c) $\dfrac{3xyz}{abc}$ (d) 1

51. If $x = y(b-c), y = z(c-a)$ and $z = x(a-b)$, then the value of $\dfrac{x^3}{y^3} + \dfrac{y^3}{z^3} + \dfrac{z^3}{x^3}$ is
(a) -9 (b) -3
(c) 1 (d) 3

52. If $a + b + c = 0$, then what is the value of $\dfrac{1}{abc}\{(a+b+c)^3 - a^3 - b^3 - c^3\}$.
(a) 9 (b) -3
(c) 3 (d) -9

53. If $b - a = 1$, then $a^3 - b^3 + 3ab = ?$
(a) 1 (b) 0
(c) 3 (d) -1

54. If $x + y + z = 10, x^2 + y^2 + z^2 = 30$, then find the value of $x^3 + y^3 + z^3 - 3xyz$.
(a) 50 (b) -50
(c) 60 (d) -60

55. If $x + y + z = 9, x^2 + y^2 + z^2 = 35$, then find the value of $x^3 + y^3 + z^3 - 3xyz$.
(a) 105 (b) 108
(c) 109 (d) 125

56. If $a = 25, b = 15, c = -10$, then find $\dfrac{(a)^3 + (b)^3 + (c)^3 - 3abc}{(a-b)^2 + (b-c)^2 + (c-a)^2}$.
(c) 30 (b) -15
(c) -30 (d) 15

57. If $a = -5, b = -6, c = 10$, then find $\dfrac{(a)^3 + (b)^3 + (c)^3 - 3abc}{(ab + bc + ca - a^2 - b^2 - c^2)}$.
(a) -1 (b) 1
(c) 21 (d) 18

58. If $a + b + c = 0$, then the value of $(a + b)(b + c)(c + a) + abc$ is equal to
(a) -1 (b) 1
(c) 0 (d) 18

59.
$$\frac{1}{(a-b)(b-c)} + \frac{1}{(b-c)(c-a)} + \frac{1}{(c-a)(a-b)} = ?$$
(a) abc (b) 1
(c) 0 (d) $-abc$

60. If $a+b+c=0$, then find the value of $(a-b)(b-c)+(b-c)(c-a)+(c-a)(a-b)$.
(a) $3abc$
(b) $3(ab+bc+ca)$
(c) $3(ab-bc-ca)$
(d) None of these

61. If $a+b+c=0$, then find the value of $\frac{1}{bc}+\frac{1}{ac}+\frac{1}{ab}$.
(a) 2 (b) -1
(c) 0 (d) $-abc$

62. If $a+b+c=0$, then find the value of $\frac{a^2+b^2+c^2}{a^2-bc}$.
(a) 0 (b) 1
(c) 2 (d) 3

63. If $a+b+c=0$, then the value of $\frac{a^2-bc}{b^2-ca}$ is
(a) 0 (b) 1
(c) 2 (d) 3

64. If $a+b+c=0$, then find the value of $\frac{(a^2+b^2+c^2)^2}{a^2b^2+b^2c^2+c^2a^2}$.
(a) 1 (b) 2
(c) 3 (d) 4

65. If $a+b+c=0$, what is the value of $\frac{a^2b^2+b^2c^2+c^2a^2}{a^4+b^4+c^4}$.
(a) $\frac{2}{3}$ (b) $\frac{3}{2}$
(c) $\frac{1}{2}$ (d) $\frac{3}{4}$

66. If $a+b+c=0$, then the value of the expression, $\frac{1}{a^2+b^2-c^2}+\frac{1}{b^2+c^2-a^2}+\frac{1}{c^2+a^2-b^2}=?$
(a) 0 (b) 1
(c) 3 (d) $a+b+c$

67. If $a(a+2)=a+b+c$, $b(b+2)=a+b+c$ & $c(c+2)=a+b+c$, then find the value of $\frac{1}{a+2}+\frac{1}{b+2}+\frac{1}{c+2}$.
(a) -1 (b) 2
(c) 1 (d) 0

68. If $a+b+c=2s$, then find the value of $\frac{(s-a)^2+(s-b)^2+(s-c)^2+s^2}{a^2+b^2+c^2}$.
(a) 1 (b) 0
(c) $a^2+b^2+c^2$ (d) 2

69. If $\frac{x}{y}+\frac{y}{x}=1,\ (x,y\neq 0)$, then $2(x^3+y^3)=?$
(a) 2 (b) 1
(c) -1 (d) 0

70. If $\frac{x}{y}+\frac{y}{x}=-2,\ (x,y\neq 0)$, then $\{x^3+y^3+3xy(x+y)\}=?$
(a) 1 (b) 0
(c) -1 (d) 2

71. If $\frac{a}{b}+\frac{b}{a}=3$, then $\frac{a^3}{b^3}+\frac{b^3}{a^3}=?$
(a) 18 (b) 36
(c) 24 (d) 12

72. If $\frac{11}{x+11}+\frac{23}{y+23}+\frac{239}{z+239}=2$, then what is the value of $\frac{x}{x+11}+\frac{y}{y+23}+\frac{z}{z+239}$
(a) 0 (b) 1
(c) 2 (d) 4

73. If x, y, z are positive integers such that $x^2+y^2=45$ and $y^2+z^2=40$, then find the value of $x+y+z$.
(a) 11 (b) 10
(c) 20 (d) 15

74. If $\frac{x-y}{3}=\frac{x+y}{7}=\frac{xy}{5}$, then the numerical value of xy is

(a) $\frac{4}{3}$ (b) $\frac{3}{4}$

(c) $\frac{5}{2}$ (d) $\frac{1}{2}$

75. If $x = 2 - 2^{1/3} + 2^{2/3}$, then find the value of $x^3 - 6x^2 + 18x + 18$.

(a) 22 (b) 40

(c) 33 (d) 45

76. If $x = b + c - 2a$, $y = c + a - 2b$, $z = a + b - 2c$, then find the value of $x^2 + y^2 - z^2 + 2xy$.

(a) $a + b + c$ (b) $a + b - c$

(c) 0 (d) $a - b + c$

77. If $pq(p + q) = 1$, then find the value of $\frac{1}{p^3q^3} - p^3 - q^3$.

(a) 1 (b) -2

(c) 3 (d) 0

78. If $\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = a - b\sqrt{6}$, then find a and b.

(a) $a = 1, b = 4$ (b) $a = 2, b = 3$

(c) $a = 4, b = 1$

(d) $a = 2, b = -5/6$

79. If $\frac{5+\sqrt{3}}{7-4\sqrt{3}} = 47a + \sqrt{3}b$, then find a and b.

(a) $a = 3, b = 5$ (b) $a = 1, b = 27$

(c) $a = 7, b = 3$ (d) $a = 2, b = 7$

80. If $x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and $y = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, then find $x^2 + y^2$.

(a) 96 (b) 98

(c) 90 (d) 100

81. If $a = 3.23$, $b = 5.95$ and $c = 2.72$, then the value of $a^3 - b^3 + c^3 + 3abc$ is

(a) 0 (b) 1

(c) 2 (d) 3

82. If $5^{\sqrt{x}} + 12^{\sqrt{x}} = 13^{\sqrt{x}}$, then x is equal to

(a) $\frac{25}{4}$ (b) 4

(c) 9 (d) 16

83. The value of $\left(x^{b+c}\right)^{b-c}.\left(x^{c+a}\right)^{c-a}.\left(x^{a+b}\right)^{a-b}$ $(x \neq 0)$ is

(a) 1 (b) 2

(c) -1 (d) 0

84. The value of $\left(\frac{x^a}{x^b}\right)^{a+b}.\left(\frac{x^b}{x^c}\right)^{b+c}.\left(\frac{x^c}{x^a}\right)^{c+a} = ?$

(a) 0 (b) 1

(c) x^{abc} (d) $x^{(a+b+c)}$

85. The value of

$$\frac{1}{1+x^{b-a}+x^{c-a}}+\frac{1}{1+x^{a-b}+x^{c-b}}+\frac{1}{1+x^{b-c}+x^{a-c}} = ?$$

(a) 0 (b) 1

(c) x^{a-b-c} (d) None of these

86. The value of

$$\left(\frac{x^b}{x^c}\right)^{b+c-a}.\left(\frac{x^c}{x^a}\right)^{c+a-b}.\left(\frac{x^a}{x^b}\right)^{a+b-c} = ?$$

(a) 1 (b) x^{abc}

(c) $x^{(a+b+c)}$ (d) $x^{(ab+bc+ca)}$

87. $\left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}}.\left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}}.\left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} = ?$

(a) 1 (b) $\frac{1}{x^{abc}}$

(c) $\frac{1}{x^{ab+bc+ca}}$ (d) None of these

88. If $a = \dfrac{\sqrt{5}+1}{\sqrt{5}-1}$ and $b = \dfrac{\sqrt{5}-1}{\sqrt{5}+1}$, then the value of $\dfrac{a^2+ab+b^2}{a^2-ab+b^2}$ is

(a) $\dfrac{3}{4}$ (b) $\dfrac{4}{3}$

(c) $\dfrac{3}{5}$ (d) $\dfrac{5}{3}$

89. If $x=\sqrt{a}+\dfrac{1}{\sqrt{a}}$ and $y=\sqrt{a}-\dfrac{1}{\sqrt{a}}$, then find the value of $x^4 + y^4 - 2x^2y^2$ is

(a) 14 (b) 16

(c) 10 (d) 18

90. If $a^4 + a^2b^2 + b^4 = 21$ and $a^2 + ab + b^2 = 7$, then the value of ab is –

(a) –1 (b) 0

(c) 2 (d) 1

91. If $a + b + c = 0$, then value of

$$\frac{1}{x^b + x^{-c} + 1} + \frac{1}{x^c + x^{-a} + 1} + \frac{1}{x^a + x^{-b} + 1}$$

is

(a) 1 (b) 0

(c) abc (d) x

92. If $\dfrac{7x-3}{x}+\dfrac{7y-3}{y}+\dfrac{7z-3}{z}=0$, then the value of $\dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}$ is

(a) 7 (b) 9

(c) 1 (d) 0

93. If $2^x = 4^y = 8^z$ and $\dfrac{1}{2x}+\dfrac{1}{4y}+\dfrac{1}{4z}=4$, then find the value of x

(a) $\dfrac{7}{16}$ (b) $\dfrac{16}{7}$

(c) $\dfrac{8}{16}$ (d) $\dfrac{6}{16}$

94. If $\dfrac{x^2}{by+cz} = \dfrac{y^2}{cz+ax} = \dfrac{z^2}{ax+by} = 4$, then the value of $\dfrac{a}{4a+x} + \dfrac{b}{4b+y} + \dfrac{c}{4c+z}$ is

(a) –1 (b) 1/4

(c) 1 (d) – 2

95. If $\sqrt{\dfrac{x-a}{x-b}} + \dfrac{a}{x} = \sqrt{\dfrac{x-b}{x-a}} + \dfrac{b}{x}$, $(b \neq a)$, then the value of x is

(a) $\dfrac{ab}{a+b}$ (b) 1

(c) $\dfrac{a}{a+b}$ (d) $\dfrac{b}{a+b}$

96. If $x^2 - 2xy + 5y^2 - 4y + 1 = 0$, then the value of x and y are :

(a) $x = 1/2, y = 2$

(b) $x = 1/2, y = 1/2$

(c) $x = 1, y = -1$

(d) $x = 2, y = 1$

97. If $a^x = b$, $b^y = c$ and $xyz = 1$, then what is the value of c^z ?

(a) a (b) b

(c) ab (d) a/b

98. If $(3.7)^x = (0.037)^y = 10000$, then what is the value of $\dfrac{1}{x}-\dfrac{1}{y}$.

(a) 1 (b) 2

(c) 1/2 (d) 1/4

99. If $p^x = r^y = m$ and $r^w = p^z = n$, then which one of the following is correct?

(a) $xw = yz$ (b) $xz = yw$

(c) $x + y = w + z$ (d) $x - y = w - z$

100. $x(y - z)(y + z) + y(z - x)(z + x) + z(x - y)(x + y)$ is equal to :

(a) $(x + y)(y + z)(z + x)$

(b) $(x - y)(x - z)(z - y)$

(c) $(x + y)(z - y)(x - z)$

(d) $(y - x)(z - y)(x - z)$

101. If $a = \dfrac{1+x}{2-x}$, then the value of $\dfrac{1}{a+1}+\dfrac{2a+1}{a^2-1}$ is equal to :

(a) $\dfrac{(1+x)(2+x)}{2x-1}$ (b) $\dfrac{(1-x)(2-x)}{x+1}$

(c) $\dfrac{(1+x)(2-x)}{2x-1}$ (d) $\dfrac{(1-x)(2-x)}{2x+1}$

102. If $\dfrac{(x-y)^3+(y-z)^3+(z-x)^3}{4(x-y)(y-z)(z-x)}$ is equal to :

(a) $-\dfrac{3}{4}$ (b) $\dfrac{1}{4}$

(c) $\dfrac{3}{4}$ (d) 0

103. If $x = (b-c)(a-d)$, $y = (c-a)(b-d)$, $z = (a-b)(c-d)$, then the value of $x^3 + y^3 + z^3$ is equal to :

(a) xyz (b) $2xyz$

(c) $3xyz$ (d) $-3xyz$

104. If $x^2 + 2 = 2x$, then the value of $x^4 - x^3 + x^2 + 2$ is :

(a) 0 (b) 1

(c) -1 (d) $\sqrt{2}$

105. If $2^x = 3^y = 6^{-z}$, then the value of $\left(\dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}\right)$ is equal to :

(a) 0 (b) 1

(c) $\dfrac{3}{2}$ (d) $-\dfrac{1}{2}$

106. If $(x^2 + y^2)(p^2 + q^2) = (xp + yq)^2$, then which one is correct?

(a) $xy = pq$ (b) $px = yq$

(c) $xq = yp$

(d) None of these

107. If $x^2 + 8y^2 + 9z^2 - 4xy - 12yz = 0$, then the value of

(a) $x = y = z$

(b) $3x = 2y = z$

(c) $x = 2y = 3z$

(d) $x + 2\sqrt{2}\,y + 3z = 0$

108. If $a = 89$, $b = -69$, $c = 8$, then the value of $9(a+b)^2 + 49c^2 - 42(a+b)c$ is :

(a) 2 (b) 4

(c) 16 (d) 0

109. If $x = \dfrac{p+q}{p-q}$ and $y = \dfrac{p-q}{p+q}$, then $\dfrac{x-y}{x+y}$ is :

(a) $\dfrac{p^2+q^2}{2pq}$ (b) $\dfrac{2pq}{p^2+q^2}$

(c) $\dfrac{2pq}{p^2-q^2}$ (d) $\dfrac{2(p^2-q^2)}{pq}$

110. $(2+1)(2^2+1)(2^4+1)(2^8+1)(2^{16}+1)(2^{32}+1)(2^{64}+1)$ is

(a) $2^{256}-1$ (b) $2^{256}+1$

(c) $2^{128}-1$ (d) $2^{128}+1$

111. If $x + y + z = 0$, then the value of $[(y-z-x)/2]^3 + [(z-x-y)/2]^3 + [(x-y-z)/2]^3$ is:

(a) $24xyz$ (b) $-24xyz$

(c) $3xyz$ (d) xyz

112. If $ax + by = 6$, $bx - ay = 2$ & $x^2 + y^2 = 4$ then the value of $(a^2 + b^2)$ is

(a) 2 (b) 4

(c) 5 (d) 10

113. If $a = 3 + 2\sqrt{2}$] then the value of $\dfrac{a^6+a^4+a^2+1}{a^3}$ is

(a) 204 (b) 212

(c) 192 (d) 240

114. If $x^2 + y^2 + z^2 + 2 = 2(y - x)$, then the value of $x^3 + y^3 + z^3$ is:

(a) 1 (b) 0

(c) 2 (d) 3

115. If $a^3b = abc = 180$ and a, b, c are positive numbers then the value of c is :

(a) 4 (b) 25

(c) 110 (d) 1

116. If $x = \sqrt{3} + \sqrt{2}$, then the value of $\left(x - \dfrac{1}{x}\right)$ is:

(a) 2 (b) 3

(c) $2\sqrt{2}$ (d) $2\sqrt{3}$

117. If $(a^2 + b^2)^3 = (a^3 + b^3)^2$, then the value of $\dfrac{a}{b}+\dfrac{b}{a}$ is :

(a) $\dfrac{1}{3}$ (b) $\dfrac{2}{3}$

(c) $-\dfrac{1}{3}$ (d) $-\dfrac{2}{3}$

118. If $x \neq 0, y \neq 0$ & $z \neq 0$ and $\dfrac{1}{x^2}+\dfrac{1}{y^2}+\dfrac{1}{z^2}=\dfrac{1}{xy}+\dfrac{1}{yz}+\dfrac{1}{zx}$, then the relation between x, y, z will be

(a) $x = y = z$ (b) $x + y + z = 0$

(c) $x + y = z$ (d) $x + y = z = 0$

119. If $p - 2q = 4$, then the value of $p^3 - 8q^3 - 24pq - 64$ is:

(a) 2 (b) 0
(c) 3 (d) –1

120. If $\frac{1}{\sqrt[3]{4}+\sqrt[3]{2}+1} = a\sqrt[3]{4}+b\sqrt[3]{2}+c$ and a, b, c rational numbers, then the value of $a + b + c$ is

(a) 0 (b) 1
(c) 2 (d) 3

121. If $a^3 - b^3 - c^3 = 0$, then $a^9 - b^9 - c^9 - 3a^3b^3c^3$ is

(a) 1 (b) 2
(c) 0 (d) –1

122. If $\frac{4x-3}{x}+\frac{4y-3}{y}+\frac{4z-3}{z}=0$, then the value of $\frac{2}{x}+\frac{2}{y}+\frac{2}{z}$ is

(a) 18 (b) 6
(c) 8 (d) 12

123. If $(x-3)^2+(y-5)^2+(z-4)^2=0$, then the value of $\frac{x^2}{9}+\frac{y^2}{25}+\frac{z^2}{16}$ is

(a) 12 (b) 9
(c) 3 (d) 1

124. If $x = \sqrt[3]{5}+2$, then the value of $x^3 - 6x^2 + 12x - 13$ is

(a) –1 (b) 1
(c) 2 (d) 0

125. If $(x + a)(x + b) = 1$ and $a - b + 5 = 0$, then the value of $(x+a)^3 - \frac{1}{(x+a)^3}$ is

(a) – 140 (b) 110
(c) – 110 (d) 140

126. If $\sqrt{x}=\sqrt{3}-\sqrt{5}$, then the value of $x^2 - 16x + 6$ is

(a) 0 (b) –2
(c) 2 (d) 4

127. $\sqrt{2\sqrt[3]{4\sqrt{2\sqrt[3]{4\sqrt{2\sqrt[3]{4\ldots\ldots\ldots\ldots}}}}}}$ will be equal to

(a) 2 (b) 2^2
(c) 2^3 (d) 2^5

128. One factor of $(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3$

(a) $(a + b)(a - b)$
(b) $(a + b)(a + b)$
(c) $(a - b)(a - b)$
(d) $(b - c)(b - c)$

129. If $a - b = 3$ and $a^3 - b^3 = 117$, then the value of $|a + b|$ is equal to :

(a) 3 (b) 5
(c) 7 (d) 9

130. If $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{1}{a+b+c}$ where $a + b + c \neq 0$, $abc \neq 0$, then $(a + b)(b + c)(c + a) = ?$

(a) 0 (b) 1
(c) –1 (d) 2

131. Find the value of x for which the expression $2 - 3x - 4x^2$ has the greatest value :

(a) $\frac{41}{46}$ (b) $-\frac{41}{16}$
(c) $\frac{3}{8}$ (d) $-\frac{3}{8}$

132. Find the minimum value of $(8 - x)(4 - x)$

(a) – 4 (b) 4
(c) 6 (d) –6

133. $(a + b - 2c)^3 + (b + c - 2a)^3 + (c + a - 2b)^3$ is equal to :

(a) $(a + b - 2c)(b + c - 2a)(c + a - 2b)$
(b) $2(a + b - 2c)(b + c - 2a)(c + a - 2b)$
(c) $3(a + b - 2c)(b + c - 2a)(c + a - 2b)$
(d) $-3(a + b - 2c)(b + c - 2a)(c + a - 2b)$

134. If $\frac{5x}{2x^2+5x+1}=\frac{1}{3}$, then the value of $\left(x+\frac{1}{2x}\right)$ is

(a) 10 (b) 20
(c) 5 (d) 15

135. If $x^3+\frac{3}{x}=4(a^3+b^3)$ and $3x+\frac{1}{x^3}=4(a^3-b^3)$, then $4(a^2 - b^2)$ is equal to

(a) 0 (b) 1
(c) 2 (d) 4

136. If $x + y + z = 0$, then the value of $\frac{(x+y)^3+(y+z)^3+(z+x)^3-17xyz}{10(x+y)(y+z)(z+x)}$ is

(a) $3xyz$ (b) 4
(c) 0 (d) 2

137. If $x+y+z=1, xy+yz+zx=-1, xyz=-1$, then $x^3+y^3+z^3=?$

(a) 1 (b) 0
(c) -2 (d) -1

138. If $a^2+4b^2+\frac{1}{4a^2}+\frac{1}{b^2}=5$, then the value of a^2+b^2 will be

(a) 1 (b) -1
(c) 2 (d) 0

139. The expression $\frac{\left(x+\frac{1}{y}\right)^a\left(x-\frac{1}{y}\right)^b}{\left(y+\frac{1}{x}\right)^a\left(y-\frac{1}{x}\right)^b}$ reduces to :

(a) $\left(\frac{y}{x}\right)^{a+b}$ (b) $\left(\frac{x}{y}\right)^{a+b}$
(c) $\left(\frac{y}{x}\right)^{a-b}$ (d) $\left(\frac{x}{y}\right)^{a-b}$

140. If $a^b = b^a$, then $\left(\frac{a}{b}\right)^{a/b}$ equal to :

(a) $a^{\left(\frac{b}{a}-1\right)}$ (b) $a^{\left(\frac{a}{b}-1\right)}$
(c) $a^{\frac{a}{b}}$ (d) $a^{\frac{b}{a}}$

141. $\frac{a^{1/2}+a^{-1/2}}{1-a}+\frac{1-a^{-1/2}}{1+\sqrt{a}}=?$

(a) $\frac{a}{a-1}$ (b) $\frac{3+a}{(1-a)}$
(c) $\frac{2}{a-1}$ (d) $\frac{2}{1-a}$

142. If $x=\frac{2pq}{1+q^2}$ then $\frac{\sqrt{p+x}+\sqrt{p-x}}{\sqrt{p+x}-\sqrt{p-x}}=?$

(a) p (b) $p-q$
(c) $1/q$ (d) $2q$

143. If $x=\frac{\sqrt{2}+1}{\sqrt{2}-1}$ and $x-y=4\sqrt{2}$, then the value of (x^2+y^2) is :

(a) 30 (b) 32
(c) 34 (d) 38

144. If $a=\left(\sqrt{3}+\sqrt{2}\right)^{-3}$ & $b=\left(\sqrt{3}-\sqrt{2}\right)^{-3}$, then $(a+1)^{-1}+(b+1)^{-1}=?$

(a) $48\sqrt{2}$ (b) $50\sqrt{3}$
(c) 1 (d) 5

145. If $a=x+y, b=x-y, c=x+2y$, then $a^2+b^2+c^2-ab-bc-ca=?$

(a) $4y^2$ (b) $7y^2$
(c) $6y^2$ (d) $5y^2$

146. If $x+y+z=13$, then maximum value of $(x-2)(y+1)(z-3)=?$

(a) 54 (b) 25
(c) 27 (d) 30

147. If $\sqrt{28-6\sqrt{3}}=\sqrt{3}a+b$, then $(a-b)=?$ (where a, b are rational number)

(a) -2 (b) 2
(c) 4 (d) -1

148. If $a \neq b$ which one of the following statement is correct ?

(a) $\frac{a+b}{2}=\sqrt{ab}$

(b) $\frac{a+b}{2}<\sqrt{ab}$

(c) $\frac{a+b}{2}>\sqrt{ab}$

(d) All of the above

149. If $a + b = 1$, $b + c = 2$ and $c + a = 3$, then $(a^2+b^2+c^2+ab+bc+ca)=?$

(a) 3.5 (b) 18
(c) 7 (d) 9

150. If $(a+b+2c+3d)(a-b-2c+3d)=(a-b+2c-3d)(a+b-2c-3d)$, then $2bc$ is equal to

(a) $3ad$ (b) $\frac{3}{2}$

(c) a^2d^2 (d) $\frac{3a}{2d}$

151. If $2s = a + b + c$, then the value of $(s-a)^2 + (s-b)^2 + (s-c)^2 + s^2 - a^2 - b^2 - c^2$ will be :

(a) –1 (b) 1

(c) 2 (d) 0

152. If $a^{1/m} = b^{1/n} = c^{1/p}$ and $abc = 1$, then $(m + n + p)$ is equal to :

(a) 0 (b) 2

(c) 1 (d) –2

153. If $2s = 9$, then the value of $s^2 + (s-1)^2 + (s-3)^2 + (s-5)^2$ is :

(a) 9 (b) 25

(c) 45 (d) 35

154. If p, q and r be such that $p + q = r$ and $pqr = 30$, then what is the value of $p^3 + q^3 - r^3$

(a) 0 (b) 90

(c) –90

(d) Cannot be determined

155. If $x+y=\sqrt{3}$ and $x-y=\sqrt{2}$, then find $8xy(x^2+y^2)$.

(a) $\frac{5}{9}$ (b) 5

(c) $\frac{5}{2}$ (d) $\sqrt{6}$

156. If $2a + 3b = 4$, then find $8a^3 + 27b^3 + 72ab$.

(a) 54 (b) 48

(c) 72 (d) 64

157. If $x + \frac{2}{x} = 3$, then find $\frac{x^2+x+2}{x^2(3-x)}$.

(a) 0 (b) 1

(c) 2 (d) 3

158. If $x = ay$ and $y = bx$, then find $\frac{1}{1+a} + \frac{1}{1+b}$

(a) 0 (b) 1

(c) – 1 (d) 2

159. If $a = \frac{4}{3}$, then find $27a^3 - 108a^2 + 144a - 317$.

(a) 261 (b) – 253

(c) – 245 (d) 0

160. If $a + b = 5$ and $a^2 + b^2 = 13$, then find $a - b$, where $(a > b)$.

(a) 1 (b) – 3

(c) 2 (d) – 1

161. If $x^2 + x - 6 = 0$ and $x^2 + 6x + 9 = 0$, then find x.

(a) 2 (b) 3

(c) – 2 (d) – 3

162. If $x + \frac{1}{x} = 3$, then find $\frac{x^3+\frac{1}{x}}{x^2-x+1}$.

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$

(c) $\frac{7}{2}$ (d) $\frac{11}{2}$

163. If $ax + by = 3$, $bx - ay = 4$ & $x^2 + y^2 = 1$, then find $a^2 + b^2$.

(a) 17 (b) 16

(c) 9 (d) 25

164. If $a + b + c = 15$ and $a^2 + b^2 + c^2 = 83$, then find $a^3 + b^3 + c^3 - 3abc$.

(a) 200 (b) 180

(c) 190 (d) 210

165. If $\frac{1}{a} - \frac{1}{b} = \frac{1}{a-b}$, then find $a^3 + b^3$.

(a) 0 (b) –1

(c) 1 (d) 2

166. If $2x = a + \sqrt{\frac{4b^3-a^3}{3a}}$ and $2y = a - \sqrt{\frac{4b^3-a^3}{3a}}$, then find $x^3 + y^3$.

(a) a (b) b

(c) a^3 (d) b^3

167. If $a - b = 2$ and $ab = 15$, then find $(a^2 - b^2)(a^3 - b^3)$.

(a) 1450 (b) 1500

(c) 1528 (d) 1568

168. If $4x-5z=16$ and $xz=12$, then find $64x^3-125z^3$.
(a) 15610 (b) 15616
(c) 15618 (d) 15620

169. If $x+y+z=15$ and $xy+yz+zx=75$, then find $\frac{x+4y+z}{3z}$.
(a) 1 (b) 0
(c) 2 (d) – 1

170. If $a^2+b^2=2$ and $c^2+d^2=1$, then find $(ad-bc)^2+(ac+bd)^2$.
(a) $\frac{4}{9}$ (b) $\frac{1}{2}$
(c) 1 (d) 2

171. If $3(a^2+b^2+c^2)=(a+b+c)^2$, then find the relation between a,b & c.
(a) $a=b=c$ (b) $a=b\neq c$
(c) $a<b<c$ (d) $a>b>c$

172. If $x+\frac{1}{x}=-\sqrt{3}$, then find $x^{26}+\frac{1}{x^{26}}$.
(a) $\frac{4}{9}$ (b) $\frac{1}{2}$
(c) 1 (d) 2

173. If $\sqrt{2\sqrt{4\sqrt{2\sqrt{4\sqrt{2}}}}}=32^a$, then find the value of a.
(a) $\frac{41}{160}$ (b) $\frac{41}{80}$
(c) 1 (d) 2

174. If $a^2=2$, then find $a+1$.
(a) $\frac{a-1}{3-2a}$ (b) $\frac{a-1}{3+2a}$
(c) $\frac{a+1}{3+2a}$ (d) $\frac{a+1}{3+2a}$

175. If $(a+b+c)p=(b+c-a)q=(c+a-b)r=(a+b-c)s$, then find $\frac{1}{q}+\frac{1}{r}+\frac{1}{s}-\frac{1}{p}$.
(a) 1 (b) 2
(c) 0 (d) $a+b+c$

176. If $x(x-3)=-1$, then find $x^3(x^3-18)$.
(a) 1 (b) 2
(c) 0 (d) –1

177. If $x=\frac{p+q+r}{3}$, then find $(x-p)^3+(x-q)^3+(x-r)^3-3(x-p)(x-q)(x-r)$.
(a) pqr (b) $p+q+r$
(c) 0 (d) 3

178. If $a^2+b^2=2(a-2b)-5$, then find a^3+b^3+3ab.
(a) –13 (b) 13
(c) 0
(d) can't determined

179. If $x+\frac{1}{x}=-1$ & $y+\frac{1}{y}=2$, then find $(x)^{3y}+(y)^{3x}$
(a) 2 (b) 1
(c) 0 (d) –2

180. If $a^3+b^3+c^3=3abc$ and a,b,c are positive numbers, then find $\frac{2a+7b+9c}{a+2b+3c}$.
(a) $\frac{4}{9}$ (b) 1
(c) 3
(d) can't determined

181. If $\frac{p}{a}+\frac{q}{b}+\frac{r}{c}=1$, $\frac{a}{p}+\frac{b}{q}+\frac{c}{r}=1$, $pqr=-1$ and $abc=1$ and p, q, r & a, b, c are non-zero, then find $\frac{p^2}{a^2}+\frac{q^2}{b^2}+\frac{r^2}{c^2}$.
(a) – 1 (b) 0
(c) 3 (d) 2

182. If $x=\frac{4\sqrt{15}}{\sqrt{5}+\sqrt{3}}$, then $\frac{x+\sqrt{20}}{x-\sqrt{20}}+\frac{x+\sqrt{12}}{x-\sqrt{12}}=?$

(a) 1 (b) 2
(c) 3 (d) 4

183. If $x=\frac{2\sqrt{6}}{\sqrt{3}+\sqrt{2}}$, then $\frac{x+\sqrt{2}}{x-\sqrt{2}}+\frac{x+\sqrt{3}}{x-\sqrt{3}}=?$

(a) 1 (b) 2
(c) 3 (d) 4

184. If $x=\frac{2\sqrt{24}}{\sqrt{3}+\sqrt{2}}$, then $\frac{x+\sqrt{8}}{x-\sqrt{8}}+\frac{x+\sqrt{12}}{x-\sqrt{12}}=?$

(a) 1 (b) 2
(c) 3 (d) 4

185. If $x=\frac{4\sqrt{6}}{\sqrt{2}+\sqrt{3}}$, then $\frac{x+2\sqrt{2}}{x-2\sqrt{2}}+\frac{x+2\sqrt{3}}{x-2\sqrt{3}}=?$

(a) 1 (b) 2
(c) 3 (d) 4

186. If $x=\frac{4ab}{a+b}$, then $\frac{x+2a}{x-2a}+\frac{x+2b}{x-2b}=?$

(a) 1 (b) 2
(c) 3 (d) 4

187. If $x=\frac{12pq}{p+q}$, then $\frac{x+6p}{x-6p}+\frac{x+6q}{x-6q}=?$

(a) 1 (b) 2
(c) 3 (d) 4

188. If $x=\frac{6pq}{p+q}$, then $\frac{x+3p}{x-3p}+\frac{x+3q}{x-3q}=?$

(a) 1 (b) 2
(c) 3 (d) 4

189. If $x=\frac{8ab}{a+b}$, then $\frac{x+4a}{x-4a}+\frac{x+4b}{x-4b}=?$

(a) 1 (b) 2
(c) 3 (d) 4

190. If $(x+y)^2-z^2=4$, $(y+z)^2-x^2=9$, $(z+x)^2-y^2=36$, then find the value of $(x+y+z)$.

(a) 7 (b) –7
(c) ± 7 (d) 49

191. If $x=\sqrt{\frac{\sqrt{5}+1}{\sqrt{5}-1}}$, then find the value of $5x^2-5x-1$.

(a) 0 (b) 5
(c) 3 (d) 4

192. If $x=\frac{\sqrt{3}}{2}$, then the value of $\frac{\sqrt{1+x}}{1+\sqrt{1+x}}+\frac{\sqrt{1-x}}{1-\sqrt{1-x}}$ is

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{2}{\sqrt{3}}$
(c) $\sqrt{3}$ (d) 2

193. If $x=\frac{\sqrt{3}}{2}$, then $\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}=?$

(a) $-\sqrt{3}$ (b) $\frac{\sqrt{3}}{2}$
(c) $-\sqrt{2}$ (d) $\sqrt{3}$

194. If $\frac{x^2}{by+cz}=\frac{y^2}{cz+ax}=\frac{z^2}{ax+by}=\frac{1}{5}$, then find $\frac{5a}{5x+a}+\frac{5b}{5y+b}+\frac{5c}{5z+c}$.

(a) 5 (b) 2
(c) 1 (d) $\frac{1}{5}$

195. If $a+b+c=0$ $(a \neq b \neq c)$, then find $\frac{a^2}{2a^2+bc}+\frac{b^2}{2b^2+ac}+\frac{c^2}{2c^2+ab}$.

(a) 0 (b) $a+b+c$
(c) 1 (d) abc

196. If $xy=r$, $xz=r^2$, $yz=r^3$, $x+y+z=13$ and $x^2+y^2+z^2=91$, then find $\frac{z}{y}$.

(a) 3 (b) $\frac{7}{3}$
(c) 4 (d) $\frac{13}{3}$

197. If $xy + yz + zx = xyz$, then find the value of $\frac{x+y}{xy(z-1)} + \frac{y+z}{yz(x-1)} + \frac{z+x}{zx(y-1)}$.

(a) $-\frac{3}{2}$ (b) $\frac{1}{2}$

(c) 0 (d) 1

198. If $a + b + c = 2$ and $ab + bc + ca = -1$, then $(a + b)^2 + (b + c)^2 + (c + a)^2 = ?$

(a) 5 (b) 10

(c) 15 (d) 25

199. If $\frac{b}{y} + \frac{z}{c} = 1$ and $\frac{c}{z} + \frac{x}{a} = 1$, then $\frac{a}{x} + \frac{y}{b} = ?$

(a) 0 (b) 1

(c) –1 (d) 2

200. If $x + \frac{2}{y} = 1$ and $y + \frac{1}{z} = 2$, then $z + \frac{1}{2x} = ?$

(a) $\frac{1}{2}$ (b) 2

(c) 1 (d) 6

201. If $a^x = m, a^y = n$ and $a^2 = (m^y.n^x)^z$, then $xyz = ?$

(a) 0 (b) 2

(c) 1 (d) $\frac{1}{2}$

202. If $\left(\frac{1}{2}\right)^k = \sqrt{3}$ and $\left(\frac{1}{3}\right)^m = \sqrt{2}$, then $\frac{mk}{2} = ?$

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$

(c) $\frac{1}{2}$ (d) 1

203. If b = 36, then the value of $\frac{a-b}{\sqrt{a}-\sqrt{b}} + \frac{b-a}{\sqrt{a}+\sqrt{b}}$ is

(a) Data insufficient (b) 12

(c) 6 (d) 0

204. If $x \in \mathrm{R}$, then maximum value of $\left(\sqrt{3} - x + 5\right)\left(\sqrt{3} + x - 5\right)$ is :

(a) $\sqrt{3}$ (b) 5

(c) 3 (d) $\sqrt{3} + 5$

205. If $x = \frac{\sqrt{93}+\sqrt{19}}{\sqrt{97}-\sqrt{23}}$ and $y = \frac{\sqrt{93}-\sqrt{19}}{\sqrt{97}+\sqrt{23}}$, then $\frac{1}{x+1} + \frac{1}{y+1} = ?$

(a) 93 (b) 2

(c) 1 (d) 0

206. If $x = 1 + \sqrt{2} + \sqrt{3}$, then the value of $(2x^4 - 8x^3 - 5x^2 + 26x - 28)$ is

(a) 0 (b) $6\sqrt{6}$

(c) $3\sqrt{6}$ (d) $\sqrt{6}$

207. If $\frac{5}{3^{2/3} - 6^{1/3} + 2^{2/3}} = a\sqrt[3]{3} + b\sqrt[3]{2} + c\sqrt[3]{6}$, then find $a + b + c$.

(a) 0 (b) 1

(c) 2 (d) 3

208. If a, b are real numbers and $\frac{\sqrt{a}+\sqrt{b}}{a-b} - \frac{\sqrt{b}-\sqrt{a}}{a+b} = 4$, then find the value of x if $\frac{a^x + b^x}{a^2 - b^2} = 2$.

(a) $\frac{1}{2}$ (b) 2

(c) $\frac{3}{2}$ (d) 1

209. If $\frac{76}{4+\sqrt{7}+\sqrt{11}} = p+q\sqrt{7}+r\sqrt{11}+s\sqrt{77}$,

then $p+q+r+s$ =?

(a) 6 (b) 4
(c) 9 (d) 8

210. If $x = 4+\sqrt{11}+\sqrt{7}$, then find $x^4 - 16x^3 + 60x^2 + 32x$.
(a) 244 (b) 264
(c) 280 (d) 304

211. If $5^a + 2^{b+1} = 189$ & $5^{a+1} + 2^{b-2} = 633$, then find $a + b$.
(a) 8 (b) 7
(c) 10 (d) 9

212. If $\frac{x-a^2}{b+c} + \frac{x-b^2}{c+a} + \frac{x-c^2}{a+b} = 4(a+b+c)$, then the value of x is equal to :
(a) $(a+b+c)^2$
(b) $a^2 + b^2 + c^2$
(c) $ab + bc + ca$
(d) $a^2 + b^2 + c^2 - ab - bc - ca$

213. Find the value of

$$\frac{1}{1+p^{a-b}+p^{a-c}} + \frac{1}{1+p^{b-a}+p^{b-c}} + \frac{1}{1+p^{c-a}+p^{c-b}}.$$

(a) $\frac{1}{2}$ (b) 2
(c) 1 (d) –3

214. The value of $p+\sqrt{p^2+\sqrt{p^4+\sqrt{p^8+\sqrt{p^{16}}}}}$ ...∞

(a) $p\left(\frac{\sqrt{5}+2}{2}\right)$ (b) $p\left(\frac{3+\sqrt{5}}{2}\right)$

(c) $\frac{p}{1+\sqrt{p}}$ (d) $p\left(\frac{\sqrt{5}+1}{2}\right)$

215. If $x = \sqrt[4]{4\sqrt[4]{4\sqrt[4]{4\sqrt[4]{4}}}}$ ..∞ $= 32^a$, then a = ?

(a) $\frac{2}{15}$ (b) $\frac{4}{15}$

(c) $\frac{2}{5}$ (d) $\frac{1}{5}$

216. If $y = \frac{x^2-10x+64}{x^2+10x+64}$, then minimum value of y is

(a) $\frac{13}{3}$ (b) $\frac{13}{4}$

(c) $\frac{3}{13}$ (d) $\frac{4}{13}$

217. If $p = \sqrt{5} - 2$, then $p^4 + 16p^2 + 8p^3 + 4 = ?$
(a) 3 (b) 5
(c) 1 (d) 0

218. Find the value of
$x^3 - 8y^3 - 36xy - 216$, when $x = 2y + 6$
(a) 0 (b) 1
(c) 2 (d) –1

219. If $p = 2 - a$, then $a^3 + 6ap + p^3 - 6 = ?$
(a) 1 (b) 0
(c) 2 (d) 4

220. If $x^4 + y^4 = 17$ and $x + y = 1$, then $x^2y^2 - 2xy = ?$
(a) 8 (b) 10
(c) 12 (d) 16

221. If $\frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{a+b} = k$, then $k = ?$
(a) 1 (b) –1
(c) 2
(d) None of these

222. If $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$, then

$$\frac{ax-by}{(a+b)(x-y)} + \frac{by-cz}{(b+c)(y-z)} + \frac{cz-ax}{(c+a)(z-x)} = ?$$

(a) 0 (b) 2
(c) 1 (d) 3

223. Find the value of

$$\frac{a^3(b+c)}{(a-b)(a-c)}+\frac{b^3(c+a)}{(b-c)(b-a)}+\frac{c^3(a+b)}{(c-a)(c-b)}.$$

(a) abc (b) $a+b+c$

(c) $ab+bc+ca$ (d) 3

224. Find the value of

$$\frac{a^2-b^2-c^2}{(a-b)(a-c)}+\frac{b^2-c^2-a^2}{(b-c)(b-a)}+\frac{c^2-a^2-b^2}{(c-a)(c-b)}.$$

(a) abc (b) $a+b+c$

(c) 2 (d) 0

225. If $x^2+y^2=z+1, y^2+z^2=x+1$, $z^2+x^2=y+1$, then find xyz.

(a) 1 (b) $-\frac{1}{8}$

(c) 1 or $\frac{1}{8}$ (d) 1 or $-\frac{1}{8}$

226. Find the value of $\frac{\sqrt{26-15\sqrt{3}}}{5\sqrt{2}-\sqrt{38+5\sqrt{3}}}$.

(a) $\sqrt{3}$ (b) $\sqrt{2}$

(c) $\frac{1}{\sqrt{3}}$ (d) 3

227. The maximum value of $\frac{x+2}{2x^2+3x+6}$ is

(a) 3 (b) $\frac{1}{3}$

(c) 2 (d) $\frac{1}{2}$

228. Find the value of

$$\frac{(a+b+c)^4-(b+c)^4-(c+a)^4-(a+b)^4+a^4+b^4+c^4}{a+b+c}$$

(a) $3abc$ (b) $4abc$

(c) $6abc$ (d) $12abc$

229. Find the value of

$$\frac{a(b-c)^2}{(c-a)(a-b)}+\frac{b(c-a)^2}{(a-b)(b-c)}+\frac{c(a-b)^2}{(b-c)(c-a)}.$$

(a) $a+b+c$ (b) 3

(c) $a^2+b^2+c^2$ (d) abc

230. Find the value of

$$\frac{2a}{a+b}+\frac{2b}{b+c}+\frac{2c}{c+a}+\frac{(b-c)(c-a)(a-b)}{(b+c)(c+a)(a+b)}.$$

(a) 1 (b) 2

(c) 3 (d) 4

231. If $x+\frac{1}{x}=-\sqrt{3}$, then the value of $x^{67}+x^{53}+x^{43}+x^{29}+x^{24}+x^{12}+x^6+3$ is :

(a) $\sqrt{3}$ (b) 0

(c) $2(2-\sqrt{3})$ (d) $2(2+\sqrt{3})$

232. If $x+\frac{1}{x}=1$, then $x^{52}+x^{46}+x^{32}+x^{26}+x^{21}+x^{15}+x^6+x^3+4$ is equal to

(a) 0 (b) 3

(c) 4 (d) 2

233. If $(ab-b+1)=0$ and $(bc-c+1)=0$, then $(a-ac)$ is equal to

(a) –1 (b) 0

(c) 1 (d) 2

234. If $a+b+c=20$ and $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=30$, then find the value of $\left(\frac{a}{b}+\frac{b}{a}+\frac{b}{c}+\frac{c}{b}+\frac{c}{a}+\frac{a}{c}\right)$.

(a) 597 (b) 600

(c) 599 (d) can't find

235. If $\frac{b-c}{a}+\frac{a+c}{b}+\frac{a-b}{c}=1$ and $a-b+c\neq 0$, then which one of the following relation is true ?

(a) $\frac{1}{a}=\frac{1}{b}+\frac{1}{c}$ (b) $\frac{1}{b}=\frac{1}{a}-\frac{1}{c}$

(c) $\frac{1}{b}=\frac{1}{a}+\frac{1}{c}$ (d) $\frac{1}{c}=\frac{1}{a}+\frac{1}{b}$

236. If $\left(\frac{x}{y}\right)=\left(\frac{z}{w}\right)$, then what is $(xy+zw)^2$ equal to

(a) $(x^2+z^2)(y^2+w^2)$ (b) $x^2y^2+z^2w^2$

(c) $x^2w^2+y^2z^2$

(d) $(x^2+w^2)(y^2+z^2)$

237. If $x=\frac{a}{b+c}$, $y=\frac{b}{a+c}$ and $z=\frac{c}{a+b}$, then $xy+yz+zx+2xyz=?$

(a) 2

(b) $(a+b)(b+c)(c+a)$

(c) 0 (d) 1

238. Find the value of x if

$$\sqrt{x+2\sqrt{x+2\sqrt{x+2\sqrt{x+2}}}}=x.$$

(a) 1 (b) 3

(c) 6 (d) 12

239. Find the maximum value of the expression

$$\frac{1}{x^2+5x+10}$$

(a) $\frac{15}{2}$ (b) 1

(c) $\frac{4}{15}$ (d) 2

240. Let x, y be two positive numbers that $x+y=1$, then the minimum value of

$\left(x+\frac{1}{x}\right)^2+\left(y+\frac{1}{y}\right)^2$ is

(a) 12 (b) 20

(c) 12.5 (d) 13.3

241. If $p^a=q^b=r^c$ and $\frac{p}{q}=\frac{q}{r}$, then $\left(\frac{1}{a}+\frac{1}{c}\right)b=?$

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{3}{4}$ (d) 2

242. If $x^2(x+y+z)=36$, $y^2(x+y+z)=46$, $z^2(x+y+z)=63$, $xy(x+y+z)=111$, $yz(x+y+z)=99$, $zx(x+y+z)=82$, then $x=?$

(a) 6 (b) 7

(c) 2 (d) 4

243. If $x=\frac{a-b}{a+b}$, $y=\frac{b-c}{b+c}$, $z=\frac{c-a}{c+a}$, then $\frac{1+x}{1-x}\cdot\frac{1+y}{1-y}\cdot\frac{1+z}{1-z}=?$

(a) 1 (b) 2

(c) 3 (d) 4

244. If $x=5$ & $y=z$ and x, y & z are positive numbers, then the maximum value of $x^2+y^2+z^2-(xy+yz+zx)$ is

(a) 0 (b) 25

(c) 50 (d) –5

245. If x, y, z are real number and $x+2y+z=-6$, then find the maximum value of $x^2+2y^2+z^2+2yz$.

(a) 12 (b) 8

(c) –12 (d) 6

246. If $a^x=bc$, $b^y=ac$, $c^z=ab$, then $\frac{x}{1+x}+\frac{y}{1+y}+\frac{z}{1+z}=?$

(a) 0 (b) 1

(c) 2 (d) 3

247. If $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}$, then $xy+yz+zx=?$

(a) $\frac{(a+b+c)^2}{x^2+y^2+z^2}$

(b) $\frac{x^2(a+b+c)^2-a^2(x^2+y^2+z^2)}{2a^2}$

(c) $\frac{ax+by+cz}{(a+b+c)^2}$ (d) 3

248. If $\frac{x}{y} = \frac{z}{w}$, then $\frac{x^m + y^m + z^m + w^m}{x^{-m} + y^{-m} + z^{-m} + w^{-m}} = ?$

(a) $\frac{x}{y}$ (b) 1
(c) $(xyzw)^{m/2}$ (d) $(xyzw)^m$

249. If $\sqrt{13x^3 - 14x + 29} + \sqrt{13x^3 - 14x - 21} = 10$, then

$\sqrt{13x^3 - 14x + 29} - \sqrt{13x^3 - 14x - 21} = ?$

(a) 3 (b) 4
(c) 5 (d) 6

250. If $\frac{x^2}{by + cz} = \frac{y^2}{cz + ax} = \frac{z^2}{ax + by} = \frac{1}{3}$, then

find $\frac{a}{a + 3x} + \frac{b}{b + 3y} + \frac{c}{c + 3z}$.

(a) 1 (b) 2
(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

251. If $x = \left(a + \sqrt{a^2 + b^3}\right)^{1/3} + \left(a - \sqrt{a^2 + b^3}\right)^{1/3}$,

then what is the value of $x^3 + 3bx - 2a$.

(a) $2a^3$ (b) $-2a^3$
(c) 0 (d) 1

252. If $a = \frac{xy}{x + y}$, $b = \frac{xz}{x + z}$ and $c = \frac{yz}{y + z}$ and a, b and c equal to non-zero, then x is equal to:

(a) $\frac{2abc}{ac + bc - ab}$ (b) $\frac{2abc}{ab - ac + bc}$

(c) $\frac{2abc}{ab + bc + ac}$

(d) $\frac{2abc}{ab - ac - bc}$

253. If $\frac{x - a^2}{b^2 + c^2} + \frac{x - b^2}{c^2 + a^2} + \frac{x - c^2}{a^2 + b^2} = 3$, then the value of x is

(a) $a^2 + b^2$ (b) $a^2 + b^2 + c^2$
(c) $a^2 - b^2 - c^2$ (d) $a^2 + b^2 - c^2$

254. If $(x - 5)(y + 6)(z - 8) = 1331$, then the minimum value of $(x + y + z)$ is :

(a) 40 (b) 33
(c) 19 (d) not unique

255. If a, b, c and d are four positive number such that $a + b + c + d = 4$, then what is the maximum value of $(a + 1)(b + 1)(c + 1)(d + 1)$.

(a) 32 (b) 8
(c) 16 (d) 81

256. If x, y, z are three positive real numbers such that $x + y + z = 1$, then the minimum value of

$\left(\frac{1}{x} - 1\right)\left(\frac{1}{y} - 1\right)\left(\frac{1}{z} - 1\right)$ is :

(a) 16 (b) 8
(c) 4 (d) 1

257. If a, b, c are positive real numbers, then the least value of $(a + b + c)\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$ is :

(a) 1 (b) 9
(c) 12
(d) None of these

258. If a, b, c are all positive, then the minimum value of the expression

$\frac{(a^2 + a + 1)(b^2 + b + 1)(c^2 + c + 1)}{abc}$ is:

(a) 3 (b) 9
(c) 27 (d) 1

259. If $x = \frac{1}{2}\left(\sqrt{\frac{9}{8}} - \sqrt{\frac{8}{9}}\right)$, then $\frac{18\sqrt{1 + x^2}}{x + \sqrt{1 + x^2}} = ?$

(a) 16 (b) 17
(c) 19 (d) 20

260. If $\sqrt{10+\sqrt{24}+\sqrt{40}+\sqrt{60}} = \sqrt{a}+\sqrt{b}+\sqrt{c}$, then the value of $a + b + c$ is
(a) $\sqrt{10}$ (b) 10
(c) 11 (d) $\sqrt{11}$

261. If $x + \frac{1}{x} = 2a$, $y + \frac{1}{y} = 2c$, $x - \frac{1}{x} = 2b$ and $y - \frac{1}{y} = 2d$ then the value of $xy + \frac{1}{xy}$ is.
(a) $ac + bd$ (b) $ac - bd$
(c) $2(ac - bd)$ (d) $2(ac + bd)$

262. Find the value of $a^3 + b^3 + c^3 - 3abc$ if we subsititute $(s-a), (s-b), (s-c)$ for a, b, c respectively, where $3s = (a+b+c)$
(a) 0 (b) $3abc$
(c) $a^3 + b^3 + c^3 - 3abc$
(d) None of these

263. Find the value of $a(b-c)^3 + b(c-a)^3 + c(a-b)^3$.
(a) $3abc$
(b) $(a-b)(b-c)(c-a)$
(c) $(a-b)(b-c)(c-a)(a+b+c)$
(d) $(a+b)(b+c)(c+a)(a+b+c)$

264. Find the value of $a^4(b^2 - c^2) + b^4(c^2 - a^2) + c^4(a^2 - b^2)$.
(a) $3a^2b^2c^2$
(b) $(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)$
(c) $-(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)$
(d) $(a^2+b^2)(b^2+c^2)(c^2+a^2)$

265. Find the value of $a(b-c)^2 + b(c-a)^2 + c(a-b)^2 + 8abc$.
(a) $(a+b)(b+c)(c+a)$
(b) $(a-b)(b-c)(c-a)$
(c) 0
(d) abc

266. Find the value of $(bc + ca + ab)^3 - b^3c^3 - c^3a^3 - a^3b^3$.
(a) $3abc(a+b)(b+c)(c+a)$
(b) $(a+b)(b+c)(c+a)$
(c) $(a-b)(b-c)(c-a)$
(d) $24abc$

267. Find the value of $(a+b+c)^4 - (b+c)^4 - (c+a)^4 - (a+b)^4 + a^4 + b^4 + c^4$.
(a) $12abc(a+b+c)$
(b) $abc(a+b+c)$
(c) 2 (d) abc

268. Find the value of
$$\frac{a^2-b^2-c^2}{(a-b)(a-c)}+\frac{b^2-c^2-a^2}{(b-c)(b-a)}+\frac{c^2-a^2-b^2}{(c-a)(c-b)}$$
(a) 2 (b) 1
(c) 3 (d) 4

269. If $a+b+c=0$, then $\frac{2(a^4+b^4+c^4)}{(a^2b^2+b^2c^2+c^2a^2)}=?$
(a) 1 (b) 2
(c) 3 (d) 4

270. If $x+\frac{1}{x}=5$ and $x^2+\frac{1}{x^3}=8$, then the value of $x^3+\frac{1}{x^2}$ is
(a) 125 (b) 215
(c) 256 (d) 525

271. If $a=\frac{1}{2-\sqrt{3}}, b=\frac{1}{2+\sqrt{3}}$, then find the value of $7a^2+11ab-7b^2$
(a) 109 (b) $11+56\sqrt{3}$
(c) $56\sqrt{3}$ (d) $11-56\sqrt{3}$

272. Find $\frac{\sqrt{x^2-1}}{x-\sqrt{x^2-1}}$, when $2x=\sqrt{a}+\frac{1}{\sqrt{a}}$
(a) $\frac{a-1}{2}$ (b) $\frac{a+1}{2}$
(c) $\frac{a}{2}$ (d) $\frac{a-2}{2}$

273. If $\frac{x^3+1}{x^2-1} = x + \sqrt{\frac{6}{x}}$, then find $x + \frac{1}{x}$.

(a) $\frac{13}{6}$ (b) $\frac{6}{13}$

(c) 3 (d) $\frac{4}{3}$

274. If $\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = \frac{10}{3}$ and $x + y = 10$, then x & y = ?

(a) $x = 3, y = 3$ (b) $x = 9, y = 1$

(c) $x = \frac{9}{2}, y = 1$ (d) $x = 3, y = 2$

275. If $x = \frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}$, then find $\frac{x+\sqrt{3}}{x-\sqrt{3}} - \frac{x+\sqrt{2}}{x-\sqrt{2}}$.

(a) 2 (b) 1

(c) $\sqrt{\frac{2}{3}}$ (d) $\sqrt{\frac{3}{2}}$

276. If $(x+1)(x+2) + \frac{1}{x(x-1)} = 0$, then find $x^2 + x$.

(a) 4 (b) 1

(c) 9 (d) 0

277. If $x^2 + xy + y^2 = 84$ & $x - \sqrt{xy} + y = 6$, then find $x^3 + y^3$.

(a) 72 (b) 520

(c) 512 (d) 600

278. If $x^4 + x^2y^2 + y^4 = 931$ & $x^2 - xy + y^2 = 19$, then find $2x^2 + 3y^2$.

(a) 52 (b) 50

(c) 62 (d) 77

279. If $x^3 + y^3 + z^3 = a^3, x^2 + y^2 + z^2 = a^2$ & $x + y + z = a$, then find xyz.

(a) a (b) 0

(c) a^2 (d) a^3

******

Answer

1.	(c)	2.	(b)	3.	(d)	4.	(a)	5.	(d)	6.	(b)	7.	(a)	8.	(a)	9.	(a)
10.	(c)	11.	(c)	12.	(c)	13.	(c)	14.	(a)	15.	(a)	16.	(c)	17.	(a)	18.	(a)
19.	(d)	20.	(a)	21.	(c)	22.	(c)	23.	(b)	24.	(b)	25.	(c)	26.	(c)	27.	(d)
28.	(c)	29.	(a)	30.	(b)	31.	(b)	32.	(c)	33.	(a)	34.	(c)	35.	(d)	36.	(c)
37.	(c)	38.	(c)	39.	(d)	40.	(d)	41.	(c)	42.	(d)	43.	(d)	44.	(d)	45.	(a)
46.	(d)	47.	(c)	48.	(c)	49.	(b)	50.	(c)	51.	(d)	52.	(b)	53.	(d)	54.	(b)
55.	(b)	56.	(d)	57.	(b)	58.	(c)	59.	(c)	60.	(b)	61.	(c)	62.	(c)	63.	(b)
64.	(d)	65.	(c)	66.	(a)	67.	(c)	68.	(a)	69.	(d)	70.	(b)	71.	(a)	72.	(b)
73.	(a)	74.	(c)	75.	(b)	76.	(c)	77.	(c)	78.	(d)	79.	(b)	80.	(b)	81.	(a)
82.	(b)	83.	(a)	84.	(b)	85.	(b)	86.	(a)	87.	(a)	88.	(b)	89.	(b)	90.	(c)
91.	(a)	92.	(a)	93.	(a)	94.	(b)	95.	(a)	96.	(b)	97.	(a)	98.	(c)	99.	(a)
100.	(b)	101.	(c)	102.	(c)	103.	(c)	104.	(a)	105.	(a)	106.	(c)	107.	(c)	108.	(c)
109.	(b)	110.	(c)	111.	(c)	112.	(d)	113.	(a)	114.	(b)	115.	(d)	116.	(c)	117.	(b)
118.	(a)	119.	(b)	120.	(a)	121.	(c)	122.	(c)	123.	(c)	124.	(d)	125.	(c)	126.	(c)
127.	(a)	128.	(a)	129.	(c)	130.	(a)	131.	(d)	132.	(a)	133.	(c)	134.	(c)	135.	(d)
136.	(d)	137.	(a)	138.	(a)	139.	(b)	140.	(b)	141.	(d)	142.	(c)	143.	(c)	144.	(c)
145.	(b)	146.	(c)	147.	(c)	148.	(c)	149.	(c)	150.	(a)	151.	(d)	152.	(a)	153.	(d)
154.	(c)	155.	(b)	156.	(d)	157.	(c)	158.	(b)	159.	(b)	160.	(a)	161.	(d)	162.	(c)
163.	(d)	164.	(b)	165.	(a)	166.	(d)	167.	(d)	168.	(b)	169.	(c)	170.	(d)	171.	(a)
172.	(c)	173.	(a)	174.	(a)	175.	(c)	176.	(d)	177.	(c)	178.	(a)	179.	(a)	180.	(c)
181.	(c)	182.	(b)	183.	(b)	184.	(b)	185.	(b)	186.	(b)	187.	(b)	188.	(b)	189.	(b)
190.	(c)	191.	(d)	192.	(b)	193.	(d)	194.	(a)	195.	(c)	196.	(a)	197.	(d)	198.	(b)
199.	(b)	200.	(a)	201.	(c)	202.	(a)	203.	(b)	204.	(c)	205.	(c)	206.	(b)	207.	(c)
208.	(c)	209.	(d)	210.	(d)	211.	(a)	212.	(a)	213.	(c)	214.	(b)	215.	(a)	216.	(c)
217.	(b)	218.	(a)	219.	(c)	220.	(a)	221.	(b)	222.	(d)	223.	(c)	224.	(c)	225.	(d)
226.	(c)	227.	(b)	228.	(d)	229.	(a)	230.	(c)	231.	(d)	232.	(a)	233.	(c)	234.	(a)
235.	(a)	236.	(a)	237.	(d)	238.	(b)	239.	(c)	240.	(c)	241.	(d)	242.	(c)	243.	(a)
244.	(b)	245.	(a)	246.	(c)	247.	(b)	248.	(c)	249.	(c)	250.	(a)	251.	(c)	252.	(a)
253.	(b)	254.	(a)	255.	(c)	256.	(b)	257.	(b)	258.	(c)	259.	(b)	260.	(b)	261.	(d)
262.	(a)	263.	(c)	264.	(c)	265.	(a)	266.	(a)	267.	(a)	268.	(a)	269.	(d)	270.	(a)
271.	(b)	272.	(a)	273.	(a)	274.	(b)	275.	(c)	276	(b)	277	(b)	278.	(d)	279.	(b)

Solution & Hints

Sol^n (Hint): Q. 1 to Q. 4 (See Type – 2)

Sol^n 5. $x^2 - 2x + 1 = 0$

$(x-1)^2 = 0 \Rightarrow x = 1$, then

$$\frac{x^5 + x^4 + x^3 + x^2}{x} = \frac{1+1+1+1}{1} = 4$$

Sol^n 6. (Hint): Type – 2 (Put $x = 1$)

Sol^n 7. $$\frac{x^6 + x^4 + x^2 + 1}{x^3} = \frac{x^6}{x^3} + \frac{x^4}{x^3} + \frac{x^2}{x^3} + \frac{1}{x^3}$$

$$= x^3 + x + \frac{1}{x} + \frac{1}{x^3} = 110 + 5 = 115$$

$$\because x + \frac{1}{x} = 5 \text{ and } \left(x + \frac{1}{x}\right)^3 = (5)^3$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3x.\frac{1}{x}\left(x + \frac{1}{x}\right) = 125$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 125 - 3 \times 5 = 110$$

Sol^n 8. $x^2 - 7x + 1 = 0$

$\Rightarrow x^2 + 1 = 7x$

$$\Rightarrow \frac{20x}{5x^2 - 15x + 5} = \frac{20x}{5(x^2+1) - 15x}$$

$$\Rightarrow \frac{20x}{5 \times 7x - 15x} = \frac{20x}{35x - 15x} = \frac{20x}{20x} = 1$$

Sol^n 9. $2x + \frac{1}{3x} = 5$

[multiply by 2 both sides]

$4x + \frac{2}{3x} = 10$

[now squaring both sides]

$$\Rightarrow \left(4x + \frac{2}{3x}\right)^2 = (10)^2$$

$$\Rightarrow 16x^2 + \frac{4}{9x^2} + \frac{16}{3} = 100$$

$$\Rightarrow 16x^2 + \frac{4}{9x^2} = 100 - \frac{16}{3} = \frac{284}{3}$$

Sol^{n}10. (Hint): Put $x = -1$

Sol^n 11. (Hint): $5a + \frac{1}{3a} = 5$

(multiply by $\frac{3}{5}$ both sides and then squaring both sides)

Sol^n 12. $x - \frac{1}{x-3} = 0$

$$(x-3) - \frac{1}{(x-3)} = -3$$

Squaring both sides

$$(x-3)^2 + \frac{1}{(x-3)^2} - 2 = 9$$

$$(x-3)^2 + \frac{1}{(x-3)^2} = 9 + 2 = 11$$

Sol^n 13. (Hint): (Same as in 12)

Sol^n 14. $x^2 + y^2 + \frac{1}{x^2} + \frac{1}{y^2} = 4$

$$\left(x^2 + \frac{1}{x^2} - 2\right) + \left(y^2 + \frac{1}{y^2} - 2\right) = 0$$

$$\left(x - \frac{1}{x}\right)^2 + \left(y - \frac{1}{y}\right)^2 = 0$$

(study type :3)

Hence,

$$x - \frac{1}{x} = 0 \Rightarrow x = \frac{1}{x} \Rightarrow x^2 = 1$$

$$y - \frac{1}{y} = 0 \Rightarrow \text{y} = \frac{1}{y} \Rightarrow y^2 = 1$$

$\therefore x^2 + y^2 = 2$

Sol^n 15. $x^2+y^2+\frac{1}{x^2}+\frac{1}{y^2}=0$

$$\left(x^2+\frac{1}{x^2}+2\right)+\left(y^2+\frac{1}{y^2}-2\right)=0$$

$$\left(x+\frac{1}{x}\right)^2+\left(y-\frac{1}{y}\right)^2=0 \qquad \textbf{(Study type 3)}$$

Hence,

$$x+\frac{1}{x}=0 \Rightarrow x=-\frac{1}{x} \Rightarrow x^2=-1$$

$$y-\frac{1}{y}=0 \Rightarrow y=\frac{1}{y} \Rightarrow y^2=1$$

$\therefore\ x^2+y^2=0$

Sol^{n}16. $\frac{2p}{p^2-2p+1}=\frac{1}{4} \Rightarrow p^2-2p+1=4\times 2p=8p$

divide by p both sides

$$p-2+\frac{1}{p}=8 \Rightarrow p+\frac{1}{p}=10$$

Sol^n 17. (Hint): See Type - 1 (E)

Sol^{n}18. $x+\frac{1}{x}=5 \Rightarrow x^2+\frac{1}{x^2}=(5)^2-2=23$

$$=\frac{x^4+3x^3+5x^2+3x+1}{x^4+1}$$

$$=\frac{x^2\left(x^2+3x+5+\frac{3}{x}+\frac{1}{x^2}\right)}{x^2\left(x^2+\frac{1}{x^2}\right)}$$

$$=\frac{\left(x^2+\frac{1}{x^2}\right)+3\left(x+\frac{1}{x}\right)+5}{x^2+\frac{1}{x^2}}$$

$$=\frac{23+3(5)+5}{23}=\frac{43}{23}$$

Sol^{n}19. $\because\ a+b+c=0$

$\therefore\ a+b=-c,\ b+c=-a,\ \&\ c+a=-b$ then

$$\frac{3(a+b)(b+c)(c+a)}{abc}=\frac{3(-c)(-a)(-b)}{abc}=-3$$

Sol^n Q.20 to Q. 22 (Hint): See Type-3
Sol^n Q.23 to Q. 24 (Hint): See Type-1(E)
Sol^n Q.25 (Hint): See Type-1(D)
Sol^n Q.26 (Hint): See Type-1(C)
Sol^n Q.27 to Q. 28 (Hint): See Type-1(E)

Sol^{n}29.

$x=(a+b-c),\ y=(b+c-a)\ \&\ z=(c+a-b)$

here, $x+y+z=a+b+c$

$(x-a)+(y-b)+(z-c)=0$

then,

$$(x-a)^3+(y-b)^3+(z-c)^3=3(x-a)(y-b)(z-c)$$

Sol^{n}30. $a+b+c=0$

it means $a+b=-c,\ b+c=-a\ \&\ c+a=-b$

then, $\frac{a+b}{c}-\frac{2b}{c+a}+\frac{b+c}{a}=\frac{-c}{c}-\frac{2b}{-b}+\frac{-a}{a}=0$

Sol^{n}31. $a+b+c=0$

$$=\frac{1}{(a+b)(b+c)}+\frac{1}{(a+c)(b+c)}+\frac{1}{(b+a)(c+a)}$$

$$=\frac{c+a+a+b+b+c}{(a+b)(b+c)(c+a)}$$

$$=\frac{2(a+b+c)}{(a+b)(b+c)(c+a)}=0$$

Or we can put $a=2,\ b=-1,\ c=-1$

$$=\frac{1}{(2-1)(-1-1)}+\frac{1}{(2-1)(-1-1)}+\frac{1}{(-1+2)(-1+2)}$$

$$=-\frac{1}{2}-\frac{1}{2}+\frac{1}{1}=0$$

Sol^{n}32. $\frac{(a+b)^2-(a-b)^2}{a^2b-ab^2}$

$$=\frac{a^2+b^2+2ab-(a^2+b^2-2ab)}{ab(a-b)}$$

$$=\frac{4ab}{ab(a-b)}=\frac{4}{a-b}$$

Sol^{n}33. $4x = 8y \Rightarrow \frac{x}{y} = \frac{8}{4} = 2$, then $\frac{x}{y} - 1$

$= 2 - 1 = 1$

Sol^{n}34. $x^3 + x^2 + \frac{1}{x^2} + \frac{1}{x^2}$

$= \left(x^3 + \frac{1}{x^3}\right) + \left(x^2 + \frac{1}{x^2}\right)$

$= a^3 - 3a + a^2 - 2$

$\because$ On cubing and squaring of given $x + \frac{1}{x} = a$

we find $x^3 + \frac{1}{x^3} = a^3 - 3a$ and $x^2 + \frac{1}{x^2} = a^2 - 2$

or put $a = 2$, then $x + \frac{1}{x} = 2 \Rightarrow x = 1$

Hence, $x^3 + x^2 + \frac{1}{x^3} + \frac{1}{x^2} = 1 + 1 + 1 + 1 = 4$

put $a = 2$ in all option

(a) 12 (b) 2

(c) 4 (d) 2

Hence (c) is correct

Sol^n Q. 35 to Q. 36 So (Hint): See type-1(G)

Sol^{n}37. $x^3 - x^2y - xy^2 + y^3 = x^3 + y^3 - xy(x + y)$

$= x^3 + y^3 + 3xy(x + y) - 4xy(x + y)$

$= (x + y)^3 - 4xy(x + y) = a^3 - 4ab^2$

[put $x + y = a$, $xy = b^2$]

Sol^n Q. 38 to Q.39(Hint): See type -4

Sol^n Q. 40 to Q.41 (Hint): See type -5

Sol^n Q. 42 to Q.47 (Hint): See type -3

Sol^n 48.

$$\Rightarrow \frac{(a-b)^3+(b-a)^3+(c-a)^3}{(a-b)(b-c)(c-a)} = \frac{3(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)} = 3$$

Sol^{n}49. $x^a . x^b . x^c = 1$

$\Rightarrow x^{a+b+c} = x^0 \quad \Rightarrow a+b+c=0$

then, $a^3 + b^3 + c^3 = 3abc$

Sol^{n}50. $x = a(b - c)$, $y = b(c - a)$ & $z = c(a - b)$,

here, $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$

hence, $\frac{x^3}{a^3} + \frac{y^3}{b^3} + \frac{z^3}{c^3} = 3.\frac{x}{a}.\frac{y}{b}.\frac{z}{c} = \frac{3xyz}{abc}$

Sol^{n}51. Same as Q.50.

Sol^{n}52. $\frac{1}{abc}\left[(a+b+c)^3 - a^3 - b^3 - c^3\right]$

$= \frac{1}{abc}\left[(0)^3 - (a^3 + b^3 + c^3)\right]$

$= \frac{1}{abc}(-3abc) = -3$

($\because$ If $a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc$)

Sol^{n}53. $\because b - a = 1 \Rightarrow b - a - 1 = 0$, then

$(b)^3 + (-a)^3 + (-1)^3 - 3.(b)(-a)(-1) = 0$

$b^3 - a^3 - 1 - 3ab = 0$ or $a^3 - b^3 + 3ab = -1$

Sol^{n}54. $x + y + z = 10$, $x^2 + y^2 + z^2 = 30$

$\because (x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$

$(10)^2 = 30 + 2(xy + yz + zx)$

$\therefore xy + yz + zx = 35$, then

$\Rightarrow x^3 + y^3 + z^3 - 3xyz$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= (10)(30 - 35) = -50$

Sol^{n}55. (Hint): Same as Q.54

Sol^{n}56. $\frac{a^3 + b^3 + c^3 - 3abc}{(a-b)^2 + (b-c)^2 + (c-a)^2}$

$$= \frac{\frac{1}{2}(a+b+c)\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right]}{(a-b)^2 + (b-c)^2 + (c-a)^2}$$

$= \frac{1}{2}(a + b + c) = \frac{1}{2}(25 + 15 - 10) = 15$

Sol^{n}57. $\frac{(a)^3+(b)^3+(c)^3 - 3abc}{(ab+bc+ca - a^2 - b^2 - c^2)}$

$$= \frac{(a+b+c)(a^2+b^2+c^2-ab-bc-ca)}{-(a^2+b^2+c^2-ab-bc-ca)}$$

$= -(a + b + c) = -(-5 - 6 + 10) = 1$

Sol^{n}58. $(a + b)(b + c)(c + a) + abc$

$\Rightarrow (-c)(-a)(-b) + abc = -abc + abc = 0$

($\because a+b+c=0 \therefore a+b=-c, b+c=-a$, and $c+a=-b$)

Sol^{n}59. $\frac{1}{(a-b)(b-c)} + \frac{1}{(b-c)(c-a)} + \frac{1}{(c-a)(a-b)}$

$= \frac{c-a+a-b+b-c}{(a-b)(b-c)(c-a)} = 0$

Sol^{n}60. $(a-b)(b-c)+(b-c)(c-a)+(c-a)(a-b)$
$= ab-ac-b^2+bc+bc-ba-c^2+ac+ca-cb-a^2+ab$
$= -(a^2+b^2+c^2-ab-bc-ca)$
$= 3(ab+bc+ca)$
$[\because a+b+c=0 \Rightarrow a^2+b^2+c^2=-2(ab+bc+ca)]$

Sol^{n}61. $\frac{1}{bc}+\frac{1}{ac}+\frac{1}{ab}$

$= \frac{a+b+c}{abc} = 0 \quad (\because a+b+c=0)$

Sol^{n}62. $a+b+c=0 \Rightarrow b+c=-a$ (squaring both sides)
$\Rightarrow b^2+c^2+2bc=a^2$
$\Rightarrow b^2+c^2=a^2-2bc$

$$\frac{a^2+b^2+c^2}{a^2-bc}=\frac{a^2+a^2-2bc}{a^2-bc}=\frac{2(a^2-bc)}{a^2-bc}=2$$

Method - 2: choose a, b, c such that $a+b+c=0$
$a=1, b=-1, c=0$, then

$$\frac{a^2+b^2+c^2}{a^2-bc}=\frac{1+1+0}{1-0}=2$$

Sol^{n}63. $a+b+c=0$ so put $a=1, b=-1, c=0$

$$\therefore \frac{a^2-bc}{b^2-ca}=\frac{1-0}{1-0}=1$$

Sol^{n}64. $a+b+c=0$ so put $a=1, b=-1, c=0$

$$\therefore \frac{\left(a^2+b^2+c^2\right)^2}{a^2b^2+b^2c^2+c^2a^2}=\frac{(1+1+0)^2}{1+0+0}=4$$

Sol^{n}65. $a+b+c=0$ so put $a=1, b=-1, c=0$ then

$$\therefore \frac{a^2b^2+b^2c^2+c^2a^2}{a^4+b^4+c^4}=\frac{1+0+0}{1+1}=\frac{1}{2}$$

Sol^{n}66. $\because a+b+c=0 \Rightarrow a+b=-c \Rightarrow (a+b)^2=(-c)^2$
$a^2+b^2+2ab=c^2 \Rightarrow a^2+b^2-c^2=-2ab$
Similarly, $b^2+c^2-a^2=-2bc$
$c^2+a^2-b^2=-2ac$

$$\Rightarrow \frac{1}{a^2+b^2-c^2}+\frac{1}{b^2+c^2-a^2}+\frac{1}{c^2+a^2-b^2}$$

$$=\frac{1}{-2ab}+\frac{1}{-2bc}+\frac{1}{-2ca}=\frac{-c-b-c}{2abc}$$

$$=-\frac{(a+b+c)}{2abc}=0$$

Method-2:

Put $a=2, b=-1, c=1$

$$\frac{1}{4+1-1}+\frac{1}{1+1-4}+\frac{1}{1+4-1}=\frac{1}{4}+\frac{1}{-2}+\frac{1}{4}=0$$

Sol^{n}67. $\because a(a+2)=a+b+c$

$$\Rightarrow \frac{a}{a+b+c}=\frac{1}{a+2} \quad \ldots(i)$$

$\because b(b+2)=a+b+c$

$$\Rightarrow \frac{b}{a+b+c}=\frac{1}{b+2} \quad \ldots(ii)$$

$\because c(c+2)=a+b+c$

$$\Rightarrow \frac{c}{a+b+c}=\frac{1}{c+2} \quad \ldots(iii)$$

Adding equation (i), (ii) and (iii)

$$\frac{1}{a+2}+\frac{1}{b+2}+\frac{1}{c+2}$$

$$=\frac{a}{a+b+c}+\frac{b}{a+b+c}+\frac{c}{a+b+c}=\frac{a+b+c}{a+b+c}=1$$

Sol^{n}68. $\frac{(s-a)^2+(s-b)^2+(s-c)^2+s^2}{a^2+b^2+c^2}$

$$=\frac{s^2+a^2-2as+s^2+b^2-2sb+s^2+c^2-2sc+s^2}{a^2+b^2+c^2}$$

$(\because a+b+c=2s)$

$$=\frac{4s^2+(a^2+b^2+c^2)-2s(a+b+c)}{a^2+b^2+c^2}$$

$$=\frac{4s^2+a^2+b^2+c^2-2s(2s)}{a^2+b^2+c^2}$$

$$=\frac{4s^2+a^2+b^2+c^2-4s^2}{a^2+b^2+c^2}=\frac{a^2+b^2+c^2}{a^2+b^2+c^2}=1$$

Method–2:

$\because$ Answer is not depending upon s, so we can put $s=0$ then

$$=\frac{(s-a)^2+(s-b)^2+(s-c)^2+s^2}{a^2+b^2+c^2}$$

$$=\frac{a^2+b^2+c^2}{a^2+b^2+c^2}=1$$

Sol^{n}69. $\frac{x}{y}+\frac{y}{x}=1 \Rightarrow \frac{x^2+y^2}{xy}=1 \Rightarrow x^2+y^2=xy$

$\therefore 2(x^3+y^3)=2[(x+y)(x^2+y^2-xy)]=0$

Sol^{n}70. $\frac{x}{y}+\frac{y}{x}=-2 \Rightarrow \frac{x^2+y^2}{xy}=-2$

$\Rightarrow x^2+y^2=-2xy$

$\Rightarrow (x^2+y^2+2xy)=0 \Rightarrow (x+y)^2=0$

hence, $x=-y$, then

$x^3+y^3+3xy(x+y)=(-y)^3+y^3+3xy(-y+y)=0$

Sol^{n}71. $\frac{a}{b}+\frac{b}{a}=3 \Rightarrow \left(\frac{a}{b}+\frac{b}{a}\right)^3=3^3$

$\Rightarrow \frac{a^3}{b^3}+\frac{b^3}{a^3}+3\frac{a}{b}.\frac{b}{a}\left(\frac{a}{b}+\frac{b}{a}\right)=27$

$\frac{a^3}{b^3}+\frac{b^3}{a^3}+3.1\ (3)=27$

$\Rightarrow \frac{a^3}{b^3}+\frac{b^3}{a^3}=27-9=18$

Sol^{n}72. $\frac{11}{x+11}+\frac{23}{y+23}+\frac{239}{z+239}=2$

subtract 1 from each term

$\Rightarrow \frac{11}{x+11}-1+\frac{23}{y+23}-1+\frac{239}{z+239}-1=2-1-1-1$

$\Rightarrow \frac{11-x-11}{x+11}+\frac{23-y-23}{y+23}+\frac{239-z-239}{z+239}=-1$

$\Rightarrow \frac{-x}{x+11}+\frac{-y}{y+23}+\frac{-z}{z+239}=-1$

$\Rightarrow \frac{x}{x+11}+\frac{y}{y+23}+\frac{z}{z+239}=1$

Sol^{n}73. $x^2+y^2=45, y^2+z^2=40$

subtracting both equation.

$(x^2+y^2)-(y^2+z^2)=45-40$

$x^2-z^2=5$ if x, y, z are integer

then x should be 3 and z should be 2

$\Rightarrow 3^2-2^2=5$

from $x^2+y^2=45$

$3^2+y^2=45 \Rightarrow y^2=45-9=36 \Rightarrow y=6$

$\therefore x+y+z=3+6+2=11$

Sol^{n}74. $\frac{x-y}{3}=\frac{x+y}{7}=\frac{xy}{5}=k$

$x-y=3k$ & $x+y=7k$

then,

$x=5k$ & $y=2k$

$xy=5k$

$(5k)(2k)=5k \Rightarrow k=\frac{1}{2}$

then, $xy=\frac{5}{2}$

Sol^{n}75. $x=2-2^{1/3}+2^{2/3} \Rightarrow x-2=2^{2/3}-2^{1/3}$

Cubing both sides

$(x-2)^3=(2^{2/3}-2^{1/3})^3$

$x^3-8-3x.2(x-2)=2^2-2^1-3.2^{2/3}.2^{1/3}(2^{2/3}-2^{1/3})$

$x^3-8-6x^2+12x=4-2-3.2.(x-2)$

$(\because x-2=2^{2/3}-2^{1/3})$

$x^3-6x^2+12x-8=2-6x+12$

$x^3-6x^2+18x=14+8=22$

adding 18 to both sides

$x^3-6x^2+18x+18=22+18=40$

Sol^{n}76. $x=b+c-2a, y=c+a-2b$ & $z=a+b-2c$

$\Rightarrow x^2+y^2-z^2+2xy=(x+y)^2-z^2$

$\Rightarrow (x+y+z)(x+y-z)=0$

$\Rightarrow (b+c-2a+c+a-2b+a+b-2c)$

$(b+c-2a+c+a-2b-a-b+2c)$

$=(0)(4c-2a-2b)$

$=0$

Sol^{n}77. $p+q=\frac{1}{pq}$ (cubing both side)

$(p+q)^3=\frac{1}{p^3q^3} \Rightarrow p^3+q^3+3pq(p+q)=\frac{1}{p^3q^3}$

$p^3+q^3+3=\frac{1}{p^3q^3} \Rightarrow \frac{1}{p^3q^3}-p^3-q^3=3$

$(\because pq(p+q)=1)$

Sol^{n}78. $\dfrac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = a-b\sqrt{6}$

$$\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}\times\frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} = a-b\sqrt{6}$$

$$\frac{6+2\sqrt{6}+3\sqrt{6}+6}{18-12} = a-b\sqrt{6}$$

$$\frac{12+5\sqrt{6}}{6} = a-b\sqrt{6}$$

$$2+\frac{5}{6}\sqrt{6} = a-b\sqrt{6}$$

$a=2$

$b=-\dfrac{5}{6}$

Sol^{n}79. (Hint): Same as Q. 78

Sol^{n}80. $x = \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}\times\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \dfrac{\left(\sqrt{3}+\sqrt{2}\right)^2}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}$

$$= \frac{3+2+2\sqrt{6}}{3-2} = 5+2\sqrt{6}$$

Similarly,

$y = \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = 5-2\sqrt{6}$ and $xy = 1$

$\therefore x^2+y^2 = x^2+y^2+2xy-2xy = (x+y)^2-2xy = (10)^2-2 = 98$

Sol^{n}81. $a = 3.23$, $b = 5.95$ & $c = 2.72$

here, $a-b+c=0$

then, the value of $a^3-b^3+c^3+3abc = 0$

Sol^{n}82. We know that $5^2+12^2 = 13^2$

on comparing this with $5^{\sqrt{x}}+12^{\sqrt{x}} = 13^{\sqrt{x}}$,

$\sqrt{x} = 2 \Rightarrow x = 4$

Sol^{n}83. $(x^{b+c})^{b-c}.(x^{c+a})^{c-a}.(x^{a+b})^{a-b}$

$$= x^{b^2-c^2}.x^{c^2-a^2}.x^{a^2-b^2}$$

$$= x^{b^2-c^2+c^2-a^2+a^2-b^2} = x^0 = 1$$

Sol^{n}84. $\left(\dfrac{x^a}{x^b}\right)^{a+b}.\left(\dfrac{x^b}{x^c}\right)^{b+c}.\left(\dfrac{x^c}{x^a}\right)^{c+a}$

$$= \left(x^{a-b}\right)^{a+b}.\left(x^{b-c}\right)^{b+c}.\left(x^{c-a}\right)^{c+a}$$

$$= x^{a^2-b^2}.x^{b^2-c^2}.x^{c^2-a^2} = 1$$

Method–2: ($\because$ answer is not depending upon a, b, c so we can put a, b, c equal to zero.)

$$\left(\frac{x^a}{x^b}\right)^{a+b}.\left(\frac{x^b}{x^c}\right)^{b+c}.\left(\frac{x^c}{x^a}\right)^{c+a} = 1$$

Sol^{n}85.

$$\frac{1}{1+x^{b-a}+x^{c-a}}+\frac{1}{1+x^{a-b}+x^{c-b}}+\frac{1}{1+x^{b-c}+x^{a-c}}$$

$$= \frac{1}{x^{-a}\left(x^a+x^b+x^c\right)}+\frac{1}{x^{-b}\left(x^a+x^b+x^c\right)}+\frac{1}{x^{-c}\left(x^a+x^b+x^c\right)}$$

$$= \frac{x^a}{(x^a+x^b+x^c)}+\frac{x^b}{(x^a+x^b+x^c)}+\frac{x^c}{(x^a+x^b+x^c)}$$

$$= \frac{x^a+x^b+x^c}{(x^a+x^b+x^c)} = 1$$

Method–2: (put $a=0$, $b=0$, $c=0$ or $a=b=c$)

$$= \frac{1}{(1+x^0+x^0)}+\frac{1}{(1+x^0+x^0)}+\frac{1}{(1+x^0+x^0)}$$

$$= \frac{1}{1+1+1}+\frac{1}{1+1+1}+\frac{1}{1+1+1} = \frac{1}{3}+\frac{1}{3}+\frac{1}{3} = 1$$

Sol^{n}86. $\left(\dfrac{x^b}{x^c}\right)^{b+c-a}.\left(\dfrac{x^c}{x^a}\right)^{c+a-b}.\left(\dfrac{x^a}{x^b}\right)^{a+b-c}$

$$= \left(x^{b-c}\right)^{(b+c)-a}+\left(x^{c-a}\right)^{(c+a)-b}.\left(x^{a-b}\right)^{(a+b)-c}$$

$$= (x)^{(b^2-c^2)-ab+ac}.(x)^{c^2-a^2-bc+ba}.(x)^{a^2-b^2-ac+bc}$$

$$= (x)^{b^2-c^2-ab+ac+c^2-a^2-bc+ba+a^2-b^2-ac+bc}$$

$$= x^0 = 1$$

Sol^n87. $\left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}} \cdot \left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} \cdot \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}}$

$= \left(x^{a-b}\right)^{\frac{1}{ab}} \cdot \left(x^{b-c}\right)^{\frac{1}{bc}} \cdot \left(x^{c-a}\right)^{\frac{1}{ca}}$

$= (x)^{\frac{a-b}{ab}} \cdot (x)^{\frac{b-c}{bc}} \cdot (x)^{\frac{c-a}{ca}}$

$= x^{\frac{1}{b}-\frac{1}{a}+\frac{1}{c}-\frac{1}{b}+\frac{1}{a}-\frac{1}{c}} = x^0 = 1$

Sol^n88. $a = \frac{\sqrt{5}+1}{\sqrt{5}-1}$, $b = \frac{\sqrt{5}-1}{\sqrt{5}+1} \Rightarrow ab = 1$

and $a + b = \frac{\left(\sqrt{5}+1\right)^2 + \left(\sqrt{5}-1\right)^2}{\left(\sqrt{5}+1\right)\left(\sqrt{5}-1\right)}$

$= \frac{5+1+2\sqrt{5}+5+1-2\sqrt{5}}{\left(\sqrt{5}\right)^2-(1)^2} = \frac{12}{4} = 3$

$\therefore \frac{a^2+ab+b^2}{a^2-ab+b^2} = \frac{a^2+2ab+b^2-ab}{a^2+2ab+b^2-3ab}$

$= \frac{(a+b)^2-ab}{(a+b)^2-3ab}$

$= \frac{3^2-1}{3^2-3} = \frac{8}{6} = \frac{4}{3}$

Sol^n89. Put $a = 1$, then

$x = 2, y = 0$

$x^4 + y^4 - 2x^2y^2 = (2)^4 + (0)^4 - 2(2)^2(0)^2$

$= 16$

Sol^n90. $a^4 + a^2b^2 + b^4 = 21$

$a^2 + b^2 = 7 - ab$

squaring both sides

$a^4 + b^4 + 2a^2b^2 = 49 + a^2b^2 - 14ab$

$a^4 + b^4 + a^2b^2 = 49 - 14ab$

$21 = 49 - 14ab \Rightarrow 14ab = 28$

$ab = 2$

Sol^n91. Put $a = b = c = 0$

$= \frac{1}{x^0+\frac{1}{x^0}+1} + \frac{1}{x^0+\frac{1}{x^0}+1} + \frac{1}{x^0+\frac{1}{x^0}+1}$

$= \frac{1}{1+1+1} + \frac{1}{1+1+1} + \frac{1}{1+1+1}$

$= \frac{1}{3}+\frac{1}{3}+\frac{1}{3} = 1$

Sol^n92. $\frac{7x-3}{x} + \frac{7y-3}{y} + \frac{7z-3}{z} = 0$

on dividing

$\Rightarrow 7-\frac{3}{x}+7-\frac{3}{y}+7-\frac{3}{z} = 0$

$\Rightarrow 21 = 3\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$

$\Rightarrow \frac{1}{x}+\frac{1}{y}+\frac{1}{z} = 7$

Sol^n93. $2^x = 4^y = 8^z$

$\Rightarrow 2^x = 2^{2y} = 2^{3z}$

$\Rightarrow x = 2y = 3z$

$\Rightarrow \frac{1}{2x} + \frac{1}{4\left(\frac{x}{2}\right)} + \frac{1}{4\left(\frac{x}{3}\right)} = 4$

$\Rightarrow \frac{1}{2x}+\frac{1}{2x}+\frac{3}{4x} = 4$

$\Rightarrow \frac{2+2+3}{4x} = 4$

$\Rightarrow \frac{7}{4x} = 4 \quad \Rightarrow x = \frac{7}{16}$

Sol^n94. (Hint): See type -7 $\Rightarrow x = y = z = 8$

$a = b = c = 1$

Sol^n 95. $\sqrt{\frac{x-a}{x-b}}+\frac{a}{x}=\sqrt{\frac{x-b}{x-a}}+\frac{b}{x}$

$\Rightarrow \frac{\sqrt{x-a}}{\sqrt{x-b}}-\frac{\sqrt{x-b}}{\sqrt{x-a}}=\frac{b}{x}-\frac{a}{x}$

$\Rightarrow \frac{x-a-x+b}{\sqrt{x-a}.\sqrt{x-b}}=\frac{b-a}{x}$

$\Rightarrow \frac{b-a}{\sqrt{(x-a)(x-b)}}=\frac{b-a}{x}$

$\Rightarrow x=\sqrt{(x-a)(x-b)}$ (squaring both sides)

$\Rightarrow x^2=x^2-x(a+b)+ab$

$\Rightarrow x(a+b)=ab \quad \Rightarrow x=\frac{ab}{a+b}$

Sol^n 96. $x^2-2xy+y^2+4y^2-4y+1=0$

$(x-y)^2+(2y-1)^2=0$

$x-y=0 \quad \Rightarrow x=y$

$2y-1=0 \quad \Rightarrow y=\frac{1}{2}$ & $x=\frac{1}{2}$

Sol^n 97. $a^x=b, b^y=c, xyz=1$

$\therefore c^z=(b^y)^z=(b)^{yz}=(a^x)^{yz}=a^{xyz}=a$

Sol^n 98. We know that

$a^x=b$ then $a=b^{1/x}$ (e.g. $a^2=3 \Rightarrow a=3^{1/2}$)

$\therefore (3.7)^x=(0.037)^y=10000=10^4$

$\Rightarrow (3.7)=10^{4/x}$ …(i)

$\Rightarrow (0.037)=10^{4/y}$ …(ii)

divide eq^n. (i) by eq^n. (ii)

$\frac{3.7}{0.037}=\frac{10^{\frac{4}{x}}}{10^{\frac{4}{y}}} \Rightarrow 100=10^{\frac{4}{x}-\frac{4}{y}}$

$10^2=10^{4\left(\frac{1}{x}-\frac{1}{y}\right)}$ (on comparing power)

$4\left(\frac{1}{x}-\frac{1}{y}\right)=2 \quad \Rightarrow \frac{1}{x}-\frac{1}{y}=\frac{2}{4}=\frac{1}{2}$

Sol^n 99. $p^x=r^y=m \Rightarrow r=p^{x/y}$

$r^w=p^z=n \Rightarrow r=p^{z/w}. \Rightarrow r=p^{z/w}.$

on comparing power

$\frac{x}{y}=\frac{z}{w} \Rightarrow xw=yz$

Sol^n 100. $x(y-z)(y+z)+y(z-x)(z+x)+z(x-y)(x+y)$

$= x(y^2-z^2)+y(z^2-x^2)+z(x-y)(x+y)$

$= xy^2-xz^2+yz^2-yx^2+z(x-y)(x+y)$

$= xy^2-yx^2-xz^2+yz^2+z(x-y)(x+y)$

$= -xy(x-y)-z^2(x-y)+z(x-y)(x+y)$

$= (x-y)[-xy-z^2+z(x+y)]$

$= (x-y)(-xy+zx-z^2+zy)$

$= (x-y)[-x(y-z)+z(y-z)]$

$= (x-y)(y-z)(z-x)$

Method–2 :

(put $x=0, y=1, z=2$ in question and all option)

$0+1(2-0)(2+0)+2(0-1)(0+1)=4-2=2$

Option (a): $(0+1)(1+2)(2+0)=6$

Option (b): $(0-1)(0-2)(2-1)=2$

Option (c): $(0+1)(2-1)(0-2)=-2$

Option (d): $(1-0)(2-1)(0-2)=-2$

Hence option (b) is correct.

Sol^n 101. $\frac{1}{a+1}+\frac{2a+1}{a^2-1}=\frac{a-1+2a+1}{a^2-1}$

$=\frac{3a}{a^2-1}=\frac{3a}{a\left(a-\frac{1}{a}\right)}=\frac{3}{a-\frac{1}{a}}$

put, $a=\frac{1+x}{2-x}$

$=\frac{3}{\frac{1+x}{2-x}-\frac{2-x}{1+x}}=\frac{3(2-x)(1+x)}{(1+x)^2-(2-x)^2}=\frac{3(2-x)(1+x)}{6x-3}$

$=\frac{(2-x)(1+x)}{2x-1}$

Method–2: we can put, $x = 1$

$$a = \frac{1+1}{2-1} = 2$$

then, $\frac{1}{a+1} + \frac{2a+1}{a^2-1} = \frac{1}{3} + \frac{5}{3} = 2$

(put, $x = 1$ in all four option)

Option (a) $\Rightarrow \frac{(1+x)(2+x)}{2x-1} = \frac{(1+1)(2+1)}{1} = 6$

Option(b) $\Rightarrow$

$$\frac{(1-x)(2-x)}{x+1} = \frac{(1-1)(2-1)}{1+1} = 0$$

Option (c) $\Rightarrow \frac{(1+x)(2-x)}{2x-1} = \frac{(1+1)(2-1)}{1} = 2$

Option (d) $\Rightarrow \frac{(1-x)(2-x)}{2x+1} = \frac{(1-1)(2-1)}{2\times1+1} = 0$

Hence option (c) is correct

Sol^{n}102. (Hint): See type - 5

Sol^{n}103. $x=(b-c)(a-d), y=(c-a)(b-d), z=(a-b)(c-d)$

$\because x+y+z=0$

$\therefore x^3+y^3+z^3=3xyz$

Sol^{n}104. $x^2+2=2x \Rightarrow x^2=2x-2$

$\Rightarrow x^4-x^3+x^2+2=(x^2)^2-x(x^2)+(x^2+2)$

$= (2x-2)^2-x(2x-2)+2x$

$= 4x^2+4-8x-2x^2+2x+2x$

$= 2x^2+4-4x$

$= 2(x^2+2)-4x = 2\,(2x)-4x$

$= 0$

Sol^{n}105. $2^x=3^y=6^{-z}=k$

$2=k^{1/x},\ 3=k^{1/y}\ \&\ \ 6=k^{-1/z}$

$\Rightarrow 2\times3=k^{-1/z}$

$k^{1/x}\times k^{1/y}=k^{-1/z}$

$$k^{\frac{1}{x}+\frac{1}{y}}=k^{-1/z}$$

on comparing power

$$\Rightarrow \frac{1}{x}+\frac{1}{y}=-\frac{1}{z}$$

$$\Rightarrow \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0$$

Sol^{n}106. $(x^2+y^2)(p^2+q^2)=(xp+yq)^2$

$x^2p^2+y^2p^2+x^2q^2+y^2q^2=x^2p^2+y^2q^2+2xypq$

$x^2q^2+y^2p^2-2xypq=0$

$(xq-yp)^2=0 \quad \Rightarrow xq=yp$

Sol^{n}107. $x^2+8y^2+9z^2-4xy-12yz=0$

$x^2+4y^2-4xy+4y^2+9z^2-12yz=0$

$(x-2y)^2+(2y-3z)^2=0$

See type - 3

$x-2y=0$ and $2y-3z=0$

$x=2y=3z$

Sol^{n}108. $9(a+b)^2+49c^2-42(a+b)c$

$= [3(a+b)-7c]^2=[3(89-69)-7\times8]^2$

$= (3\times20-56)^2=(60-56)^2$

$= 16$

Sol^{n}109. $x=\frac{p+q}{p-q}\ \&\ y=\frac{p-q}{p+q}$

$$\Rightarrow \frac{x}{y}=\frac{(p+q)^2}{(p-q)^2}$$

Applying Component & Dividend rule.

$$\Rightarrow \frac{x+y}{x-y}=\frac{(p+q)^2+(p-q)^2}{(p+q)^2-(p-q)^2}=\frac{p^2+q^2}{2pq}$$

$$\Rightarrow \frac{x-y}{x+y}=\frac{2pq}{p^2+q^2}$$

Sol^{n}110. Using formula $(a^2-b^2)=(a-b)(a+b)$

$= (2+1)(2^2+1)(2^4+1)(2^8+1)(2^{16}+1)(2^{32}+1)(2^{64}+1)$

$= (2-1)(2+1)(2^2+1)(2^4+1)(2^8+1)(2^{16}+1)(2^{32}+1)(2^{64}+1)$

$= (2^2-1)(2^2+1)(2^4+1)(2^8+1)(2^{16}+1)(2^{32}+1)(2^{64}+1)$

$= (2^4-1)(2^4+1)(2^8+1)(2^{16}+1)(2^{32}+1)(2^{64}+1)$

$= (2^8-1)(2^8+1)(2^{16}+1)(2^{32}+1)(2^{64}+1)$

$= (2^{16}-1)(2^{16}+1)(2^{32}+1)(2^{64}+1)$

$= (2^{32}-1)(2^{32}+1)(2^{64}+1)$

$= (2^{64}-1)(2^{64}+1)$

$= 2^{128}-1$

Sol^{n}111. $x+y+z=0$

$y=-z-x,\ z=-x-y, \&\ x=-y-z,$

$[(y-z-x)/2]^3+[(z-x-y)/2]^3+[(x-y-z)/2]^3$

$=[(y+y)/2]^3+[(z+z)/2]^3+[(x+x)/2]^3$

$=y^3+z^3+x^3=3xyz \qquad (\because x+y+z=0)$

Sol^n^112. $ax + by = 6$

$a^2x^2 + b^2y^2 + 2abxy = 36$ …(i)

$bx - ay = 2$

$b^2x^2 + a^2y^2 - 2abxy = 4$ …(ii)

Adding both eq^n^. (i) & (ii)

$a^2x^2 + b^2x^2 + b^2y^2 + a^2y^2 = 40$

$x^2(a^2 + b^2) + y^2(a^2 + b^2) = 40$

$(a^2 + b^2)(x^2 + y^2) = 40$

$(a^2 + b^2) = 10 \quad (\because x^2 + y^2 = 4)$

Sol^n^113. $a = 3 + 2\sqrt{2}$

$$\frac{1}{a} = \frac{1}{3+2\sqrt{2}} = 3 - 2\sqrt{2}$$

$$a + \frac{1}{a} = 6 \Rightarrow a^3 + \frac{1}{a^3} = \left(a + \frac{1}{a}\right)^3 - 3\left(a + \frac{1}{a}\right)$$

$$= (6)^3 - 3(6) = 198$$

$$\frac{a^6 + a^4 + a^2 + 1}{a^3} = a^3 + a + \frac{1}{a} + \frac{1}{a^3} = 198 + 6 = 204$$

Sol^n^114. See Type-3

Sol^n^115. $a^3b = abc = 180 = 2^2 \times 3^2 \times 5^1 = 1^3 \times 2^2 \times 3^2 \times 5^1$

a, b, c are integers so a should be 1.

$a^3b = abc \quad \Rightarrow \quad c = a^2 = (1)^2 = 1$

here, $b = 180$

Sol^n^116. $x = \sqrt{3} + \sqrt{2}$ & $\frac{1}{x} = \sqrt{3} - \sqrt{2}$

$$\Rightarrow x - \frac{1}{x} = 2\sqrt{2}$$

Sol^n^117. $(a^2 + b^2)^3 = (a^3 + b^3)^2$

$a^6 + b^6 + 3a^2b^2(a^2 + b^2) = a^6 + b^6 + 2a^3b^3$

$3a^2b^2(a^2 + b^2) = 2a^3b^3$

$3(a^2 + b^2) = 2ab$

$$\frac{a^2 + b^2}{ab} = \frac{2}{3} \Rightarrow \frac{a}{b} + \frac{b}{a} = \frac{2}{3}$$

Sol^n^118. $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{xy} + \frac{1}{yz} + \frac{1}{zx}$

Multiply by 2 both sides

$$\frac{2}{x^2} + \frac{2}{y^2} + \frac{2}{z^2} = \frac{2}{xy} + \frac{2}{yz} + \frac{2}{zx}$$

$$\left(\frac{1}{x^2} + \frac{1}{y^2} - \frac{2}{xy}\right) + \left(\frac{1}{y^2} + \frac{1}{z^2} - \frac{2}{yz}\right) + \left(\frac{1}{z^2} + \frac{1}{x^2} - \frac{2}{zx}\right) = 0$$

$$\left(\frac{1}{x} - \frac{1}{y}\right)^2 + \left(\frac{1}{y} - \frac{1}{z}\right)^2 + \left(\frac{1}{z} - \frac{1}{x}\right)^2 = 0$$

$$\Rightarrow \left(\frac{1}{x} - \frac{1}{y}\right) = 0 \Rightarrow \frac{1}{x} = \frac{1}{y} \Rightarrow x = y \quad \text{…(i)}$$

$$\Rightarrow \left(\frac{1}{y} - \frac{1}{z}\right) = 0 \Rightarrow \frac{1}{y} = \frac{1}{z} \Rightarrow y = z \quad \text{…(ii)}$$

$$\Rightarrow \left(\frac{1}{z} - \frac{1}{x}\right) = 0 \Rightarrow \frac{1}{z} = \frac{1}{x} \Rightarrow z = x \quad \text{…(iii)}$$

from equation (i), (ii) & (iii)

$x = y = z$

Sol^n^119. $p - 2q = 4$

$p - 2q - 4 = 0$

(If $a + b + c = 0$ then, $a^3 + b^3 + c^3 = 3abc$)

$p^3 + (-2q)^3 + (-4)^3 = 3.p(-2q).(-4)$

$p^3 - 8q^3 - 64 = 24pq$

$p^3 - 8q^3 - 24pq - 64 = 0$

Sol^n^120. $\frac{1}{\sqrt[3]{4} + \sqrt[3]{2} + 1} = a\sqrt[3]{4} + b\sqrt[3]{2} + c$

Let $t = \sqrt[3]{2} \Rightarrow t^2 = \sqrt[3]{4}$ & $t^3 = 2$

$$\frac{1}{t^2 + t + 1} = at^2 + bt + c$$

$$\Rightarrow \frac{t - 1}{(t-1)(t^2 + t + 1)} = at^2 + bt + c$$

$$\Rightarrow \frac{t - 1}{(t^3 - 1)} = at^2 + bt + c \quad (\because t^3 - 1 = 2 - 1 = 1)$$

$t - 1 = at^2 + bt + c$

Comparing coefficient

$a = 0, b = 1, c = -1$

$a + b + c = 0 + 1 - 1 = 0$

Sol^n^121. $a^3 - b^3 - c^3 = 0$ or $a^3 + (-b^3) + (-c^3) = 0$

then,

$(a^3)^3 + (-b^3)^3 + (-c^3)^3 - 3.(a^3)(-b^3)(-c^3) = 0$

$a^9 - b^9 - c^9 - 3a^3b^3c^3 = 0$

Sol^{n}122. $\frac{4x-3}{x}+\frac{4y-3}{y}+\frac{4z-3}{z}=0$

$$4-\frac{3}{x}+4-\frac{3}{y}+4-\frac{3}{z}=0$$

$$\frac{3}{x}+\frac{3}{y}+\frac{3}{z}=12 \Rightarrow \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=4$$

hence, $\frac{2}{x}+\frac{2}{y}+\frac{2}{z}=8$

Sol^{n}123. $(x-3)^2+(y-5)^2+(z-4)^2=0$

$$x-3=0 \Rightarrow x=3 \Rightarrow \frac{x}{3}=1 \quad ...(i)$$

$$y-5=0 \Rightarrow y=5 \Rightarrow \frac{y}{5}=1 \quad ...(ii)$$

$$z-4=0 \Rightarrow z=4 \Rightarrow \frac{z}{4}=1 \quad ...(iii)$$

Add all three equation after squaring

$$\frac{x^2}{9}+\frac{y^2}{25}+\frac{z^2}{16}=3$$

Sol^{n}124. $x-2=\sqrt[3]{5}$

Cubing both sides

$$(x-2)^3=\left(\sqrt[3]{5}\right)^3$$

$$x^3-8-6x(x-2)=5$$

$$x^3-6x^2+12x-13=0$$

Sol^{n}125. See example 74 (Type- 1(G))

Sol^{n}126. $\sqrt{x}=\sqrt{3}-\sqrt{5}$

Squaring both sides

$$x=3+5-2.\sqrt{3}.\sqrt{5}$$

$$x=8-2\sqrt{15}$$

$$x-8=\left(-2\sqrt{15}\right)$$

Squaring both sides

$$x^2+64-16x=60$$

$$x^2-16x+4=0$$

adding 2 both sides

$$x^2-16x+6=2$$

Sol^{n}127. let $x=\sqrt{2\sqrt[3]{4\sqrt{2\sqrt[3]{4\sqrt{2\sqrt[3]{4............\infty}}}}}}$

$$x=\sqrt{2\sqrt[3]{4x}} \quad \text{(squaring both sides)}$$

$$x^2=2\sqrt[3]{4x} \quad \text{(cubing both sides)}$$

$$x^6=2^3.4x=2^5.x \Rightarrow x^5=2^5$$

hence, $x=2$

Sol^{n}128. $(a^2-b^2)+(b^2-c^2)+(c^2-a^2)=0$

$(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3=$

$3(a^2-b^2)(b^2-c^2)(c^2-a^2)$

$=3(a+b)(a-b)(b+c)(b-c)(c+a)(c-a)$

Hence, option (a) is correct.

Sol^{n}129. $a-b=3,\ a^3-b^3=117$

We can think as $a=5,\ b=2$

both equation are satisfying these values

$\Rightarrow 5-3=2$ and $(5)^3-(2)^3=125-8=117$

hence, $|a+b|=7$

or

$a^3-b^3=117$

$(a-b)(a^2+b^2+ab)=117$

$a^2+b^2+ab=39$

$(a-b)^2+3ab=39 \Rightarrow (3)^2+3ab=39$

$ab=10 \Rightarrow a=5,\ b=2$

$|a+b|=5+2=7$

Sol^{n}130. $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{1}{a+b+c}$

$$\frac{1}{b}+\frac{1}{c}=\frac{1}{a+b+c}-\frac{1}{a}$$

$$\frac{b+c}{bc}=\frac{a-a-b-c}{a(a+b+c)}=\frac{-b-c}{a(a+b+c)}$$

$$\frac{1}{bc}=\frac{-1}{a(a+b+c)} \Rightarrow a(a+b+c)=-bc$$

$$a^2+ab+ac+bc=0 \Rightarrow a(a+b)+c(a+b)=0$$

$$\Rightarrow (a+b)(c+a)=0$$

hence, $(a+b)(b+c)(c+a)=0$

Sol^{n}131. (Hint): See Example-121

Sol^{n}132. (Hint): See Example-123

Sol^{n}133. $(a+b-2c)+(b+c-2a)+(c+a-2b)=0$

$(a+b-2c)^3+(b+c-2a)^3+(c+a-2b)^3$
$=3(a+b-2c)(b+c-2a)(c+a-2b)$

Sol^{n}134. $\dfrac{5x}{2x^2+5x+1}=\dfrac{1}{3}$

$15x=2x^2+5x+1$

$2x^2-10x+1=0$

Divide by $2x$

$\Rightarrow x-5+\dfrac{1}{2x}=0 \Rightarrow x+\dfrac{1}{2x}=5$

IInd

$2x\left(x+\dfrac{1}{2x}\right)=10x=x+\dfrac{2}{2x}=5$

Sol^{n}135. $x^3+\dfrac{3}{x}=4(a^3+b^3)$ and $3x+\dfrac{1}{x^3}=4(a^3-b^3)$

Adding both equation

$x^3+\dfrac{3}{x}+3x+\dfrac{1}{x^3}=4(a^3+b^3)+4(a^3-b^3)$

$\left(x+\dfrac{1}{x}\right)^3=8a^3 \Rightarrow x+\dfrac{1}{x}=2a$

Subtracting both equation

$x^3+\dfrac{3}{x}-3x-\dfrac{1}{x^3}=4(a^3+b^3)-4(a^3-b^3)$

$\left(x-\dfrac{1}{x}\right)^3=8b^3 \Rightarrow x-\dfrac{1}{x}=2b$

$4(a^2-b^2)=(2a)^2-(2b)^2$

$=\left(x+\dfrac{1}{x}\right)^2-\left(x-\dfrac{1}{x}\right)^2$

$=x^2+\dfrac{1}{x^2}+2-x^2-\dfrac{1}{x^2}+2=4$

Method - 2

Value of $4(a^2-b^2)$ in any option is independent of x. So we can take any value of x. Put $x=1$

$x^3+\dfrac{3}{x}=4(a^3+b^3) \Rightarrow (a^3+b^3)=1$ ...(i)

$3x+\dfrac{1}{x^3}=4(a^3-b^3) \Rightarrow (a^3-b^3)=1$ ...(ii)

from equation (i) & (ii)

$a=1$ & $b=0$

hence, $4(a^2-b^2)=4$

Sol^{n}136. $x+y+z=0$

$x+y=-z,\ y+z=-x$ & $z+x=-y$

$\dfrac{(x+y)^3+(y+z)^3+(z+x)^3-17xyz}{10(x+y)(y+z)(z+x)}$

$=\dfrac{(-z)^3+(-x)^3+(-y)^3-17xyz}{10(-z)(-x)(-y)}$

$=\dfrac{-\left[(x)^3+(y)^3+(z)^3+17xyz\right]}{-10(xyz)}=\dfrac{3xyz+17xyz}{10xyz}=2$

Method -2

here, we can put $x=1$, $y=2$ & $z=-3$ in such a way so that $x+y+z=0$

$\dfrac{(x+y)^3+(y+z)^3+(z+x)^3-17xyz}{10(x+y)(y+z)(z+x)}$

$=\dfrac{(1+2)^3+(2-3)^3+(-3+1)^3-17(1)(2)(-3)}{10(1+2)(2-3)(-3+1)}$

$=\dfrac{120}{60}=2$

Sol^{n}137. $x+y+z=1,\ xy+yz+zx=-1,\ xyz=-1$

$(x+y+z)^2=x^2+y^2+z^2+2(xy+yz+zx)$

$(1)^2=x^2+y^2+z^2+2(-1)$

$x^2+y^2+z^2=3$

then,

$x^3+y^3+z^3=3xyz+(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

$=3(-1)+(1)(3+1)=-3+4=1$

Sol^{n}138. $a^2+4b^2+\frac{1}{4a^2}+\frac{1}{b^2}=5$

$$\left(a^2+\frac{1}{4a^2}-1\right)+\left(4b^2+\frac{1}{b^2}-4\right)=0$$

$$\left(a-\frac{1}{2a}\right)^2+\left(2b-\frac{1}{b}\right)^2=0 \quad \textbf{(Study type - 3)}$$

Hence,

$$a-\frac{1}{2a}=0\Rightarrow a=\frac{1}{2a}\Rightarrow a^2=\frac{1}{2}$$

$$2b-\frac{1}{b}=0\Rightarrow 2b=\frac{1}{b}\Rightarrow b^2=\frac{1}{2}$$

$$\therefore\ a^2+b^2=\frac{1}{2}+\frac{1}{2}=1$$

Sol^{n}139. $\frac{\left(x+\frac{1}{y}\right)^a\left(x-\frac{1}{y}\right)^b}{\left(y+\frac{1}{x}\right)^a\left(y-\frac{1}{x}\right)^b}=\frac{\left(\frac{xy+1}{y}\right)^a\left(\frac{xy-1}{y}\right)^b}{\left(\frac{xy+1}{x}\right)^a\left(\frac{xy-1}{x}\right)^b}$

$$=\frac{(xy+1)^a(xy-1)^b x^a x^b}{y^a y^b(xy+1)^a(xy-1)^b}=\frac{x^{a+b}}{y^{a+b}}=\left(\frac{x}{y}\right)^{a+b}$$

Sol^{n}140. $a^b=b^a\Rightarrow b=a^{b/a}$

then, $\left(\frac{a}{a^{b/a}}\right)^{a/b}=\left(a^{1-b/a}\right)^{a/b}=a^{\left(\frac{a}{b}-1\right)}$

Sol^{n}141. $\frac{a^{1/2}+a^{-1/2}}{1-a}+\frac{1-a^{-1/2}}{1+\sqrt{a}}$

$$=\frac{a^{1/2}+a^{-1/2}}{(1-a^{1/2})(1+a^{1/2})}+\frac{1-a^{-1/2}}{1+a^{1/2}}$$

$$=\frac{(a^{1/2}+a^{-1/2})+(1-a^{-1/2})(1-a^{1/2})}{(1-a^{1/2})(1+a^{1/2})}$$

$$=\frac{a^{1/2}+a^{-1/2}+1-a^{1/2}-a^{-1/2}+1}{(1-a^{1/2})(1+a^{1/2})}$$

$$=\frac{2}{(1-a)}$$

Sol^{n}142. $x=\frac{2pq}{1+q^2}\Rightarrow\frac{1+q^2}{2q}=\frac{p}{x}$

Applying Component & Dividend rule

$$\frac{p+x}{p-x}=\frac{1+q^2+2q}{1+q^2-2q}=\frac{(1+q)^2}{(1-q)^2}$$

$$\sqrt{\frac{p+x}{p-x}}=\sqrt{\frac{(1+q)^2}{(1-q)^2}}\Rightarrow\frac{\sqrt{p+x}}{\sqrt{p-x}}=\frac{1+q}{1-q}$$

Again applying Component & Dividend rule

$$\Rightarrow\frac{\sqrt{p+x}+\sqrt{p-x}}{\sqrt{p+x}-\sqrt{p-x}}=\frac{1+q+1-q}{1+q-1+q}=\frac{2}{2q}=\frac{1}{q}$$

Sol^{n}143. $x=\frac{\sqrt{2}+1}{\sqrt{2}-1}\times\frac{\sqrt{2}+1}{\sqrt{2}+1}=\left(\sqrt{2}+1\right)^2=3+2\sqrt{2}$

$$x-y=4\sqrt{2}$$

$$\Rightarrow\quad y=\left(3+2\sqrt{2}-4\sqrt{2}\right)=3-2\sqrt{2}$$

here,

$x+y=6,\ xy=9-8=1$

$$x^2+y^2=(x+y)^2-2xy$$

$$=(6)^2-2\times1=34$$

Sol^{n}144. $a=\left(\sqrt{3}+\sqrt{2}\right)^{-3}$ & $b=\left(\sqrt{3}-\sqrt{2}\right)^{-3}$

here, $ab=1$ **(See type - 8)**

$$(a+1)^{-1}+(b+1)^{-1}=\frac{1}{a+1}+\frac{1}{b+1}=1$$

Sol^{n}145. $a=x+y,\ b=x-y,\ c=x+2y$

$$\Rightarrow\ (a^2+b^2+c^2-ab-bc-ca)$$

$$=\frac{1}{2}\left\{(a-b)^2+(b-c)^2+(c-a)^2\right\}$$

$$=\frac{1}{2}\left\{(2y)^2+(-3y)^2+(y)^2\right\}$$

$$=7y^2$$

Sol^{n}146. See example 129 Hint

Sol^{n}147. $\sqrt{28-6\sqrt{3}}=\sqrt{3}a+b$

$\Rightarrow \sqrt{\left(3\sqrt{3}-1\right)^2}=\sqrt{3}a+b$

$\Rightarrow 3\sqrt{3}-1=\sqrt{3}a+b$

on comparing coefficient

$a=3, b=-1$

then, $a-b=4$

Sol^{n}148. $\left(\sqrt{a}-\sqrt{b}\right)^2>0$

$a+b-2\sqrt{ab}>0$

$a+b>2\sqrt{ab}$

$\frac{a+b}{2}>\sqrt{ab}$

Sol^{n}149. $a+b=1,\ b+c=2,\ c+a=3$

$\Rightarrow a^2+b^2+c^2+ab+bc+ca$

$=\frac{1}{2}\left(2a^2+2b^2+2c^2+2ab+2bc+2ca\right)$

$=\frac{1}{2}\left[(a+b)^2+(b+c)^2+(c+a)^2\right]$

$=\frac{1}{2}\left[(1)^2+(2)^2+(3)^2\right]=7$

Sol^{n}150. $(a+b+2c+3d)(a-b-2c+3d)$

$=(a-b+2c-3d)(a+b-2c-3d)$

$\Rightarrow \frac{a+b+2c+3d}{a+b-2c-3d}=\frac{a-b+2c-3d}{a-b-2c+3d}$

Applying Component & Dividend rule

$\Rightarrow \frac{2(a+b)}{2(2c+3d)}=\frac{2(a-b)}{2(2c-3d)}$

$\Rightarrow \frac{(a+b)}{(a-b)}=\frac{2c+3d}{2c-3d}$

Again applying Component & Dividend rule

$\frac{2a}{2b}=\frac{4c}{6d} \Rightarrow \frac{a}{b}=\frac{2c}{3d}$

$\Rightarrow 2bc=3ad$

Sol^{n}151. Answer is independent of s

so put $s=0$

$(s-a)^2+(s-b)^2+(s-c)^2+s^2-a^2-b^2-c^2$

$=a^2+b^2+c^2-a^2-b^2-c^2=0$

Sol^{n}152. $a^{1/m}=b^{1/n}=c^{1/p}=k$ (Let)

$a=k^m,\ b=k^n,\ c=k^p$

$abc=k^{m+n+p}$

$k^{m+n+p}=1=k^0$

then, $m+n+p=0$

Sol^{n}153. $2s=9$

$s^2+(s-1)^2+(s-3)^2+(s-5)^2$

$=\left(\frac{9}{2}\right)^2+\left(\frac{9}{2}-1\right)^2+\left(\frac{9}{2}-3\right)^2+\left(\frac{9}{2}-5\right)^2$

$=\frac{81}{4}+\frac{49}{4}+\frac{9}{4}+\frac{1}{4}=35$

Method - 2

here we can put $s=0$ $(\because 1+3+5=9)$

$0+(-1)^2+(-3)^2+(-5)^2=35$

Sol^{n}154. $p+q=r$ & $pqr=30$

$\Rightarrow p+q-r=0$

then,

$p^3+q^3-r^3=3pq(-r)=-3pqr=-90$

Sol^{n}155. $x+y=\sqrt{3} \Rightarrow x^2+y^2+2xy=3$ ...(1)

$x-y=\sqrt{2} \Rightarrow x^2+y^2-2xy=2$ ...(2)

Adding eq^n (1) & (2)

$2(x^2+y^2)=5$ ...(3)

Subtracting eq^n (1) & (2) $4xy=1$... (4)

multiply eq^n (3) & (4)

$8xy(x^2+y^2)=5$

Sol^{n}156. $2a+3b=4$

cubing both sides.

$(2a+3b)^3 = 4^3$

$\Rightarrow 8a^3+27b^3+3\times 2a\times 3b(2a+3b)=64$

$\Rightarrow 8a^3+27b^3+18ab(4) = 64$

$\Rightarrow 8a^3+27b^3+72ab=64$

Sol^{n}157. $x+\frac{2}{x}=3 \Rightarrow 3-x=\frac{2}{x}$

$$\frac{x^2+x+2}{x^2(3-x)} = \frac{x\left(x+\frac{2}{x}\right)+x}{x^2(3-x)}$$

$$= \frac{3x+x}{x^2(3-x)} = \frac{4x}{x^2\left(\frac{2}{x}\right)} = 2$$

Sol^{n}158. $x=ay$ & $y=bx$

multiply both equation

$xy=abxy \Rightarrow ab=1$

then,

$\frac{1}{1+a}+\frac{1}{1+b}=1$ **(Study type - 9)**

Sol^{n}159. $a=\frac{4}{3} \Rightarrow 3a=4 \Rightarrow 3a-4=0$

then, $27a^3-108a^2+144a-317$

$= (3a)^3-3\times 3a\times 4(3a-4)-64-253$

$= (3a-4)^3-253$

$= -253$

Sol^{n}160. $a+b=5 \Rightarrow a^2+b^2+2ab=25$

$a^2+b^2=13$ then, $2ab=12$

$a^2+b^2-2ab=13-2ab=13-12$

$(a-b)^2=1$

$a-b=+1$ $(a > b \text{ or } a-b > 0)$

Sol^{n}161. $x^2+x-6=0$ ...(i)

$x^2+6x+9=0$ ...(ii)

Subtract eq^n (i) from eq^n (ii)

$x^2+6x+9-x^2-x+6=0$

$5x=-15 \Rightarrow x=-3$

Sol^{n}162. $x+\frac{1}{x}=3$

$$\frac{x^3+\frac{1}{x}}{x^2-x+1} = \frac{x\left(x^2+\frac{1}{x^2}\right)}{x\left(x-1+\frac{1}{x}\right)} = \frac{\left(x+\frac{1}{x}\right)^2-2}{\left(x+\frac{1}{x}\right)-1}$$

$$= \frac{(3)^2-2}{3-1} = \frac{7}{2}$$

Sol^{n}163. $ax+by=3,\ bx-ay=4$

add both equation after squaring

$(ax+by)^2+(bx-ay)^2=3^2+4^2$

$\Rightarrow a^2x^2+b^2y^2+2axby+b^2x^2+a^2y^2-2bxay=25$

$\Rightarrow x^2(a^2+b^2)+y^2(a^2+b^2)=25$

$\Rightarrow (a^2+b^2)(x^2+y^2) = 25$

$\Rightarrow (a^2+b^2) = 25$ $(\because x^2+y^2=1)$

Sol^{n}164. $a+b+c=15,\ a^2+b^2+c^2=83$

$\Rightarrow (a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$

$\Rightarrow 225 = 83 + 2(ab+bc+ca)$

$ab+bc+ca=71$

$\Rightarrow a^3+b^3+c^3-3abc$

$=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$= 15(83-71) = 180$

Sol^{n}165. $\frac{1}{a}-\frac{1}{b}=\frac{1}{a-b}$

$\Rightarrow \frac{b-a}{ab}=\frac{1}{a-b} \Rightarrow (b-a)(a-b)=ab$

$\Rightarrow -(a^2+b^2-2ab)=ab \Rightarrow a^2+b^2-ab=0$

$\Rightarrow a^3+b^3=(a+b)(a^2+b^2-ab)=0$

Sol^{n}166. $2x=a+\sqrt{\frac{4b^3-a^3}{3a}}$ …(1)

$2y=a-\sqrt{\frac{4b^3-a^3}{3a}}$ …(2)

Add eq^n (1) & (2)

$2(x+y)=2a \Rightarrow x+y=a$

multiple both eq^n (1) & (2)

$\Rightarrow 4xy=a^2-\frac{4b^3-a^3}{3a}=\frac{4a^3-4b^3}{3a}$

$\Rightarrow xy=\frac{a^3-b^3}{3a}$

$\Rightarrow (x+y)^3=x^3+y^3+3xy(x+y)$

$\Rightarrow a^3=x^3+y^3+3\left[\frac{(a^3-b^3)}{3a}\right](a)$

$\Rightarrow a^3=x^3+y^3+a^3-b^3$

$\Rightarrow x^3+y^3=b^3$

Sol^{n}167. $a-b=2,\ ab=15$

then,

$(a+b)=\sqrt{(a-b)^2+4ab}=\sqrt{4+4\times15}=8$

$\Rightarrow (a^2-b^2)(a^3-b^3)$

$=(a+b)(a-b)(a-b)(a^2+b^2+ab)$

$=(a+b)(a-b)^2\left\{(a+b)^2-ab\right\}$

$=8(2)^2\left\{(8)^2-15\right\}=1568$

Sol^{n}168. $4x-5z=16$ & $xz=12$

cubing both sides

$(4x-5z)^3=(16)^3$

$64x^3-125z^3-3(4x)(5z)(4x-5z)=4096$

$64x^3-125z^3=60(12)(16)+4096=15616$

Sol^{n}169. $x+y+z=15$ and $xy+yz+zx=75$

$(x+y+z)^2=x^2+y^2+z^2+2(xy+yz+zx)$

$225=x^2+y^2+z^2+2(75)$

$x^2+y^2+z^2=225-150=75$

here, $x^2+y^2+z^2=xy+yz+zx$

it means $x=y=z$

then, $\frac{x+4y+z}{3z}=\frac{x+4x+x}{3x}=2$

Sol^{n}170. $a^2+b^2=2$ & $c^2+d^2=1$

$\Rightarrow (ad-bc)^2+(ac+bd)^2$

$=a^2d^2+b^2c^2-2adbc+a^2c^2+b^2d^2+2acbd$

$=a^2(c^2+d^2)+b^2(c^2+d^2)$

$=(c^2+d^2)(a^2+b^2)=2\times1=2$

Sol^{n}171. $3(a^2+b^2+c^2)=(a+b+c)^2$

$3(a^2+b^2+c^2)=a^2+b^2+c^2+2(ab+bc+ca)$

$2(a^2+b^2+c^2-ab-bc-ca)=0$

it means $a=b=c$ **(See type-4)**

Sol^{n}172. $x+\frac{1}{x}=-\sqrt{3} \Rightarrow x^2+\frac{1}{x^2}=3-2=1$

See type - 2(F) when $x+\frac{1}{x}=-\sqrt{3}$ then $x^6=-1$

hence,

$x^{26}+\frac{1}{x^{26}}=x^2.x^{24}+\frac{1}{x^2.x^{24}}=x^2+\frac{1}{x^2}=1$

$(\because x^{24}=(x^6)^4=(-1)^4=1)$

Sol^n 173. $\sqrt{2\sqrt{4\sqrt{2\sqrt{4\sqrt{2}}}}} = 32^a = 2^{5a}$

Squaring both sides

$2\sqrt{4\sqrt{2\sqrt{4\sqrt{2}}}} = 2^{10a}$

Again squaring both sides

$2^2.4\sqrt{2\sqrt{4\sqrt{2}}} = 2^{20a} \Rightarrow 2^4\sqrt{2\sqrt{4\sqrt{2}}} = 2^{20a}$

Again squaring both sides

$2^8.2\sqrt{4\sqrt{2}} = 2^{40a} \Rightarrow 2^9\sqrt{4\sqrt{2}} = 2^{40a}$

Again squaring both sides

$2^{18}.4\sqrt{2} = 2^{80a} \Rightarrow 2^{20}\sqrt{2} = 2^{80a}$

Again squaring both sides

$2^{40}.2 = 2^{160a} \Rightarrow 2^{41} = 2^{160a}$

on comparing power

$160a = 41$, then $a = \dfrac{41}{160}$

Sol^n 174. $a^2 = 2 \Rightarrow a^2 - 1 = 1 \Rightarrow (a-1)(a+1) = 1$

then, $a - 1 = \dfrac{1}{a+1}$ …(i)

$a^2 = 2 \Rightarrow a^2 + 1 = 3 \Rightarrow a^2 - 2a + 1 = 3 - 2a$

$\Rightarrow (a-1)^2 = 3 - 2a \Rightarrow (a-1)(a-1) = 3 - 2a$ ...(ii)

put the value of $(a-1)$ from eq^n (i) in equation (ii)

$\Rightarrow (a-1).\dfrac{1}{(a+1)} = 3 - 2a$

$\Rightarrow (a+1) = \dfrac{a-1}{3-2a}$

Sol^n 175. $(a+b+c)p = (b+c-a)q = (c+a-b)r$
$= (a+b-c)s = k$ (Let)

$\Rightarrow \dfrac{k}{p} = a+b+c$ .....(i)

$\Rightarrow \dfrac{k}{q} = b+c-a,$ .....(ii)

$\Rightarrow \dfrac{k}{r} = c+a-b$ .....(iii)

$\Rightarrow \dfrac{k}{s} = a+b-c$ .....(iv)

Adding eq^n (ii), (iii), (iv) and subtract eq^n (i).

$\Rightarrow k\left(\dfrac{1}{q} + \dfrac{1}{r} + \dfrac{1}{s} - \dfrac{1}{p}\right)$

$= k(b+c-a+c+a-b+a+b-c-(a+b+c))$

then, $\dfrac{1}{q} + \dfrac{1}{r} + \dfrac{1}{s} - \dfrac{1}{p} = 0$

Sol^n 176. $x(x-3) = -1$

$x - 3 = \dfrac{-1}{x} \Rightarrow x + \dfrac{1}{x} = 3$ (cubing both sides)

$\Rightarrow x^3 + \dfrac{1}{x^3} + 3x.\dfrac{1}{x}\left(x + \dfrac{1}{x}\right) = 27$

$\Rightarrow x^3 + \dfrac{1}{x^3} = 27 - 3\times 3 = 18$

$\Rightarrow x^3 - 18 = -\dfrac{1}{x^3}$

$\Rightarrow x^3(x^3 - 18) = -1$

Sol^n 177. $x = \dfrac{p+q+r}{3} \Rightarrow 3x = p+q+r$

$(x-p)+(x-q)+(x-r) = 0$

then, $(x-p)^3 + (x-q)^3 + (x-r)^3 - 3(x-p)(x-q)(x-r) = 0$

Sol^n 178. $a^2 + b^2 = 2(a - 2b) - 5$

$(a^2 - 2a + 1) + (b^2 + 4b + 4) = 0$

$(a-1)^2 + (b+2)^2 = 0$

$a - 1 = 0 \Rightarrow a = 1$

$b + 2 = 0 \Rightarrow b = -2$

then, $a^3 + b^3 + 3ab = 1 - 8 - 3\times 2 = -13$

Sol^n 179. $x + \dfrac{1}{x} = -1 \Rightarrow x^3 = 1$ **(See type-2(D))**

$y + \dfrac{1}{y} = 2 \Rightarrow y = 1$ **(See type-2(A))**

$\Rightarrow (x)^{3y} + (y)^{3x} = \left(x^3\right)^y + (y)^{3x}$

$= (1)^y + (1)^{3x} = 1 + 1 = 2$

Sol^{n}180. If $a^3+b^3+c^3=3abc \Rightarrow a=b=c$ when a,b,c are positive numbers **(See type - 5)**

$$\frac{2a+7b+9c}{a+2b+3c}=\frac{2a+7a+9a}{a+2a+3a}=\frac{18a}{6a}=3$$

Sol^{n}181. $\frac{p}{a}+\frac{q}{b}+\frac{r}{c}=1$ (squaring both sides)

$$\frac{p^2}{a^2}+\frac{q^2}{b^2}+\frac{r^2}{c^2}+2\left(\frac{pq}{ab}+\frac{qr}{bc}+\frac{pr}{ac}\right)=1$$

$$\frac{p^2}{a^2}+\frac{q^2}{b^2}+\frac{r^2}{c^2}+2\left(\frac{cpq+aqr+bpr}{abc}\right)=1$$

$(\because abc=1)$

$$\frac{p^2}{a^2}+\frac{q^2}{b^2}+\frac{r^2}{c^2}=1-2(cpq+aqr+bpr) \quad \ldots(i)$$

$$\frac{a}{p}+\frac{b}{q}+\frac{c}{r}=1 \Rightarrow \frac{aqr+bpr+cpq}{pqr}=1$$

$aqr+bpr+cpq=pqr=-1 \quad (\because pqr=-1)$

put this value in equation (i)

$$\frac{p^2}{a^2}+\frac{q^2}{b^2}+\frac{r^2}{c^2}=1-2(-1)=3$$

Sol^n 182.

$$x=\frac{4\sqrt{15}}{\sqrt{5}+\sqrt{3}}=\frac{2\sqrt{4\times15}}{\sqrt{5}+\sqrt{3}}=\frac{2\sqrt{20}.\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$\frac{x}{\sqrt{20}}=\frac{2\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

Applying Component & Dividend rule

$$\frac{x+\sqrt{20}}{x-\sqrt{20}}=\frac{2\sqrt{3}+\sqrt{5}+\sqrt{3}}{2\sqrt{3}-\sqrt{5}-\sqrt{3}}=\frac{3\sqrt{3}+\sqrt{5}}{\sqrt{3}-\sqrt{5}} \quad \ldots(1)$$

$$x=\frac{4\sqrt{15}}{\sqrt{5}+\sqrt{3}}=\frac{2\sqrt{4\times15}}{\sqrt{5}+\sqrt{3}}=\frac{2\sqrt{12}.\sqrt{5}}{\sqrt{5}+\sqrt{3}}$$

$$\frac{x}{\sqrt{12}}=\frac{2\sqrt{5}}{\sqrt{5}+\sqrt{3}}$$

Applying Component & Dividend rule

$$\frac{x+\sqrt{12}}{x-\sqrt{12}}=\frac{2\sqrt{5}+\sqrt{5}+\sqrt{3}}{2\sqrt{5}-\sqrt{5}-\sqrt{3}}=\frac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \quad \ldots(2)$$

Adding eq^n (1) & (2)

$$\frac{x+\sqrt{20}}{x-\sqrt{20}}+\frac{x+\sqrt{12}}{x-\sqrt{12}}=\frac{3\sqrt{3}+\sqrt{5}}{\sqrt{3}-\sqrt{5}}+\frac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$$

$$=\frac{3\sqrt{3}+\sqrt{5}}{\sqrt{3}-\sqrt{5}}-\frac{3\sqrt{5}+\sqrt{3}}{\sqrt{3}-\sqrt{5}}$$

$$=\frac{3\sqrt{3}+\sqrt{5}-3\sqrt{5}-\sqrt{3}}{\sqrt{3}-\sqrt{5}}=\frac{2\sqrt{3}-2\sqrt{5}}{\sqrt{3}-\sqrt{5}}=2$$

Sol^n Q183 to Q185. Same as Q.182

Sol^n 186. $x=\frac{4ab}{a+b}=\frac{2a\times2b}{a+b}$

$$\therefore \frac{x}{2a}=\frac{2b}{a+b}$$

Applying Component & Dividend rule

$$\frac{x+2a}{x-2a}=\frac{2b+a+b}{2b-a-b}=\frac{3b+a}{b-a} \quad \ldots(1)$$

$$x=\frac{4ab}{a+b}=\frac{2a\times2b}{a+b}$$

$$\therefore \frac{x}{2b}=\frac{2a}{a+b}$$

Applying Component & Dividend rule

$$\frac{x+2b}{x-2b}=\frac{2a+a+b}{2a-a-b}=\frac{3a+b}{a-b} \quad \ldots(2)$$

Adding eq^n (1) & (2)

$$\frac{x+2a}{x-2a}+\frac{x+2b}{x-2b}=\frac{3b+a}{b-a}+\frac{3a+b}{a-b}$$

$$=\frac{3b+a}{b-a}-\frac{3a+b}{b-a}=\frac{2b-2a}{b-a}=2$$

Sol^n Q187 to Q189. Same as Q.186

Sol^n 190. $(x+y)^2-z^2=4$

$\Rightarrow (x+y+z)(x+y-z)=4 \quad \ldots(i)$

$$(y+z)^2 - x^2 = 9$$
$$\Rightarrow (x+y+z)(y+z-x) = 9 \quad \ldots\text{(ii)}$$
$$(z+x)^2 - y^2 = 36$$
$$\Rightarrow (x+y+z)(z+x-y) = 36 \quad \ldots\text{(iii)}$$

Adding all three equations–

$$(x+y+z)(x+y-z+y+z-x+z+x-y) = 49$$
$$(x+y+z)^2 = 49 \Rightarrow (x+y+z) = \pm 7$$

Sol^n 191. $x = \sqrt{\dfrac{\sqrt{5}+1}{\sqrt{5}-1}} = \sqrt{\dfrac{\sqrt{5}+1}{\sqrt{5}-1} \times \dfrac{\sqrt{5}+1}{\sqrt{5}+1}} = \dfrac{\sqrt{5}+1}{2}$

$2x = \sqrt{5}+1 \Rightarrow 2x - 1 = \sqrt{5}$

squaring both sides

$$4x^2 + 1 - 4x = 5 \Rightarrow 4x^2 - 4x - 4 = 0$$
$$\Rightarrow x^2 - x - 1 = 0$$
$$\Rightarrow 5x^2 - 5x - 1 = 4$$

Sol^n 192. $x = \dfrac{\sqrt{3}}{2}$

$$\Rightarrow \sqrt{1+x} = \sqrt{1+\frac{\sqrt{3}}{2}} = \sqrt{\frac{2+\sqrt{3}}{2}} = \sqrt{\frac{4+2\sqrt{3}}{4}}$$

$$= \sqrt{\frac{\left(\sqrt{3}+1\right)^2}{4}} = \frac{\sqrt{3}+1}{2}$$

Similarly, $\sqrt{1-x} = \dfrac{\sqrt{3}-1}{2}$

$$\Rightarrow \quad \frac{\sqrt{1+x}}{1+\sqrt{1+x}} + \frac{\sqrt{1-x}}{1-\sqrt{1-x}} = \frac{\frac{\sqrt{3}+1}{2}}{1+\frac{\sqrt{3}+1}{2}} + \frac{\frac{\sqrt{3}-1}{2}}{1-\frac{\sqrt{3}-1}{2}}$$

$$= \frac{\sqrt{3}+1}{3+\sqrt{3}} + \frac{\sqrt{3}-1}{3-\sqrt{3}} = \frac{3\sqrt{3}+3-3-\sqrt{3}+3\sqrt{3}-3+3-\sqrt{3}}{(3)^2-\left(\sqrt{3}\right)^2}$$

$$= \frac{4\sqrt{3}}{6} = \frac{2}{\sqrt{3}}$$

Sol^n 193. Hint: Same as Q.192

Sol^n 194. Hint: See example 110 (Type -7)

Sol^n 195. $a+b+c = 0$

Multiply both side by *'a'*

$$a^2 + ab + ac = 0$$

then, $a^2 = -ab - ac$

$$\Rightarrow \frac{a^2}{2a^2+bc} = \frac{a^2}{a^2+a^2+bc} = \frac{a^2}{a^2-ab-ac+bc}$$

$$= \frac{a^2}{a(a-b)-c(a-b)} = \frac{a^2}{(a-b)(a-c)}$$

Similarly,

$$\Rightarrow \frac{a^2}{2a^2+bc} + \frac{b^2}{2b^2+ac} + \frac{c^2}{2c^2+ab}$$

$$= \frac{a^2}{(a-b)(a-c)} + \frac{b^2}{(b-a)(b-c)} + \frac{c^2}{(c-a)(c-b)}$$

$$= \frac{-a^2}{(a-b)(c-a)} - \frac{b^2}{(a-b)(b-c)} - \frac{c^2}{(c-a)(b-c)}$$

$$= \frac{-a^2(b-c)-b^2(c-a)-c^2(a-b)}{(a-b)(b-c)(c-a)}$$

$$= \frac{-a^2(b-c)-b^2c+b^2a-c^2a+c^2b)}{(a-b)(b-c)(c-a)}$$

$$= \frac{-a^2(b-c)-b^2c+c^2b+b^2a-c^2a}{(a-b)(b-c)(c-a)}$$

$$= \frac{-a^2(b-c)-bc(b-c)+a(b^2-c^2)}{(a-b)(b-c)(c-a)}$$

$$= \frac{(b-c)(-a^2-bc+a(b+c))}{(a-b)(b-c)(c-a)}$$

$$= \frac{-a^2-bc+ab+ac}{(a-b)(c-a)}$$

$$= \frac{-a(a-b)+c(a-b)}{(a-b)(c-a)}$$

$$= \frac{(a-b)(c-a)}{(a-b)(c-a)}$$

$$= 1$$

Method:-2

Put $a = 0, b = -1, c = 1$ such that $a + b + c = 0$

$$\Rightarrow \frac{a^2}{2a^2+bc} + \frac{b^2}{2b^2+ac} + \frac{c^2}{2c^2+ab}$$

$$= \frac{0}{0-1} + \frac{1}{2+0} + \frac{1}{2+0}$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

Sol^n 196. $x + y + z = 13,\ x^2 + y^2 + z^2 = 91$

Squaring both side.

$x^2 + y^2 + z^2 + 2\,(xy + yz + zx) = 13^2 = 169$

$91 + 2\,(r + r^2 + r^3) = 169$

$2\,(r + r^2 + r^3) = 169 - 91 = 78$

$(r + r^2 + r^3) = 39 = 3 + 9 + 27$

hence, $r = 3$

$\because\ xy = r \ldots$(i) and $xz = r^2\ \ldots$(ii)

divide eq^n (ii) by eq^n (i)

$$\frac{xz}{xy} = \frac{r^2}{r} = r$$

$$\Rightarrow \frac{z}{y} = r = 3$$

Sol^{n}197. $xy + yz + zx = xyz$

$$= \frac{x+y}{xyz-xy} + \frac{y+z}{zyz-yz} + \frac{z+x}{xyz-zx}$$

$$= \frac{x+y}{yz+zx} + \frac{y+z}{xy+zx} + \frac{z+x}{xy+yz}$$

$$= \frac{x+y}{z(y+x)} + \frac{y+z}{x(y+z)} + \frac{z+x}{y(z+x)}$$

$$= \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{yz+xz+xy}{xyz}$$

$$= \frac{xyz}{xyz} = 1$$

Sol^{n}198. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$

$\Rightarrow 4 = a^2 + b^2 + c^2 + 2(-1)$

$\Rightarrow a^2 + b^2 + c^2 = 4 + 2 = 6$

Now,

$\Rightarrow (a+b)^2 + (b+c)^2 + (c+a)^2$

$= 2\left(a^2 + b^2 + c^2 + ab + bc + ca\right)$

$= 2(6-1) = 10$

Sol^{n}199. $\frac{b}{y} = 1 - \frac{z}{c} = \frac{c-z}{c}$

$$\frac{y}{b} = \frac{c}{c-z} \qquad \ldots\text{(i)}$$

$$\frac{c}{z} + \frac{x}{a} = 1 \Rightarrow \frac{x}{a} = 1 - \frac{c}{z}$$

$$\Rightarrow \frac{x}{a} = \frac{z-c}{z} \Rightarrow \frac{a}{x} = \frac{z}{z-c} \qquad \ldots\text{(ii)}$$

Adding eq^n (i) and (ii)

$$\frac{a}{x} + \frac{y}{b} = \frac{z}{z-c} + \frac{c}{c-z}$$

$$= \frac{z}{z-c} - \frac{c}{z-c} = \frac{z-c}{z-c} = 1$$

Sol^{n}200. $x + \frac{2}{y} = 1 \Rightarrow x = 1 - \frac{2}{y} = \frac{y-2}{y}$

$$\frac{1}{x} = \frac{y}{y-2}$$

$$\frac{1}{2x} = \frac{y}{2(y-2)} \qquad \ldots\text{(i)}$$

$$y + \frac{1}{z} = 2 \Rightarrow \frac{1}{z} = 2 - y$$

$$z = \frac{1}{2-y} \qquad \ldots\text{(ii)}$$

From eq^n (i) & (ii)

$$z + \frac{1}{2x} = \frac{1}{2-y} + \frac{y}{2(y-2)}$$

$$= \frac{1}{2-y} - \frac{y}{2(2-y)}$$

$$= \frac{2-y}{2(2-y)} = \frac{1}{2}$$

Method 2 : We can put, $x = -1, y = 1, z = 1$ because these values are satisfying both given equation.

So, $z + \frac{1}{2x} = 1 - \frac{1}{2} = \frac{1}{2}$

Sol^{n}201. $a^x = m \Rightarrow a = m^{1/x}$ …(i)

$a^y = n \Rightarrow a = n^{1/y}$ …(ii)

Multiplying eq^n (i) & (ii)

$a^2 = m^{1/x}.n^{1/y}$ …(iii)

Also, $a^2 = \left(m^y.n^x\right)^z$ …(iv)

From eq^n (iii) & (iv)

$m^{1/x}.n^{1/y} = m^{yz}.n^{xz}$

$\therefore \ \frac{1}{x} = yz \ \Rightarrow \ xyz = 1$

Sol^{n}202. $\left(\frac{1}{2}\right)^k = \sqrt{3} \Rightarrow 2^{-k} = 3^{1/2}$ …(i)

$\left(\frac{1}{3}\right)^m = \sqrt{2} \Rightarrow 3^{-m} = 2^{1/2}$ …(ii)

Multiply both eq^n (i) & (ii)

$2^{-k}.3^{-m} = 3^{1/2}.2^{1/2}$

on comapring power

$m = k = -\frac{1}{2}$, then $\frac{mk}{2} = \frac{1}{8}$

Sol^{n}203. $\frac{a-b}{\sqrt{a}-\sqrt{b}} + \frac{b-a}{\sqrt{a}+\sqrt{b}} = \frac{a-b}{\sqrt{a}-\sqrt{b}} - \frac{a-b}{\sqrt{a}+\sqrt{b}}$

$= (a-b)\left(\frac{\sqrt{a}+\sqrt{b}-\sqrt{a}+\sqrt{b}}{\left(\sqrt{a}-\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\right)}\right)$

$= (a-b)\left(\frac{2\sqrt{b}}{a-b}\right)$

$= 2\sqrt{b} = 2\sqrt{36}$

$= 12$

Sol^{n}204. $\left[\sqrt{3}-(x-5)\right]\left[\sqrt{3}+(x-5)\right]$

$= \left(\sqrt{3}\right)^2 - (x-5)^2$

$= 3 - (x-5)^2$

$\therefore$ Maximum Value $= 3$

Sol^{n}205. See type-9

If $xy = \left(\frac{\sqrt{93}+\sqrt{19}}{\sqrt{97}-\sqrt{23}}\right)\left(\frac{\sqrt{93}-\sqrt{19}}{\sqrt{97}+\sqrt{23}}\right)$

$xy = \frac{93-19}{97-23} = \frac{74}{74} = 1$

then, $\frac{1}{x+1} + \frac{1}{y+1} = 1$

Sol^{n}206. $x = 1+\sqrt{2}+\sqrt{3} \Rightarrow (x-1) = \sqrt{2}+\sqrt{3}$

Squaring both sides

$\Rightarrow (x-1)^2 = \left(\sqrt{2}+\sqrt{3}\right)^2$

$\Rightarrow x^2 - 2x + 1 = 2 + 3 + 2\sqrt{2}.\sqrt{3}$

$\Rightarrow x^2 - 2x - 4 = 2\sqrt{6}$ …(i)

Again squaring both sides

$\Rightarrow \left(x^2-2x-4\right)^2 = \left(2\sqrt{6}\right)^2$

$\Rightarrow x^4 + 4x^2 + 16 - 4x^3 + 16x - 8x^2 = 24$

$\Rightarrow x^4 - 4x^3 - 4x^2 + 16x - 8 = 0$

Multiply by 2 both sides

$\Rightarrow 2x^4 - 8x^3 - 8x^2 + 32x - 16 = 0$

$\Rightarrow 2x^4 - 8x^3 - 5x^2 - 3x^2 + 26x + 6x - 28 + 12 = 0$

$\Rightarrow 2x^4 - 8x^3 - 5x^2 + 26x - 28 = 3x^2 - 6x - 12$

$= 3\left(x^2-2x-4\right) = 3\left(2\sqrt{6}\right)$

$= 6\sqrt{6}$

Sol^{n}207. $\frac{5}{3^{\frac{2}{3}} - 6^{\frac{1}{3}} + 2^{\frac{2}{3}}} = \frac{5}{3^{\frac{2}{3}} - 6^{\frac{1}{3}} + 2^{\frac{2}{3}}} \times \frac{3^{\frac{1}{3}} + 2^{\frac{1}{3}}}{3^{\frac{1}{3}} + 2^{\frac{1}{3}}}$

$= \frac{5\left(3^{\frac{1}{3}} + 2^{\frac{1}{3}}\right)}{\left(3^{\frac{1}{3}} + 2^{\frac{1}{3}}\right)\left((3^{\frac{1}{3}})^2 - (3^{\frac{1}{3}}.2^{\frac{1}{3}}) + (2^{\frac{1}{3}})^2\right)}$

$$\left(\because a^3+b^3=(a+b)(a^2-ab+b^2)\right)$$

$$=\frac{5\left(3^{\frac{1}{3}}+2^{\frac{1}{3}}\right)}{\left(3^{\frac{1}{3}}\right)^3+\left(2^{\frac{1}{3}}\right)^3}$$

$$=\frac{5\left(3^{\frac{1}{3}}+2^{\frac{1}{3}}\right)}{3+2}$$

$$=3^{\frac{1}{3}}+2^{\frac{1}{3}}$$

$$=\sqrt[3]{3}+\sqrt[3]{2}$$

$$\Rightarrow \quad a\sqrt[3]{3}+b\sqrt[3]{2}+c\sqrt[3]{6}=\sqrt[3]{3}+\sqrt[3]{2}$$

Now comparing coefficient

$a=1, b=1, c=0$

$\Rightarrow \quad a+b+c=1+1+0=2$

Sol^{n}208.

$$\frac{\sqrt{a}+\sqrt{b}}{a-b}-\frac{\sqrt{b}-\sqrt{a}}{a+b}=4,$$

$$\frac{(a+b)\left(\sqrt{a}+\sqrt{b}\right)-\left(\sqrt{b}-\sqrt{a}\right)(a-b)}{(a-b)(a+b)}=4$$

$$\frac{a\sqrt{a}+a\sqrt{b}+b\sqrt{a}+b\sqrt{b}-a\sqrt{b}+a\sqrt{a}+b\sqrt{b}-b\sqrt{a}}{(a-b)(a+b)}=4$$

$$\Rightarrow \frac{2\left(a\sqrt{a}+b\sqrt{b}\right)}{a^2-b^2}=4$$

$$\Rightarrow \frac{a^{3/2}+b^{3/2}}{a^2-b^2}=2=\frac{a^x+b^x}{a^2-b^2}$$

On comparing power $x=3/2$

Sol^{n}209. $\frac{76}{4+\sqrt{7}+\sqrt{11}}$

$$=\frac{76\left(4+\sqrt{7}-\sqrt{11}\right)}{\left(4+\sqrt{7})+\sqrt{11}\right)\left(4+\sqrt{7})-\sqrt{11}\right)}$$

$$=\frac{76\left(4+\sqrt{7}-\sqrt{11}\right)}{\left(4+\sqrt{7}\right)^2-\left(\sqrt{11}\right)^2}=\frac{76\left(4+\sqrt{7}-\sqrt{11}\right)}{8\sqrt{7}+12}$$

$$=\frac{19\left(4+\sqrt{7}-\sqrt{11}\right)}{2\sqrt{7}+3}$$

Now, again rationalization

$$=\frac{19\left(4+\sqrt{7}-\sqrt{11}\right)}{\left(2\sqrt{7}\right)^2-(3)^2}\times\left(2\sqrt{7}-3\right)$$

$$=\frac{19\left(2+5\sqrt{7}+3\sqrt{11}-2\sqrt{77}\right)}{28-9}$$

$$=2+5\sqrt{7}+3\sqrt{11}-2\sqrt{77}$$

$$=p+q\sqrt{7}+r\sqrt{11}+s\sqrt{77}$$

$\therefore\ p=2, q=5, r=3, s=-2$

$\therefore\ p+q+r+s=8$

Sol^{n}210. $x=4+\sqrt{11}+\sqrt{7}$

$x-4=\sqrt{11}+\sqrt{7}$

Squaring both sides

$(x-4)^2=\left(\sqrt{11}+\sqrt{7}\right)^2$

$x^2+16-8x=11+7+2\sqrt{77}$

$x^2-8x-2=2\sqrt{77}$

Again squaring

$x^4-16x^3+60x^2+32x+4=308$

$\therefore x^4-16x^3+60x^2+32x=304$

Sol^{n}211. $5^a+2^{b+1}=189 \Rightarrow 5^a+2.2^b=189$

$$\Rightarrow 5^{a+1}+2^{b-2}=633 \Rightarrow 5.5^a+\frac{2^b}{2^2}=633$$

Let $5^a=x, \qquad 2^b=y$

Then, $x+2y=189$ …(i)

$5x+\frac{y}{4}=633$ …(ii)

After solution both equation,

$x=125, \quad y=32$

$5^a=125=5^3 \Rightarrow a=3$

$2^b=32=2^5 \Rightarrow b=5$

hence, $a+b=3+5=8$

Method : 2

a and b integer. 2^{b+1} must be a power of 2.

$5^a+2^{b+1}=189$

If $a=0$ or 1 or 2, then 2^{b+1} is not a power of 2.

but if $a=3$, 2^{b+1} is a power of 2.

$5^3+2^{b+1}=189 \Rightarrow 2^{b+1}=64$

On comparing power

$b+1=6 \Rightarrow b=5$

$a=3$ & $b=5$ also satisfy the second equation.

hence, $a=3, b=5 \Rightarrow a+b=8$

Sol^{n}212. From option

$x=(a+b+c)^2$

$$\frac{(a+b+c)^2-a^2}{b+c}+\frac{(a+b+c)^2-b^2}{c+a}+\frac{(a+b+c)^2-c^2}{a+b}$$

$=4(a+b+c)$

$$\frac{(2a+b+c)(b+c)}{(b+c)}+\frac{(a+2b+c)(a+c)}{(c+a)}+\frac{(a+b+2c)(a+b)}{(a+b)}$$

$=4(a+b+c)$

$(2a+b+c)+(a+2b+c)+(a+b+2c)=4(a+b+c)$

$4(a+b+c)=4(a+b+c)$

L.H.S. = R.H.S.

hence, $x=(a+b+c)^2$

Method-2

Let $a=b=c=1$

$$\frac{x-1}{2}+\frac{x-1}{2}+\frac{x-1}{2}=4\times 3$$

$3x-3=4\times 3\times 2$

$x=9$

put the value of a, b & c in all options

(i) $(1+1+1)^2=9$ (ii) $(1+1+1)=3$

(iii) $(1+1+1)=3$ (iv) $1+1+1-1-1-1=0$

hence option (a) is correct.

Sol^{n}213. Here anwser is independent of a, b, c then we can put any value of a, b & c, so we can put $a=b=c=0$

$$\frac{1}{1+p^{a-b}+p^{a-c}}+\frac{1}{1+p^{b-a}+p^{b-c}}+\frac{1}{1+p^{c-a}+p^{c-b}}$$

$$=\frac{1}{1+p^0+p^0}+\frac{1}{1+p^0+p^0}+\frac{1}{1+p^0+p^0}$$

$$=\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=1$$

Sol^{n}214. $P+\sqrt{P^2+\sqrt{P^4+\sqrt{P^8+\sqrt{P^{16}+.....\infty}}}}$

$$=P+\sqrt{P^2+\sqrt{P^4+\sqrt{P^8+P^8\sqrt{1+.....\infty}}}}$$

$$=P+\sqrt{P^2+\sqrt{P^4+\sqrt{P^8\left(1+\sqrt{1+.....\infty}\right)}}}$$

$$=P+\sqrt{P^2+\sqrt{P^4+P^4\sqrt{1+\sqrt{1........\infty}}}}$$

$$=P+\sqrt{P^2+\sqrt{P^4\left(1+\sqrt{1+\sqrt{1........\infty}}\right)}}$$

$$=P+\sqrt{P^2+P^2\sqrt{1+\sqrt{1+\sqrt{1........\infty}}}}$$

$$=P+\sqrt{P^2\left(1+\sqrt{1+\sqrt{1+\sqrt{1........\infty}}}\right)}$$

$$=P+P\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1........\infty}}}}}$$

$$=P\left(1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1........\infty}}}}}\right)$$

$$\Rightarrow 1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1........\infty}}}}}$$

$=x$(let)

$1+\sqrt{x}=x \quad \Rightarrow \sqrt{x}=x-1$

Squaring both sides

$\Rightarrow x = x^2 - 2x + 1$

$x^2 - 3x + 1 = 0$

Then, $x = \dfrac{+3 \pm \sqrt{9-4}}{2}$

$= \dfrac{3 \pm \sqrt{5}}{2}$

But $1+\sqrt{1+\sqrt{1.....\infty}}$ will be positive and equal to $\dfrac{3+\sqrt{5}}{2}$ so ,

$\Rightarrow \; P\left(1+\sqrt{1+\sqrt{1+....\infty}}\right) = P\left(\dfrac{3+\sqrt{5}}{2}\right)$

Sol^{n}215. $x = \sqrt[4]{4\sqrt[4]{4\sqrt[4]{4\sqrt[4]{4....\infty}}}}$

$\Rightarrow x = \sqrt[4]{4x} \;\Rightarrow x^4 = 4x \Rightarrow x^3 = 4 = 2^2$

then $x = 2^{2/3} = 32^a$

$\Rightarrow 2^{2/3} = \left(2^5\right)^a = 2^{5a}$

$\Rightarrow 5a = \dfrac{2}{3} \;\Rightarrow a = \dfrac{2}{15}$

Sol^{n}216. $y = \dfrac{x^2 - 10x + 64}{x^2 + 10x + 64}$

$x^2 y + 10xy + 64y = x^2 - 10x + 64$

$x^2(y-1) + x(10y+10) + 64(y-1) = 0$

hence, x is real number so $b^2 - 4ac \ge 0$

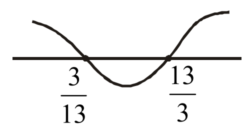
$(10y+10)^2 - 4\times(y-1)\times 64(y-1) \ge 0$

$[10(y+1)]^2 - [16(y-1)]^2 \ge 0$

$[10(y+1)+16(y-1)][10(y+1)-16(y-1)] \ge 0$

$(26y-6)(-6y+26) \ge 0$

$(26y-6)(6y-26) \le 0$



critical points are $y = \dfrac{3}{13}$ or $\dfrac{13}{3}$

Minimum value $= \dfrac{3}{13}$ & Maximum value $= \dfrac{13}{3}$

Sol^{n}217. $p = \sqrt{5} - 2 \Rightarrow p + 2 = \sqrt{5}$

Squaring both sides

$(p+2)^2 = \left(\sqrt{5}\right)^2$

$p^2 + 4 + 4p = 5$

$\Rightarrow p^2 + 4p = 1$

Again squaring both sides

$\Rightarrow \; p^4 + 16p^2 + 8p^3 + 4 = 1 + 4$

$\Rightarrow \; p^4 + 16p^2 + 8p^3 + 4 = 5$

Sol^{n}218. $x = 2y + 6 \quad \Rightarrow x - 2y = 6 \quad$ ...(i)

Cubing both sides

$x^3 - 8y^3 - 3x \times 2y(x-2y) = 6^3$

$\Rightarrow \; x^3 - 8y^3 - 6xy \times 6 = 216$

$\Rightarrow \; x^3 - 8y^3 - 36xy - 216 = 0$

Sol^{n}219. $p = 2 - a \Rightarrow p + a = 2$

cubing both sides

$a^3 + p^3 + 3ap(a+p) = 8$

$a^3 + p^3 + 3ap(2) = 8$

then, $a^3 + 6ap + p^3 - 6 = 2$

Sol^{n}220. $x^4 + y^4 = 17 \quad \& \quad x + y = 1$

Put $x = 2 \quad \& \quad y = -1$

$\therefore \quad x^2y^2 - 2xy = (2)^2(-1)^2 - 2(2)(-1)$

$= 4 + 4 = 8$

Sol^{n}221. $\dfrac{a}{b+c}=\dfrac{b}{c+a}=\dfrac{c}{a+b}=k$

$a = k(b+c)$ ...(i)

$b=k(c+a)$ ...(ii)

$c=k(a+b)$ ...(ii)

Adding (i), (ii) & (iii)

$a+b+c=k(2a+2b+2c)$

$\Rightarrow (a+b+c)=2k(a+b+c)$

$\Rightarrow k=\dfrac{1}{2}$

Subtracting (i) & (ii)

$a-b=k(b-a)$

$\Rightarrow k=-1$

Sol^{n}222. Method 1 :

$\dfrac{x}{a}=\dfrac{y}{b}=\dfrac{z}{c}=k$ (let)

$\Rightarrow$ $x=ak$, $y=bk$ & $z=ck$

$$\Rightarrow \frac{ax-by}{(a+b)(x-y)}+\frac{by-cz}{(b+c)(y-z)}+\frac{cz-ax}{(c+a)(z-x)}$$

$$=\frac{a^2k-b^2k}{(a+b)(ak-bk)}+\frac{b^2k-c^2k}{(b+c)(bk-ck)}+\frac{c^2k-a^2k}{(c+a)(ck-ak)}$$

$$=\frac{a^2-b^2}{(a+b)(a-b)}+\frac{b^2-c^2}{(b+c)(b-c)}+\frac{c^2-a^2}{(c+a)(c-a)}$$

$$=\frac{a^2-b^2}{a^2-b^2}+\frac{b^2-c^2}{b^2-c^2}+\frac{c^2-a^2}{c^2-a^2}=1+1+1=3$$

Method 2 :

We can take $\dfrac{x}{a}=\dfrac{y}{b}=\dfrac{z}{c}=1$

$x=a, y=b, z=c$

$$\frac{ax-by}{(a+b)(x-y)}+\frac{by-cz}{(b+c)(y-z)}+\frac{cz-ax}{(c+a)(z-x)}$$

$$=\frac{a^2-b^2}{(a+b)(a-b)}+\frac{b^2-c^2}{(b+c)(b-c)}+\frac{c^2-a^2}{(c+a)(c-a)}$$

$$=\frac{a^2-b^2}{a^2-b^2}+\frac{b^2-c^2}{b^2-c^2}+\frac{c^2-a^2}{c^2-a^2}=1+1+1=3$$

Method 3 :

$x=a=1, y=b=2, z=c=3$

$$=\frac{1-4}{3(-1)}+\frac{4-9}{5(-1)}+\frac{9-1}{4(2)}$$

$$=\frac{-3}{-3}+\frac{-5}{-5}+\frac{8}{8}=1+1+1=3$$

Sol^{n}223.

$$\frac{a^3(b+c)}{(a-b)(a-c)}+\frac{b^3(c+a)}{(b-c)(b-a)}+\frac{c^3(a+b)}{(c-a)(c-b)}$$

$$=-\frac{a^3(b+c)}{(a-b)(c-a)}-\frac{b^3(c+a)}{(b-c)(a-b)}-\frac{c^3(a+b)}{(c-a)(b-c)}$$

$$=\frac{-a^3(b+c)(b-c)-b^3(c+a)(c-a)-c^3(a+b)(a-b)}{(a-b)(b-c)(c-a)}$$

$$=\frac{-a^3(b+c)(b-c)-b^3(c^2-a^2)-c^3(a^2-b^2)}{(a-b)(b-c)(c-a)}$$

$$=\frac{-a^3(b+c)(b-c)-b^3c^2+b^3a^2-c^3a^2+c^3b^2}{(a-b)(b-c)(c-a)}$$

$$=\frac{-a^3(b+c)(b-c)-b^3c^2+c^3b^2+b^3a^2-c^3a^2}{(a-b)(b-c)(c-a)}$$

$$=\frac{-a^3(b+c)(b-c)-b^2c^2(b-c)+a^2(b^3-c^3)}{(a-b)(b-c)(c-a)}$$

$$=\frac{-a^3(b+c)(b-c)-b^2c^2(b-c)+a^2(b-c)(b^2+c^2+bc)}{(a-b)(b-c)(c-a)}$$

$$= \frac{(b-c)\left\{-a^3(b+c)-b^2c^2+a^2(b^2+c^2+bc)\right\}}{(a-b)(b-c)(c-a)}$$

$$= \frac{-a^3b-a^3c-b^2c^2+a^2b^2+a^2c^2+a^2bc}{(a-b)(c-a)}$$

$$= \frac{-a^3b+a^2bc-b^2c^2+a^2b^2-a^3c+a^2c^2}{(a-b)(c-a)}$$

$$= \frac{a^2b(c-a)-b^2(c^2-a^2)+a^2c(c-a)}{(a-b)(c-a)}$$

$$= \frac{a^2b(c-a)-b^2(c+a)(c-a)+a^2c(c-a)}{(a-b)(c-a)}$$

$$= \frac{(c-a)\left[a^2b-b^2(c+a)+a^2c\right]}{(a-b)(c-a)}$$

$$= \frac{a^2b-b^2a-b^2c+a^2c}{(a-b)}$$

$$= \frac{ab(a-b)+c(a-b)(a+b)}{(a-b)}$$

$$= \frac{(a-b)\left[ab+c(a+b)\right]}{(a-b)}$$

$=ab + bc + ca$

Method 2 :

Put $a=1$, $b=2$, $c=3$

$$= \frac{a^3(b+c)}{(a-b)(a-c)} + \frac{b^3(c+a)}{(b-c)(b-a)} + \frac{c^3(a+b)}{(c-a)(c-b)}$$

$$= \frac{1(5)}{(-1)(-2)} + \frac{8(4)}{(-1)(1)} + \frac{27(3)}{2(1)} = \frac{5}{2} - 32 + \frac{81}{2} = 11$$

Put, $a=1, b=2, c=3$ in all option

(a) $abc = 6$

(b) $a+b+c = 6$

(c) $ab+bc+ca = 2+6+3 = 11$

(d) 3

hence, option (c) is correct.

Sol^n 224. Put, $a=1, b=2, c=3$

$$\frac{a^2-b^2-c^2}{(a-b)(a-c)} + \frac{b^2-c^2-a^2}{(b-c)(b-a)} + \frac{c^2-a^2-b^2}{(c-a)(c-b)}$$

$$= \frac{1-4-9}{(-1)(-2)} + \frac{4-9-1}{(-1)(1)} + \frac{9-1-4}{(2)(1)}$$

$$= \frac{-12}{2} + \frac{-6}{-1} + \frac{4}{2} = -6+6+2 = 2$$

Put these values in all option-

Option :

(a) $abc = 6$

(b) $a+b+c = 1+2+3 = 6$

(c) 2

(d) 0

hence, option (c) is correct.

Sol^n 225. $x^2 + y^2 = z + 1$ …(i)

$y^2 + z^2 = x + 1$ …(ii)

$z^2 + x^2 = y + 1$ …(iii)

subtraction eq^n. (i) from eq. (ii)

$z^2 - x^2 = x - z$

$\Rightarrow$ $(z+x)(z-x) + (z-x) = 0$

$\Rightarrow$ $(z-x)(z+x+1) = 0$

If $z-x=0 \Rightarrow z=x$

or $z+x=-1$

Similarly:

$x=y=z$

$z+x=-1, x+y=-1$ & $y+z=-1$

but $x=y=z$ in eq^n (i)

$x^2+x^2 = x+1$

$2x^2 - x - 1 = 0$

$2x^2 - 2x + x - 1 = 0$

$2x(x-1) + 1(x-1) = 0$

$(x-1)(2x+1) = 0$

$x = 1$, or $\frac{-1}{2} = y = z$

There values are satisfying all three eq^n.

hence, $xyz = 1 \times 1 \times 1 = 1$

or $xyz = \left(\frac{-1}{2}\right)\left(\frac{-1}{2}\right)\left(\frac{-1}{2}\right) = \frac{-1}{8}$

Sol^{n}226. $26 - 15\sqrt{3} = \dfrac{52-30\sqrt{3}}{2}$

$= \dfrac{25+27-2\times5\times3\sqrt{3}}{2}$

$= \dfrac{\left(3\sqrt{3}-5\right)^2}{2}$

Now,

$38 + 5\sqrt{3} = \dfrac{76+2.5\sqrt{3}}{2}$

$= \dfrac{75+1+2\times1\times5\sqrt{3}}{2}$

$= \dfrac{\left(5\sqrt{3}+1\right)^2}{2}$

$$\Rightarrow \frac{\sqrt{26-15\sqrt{3}}}{5\sqrt{2}-\sqrt{38+5\sqrt{3}}} = \frac{\sqrt{\dfrac{\left(3\sqrt{3}-5\right)^2}{2}}}{5\sqrt{2}-\sqrt{\dfrac{\left(5\sqrt{3}+1\right)^2}{2}}}$$

$$\Rightarrow \frac{\dfrac{3\sqrt{3}-5}{\sqrt{2}}}{5\sqrt{2}-\dfrac{\left(5\sqrt{3}+1\right)}{\sqrt{2}}} = \frac{3\sqrt{3}-5}{5\sqrt{2}.\sqrt{2}-\left(5\sqrt{3}+1\right)}$$

$$\Rightarrow \frac{3\sqrt{3}-5}{10-5\sqrt{3}-1} = \frac{3\sqrt{3}-5}{9-5\sqrt{3}} = \frac{3\sqrt{3}-5}{\sqrt{3}\left(3\sqrt{3}-5\right)} = \frac{1}{\sqrt{3}}$$

Sol^{n}227. let $y = \dfrac{x+2}{2x^2+3x+6}$

$y\,(2x^2 + 3x + 6) = x + 2$

$x^2.\,(2y) + x\,(3y - 1) + 6y - 2 = 0$

$\because$ x is real, hence $b^2 - 4\,ac \geq 0$

$(3y - 1)^2 - 4\,(2y)\,(6y - 2) \geq 0$

$9y^2 + 1 - 6y - 8y\,(6y - 2) \geq 0$

$9y^2 + 1 - 6y - 48y^2 + 16y \geq 0$

$-39y^2 + 10y + 1 \geq 0$

$39y^2 - 10y - 1 \leq 0$

$39y^2 - 13y + 3y - 1 \leq 0$

$13y\,(3y - 1) + 1\,(3y - 1) \leq 0$

$(3y - 1)\,(13y + 1) \leq 0$

$y \in \left(-\dfrac{1}{13}, \dfrac{1}{3}\right)$

minimum value $= -\dfrac{1}{13}$

maximum value $= \dfrac{1}{3}$

Sol^{n}228. Put $a = b = c = 1$

$$= \frac{(a+b+c)^4-(b+c)^4-(c+a)^4-(a+b)^4+a^4+b^4+c^4}{a+b+c}$$

$$= \frac{(3)^4-(2)^4-(2)^4-(2)^4+1+1+1}{1+1+1} = \frac{36}{3} = 12$$

Put $a = b = c = 1$ in all option –

option (a) 3 abc = 3

(b) 4 abc = 4

(c) 6 abc = 6

(d) 12 abc = 12

hence, option (d) is correct

Sol^{n}229. put $a = 0$, $b = 2$, $c = 3$

$$= \frac{a(b-c)^2}{(c-a)(a-b)} + \frac{b(c-a)^2}{(a-b)(b-c)} + \frac{c(a-b)^2}{(b-c)(c-a)}$$

$$= 0 + \frac{2.(3)^2}{(-2)(-1)} + \frac{3(-2)^2}{(-1)(3)}$$

$$= \frac{18}{2} - \frac{12}{3} = 9 - 4 = 5$$

put these value in all option.

option (a) $a + b + c = 5$

option (b) 3

option (c) $a^2 + b^2 + c^2 = 13$

option (d) $abc = 0$

hence, option (a) is correct

Sol^{n}230. All option are independent of a, b & c.

put $a = b =$

$$\frac{2a}{a+b}+\frac{2b}{b+c}+\frac{2c}{c+a}+\frac{(b-c)(c-a)(a-b)}{(b+c)(c+a)(a+b)}$$

$$=\frac{2}{2}+\frac{2}{2}+\frac{2}{2}+0$$

$$=1+1+1=3$$

Sol^{n}231. $x+\frac{1}{x}=-\sqrt{3} \quad \Rightarrow \quad x^6=-1$

See type 2(f)

$x^{67}+x^{53}+x^{43}+x^{29}+x^{24}+x^{12}+x^6+3$

$$=x.x^{66}+\frac{x^{54}}{x}+x.x^{42}+\frac{x^{30}}{x}+\left(x^6\right)^4+\left(x^6\right)^2+x^6+3$$

$$\Rightarrow x^{66}=\left(x^6\right)^{11}=-1$$

$$\Rightarrow x^{54}=\left(x^6\right)^9=-1$$

$$\Rightarrow x^{42}=\left(x^6\right)^7=-1$$

$$\Rightarrow x^{30}=\left(x^6\right)^5=-1$$

$$=x.(-1)^{11}+\frac{1}{x}(-1)^9+x.(-1)^7+\frac{1}{x}(-1)^5+(-1)^4$$

$$+(-1)^2+(-1)+3$$

$$=-x-\frac{1}{x}-x-\frac{1}{x}+1+1-1+3$$

$$=-\sqrt{3}-\sqrt{3}+4$$

$$=-2\sqrt{3}+4 \quad =2\left(2-\sqrt{3}\right)$$

Sol^{n}232. If $x+\frac{1}{x}=1$ **[See type 2(C)]**

$$\Rightarrow x^3=-1$$

$$\Rightarrow x^{52}+x^{46}+x^{32}+x^{26}+x^{21}+x^{15}+x^6+x^3+4$$

$$\Rightarrow x.\left(x^3\right)^{17}+x.\left(x^3\right)^{15}+\frac{1}{x}\left(x^3\right)^{11}+\frac{1}{x}\left(x^3\right)^9+\left(x^3\right)^7$$

$$+\left(x^3\right)^5+\left(x^3\right)^2+x^3+4$$

$$\Rightarrow x.(-1)^{17}+x.(-1)^{15}+\frac{1}{x}(-1)^{11}+\frac{1}{x}(-1)^9+(-1)^7+$$

$$(-1)^5+(-1)^2+(-1)+4$$

$$=-x-x-\frac{1}{x}-\frac{1}{x}-1-1+1-1+4$$

$$=-1-1+2=0$$

Sol^{n}233. Method 1 :

$ab-b+1=0$

Multiply by c

$abc-bc+c=0 \Rightarrow abc=bc-c$ ...(I)

$bc-c+1=0 \Rightarrow bc-c=-1$ ...(II)

From (i) & (ii), abc = –1

Multiply by a in eq. (ii)

$abc-ac+a=0 \;\Rightarrow\; a-ac=-abc$

$=-(-1)$

$=1$

Method 2 :

$ab-b+1=0 \;\Rightarrow\; b-ab=1$

$bc-c+1=0 \;\Rightarrow\; c-bc=1$

using similarty

$a-ac=1$

Sol^{n}234. $a+b+c=20$ .... (i)

and, $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=30$ .... (ii)

multiply eqn (i) & (ii)

$$(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)=20\times30$$

$$1+\frac{a}{b}+\frac{a}{c}+\frac{b}{a}+1+\frac{b}{c}+\frac{c}{a}+\frac{c}{b}+1=600$$

$$\frac{a}{b}+\frac{b}{a}+\frac{b}{c}+\frac{c}{b}+\frac{c}{a}+\frac{a}{c}=597$$

Sol^n^235. $\frac{b-c}{a}+\frac{a+c}{b}+\frac{a-b}{c}=1$

$\frac{b-c}{a}+\frac{a-b}{c}=1-\frac{a+c}{b}$

$\frac{bc-c^2+a^2-ab}{ac}=\frac{b-a-c}{b}$

$\frac{b(c-a)-(c+a)(c-a)}{ac}=\frac{b-a-c}{b}$

$\frac{(c-a)-(b-c-a)}{ac}=\frac{(b-a-c)}{b}$

$\frac{c-a}{ac}=\frac{1}{b}$

$\frac{1}{a}-\frac{1}{c}=\frac{1}{b}$

$\frac{1}{a}=\frac{1}{b}+\frac{1}{c}$

Sol^n^236. $\frac{x}{y}=\frac{z}{w}\Rightarrow xw=yz$

$=(xy+zw)^2=x^2y^2+z^2w^2+2xyzw$

$=x^2y^2+xyzw+z^2w^2+xyzw$

$\{\because xyzw=(xw)(yz)=(y^2z^2)\}$

$\{\because xyzw=(xw)(yz)=(x^2w^2)\}$

$=x^2y^2+y^2z^2+z^2w^2+x^2w^2$

$=y^2(x^2+z^2)+w^2(x^2+z^2)$

$=(x^2+z^2)(y^2+w^2)$

Sol^n^237. Put $a=b=c=1$ then, $x=\frac{a}{b+c}=\frac{1}{2}$

$y=\frac{b}{a+c}=\frac{1}{2}$

$z=\frac{c}{a+b}=\frac{1}{2}$

then, $xy+yz+zx+2xyz=\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+2\times\frac{1}{8}=1$

hence, option (d) is correct.

Sol^n^238. $\sqrt{x+2\sqrt{x+2\sqrt{x+2\sqrt{x+2}}}}=x$

$\sqrt{x+2x}=x$

$\sqrt{3x}=x$

$3x=x^2$ (Squaring both sides)

$x=3$

Sol^n^239. $\frac{1}{x^2+5x+10}$ Will be maximum when $x^2+5x+10$ minimum, we will find minimum value of $x^2+5x+10$

differentiation = 0 $\Rightarrow$ $2x+5=0$

$=\left(\frac{-5}{2}\right)^2+5\left(\frac{-5}{2}\right)+10$

$=\frac{25}{4}-\frac{25}{2}+10$

$=\frac{15}{4}$

hence, maximum value of $\frac{1}{x^2+5x+10}=\frac{1}{15/4}$

$=\frac{4}{15}$

Sol^n^240. $x+y=1$

$\left(x+\frac{1}{x}\right)^2+\left(y+\frac{1}{y}\right)^2$ will be minimum when

$x=y=\frac{1}{2}$ minimum value of

$\left(x+\frac{1}{x}\right)^2+\left(y+\frac{1}{y}\right)^2$

$=\left(\frac{1}{2}+2\right)^2+\left(\frac{1}{2}+2\right)^2$

$=(2.5)^2+(2.5)^2=6.25+6.25=12.5$

Sol^{n}241. $p^a = q^b = r^c = k$ (let)

$p = k^{1/a}, \quad q = k^{1/b}, r = k^{1/c}$

$\frac{p}{q} = \frac{q}{r} \Rightarrow pr = q^2$

$k^{1/a} . k^{1/c} = k^{2/b} \Rightarrow k^{1/a+1/c} = k^{2/b}$

on comparing power.

$\frac{1}{a} + \frac{1}{c} = \frac{2}{b}$

$\left(\frac{1}{a}+\frac{1}{c}\right) \cdot b = 2$

Sol^{n}242. $x^2 (x+y+z) = 36$ …(i)

$y^2 (x+y+z) = 46$ …(ii)

$z^2 (x+y+z) = 63$ …(iii)

$xy (x+y+z) = 111 \Rightarrow 2xy (x+y+z) = 222$ ... (iv)

$yz (x+y+z) = 99 \Rightarrow 2yz (x+y+z) = 198$ …(v)

$zx (x+y+z) = 82 \Rightarrow 2zx (x+y+z) = 164$ …(vi)

Adding all six equation -

$\Rightarrow (x+y+z)(x^2+y^2+z^2+2xy+2yz+2zx) = 729$

$\Rightarrow (x+y+z)(x+y+z)^2 = 729$

$\Rightarrow (x+y+z)^3 = 729$

$\Rightarrow x+y+z = 9$

Put the value in eq^n. (i)

$9x^2 = 36$

$x^2 = 4$

$x = 2$

Sol^{n}243. Method - 1 :

$x = \frac{a-b}{a+b}$ or $\frac{1}{x} = \frac{a+b}{a-b}$

using component & dividend rule

$\frac{1+x}{1-x} = \frac{a+b+a-b}{a+b-a+b} = \frac{a}{b}$

similarly, $\frac{1+y}{1-y} = \frac{b}{c}$ & $\frac{1+z}{1-z} = \frac{c}{a}$

hence, $\frac{1+x}{1-x} \cdot \frac{1+y}{1-y} \cdot \frac{1+z}{1-z} = \frac{a}{b} \times \frac{b}{c} \times \frac{c}{a} = 1$

Method - 2 :

put $a = -1, b = 2, c = -3$

hence, $x = -3, y = -5$ & $z = \frac{1}{2}$

$\frac{1+x}{1-x} \cdot \frac{1+y}{1-y} \cdot \frac{1+z}{1-z} = \frac{1-3}{1+3} \cdot \frac{1-5}{1+5} \cdot \frac{1+\frac{1}{2}}{1-\frac{1}{2}}$

$\left(-\frac{2}{4}\right)\left(\frac{-4}{6}\right)\left(\frac{3}{1}\right) = 1$

Sol^{n}244. $x = 5$ and $y = z$

then the maximum value of

$x^2 + y^2 + z^2 - (xy + yz + zx)$

$= \frac{1}{2}[(x-y)^2 + (y-z)^2 + (z-x)^2]$

put, $y = z$

$= \frac{1}{2}[(x-z)^2 + (z-x)^2]$

$= \frac{1}{2}\left[2(x-z)^2\right]$

$= (x-z)^2$

put, $z = 0$ (for maximum value)

$= (x-z)^2 = (5-0)^2 = 25$

Sol^{n}245. $x + 2y + z = -6 \Rightarrow x + y + (y+z) = -6$

then, $x^2 + 2y^2 + z^2 + 2yz$

$= x^2 + y^2 + (y+z)^2$ (for maximum value)

$= (-2)^2 + (-2)^2 + (-2)^2$ ($\because x = y = y+z = -2$)

$= 12$

Sol^{n}246. $a^x = bc$ …(i)

$b^y = ac$ …(ii)

$c^z = ab$ …(iii)

Multiply all three equation–

$a^x b^y c^z = a^2 b^2 c^2$

on comparing power–

$x = 2, y = 2, z = 2$

$\frac{x}{1+x} + \frac{y}{1+y} + \frac{z}{1+z} = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = 2$

Sol^{n}247. $\frac{x}{a} = \frac{y}{b} = \frac{z}{c} = k$

$\Rightarrow x = ak, y = bk, z = ck$

adding all three eq^n.

$x + y + z = k(a+b+c)$

squaring both sides.

$x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = k^2 (a+b+c)^2$

$2xy + 2yz + 2zx = k^2 (a+b+c)^2 - (x^2+y^2+z^2)$

put, $k = \frac{x}{a}$

$2(xy + yz + zx) = \frac{x^2}{a^2}(a+b+c)^2 - (x^2+y^2+z^2)$

$xy + yz + zx = \frac{x^2 (a+b+c)^2 - a^2 (x^2+y^2+z^2)}{2a^2}$

Sol^n 248. $\frac{x}{y} = \frac{z}{w} = k$ (let)

$x = ky$ & $z = kw$

$$\Rightarrow \frac{x^m + y^m + z^m + w^m}{x^{-m} + y^{-m} + z^{-m} + w^{-m}}$$

$$= \frac{k^m y^m + y^m + k^m w^m + w^m}{k^{-m} y^{-m} + y^{-m} + k^{-m} w^{-m} + w^{-m}}$$

$$= \frac{y^m (k^m + 1) + w^m (k^m + 1)}{y^{-m} (k^{-m} + 1) + w^{-m} (k^{-m} + 1)}$$

$$= \frac{(k^m + 1)(y^m + w^m)}{(k^{-m} + 1)(y^{-m} + w^{-m})}$$

$$= \frac{(k^m + 1)(y^m + w^m)}{\left(\frac{1}{k^m} + 1\right)\left(\frac{1}{y^m} + \frac{1}{w^m}\right)}$$

$$= \frac{(k^m + 1)(y^m + w^m)}{\frac{(k^m + 1)}{k^m} \cdot \frac{(y^m + w^m)}{y^m . w^m}}$$

$= k^m y^m w^m = (kyw)^m = (k^2 y^2 w^2)^{m/2}$

$= (ky.\ y.\ w.\ kw)^{m/2} = (xyzw)^{m/2}$

Sol^n 249. $\sqrt{13x^3 - 14x + 29} + \sqrt{13x^3 - 14x - 21} = 10$ ...(i)

$\sqrt{13x^3 - 14x + 29} - \sqrt{13x^3 - 14x - 21} = t$ ... (ii)

multiplying both equation (i) & (ii)

$\Rightarrow \left(\sqrt{13x^3 - 14x + 29}\right)^2 - \left(\sqrt{13x^3 - 14x - 21}\right)^2 = 10\,t$

$\Rightarrow 13x^3 - 14x + 29 - 13x^3 + 14x + 21 = 10\,t$

$\Rightarrow 50 = 10\,t$

$\Rightarrow t = 5$

Sol^n 250. Method 1:–

$3x^2 = by + cz$ ... (i)

$3y^2 = cz + ax$ ...(ii)

$3z^2 = ax + by$ ...(iii)

adding ax, bx and cz to eq^n. (i), (ii) & (iii) respectively

$ax + 3x^2 = ax + by + cz \Rightarrow x(a + 3x) = k$ (let)

$by + 3y^2 = ax + by + cz \Rightarrow y(b + 3y) = k$ (let)

$cz + 3z^2 = ax + by + cz \Rightarrow z(c + 3z) = k$ (let)

$$\Rightarrow \frac{a}{a + 3x} + \frac{b}{b + 3y} + \frac{c}{c + 3z}$$

$$= \frac{ax}{ax + 3x^2} + \frac{by}{by + 3y^2} + \frac{cz}{cz + 3z^2}$$

$$= \frac{ax}{k} + \frac{by}{k} + \frac{cz}{k} = \frac{ax + by + cz}{k} = \frac{k}{k} = 1$$

Method 2:–

put, $x = y = z = 2$ and $a = b = c = 3$ which all three equation.

$$\Rightarrow \frac{a}{a + 3x} + \frac{b}{b + 3y} + \frac{c}{c + 3z} = \frac{3}{9} + \frac{3}{9} + \frac{3}{9} = \frac{9}{9} = 1$$

Sol^n 251. $x = \left(a + \sqrt{a^2 + b^3}\right)^{1/3} + \left(a - \sqrt{a^2 + b^3}\right)^{1/3}$

we can solve this question after cubing both side but we can see option is independent of b.

So, we put $b = 0$

then question becomes–

$x = \left(a + \sqrt{a^2}\right)^{1/3} + \left(a - \sqrt{a^2}\right)^{1/3} = (2a)^{1/3}$,

then $x^3 - 2a = ?$

$x = (2a)^{1/3}$

$x^3 = 2a$

$x^3 - 2a = 0$

Sol^n 252. $a = \frac{xy}{x + y} \Rightarrow \frac{1}{a} = \frac{1}{x} + \frac{1}{y}$ ...(i)

$b = \frac{xz}{x + z} \Rightarrow \frac{1}{b} = \frac{1}{z} + \frac{1}{x}$ ...(ii)

$c = \frac{yz}{y + z} \Rightarrow \frac{1}{c} = \frac{1}{y} + \frac{1}{z}$ ...(iii)

Adding all three equation–

$$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 2\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$$

but $\frac{1}{y} + \frac{1}{z} = \frac{1}{c}$

$$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 2\left(\frac{1}{x} + \frac{1}{c}\right) = \frac{2}{x} + \frac{2}{c}$$

$$\Rightarrow \frac{2}{x} = \frac{1}{a} + \frac{1}{b} - \frac{1}{c} = \frac{bc + ca - ab}{abc}$$

$$\Rightarrow x = \frac{2abc}{bc+ca-ab}$$

Sol^{n}253. $\frac{x-a^2}{b^2+c^2}+\frac{x-b^2}{c^2+a^2}+\frac{x-c^2}{a^2+b^2}=3$

put, $a = b = c = 1$

then, $\frac{x-1}{2}+\frac{x-1}{2}+\frac{x-1}{2}=3$

hence, $x = 3$

put, $a = b = c = 1$ in all four option

(a) $a^2 + b^2 = 2$ (b) $a^2 + b^2 + c^2 = 3$

(c) $a^2 - b^2 - c^2 = -1$

(d) $a^2 + b^2 - c^2 = 1$

hence, option (b) is correct.

Sol^{n}254. Hint: (See Example –131)

Sol^{n}255. $a + b + c + d = 4$

$\Rightarrow (a + 1) + (b + 1) + (c + 1) + (d + 1) = 8$

when, $a + 1 = b + 1 = c + 1 = d + 1 = 2$

then, maximum value of $(a+1)(b+1)(c+1)(d+1)=2^4$
$= 16$

Sol^{n}256. $x + y + z = 1$

$\left(\frac{1}{x}-1\right)\left(\frac{1}{y}-1\right)\left(\frac{1}{z}-1\right)$ will be minimum when

$x = y = z = \frac{1}{3}$

minimum value of $\left(\frac{1}{x}-1\right)\left(\frac{1}{y}-1\right)\left(\frac{1}{z}-1\right)$

$= (2)(2)(2) = 8$

Sol^{n}257. $(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

$= 1+\frac{a}{b}+\frac{a}{c}+\frac{b}{a}+1+\frac{b}{c}+\frac{c}{a}+\frac{c}{b}+1$

$= 3+\left(\frac{a}{b}+\frac{b}{a}\right)+\left(\frac{b}{c}+\frac{c}{b}\right)+\left(\frac{c}{a}+\frac{a}{c}\right)$

let, $\frac{a}{b}=x$, $\frac{b}{c}=y$, $\frac{c}{a}=z$

$= 3+\left(x+\frac{1}{x}\right)+\left(y+\frac{1}{y}\right)+\left(z+\frac{1}{z}\right)$

hence, least value $= 3 + 2 + 2 + 2 = 9$

($\because$ least value of $x+\frac{1}{x}=2$ If x is positive integer)

Method:–

We can say that

$(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ will be least when $a = b = c$

least value $= (3a)\left(\frac{3}{a}\right) = 9$

Sol^{n}258. $\frac{(a^2+a+1)(b^2+b+1)(c^2+c+1)}{abc}$

$= \frac{(a^2+a+1)}{a}\cdot\frac{(b^2+b+1)}{b}\cdot\frac{(c^2+c+1)}{c}$

$= \left(a+\frac{1}{a}+1\right)\cdot\left(b+\frac{1}{b}+1\right)\cdot\left(c+\frac{1}{c}+1\right)$

Minimum value $= (2 + 1)(2 + 1)(2 + 1) = 27$

Method : 2

Put, $a = b = c = 1$,

then, minimum value $= 27$

Sol^{n}259. $x=\frac{1}{2}\left(\sqrt{\frac{9}{8}}-\sqrt{\frac{8}{9}}\right)=\frac{1}{2}\left(\frac{9-8}{\sqrt{8}.\sqrt{9}}\right)$

$=\frac{1}{2}\left(\frac{1}{2\sqrt{2}.3}\right)=\frac{1}{12\sqrt{2}}$

$$\Rightarrow \frac{18\sqrt{1+x^2}}{x+\sqrt{1+x^2}}=\frac{18\sqrt{1+\frac{1}{288}}}{\frac{1}{12\sqrt{2}}+\sqrt{1+\frac{1}{288}}}$$

$$\Rightarrow \frac{18\sqrt{\frac{289}{288}}}{\frac{1}{12\sqrt{2}}+\sqrt{\frac{289}{288}}}=\frac{18\times\frac{17}{12\sqrt{2}}}{\frac{1}{12\sqrt{2}}+\frac{17}{12\sqrt{2}}}$$

$=\frac{18\times17}{18}=17$

Sol^{n}260. $\sqrt{10+\sqrt{24}+\sqrt{40}+\sqrt{60}} = \sqrt{a}+\sqrt{b}+\sqrt{c}$

squaring both sides–

$\Rightarrow 10+\sqrt{24}+\sqrt{40}+\sqrt{60} = a+b+c+2\sqrt{ab}+2\sqrt{bc}+2\sqrt{ca}$

$\Rightarrow 10+2\sqrt{2}\,.\,\sqrt{3}+2\sqrt{2}\,.\,\sqrt{5}+2\sqrt{5}\,.\,\sqrt{3} = a+b+c+2\sqrt{a}\,.\,\sqrt{b}+2\sqrt{a}\,.\sqrt{c}+2\sqrt{c}\,.\sqrt{b}$

on comparing rational or irrational part.

$\sqrt{a} = \sqrt{2} \Rightarrow a = 2$

$\sqrt{b} = \sqrt{3} \Rightarrow b = 3$

$\sqrt{c} = \sqrt{5} \Rightarrow c = 5$

$\therefore a+b+c = 10$

Sol^{n}261. $x+\frac{1}{x} = 2a$ & $y+\frac{1}{y} = 2c$

multiply both equation–

$\left(x+\frac{1}{x}\right)\left(y+\frac{1}{y}\right) = 4ac$

$xy+\frac{x}{y}+\frac{y}{x}+\frac{1}{xy} = 4ac$ …(i)

$x-\frac{1}{x} = 2b$ & $y-\frac{1}{y} = 2d$

Multiply both equation–

$\left(x-\frac{1}{x}\right)\left(y-\frac{1}{y}\right) = 4bd$

$xy-\frac{x}{y}-\frac{y}{x}+\frac{1}{xy} = 4bd$ …(ii)

adding eq^n. (i) & (ii)

$2xy+\frac{2}{xy} = 4ac+4bd$

$xy+\frac{1}{xy} = 2(ac+bd)$

Sol^{n}262. $3s = (a+b+c)$

$(s-a)+(s-b)+(s-c) = 0$

then,

$(s-a)^3+(s-b)^3+(s-c)^3-3(s-a)(s-b)(s-c) = 0$ ..(i)

when we will substitute $(s-a), (s-b), (s-c)$ for a, b, c respectively in $a^3+b^3+c^3-3abc$

then, from eq^n. (i)–

$(s-a)^3+(s-b)^3+(s-c)^3-3(s-a)(s-b)(s-c) = 0$

Method 2:

we can put, $s = 0$

it means. $a+b+c = 0$

then, $a^3+b^3+c^3-3abc = 0$

put $-a, -b, -c$ for a, b, c respectively

$(-a)^3+(-b)^3+(-c)^3-3(-a)(-b)(-c) = 0$

$\Rightarrow -a^3-b^3-c^3+3abc = 0$

$\Rightarrow -(a^3+b^3+c^3-3abc) = 0$

Sol^{n}263. put, $a = 1, b = 2, c = 3$

$\Rightarrow a(b-c)^3+b(c-a)^3+c(a-b)^3$

$= 1(2-3)^3+2(3-1)^3+3(1-2)^3$

$= -1+16-3 = 12$

put there value in all option–

option (a) $\Rightarrow 3abc = 18$

option (b) $\Rightarrow (a-b)(b-c)(c-a) = 2$

option (c) $\Rightarrow (a-b)(b-c)(c-a)(a+b+c) = 12$

option (d) $\Rightarrow (a+b)(b+c)(c+a)(a+b+c) = 360$

hence, option (c) is correct.

Sol^{n}264. put $a = 0, b = 1, c = 2$

$a^4(b^2-c^2)+b^4(c^2-a^2)+c^4(a^2-b^2)$

$= 0+1(4)+16(-1) = -12$

put there value in all option –

option (a) $\Rightarrow 3a^2b^2c^2 = 0$

option (b) $\Rightarrow (a^2-b^2)(b^2-c^2)(c^2-a^2) = 12$

option(c) $\Rightarrow -(a^2-b^2)(b^2-c^2)(c^2-a^2) = -12$

option(d) $\Rightarrow (a^2+b^2)(b^2+c^2)(c^2+a^2) = 20$

option (c) is correct.

Sol^{n}265. put, $a = 1, b = 2, c = 3$

$a(b-c)^2+b(c-a)^2+c(a-b)^2+8abc$

$= 1(-1)^2+2(2)^2+3(-1)^2+8(1)(2)(3)$

$= 1+8+3+48 = 60$

put there value in all option.

option (a) $\Rightarrow (a+b)(b+c)(c+a) = 60$

option (b) $\Rightarrow (a-b)(b-c)(c-a) = 2$

option (c) $\Rightarrow 0$

option (d) $\Rightarrow abc = 6$

hence, option (a) is correct.

Sol^{n}266. put, $a = 1, b = -2, c = 3$

$(bc + ca + ab)^3 - b^3c^3 - c^3a^3 - a^3b^3$

$= (-6+3-2)^3-(-8)(27)-(27)(1)-(1)(-8)$

$= -125+216-27+8 = 72$

put these value in all option

option (a) $\Rightarrow$ $3abc\,(a+b)(b+c)(c+a) = 72$

option (b) $\Rightarrow$ $(a+b)(b+c)(c+a) = -4$

option (c) $\Rightarrow$ $(a-b)(b-c)(c-a) = -30$

option (c) $\Rightarrow$ $24\,abc = -144$

hence, option (a) is correct.

Method 2 :

$\because$ $(x+y+z)^3 = x^3+y^3+z^3+3\,(x+y)(y+z)(z+x)$

$\therefore$ $(bc+ca+ab)^3 - b^3c^3 - c^3a^3 - a^3b^3$

$\Rightarrow$ $b^3c^3 + c^3a^3 + a^3b^3 + 3\,(bc+ca)(ca+ab)(bc+ab) - b^3c^3 - c^3a^3 - a^3b^3$

$\Rightarrow$ $3\,(bc+ca)(ca+ab)(bc+ab)$

$\Rightarrow$ $3abc\,(a+b)(b+c)(c+a)$

Sol^{n}267. put $a = 1, b = -2, c = 3$

$= (a+b+c)^4 - (b+c)^4 - (c+a)^4 - (a+b)^4 + a^4 + b^4 + c^4$

$= (1-2+3)^4-(-2+3)^4-(3+1)^4-(1-2)^4+(1)^4+(-2)^4+(3)^4$

$= 16-1-256-1+1+16+81 = -144$

put there value in all option

option (a) $\Rightarrow$ $12\,abc\,(a+b+c) = -144$

option (b) $\Rightarrow$ $abc\,(a+b+c) = -12$

option (c) $\Rightarrow$ 2

opiton (d) $\Rightarrow$ $abc = -6$

hence, option (a) is correct.

Sol^{n}268. put $a = 1, b = 2, c = 3$

$$\frac{a^2-b^2-c^2}{(a-b)(a-c)} + \frac{b^2-c^2-a^2}{(b-c)(b-a)} + \frac{c^2-a^2-b^2}{(c-a)(c-b)}$$

$$= \frac{1-4-9}{(-1)(-2)} + \frac{4-9-1}{(-1)(+1)} + \frac{9-1-4}{(2)(1)}$$

$$= \frac{-12}{2} + \frac{-6}{-1} + \frac{4}{2}$$

$= -6+6+2 = 2$

Sol^{n}269. put, $a = 1, b = -1, c = 0$ such that $a+b+c = 0$

$$\Rightarrow \frac{2\left(a^4+b^4+c^4\right)}{\left(a^2b^2+b^2c^2+c^2a^2\right)} = \frac{2(1+1+0)}{(1+0+0)} = 4$$

Sol^{n}270. $x+\frac{1}{x} = 5 \Rightarrow x^2+\frac{1}{x^2} = (5)^2-2 = 23$ …(i)

$x+\frac{1}{x} = 5 \Rightarrow x^3+\frac{1}{x^3} = (5)^3-3\times5 = 110$ …(ii)

Adding eq^n. (i) and (ii)

$$x^2+\frac{1}{x^2}+x^3+\frac{1}{x^3} = 23+110$$

$$\left(x^2+\frac{1}{x^3}\right)+\left(x^3+\frac{1}{x^2}\right) = 133$$

$$8+x^3+\frac{1}{x^2} = 133$$

$$x^3+\frac{1}{x^2} = 133-8 = 125$$

Sol^{n}271. $a = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = 2+\sqrt{3}$

$$b = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = 2-\sqrt{3}$$

here, $a+b = 4, a-b = 2\sqrt{3}$ & $ab = 1$

$7a^2+11ab-7b^2$

$= 7\left(a^2-b^2\right)+11ab$

$= 7(a+b)(a-b)+11ab$

$= 7(4)\left(2\sqrt{3}\right)+11 = 11+56\sqrt{3}$

Sol^{n}272. $2x = \sqrt{a}+\frac{1}{\sqrt{a}}$

$$4x^2 = a+\frac{1}{a}+2$$

$$4x^2-4 = a+\frac{1}{a}+2-4$$

$$4\left(x^2-1\right)=\left(\sqrt{a}-\frac{1}{\sqrt{a}}\right)^2$$

$$\sqrt{\left(x^2-1\right)}=\frac{1}{2}\left(\sqrt{a}-\frac{1}{\sqrt{a}}\right)$$

$$\frac{\sqrt{x^2-1}}{x-\sqrt{x^2-1}}=\frac{\frac{1}{2}\left(\sqrt{a}-\frac{1}{\sqrt{a}}\right)}{\frac{1}{2}\left(\sqrt{a}+\frac{1}{\sqrt{a}}\right)-\frac{1}{2}\left(\sqrt{a}-\frac{1}{\sqrt{a}}\right)}$$

$$=\frac{\sqrt{a}-\frac{1}{\sqrt{a}}}{\sqrt{a}+\frac{1}{\sqrt{a}}-\sqrt{a}+\frac{1}{\sqrt{a}}}=\frac{\sqrt{a}-\frac{1}{\sqrt{a}}}{\frac{2}{\sqrt{a}}}=\frac{a-1}{2}$$

Method : 2

Put, $a = 1$ in question and all options

$$2x=1+1 \Rightarrow x=1$$

then, $\frac{\sqrt{x^2-1}}{x-\sqrt{x^2-1}}=\frac{0}{1-0}=0$

Option (a) $\frac{a-1}{2}=0$ (on putting $a = 1$)

hence (a) is answer.

Sol^{n}273. $\frac{x^3+1}{x^2-1}=x+\sqrt{\frac{6}{x}}$

$$\Rightarrow \frac{x^3+1}{x^2-1}-x=\sqrt{\frac{6}{x}}$$

$$\Rightarrow \frac{x^3+1-x^3+x}{x^2-1}=\sqrt{\frac{6}{x}}$$

$$\Rightarrow \frac{1+x}{(x-1)(x+1)}=\sqrt{\frac{6}{x}}$$

$$\frac{1}{x-1}=\sqrt{\frac{6}{x}}$$

Squaring both sides

$$\frac{1}{x^2+1-2x}=\frac{6}{x}$$

$$6x^2+6-12x=x$$

$$6x^2-13x+6=0$$

divide by x

$$6x-13+\frac{6}{x}=0$$

$$6\left(x+\frac{1}{x}\right)=13$$

$$x+\frac{1}{x}=\frac{13}{6}$$

Sol^{n}274. $\sqrt{\frac{x}{y}}+\sqrt{\frac{y}{x}}=\frac{10}{3}$

$$\frac{x+y}{\sqrt{xy}}=\frac{10}{3} \Rightarrow \frac{10}{\sqrt{xy}}=\frac{10}{3}$$

$$\sqrt{xy}=3 \Rightarrow xy=9$$

By option we can check

$x=9, y=1$

OR, $\Rightarrow$

$x+y=10$

$$x-y=\sqrt{(x+y)^2-4xy}=\sqrt{100-36}=8$$

hence, $x=9,\ y=1$

Sol^{n}275. $x=\frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}=\frac{\sqrt{3}.\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$$\frac{x}{\sqrt{3}}=\frac{\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

Applying component & devidend rule

$$\frac{x+\sqrt{3}}{x-\sqrt{3}}=\frac{2\sqrt{2}+\sqrt{3}}{-\sqrt{3}} \quad ...(i)$$

if $\frac{x}{\sqrt{2}}=\frac{\sqrt{3}}{\sqrt{3}+\sqrt{2}}$

then applying component & devidend rule

$$\frac{x+\sqrt{2}}{x-\sqrt{2}}=\frac{2\sqrt{3}+\sqrt{2}}{-\sqrt{2}} \quad ....(ii)$$

Subtracting eq^n. (ii) from eq^n (i)

$$\frac{x+\sqrt{3}}{x-\sqrt{3}}-\frac{x+\sqrt{2}}{x-\sqrt{2}}=\frac{2\sqrt{2}+\sqrt{3}}{-\sqrt{3}}-\frac{2\sqrt{3}+\sqrt{2}}{-\sqrt{2}}$$

$$= -\frac{2\sqrt{2}+\sqrt{3}}{\sqrt{3}} + \frac{2\sqrt{3}+\sqrt{2}}{\sqrt{2}}$$

$$= \frac{-4-\sqrt{6}+6+\sqrt{6}}{\sqrt{6}}$$

$$= \frac{2}{\sqrt{6}} = \sqrt{\frac{2}{3}}$$

Sol^n 276. $(x+1)(x+2)+\frac{1}{x(x-1)}=0$

$(x^2+x)(x+2)(x-1)+1=0$
$(x^2+x)(x^2+x-2)+1=0$
let $'t' = (x^2+x)$ then
$(t)(t-2)+1=0 \Rightarrow t^2-2t+1=0$
$(t-1)^2=0 \Rightarrow t=1$
hence $x^2+x=1$

Sol^n 277. $x^2 + xy + y^2 = 84$ ......(1)

$x - \sqrt{xy} + y = 6$ ......(2)

$\Rightarrow x + y = 6 + \sqrt{xy}$

Squaring both side

$x^2 + y^2 + 2xy = 36 + xy + 12\sqrt{xy}$

$x^2 + y^2 + xy = 36 + 12\sqrt{xy}$

$84 = 36 + 12\sqrt{xy}$

$12\sqrt{xy} = 84 - 36 = 48$

$\sqrt{xy} = 4 \quad \Rightarrow xy = 16$

put $\sqrt{xy} = 4$ in equation (2)

$x - 4 + y = 6$

$x + y = 10$ ......(3) hence

$(x+1)(x+2) = \frac{1}{x(1-x)}$

$(x2+x) = \frac{1}{(x+2)(1-x)}$

$x^2+x = \frac{1}{(-x^2-x+2)}$

$t = (2-1) = 1$
$t = 1$

$(x - y) = \sqrt{(x+y)^2 - 4xy} = \sqrt{100-64} = 6$ ...(4)

on solving equation (3) & (4)

$x = 8, \quad y = 2$
$x^3 + y^3 = 512 + 8 = 520$

Sol^n 278. $x^2 - xy + y^2 = 19$ ...(1)

$x^4 + x^2y^2 + y^4 = 931$ ...(2)

$x^4 + 2x^2y^2 + y^4 - x^2y^2 = 931$

$(x^2 + y^2)^2 - (xy)^2 = 931$

$(x^2 + y^2 + xy)(x^2 - xy + y^2) = 931$

then $x^2 + y^2 + xy = \frac{931}{19} = 49$ ...(3)

adding eqn (1) and (3)

$2(x^2+y^2) = 19+49 = 68$

$x^2 + y^2 = 34$ ...(4)

and subtracting eqn (1) from eqn (3)

$2xy = 49 - 19 = 30$ ...(5)

adding eqn (4) and (5)

$(x^2+y^2+2xy) = 30+34$

$(x+y)^2 = 64 \Rightarrow x+y=8$ ...(6)

subtracting eqn (5) from eqn (4)

$x^2+y^2-2xy = 34-30$

$(x-y)^2 = 4 \Rightarrow x-y=2$ .....(7)

On solving eqn (6) & eqn (7)

$x=5, y=3$

then $2x^2+3y^2 = 2\times25+3\times9 = 77$

Sol^n 279.

$(x+y+z)^2 = x^2+y^2+z^2+2(xy+yz+zx)$

$a^2 = a^2 + 2(xy+yz+zx)$

$\Rightarrow xy+yz+zx = 0$

$x^3+y^3+z^3-3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

$a^3 - 3xyz = a(a^2-0) = a^3$

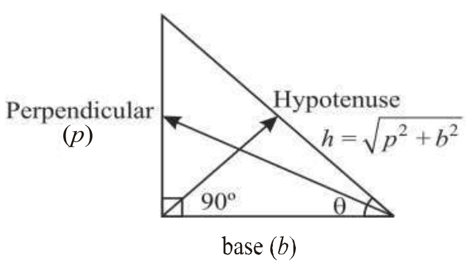
$3xyz = 0 \Rightarrow xyz = 0$

2 Trigonometry

Trigonometry

↓

- Triangle → Right angle triangle
- Geometry → Quadrant



$h > p, b$

Some assumptions :

sine ratio $\Rightarrow \sin\theta = \dfrac{p}{h} \leftrightarrow \text{cosec}\,\theta = \dfrac{h}{p} \geq 1$

cosine ratio $\Rightarrow \cos\theta = \dfrac{b}{h} \leftrightarrow \sec\theta = \dfrac{h}{b} \geq 1$

tangent ratio $\Rightarrow \tan\theta = \dfrac{p}{b} \leftrightarrow \cot\theta = \dfrac{b}{p}$

$\Rightarrow \sin\theta \,.\, \text{cosec}\theta = 1$

$\Rightarrow \cos\theta . \sec\theta = 1$

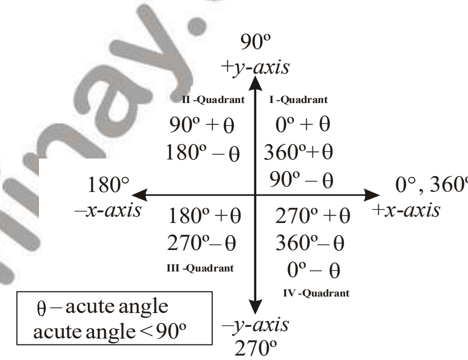
$\Rightarrow \tan\theta \,.\, \cot\theta = 1$

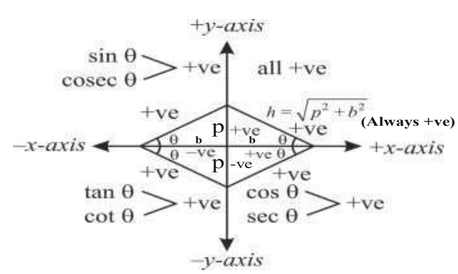
$\sin\theta = \dfrac{1}{\text{cosec}\,\theta} \Rightarrow \text{cosec}\,\theta = \dfrac{1}{\sin\theta}$

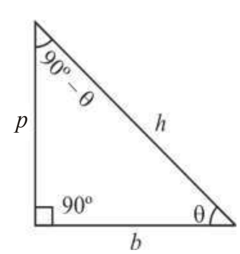
$\cos\theta = \dfrac{1}{\sec\theta} \Rightarrow \sec\theta = \dfrac{1}{\cos\theta}$

$\tan\theta = \dfrac{1}{\cot\theta} \Rightarrow \cot\theta = \dfrac{1}{\tan\theta}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} \Rightarrow \cot\theta = \dfrac{\cos\theta}{\sin\theta}$







In above triangle, $\sin\theta = \frac{p}{h}$ & $\cos(90^o - \theta) = \sin\theta$

We can see here subtracting 90°, cosine is converted into sine, because for angle (90° – θ) base is converted into perpendicular.

If we add another 90° again it will be changed and converted into in its own form. So we can say –

⇒ 90º, 270º .... (odd multiple of 90°) will be changed.

⇒ 0º, 180º, 360º ....(even multiple of 90°) will not be changed.

Change will be in following manner :

$\sin\theta \to \cos\theta$ & $\cos\theta \to \sin\theta$

$\tan\theta \to \cot\theta$ & $\cot\theta \to \tan\theta$

$\sec\theta \to \text{cosec}\theta$ & $\text{cosec}\theta \to \sec\theta$

e.g. (i) $\sin(-\theta) = \sin(0 - \theta) = -\sin\theta$

(∵ (0°–θ) is in IV quadrant and sin is –ve in IV quadrant)

(ii) $\cos(-\theta) = \cos(0 - \theta) = \cos\theta$

(∵ (0°–θ) is in IV quadrant and cos is +ve in IV quadrant)

(iii) $\tan(-\theta) = \tan(0 - \theta) = -\tan\theta$

(∵ (0°–θ) is in IV quadrant and tan is –ve in IV quadrant)

Similarly, $\text{cosec}(-\theta) = -\text{cosec}\,\theta$

$\sec(-\theta) = \sec\theta$

$\cot(-\theta) = -\cot\theta$

$\sin(90^o - \theta) = \cos\theta$ & $\cos(90^o - \theta) = \sin\theta$

$\tan(90^o - \theta) = \cot\theta$ & $\cot(90^o - \theta) = \tan\theta$

$\sec(90^o - \theta) = \text{cosec}\theta$ & $\text{cosec}(90^o - \theta) = \sec\theta$

$\sin(90^o + \theta) = \cos\theta$ & $\cos(90^o + \theta) = -\sin\theta$

$\tan(90^o + \theta) = -\cot\theta$ & $\cot(90^o + \theta) = -\tan\theta$

$\sec(90^o + \theta) = -\text{cosec}\theta$ & $\text{cosec}(90^o + \theta) = \sec\theta$

$\sin(180^o - \theta) = \sin\theta$ & $\cos(180^o - \theta) = -\cos\theta$

$\tan(180^o - \theta) = -\tan\theta$ & $\cot(180^o - \theta) = -\cot\theta$

$\sec(180^o - \theta) = -\sec\theta$ & $\text{cosec}(180^o - \theta) = \text{cosec}\theta$

$\sin(180^o + \theta) = -\sin\theta$ & $\cos(180^o + \theta) = -\cos\theta$

$\tan(180^o + \theta) = \tan\theta$ & $\cot(180^o + \theta) = \cot\theta$

$\sec(180^o + \theta) = -\sec\theta$ & $\text{cosec}(180^o + \theta) = -\text{cosec}\theta$

$\sin(270^o - \theta) = \cos\theta$ & $\cos(270^o - \theta) = -\sin\theta$

$\tan(270^o - \theta) = \cot\theta$ & $\cot(270^o - \theta) = \tan\theta$

$\text{cosec}(270^o - \theta) = -\sec\theta$ & $\sec(270^o - \theta) = -\text{cosec}\theta$

$\sin(270^o + \theta) = -\cos\theta$ & $\cos(270^o + \theta) = \sin\theta$

$\tan(270^o + \theta) = -\cot\theta$ & $\cot(270^o + \theta) = -\tan\theta$

$\text{cosec}(270^o + \theta) = -\sec\theta$ & $\sec(270^o + \theta) = \text{cosec}\theta$

$\sin(360^o - \theta) = -\sin\theta$ & $\cos(360^o - \theta) = \cos\theta$

$\tan(360^o - \theta) = -\tan\theta$ & $\cot(360^o - \theta) = -\cot\theta$

$\text{cosec}(360^o - \theta) = -\text{cosec}\theta$ & $\sec(360^o - \theta) = \sec\theta$

$\sin(360^o + \theta) = \sin\theta$ & $\cos(360^o + \theta) = \cos\theta$

$\tan(360^o + \theta) = \tan\theta$ & $\cot(360^o + \theta) = \cot\theta$

$\text{cosec}(360^o + \theta) = \text{cosec}\theta$ & $\sec(360^o + \theta) = \sec\theta$

θ	**0°**	**30°**	**45°**	**60°**	**90°**
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
cot	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
cosec	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Ex 1. $\dfrac{\sin(270°+\theta).\cos(360°+\theta).\tan(170°+\theta)}{\cos(180°+\theta).\sin(270°-\theta).\cot(260°+\theta)} = ?$

Sol. $= \dfrac{-\cos\theta.\cos\theta.\tan(180^o+\theta-10^o)}{-\cos\theta.-\cos\theta.\cot(270^o+\theta-10^o)}$

$= \dfrac{(-1)\tan(\theta-10^o)}{-\tan(\theta-10^o)} = 1$

Ex 2. $\sin 1125^o = \sin(3\times 360^o + 45^o)$

$= \sin 45^o = \dfrac{1}{\sqrt{2}}$

Ex 3. $\cot 780^o = \cot(2\times 360^o + 60^o)$

$= \cot 60^o = \dfrac{1}{\sqrt{3}}$

Ex 4. $\tan 225^o = \tan(180^o + 45^o)$
$= \tan 45^o = 1$

Ex 5. $\cos 120^o = \cos(90^o + 30^o)$

$= -\sin 30^o = -\dfrac{1}{2}$

Ex 6. $\text{cosec}(1020^o) = \text{cosec}(3\times 360^o - 60^o)$

$= -\text{cosec}60^o = -\dfrac{2}{\sqrt{3}}$

0^o 30^o 45^o 60^o 90^o

sin 0 ....increases to........1

cos 1 ....decreases to........0

α, β – acute angle

(1) if $\alpha > \beta$

$\sin\alpha > \sin\beta$ & $\cos\alpha < \cos\beta$

(2) if $\alpha < \beta$

$\sin\alpha < \sin\beta$ & $\cos\alpha > \cos\beta$

(3) $0 \le \theta < 45^o$ & $0 \le \theta \le 45^o$

then $\cos\theta > \sin\theta$ & then $\cos\theta \ge \sin\theta$

(4) $45^o < \theta \le 90^o$ & $45^o \le \theta \le 90^o$

then $\cos\theta < \sin\theta$ then $\cos\theta \le \sin\theta$

Ex 7. $\cos 10^o - \sin 10^o$

(i) +ve (ii) –ve

(iii) 0 (iv) 1

Sol. 10^o is in the range $0^o \le \theta < 45^o$ then

$\cos\theta > \sin\theta \Rightarrow \cos 10^o > \sin 10^o$

$\text{Cos}10° - \sin 10° > 0$

hence, $\cos 10^o - \sin 10^o$ will be positive.

Ex 8. If α, β is obtuse angle & $\alpha > \beta$.

$(90^o < (\alpha \,\&\, \beta) < 180^o)$ then which options are correct?

(a) $\sin\alpha > \sin\beta$ (b) $\sin\alpha < \sin\beta$

(c) $\cos\alpha > \cos\beta$ (d) $\cos\alpha < \cos\beta$

Sol. let $\alpha = 90^o + \theta_1$ & $\beta = 90^o + \theta_2$

$\because \alpha > \beta$

$\therefore 90^o + \theta_1 > 90^o + \theta_2$

$\Rightarrow \theta_1 > \theta_2$

$\Rightarrow \cos\theta_1 < \cos\theta_2$

$\Rightarrow \cos(\alpha - 90^o) < \cos(\beta - 90^o)$

$\Rightarrow \sin\alpha < \sin\beta$

Now,

$\theta_1 > \theta_2$

$\Rightarrow \sin\theta_1 > \sin\theta_2$

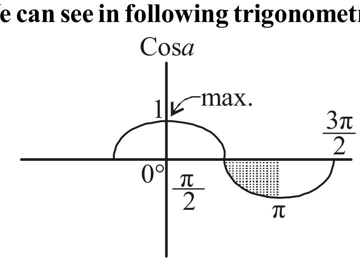
$\Rightarrow \sin(\alpha - 90^o) > \sin(\beta - 90^o)$

$\Rightarrow -\cos\alpha > -\cos\beta$ $(\because -x > -y \Rightarrow x < y)$

$\Rightarrow \cos\alpha < \cos\beta$

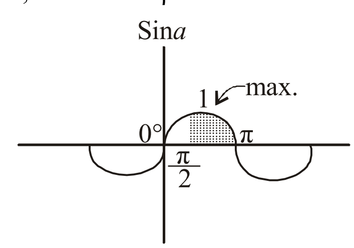
Ans. (b) & (d) option

We can see in following trigonometric graph



$\alpha > \beta$ $(\frac{\pi}{2} < \alpha, \beta < \pi)$

then, $\text{Cos}\alpha < \text{Cos}\beta$



$\alpha > \beta$ $(\frac{\pi}{2} < \alpha, \beta < \pi)$

then, $\text{Sin}\alpha < \text{Sin}\beta$

Type – 1

1. If $\tan\alpha.\tan\beta = 1$ then $\alpha + \beta = 90^o$
2. If $\sin\alpha.\sec\beta = 1$ then $\alpha + \beta = 90^o$
3. If $\cos\alpha.\text{cosec}\beta = 1$ then $\alpha + \beta = 90^o$
4. If $\cot\alpha.\cot\beta = 1$ then $\alpha + \beta = 90^o$
5. If $\sin\alpha.\text{cosec}\beta = 1$ then $\alpha + \beta = 180^o$

Proof: 1. $\tan\alpha.\tan\beta = 1$

$\Rightarrow \quad \tan\alpha = \dfrac{1}{\tan\beta} \Rightarrow \tan\alpha = \cot\beta$

$\Rightarrow \quad \tan\alpha = \tan(90^o - \beta)$

$\Rightarrow \quad \alpha = 90^o - \beta \Rightarrow \alpha + \beta = 90^o$

Ex 9. If $\tan 2\theta.\ \tan 4\theta = 1$, find $\sin 2\theta + \cos 4\theta = ?$

Sol. $\Rightarrow 2\theta + 4\theta = 90^o$

$\Rightarrow 6\theta = 90^o$

$\Rightarrow \theta = 15^o$

$\sin 2\theta + \cos 4\theta = \sin 30^o + \cos 60^o = \dfrac{1}{2} + \dfrac{1}{2} = 1$

Ex 10. If $\sin(x + y).\sec(x - y) = 1$ then $\tan^2 x + \sin^2 x + \sec^2 x = ?$

Sol. $\Rightarrow x + y + x - y = 90^o$

$\Rightarrow 2x = 90^o$

$\Rightarrow x = 45^o$

$\tan^2 x + \sin^2 x + \sec^2 x = \tan^2 45^o + \sin^2 45^o + \sec^2 45^o$

$= 1 + \dfrac{1}{2} + 2 = \dfrac{7}{2}$

Ex 11. If $\cos(2\alpha + 10^o).\text{cosec}(\alpha - 40^o) = 1$ find α.

Sol. $\Rightarrow 2\alpha + 10^o + \alpha - 40 = 90^o$

$\Rightarrow 3\alpha = 90^o + 30^o$

$\Rightarrow \alpha = 40^o$

Ex 12. Find $\tan 15^o.\tan 25^o.\tan 45^o.\tan 65^o.\tan 75^o$.

Sol. $\tan 15°.\tan 25°.\tan 45°.\tan 65°.\tan 75°$ (with $\tan 25°.\tan 65° = 1$ and $\tan 15°.\tan 75° = 1$)

$= \tan 45^o = 1$

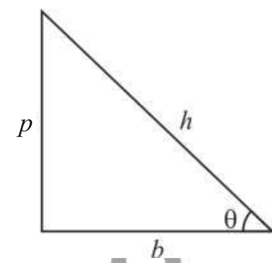
Ex 13. Find $\tan 1^o.\tan 2^o....................\tan 89^o$

Sol. $\Rightarrow \tan 1^o.\tan 89^o = 1$

$\Rightarrow \tan 2^o.\tan 88^o = 1$ similarly all pair should be 1 and in the middle $\tan 45^o = 1$

hence, $\tan 1^o.\tan 2^o....................\tan 89^o = 1$

Basic Identity :



Proof of identity 1: using pythagoras theorem,

$\Rightarrow h^2 = p^2 + b^2$ (dividing by h^2 both side)

$\Rightarrow \dfrac{h^2}{h^2} = \dfrac{p^2}{h^2} + \dfrac{b^2}{h^2}$

$\Rightarrow \quad 1 = \sin^2\theta + \cos^2\theta$

Proof of identity 2: using pythagoras theorem,

$\Rightarrow h^2 = p^2 + b^2$ (dividing by b^2 both side)

$\Rightarrow \dfrac{h^2}{b^2} = \dfrac{p^2}{b^2} + \dfrac{b^2}{b^2}$

$\Rightarrow \quad \sec^2\theta = \tan^2\theta + 1$

Proof of identity 3: using pythagoras theorem,

$\Rightarrow h^2 = p^2 + b^2$ (dividing by p^2 both side)

$\Rightarrow \quad \dfrac{h^2}{p^2} = \dfrac{p^2}{p^2} + \dfrac{b^2}{p^2}$

$\Rightarrow \quad \text{cosec}^2\theta = 1 + \cot^2\theta$

(1). $\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$ & $\cos^2\theta = 1 - \sin^2\theta$

(2). $\sec^2\theta - \tan^2\theta = 1$

$\sec^2\theta = 1 + \tan^2\theta$ & $\tan^2\theta = \sec^2\theta - 1$

(3). $\text{cosec}^2\theta - \cot^2\theta = 1$

$\text{cosec}^2\theta = 1 + \cot^2\theta$ & $\cot^2\theta = \text{cosec}^2\theta - 1$

Question based on Identity 1: $\sin^2\theta + \cos^2\theta = 1$

Ex.14. If $\sin\theta + \cos\theta = \dfrac{7}{5}$, find $\sin\theta.\cos\theta$.

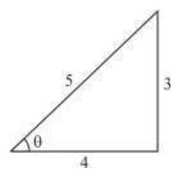
Sol. $\sin\theta + \cos\theta = \dfrac{7}{5}$ (squaring both sides)

$$\sin^2\theta + \cos^2\theta + 2\sin\theta.\cos\theta = \frac{49}{25}$$

$$2\sin\theta.\ \cos\theta = \frac{49}{25} - 1 = \frac{24}{25}$$

$$\sin\theta.\cos\theta = \frac{12}{25}$$

or



We can see in the right triangle in which hypotenuse is 5 then base is 4 & perpendicular is 3.

then check $\Rightarrow$ $\sin\theta + \cos\theta = \frac{3}{5} + \frac{4}{5} = \frac{7}{5}$

hence, $\sin\theta.\cos\theta = \frac{3}{5} \times \frac{4}{5} = \frac{12}{25}$

Ex 15. If $\sin\theta + \cos\theta = \frac{17}{13}$, then $\sin\theta.\cos\theta$.

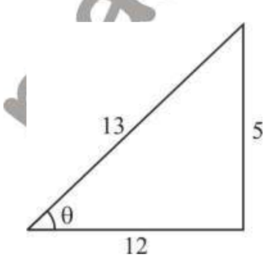
Sol. $\sin\theta + \cos\theta = \frac{17}{13}$ (squaring both sides)

$$\sin^2\theta + \cos^2\theta + 2\sin\theta.\cos\theta = \frac{289}{169}$$

$$2\sin\theta.\cos\theta = \frac{289}{169} - 1 = \frac{120}{169}$$

$$\sin\theta.\cos\theta = \frac{60}{169}$$

or



We can see in the right triangle in which hypotenuse is 13 then base is 12 & perpendicular is 5.

then check $\Rightarrow \sin\theta + \cos\theta = \frac{5}{13} + \frac{12}{13} = \frac{17}{13}$

hence, $\sin\theta\ .\ \cos\theta = \frac{5}{13} \times \frac{12}{13} = \frac{60}{169}$

Ex 16. If $\cos^4\theta - \sin^4\theta = \frac{2}{13}$, find $\cos^2\theta - \sin^2\theta + 1$.

Sol. $\because\ \cos^4\theta - \sin^4\theta = \frac{2}{13}$

$$\Rightarrow (\cos^2\theta - \sin^2\theta).(\cos^2\theta + \sin^2\theta) = \frac{2}{13}$$

$$\Rightarrow \cos^2\theta - \sin^2\theta = \frac{2}{13} \quad \text{(adding 1 both side)}$$

$$\Rightarrow \cos^2\theta - \sin^2\theta + 1 = \frac{15}{13}$$

Ex 17. If $k = (1 - \sin\alpha).(1 - \sin\beta).(1 - \sin\gamma) = (1 + \sin\alpha).(1 + \sin\beta).(1 + \sin\gamma)$ then find k.

Sol. $k = (1 - \sin\alpha).(1 - \sin\beta).(1 - \sin\gamma)$ ....(i)

$k = (1 + \sin\alpha).(1 + \sin\beta).(1 + \sin\gamma)$ ....(ii)

Multiply both equation (i) & (ii)

$\Rightarrow k^2 = [(1 - \sin\alpha)(1 + \sin\alpha)].[(1 - \sin\beta)(1 + \sin\beta)][(1 - \sin\gamma)(1 + \sin\gamma)]$

$\Rightarrow k^2 = (1 - \sin^2\alpha)(1 - \sin^2\beta)(1 - \sin^2\gamma)$

$\Rightarrow k^2 = \cos^2\alpha.\cos^2\beta.\cos^2\gamma$

$\Rightarrow k = \pm \cos\alpha.\cos\beta.\cos\gamma$

if α, β, γ are acute angle or $\alpha, \beta, \gamma < 90^\circ$ or α, β, γ are in the Ist quadrant.

then, k = (+ve) $\cos\alpha.\cos\beta.\cos\gamma$

Ex 18. If $\frac{\cos\alpha}{\cos\beta} = m$ and $\frac{\cos\alpha}{\sin\beta} = n$, find $(m^2 + n^2)\cos^2\beta$

Sol. $\Rightarrow \cos\alpha = m\cos\beta$ …(i)

$\Rightarrow \cos\alpha = n\sin\beta$ …(ii)

From (i) & (ii)

$m\cos\beta = n\sin\beta$

squaring both sides

$\Rightarrow m^2\cos^2\beta = n^2\sin^2\beta$

$\Rightarrow m^2\cos^2\beta = n^2(1 - \cos^2\beta)$

$\Rightarrow (m^2 + n^2)\cos^2\beta = n^2$

Ex 19. If $\frac{\cos\alpha}{\cos\beta} = m$ and $\frac{\sin\alpha}{\sin\beta} = n$, find $(m^2 - n^2)\cos^2\beta$

Sol. $\Rightarrow \cos\alpha = m\cos\beta$ ...(i)

$\Rightarrow \sin\alpha = n\sin\beta$ ...(ii)

After Squaring then add eqn. (i) & (ii)

$\Rightarrow \cos^2\alpha + \sin^2\alpha = m^2\cos^2\beta + n^2\sin^2\beta$

$\Rightarrow 1 = m^2\cos^2\beta + n^2(1 - \cos^2\beta)$

$\Rightarrow 1 = m^2\cos^2\beta + n^2 - n^2\cos^2\beta$

$\Rightarrow (m^2 - n^2)\cos^2\beta = 1 - n^2$

Ex 20. If $\tan\alpha = n\tan\beta$ and $\sin\alpha = m\sin\beta$, find $\cos^2\alpha$.

Sol. $\because \tan\alpha = n\tan\beta$

$\Rightarrow \frac{\sin\alpha}{\cos\alpha} = n\frac{\sin\beta}{\cos\beta}$ (put $\sin\alpha = m\sin\beta$)

$\Rightarrow \frac{m\sin\beta}{\cos\alpha} = \frac{n\sin\beta}{\cos\beta}$

$\Rightarrow m\cos\beta = n\cos\alpha$ ......(i)

Given : $\Rightarrow m\sin\beta = \sin\alpha$ ......(ii)

After squaring add eqn. (i) and (ii)

$\Rightarrow m^2\cos^2\beta + m^2\sin^2\beta = n^2\cos^2\alpha + \sin^2\alpha$

$\Rightarrow m^2(\cos^2\beta + \sin^2\beta) = n^2\cos^2\alpha + (1 - \cos^2\alpha)$

$\Rightarrow m^2 = n^2\cos^2\alpha + 1 - \cos^2\alpha$

$\Rightarrow m^2 - 1 = \cos^2\alpha\,(n^2 - 1)$

$\Rightarrow \cos^2\alpha = \frac{m^2 - 1}{n^2 - 1}$

Ex 21. If $\frac{\sin A}{\sin B} = \frac{\sqrt{3}}{2}$ and $\frac{\cos A}{\cos B} = \frac{\sqrt{5}}{2}$ then, $5\tan^2 A + \tan^2 B = ?$

Sol. $\frac{\sin A}{\sin B} = \frac{\sqrt{3}}{2} \Rightarrow 2\sin A = \sqrt{3}\sin B$ ...(i)

$\frac{\cos A}{\cos B} = \frac{\sqrt{5}}{2} \Rightarrow 2\cos A = \sqrt{5}\cos B$ ...(ii)

After squaring Add eq. (i) & (ii)

$\Rightarrow 4\sin^2 A + 4\cos^2 A = 3\sin^2 B + 5\cos^2 B$

$\Rightarrow 4(\sin^2 A + \cos^2 A) = 3\sin^2 B + 5(1 - \sin^2 B)$

$\Rightarrow 4 = 3\sin^2 B + 5 - 5\sin^2 B$

$\Rightarrow 4 = 5 - 2\sin^2 B$

$\Rightarrow 2\sin^2 B = 1$

$\Rightarrow \sin B = \frac{1}{\sqrt{2}} = \sin 45^o$

$\Rightarrow B = 45^o$

$\Rightarrow \frac{\tan A}{\tan B} = \frac{\sqrt{3}}{\sqrt{5}}$ (on dividing eqn. (i) & (ii)

$\Rightarrow \frac{\tan A}{\tan 45^o} = \frac{\sqrt{3}}{\sqrt{5}} \Rightarrow \tan A = \frac{\sqrt{3}}{\sqrt{5}}$

$\therefore 5\tan^2 A + \tan^2 B = 5\left(\frac{\sqrt{3}}{\sqrt{5}}\right)^2 + 1 = 5 \times \frac{3}{5} + 1 = 4$

(B). $a\sin\theta + b\cos\theta = c$

$b\sin\theta - a\cos\theta = d$ or $a\cos\theta - b\sin\theta = d$

$\boldsymbol{a^2 + b^2 = c^2 + d^2}$ (add after squaring)

Proof:

$a^2\sin^2\theta + b^2\cos^2\theta + 2ab\sin\theta.\cos\theta + b^2\sin^2\theta + a^2\cos^2\theta - 2ab\sin\theta.\cos\theta = c^2 + d^2$

$\Rightarrow a^2(\sin^2\theta + \cos^2\theta) + b^2(\sin^2\theta + \cos^2\theta) = c^2 + d^2$

$\Rightarrow a^2 + b^2 = c^2 + d^2$

Ex 22. If $a\sin\theta + b\cos\theta = c$,find $a\cos\theta - b\sin\theta = ?$

(i) $\pm\sqrt{a^2 + b^2 + c^2}$ (ii) $\pm\sqrt{a^2 - b^2 + c^2}$

(iii) $\pm\sqrt{a^2 + b^2 - c^2}$ (iv) none of these

Sol. $a\sin\theta + b\cos\theta = c$ ...(i)

$a\cos\theta - b\sin\theta = d$ (let) ...(ii)

After Squaring then Add eqn. (i) & (ii)

We get,

$\Rightarrow a^2 + b^2 = c^2 + d^2$

$\Rightarrow a^2 + b^2 - c^2 = d^2$

$\Rightarrow d = \pm\sqrt{a^2 + b^2 - c^2}$

Ex 23. If $p\sin\theta + q\cos\theta = 3$ & $q\sin\theta - p\cos\theta = 2$, then $p^2 + q^2 = ?$

Sol. $p\sin\theta + q\cos\theta = 3$

$q\sin\theta - p\cos\theta = 2$

Using above identity

$\Rightarrow p^2 + q^2 = 3^2 + 2^2$

$\Rightarrow p^2 + q^2 = 13$

Ex 24. If $3\sin\theta + 5\cos\theta = 3$, then $5\sin\theta - 3\cos\theta$ = ?

(a) ± 5 (b) ± 3
(c) ± 4 (d) ± 16

Sol. $3\sin\theta + 5\cos\theta = 3$
$5\sin\theta - 3\cos\theta = x$
Using above identity
$\Rightarrow 3^2 + 5^2 = 3^2 + x^2 \Rightarrow x^2 = 5^2 = 25$
$\Rightarrow x = \pm 5$

(C). $\sin\theta + \cos\theta = x$
$\underline{\sin\theta - \cos\theta = y}$
$\Rightarrow 2 = x^2 + y^2$ (add after squaring)

Ex 25. If $\sin\theta + \cos\theta = x$ then $\sin\theta - \cos\theta$ = ?

(a) $\pm\sqrt{2-x^2}$ (b) $\pm\sqrt{2+x^2}$
(c) $\pm\sqrt{4-x^2}$ (d) $\pm\sqrt{4+x^2}$

Sol. $\sin\theta + \cos\theta = x$
$\sin\theta - \cos\theta = y$ (let)
Using above identity
$\Rightarrow 2 = x^2 + y^2 \quad \Rightarrow 2 - x^2 = y^2$
$\Rightarrow y = \pm\sqrt{2-x^2}$

Ex 26. If $\sin\theta + \cos\theta = \sqrt{2}\sin\theta$, then $\sin\theta - \cos\theta$ = ?

Sol. $\sin\theta + \cos\theta = \sqrt{2}\sin\theta$
$\sin\theta - \cos\theta = x$ (let)
Using above identity
$\Rightarrow 2 = (\sqrt{2}\sin\theta)^2 + y^2$
$\Rightarrow 2 = 2\sin^2\theta + y^2$
$\Rightarrow 2 - 2\sin^2\theta = y^2$
$\Rightarrow y^2 = 2(1 - \sin^2\theta)$
$\Rightarrow y^2 = 2\cos^2\theta$
$\Rightarrow y = \pm\sqrt{2}\cos\theta$

Ex 27. If $\sin\theta + \cos\theta = \frac{17}{13}$, then $\sin\theta - \cos\theta$ = ?

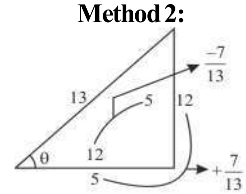
Sol. **Method:1** Using Identity

$$\Rightarrow 2 = \left(\frac{17}{13}\right)^2 + y^2$$

$$\Rightarrow 2 - \frac{289}{169} = y^2$$

$$\Rightarrow y^2 = \frac{49}{169} \Rightarrow y = \pm\frac{7}{13}$$

Method 2:

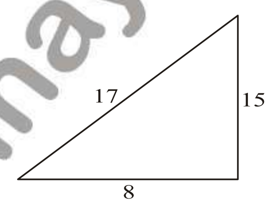


$$\Rightarrow \sin\theta - \cos\theta = \frac{5}{13} - \frac{12}{13} \text{ or } \frac{12}{13} - \frac{5}{13} = \pm\frac{7}{13}$$

(If in the question (-) is asked then two answers will be possible ± thats why in the diagram we have mentioned perpendicular and base 5, 12 and 12, 5 respectively.)

Ex 28. If $\sin\theta - \cos\theta = \frac{7}{17}$ then $\sin\theta + \cos\theta$ = ?

Sol.



$$\therefore \sin\theta + \cos\theta = \frac{15}{17} + \frac{8}{17} = \frac{23}{17}$$

(Here (+) was asked thats why only one answer will be possible. There will be no difference by taking values 15, 8 or 8, 15)

Ex 29. If $x\sin\theta + y\cos\theta = 2$ & $x\sin\theta - y\cos\theta = 0$ then which option is correct ?

(i) $x^2 + y^2 = 4$ (ii) $x^2 + y^2 = 1$
(iii) $\frac{1}{x^2} + \frac{1}{y^2} = 1$ (iv) $\frac{1}{x^2} + \frac{1}{y^2} = 4$

Sol. $x\sin\theta - y\cos\theta = 0 \Rightarrow x\sin\theta = y\cos\theta$
$\Rightarrow x\sin\theta + x\sin\theta = 2$
$\Rightarrow 2x\sin\theta = 2 \Rightarrow x\sin\theta = 1 \Rightarrow \sin\theta = \frac{1}{x}$ …(i)
$\Rightarrow y\cos\theta = x\sin\theta = 1$
$\Rightarrow \cos\theta = \frac{1}{y}$ …(ii)
After squaring then add (i) & (ii)
$\Rightarrow \frac{1}{x^2} + \frac{1}{y^2} = 1$
Option (iii) is correct.

Ex 30. If $x \sin^3\theta + y \cos^3\theta = \sin\theta.\cos\theta$ and $x \sin\theta - y \cos\theta = 0$ find $x^2 + y^2$

Sol. $x \sin\theta - y \cos\theta = 0$

$\Rightarrow \quad x \sin\theta = y \cos\theta$ ......(i)

$x \sin\theta \,.\, \sin^2\theta + y \cos^3\theta = \sin\theta \,.\, \cos\theta$ ....(ii)

put $x \sin\theta = y \cos\theta$ in equation (ii)

$\Rightarrow y \cos\theta \,.\, \sin^2\theta + y \cos^3\theta = \sin\theta \,.\, \cos\theta$

$\Rightarrow y \cos\theta\, [\sin^2\theta + \cos^2\theta] = \sin\theta \,.\, \cos\theta$

$\Rightarrow y \cos\theta = \sin\theta \cos\theta$

$\Rightarrow y = \sin\theta$ [put it in equation (i)]

then $x = \cos\theta$

$\therefore\ x^2 + y^2 = \cos^2\theta + \sin^2\theta$

$\Rightarrow \quad x^2 + y^2 = 1$

Ex 31. If $\dfrac{\sin\theta}{x} = \dfrac{\cos\theta}{y}$ then $\sin\theta - \cos\theta = ?$

(i) $x - y$ (ii) $x + y$

(iii) $\dfrac{x-y}{\sqrt{x^2+y^2}}$ (iv) $\dfrac{x-y}{\sqrt{x^2-y^2}}$

Sol. $\Rightarrow \dfrac{\sin\theta}{\cos\theta} = \dfrac{x}{y}$

Let $\sin\theta = kx$ ...(i)

& $\cos\theta = ky$ ...(ii)

After squaring add both equation (i) & (ii)

$\Rightarrow \sin^2\theta + \cos^2\theta = k^2x^2 + k^2y^2$

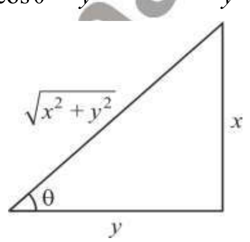
$\Rightarrow 1 = k^2(x^2 + y^2)$

$\Rightarrow k = \dfrac{1}{\sqrt{x^2+y^2}}$

then, $\sin\theta - \cos\theta = k(x-y) = \dfrac{x-y}{\sqrt{x^2+y^2}}$

or

$\Rightarrow \dfrac{\sin\theta}{\cos\theta} = \dfrac{x}{y} \Rightarrow \tan\theta = \dfrac{x}{y}$



$$\sin\theta - \cos\theta = \frac{x}{\sqrt{x^2+y^2}} - \frac{y}{\sqrt{x^2+y^2}} = \frac{x-y}{\sqrt{x^2+y^2}}$$

Ex 32. If $10 \sin^4\alpha + 15 \cos^4\alpha = 6$, find $27 \operatorname{cosec}^6\alpha + 8\sec^6\alpha$.

Sol. $10 \sin^4\alpha + 15 \cos^4\alpha = 6$

Divide both side by $\cos^4\alpha$

$10\tan^4\alpha + 15 = 6 \sec^4\alpha$

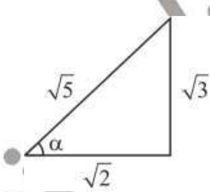
$10\tan^4\alpha + 15 = 6(1 + \tan^2\alpha)^2$

$10\tan^4\alpha + 15 = 6 + 6\tan^4\alpha + 12\tan^2\alpha$

$4\tan^4\alpha - 12\tan^2\alpha + 9 = 0$

$(2\tan^2\alpha - 3)^2 = 0 \Rightarrow 2\tan^2\alpha = 3$

$\Rightarrow \tan\alpha = \sqrt{\dfrac{3}{2}}$



$\operatorname{cosec}\alpha = \sqrt{\dfrac{5}{3}} \quad \Rightarrow \operatorname{cosec}^2\alpha = \dfrac{5}{3}$

$\sec\alpha = \sqrt{\dfrac{5}{2}} \quad \Rightarrow \sec^2\alpha = \dfrac{5}{2}$

Now, $27(\operatorname{cosec}^2\alpha)^3 + 8(\sec^2\alpha)^3$

$= 27 \times \left(\dfrac{5}{3}\right)^3 + 8\left(\dfrac{5}{2}\right)^3 = 27 \times \dfrac{125}{27} + 8 \times \dfrac{125}{8}$

$= 125 + 125 = 250$

IInd method

$\because\ \sin^2\alpha + \cos^2\alpha = 1$ we break question in the form of this identity.

$\Rightarrow\ 10 \sin^4\alpha + 15 \cos^4\alpha = 6$

$\Rightarrow\ \dfrac{10}{6}\sin^4\alpha + \dfrac{15}{6}\cos^4\alpha = 1$

$\Rightarrow \left(\dfrac{5}{3}\sin^2\alpha\right).\sin^2\alpha + \left(\dfrac{5}{2}\cos^2\alpha\right).\cos^2\alpha = 1$

$\Downarrow \qquad \Downarrow$

$1 \qquad 1$

If these two values becomes 1 then identity $\sin^2\alpha + \cos^2\alpha = 1$ will be obtained.

For that: $\sin^2\alpha$ should be $\dfrac{3}{5}$ & $\cos^2\alpha$ should be $\dfrac{2}{5}$

$\sin^2\alpha = \frac{3}{5}$ & $\cos^2\alpha = \frac{2}{5}$

In these value we can see sum of $\sin^2\alpha$ & $\cos^2\alpha$ is 1.

Hence our an option astive.

Now $27\text{cosec}^6\alpha + 8\sec^6\alpha =$

$27\left(\text{cosec}^2\alpha\right)^3 + 8\left(\sec^2\alpha\right)^3$

$= 27\left(\frac{5}{3}\right)^3 + 8\left(\frac{5}{2}\right)^3 = 27 \times \frac{125}{27} + 8 \times \frac{125}{8}$

$= 125 + 125 = 250$

Ex 33. If $15\sin^3\alpha + 20\cos^3\alpha = 12$ find $10\sin\alpha + 15\cos\alpha$.

Sol. $15\sin^3\alpha + 20\cos^3\alpha = 12$

$\Rightarrow \frac{15}{12}\sin^3\alpha + \frac{20}{12}\cos^3\alpha = 1$

$\left(\frac{5}{4}\sin\alpha\right)\sin^2\alpha + \left(\frac{5}{3}\cos\alpha\right)\cos^2\alpha = 1$

$\Downarrow \qquad \Downarrow$

$1 \qquad 1$

If these two values becomes 1 then identity $\sin^2\alpha + \cos^2\alpha = 1$ will be obtained.

For that: $\sin\alpha$ should be $\frac{4}{5}$ & $\cos\alpha$ should be $\frac{3}{5}$

$\Rightarrow \sin^2\alpha = \frac{16}{25}$ & $\cos^2\alpha = \frac{9}{25}$

From these value we can see sum of $\sin^2\alpha$ & $\cos^2\alpha$ is 1. Hence our an option astive

hence $\sin\alpha = \frac{4}{5}$ and $\cos\alpha = \frac{3}{5}$

$10\sin\alpha + 15\cos\alpha = 10 \times \frac{4}{5} + 15 \times \frac{3}{5}$

$= 8 + 9 = 17$

Ex 34. If $\sin\theta + \sin^2\theta = 1$ then find $\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta + 4 = ?$

Sol. $\sin\theta + \sin^2\theta = 1 \Rightarrow \sin\theta = 1 - \sin^2\theta$

$\Rightarrow \sin\theta = \cos^2\theta$ (put this in question)

$\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta + 4$

$= (\cos^2\theta)^6 + 3(\cos^2\theta)^5 + 3(\cos^2\theta)^4 + (\cos^2\theta)^3 + 4$

$= \sin^6\theta + 3\sin^5\theta + 3\sin^4\theta + \sin^3\theta + 4$

$= \sin^6\theta + 3\sin^2\theta.\sin\theta(\sin^2\theta + \sin\theta) + \sin^3\theta + 4$

$= \left(\sin^2\theta + \sin\theta\right)^3 + 4 = 1 + 4 = 5$

Ex 35. If $\sin\theta + \sin^2\theta + \sin^3\theta = 1$ find $\cos^6\theta - 4\cos^4\theta + 8\cos^2\theta$.

Sol. $\sin\theta + \sin^2\theta + \sin^3\theta = 1$

$\Rightarrow \sin\theta + \sin^3\theta = 1 - \sin^2\theta$

$\Rightarrow \sin\theta\,(1 + \sin^2\theta) = \cos^2\theta$

$\Rightarrow \sin\theta\,(1 + 1 - \cos^2\theta) = \cos^2\theta$

$\Rightarrow \sin\theta\,(2 - \cos^2\theta) = \cos^2\theta$

squaring both sides

$\Rightarrow \sin^2\theta\,(2 - \cos^2\theta)^2 = \cos^4\theta$

$\Rightarrow \sin^2\theta\,(4 + \cos^4\theta - 4\cos^2\theta) = \cos^4\theta$

$\Rightarrow (1 - \cos^2\theta)\,(4 + \cos^4\theta - 4\cos^2\theta) = \cos^4\theta$

$4 + \cos^4\theta - 4\cos^2\theta - 4\cos^2\theta - \cos^6\theta + 4\cos^4\theta = \cos^4\theta$

$\Rightarrow 4 - 8\cos^2\theta - \cos^6\theta + 4\cos^4\theta = 0$

$\Rightarrow \cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = 4$

Question based on Identity 2:

(A) $\sec^2\theta = 1 + \tan^2\theta$ or $\sec^2\theta - \tan^2\theta = 1$

Ex 36. If $\sec^2\theta + \tan^2\theta = 7$ find $\sin\theta$, $\cos\theta$ & $\tan\theta$ (θ in 1st qudrant).

Sol. $\sec^2\theta + \tan^2\theta = 7$

$\sec^2\theta - \tan^2\theta = 1$ (from Identity)

Adding above both equations:

$\Rightarrow 2\sec^2\theta = 8 \Rightarrow \sec^2\theta = 4 \Rightarrow \sec\theta = 2 = \sec 60^\circ$

$\theta = 60^\circ$

$\Rightarrow \cos\theta = \cos 60^\circ = \frac{1}{2}$

$\Rightarrow \sin\theta = \sin 60^\circ = \frac{\sqrt{3}}{2}$

$\Rightarrow \tan\theta = \tan 60^\circ = \sqrt{3}$

Ex 37. If $2\sec^2\theta + \tan^2\theta = 17$, then find $\cot\theta = ?$

Sol. $2\sec^2\theta + \tan^2\theta = 17$

$\Rightarrow 2(1 + \tan^2\theta) + \tan^2\theta = 17$

$\Rightarrow 3\tan^2\theta = 17 - 2 = 15$

$\Rightarrow \tan^2\theta = 5$

$\Rightarrow \tan\theta = \sqrt{5}$

$\Rightarrow \cot\theta = \frac{1}{\tan\theta} = \frac{1}{\sqrt{5}}$

Ex 38. If $\tan^2\alpha = 1 + 2\tan^2\beta$, then $\sqrt{2}\cos\alpha - \cos\beta = ?$

Sol. $\tan^2\alpha = 1 + 2\tan^2\beta$ (Using Identity)

$\Rightarrow \sec^2\alpha - 1 = 1 + 2(\sec^2\beta - 1)$

$\Rightarrow \sec^2\alpha - 1 = 1 + 2\sec^2\beta - 2$

$\Rightarrow \sec^2\alpha - 1 = 2\sec^2\beta - 1$

$\Rightarrow \sec^2\alpha = 2\sec^2\beta$

$\Rightarrow \sec\alpha = \sqrt{2}\sec\beta$

$\Rightarrow \dfrac{1}{\cos\alpha} = \sqrt{2}\left(\dfrac{1}{\cos\beta}\right)$

$\Rightarrow \cos\beta = \sqrt{2}\cos\alpha$

$\Rightarrow \sqrt{2}\cos\alpha - \cos\beta = 0$

Method 2:

$\alpha = 45^o$ & $\beta = 0^o$ satisfies $\tan^2\alpha = 1 + 2\tan^2\beta$

put $\alpha = 45^o$ & $\beta = 0^o$ in $\sqrt{2}\cos\alpha - \cos\beta$

$= \sqrt{2}\cos 45^o - \cos 0^o = 1 - 1 = 0$

Ex 39. If $\tan^2\theta + \cot^2\theta = 14$, then find $\sec\theta \,.\, \text{cosec}\theta = ?$

Sol. $\tan^2\theta + \cot^2\theta = 14$ (adding 2 both sides)

$\Rightarrow \tan^2\theta + \cot^2\theta + 2 = 16$

$\Rightarrow (\tan\theta + \cot\theta)^2 = 16$

$\Rightarrow \tan\theta + \cot\theta = 4$

$\Rightarrow \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} = 4 \Rightarrow \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta.\cos\theta} = 4$

$\Rightarrow \dfrac{1}{\sin\theta.\cos\theta} = 4 \Rightarrow \sec\theta.\text{cosec}\theta = 4$

Method 2:

$\tan^2\theta + \cot^2\theta = 14$

$\Rightarrow \sec^2\theta - 1 + \text{cosec}^2\theta - 1 = 14$

$\Rightarrow \sec^2\theta + \text{cosec}^2\theta = 16$

$\Rightarrow \dfrac{1}{\cos^2\theta} + \dfrac{1}{\sin^2\theta} = 16$

$\Rightarrow \dfrac{\sin^2\theta + \cos^2\theta}{\sin^2\theta\cos^2\theta} = 16$

$\Rightarrow \dfrac{1}{\sin^2\theta\cos^2\theta} = 16$

$\Rightarrow \dfrac{1}{\sin\theta\cos\theta} = 4$

$\Rightarrow \sec\theta.\text{cosec}\theta = 4$

(B). $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\Rightarrow \sec\theta + \tan\theta = \dfrac{1}{\sec\theta - \tan\theta}$ or

$\Rightarrow \sec\theta - \tan\theta = \dfrac{1}{\sec\theta + \tan\theta}$

$\sec\theta + \tan\theta = x$, then $\sec\theta - \tan\theta = \dfrac{1}{x}$

On Addition **on subtraction**

$\Rightarrow 2\sec\theta = x + \dfrac{1}{x}$ & $\Rightarrow 2\tan\theta = x - \dfrac{1}{x}$

$\Rightarrow \sec\theta = \dfrac{x^2+1}{2x}$ & $\Rightarrow \tan\theta = \dfrac{x^2-1}{2x}$

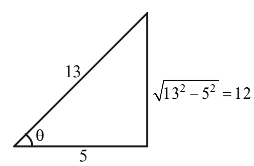
Ex 40. If $\sec\theta + \tan\theta = 5$, then find $\sin\theta = ?$

Sol. $\because \sec\theta + \tan\theta = 5$

$\therefore \sec\theta - \tan\theta = \dfrac{1}{5}$ **(Adding)**

$2\sec\theta = \dfrac{26}{5} \Rightarrow \sec\theta = \dfrac{13}{5} = \dfrac{h}{b}$

hence, $\sin\theta = \dfrac{12}{13}$



Ex 41. If $\tan^2\theta = 1 - e^2$, then find $\sec\theta + \tan^3\theta.\text{cosec}\theta = ?$

(a) $(1-e^2)^{3/2}$ (b) $(1-e^2)^{1/2}$

(c) $(2-e^2)^{3/2}$ (d) $(2+e^2)^{1/2}$

Sol. $\tan^2\theta = 1 - e^2$

$\Rightarrow \sec\theta + \tan^3\theta \,.\, \text{cosec}\,\theta = \sec\theta + \dfrac{\sin^3\theta}{\cos^3\theta}.\text{cosec}\theta$

$= \sec\theta + \dfrac{\sin^2\theta}{\cos^2\theta}.\dfrac{\sin\theta}{\cos\theta}.\text{cosec}\theta$

$= \sec\theta + \tan^2\theta.\sec\theta \quad (\because \sin\theta.\text{cosec}\theta = 1)$

$= \sec\theta\,[1 + \tan^2\theta]\ (\because \sec\theta = \sqrt{1+\tan^2\theta})$

$= \sqrt{1+\tan^2\theta}\,.[1 + \tan^2\theta]$

$= (1 + \tan^2\theta)^{3/2}$

$= (1 + 1 - e^2)^{3/2}$

$= (2 - e^2)^{3/2}$

(C). $a\sec\theta + b\tan\theta = c$

$b\sec\theta + a\tan\theta = d$

$a^2 - b^2 = c^2 - d^2$ (subtract after squaring)

OR

$a\sec\theta - b\tan\theta = c$

$b\sec\theta - a\tan\theta = d$ **or** $a\tan\theta - b\sec\theta = d$

$a^2 - b^2 = c^2 - d^2$ (subtract after squaring)

Proof:

$a^2\sec^2\theta + b^2\tan^2\theta + 2ab\sec\theta.\tan\theta - b^2\sec^2\theta - a^2\tan^2\theta - 2ab\sec\theta.\tan\theta = c^2 - d^2$

$\Rightarrow a^2(\sec^2\theta - \tan^2\theta) - b^2(\sec^2\theta - \tan^2\theta) = c^2 - d^2$

$\Rightarrow a^2 - b^2 = c^2 - d^2$

Ex 42. If $p\sec\theta - q\tan\theta = 10$ & $p\tan\theta - q\sec\theta = 8$. find $p^2 - q^2 + 4$.

Sol. $p\sec\theta - q\tan\theta = 10$

$p\tan\theta - q\sec\theta = 8$ (Using above identity)

$\Rightarrow p^2 - q^2 = 10^2 - 8^2$

$\Rightarrow p^2 - q^2 = 36$ (add 4 to both side)

$\Rightarrow p^2 - q^2 + 4 = 40$

Ex 43. If $\sec\theta = a + \frac{1}{4a}$, find $\frac{1}{\cos\theta} + \tan\theta$.

(i) a (ii) $2a$
(iii) $3a$ (iv) $4a$

Sol. $\sec\theta = a + \frac{1}{4a}$

$$\Rightarrow \tan\theta = \sqrt{\sec^2\theta - 1} = \sqrt{\left(a + \frac{1}{4a}\right)^2 - 1}$$

$$= \sqrt{\left(a^2 + \frac{1}{16a^2} + \frac{1}{2}\right) - 1} = \sqrt{a^2 + \frac{1}{16a^2} - \frac{1}{2}}$$

$$= \sqrt{\left(a - \frac{1}{4a}\right)^2} = a - \frac{1}{4a}$$

$$\Rightarrow \sec\theta + \tan\theta = a + \frac{1}{4a} + a - \frac{1}{4a} = 2a$$

Method: 2

put $a = 1$, then, $\sec\theta = 1 + \frac{1}{4} = \frac{5}{4}$

$$\sec\theta + \tan\theta = \frac{5}{4} + \frac{3}{4} = \frac{8}{4} = 2$$

Put $a = 1$ in all option hence option(ii) is correct

Method -3

$2\sec\theta = 2a + \frac{1}{2a}$ it means $\sec\theta + \tan\theta = 2a$

$4\sec\theta - \tan\theta = \frac{1}{2\theta}$

Ex 44. $\frac{\tan A - \sec A + 1}{\tan A + \sec A - 1} = ?$

(i) $\frac{1 + \sin A}{\cos A}$ (ii) $\frac{1 - \sin A}{\cos A}$

(iii) $\frac{1 + \cos A}{\sin A}$ (iv) $\frac{1 - \cos A}{\sin A}$

Sol. $\frac{\tan A - \sec A + 1}{\tan A + \sec A - 1}$

$$= \frac{\tan A - \sec A + (\sec^2 A - \tan^2 A)}{\tan A + \sec A - 1}$$

$$= \frac{(\sec A - \tan A) + (\sec A + \tan A)(\sec A - \tan A)}{\tan A + \sec A - 1}$$

$$= \frac{(\sec A - \tan A)(\sec A + \tan A - 1)}{\tan A + \sec A - 1} = \sec A - \tan A$$

$$= \frac{1}{\cos A} - \frac{\sin A}{\cos A} = \frac{1 - \sin A}{\cos A}$$

Question based on Identity 3: $\operatorname{cosec}^2\theta = \cot^2\theta + 1$ (same as Identity 2)

Ex 45. If $\operatorname{cosec}^2\theta + \cot^2\theta = 7$ find $\sin\theta$ & $\cos\theta$

Sol. $\operatorname{cosec}^2\theta + \cot^2\theta = 7$ (given)

$\operatorname{cosec}^2\theta - \cot^2\theta = 1$ (identity 3)

on adding $\Rightarrow 2\operatorname{cosec}^2\theta = 8$

$\Rightarrow \operatorname{cosec}^2\theta = 4$

$\Rightarrow \operatorname{cosec}\theta = 2$ then, $\theta = 30^o$

$\Rightarrow \sin 30^o = \frac{1}{2}$ & $\cos 30^o = \frac{\sqrt{3}}{2}$

Ex 46 . If $\text{cosec}^4\alpha = 17 + \cot^4\alpha$, then find $\sin\alpha$, $\cos\alpha$.

Sol. $\text{cosec}^4\alpha = 17 + \cot^4\alpha \Rightarrow \text{cosec}^4\alpha - \cot^4\alpha = 17$

$\Rightarrow (\text{cosec}^2\alpha + \cot^2\alpha).(\text{cosec}^2\alpha - \cot^2\alpha) = 17$

$\Rightarrow \text{cosec}^2\alpha + \cot^2\alpha = 17$ .....(i)

$\text{cosec}^2\alpha - \cot^2\alpha = 1$ .....(ii)

$2\text{cosec}^2\alpha = 18 \Rightarrow \text{cosec}^2\alpha = 9$

[add equation (i) & (ii)]

$$\sin^2\alpha = \frac{1}{9} \Rightarrow \sin\alpha = \frac{1}{3}$$

$$\cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-\frac{1}{9}} = \frac{2\sqrt{2}}{3}$$

(B). $\text{cosec}^2\theta - \cot^2\theta = 1$

$\Rightarrow (\text{cosec}\theta + \cot\theta)(\text{cosec}\theta - \cot\theta) = 1$

$$\Rightarrow \text{cosec}\theta - \cot\theta = \frac{1}{\text{cosec}\theta + \cot\theta}$$

If $\text{cosec}\theta + \cot\theta = x$

then $\text{cosec}\theta - \cot\theta = \dfrac{1}{x}$

Addition & Subtraction

$\Rightarrow 2\text{cosec}\theta = x + \dfrac{1}{x}$ & $2\cot\theta = x - \dfrac{1}{x}$

$\Rightarrow \text{cosec}\theta = \dfrac{x^2+1}{2x}$ & $\cot\theta = \dfrac{x^2-1}{2x}$

Ex 47. If $\text{cosec}\theta + \cot\theta = 7$, then find $\tan\theta$.

Sol. $\because \text{cosec}\theta + \cot\theta = 7$

$\therefore \text{cosec}\theta - \cot\theta = \dfrac{1}{7}$

$\Rightarrow 2\cot\theta = 7 - \dfrac{1}{7}$ (on subtracting)

$$\Rightarrow 2\cot\theta = \frac{48}{7}$$

$$\Rightarrow \cot\theta = \frac{24}{7} \Rightarrow \tan\theta = \frac{7}{24}$$

(C). $a\,\text{cosec}\theta + b\cot\theta = c$

$b\,\text{cosec}\theta + a\cot\theta = d$

$a^2 - b^2 = c^2 - d^2$ (subtract after squaring)

OR

$a\,\text{cosec}\theta - b\cot\theta = c$

$a\cot\theta - b\,\text{cosec}\theta = d$ **or** $b\,\text{cosec}\theta - a\cot\theta = d$

$a^2 - b^2 = c^2 - d^2$ (subtract after squaring)

Proof:

$a^2\text{cosec}^2\theta + b^2\cot^2\theta + 2ab\,\text{cosec}\theta.\cot\theta - b^2\text{cosec}^2\theta - a^2\cot^2\theta - 2ab\,\text{cosec}\theta.\cot\theta = c^2 - d^2$

$\Rightarrow a^2(\text{cosec}^2\theta - \cot^2\theta) - b^2(\text{cosec}^2\theta - \cot^2\theta) = c^2 - d^2$

$\Rightarrow a^2 - b^2 = c^2 - d^2$

Ex 48. If $3\text{cosec}\theta - 7\cot\theta = 3$, then find $3\cot\theta - 7\text{cosec}\theta = ?$

Sol. $3\text{cosec}\theta - 7\cot\theta = 3$

$3\cot\theta - 7\text{cosec}\theta = d$ (let)

Using previous identity ($a^2 - b^2 = c^2 - d^2$)

$\Rightarrow 3^2 - 7^2 = 3^2 - d^2$

$\Rightarrow d^2 = 7^2 = 49$

$\Rightarrow d = \pm 7$

Type – 2

(A)

1. $\sin^2\alpha + \sin^2\beta = 2$ or $\sin\alpha + \sin\beta = 2$

→ $\alpha = \beta = 90^\circ$ ←

2. $\sin^2\alpha + \cos^2\beta = 2$ or $\sin\alpha + \cos\beta = 2$

→ $\alpha = 90^\circ$ & $\beta = 0^\circ$ ←

3. $\cos^2\alpha + \cos^2\beta = 2$ or $\cos\alpha + \cos\beta = 2$

→ $\alpha = \beta = 0^\circ$ ←

(B)

1. $\sin^2\alpha + \sin^2\beta = 0$ when $\alpha = \beta = 0$

2. $\sin^2\alpha + \cos^2\beta = 0$ when $\alpha = 0$ & $\beta = 90^\circ$

3. $\cos^2\alpha + \cos^2\beta = 0$ when $\alpha = \beta = 90^\circ$

Ex 49. $\sin^2\alpha + \cos^2\beta = 2$ then $\tan\left(\dfrac{\alpha+\beta}{2}\right) = ?$

Sol. $\sin^2\alpha + \cos^2\beta = 2 \Rightarrow \alpha = 90^\circ, \beta = 0^\circ$

$$\Rightarrow \tan\left(\frac{\alpha+\beta}{2}\right) = \tan\left(\frac{90^\circ}{2}\right) = \tan 45^\circ = 1$$

Ex 50. If $\sin^2\alpha + \sin^2\beta + \cos^2\gamma = 3$

then find $\tan\left(\frac{\alpha+\beta+\gamma}{4}\right)+\cot\left(\frac{\alpha+\beta+\gamma}{4}\right)$.

Sol. $\sin^2\alpha + \sin^2\beta + \cos^2\gamma = 3$

All $\sin\alpha$, $\sin\beta$ & $\cos\gamma$ should be 1 so

$\alpha = \beta = 90^o$ & $\gamma = 0^o$

$$\tan\left(\frac{\alpha+\beta+\gamma}{4}\right)+\cot\left(\frac{\alpha+\beta+\gamma}{4}\right)$$

$$= \tan\left(\frac{180^o}{4}\right)+\cot\left(\frac{180^o}{4}\right)$$

$= \tan45^o + \cot45^o$

$= 1 + 1 = 2$

Ex 51. If $\sin^2\alpha + \cos^2\beta = 2$, then find $\sin\left(\frac{2\alpha+\beta}{3}\right)$.

(a) $\sin\frac{\alpha}{3}$ (b) $\cos\frac{2\alpha}{3}$

(c) $\cos\frac{\alpha}{3}$ (d) none of these

Sol. $\sin^2\alpha + \cos^2\beta = 2 \Rightarrow \alpha = 90^o$ & $\beta = 0^o$

$$\Rightarrow \sin\left(\frac{2\alpha+\beta}{3}\right) = \sin\left(\frac{180^o}{3}\right) = \sin 60^o$$

$\sin60^o = \cos30^o = \cos\frac{\alpha}{3}$

Formula :

1. $\sin(A + B) = \sin A \cos B + \cos A \sin B$

2. $\sin(A - B) = \sin A \cos B - \cos A \sin B$

3. $\cos(A + B) = \cos A \cos B - \sin A \sin B$

4. $\cos(A - B) = \cos A \cos B + \sin A \sin B$

5. $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

6. $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

Put A = B in formula 1, 3 & 5 respectively

7. $\sin 2A = \boxed{2\sin A \cos A}$

$$= \frac{2\sin A}{\cos A}.\cos^2 A = \frac{2\tan A}{\sec^2 A} = \boxed{\frac{2\tan A}{1+\tan^2 A}}$$

8. $\cos 2A = \boxed{\cos^2 A - \sin^2 A}$

$= \boxed{2\cos^2 A - 1} = \boxed{1 - 2\sin^2 A}$

$$= \cos^2 A\left(1 - \frac{\sin^2 A}{\cos^2 A}\right) = \boxed{\frac{1-\tan^2 A}{1+\tan^2 A}}$$

9. $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

Adding & subtracting formula (1) & (2) and (3) & (4)

10. $2\sin A \cos B = \sin(A + B) + \sin(A - B)$

11. $2\cos A \sin B = \sin(A + B) - \sin(A - B)$

12. $2\cos A \cos B = \cos(A + B) + \cos(A - B)$

13. $2\sin A \sin B = \cos(A - B) - \cos(A + B)$

let $A + B = C$ & $A - B = D$

then $A = \frac{C+D}{2}$ & $B = \frac{C-D}{2}$, put these values in formula (10), (11), (12) & (13)

14. $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)$

15. $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right).\sin\left(\frac{C-D}{2}\right)$

16. $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)$

17. $\cos C - \cos D = 2\sin\left(\frac{C+D}{2}\right).\sin\left(\frac{D-C}{2}\right)$

18. $\sin^2 A - \sin^2 B = \sin(A + B).\sin(A - B)$

19. $\cos^2 A - \sin^2 B = \cos(A + B).\cos(A - B)$

20. $\sin 3A = 3\sin A - 4\sin^3 A$

21. $\cos 3A = 4\cos^3 A - 3\cos A$

22. $\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

Questions based on formulas :

Ex 52. Find $\frac{1}{\sin 10^o} - \frac{\sqrt{3}}{\cos 10^o}$.

Sol. $\frac{1}{\sin 10^o} - \frac{\sqrt{3}}{\cos 10^o} = \frac{\cos 10^o - \sqrt{3}\sin 10^o}{\sin 10^o \cos 10^o}$

$$= \frac{2\left(\frac{1}{2}\cos 10^o - \frac{\sqrt{3}}{2}\sin 10^o\right)}{\sin 10^o \cos 10^o}$$

$$= \frac{2\left[\sin 30^o \cos 10^o - \cos 30^o \sin 10^o\right]}{\sin 10^o \cos 10^o}$$

$$= \frac{2.2\sin\left(30^o - 10^o\right)}{2\sin 10^o \cos 10^o} = \frac{4\sin 20^o}{\sin 20^o} = 4$$

Ex 53. If $\cos x = \frac{2\cos y - 1}{2 - \cos y}$ then $\tan\frac{x}{2}.\cot\frac{y}{2} = ?$

Sol. $\Rightarrow \frac{\cos x}{1} = \frac{2\cos y - 1}{2 - \cos y}$

Applying Component-Dividend rule

$$\Rightarrow \frac{\cos x + 1}{\cos x - 1} = \frac{2\cos y - 1 + 2 - \cos y}{2\cos y - 1 - 2 + \cos y}$$

$$\Rightarrow \frac{\cos x + 1}{\cos x - 1} = \frac{1 + \cos y}{3\left(\cos y - 1\right)}$$

$$\Rightarrow \frac{2\cos^2\frac{x}{2} - 1 + 1}{1 - 2\sin^2\frac{x}{2} - 1} = \frac{1 + 2\cos^2\frac{y}{2} - 1}{3\left(1 - 2\sin^2\frac{y}{2} - 1\right)}$$

$$\Rightarrow \frac{2\cos^2\frac{x}{2}}{-2\sin^2\frac{x}{2}} = \frac{2\cos^2\frac{y}{2}}{3\left(-2\sin^2\frac{y}{2}\right)}$$

$$\Rightarrow \cot^2\frac{x}{2} = \frac{1}{3}\cot^2\frac{y}{2}$$

$$\Rightarrow 3 = \tan^2\frac{x}{2}.\cot^2\frac{y}{2}$$

$$\Rightarrow \tan\frac{x}{2}.\cot\frac{y}{2} = \sqrt{3}$$

Method : 2

$$\text{Cos } x = \frac{2\cos y - 1}{2 - \cos y}$$

put y = 90°

$$\cos x = \frac{2\times 0 - 1}{2 - 0} = \frac{-1}{2} = \cos 120^o$$

x= 120°

hence $\tan\frac{x}{2}.\cot\frac{y}{2}$

= tan60°. cot45°

$= \sqrt{3}.1 = \sqrt{3}$

Ex 54. Find $\frac{1}{2}\text{cosec}10^o - 2\sin 70^o$.

Sol. $\frac{1}{2}\text{cosec}10^o - 2\sin 70^o = \frac{1}{2\sin 10^o} - 2\sin 70^o$

$$= \frac{1 - 2.2\sin 70^o \sin 10^o}{2\sin 10^o} = \frac{1 - 2\left[\cos 60^o - \cos 80^o\right]}{2\sin 10^o}$$

[Using formula: 2sinA.sinB = cos(A – B) – cos(A + B)]

$$= \frac{1 - 2\left(\frac{1}{2} - \cos 80^o\right)}{2\sin(90^o - 80^o)} = \frac{1 - 1 + 2\cos 80^o}{2\sin 10^o}$$

$$= \frac{2\cos 80^o}{2\cos 80^o} = 1$$

Ex 55. Find $\cot 70^o + 4\cos 70^o$.

Sol. $\cot 70^o + 4\cos 70^o = \tan 20^o + 4\sin 20^o$

$$= \frac{\sin 20^o}{\cos 20^o} + 4\sin 20^o$$

$$= \frac{\sin 20^o + 4\sin 20^o.\cos 20^o}{\cos 20^o} = \frac{\sin 20^o + 2\sin 40^o}{\cos 20^o}$$

$$= \frac{\left(\sin 20^o + \sin 40^o\right) + \sin 40^o}{\cos 20^o}$$

$$= \frac{\left(2\sin 30^o.\cos 10^o\right) + \cos 50^o}{\cos 20^o}$$

$$= \frac{\cos 10^o + \cos 50^o}{\cos 20^o} = \frac{2\cos 30^o.\cos 20^o}{\cos 20^o} = \sqrt{3}$$

Ex 56. Find $4\cos 20^\circ - \sqrt{3}\cot 20^\circ$.

Sol. $4\cos 20^\circ - \sqrt{3}\cot 20^\circ = 4\cos 20^\circ - \sqrt{3}.\dfrac{\cos 20^\circ}{\sin 20^\circ}$

$$= \frac{4\sin 20^\circ \cos 20^\circ - \sqrt{3}\cos 20^\circ}{\sin 20^\circ}$$

$$= \frac{2.2\sin 20^\circ \cos 20^\circ - \sqrt{3}\cos 20^\circ}{\sin 20^\circ}$$

$$= \frac{2\sin 40^\circ - \sqrt{3}\cos 20^\circ}{\sin 20^\circ}$$

$$= \frac{2\left[\sin 40^\circ - \frac{\sqrt{3}}{2}\cos 20^\circ\right]}{\sin 20^\circ}$$

$$= \frac{2[\sin 40^\circ - \sin 60^\circ \cos 20^\circ]}{\sin 20^\circ}$$

$$= \frac{2\sin 40^\circ - 2\sin 60^\circ \cos 20^\circ}{\sin 20^\circ}$$

$$= \frac{2\sin 40^\circ - \sin 80^\circ - \sin 40^\circ}{\sin 20^\circ}$$

$$= \frac{\sin 40^\circ - \sin 80^\circ}{\sin 20^\circ} = \frac{2\cos 60^\circ.(-\sin 20^\circ)}{\sin 20^\circ} = -1$$

Type – 3

Advanced Identity :

(A). If $A + B + C = 180^\circ$ or π (or ABC is a triangle)

$\Rightarrow$ $A + B = 180^\circ - C$ (taking tan both side)

$\Rightarrow$ $\tan(A + B) = \tan(180^\circ - C)$

$\Rightarrow$ $\dfrac{\tan A + \tan B}{1 - \tan A.\tan B} = -\tan C$

$\Rightarrow$ $\tan A + \tan B = -\tan C + \tan A.\tan B.\tan C$

1. $\boxed{\tan A + \tan B + \tan C = \tan A.\tan B.\tan C}$

$\Rightarrow$ $\dfrac{1}{\cot A} + \dfrac{1}{\cot B} + \dfrac{1}{\cot C} = \dfrac{1}{\cot A.\cot B.\cot C}$

$$\frac{\cot B.\cot C + \cot C.\cot A + \cot A.\cot B}{\cot A.\cot B.\cot C}$$

$$= \frac{1}{\cot A.\cot B.\cot C}$$

2. $\boxed{\cot A.\cot B + \cot B.\cot C + \cot C.\cot A = 1}$

Ex 57. $\tan 20^\circ + \tan 72^\circ + \tan 88^\circ = ?$

Sol. Sum of Angle $= 20^\circ + 72^\circ + 88^\circ = 180^\circ$

So, $\tan 20^\circ + \tan 72^\circ + \tan 88^\circ$

$= \tan 20^\circ.\tan 72^\circ.\tan 88^\circ$

Ex 58. Find $\tan(2x - y) + \tan(y - x) + \tan(\pi - x)$

Sol. Sum of Angle $= 2x - y + y - x + \pi - x = \pi$

So, $\tan(2x - y) + \tan(y - x) + \tan(\pi - x)$

$= \tan(2x - y).\tan(y - x).\tan(\pi - x)$

Ex 59. In a triangle ABC tanA+tanB+tanC = 6 and tanA. tan B = 3, then find the nature of triangle.

Sol. tanA + tanB + tanC = 6 = tanA.tanB.tanC

$\Rightarrow$ 6 = 3.tan C $\Rightarrow$ tan C = 2

$\because$ tan C is +ve, so angle C will be in Ist quadrant or IIIrd quadrant. In IIIrd quardrant any angle is greater than 180° so angle C will be in Ist quadrant and will be acute ($\angle C < 90^\circ$).

for $\angle A$ and $\angle B$

$\Rightarrow$ tanA.tanB = 3

(–ve) & (–ve)

or (+ve) & (+ve)

In this case, both tan A & tan B will be positve or both will be negative.

But when both will be negative it means $\angle A$ and $\angle B$ will be in IInd quadrant or in IVth quadrant which is not possible in a triangle. Hence both tan A & tan B will be positve and will be in 1st quadrant ($\angle A$ & $\angle B < 90^\circ$)

$\Rightarrow$ $\angle A, \angle B, \angle C$ are acute angle, so triangle ABC will be acute angle triangle.

(B). If $A + B + C = 90^\circ$

1. $\boxed{\cot A + \cot B + \cot C = \cot A.\cot B.\cot C}$

2. $\boxed{\tan A \tan B + \tan B \tan C + \tan C \tan A = 1}$

Ex 60. In a quadrilateral $\angle D = 270^\circ$ find tan A.tanB + tanB.tanC + tanC.tan A .

Sol. In a quadrilateral $\angle A + \angle B + \angle C + \angle D = 360^\circ$

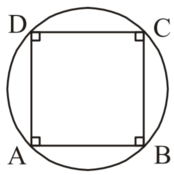
$\angle A + \angle B + \angle C = 360^\circ - 270^\circ$

$\angle A + \angle B + \angle C = 90^\circ$

hence tanA.tanB + tanB.tanC + tanC.tanA = 1

Ex 61. In a cyclic quadrilateral ABCD, then find cosA + cosB + cosC + cosD = ?

Sol. In a cyclic Quadrilateral,



$\angle A + \angle C = 180^\circ$

$\angle B + \angle D = 180^\circ$

$\Rightarrow$ $\cos A + \cos B + \cos(180^\circ - A) + \cos(180^\circ - B)$

$= \cos A + \cos B - \cos A - \cos B = 0$

Ex 62. In a triangle ABC, find

$$\tan\frac{A}{2}.\tan\frac{B}{2} + \tan\frac{B}{2}.\tan\frac{C}{2} + \tan\frac{C}{2}.\tan\frac{A}{2}.$$

Sol. $\angle A + \angle B + \angle C = 180^\circ$

$$\Rightarrow \frac{\angle A}{2} + \frac{\angle B}{2} + \frac{\angle C}{2} = \frac{180^\circ}{2} = 90^\circ$$

[Using Identity B.(2)]

$$\Rightarrow \tan\frac{A}{2}.\tan\frac{B}{2} + \tan\frac{B}{2}.\tan\frac{C}{2} + \tan\frac{C}{2}.\tan\frac{A}{2} = 1$$

(C). If $A + B = 45^\circ$ or 225°

taking tan both sides

$\Rightarrow$ $\tan(A + B) = \tan 45^\circ$

$\Rightarrow$ $\dfrac{\tan A + \tan B}{1 - \tan A.\tan B} = 1$

$\Rightarrow$ tanA + tanB = 1 – tanA.tanB

$\Rightarrow$ tanA + tanB + tanA.tanB = 1

Adding 1 both side

$\Rightarrow$ (1 + tanA) + tanB(1 + tanA) = 1 + 1

$\Rightarrow$ (1 + tanA).(1 + tanB) = 2

Similarly we can prove,

$\Rightarrow$ (1 – cotA).(1 – cotB) = 2

(1). $\boxed{(1 + \tan A)(1 + \tan B) = 2}$

(2). $\boxed{(1 - \cot A)(1 - \cot B) = 2}$

or

$\boxed{(\cot A - 1)(\cot B - 1) = 2}$

Ex 63. Find $(1 + \tan 2^\circ)(1 + \tan 43^\circ)$.

Sol. $\angle A + \angle B = 2^\circ + 43^\circ = 45^\circ$

hence, $(1 + \tan 2^\circ)(1 + \tan 43^\circ) = 2$

Ex 64. Find $(1 + \tan 1^\circ).(1 + \tan 2^\circ) \,..........\, (1 + \tan 45^\circ) = 2^n$ find the value of n.

Sol. $(1 + \tan 1^\circ)(1 + \tan 2^\circ) + ...(1 + \tan 44^\circ)(1 + \tan 45^\circ) = 2^n$

$\Rightarrow$ $(1 + \tan 1^\circ).(1 + \tan 44^\circ) = 2$

$\Rightarrow$ $(1 + \tan 2^\circ).(1 + \tan 43^\circ) = 2$

Similarly till 44°, total there will be 44 terms and product of two terms will be 2 & total pair of such term is 22.

last term $(1 + \tan 45^\circ)$ will also be 2. So,

$(1 + \tan 1^\circ)(1 + \tan 2^\circ) + ...(1 + \tan 44^\circ)(1 + \tan 45^\circ)$

$= 2^{22}.2 = 2^{23} = 2^n$

on comparing power $\Rightarrow n = 23$

Ex 65. Find

$$\left[1 + \tan\left(22\frac{1}{2}^\circ + x - y\right)\right]\left[1 + \tan\left(22\frac{1}{2}^\circ + y - x\right)\right].$$

Sol. $\angle A + \angle B = 22\frac{1}{2}^\circ + x - y + 22\frac{1}{2}^\circ + y - x = 45^\circ$

$$\Rightarrow \left[1 + \tan\left(22\frac{1}{2}^\circ + x - y\right)\right]\left[1 + \tan\left(22\frac{1}{2}^\circ + y - x\right)\right] = 2$$

Ex 66. If $A + B = 45^\circ$, then $\dfrac{\tan A}{1 - \tan A}.\dfrac{\tan B}{1 - \tan B} = ?$

Sol. $$\frac{\tan A}{1 - \tan A}.\frac{\tan B}{1 - \tan B} = \frac{\frac{1}{\cot A}}{1 - \frac{1}{\cot A}}.\frac{\frac{1}{\cot B}}{1 - \frac{1}{\cot B}}$$

$$= \frac{1}{\cot A - 1}.\frac{1}{\cot B - 1} = \frac{1}{(\cot A - 1)(\cot B - 1)} = \frac{1}{2}$$

(D). If $A - B = 45^\circ$ or 225°

we put in previous identity B = – B then we obtain

1. $\boxed{(1 + \tan A)(1 - \tan B) = 2}$

2. $\boxed{(1 - \cot A)(1 + \cot B) = 2}$

(E). Identity

(1). $\sin\theta \,.\, \sin(60^o - \theta).\, \sin(60^o + \theta) = \frac{1}{4}\sin 3\theta$

(2). $\cos\theta \,.\, \cos(60^o - \theta).\, \cos(60^o + \theta) = \frac{1}{4}\cos 3\theta$

(3). $\tan\theta \,.\, \tan(60^o - \theta).\, \tan(60^o + \theta) = \tan 3\theta$

(4). $\cot\theta \,.\, \cot(60^o - \theta).\, \cot(60^o + \theta) = \cot 3\theta$

Proof of (1):

$\Rightarrow \sin\theta.\, \sin(60^o - \theta).\, \sin(60^o + \theta)$

$= \frac{1}{2}\left(\sin\theta.2\sin(60^o + \theta).\sin(60^o - \theta)\right)$

$= \frac{1}{2}\sin\theta.[\cos(60^o + \theta - 60^o + \theta) - \cos(60^o + \theta + 60^o - \theta)]$

$= \frac{1}{2}\sin\theta.(\cos 2\theta - \cos 120^o)$

$= \frac{1}{2}\sin\theta.\left(1 - 2\sin^2\theta + \frac{1}{2}\right)$

$= \frac{1}{2}\sin\theta.\left(\frac{3}{2} - 2\sin^2\theta\right) = \frac{1}{2}\sin\theta.\left(\frac{3 - 4\sin^2\theta}{2}\right)$

$= \frac{1}{4}\left(3\sin\theta. - 4\sin^3\theta\right) = \frac{1}{4}\sin 3\theta$

Question based on above four Identity:

Ex 67. Find $\sin 20^o \sin 40^o \sin 60^o \sin 80^o$.

Sol. put $\theta = 20^o$ in identity 1

$\Rightarrow \sin\theta.\, \sin(60^o - \theta).\, \sin(60^o + \theta) = \frac{1}{4}\sin 3\theta$

$\Rightarrow \sin 20^o.\, \sin(60^o - 20^o).\, \sin(60^o + 20^o) = \frac{1}{4}\sin 3\times 20$

$\Rightarrow \sin 20^o.\, \sin 40^o.\, \sin 80^o = \frac{1}{4}\sin 60^o$

hence, $\sin 60^o.\, \sin 20^o \sin 40^o \sin 80^o$

$= \sin 60^o.\left(\frac{1}{4}\sin 60^o\right)$

$= \frac{\sqrt{3}}{2}\times\frac{1}{4}\times\sin 60^o = \frac{\sqrt{3}}{2}\times\frac{1}{4}\times\frac{\sqrt{3}}{2} = \frac{3}{16}$

Ex 68. Find $1 - \sin 10^o \sin 50^o \sin 70^o$.

Sol. $1 - \sin 10^o \sin 50^o \sin 70^o$

$= 1 - \left(\frac{1}{4}\sin 3\times 10^o\right)$

$= 1 - \frac{1}{4}\times\sin 30^o = 1 - \frac{1}{4}\times\frac{1}{2} = \frac{7}{8}$

Ex 69. Find $\sin 12^o \sin 48^o \sin 54^o$.

Sol. $\underset{\Downarrow}{\sin 12^o \sin 48^o \sin 72^o}.\left(\frac{\sin 54^o}{\sin 72^o}\right)$

$= \left(\frac{1}{4}\times\sin 3\times 12^o\right)\times\frac{\cos 36^o}{2\sin 36^o\cos 36^o}$

$= \frac{1}{4}\times\sin 36^o\times\frac{1}{2\sin 36^o} = \frac{1}{8}$

Ex 70. Find $\sin 6^o \sin 42^o \sin 66^o \sin 78^o$.

Sol. $\underset{\Downarrow}{\sin 6^o \sin 54^o \sin 66^o}\,.\,\frac{\sin 42^o \sin 78^o}{\sin 54^o}$

$= \frac{\left(\frac{1}{4}\sin 18^o\right).\sin 42^o.\sin 78^o}{\sin 54^o}$

$= \frac{1}{4}.\frac{1}{4}\sin 54^o.\frac{1}{\sin 54^o} = \frac{1}{16}$

maximum & minimum value of trigonometric :

	max^m	**min^m**
$\sin\theta$ or $\cos\theta$	1	-1
$\sin^2\theta$ or $\cos^2\theta$	1	0
$\sin^3\theta$ or $\cos^3\theta$	1	-1
$\tan\theta$ or $\cot\theta$	∞	$-\infty$
$\tan^2\theta$ or $\cot^2\theta$	∞	0
$\tan^3\theta$ or $\cot^3\theta$	∞	$-\infty$
$\sec\theta$ or $\mathrm{cosec}\,\theta$	∞	$-\infty$
$\sec^2\theta$ or $\mathrm{cosec}^2\theta$	∞	1
$\sec^3\theta$ or $\mathrm{cosec}^3\theta$	∞	$-\infty$

Note: The value of $\sec\theta$ & $\mathrm{cosec}\,\theta$ can be anything $-\infty$ to ∞ but value of $\sec\theta$ & $\mathrm{cosec}\,\theta$ can't be between -1 & 1 but it can be -1 & 1.

Ex 71. Which one is incorrect ?

(a) $\sin\theta = \frac{3}{4}$ (b) $\cos\theta = \frac{3}{4}$

(c) $\sec\theta = \frac{1}{2}$ (d) $\tan\theta = 9$

Sol. (c) $\sec\theta = \frac{1}{2}$ (it can't be between – 1 & 1)

Type-1

Ex 72. Find max^m & min^m value of $10 + \sin\alpha$.

Sol. max^m value $= 10 + 1 = 11$

min^m value $= 10 - 1 = 9$

Ex 73. Find max^m & min^m value of $10 + \sin\alpha + \cos\beta$.

Sol. max^m value $= 10 + 1 + 1 = 12$

min^m value $= 10 - 1 - 1 = 8$

(α & β are independent to each other)

Ex 74. Find max^m & min^m value of $10 + \sin^2\alpha$.

Sol. max^m value $= 10 + 1 = 11$

min^m value $= 10 + 0 = 10$

Ex 75. Find max^m & min^m value of $10 + \tan^2\alpha$.

Sol. max^m value will not be asked & equal to ∞.

min^m value $= 10 + 0 = 10$

Ex 76. Find max^m & min^m value of $11 + \sec^2\alpha$.

Sol. max^m value will not be asked & equal to ∞

min^m value $= 11 + 1 = 12$

Ex 77. Find max^m & min^m value of $10\sin^2\alpha + 12\cos^2\alpha$.

Sol. here angle is same so value of $\sin^2\alpha$ & $\cos^2\alpha$ can't be directly put.

Note: If $a\sin^2\alpha + b\cos^2\alpha$ is given then out of a & b greater value is max^m and smaller value is min^m

$10\sin^2\alpha + 12\cos^2\alpha$

$= 10\sin^2\alpha + 10\cos^2\alpha + 2\cos^2\alpha$

$= 10(\sin^2\alpha + \cos^2\alpha) + 2\cos^2\alpha$

$= 10 + 2\cos^2\alpha$

$\Rightarrow max^m$ value $= 10 + 2(1) = 12$

$\Rightarrow min^m$ value $= 10 + 0 = 10$

e.g. if $a > b$ | if $a < b$

$max^m = a$ | $max^m = b$

$min^m = b$ | $min^m = a$

Ex 78. Find max^m & min^m value of $32\sin^2\alpha + 43\cos^2\alpha$.

Sol. max^m value $= 43$

min^m value $= 32$

Ex 79. Find max^m & min^m value of $10\sin^2\alpha - 23\cos^2\alpha$.

Sol. max^m value $= 10$

min^m value $= -23$

Ex 80. Find max^m & min^m value of $-11\sin^2\alpha - 3\cos^2\alpha$.

Sol. max^m value $= -3$

min^m value $= -11$

Ex 81. Find max^m & min^m value of $10\sin^2\alpha + 101\cos^2\beta$

Sol. angle are different & independent then seperate max^m & min^m value of $\sin^2\alpha$ & $\cos^2\beta$ can be put.

max^m value $= 10 \times 1 + 101 \times 1 = 111$

min^m value $= 10 \times 0 + 101 \times 0 = 0$

Ex 82. Find max^m & min^m value of $23\sec^2\theta + 20\tan^2\theta$

Sol. max^m value will not be asked & equal to ∞.

$\Rightarrow 23\sec^2\theta + 20\tan^2\theta$

$= 23(1 + \tan^2\theta) + 20\tan^2\theta$

$= 23 + 43\tan^2\theta$

min^m value $= 23 + 0 = 23$

or

$\Rightarrow 23\sec^2\theta + 20(\sec^2\theta - 1)$

$= 43\sec^2\theta - 20$

min^m value $= 43 - 20 = 23$

Note: If $a\sec^2\alpha + b\tan^2\alpha$ is given then coefficient of $\sec^2\alpha$ is value min^m is equal to a.

Ex 83. Find max^m & min^m value of $17\sec^2\theta + 15\tan^2\theta$.

Sol. min^m value $= 17$

Type –2

$a\sin\theta + b\cos\theta$

$\Rightarrow$ max^m value $= \sqrt{a^2 + b^2}$

$\Rightarrow$ min^m value $= -\sqrt{a^2 + b^2}$

Proof:

$a\sin\theta + b\cos\theta$

$$= \sqrt{a^2+b^2}\left[\frac{a}{\sqrt{a^2+b^2}}\sin\theta + \frac{b}{\sqrt{a^2+b^2}}\cos\theta\right]$$

$$= \sqrt{a^2+b^2}\,[\sin(\alpha+\theta)] = \pm\sqrt{a^2+b^2}$$

$$\left(\text{let}, \cos\alpha = \frac{a}{\sqrt{a^2+b^2}} \text{ then } \sin\alpha = \frac{b}{\sqrt{a^2+b^2}}\right)$$

Ex 84. Find max^m & min^m value of $8\sin\theta + 15\cos\theta$.

Sol. max^m value $= \sqrt{8^2 + 15^2} = 17$

min^m value $= -\sqrt{8^2 + 15^2} = -17$

Ex 85. Find max^m & min^m value of $27^{\sin x} . 81^{\cos x}$

Sol. $27^{\sin x} . 81^{\cos x}$

$= (3^3)^{\sin x} . (3^4)^{\cos x}$

$= 3^{3\sin x} . 3^{4\cos x}$

$= 3^{3\sin x + 4\cos x}$

max^m & min^m value of $3\sin x + 4\cos x$

$= \pm\sqrt{a^2 + b^2} = \pm\sqrt{3^2 + 4^2} = \pm 5$

$\Rightarrow$ max^m value of $27^{\sin x} . 81^{\cos x} = 3^5 = 243$

$\Rightarrow$ min^m value of $27^{\sin x} . 81^{\cos x} = 3^{-5} = \dfrac{1}{3^5} = \dfrac{1}{243}$

Ex 86. Find max^m & min^m value of

$$5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3$$

Sol. $5\cos\theta + 3\left(\cos\theta.\cos\dfrac{\pi}{3} - \sin\theta.\sin\dfrac{\pi}{3}\right) + 3$

$= 5\cos\theta + \dfrac{3}{2}\cos\theta - \dfrac{3\sqrt{3}}{2}\sin\theta + 3$

max & min value of $\left(\dfrac{13}{2}\cos\theta - \dfrac{3\sqrt{3}}{2}\sin\theta\right) + 3$

$= \pm\sqrt{\left(\dfrac{13}{2}\right)^2 + \left(\dfrac{-3\sqrt{3}}{2}\right)^2}$

$= \pm\sqrt{\dfrac{169}{4} + \dfrac{27}{4}} = \pm\sqrt{49} = \pm 7$

$\Rightarrow$ max^m value $= +7 + 3 = 10$

$\Rightarrow$ min^m value $= -7 + 3 = -4$

Ex 87. Find max^m & min^m value of $10\cos^2 x - 6\sin x.\cos x + 2\sin^2 x$

Sol. Ist method

$9\cos^2 x - 6\sin x.\cos x + \sin^2 x + (\sin^2 x + \cos^2 x)$

$\Downarrow$ 1

$= (3\cos x - \sin x)^2 + 1$

$\Downarrow$ 0 $\quad$ [$\because$ min^m value of $10 + x^2 = 10$]

$\Rightarrow$ then min^m value = 1

max^m value of $3\cos x - \sin x = \sqrt{3^2 + 1^2} = \sqrt{10}$

$\Rightarrow$ then max^m value $= \left(\sqrt{10}\right)^2 + 1 = 10 + 1 = 11$

IInd method

$10\cos^2 x - 6\sin x.\cos x + 2\sin^2 x$

$= 8\cos^2 x - 6\sin x.\cos x + 2(\sin^2 x + \cos^2 x)$

$= 8\cos^2 x - 6\sin x.\cos x + 2$

$= 8.\left(\dfrac{\cos 2x + 1}{2}\right) - 3.2\sin x\cos x + 2$

$= 4\cos 2x - 3\sin 2x + 6$

($\because \cos^2 x = \dfrac{\cos 2x + 1}{2}$)

max^m & min^m value of $4\cos 2x - 3\sin 2x$

$= \pm\sqrt{4^2 + (-3)^2} = \pm 5$

$\Rightarrow$ max^m value $= +5 + 6 = 11$

$\Rightarrow$ min^m value $= -5 + 6 = 1$

<u>Type – 3</u>

max^m & min^m value of $\sin^n\theta.\cos^n\theta$

$\Rightarrow (\sin\theta.\cos\theta)^n$

$= \left(\dfrac{2\sin\theta.\cos\theta}{2}\right)^n = \left(\dfrac{\sin 2\theta}{2}\right)^n = \dfrac{\sin^n 2\theta}{2^n}$

$\Rightarrow$ max^m value $= \dfrac{1^n}{2^n} = \dfrac{1}{2^n}$ [$\because$ max^m value of $\sin 2\theta = 1$]

$\Rightarrow$ min^m value $= \dfrac{(-1)^n}{2^n} \begin{cases} \dfrac{-1}{2^n} & \text{n – odd} \\ 0 & \text{n – even} \end{cases}$

[$\because$ min^m value of $\sin^n 2\theta$ is 0 if n is even & is -1 if n is odd]

Ex 88. Find max^m & min^m value of $\sin^4\theta.\cos^4\theta$

Sol. max^m value $= \frac{1}{2^n} = \frac{1}{2^4} = \frac{1}{16}$ and min^m value = 0

Ex 89. Find max^m & min^m value of $\sin^3\theta.\cos^3\theta$

Sol. max^m value $= \frac{1}{2^n} = \frac{1}{2^3} = \frac{1}{8}$

min^m value $= \frac{-1}{2^n} = \frac{-1}{2^3} = \frac{-1}{8}$

Type – 4

$\sin^{2m}\theta \le$ .... $\sin^6\theta \le \sin^4\theta \le \sin^2\theta$ ....(i)

$\cos^{2n}\theta \le$ .... $\cos^6\theta \le \cos^4\theta \le \cos^2\theta$ ......(ii)

Adding above both equation

$\Rightarrow$ $\sin^{2m}\theta + \cos^{2n}\theta \le \sin^2\theta + \cos^2\theta$

$\Rightarrow$ $\sin^{2m}\theta + \cos^{2n}\theta \le 1$

max^m value of $\sin^{2m}\theta + \cos^{2n}\theta$ is equal to 1

In this case power of sin & cos should be even.

Ex 90. Find maximum value of $\sin^8\theta + \cos^{14}\theta$.

Sol. max^m value = 1

Ex 91. Find max^m & min^m value of $\sin^6\theta + \cos^6\theta$

Sol. max^m value = 1

($\because \sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$)

$\Rightarrow$ $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$ should be minimum when $\sin^2\theta.\cos^2\theta$ is maximum

$\Rightarrow$ min^m value $= 1 - 3\left(\frac{1}{2^2}\right) = 1 - \frac{3}{4} = \frac{1}{4}$

Ex 92. Find max^m & min^m value of $\sin^4\theta + \cos^4\theta$

Sol. max^m value = 1

($\because \sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta.\cos^2\theta$)

$\Rightarrow$ $\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta.\cos^2\theta$ should be minimum when $\sin^2\theta.\cos^2\theta$ is maximum

$\Rightarrow$ min^m value $= 1 - 2\left(\frac{1}{2^2}\right) = 1 - \frac{1}{2} = \frac{1}{2}$

Ex 93. Find max^m & min^m value of $\sin^2\theta + \cos^4\theta$

Sol. max^m value = 1

$\because \sin^2\theta + \cos^4\theta = 1 - \cos^2\theta + \cos^4\theta$

$= 1 - \cos^2\theta.(1 - \cos^2\theta) = 1 - \cos^2\theta.\sin^2\theta$

$\Rightarrow$ $\sin^2\theta + \cos^4\theta = 1 - \cos^2\theta.\sin^2\theta$ should be minimum when $\sin^2\theta.\cos^2\theta$ is maximum.

$\Rightarrow$ min^m value $= 1 - \left(\frac{1}{2^2}\right) = 1 - \frac{1}{4} = \frac{3}{4}$

Ex 94. Find max^m & min^m value of $\cos^2\theta + \sin^4\theta$.

Sol. max^m value = 1

$\because \cos^2\theta + \sin^4\theta = 1 - \sin^2\theta + \sin^4\theta$

$= 1 - \sin^2\theta.(1 - \sin^2\theta) = 1 - \sin^2\theta.\cos^2\theta$

$\Rightarrow \cos^2\theta + \sin^4\theta = 1 - \sin^2\theta.\cos^2\theta$ should be minimum when $\sin^2\theta.\cos^2\theta$ is maximum.

$\Rightarrow$ min^m value $= 1 - \left(\frac{1}{2^2}\right) = 1 - \frac{1}{4} = \frac{3}{4}$

Ex 95. Find maximum value of $\sin^{88}\theta + \cos^{114}\theta$.

Sol. max^m value = 1 (power 88 & 114 is even number)

Ex 96. Find max^m & min^m value of $\sin^2\theta + \cos\theta$

Sol. $\sin^2\theta + \cos\theta = 1 - \cos^2\theta + \cos\theta$

$$= 1 - \left(\cos^2\theta - \cos\theta + \frac{1}{4} - \frac{1}{4}\right) = 1 - \left(\cos\theta - \frac{1}{2}\right)^2 + \frac{1}{4}$$

$$= \frac{5}{4} - \left(\cos\theta - \frac{1}{2}\right)^2$$

max^m value $= \frac{5}{4}$ [$\because$ max^m value of $10 - x^2 = 10$]

$\Rightarrow$ $\sin^2\theta + \cos\theta = \frac{5}{4} - \left(\cos\theta - \frac{1}{2}\right)^2$ should be minimum when $\left(\cos\theta - \frac{1}{2}\right)^2$ is maximum.

$\Rightarrow$ min^m value $= \frac{5}{4} - \left(-1 - \frac{1}{2}\right)^2 = \frac{5}{4} - \left(-\frac{3}{2}\right)^2$

$$= \frac{5}{4} - \frac{9}{4} = -1$$

Ex 97. Find max^m & min^m value of $\cos^2\theta + \sin\theta$

Sol. $\cos^2\theta + \sin\theta = 1 - \sin^2\theta + \sin\theta$

$$= 1 - \left(\sin^2\theta - \sin\theta + \frac{1}{4} - \frac{1}{4}\right) = 1 - \left(\sin\theta - \frac{1}{2}\right)^2 + \frac{1}{4}$$

$$= \frac{5}{4} - \left(\sin\theta - \frac{1}{2}\right)^2$$

max^{m} value $= \frac{5}{4}$ $\quad [\because \text{max}^{m}$ value of $10 - x^2 = 10]$

$\Rightarrow \cos^2\theta + \sin\theta = \frac{5}{4} - \left(\sin\theta - \frac{1}{2}\right)^2$ should be minimum when $\left(\sin\theta - \frac{1}{2}\right)^2$ is maximum.

$\Rightarrow \text{min}^{m}$ value $= \frac{5}{4} - \left(-1 - \frac{1}{2}\right)^2 = \frac{5}{4} - \left(-\frac{3}{2}\right)^2$

$$= \frac{5}{4} - \frac{9}{4} = -1$$

Type – 5

(only minimum value will be asked in this type because maximum value is equal to ∞)

(A) $a\tan^2\theta + b\cot^2\theta$

$= \left(\sqrt{a}\tan\theta\right)^2 + \left(\sqrt{b}\cot\theta\right)^2 - 2\sqrt{a}\tan\theta.\sqrt{b}\cot\theta + 2\sqrt{a}\tan\theta.\sqrt{b}\cot\theta$

$= \left(\sqrt{a}\tan\theta - \sqrt{b}\cot\theta\right)^2 + 2\sqrt{a}\tan\theta.\sqrt{b}\cot\theta$

$= \left(\sqrt{a}\tan\theta - \sqrt{b}\cot\theta\right)^2 + 2\sqrt{ab}$

Min^{m} value will be $2\sqrt{ab}$ when $\sqrt{a}\tan\theta - \sqrt{b}\cot\theta = 0$

Min^{m} value $= 2\sqrt{ab} \Rightarrow \tan^2\theta = \sqrt{\frac{b}{a}}$

$[\because \text{min}^{m}$ value of $10 + x^2 = 10$ when $x = 0]$

(B) $a\sin^2\theta + b\,\text{cosec}^2\theta$

on the basis of previous proof we can say minimum

value $= 2\sqrt{ab}$ $\quad$ when $\Rightarrow \sin^2\theta = \sqrt{\frac{b}{a}}$

but here it is necessary $b \le a$ otherwise value of $\sin^2\theta$ will be gretaer than 1 which is not possible

If $b \ge a$ then minimum value of $a\sin^2\theta + b\,\text{cosec}^2\theta$ will be $a + b$

(C) Same as in $a\cos^2\theta + b\sec^2\theta$

Conclusion: (i) $a\tan^2\theta + b\cot^2\theta$

$\Rightarrow$ **minimum value** $= 2\sqrt{ab}$

(ii) $a\sin^2\theta + b\,\text{cosec}^2\theta$

$\Rightarrow$ **minimum value** $= 2\sqrt{ab}$ **when** $b \le a$

$= a + b$ **when** $b \ge a$

(iii) $a\cos^2\theta + b\sec^2\theta$

$\Rightarrow$ **minimum value** $= 2\sqrt{ab}$ **when** $b \le a$

$= a + b$ **when** $b \ge a$

(D) $a\sec^2\theta + b\,\text{cosec}^2\theta$

Minimum value $= \left(\sqrt{a} + \sqrt{b}\right)^2$

Proof: $a\sec^2\theta + b\,\text{cosec}^2\theta$

$= a\left(1 + \tan^2\theta\right) + b\left(1 + \cot^2\theta\right)$

$= a + b + \left(a\tan^2\theta + b\cot^2\theta\right)$

$\Rightarrow$ minimum value $= a + b + 2\sqrt{ab}$

$$= \left(\sqrt{a} + \sqrt{b}\right)^2$$

Ex 98. Find minimum value of $9\tan^2\theta + 4\cot^2\theta$.

Sol. min^{m} value $= 2\sqrt{ab} = 2\sqrt{9 \times 4} = 12$

Ex 99. Find minimum value of $4\cos^2\theta + 9\sec^2\theta$

Sol. min^{m} value $= a + b$ when $b \ge a$

$= 4 + 9 = 13$

Ex 100. Find minimum value of $4\sec^2\theta + 9\,\text{cosec}^2\theta$.

Sol. min^{m} value $= \left(\sqrt{a} + \sqrt{b}\right)^2 = \left(\sqrt{4} + \sqrt{9}\right)^2 = 25$

Ex 101. Find minimum value of $2\sin^2\theta + 32\cot^2\theta$.

Sol. $=2\sin^2\theta+32\cot^2\theta$

$=2\sin^2\theta+32(\operatorname{cosec}^2\theta-1)$

$=2\sin^2\theta+32\operatorname{cosec}^2\theta-32$

$\Rightarrow$ min^m value $=a+b$ when $b\ge a$

$=(2+32)-32=2$

Ex 102. Find minimum value of $32\cos^2\theta+2\tan^2\theta$.

Sol. $=32\cos^2\theta+2\tan^2\theta$

$=32\cos^2\theta+2(\sec^2\theta-1)$

$=32\cos^2\theta+2\sec^2\theta-2$

$\Rightarrow$ min^m value $=2\sqrt{ab}-2=2\sqrt{32\times 2}-2=14$

Ex 103. Find minimum value of

$\sin^2\theta+\operatorname{cosec}^2\theta+\cos^2\theta+\sec^2\theta+\tan^2\theta+\cot^2\theta=?$

Sol. $\sin^2\theta+\operatorname{cosec}^2\theta+\cos^2\theta+\sec^2\theta+\tan^2\theta+\cot^2\theta$

$=\sin^2\theta+\cos^2\theta+1+\tan^2\theta+1+\cot^2\theta+\tan^2\theta+\cot^2\theta$

$=3+2\tan^2\theta+2\cot^2\theta$

$\Rightarrow$ min^m value $=3+2\sqrt{ab}=3+2\sqrt{2\times 2}=7$

Ex 104. Find min^m value of

$\sin^2\alpha+\operatorname{cosec}^2\alpha+\cos^2\beta+\sec^2\beta+\tan^2\gamma+\cot^2\gamma=?$

Sol.

$\sin^2\alpha+\operatorname{cosec}^2\alpha+\cos^2\beta+\sec^2\beta+\tan^2\gamma+\cot^2\gamma$

$\Rightarrow$ min^m value

$=2\sqrt{ab}+2\sqrt{ab}+2\sqrt{ab}=2\sqrt{1\times 1}+2\sqrt{1\times 1}+2\sqrt{1\times 1}=6$

****

Exercise-(Basic Identity)

1. Find the value of $\frac{\sin 43^\circ}{\cos 47^\circ}$.

(a) 0 (b) 1
(c) sin43° (d) cos47°

2. If $\sin\theta + \operatorname{cosec}\theta = 2$, then find the value of $\sin^5\theta + \operatorname{cosec}^5\theta$.

(a) $\frac{1}{2}$ (b) 1
(c) 0 (d) 2

3. If $\tan\theta + \cot\theta = 2$, then find the value of $\tan^2\theta + \cot^2\theta$.

(a) 2 (b) 1
(c) $\sqrt{2}$ (d) 0

4. If $\cos\theta + \sec\theta = 2$, then find value of $\sin^{100}\theta + \sec^{100}\theta$.

(a) 1 (b) 2
(c) 3 (d) 100

5. If $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{5}{4}$, then the value of $\frac{\tan^2\theta + 1}{\tan^2\theta - 1}$ is

(a) $\frac{25}{16}$ (b) $\frac{41}{40}$
(c) $\frac{9}{41}$ (d) $\frac{40}{41}$

6. $\frac{\tan\theta}{1 - \cot\theta} + \frac{\cot\theta}{1 - \tan\theta}$ is equal to -

(a) $1 - \tan\theta - \cot\theta$ (b) $1 + \tan\theta - \cot\theta$
(c) $1 - \tan\theta + \cot\theta$ (d) $1 + \tan\theta + \cot\theta$

7. If $\tan\theta + \cot\theta = 2$, then the value of $\tan^n\theta + \cot^n\theta$ $(0^\circ < \theta < 90^\circ$, n is an integer) is

(a) 2 (b) 2^n
(c) $2n$ (d) 2^{n+1}

8. If $\frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta} = \frac{5}{3}$, then $\sin\theta$ is equal to

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$
(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

9. If $\cos^4\theta - \sin^4\theta = \frac{2}{3}$, then the value of $1 - 2\sin^2\theta$ is

(a) $\frac{4}{3}$ (b) 0
(c) $\frac{2}{3}$ (d) $\frac{1}{3}$

10. $\tan 46^\circ - \cot 44^\circ = ?$

(a) 0 (b) 1
(c) $\tan 46^\circ$ (d) $\cot 44^\circ$

11. $\cos 51^\circ - \sin 39^\circ + \sin 37^\circ - \cos 53^\circ = ?$

(a) 0 (b) 1
(c) cos104° (d) sin39°

12. If $x + \frac{1}{x} = 2\cos\theta$, then find the value of $x^3 + \frac{1}{x^3}$.

(a) $2\cos 3\theta$ (b) $2\sin 3\theta$
(c) $\sin 3\theta$ (d) $\cos 3\theta$

13. If $\sec\theta + \tan\theta = 3$, then find the value of $\sec\theta$.

(a) $\frac{3}{4}$ (b) $\frac{5}{3}$
(c) $\frac{1}{6}$ (d) 9

14. If $\cos\theta = \frac{5}{13}$, then find the value of $\tan^2\theta + \sec^2\theta$.

(a) $\frac{39}{23}$ (b) $\frac{39}{25}$
(c) $\frac{313}{25}$ (d) $\frac{329}{21}$

15. If

$0° < \theta < 90° \;\&\; \tan\theta = \sqrt{6+\sqrt{6+\sqrt{6+.....\infty}}}$,

then find the value of $\sec^2\theta$.

(a) 10 (b) 12
(c) 8 (d) 6

16. If $\alpha + \beta = 90°, \alpha = 2\beta$, then find the value of $\cos^2\alpha + \sin^2\beta$.

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$
(c) $\frac{3}{2}$ (d) $\frac{4}{3}$

17. Find the value of $\dfrac{1-\tan^2 22\frac{1}{2}^\circ}{1+\tan^2 22\frac{1}{2}^\circ}$.

(a) $\frac{1}{\sqrt{2}}$ (b) 1
(c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$

18. $\sin^2 88° + \cos^2 88° = ?$

(a) $\frac{1}{2}$ (b) $\sqrt{3}$
(c) 1 (d) $\frac{1}{3}$

19. If $\tan\theta = \frac{1}{2}$ and $\tan\phi = \frac{1}{3}$, then $\theta + \phi = ?$

(a) $\frac{\pi}{6}$ (b) 0
(c) $\frac{\pi}{4}$ (d) π

20. Find the value of (tanA + sec A–1) cos A.

(a) 1 + sinA
(b) (1 + sinA) (tanA – secA+1)
(c) 1 – sinA
(d) None of these

21. If $2\cos\theta = x + \frac{1}{x}$, then find the value of $2\cos^2\theta$.

(a) $\frac{1}{2}\left(x^2+\frac{1}{x^2}\right)+1$ (b) $\left(x^2+\frac{1}{x^2}\right)-1$
(c) $\frac{3}{x^3}$ (d) 1

22. Find the value of $\frac{1-\cos A}{1+\cos A}$.

(a) $\left(\frac{1-\cos A}{\sin A}\right)^2$ (b) $\frac{1-\cos A}{2\sin A}$
(c) $\left(\frac{\sin A}{1-\cos A}\right)^2$ (d) $\frac{\sin A}{1+\cos A}$

23. Find the value of $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$.

(a) $\sec\theta - \tan\theta$ (b) $\cos\theta - \cot\theta$
(c) $\sin\theta - \operatorname{cosec}\theta$ (d) $\operatorname{cosec}\theta - \cot\theta$

24. Find the value of $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} + \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$.

(a) $2\sec\theta$ (b) $\sec\theta$
(c) $2\operatorname{cosec}\theta$ (d) None of these

25. Find the value of $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} + \sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$.

(a) $-2\sec\theta$ (b) $2\sec\theta$
(c) $2\operatorname{cosec}\theta$ (d) $2\tan\theta$

26. If $\sec\theta = A$, $\operatorname{cosec}\theta = B$, then

(a) $A^2 + B^2 = AB$
(b) $A^2 + B^2 = A^2B^2$
(c) $A^2 - B^2 = A^2B^2$
(d) $A^2 + B^2 = -A^2B^2$

27. $\sin^6 A + \cos^6 A$ is equal to

(a) $1-3\sin^2 A\cos^2 A$

(b) $3\sin^2 A\cos^2 A-1$

(c) $1+3\sin^2 A\cos^2 A$

(d) 1

28. Find the value of $\sin\theta$ in terms of $\sec\theta$.

(a) $\frac{1}{\sqrt{\sec^2-1}}$ (b) $\frac{\sec^2\theta}{\sqrt{\sec^2-1}}$

(c) $\frac{\sqrt{\sec^2\theta-1}}{\sec\theta}$ (d) $\sqrt{\sec^2\theta-1}$

29. If $\sin\alpha\sec(30°+\alpha)=1$ $(0° < \alpha < 60°)$, then find the value of $\sin\alpha+\cos 2\alpha$.

(a) 1 (b) $\frac{2+\sqrt{3}}{2\sqrt{3}}$

(c) 0 (d) $\sqrt{2}$

30. Find the value of

$(\sec\theta-\cos\theta)^2 + (\operatorname{cosec}\theta-\sin\theta)^2 - (\cot\theta-\tan\theta)^2$.

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) 2

31. If $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}=3$, then find the value of $\sin^4\theta-\cos^4\theta$.

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$

(c) $\frac{3}{5}$ (d) $\frac{4}{5}$

32. If $\sin 17°=\frac{x}{y}$, then the value of $\sec 17°-\sin 73°$ is.

(a) $\frac{y^2-x^2}{xy}$ (b) $\frac{x^2}{\sqrt{y^2-x^2}}$

(c) $\frac{x^2}{y\sqrt{y^2+x^2}}$ (d) $\frac{x^2}{y\sqrt{y^2-x^2}}$

33. If $\cos 43° = \frac{x}{\sqrt{x^2+y^2}}$, then the value of $\tan 47°$ is.

(a) $\frac{y}{x}$ (b) $\frac{x}{y}$

(c) $\frac{y}{\sqrt{x^2+y^2}}$ (d) x

34. If $\tan\theta=\frac{x}{y}$, then $\frac{x\sin\theta+y\cos\theta}{x\sin\theta-y\cos\theta}$ is equal to

(a) $\frac{x^2+y^2}{x^2-y^2}$ (b) $\frac{x^2-y^2}{x^2+y^2}$

(c) $\frac{x}{\sqrt{x^2+y^2}}$ (d) $\frac{y}{\sqrt{x^2+y^2}}$

35. Find the value of $\left(\frac{1}{\cos\theta}+\frac{1}{\cot\theta}\right)\left(\frac{1}{\cos\theta}-\frac{1}{\cot\theta}\right)$.

(a) 0 (b) −1

(c) 1 (d) 2

36. If $\sin 61° = \frac{a}{\sqrt{a^2+b^2}}$, then find the value of $\tan 61°+\tan 29°$.

(a) $\frac{a}{b}$ (b) $\frac{(a+b)^2}{ab}$

(c) $\frac{a}{b}+1$ (d) $\frac{a}{b}+\frac{b}{a}$

37. If $\frac{\cos^2\theta}{\cot^2\theta - \cos^2\theta} = 3$ and $0^o < \theta < 90^o$, then find the value of θ.

(a) 30° (b) 45°
(c) 60° (d) None of these

38. $\frac{2\sin 68°}{\cos 22°} - \frac{2\cot 15°}{5\tan 75°} - \frac{3\tan 45°.\tan 20°.\tan 40°.\tan 50°.\tan 70°}{5}$ is equal to

(a) –1 (b) 0
(c) 1 (d) 2

39. If $\text{cosec}39° = x$, then the value of $\frac{1}{\text{cosec}^2 51°} + \sin^2 39° + \tan^2 51° - \frac{1}{\sin^2 51° \sec^2 39°}$ is.

(a) $\sqrt{x^2 - 1}$ (b) $\sqrt{1 - x^2}$
(c) $x^2 - 1$ (d) $1 - x^2$

40. $\frac{\sin 39°}{\cos 51°} + 2\tan 11°\tan 31°\tan 45°\tan 59°\tan 79° - 3\left(\sin^2 21° + \sin^2 69°\right) = ?$

(a) 2 (b) –1
(c) 1 (d) 0

41. Find the value of $\frac{1}{(1 + \tan^2\theta)} + \frac{1}{(1 + \cot^2\theta)}$.

(a) 2 (b) $\frac{1}{2}$
(c) $\frac{1}{4}$ (d) 1

42. The value of $\frac{\cos^3 20° - \cos^3 70°}{\sin^3 70° - \sin^3 20°} = ?$

(a) 5 (b) 7
(c) 1 (d) 3

43. The value of $\frac{\cos^n 38° - \cot^n 52°}{\sin^n 52° - \tan^n 38°} = ?$

(a) 5 (b) 7
(c) 1 (d) 3

44. The value of $\frac{\cot^n 29° - \cot^n 61°}{\tan^n 61° - \tan^n 29°} = ?$

(a) 5 (b) 7
(c) 1 (d) 3

45. If $x = \tan 15°$, then find the value of $x^2 + \frac{1}{x^2}$.

(a) 16 (b) 15
(c) 14 (d) 18

46. If $x = \cot 75°$, then find the value of $\sqrt{x} + \frac{1}{\sqrt{x}}$.

(a) $\sqrt{6}$ (b) $\sqrt{7}$
(c) $\sqrt{5}$ (d) $\sqrt{3}$

47. If tan (A+B) $= \frac{1}{2}$, tan (A – B) $= \frac{1}{3}$, then find the value of tan 2A.

(a) 5 (b) 7
(c) 1 (d) 3

48. If tan (A+2B) $= \frac{1}{2}$, tan 2 (A – B) $= \frac{1}{3}$, then find the value of $\angle A$.

(a) 45° (b) 30°
(c) 60° (d) 15°

49. If $\cos(A - B) = \frac{1}{2}$ and $\sin(A + B) = \frac{1}{2}$, then find the minimum positive value of $\angle A$.

(a) 135° (b) 45°
(c) 30° (d) 105°

50. If $\sin(x + y) = 1$ and $\tan(x - y) = \frac{1}{\sqrt{3}}$, then the value of $\sin x + \tan y$ is. $(0^o < x, y < 90°)$

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{5}{2\sqrt{3}}$
(c) $\frac{4}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{4}$

51. If $\sin A + \cos A = \frac{3}{5}$, then find the value of of $\sin A - \cos A$.

(a) $\frac{\sqrt{41}}{5}$ (b) $\frac{9}{4}$

(c) $\frac{5}{\sqrt{41}}$ (d) $\frac{7}{\sqrt{41}}$

52. If $\sec^2\theta + \tan^2\theta = \frac{7}{12}$, then $\sec^4\theta - \tan^4\theta = ?$

(a) $\frac{7}{12}$ (b) $\frac{1}{2}$

(c) $\frac{5}{12}$ (d) 1

53. If $\operatorname{cosec}^2\theta + \cot^2\theta = \frac{5}{7}$, then find the value of $\operatorname{cosec}^4\theta - \cot^4\theta$.

(a) $\frac{5}{7}$ (b) $\frac{7}{5}$

(c) $\frac{1}{7}$ (d) None of these

54. If $\sec x + \tan x = a$, then find the value of $\sin x$.

(a) $\frac{a^2-1}{a^2+1}$ (b) $\frac{a^2+1}{a^2-1}$

(c) $\frac{a-1}{a^2+1}$ (d) $\frac{a^2-1}{a+1}$

55. If $\operatorname{cosec} x - \cot x = a$, then find the value of $\cos x$.

(a) $\frac{1-a^2}{1+a^2}$ (b) $\frac{a^2-1}{a^2+1}$

(c) $\frac{a-1}{a^2+1}$ (d) $\frac{a^2-1}{a+1}$

56. Find the value of $(\sin x + \operatorname{cosec} x)^2 + (\cos x + \sec x)^2 - (\tan^2 x + \cot^2 x)$.

(a) 8 (b) 7

(c) 9 (d) 10

57. Find the value of $(\sin x + \operatorname{cosec} x)^2 + (\cos + \sec x)^2 - (\tan x + \cot x)^2$.

(a) 7 (b) 5

(c) 9 (d) 4

58. Find the value of $(\sin x - \operatorname{cosec} x)^2 + (\cos - \sec x)^2 - (\tan x - \cot x)^2$.

(a) –7 (b) 5

(c) 1 (d) 4

59. Find the value of $2(\sin\theta^6 + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta)$

(a) 1 (b) 5

(c) 9 (d) –1

60. Find the value of $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) - (\sin^2\theta + \cos^2\theta)^2$

(a) 2 (b) 5

(c) 0 (d) –2

61. If $u_n = \cos^n\delta + \sin^n\delta$, then the value of $2u_6 - 3u_4 + 2$ is

(a) 1 (b) 5

(c) 0 (d) –1

62. If $\frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta} = 2\frac{51}{79}$, then $\sin\theta$ will be -

(a) $\frac{35}{72}$ (b) $\frac{65}{144}$

(c) $\frac{91}{144}$ (d) $\frac{39}{72}$

63. If $\sin\theta + \sin^2\theta = 1$, then find the value of $\cos^2\theta + \cos^4\theta$.

(a) 1 (b) 5

(c) 0 (d) –1

64. If $\cos\theta + \cos^2\theta = 1$, then find the value of $\sin^4\theta + \sin^2\theta$.

(a) –1 (b) 5

(c) 0 (d) 1

65. If $\cos A + \cos^2 A = 1$, then find the value of $\sin^8 A + 2\sin^6 A + \sin^4 A$.

(a) –1 (b) 5

(c) 0 (d) 1

66. If $\cos A + \cos^2 A = 1$, then find the value of $\sin^{12} A + 3\sin^{10} A + 3\sin^8 A + \sin^6 A + \sin^4 A + \sin^2 A$.

(a) –1 (b) 5

(c) 2 (d) 1

67. If $\sin A + \sin^2 A = 1$, then find the value of $\cos^{12}A + 3\cos^{10}A + 3\cos^8 A + \cos^6 A + \cos^4 A + \cos^2 A$.

(a) -1 (b) 5
(c) 2 (d) 1

68. If $\cos^2 x + \cos^4 x = 1$, then find the value of $\tan^2 x + \tan^4 x$.

(a) 0 (b) 3
(c) 1 (d) 4

69. If $3\sin x + 4\cos x = 2$, then find the value of $3\cos x - 4\sin x$.

(a) $\sqrt{21}$ (b) 5
(c) 0 (d) $\sqrt{29}$

70. If $\cos\theta - \sin\theta = \sqrt{2}\cos\theta$, then find the value of $\cos\theta + \sin\theta$.

(a) $\sqrt{2}\cos\theta$ (b) $\sqrt{2}\sin\theta$
(c) $2\sin\theta$ (d) $\sqrt{2}\tan\theta$

71. If $\sin\theta + \cos\theta = \sqrt{2}$, then find the value of $\sin\theta - \cos\theta$.

(a) $\sqrt{2}$ (b) 0
(c) $2\sqrt{2}$ (d) 2

72. If $\sin\theta + \cos\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$, then $q(p^2 - 1) = ?$

(a) p (b) $2p$
(c) $3p$ (d) $2p^2$

73. If $T_n = \sin^n\theta + \cos^n\theta$ then $\dfrac{T_3 - T_5}{T_1} = ?$

(a) $\sin\theta.\cos\theta$ (b) $\sin^2\theta.\cos^2\theta$
(c) $\sin^2\theta.\cos\theta$ (d) $\sin\theta.\cos^2\theta$

74. If $x = a\cos^3\theta$, $y = b\sin^3\theta$

then $\left(\dfrac{x}{a}\right)^{\frac{2}{3}} + \left(\dfrac{y}{b}\right)^{\frac{2}{3}} = ?$

(a) 1 (b) 0
(c) 2 (d) 4

75. If $x = a\sec^n\theta$ and $y = b\tan^n\theta$, then find the value of θ.

(a) $\left(\dfrac{x}{a}\right)^{\frac{1}{n}} - \left(\dfrac{y}{b}\right)^{\frac{1}{n}} = 1$

(b) $\left(\dfrac{x}{a}\right)^{\frac{2}{n}} - \left(\dfrac{y}{b}\right)^{\frac{2}{n}} = 1$

(c) $\left(\dfrac{x}{a}\right)^2 - \left(\dfrac{y}{b}\right)^2 = 1$

(d) None of these

76. If $\tan^5\theta\tan^5 5\theta = 1$, then find the value of $\tan^n 3\theta$.

(a) 5 (b) -1
(c) 1 (d) 3

77. If $\tan\theta.\tan 2\theta = 1$, then find the value of $\sin^2 2\theta + \tan^2 2\theta$.

(a) $\dfrac{3}{4}$ (b) $\dfrac{10}{3}$
(c) $3\dfrac{3}{4}$ (d) 3

78. If $\cos\theta \operatorname{cosec} 23° = 1$, the value of θ is

(a) 23° (b) 37°
(c) 63° (d) 67°

79. If $\sin(x + y) = \cos(x - y)$, then find the value of $\cos^2 x$.

(a) 5 (b) $-\dfrac{1}{2}$
(c) $\dfrac{1}{2}$ (d) 3

80. If x, y are positive acute angles, $x + y < 90°$ and $\sin(2x - 20°) = \cos(2y + 20°)$, then value of $\sec(x + y)$ is,

(a) 5 (b) $\sqrt{2}$
(c) 2 (d) 3

81. If A and B are complementary angles, find the value of $\sqrt{\dfrac{\tan A \tan B + \cot A \cot B}{\sin A \sec B} - \dfrac{\sin^2 B}{\cos^2 A}}$

(a) 5 (b) $\sqrt{2}$
(c) 1 (d) 3

82. A and B are complementary angles , then find the value of sin A cos B + cos A sin B + 2tan A tan B – $\sec^2 A + \cot^2 B$.

(a) 5 (b) $\sqrt{2}$
(c) 2 (d) 3

83. If θ is an acute angle and $\sin\theta = \cos\theta$, then find the value of $2\tan^2\theta + \sin^2\theta - 1$.

(a) $1\frac{1}{2}$ (b) – 7
(c) 1 (d) 3

84. If $\sin(x + y) = \cos[3(x + y)]$, then the value of $\tan[2(x + y)]$ is.

(a) 5 (b) 7
(c) 1 (d) 3

85. If $\sec(5\theta - 50°) = \text{cosec}(\theta + 32°)$, then the value of θ is: $(0° < \theta < 90°)$

(a) $33\frac{1}{3}^{\circ}$ (b) 18°
(c) $3\frac{1}{3}^{\circ}$ (d) 30°

86. If $x = a\sin\theta$ and $y = b\tan\theta$, then the value of $\dfrac{a^2}{x^2} - \dfrac{b^2}{y^2} = ?$

(a) 5 (b) – 1
(c) 1 (d) 3

87. If $x = a\cos\theta$ and $y = b\cot\theta$, then the value of $\dfrac{a^2}{x^2} - \dfrac{b^2}{y^2} = ?$

(a) 5 (b) – 1
(c) 1 (d) 3

88. If $x = a\sec\theta\cos\theta$, $y = b\sec\theta\sin\theta$ and $z = c\tan\theta$, then find the value of $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} - \dfrac{z^2}{c^2}$.

(a) 1 (b) –10
(c) 0 (d) 2

89. Find the value of $3\sin 20° - 4\sin^3 20°$.

(a) $\dfrac{\sqrt{3}}{2}$ (b) $\dfrac{1}{2}$
(c) $\dfrac{1}{\sqrt{2}}$ (d) $\dfrac{2}{\sqrt{3}}$

90. Find the value of $3\cos 20° - 4\cos^3 20°$.

(a) $\dfrac{\sqrt{3}}{2}$ (b) $-\dfrac{1}{2}$
(c) $\dfrac{1}{\sqrt{2}}$ (d) $\dfrac{2}{\sqrt{3}}$

91. If $\cos^2\alpha + \cos^2\beta = 2$, the value of $\tan^3\alpha + \sin^5\beta$ is.

(a) 0 (b) 1
(c) –1 (d) $\dfrac{1}{\sqrt{3}}$

92. $\tan 1° \tan 2° \tan 3° \ldots\ldots\ \tan 88° \tan 89° = ?$

(a) 0 (b) 3
(c) 1 (d) 4

93. Find the value of

$\tan^n 1° \tan^n 2° \tan^n 3° \ldots\ldots \tan^n 88° \tan^n 89°$.

(a) 0 (b) 3
(c) 1 (d) 4

94. The product cos1° cos2° cos3° cos4° ...... cos100°. is equal to

(a) –1 (b) $\dfrac{1}{4}$
(c) 1 (d) 0

95. Find the value of $\cos 1°.\cos 2°.\cos 3° \ldots \cos 90°$.

(a) –1 (b) 0
(c) 1 (d) α

96. Find the value of $\sin 1°.\sin 2°.\sin 3°...\sin 180°$.

(a) –1 (b) 0

(c) 1 (d) α

97. Find the value of

$\sin^2 1° + \sin^2 2° + \sin^2 3° +$ ----- $+ \sin^2 87° +$ $\sin^2 88° + \sin^2 89° + \sin^2 90°$.

(a) $44\frac{1}{2}$ (b) 44

(c) $45\frac{1}{2}$ (d) N.O.T.

98. Find the value of

$\cos^2 1° + \cos^2 2° + \cos^2 3°$ +-----+$\cos^2 87° +$ $\cos^2 88° + \cos^2 89° + \cos^2 90°$.

(a) $44\frac{1}{2}$ (b) 44

(c) $45\frac{1}{2}$ (d) N.O.T.

99. $\sin^2 5° + \sin^2 10° + \sin^2 15° + ... + \sin^2 85° + \sin^2 90°$ is equal to

(a) $7\frac{1}{2}$ (b) $8\frac{1}{2}$

(c) 9 (d) $9\frac{1}{2}$

100. Find the value of $\sin 10°.\sin 30°.\sin 50°.\sin 70°$.

(a) 8 (b) $\frac{1}{16}$

(c) 16 (d) $\frac{1}{2}$

101. Find the value of $\tan 4°.\tan 43°.\tan 47°.\tan 86°$.

(a) 0 (b) 1

(c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

102. Find the value of $\cot 10°.\cot 20°.\cot 60°.\cot 70°.\cot 80°$.

(a) 1 (b) –1

(c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

103. The value of $\tan 10° \tan 15° \tan 75° \tan 80°$ is

(a) 0 (b) 1

(c) – 1 (d) 2

104. $\tan\frac{\pi}{8}\tan\frac{\pi}{12}\tan\frac{3\pi}{8}\tan\frac{5\pi}{12} - \sin^2\frac{\pi}{6} = ?$

(a) $\frac{1}{2}$ (b) $\frac{2-\sqrt{3}}{2}$

(c) $\frac{1}{4}$ (d) $\frac{3}{4}$

105. $\cot\frac{\pi}{20}\cot\frac{3\pi}{20}\cot\frac{5\pi}{20}\cot\frac{7\pi}{20}\cot\frac{9\pi}{20} = ?$

(a) –1 (b) 1

(c) $\frac{1}{2}$ (d) 0

106. Solve $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+2\cos 16\theta}}}}$

(a) $2\cos\theta$ (b) $\sqrt{2}\cos\theta$

(c) $2\sin\theta$ (d) $\sqrt{2}\sin\theta$

107. Find the value of tan 70°.

(a) tan 50° + tan 20°

(b) tan 50° + 2 tan 20°

(c) 2 tan 50° + tan 20°

(d) 2 tan 50° + 2 tan 20°

108. Find the value of tan 80°.

(a) tan 70° + tan 10°

(b) tan 70° + 2 tan 10°

(c) 2 tan 70° + tan 10°

(d) 2 tan 70° + 2 tan 10°

109. If $\tan\left(\frac{\pi}{2} - \frac{\theta}{2}\right) = \sqrt{3}$, then the value of $\cos\theta$ is.

(a) 0 (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{1}{2}$ (d) 1

110. If A, B and C be the angles of a triangle, then of the following the incorrect relation is :

(a) $\sin\frac{A+B}{2} = \cos\frac{C}{2}$

(b) $\cos\left(\frac{A+B}{2}\right) = \sin\frac{C}{2}$

(c) $\tan\left(\frac{A+B}{2}\right) = \sec\frac{C}{2}$

(d) $\cot\left(\frac{A+B}{2}\right) = \tan\frac{C}{2}$

111. If cosec A = 2, then $\frac{1}{\tan A} + \frac{\sin A}{1+\cos A} = ?$

(a) 2 (b) 3
(c) 1 (d) 4

112. If $\tan A = \sqrt{2} - 1$, then $\sin 2A = ?$

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$
(c) 2 (d) $2\sqrt{2}$

113. $\frac{5\sin^2 30° + \cos^2 45° - 4\tan^2 30°}{2\sin 30°.\cos 30° + \tan 45°} = ?$

(a) $\frac{5}{6}(2-\sqrt{3})$ (b) $\frac{5}{6}(2+\sqrt{3})$
(c) $2-\sqrt{3}$ (d) $\frac{1}{6}(2-\sqrt{3})$

114. $\frac{\cos(90°-\theta).\sec(90°-\theta).\tan\theta}{\text{cosec}(90°-\theta).\sin(90°-\theta).\cot(90°-\theta)} = ?$

(a) 1 (b) 2
(c) 3 (d) –1

115. If α is in first quadrant such that $\tan^2\alpha = \frac{8}{7}$, then the value of $\frac{(1+\sin\alpha)(1-\sin\alpha)}{(1+\cos\alpha)(1-\cos\alpha)}$

(a) $\frac{7}{8}$ (b) $\frac{8}{7}$
(c) $\frac{7}{4}$ (d) $\frac{64}{49}$

116. $\frac{k\,\text{cosec}^2 30°.\sec^2 45}{8\cos^2 45°.\sin^2 60°} = \tan^2 60° - \tan^2 30°$, then $k = ?$

(a) 1 (b) –1
(c) 2 (d) 0

117. If α is in Ist quadrant such that $\sec^2\alpha = 3$, then $\frac{\tan^2\alpha - \text{cosec}^2\alpha}{\tan^2\alpha + \text{cosec}^2\alpha} = ?$

(a) $\frac{4}{7}$ (b) $\frac{3}{7}$
(c) $\frac{2}{7}$ (d) $\frac{1}{7}$

118. If 5α and 4α are in Ist quadrant such that $\sin 5\alpha = \cos 4\alpha$, then $2\sin 3\alpha - \sqrt{3}\tan 3\alpha = ?$

(a) 1 (b) 0
(c) –1 (d) $1+\sqrt{3}$

119. If $(\sin\theta + \text{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = K + \tan^2\theta + \cot^2\theta$, then K = ?

(a) 6 (b) 3
(c) 0 (d) 7

120. $(\sin\alpha + \sec\alpha)^2 + (\cos\alpha + \text{cosec}\alpha)^2 = (K + \sec\alpha\,\text{cosec}\alpha)^2$, then K = ?

(a) 1 (b) 2
(c) 3 (d) 4

121. $\frac{\sin\theta}{\cot\theta + \text{cosec}\theta} - \frac{\sin\theta}{\cot\theta - \text{cosec}\theta} = ?$

(a) 1 (b) 2
(c) 3 (d) 0

122. If $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A} = K + \tan A + \cot A$, then K =?

(a) 1 (b) 2
(c) 0 (d) 3

123. If $\frac{\cos^2\theta}{1-\tan\theta} + \frac{\sin^3\theta}{\sin\theta - \cos\theta} = K + \sin\theta\cos\theta$, then K =?

(a) 1 (b) 2
(c) 3 (d) 4

124. $\dfrac{(1-\sin\theta+\cos\theta)^2}{(1+\cos\theta)(1-\sin\theta)}=?$

(a) 2 (b) 1
(c) 3 (d) 0

125. $\sec^6\theta - \tan^6\theta - 3\tan^2\theta.\sec^2\theta = ?$

(a) 1 (b) 3
(c) 2 (d) –1

126. $\text{cosec}^6\theta - \cot^6\theta - 3\cot^2\theta.\ \text{cosec}^2\theta = ?$

(a) 2 (b) –1
(c) 1 (d) 4

127. $\dfrac{(\text{cosec}\theta - \sec\theta)(\cot\theta - \tan\theta)}{(\text{cosec}\theta + \sec\theta)(\sec\theta.\text{cosec}\theta - 2)}=?$

(a) 2 (b) 1
(c) 3 (d) –1

128. $\sec^4\alpha\,(1-\sin^4\alpha) - 2\tan^2\alpha = ?$

(a) 1 (b) 2
(c) 3 (d) 0

129. If $\sin\theta+\cos\theta=\sqrt{2}\sin(90^\circ-\theta)$, then $\cot\theta = ?$

(a) $\sqrt{2}-1$ (b) $\sqrt{2}+1$
(c) $\sqrt{2}$ (d) 1

130. If $\cot\alpha=\dfrac{15}{8}$, then $\dfrac{(2+2\sin\alpha)(1-\sin\alpha)}{(1+\cos\alpha)(2-2\cos\alpha)}=?$

(a) $\dfrac{125}{8}$ (b) $\dfrac{225}{64}$
(c) $\dfrac{64}{225}$ (d) $\dfrac{8}{125}$

131. If $x = a\sin\alpha$ and $y = b\cos\alpha$, then $b^2x^2 + a^2y^2 = ?$

(a) 1 (b) 2
(c) a^2b^2 (d) $a^2 + b^2$

132. $\dfrac{\cot\theta}{\cot\theta-\cot 3\theta}+\dfrac{\tan\theta}{\tan\theta-\tan 3\theta}=?$

(a) 1 (b) –1
(c) 2 (d) 0

133. If $\tan\theta-\cot\theta=\dfrac{119}{60}$ for $0^\circ<\theta<\pi/2$, then the value of $\sin\theta+\cos\theta$.

(a) $\dfrac{17}{13}$ (b) $\dfrac{21}{13}$
(c) $\dfrac{19}{13}$ (d) $\dfrac{23}{13}$

134. If $x = 2\tan\alpha$, $y = 2\cot\alpha$, then $16\left(\dfrac{1}{4+x^2}+\dfrac{1}{4+y^2}\right)=?$

(a) 1 (b) 2
(c) 4 (d) 8

135. $\cos^2\dfrac{\pi}{16}+\cos^2\dfrac{3\pi}{16}+\cos^2\dfrac{5\pi}{16}+\cos^2\dfrac{7\pi}{16}=?$

(a) 0 (b) 1
(c) 2 (d) 3

136. $\cos^2(A-B)+\cos^2B-2\cos(A-B).\cos A.\cos B = ?$

(a) $\cos^2A$ (b) $\sin^2A$
(c) $\tan^2A$ (d) $\cot^2A$

137. $\dfrac{\cot^2\frac{\theta}{2}-\tan^2\frac{\theta}{2}}{\cot\theta.\text{cosec}\theta}=?$

(a) 1 (b) – 4
(c) 4 (d) 1

138. $\cos^2\theta+\cos^2(\alpha+\theta)-2\cos\alpha.\cos\theta.\cos(\theta+\alpha)=?$

(a) $\sin^2\alpha$ (b) $\cos^2\alpha$
(c) $\tan^2\alpha$ (d) $\sec^2\alpha$

139. If $\sin\theta = 3\sin(\theta+2\alpha)$, then $\tan(\theta+\alpha)+2\tan\alpha = ?$

(a) 1 (b) –1
(c) 0 (d) 2

140. If $\tan x + \sec x = 2\cot(90^\circ + x)$, then $\text{cosec}\,x = ?$

(a) – 1 (b) – 3
(c) – 5 (d) – 9

141. If $\dfrac{1}{\text{cosec}\theta+\cot\theta}-\text{cosec}\theta-\tan\theta=3k\sec\theta\,\text{cosec}\theta$, then value of k.

(a) $\dfrac{1}{3}$ (b) $-\dfrac{1}{2}$
(c) $-\dfrac{1}{3}$ (d) $\dfrac{1}{2}$

142. If $\tan\theta=\dfrac{11}{13}$, then find $\dfrac{5\sin\theta-3\cos\theta}{5\sin\theta+2\cos\theta}$.

(a) $\dfrac{81}{16}$ (b) $\dfrac{9}{4}$
(c) $\dfrac{16}{81}$ (d) $\dfrac{4}{9}$

143. If $7\sin^2\theta + 3\cos^2\theta = 4$, then value of $\tan\theta$.

(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$

(c) $2\sqrt{3}$ (d) $\frac{1}{2\sqrt{3}}$

144. $(1+\cot\theta - \text{cosec}\theta)(1+\tan\theta+\sec\theta) = ?$

(a) 1 (b) 2

(c) 0 (d) –1

145. $(\text{cosec}x - \sin x)(\sec x - \cos x)(\tan x + \cot x) = ?$

(a) 0 (b) – 1

(c) 1 (d) 2

146. $\left(\frac{\cot\theta}{\cot\theta - \cot 3\theta} + \frac{\tan\theta}{\tan\theta - \tan 3\theta}\right) = ?$

(a) 0 (b) 1

(c) 2 (d) 4

147. $\frac{\cot A + \text{cosec}A - 1}{\cot A - \text{cosec}A + 1} = ?$

(a) $\frac{\sin A}{1+\cos A}$ (b) $\frac{1+\cos A}{\sin A}$

(c) 2cotA (d) $\frac{1+\text{cosec}A}{\cos A}$

148. $\frac{\sin\theta + \sin 2\theta}{1+\cos\theta+\cos 2\theta} = ?$

(a) $\tan\theta$ (b) $\sin\theta$

(c) $\cos\theta$ (d) $\tan^2\theta$

149. For which values of x between 0 and 2π, then $2\,\text{cosec}2x\cot x - \cot^2 x = 1$ is true :

(a) 0 (b) 2

(c) 1 (d) for all x.

150. Which of the following is not true?

(a) $\sin\theta = -\frac{1}{5}$ (b) $\cos\theta = 1$

(c) $\sec\theta = \frac{1}{2}$ (d) $\tan\theta = 20$

151. If $\sin A = \frac{1}{\sqrt{10}}$ and $\sin B = \frac{1}{\sqrt{5}}$, where A and B are positive acute angles, then A + B =?

(a) π (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

152. $\sqrt{\frac{1-\sin 2A}{1+\sin 2A}} = ?$

(a) sec A + tan A (b) $\tan\left(\frac{\pi}{4} - A\right)$

(c) $\tan\left(\frac{\pi}{4} + \frac{A}{2}\right)$ (d) $\tan\left(\frac{\pi}{4} - \frac{A}{2}\right)$

153. If $\tan\theta = \frac{4}{3}$, then $\sin\theta$ = ?

(a) $-\frac{4}{5}$ but not $\frac{4}{5}$ (b) $-\frac{4}{5}$ or $\frac{4}{5}$

(c) $\frac{4}{5}$ but not $-\frac{4}{5}$ (d) $\frac{4}{5}$

154. $\frac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = ?$

(a) $\frac{1+\cos A}{\sin A}$ (b) $\frac{1+\sin A}{\cos A}$

(c) $\frac{1-\cos A}{1+\cos A}$ (d) $\frac{1+\sin A}{1-\sin A}$

155. If $x = \sec\theta - \tan\theta$, and $y = \text{cosec}\theta + \cot\theta$, then

(a) $xy + 1 = x - y$ (b) $xy + 1 = x - 2y$

(c) $xy + 1 = x + y$ (d) $xy + 1 = y - x$

156. If $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ & $\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta = 1$, then

(a) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 0$ (b) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(c) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = -1$ (d) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$

157. For any real numbers x and y, $\sec^2\theta = \frac{4xy}{(x+y)^2}$ is possible when

(a) $x = y$ (b) $x \neq y$

(c) $x > 2y$ (d) $2x < y$

158. $(\text{cosec}\,\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta) = ?$
(a) -1 (b) 1
(c) 2 (d) 0

159. If $\text{cosec}\,\theta - \cot\theta = q$, then $\text{cosec}\,\theta = ?$
(a) $\frac{1}{q}$ (b) $q + \frac{1}{q}$
(c) $\frac{1}{2q}$ (d) $\frac{1}{2}\left(q + \frac{1}{q}\right)$

160. If $\sin\theta = \frac{m^2 - n^2}{m^2 + n^2}$, then $\tan\theta = ?$
(a) $\frac{m^2+n^2}{2mn}$ (b) $\frac{m^2-n^2}{2mn}$
(c) $\frac{m^2+n^2}{m^2-n^2}$ (d) $\frac{m^2-n^2}{m^2+n^2}$

161. If $\sec\theta + \tan\theta = p$, then $\sec\theta = ?$
(a) $\frac{1}{2}\left(p + \frac{1}{p}\right)$ (b) $p - \frac{1}{p}$
(c) $2\left(p + \frac{1}{p}\right)$ (d) $-p + \frac{1}{p}$

162. If $\tan\theta = \frac{p}{q}$, then $\frac{p\sin\theta - q\cos\theta}{p\sin\theta + q\cos\theta} = ?$
(a) $\frac{p^2 - q^2}{p^2 + q^2}$ (b) $\frac{p^2 + q^2}{2p}$
(c) $\frac{p^2 + q^2}{2q}$ (d) $\frac{p^2 + q^2}{p^2 - q^2}$

163. $\cos 15° = ?$
(a) $\sqrt{\frac{1 + \cos 30°}{2}}$ (b) $\sqrt{\frac{1 - \cos 30°}{2}}$
(c) $\pm\sqrt{\frac{1 + \cos 30°}{2}}$ (d) $\pm\sqrt{\frac{1 - \cos 30°}{2}}$

164. If sinA = sinB and cosA = cosB then
(a) $\sin\left(\frac{A-B}{2}\right) = 0$ (b) $\sin\left(\frac{A+B}{2}\right) = 0$
(c) $\cos\left(\frac{A-B}{2}\right) = 0$ (d) $\cos\left(\frac{A+B}{2}\right) = 0$

165. tan 3A – tan 2A – tanA = ?
(a) tan 3A tan 2A tanA
(b) – tan 3A tan 2A tanA
(c) tanA tan 2A – tan 2A tan 3A – tan3A tanA
(d) None of these

166. $\frac{1 + \sin A - \cos A}{1 + \sin A + \cos A} = ?$
(a) $\tan\frac{A}{2}$ (b) $\cot\frac{A}{2}$
(c) $\sin\frac{A}{2}$ (d) $\cos\frac{A}{2}$

167. $\frac{1 + \cos\theta}{\sin\theta} = ?$
(a) $\tan\frac{\theta}{2}$ (b) $\cot\frac{\theta}{2}$
(c) $\tan\theta$ (d) $\cot\theta$

168. $\cot x - \tan x = ?$
(a) $\cot 2x$ (b) $2\cot^2 x$
(c) $2\cot 2x$ (d) $\cot^2 2x$

169. $\sin 50° - \sin 70° + \sin 10° = ?$
(a) 1 (b) 0
(c) $\frac{1}{2}$ (d) 2

170. $\sin 75° = ?$
(a) $\frac{2 - \sqrt{3}}{2}$ (b) $\frac{\sqrt{3} + 1}{2\sqrt{2}}$
(c) $\frac{\sqrt{3} - 1}{-2\sqrt{2}}$ (d) $\frac{\sqrt{3} - 1}{2\sqrt{2}}$

171. $\cos^2 A\,(3-4\cos^2 A)^2+\sin^2 A(3-4\sin^2 A)^2=?$
(a) 1 (b) sin4A
(c) cos4A (d) 0

172. If $\tan A=\frac{3}{2}$, then $\frac{1+\cot A}{1-\cot A}=?$
(a) –5 (b) 5
(c) $\frac{9}{4}$ (d) $\frac{4}{9}$

173. $\tan 75°-\cot 75°=?$
(a) $2\sqrt{3}$ (b) $2+\sqrt{3}$
(c) $2-\sqrt{3}$ (d) $-2+\sqrt{3}$

174. If $\tan\alpha=\frac{m}{m+1}$ and $\tan\beta=\frac{1}{2m+1}$, then, $(\alpha+\beta)=?$
(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

175. $\tan 20°\tan 40°\tan 60°\tan 80°=?$
(a) 1 (b) 2
(c) 3 (d) $\frac{\sqrt{3}}{2}$

176. The value of cos A – sin A, when $A=\frac{5\pi}{4}$, is
(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$
(c) 0 (d) 1

177. If $a\tan\theta=b$, then $a\cos 2\theta+b\sin 2\theta=?$
(a) a (b) b
(c) $-a$ (d) $-b$

178. $\cos A+\sin(270°+A)-\sin(270°-A)+\cos(180°+A)=?$
(a) –1 (b) 0
(c) 1 (d) $\frac{1}{2}$

179. $\sin 15°+\cos 105°=?$
(a) 0
(b) $2\sin 15°$
(c) $\cos 15°+\sin 15°$
(d) $\sin 15°-\cos 15$

180. The value of $\cos 105°+\sin 105°$ is
(a) $\frac{1}{2}$ (b) 1
(c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

181. The value of $\cos 15°-\sin 15°$ is
(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$
(c) $-\frac{1}{\sqrt{2}}$ (d) 0

182. If $x$cosq – sinq = 1, then $x^2+(1+x^2)$ sinq equals—
(a) –1 (b) 0
(c) 1 (d) 2

183. In ΔABC, $\operatorname{cosec}A(\sin B\cos C+\cos B\sin C)=?$
(a) $\frac{c}{a}$ (b) $\frac{a}{c}$
(c) 1 (d) –1

184. Real values of x, if $\cos\theta=x+\frac{1}{x}$, then
(a) θ is acute angle
(b) θ is right angle
(c) θ is obtuse angle
(d) The value of θ is not possible

185. If $\cos A=\frac{3}{4}$, then $32\sin\frac{A}{2}\cos\frac{5A}{2}=?$
(a) $\sqrt{7}$ (b) $-\sqrt{7}$
(c) 7 (d) –7

186. If $\sin\theta_1+\sin\theta_2+\sin\theta_3=3$, then $\cos\theta_1+\cos\theta_2+\cos\theta_3=?$
(a) 3 (b) 2
(c) 1 (d) 0

187. If $\sin\theta = \frac{24}{25}$ and θ is in second quadrant, then $\sec\theta + \tan\theta = ?$

(a) -3 (b) -5
(c) -7 (d) -9

188. $\frac{\cos 17^\circ + \sin 17^\circ}{\cos 17^\circ - \sin 17^\circ} = ?$

(a) $\tan 62^\circ$ (b) $\tan 56^\circ$
(c) $\tan 54^\circ$ (d) $\tan 73^\circ$

189. If $\sin\alpha = \frac{-3}{5}$, where $\pi < \alpha < \frac{3\pi}{2}$, then $\cos\frac{\alpha}{2} = ?$

(a) $-\frac{1}{\sqrt{10}}$ (b) $\frac{1}{\sqrt{10}}$
(c) $\frac{3}{\sqrt{10}}$ (d) $\frac{-3}{\sqrt{10}}$

190. The greatest value of the function $\sqrt{3}\sin x + \cos x$ is

(a) 4 (b) 2
(c) 1 (d) $\sqrt{3}$

191. $\cos\theta(\tan\theta + 2)(2\tan\theta + 1) = ?$

(a) 0 (b) 1
(c) $2\sec\theta + 5\sin\theta$ (d) $5\sec\theta + 2\sin\theta$

192. If x and y are the angles lying in the second quadrant and $x < y$, then which one of the following is true?

(a) $\sin x = \sin y$ (b) $\sin x < \sin y$
(c) $\sin x > \sin y$ (d) None of these

193. If $0^\circ < \theta < 90^\circ$, then $\left(\frac{5\cos\theta - 4}{3 - 5\sin\theta} - \frac{3 + 5\sin\theta}{4 + 5\cos\theta}\right)$ is equal to

(a) 0 (b) 1
(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

194. If α is a positive acute angle and $2\sin\alpha + 15\cos^2\alpha = 7$, then the value of $\cot\alpha$ is

(a) $\frac{3}{4}$ (b) $\frac{2}{3}$
(c) $\frac{\sqrt{5}}{2}$ (d) $\frac{2}{\sqrt{5}}$

195. If $3\tan\theta + 4 = 0$ where $\frac{\pi}{2} < \theta < \pi$, then the value of $2\cot\theta - 5\cos\theta - \sin\theta$.

(a) $-\frac{53}{10}$ (b) $\frac{7}{10}$
(c) $\frac{23}{10}$ (d) $\frac{37}{10}$

196. If $\sec^2\theta = 3$, $0^\circ < \theta < \frac{\pi}{2}$, then the value of $\frac{\tan^2\theta - \operatorname{cosec}^2\theta}{\tan^2\theta + \operatorname{cosec}^2\theta}$ is.

(a) $\frac{4}{7}$ (b) $\frac{2}{7}$
(c) $\frac{1}{7}$ (d) $\frac{3}{7}$

197. If $3\cos\theta - 2\sin\theta = \frac{1}{\sqrt{2}}$, $(0^\circ < \theta < 90^\circ)$, then the value of $3\sin\theta + 2\cos\theta$.

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$
(c) $\frac{3}{\sqrt{2}}$ (d) $\frac{5}{\sqrt{2}}$

198. If $\tan\theta - \cot\theta = 0$ $(0^\circ < \theta < 90^\circ)$, then the value of $\sin\theta - \cos\theta$ is

(a) 1 (b) 2
(c) -2 (d) 0

199. If $\sin A + \operatorname{cosec} A = 3$, then find the value of $\frac{\sin^4 A + 1}{\sin^2 A}$.

(a) 1 (b) 0
(c) 7 (d) 11

200. $\cos 7^\circ \cos 23^\circ \cos 45^\circ \text{cosec} 83^\circ \text{cosec} 67^\circ = ?$

(a) 0 (b) 1

(c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{2}}$

201. If $\frac{\tan\theta + \cot\theta}{\tan\theta - \cot\theta} = 2$, $(0^\circ \le \theta \le 90^\circ)$, then the value of $\sin\theta$ is

(a) $\frac{2}{\sqrt{3}}$ (b) $\frac{\sqrt{3}}{2}$

(c) $\frac{1}{2}$ (d) 1

202. If $\sin x + \cos x = c$, then $\sin^6 x + \cos^6 x$ is equal to

(a) $\frac{1+6c^2-3c^4}{16}$ (b) $\frac{1+6c^2-3c^4}{4}$

(c) $\frac{1+6c^2+3c^4}{16}$ (d) $\frac{1+6c^2+3c^4}{4}$

203. $\frac{1-\sin A \cos A}{\cos A(\sec A - \text{cosec} A)} \cdot \frac{\sin^2 A - \cos^2 A}{\sin^3 A + \cos^3 A} = ?$

(a) sinA (b) cosA

(c) tanA (d) cosecA

204. The minimum value of $12\sin^2\theta + 23\cos^2\theta$ is

(a) 0 (b) 23

(c) 12 (d) 1

205. Find the minimum value of $(\sin\theta + \text{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2$

(a) 8 (b) 7

(c) 9 (d) 4

206. Find the minimum and maximum value of $4\tan^2\theta + 9\cos^2\theta$.

(a) 8 (b) 6

(c) 13 (d) 12

207. Find the minimum and maximum value of $7\cos\alpha + 24\sin\beta$.

(a) –7 and 7 (b) – 25 and 25

(c) – 24 and 24 (d) –31 and 31

208. Find the minimum and maximum value of $5\sin^2\theta + 10\cos^2\theta + 12\sin\theta\cos\theta$.

(a) (1, 12) (b) (0, 14)

(c) (1, 14) (d) (–1, 14)

209. If $A - B = \frac{\pi}{4}$, then $(1 + \tan A)(1 - \tan B) = ?$

(a) 1 (b) – 1

(c) – 2 (d) 2

210. If $A + B = 135^\circ$, then $(1 + \cot A)(1 + \cot B) = ?$

(a) 1 (b) 2

(c) 3 (d) 4

211. If $\sec\theta = x + \frac{1}{4x}$, then find the value of $\sec\theta + \tan\theta$.

(a) x (b) $2x$

(c) $4x$ (d) $5x$

212. If $\text{cosec}\,\theta = x + \frac{1}{4x}$, then find the value of $\text{cosec}\,\theta + \cot\theta$.

(a) x (b) $2x$

(c) $4x$ (d) $5x$

213. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, then find the value of $m^2 - n^2$.

(a) $\sqrt{mn}$ (b) $2\sqrt{mn}$

(c) $3\sqrt{mn}$ (d) $4\sqrt{mn}$

214. If $\cot\theta + \cos\theta = m$ and $\cot\theta - \cos\theta = n$, then find the value of $m^2 - n^2$.

(a) $\sqrt{mn}$ (b) $2\sqrt{mn}$

(c) $3\sqrt{mn}$ (d) $4\sqrt{mn}$

215. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, then find the value of $\sqrt{mn}$.

(a) $\frac{1}{2}(m^2 - n^2)$ (b) $2(m^2 - n^2)$

(c) $\frac{1}{4}(m^2 - n^2)$ (d) $\frac{1}{4}(m^2 + n^2)$

216. If $\operatorname{cosec}\theta - \sin\theta = l$ and $\sec\theta - \cos\theta = m$, then $l^2m^2(l^2+m^2+3) = ?$

(a) 1 (b) – 1
(c) 2 (d) 4

217. If $\operatorname{cosec}\theta - \sin\theta = m$ and $\sec\theta - \cos\theta = n$, then $(m^2n)^{\frac{2}{3}} + (mn^2)^{\frac{2}{3}} = ?$

(a) 1 (b) 2
(c) 3 (d) 4

218. If $\cot\theta + \tan\theta = x$ and $\sec\theta - \cos\theta = y$, then $(x^2y)^{\frac{2}{3}} - (xy^2)^{\frac{2}{3}} = ?$

(a) 4 (b) 3
(c) 2 (d) 1

219. If $\sin\theta + \sin^2\theta + \sin^3\theta = 1$, then find the value of $\cos^6\theta - 4\cos^4\theta + 8\cos^2\theta$.

(a) 0 (b) 3
(c) 1 (d) 4

220. $\dfrac{\sin^8\theta - \cos^8\theta}{\cos 2\theta(1+\cos^2 2\theta)} = ?$

(a) 1 (b) $-\dfrac{1}{2}$
(c) –1 (d) 2

221. If $(\sec\alpha + \tan\alpha)(\sec\beta + \tan\beta)(\sec\gamma + \tan\gamma) = (\sec\alpha - \tan\alpha)(\sec\beta - \tan\beta)(\sec\gamma - \tan\gamma)$, then each of the side is equal to

(a) ± 1 (b) –1
(c) +1 (d) 4

222. If $a\sec\theta + b\tan\theta + c = 0$ and $p\sec\theta + q\tan\theta + r = 0$, then $(br - qc)^2 - (pc - ar)^2 = ?$

(a) $(bp - aq)$ (b) $(aq - bp)^2$
(c) $(aq + bp)^2$ (d) None of these

223. If $P = a\cos^3 x + 3a\cos x.\sin^2 x$ and $Q = a\sin^3 x + 3a\cos^2 x.\sin x$, then $(P + Q)^{2/3} + (P - Q)^{2/3} = ?$

(a) $2a^{2/3}$ (b) $a^{2/3}$
(c) $2a^{1/3}$ (d) $a^{1/3}$

224. If $\sin A + \sin B = -\dfrac{21}{65}$, $\cos A + \cos B = -\dfrac{27}{65}$ and $\pi < (A - B) < 3\pi$, then $\cos\left(\dfrac{A-B}{2}\right) = ?$

(a) $\dfrac{-3}{\sqrt{130}}$ (b) $\dfrac{3}{\sqrt{130}}$
(c) $\dfrac{6}{65}$ (d) $\dfrac{-6}{65}$

225. If $8\cos^2\theta + 8\sec^2\theta = 65$ and $0° < \theta < \dfrac{\pi}{2}$, then $4\cos 2\theta$ is equal to

(a) 3 (b) –3
(c) $-\dfrac{31}{8}$ (d) $\dfrac{4}{3}$

226. $\cos^4\dfrac{\pi}{8} + \cos^4\dfrac{3\pi}{8} + \cos^4\dfrac{5\pi}{8} + \cos^4\dfrac{7\pi}{8}$ is equal to

(a) $\dfrac{3}{2}$ (b) $\dfrac{-2}{3}$
(c) 1 (d) –1

227. $\cos\dfrac{\pi}{15} \cdot \cos\dfrac{2\pi}{15} \cdot \cos\dfrac{4\pi}{15} \cdot \cos\dfrac{8\pi}{15}$ is equal to

(a) $-\dfrac{1}{16}$ (b) $\dfrac{1}{16}$
(c) 1 (d) 0

228. $\left(1+\cos\dfrac{\pi}{8}\right)\left(1+\cos\dfrac{3\pi}{8}\right)\left(1+\cos\dfrac{5\pi}{8}\right)\left(1+\cos\dfrac{7\pi}{8}\right) = ?$

(a) $\dfrac{1}{4}$ (b) $\dfrac{1}{2}$
(c) $\dfrac{1}{6}$ (d) $\dfrac{1}{8}$

229. If $x = y\cos\dfrac{2\pi}{3} = z\cos\dfrac{4\pi}{3}$, then $xy + yz + zx$ is equal to

(a) –1 (b) 1
(c) 0 (d) 2

230. If A, B ∈ (0, π/2), sinA = $\frac{4}{5}$ and $\cos(A+B) = -\frac{12}{13}$, then sinB =?

(a) $\frac{61}{65}$ (b) $\frac{8}{65}$

(c) $\frac{63}{65}$ (d) $\frac{5}{13}$

231. If cos (θ – A) = a, cos (θ – B) = b, then $\sin^2(A-B) + 2ab\cos(A-B)$ is equal to

(a) $a^2 - b^2$ (b) $a^2 + b^2$

(c) $b^2 - a^2$ (d) $2ab$

232. $\sin^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8} + \sin^4\frac{5\pi}{8} + \sin^4\frac{7\pi}{8} = ?$

(a) $\frac{3}{2}$ (b) 1

(c) $\frac{1}{2}$ (d) 0

233. If α is acute angle and $2\sin^2\alpha + 15\cos^2\alpha = 7$, then cotα equal to

(a) $\frac{4}{3}$ (b) $\frac{\sqrt{5}}{2\sqrt{2}}$

(c) $\frac{2}{\sqrt{5}}$ (d) $\frac{3}{4}$

234. If $3x\sin\theta + 2y\cos\theta = 4$ and $2x\sin\theta - 3y\cos\theta = 2$, then relation between x and y.

(a) $\frac{81}{x^2} + \frac{49}{y^2} = 144$

(b) $\frac{36}{x^2} + \frac{64}{y^2} = 121$

(c) $\frac{289}{x^2} + \frac{4}{y^2} = 16$

(d) $\frac{256}{x^2} + \frac{4}{y^2} = 169$

235. If $\sin\theta + \cos\theta = a$ and $\sec\theta + \csc\theta = b$, then the value of $b(a^2 - 1)$ is equal to

(a) $2a$ (b) $3a$

(c) 0 (d) $2ab$

236. If $a\sec\theta + b\tan\theta = 1$ and $a^2\sec^2\theta - b^2\tan^2\theta = 5$, then $a^2b^2 + 4a^2$ is equal to

(a) $9b^2$ (b) $\frac{9}{a^2}$

(c) $\frac{-2}{b}$ (d) 9

237. If $\frac{\sec^4\alpha}{\sec^2\beta} - \frac{\tan^4\alpha}{\tan^2\beta} = 1$ where $\alpha, \beta \neq \frac{\pi}{2}$, then find the value of $\frac{\sec^4\beta}{\sec^2\alpha} - \frac{\tan^4\beta}{\tan^2\alpha}$.

(a) –1 (b) 0

(c) 1 (d) 2

238. If $\frac{\cos^4\alpha}{\cos^2\beta} + \frac{\sin^4\alpha}{\sin^2\beta} = 1$ then $\frac{\cos^4\beta}{\cos^2\alpha} + \frac{\sin^4\beta}{\sin^2\alpha} = ?$

(a) 4 (b) 0

(c) $\frac{1}{8}$ (d) 1

239. $7\csc\theta + 24\sec\theta = 25\csc\theta\sec\theta$, then $\cot\theta$ =?

(a) $\frac{5}{24}$ (b) $\frac{7}{24}$

(c) $\frac{11}{24}$ (d) $\frac{13}{24}$

240. If $8\sec\theta + 6\csc\theta = 20$, then $\cot\theta$ = ?

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$

(c) None of these (d) cant determine

241. $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7} = ?$

(a) $\frac{1}{2}$ (b) 0

(c) $-\frac{1}{2}$ (d) 1

242. $\cos 15° \cos 7\frac{1}{2}°.\cos 82\frac{1}{2}° = ?$

(a) $\frac{1}{2}$ (b) $\frac{1}{8}$

(c) $\frac{1}{4}$ (d) $\frac{1}{16}$

243. $\tan^2\theta = 1 - e^2$, then $\sec\theta + \tan^3\theta.\text{cosec}\theta = ?$

(a) $(e^2-1)^{3/2}$ (b) $1-e$

(c) $(2-e^2)^{3/2}$ (d) $(e^2-2)^{1/2}$

244. $3\tan\theta\tan\phi = 1$, then $\frac{\cos(\theta-\phi)}{\cos(\theta+\phi)} = ?$

(a) $\frac{1}{2}$ (b) 2

(c) $\frac{1}{3}$ (d) 3

245. $\tan 20° + \tan 40° + \sqrt{3}\tan 20°.\tan 40° = ?$

(a) $\frac{\sqrt{3}}{4}$ (b) $\frac{\sqrt{3}}{2}$

(c) $\sqrt{3}$ (d) 1

246. $\sin 36°.\sin 72°.\sin 108°.\sin 144° = ?$

(a) $\frac{5}{16}$ (b) $\frac{16}{5}$

(c) $\frac{13}{5}$ (d) $\frac{13}{16}$

247. $\frac{2\cos 40° - \cos 20°}{\sin 20°} = ?$

(a) 0 (b) $\sqrt{3}$

(c) –1 (d) 2

248. $2\sqrt{2}\sin 10°\left(\frac{\sec 5°}{2} + \frac{\cos 40°}{\sin 5°} - 2\sin 35°\right) = ?$

(a) 1 (b) 4

(c) $\sqrt{2}$ (d) –1

249. $\frac{\cos x}{\cos y} = n$, $\frac{\sin x}{\sin y} = m$,then

$(m^2-n^2)\sin^2 y = ?$

(a) $1-n^2$ (b) $1+n^2$

(c) m^2 (d) n^2

250. If $x\cos\theta + y\sin\theta = 4$ & $x\cos\theta - y\sin\theta = 0$, then which one is correct?

(a) $x^2+y^2=4$ (b) $x^2+y^2=16$

(c) $\frac{1}{x^2}+\frac{1}{y^2}=\frac{1}{4}$ (d) $\frac{1}{x^2}+\frac{1}{y^2}=4$

251. If $\tan^2\theta + \cot^2\theta = 14$, then $\sec\theta.\text{cosec}\theta = ?$

(a) 7 (b) 4

(c) $\sqrt{7}$ (d) 3

252. If $\cos\alpha+\beta = \frac{4}{5}$ & $\sin\alpha-\beta = \frac{5}{13}$ α,β -lie between 0 and $\frac{\pi}{4}$, then $\tan 2\alpha = ?$

(a) $\frac{56}{33}$ (b) $\frac{36}{33}$

(c) $\frac{33}{56}$ (d) $\frac{49}{36}$

253. If $\tan\theta - \tan\phi = x$ and $\cot\phi - \cot\theta = y$, then $\cot(\theta-\phi) = ?$

(a) $\frac{1}{x}-\frac{1}{y}$ (b) $\frac{1}{x}+\frac{1}{y}$

(c) $x+y$ (d) $\frac{1}{y}-\frac{1}{x}$

254. If $3\cos\theta = 5\sin\theta$, then

$\left(\frac{5\sin\theta - 2\sec^3\theta + 2\cos\theta}{5\sin\theta + 2\sec^3\theta - 2\cos\theta}\right) = ?$

(a) $\frac{371}{799}$ (b) $\frac{171}{979}$

(c) $\frac{271}{979}$ (d) $\frac{979}{271}$

255. $\sec\theta + \tan\theta = 2 + \sqrt{5}$, then $\sin\theta$ will be

(a) $\frac{1}{\sqrt{5}}$ (b) $\frac{\sqrt{3}}{2}$

(c) $\frac{4}{5}$ (d) $\frac{2}{\sqrt{5}}$

256. If $3\tan\theta + 4 = 0$, $\left(\frac{\pi}{2} < \theta < \pi\right)$, then find $2\cot\theta - 5\cos\theta - \sin\theta$.

(a) $-\frac{53}{10}$ (b) $\frac{7}{10}$

(c) $\frac{23}{10}$ (d) $\frac{37}{10}$

257. If $\frac{1+\sin x}{\cos x} + \frac{\cos x}{1+\sin x} = 4$, then find x.

(a) 60° (b) 30°

(c) 0° (d) 90°

258. $\cos\theta(\tan\theta + 2)(2\tan\theta + 1) = ?$

(a) $2\sec\theta + 5\sin\theta$ (b) $3\sec\theta + 4\sin\theta$

(c) $\sec\theta + \sin\theta$ (d) $4\sec\theta + 5\sin\theta$

259. $\cos\frac{\pi}{7} + \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} + \ldots\ldots\ldots\ldots\cos\frac{6\pi}{7} = ?$

(a) 0 (b) 1

(c) –1 (d) 2

260. $\tan 9° - \tan 27° - \tan 63° + \tan 81° = ?$

(a) 1 (b) 2

(c) 3 (d) 4

261. If $2\cos x + \sin x = 1$, then find $7\cos x + 6\sin x$.

(a) 6 (b) 2

(c) 7 (d) 1

262. If $\operatorname{cosec}\theta - \sin\theta = a^3$ & $\sec\theta - \cos\theta = b^3$, then $a^2b^2(a^2 + b^2) = ?$

(a) 1 (b) –1

(c) –2 (d) 2

263. If sinA + sinB = C, & cosA + cosB = D, then sin(A+B) = ?

(a) CD (b) $\frac{CD}{C^2 + D^2}$

(c) $\frac{C^2 + D^2}{2CD}$ (d) $\frac{2CD}{C^2 + D^2}$

264. If $\cos\theta = \frac{3}{5}$ and $\cos\phi = \frac{4}{5}$, where θ and ϕ are positive acute angles, then $\cos\left(\frac{\theta - \phi}{2}\right) = ?$

(a) $\frac{7}{\sqrt{2}}$ (b) $\frac{7}{5\sqrt{2}}$

(c) $\frac{7}{\sqrt{5}}$ (d) $\frac{7}{2\sqrt{5}}$

265. $\cos^2 48° - \sin^2 12° = ?$

(a) $\frac{\sqrt{5}-1}{4}$ (b) $\frac{\sqrt{5}+1}{8}$

(c) $\frac{\sqrt{3}-1}{4}$ (d) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

266. $\sin\left(\frac{\pi}{10}\right)\sin\left(\frac{3\pi}{10}\right) = ?$

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$

(c) $\frac{1}{4}$ (d) 1

267. If tanA – tanB = x and cotB – cotA = y, then cot(A–B) = ?

(a) $\frac{1}{x} + y$ (b) $\frac{1}{xy}$

(c) $\frac{1}{x} - \frac{1}{y}$ (d) $\frac{1}{x} + \frac{1}{y}$

268. $\tan\left(\frac{\pi}{4} + \theta\right) - \tan\left(\frac{\pi}{4} - \theta\right) = ?$

(a) $2\tan 2\theta$ (b) $2\cot 2\theta$

(c) $\tan 2\theta$ (d) $\cot 2\theta$

269. $\cot\left(\frac{\pi}{4} + \theta\right)\cot\left(\frac{\pi}{4} - \theta\right) = ?$

(a) –1 (b) 1

(c) 0 (d) ∞

270. If $\sin(\alpha-\beta)=\frac{1}{2}$ & $\cos(\alpha+\beta)=\frac{1}{2}$, where α,β are positive acute angles, then $\alpha\,\&\,\beta$ are –

(a) 45° and 15° (b) 60° and 15°
(c) 15° and 45° (d) 45° and 60°

271. If $\alpha+\beta-\gamma=\pi$, then $\sin^2\alpha+\sin^2\beta-\sin^2\gamma=?$

(a) $2\sin\alpha\sin\beta\cos\gamma$
(b) $2\cos\alpha\cos\beta\cos\gamma$
(c) $2\sin\alpha\sin\beta\sin\gamma$
(d) $2\cos\alpha\sin\beta\sin\gamma$

272. $1+\cos 2x+\cos 4x+\cos 6x=?$

(a) $2\cos x\cos 2x\cos 3x$
(b) $4\sin x\cos 2x\cos 3x$
(c) $4\cos x\cos 2x\cos 3x$
(d) $\cos x\cos 2x\cos 3x$

273. $\sin 12°\sin 48°\sin 54°=?$

(a) $\frac{1}{16}$ (b) $\frac{1}{8}$
(c) $\frac{1}{4}$ (d) $\frac{1}{32}$

274. $\frac{1}{4}\left[\sqrt{3}\cos 23°-\sin 23°\right]=?$

(a) cos 43° (b) cos 7°
(c) cos 53° (d) $\frac{1}{2}\cos 53°$

275. $2\cos x-\cos 3x-\cos 5x=?$

(a) $16\cos^3x\sin^2x$ (b) $\sin^3x\cos^2x$
(c) $4\cos^3x\sin^2x$ (d) $4\sin^3x\cos^2x$

276. $\sqrt{3}\,\text{cosec}\,20°-\sec 20°=?$

(a) 2 (b) $\frac{2\sin 20°}{\sin 40°}$
(c) 4 (d) $\frac{4\sin 20°}{\sin 40°}$

277. $\frac{\cos 12°-\sin 12°}{\cos 12°+\sin 12°}+\frac{\sin 147°}{\cos 147°}=?$

(a) 1 (b) –1
(c) 0 (d) $\frac{1}{2}$

278. $\cos 24°+\cos 55°+\cos 125°+\cos 204°+\cos 300°=?$

(a) 2 (b) $\frac{-1}{2}$
(c) 1 (d) $\frac{1}{2}$

279. $\frac{2\sin\theta\tan\theta(1-\tan\theta)+2\sin\theta\sec^2\theta}{(1+\tan\theta)^2}=?$

(a) $\frac{\sin\theta}{1+\tan\theta}$ (b) $\frac{2\sin\theta}{1+\tan\theta}$
(c) $\frac{2\sin\theta}{(1+\tan\theta)^2}$ (d) $\frac{2\sin\theta}{(1-\tan\theta)^2}$

280. If $m\tan(\theta-30°)=n\tan(\theta+120°)$, then $\frac{m+n}{m-n}=?$

(a) $2\cos 2\theta$ (b) $\cos 2\theta$
(c) $2\sin 2\theta$ (d) $\sin 2\theta$

281. $\cos A+\cos(240°+A)+\cos(240°-A)=?$

(a) $\cos A$ (b) 0
(c) $\sqrt{3}\sin A$ (d) $\sqrt{3}\cos A$

282. $1+\cos 56°+\cos 58°-\cos 66°=?$

(a) 2 cos 28°cos 29°sin 33°
(b) 4 cos 28°cos 29°cos 33°
(c) 4 cos 28°cos 29°sin 33°
(d) 2 cos 28°cos 29°cos 33°

283. If $A+B=225°$, then $\frac{\tan A}{1-\tan A}.\frac{\tan B}{1-\tan B}=?$

(a) 1 (b) –1
(c) 0 (d) $\frac{1}{2}$

284. $\frac{\sin(B+A)+\cos(B-A)}{\sin(B-A)+\cos(B+A)}=?$

(a) $\frac{\cos B+\sin B}{\cos B-\sin B}$ (b) $\frac{\cos A+\sin A}{\cos A-\sin A}$
(c) $\frac{\cos A-\sin A}{\cos A+\sin A}$ (d) $\frac{\cos B-\sin B}{\cos B+\sin B}$

285. If $\tan\theta = \dfrac{x\sin\phi}{1 - x\cos\phi}$ & $\tan\phi = \dfrac{y\sin\theta}{1 - y\cos\theta}$,

then $\dfrac{x}{y} = ?$

(a) $\dfrac{\sin\theta}{\sin\phi}$ (b) $\dfrac{\sin\phi}{\sin\theta}$

(c) $\dfrac{\sin\phi}{1 - \cos\theta}$ (d) $\dfrac{\sin\theta}{1 - \cos\phi}$

286. If $\tan x = \dfrac{b}{a}$, then $\sqrt{\dfrac{a+b}{a-b}} + \sqrt{\dfrac{a-b}{a+b}} = ?$

(a) $\dfrac{2\sin x}{\sqrt{\sin 2x}}$ (b) $\dfrac{2\cos x}{\sqrt{\cos 2x}}$

(c) $\dfrac{2\cos x}{\sqrt{\sin 2x}}$ (d) $\dfrac{2\cos x}{\sqrt{\cos 2x}}$

287. If $\cos(A+B) = \alpha\cos A\cos B + \beta\sin A\sin B$, then $(\alpha, \beta) = ?$

(a) $(-1, -1)$ (b) $(1, -1)$
(c) $(-1, 1)$ (d) $(1, 1)$

288. $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha = ?$

(a) $\tan\alpha$ (b) $\tan 2\alpha$
(c) $\cot\alpha$ (d) $\cot 2\alpha$

289. The greatest value of

$\cos^2\left(\dfrac{\pi}{3} - x\right) - \cos^2\left(\dfrac{\pi}{3} + x\right)$ is

(a) $-\dfrac{\sqrt{3}}{2}$ (b) $\dfrac{1}{2}$

(c) $\dfrac{\sqrt{3}}{2}$ (d) $\dfrac{3}{2}$

290. $\dfrac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}} = ?$ (x is in IV quadrant)

(a) $\dfrac{x}{2}$ (b) $\tan\dfrac{x}{2}$

(c) $\sec\dfrac{x}{2}$ (d) $\operatorname{cosec}\dfrac{x}{2}$

291. If $\sin A = n\sin B$, then $\left(\dfrac{n-1}{n+1}\right)\tan\left(\dfrac{A+B}{2}\right) = ?$

(a) $\sin\left(\dfrac{A-B}{2}\right)$ (b) $\tan\left(\dfrac{A-B}{2}\right)$

(c) $\cot\left(\dfrac{A-B}{2}\right)$ (d) $\tan\left(\dfrac{A+B}{2}\right)$

292. $\dfrac{\sin^2 A - \sin^2 B}{\sin A\cos A - \sin B\cos B} = ?$

(a) $\tan(A-B)$ (b) $\tan(A+B)$

(c) $\cot(A-B)$ (d) $\cot(A+B)$

293. If $\cos A = m\cos B$, then

(a) $\cot\dfrac{A+B}{2} = \dfrac{m+1}{m-1}\tan\dfrac{B-A}{2}$

(b) $\tan\dfrac{A+B}{2} = \dfrac{m+1}{m-1}\cot\dfrac{B-A}{2}$

(c) $\cot\dfrac{A+B}{2} = \dfrac{m+1}{m-1}\tan\dfrac{A-B}{2}$

(d) $\cot\dfrac{A+B}{2} = \dfrac{m-1}{m+1}\tan\dfrac{A-B}{2}$

294. If $x\cos\theta = y\cos\left(\theta + \dfrac{2\pi}{3}\right) = z\cos\left(\theta + \dfrac{4\pi}{3}\right)$, then

$\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z} = ?$

(a) 1 (b) 2

(c) 0 (d) $3\cos\theta$

295. $2\sin A\cos^3 A - 2\sin^3 A\cos A = ?$

(a) $\sin 4A$ (b) $\dfrac{1}{2}\sin 4A$

(c) $\dfrac{1}{4}\sin 4A$ (d) $\dfrac{1}{8}\sin 4A$

296. $\tan A + \tan(180° + A) + \cot(90° + A) + \cot(360° - A) = ?$

(a) 0 (b) 2 tan A
(c) 2 cot A (d) tan A – cot A

297. $\dfrac{\sin 3\theta + \sin 5\theta + \sin 7\theta + \sin 9\theta}{\cos 3\theta + \cos 5\theta + \cos 7\theta + \cos 9\theta} = ?$

(a) $\cot 6\theta$ (b) $\tan 6\theta$
(c) $\cot 3\theta$ (d) $\tan 3\theta$

298. If ABCD is a cyclic quadrilateral then cosA + cosB + cosC + cosD = ?

(a) 2 (cosA + cosC)
(b) 2 (cosA + cosB)
(c) 2 (cosA + cosD)
(d) 0

299. If $\dfrac{2\sin\alpha}{1+\cos\alpha+\sin\alpha} = y$, then

$\dfrac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha} = ?$

(a) y (b) $1/y$
(c) $1+y$ (d) $\dfrac{1}{1+y}$

300. $1-\dfrac{\sin^2 y}{1+\cos y}+\dfrac{1+\cos y}{\sin y}-\dfrac{\sin y}{1-\cos y} = ?$

(a) 0 (b) 1
(c) $\sin y$ (d) $\cos y$

301. $\dfrac{\sin 70° + \cos 40°}{\cos 70° + \sin 40°} = ?$

(a) 1 (b) $\dfrac{1}{\sqrt{3}}$
(c) $\sqrt{3}$ (d) $-\sqrt{3}$

302. If $x = \sec\phi - \tan\phi$ & $y = \operatorname{cosec}\phi + \cot\phi$, then

(a) $x = \dfrac{y+1}{y-1}$ (b) $y = \dfrac{1-x}{1+x}$
(c) $x = \dfrac{y-1}{y+1}$ (d) $y = \dfrac{1+x}{1-x}$

303. If $\dfrac{\sin^4 A}{a} + \dfrac{\cos^4 A}{b} = \dfrac{1}{a+b}$, then

$\dfrac{\sin^8 A}{a^3} + \dfrac{\cos^8 A}{b^3} = ?$

(a) $\dfrac{1}{(a+b)^3}$ (b) $\dfrac{a^2b^2}{(a+b)^2}$
(c) $\dfrac{a^3b^3}{(a+b)^3}$ (d) $\dfrac{ab}{a+b}$

304. If $2y\cos\theta = x\sin\theta$ and $2x\sec\theta - y\operatorname{cosec}\theta = 3$, then $x^2 + 4y^2 = ?$

(a) 4 (b) -4
(c) ± 4 (d) 0

305. If $\tan\theta - \cot\theta = a$ and $\cos\theta + \sin\theta = b$, then $\left(b^2-1\right)^2 (a^2+4) = ?$

(a) 2 (b) -4
(c) ± 4 (d) 4

306. If $\tan\theta = \dfrac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$, then $\sin\alpha + \cos\alpha$ and $\sin\alpha - \cos\alpha$ are equal to

(a) $\sqrt{2}\cos\theta, \sqrt{2}\sin\theta$
(b) $\sqrt{2}\sin\theta, \sqrt{2}\cos\theta$
(c) $\sqrt{2}\sin\theta, \sqrt{2}\sin\theta$
(d) $\sqrt{2}\cos\theta, \sqrt{2}\cos\theta$

307. If $\sin\theta + \sin\phi = a$ and $\cos\theta + \cos\phi = b$, then $\tan\left(\dfrac{\theta-\phi}{2}\right) = ?$

(a) $\sqrt{\dfrac{a^2+b^2}{4-a^2-b^2}}$ (b) $\sqrt{\dfrac{4-a^2-b^2}{a^2+b^2}}$
(c) $\sqrt{\dfrac{a^2+b^2}{4+a^2+b^2}}$ (d) $\sqrt{\dfrac{4+a^2+b^2}{a^2+b^2}}$

308. $\cos^2\alpha + \cos^2(\alpha + 120°) + \cos^2(\alpha - 120°) = ?$

(a) $\frac{3}{2}$ (b) 1

(c) $\frac{1}{2}$ (d) 0

309. The value of $\sin\frac{\pi}{14}.\sin\frac{3\pi}{14}.\sin\frac{5\pi}{14}.\sin\frac{7\pi}{14}$ $\sin\frac{9\pi}{14}.\sin\frac{11\pi}{14}.\sin\frac{13\pi}{14}$ is

(a) $\frac{1}{8}$ (b) $\frac{1}{16}$

(c) $\frac{1}{32}$ (d) $\frac{1}{64}$

310. If $\tan A = \frac{1-\cos B}{\sin B}$, express tan 2A in terms of tan B

(a) $\tan 2A = \tan B$ (b) $\tan 2A = \tan^2 B$

(c) $\tan 2A = \tan^2 A + \tan^2 B$

(d) $\tan 2A = \tan^2 A - \tan^2 B$

311. The value of sin 600° cos 330° + cos 120° sin 150° is

(a) 1 (b) –1

(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

312. If $\tan(A + B) = p$ & $\tan(A - B) = q$, then the value of tan 2A is

(a) $\frac{p+q}{p-q}$ (b) $\frac{p-q}{1+pq}$

(c) $\frac{p+q}{1-pq}$ (d) $\frac{1+pq}{p-q}$

313. $\frac{\sec 8A - 1}{\sec 4A - 1} = ?$

(a) $\frac{\tan 2A}{\tan 8A}$ (b) $\frac{\tan 8A}{\tan 2A}$

(c) $\frac{\cot 8A}{\cot 2A}$ (d) $\frac{\cot 2A}{\cot 8A}$

314. In a $\Delta ABC, \angle C = 90°$, then the equation whose roots are tan A & tan B is

(a) $abx^2 + c^2x + ab = 0$

(b) $abx^2 + c^2x - ab = 0$

(c) $abx^2 - c^2x - ab = 0$

(d) $abx^2 - c^2x + ab = 0$

315. If $\sin A + \sin 2A = x$ and $\cos A + \cos 2A = y$, then $(x^2 + y^2)(x^2 + y^2 - 3) = ?$

(a) $2y$ (b) $3y$

(c) $3y$ (d) $4y$

316. If $\cos(A - B) = \frac{3}{5}$ and tan A tan B = 2, then

(a) $\cos A \cos B = -\frac{2}{5}$

(b) $\sin A \sin B = \frac{2}{5}$

(c) $\cos A \cos B = -\frac{1}{5}$

(d) $\sin A \sin B = -\frac{1}{5}$

317. $\frac{\cos 9° + \sin 9°}{\cos 9° - \sin 9°} = ?$

(a) tan 54° (b) tan 36°

(c) tan 18° (d) cot 18°

318. If $\tan\alpha = \frac{1}{7}$ & $\tan\beta = \frac{1}{3}$, then $\cos 2\alpha = ?$

(a) $\sin 2\beta$ (b) $\sin 4\beta$

(c) $\sin 3\beta$ (d) $\cos 3\beta$

319. If A = 130° and $x = \sin A + \cos A$, then

(a) $x > 0$ (b) $x < 0$

(c) $x = 0$ (d) $x \le 0$

320. If $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$, then cos 2A = ?

(a) sin B (b) sin 2B

(c) sin 3B (d) cos 3B

321. If $\sin(120°-A) = \sin(120°-B)$, $0 < A, B < \pi$, then the values A and B are

(a) $A = B$

(b) $A = B$ or $A + B = \dfrac{\pi}{3}$

(c) $A + B = 0, A + B = \dfrac{\pi}{3}$

(d) None of these

322. $2\sin^2\beta + 4\cos(\alpha+\beta)\sin\alpha\sin\beta + \cos 2(\alpha+\beta) = ?$

(a) $\sin 2\alpha$ (b) $\cos 2\beta$

(c) $\cos 2\alpha$ (d) $\sin 2\beta$

323. The value of $\cos 12° + \cos 84° + \cos 156° + \cos 132°$ is

(a) $\dfrac{1}{2}$ (b) 1

(c) $-\dfrac{1}{2}$ (d) $\dfrac{1}{8}$

324. If $A + C = B$, then $\tan A \tan B \tan C = ?$

(a) $\tan A \tan B - \tan C$

(b) $\tan B - \tan C - \tan A$

(c) $\tan A + \tan B - \tan C$

(d) $-\tan A \tan B + \tan C$

325. $\tan\theta \sin\left(\dfrac{\pi}{2}+\theta\right)\cos\left(\dfrac{\pi}{2}-\theta\right) = ?$

(a) 1 (b) 0

(c) $\dfrac{1}{\sqrt{2}}$ (d) $\sin^2\theta$

326. If $x = \cos 10° \cos 20° \cos 40°$, then $x = ?$

(a) $\dfrac{1}{4}\tan 10°$ (b) $\dfrac{1}{8}\tan 10°$

(c) $\dfrac{1}{4}\cot 10°$ (d) $\dfrac{1}{8}\cot 10°$

327. $\sin 12° \sin 24° \sin 48° \sin 84° = ?$

(a) $\cos 20° \cos 40° \cos 60° \cos 80°$

(b) $\sin 20° \sin 40° \sin 60° \sin 80°$

(c) $\dfrac{3}{16}$ (d) $\dfrac{1}{16}$

328. $\tan 5x \tan 3x \tan 2x = ?$

(a) $\tan 5x - \tan 3x - \tan 2x$

(b) 0

(c) $\dfrac{\sin 6x - \sin 3x - \sin 2x}{\cos 5x - \cos 3x - \cos 2x}$

(d) $\tan 9x$

329. If $\cos\alpha + \cos\beta = 0 = \sin\alpha + \sin\beta$, then $\cos 2\alpha + \cos 2\beta = ?$

(a) $-2\sin(\alpha+\beta)$

(b) $2\cos(\alpha+\beta)$

(c) $2\sin(\alpha+\beta)$

(d) $-2\cos(\alpha+\beta)$

330. If $A + B + C = 180°$, then

$\dfrac{\tan A + \tan B + \tan C}{\tan A \tan B \tan C} = ?$

(a) 0 (b) 2

(c) 1 (d) –1

331. If $\cos A = a\cos B$ and $\sin A = b\sin B$, then $(b^2 - a^2)\sin^2 B = ?$

(a) $1 + a^2$ (b) $2 + a^2$

(c) $1 - a^2$ (d) $2 - a^2$

332. If $A + B + C = \pi$, then $\cos 2A + \cos 2B + \cos 2C = ?$

(a) $1 + 4\cos A \cos B \cos C$

(b) $-1 + 4\sin A \sin B \cos C$

(c) $-1 - 4\cos A \cos B \cos C$

(d) $1 + 4\sin A \sin B \sin C$.

333. If A, B, C are angles of a triangle, then $\sin^2 A + \sin^2 B + \sin^2 C - 2\cos A \cos B \cos C = ?$

(a) 1 (b) 2

(c) 3 (d) 4

334. If $A+B+C=180°$, then $\sin 2A+\sin 2B+\sin 2C=?$
(a) $4\sin A \sin B \cos C$
(b) $4\cos A \cos B \cos C$
(c) $4\sin A \sin B \sin C$
(d) $8\sin A \sin B \sin C$

335. $\cos 52° + \cos 68° + \cos 172° = ?$
(a) 0 (b) 1
(c) 2 (d) $\frac{3}{2}$

336. If $\cos 2B = \frac{\cos(A+C)}{\cos(A-C)}$, then tan A, tan B, tan C are in
(a) A.P. (b) GP.
(c) H.P. (d) None of these

337. The equation $(a+b)^2 = 4ab\sin^2\theta$ is true if and only if
(a) $2a=b$ (b) $a=b$
(c) $a=2b$ (d) $a>b$

338. If $\frac{\sin A - \sin C}{\cos C - \cos A} = \cot B$, then A, B, C are in
(a) A.P. (b) GP.
(c) H.P. (d) None of these

339. If $b\sin\alpha = a\sin(\alpha+2\beta)$, then $\frac{a+b}{a-b}$ =?
(a) $\frac{\tan\beta}{\tan(\alpha+\beta)}$ (b) $\frac{\cot\beta}{\cot(\alpha-\beta)}$
(c) $-\frac{\cot\beta}{\cot(\alpha+\beta)}$ (d) $\frac{\tan\beta}{\tan(\alpha-\beta)}$

340. $\tan 10° - \tan 50° + \tan 70° = ?$
(a) 0 (b) 1
(c) $\sqrt{3}$ (d) 3

341. $\cos^3 10° + \cos^3 110° + \cos^3 130° = ?$
(a) $\frac{3}{4}$ (b) $\frac{3}{8}$
(c) $\frac{3\sqrt{3}}{8}$ (d) $\frac{3\sqrt{3}}{4}$

342. For what value of α does the equation $4\cos\alpha + 3\cos 2\alpha - 2\sin 3\alpha + \cos 4\alpha = 2\sqrt{3}-1$ hold?
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

343. If $(\sin A+\sin B+\sin C)^2 = \sin^2 A+\sin^2 B+\sin^2 C$, then which one is true?
(a) $\sin A+\sin B+\sin C=0$
(b) $\cos A+\cos B+\cos C=0$
(c) $\frac{1}{\sin A}+\frac{1}{\sin B}+\frac{1}{\sin C}=0$
(d) None of these

344. If $\frac{\sin x}{\sin y} = p$ and $\frac{\cos x}{\cos y} = q$, then $\tan x = ?$
(a) $\frac{p}{q}\sqrt{\frac{q^2-2}{1-p^2}}$ (b) $\frac{p}{q}\sqrt{\frac{q^2-1}{1-p^2}}$
(c) $\frac{p}{q}\sqrt{\frac{1-q^2}{1-p^2}}$ (d) $\frac{p}{q}\sqrt{\frac{q^2-1}{1-p^2}}$

345. If $\cos A = \tan B$, $\cos B = \tan C$ and $\cos C = \tan A$, then sin A is equal to
(a) $\frac{\sqrt{5}-1}{4}$ (b) $\frac{\sqrt{5}-1}{2}$
(c) $\frac{\sqrt{3}-1}{4}$ (d) $\frac{\sqrt{3}-1}{2}$

346. If $\frac{3-\tan^2 A}{1-3\tan^2 A} = k$, where k is a real number, then $\operatorname{cosec} A(3\sin A - 4\sin^3 A)$ is equal to
(a) $\frac{2k}{k-1}$ where $k \ge \frac{1}{3}$ or $k \ge 3$
(b) $\frac{2k}{k-1}$, where $\frac{1}{3} \le k \le 3$
(c) $\frac{2k}{k-1}$, where $k < \frac{1}{3}$ or $k > 3$
(d) $\frac{2k}{k+1}$

347. If $a\cos^3\alpha + 3\cos\alpha\sin^2\alpha = m$ and $a\sin^3\alpha + 3\cos^2\alpha\sin\alpha = n$, then $(m+n)^{2/3} + (m-n)^{2/3} = ?$

(a) a (b) $2a$
(c) $2a^2$ (d) $2a^{2/3}$

348. If $y = a\cos^2 x + 2b\sin x\cos x + c\sin^2 x$ and $z = a\sin^2 x - 2b\sin x\cos x + c\cos^2 x$, then

(a) $y + z = a + c$ (b) $y + z = a + b$
(c) $y + a = x + b$ (d) None of these

349. If $A + B + C = \pi$, then $\frac{\cos A}{\sin B\sin C} + \frac{\cos B}{\sin C\sin A} + \frac{\cos C}{\sin A\sin B} = ?$

(a) 0 (b) 1
(c) 2 (d) 3

350. If sinA, cosA and tanA are in G.P., then $\cos^3 A + \cos^2 A = ?$

(a) 1 (b) 2
(c) 3 (d) 4

351. If $\cos(\theta-\alpha), \cos\theta, \cos(\theta+\alpha)$ are in H.P., then $\cos\theta\sec\frac{\alpha}{2} = ?$

(a) 0 (b) ± 1
(c) 2 (d) $\pm\sqrt{2}$

352. If $A + B = C$ and $\tan A = k\tan B$, and $A - B = \phi$, then $\sin C = ?$

(a) 0 (b) 1
(c) $\frac{k+1}{k-1}$ (d) $\frac{k+1}{k-1}\sin\phi$

353. If $\tan\alpha, \tan\beta$ are the roots of $x^2 + px + q = 0$ $(p \neq q)$ then $\tan(\alpha+\beta) = ?$

(a) $p - 1$ (b) $\frac{p}{q-1}$
(c) $2q - p$ (d) None of these

354. If $A + B + C = \frac{3\pi}{2}$, then $\cos 2A + \cos 2B + \cos 2C = ?$

(a) 1 – 4 sinA sinB sinC
(b) 1 – sin A sin B sin C
(c) 1 – 2sin A sin B sin C
(d) None of these

355. If $A + B + C = \pi$ and A, B,C are acute positive angles and cotA cotB cotC = k, then

(a) $k < 1$ (b) $k > \frac{1}{\sqrt{3}}$
(c) $k = \frac{1}{2\sqrt{3}}$ (d) $k \le \frac{1}{3\sqrt{3}}$

356. $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$, then $\frac{\tan x}{\tan y} = ?$

(a) a (b) b
(c) a/b (d) b/a

357. $\tan\frac{2\pi}{5} - \tan\frac{\pi}{15} - \sqrt{3}\tan\frac{2\pi}{5}\tan\frac{\pi}{15} = ?$

(a) 0 (b) 1
(c) $\sqrt{2}$ (d) $\sqrt{3}$

358. If $\sin A + \cos A = x$.

$\sin^6 A + \cos^6 A = \frac{1}{4}\left[4 - 3\left(x^2-1\right)^2\right]$ is true for all $x^2 = ?$

(a) < 0 (b) < 1
(c) ≤ -2 (d) ≤ 2

359. $\cos^2 10^\circ + \cos^2 50^\circ + \cos^2 70^\circ = ?$

(a) $\frac{1}{2}$ (b) 1
(c) $\frac{3}{2}$ (d) 2

360. If $A + B + C = 180^\circ$, then $\tan^2\frac{A}{2} + \tan^2\frac{B}{2} + \tan^2\frac{C}{2} = ?$

(a) 1 (b) 3
(c) 2 (d) 0

361. $\frac{\cot^2 15^\circ - 1}{\cot^2 15^\circ + 1} = ?$

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{2}{\sqrt{3}}$
(c) 1 (d) 0

Answer

1. (b)	**2.** (d)	**3.** (a)	**4.** (a)	**5.** (b)	**6.** (d)	**7.** (a)	**8.** (a)	**9.** (c)
10. (a)	**11.** (a)	**12.** (a)	**13.** (b)	**14.** (c)	**15.** (a)	**16.** (a)	**17.** (a)	**18.** (c)
19. (c)	**20.** (b)	**21.** (a)	**22.** (a)	**23.** (a)	**24.** (c)	**25.** (b)	**26.** (b)	**27.** (a)
28. (c)	**29.** (a)	**30.** (c)	**31.** (c)	**32.** (d)	**33.** (b)	**34.** (a)	**35.** (c)	**36.** (d)
37. (c)	**38.** (c)	**39.** (c)	**40.** (d)	**41.** (d)	**42.** (c)	**43.** (c)	**44.** (c)	**45.** (c)
46. (a)	**47.** (c)	**48.** (d)	**49.** (d)	**50.** (b)	**51.** (a)	**52.** (a)	**53.** (a)	**54.** (a)
55. (b)	**56.** (b)	**57.** (b)	**58.** (c)	**59.** (d)	**60.** (d)	**61.** (a)	**62.** (b)	**63.** (a)
64. (d)	**65.** (d)	**66.** (c)	**67.** (c)	**68.** (c)	**69.** (a)	**70.** (b)	**71.** (b)	**72.** (b)
73. (b)	**74.** (a)	**75.** (b)	**76.** (c)	**77.** (c)	**78.** (d)	**79.** (c)	**80.** (b)	**81.** (c)
82. (c)	**83.** (a)	**84.** (c)	**85.** (b)	**86.** (c)	**87.** (c)	**88.** (a)	**89.** (a)	**90.** (b)
91. (a)	**92.** (c)	**93.** (c)	**94.** (d)	**95.** (b)	**96.** (b)	**97.** (c)	**98.** (a)	**99.** (d)
100. (b)	**101.** (b)	**102.** (c)	**103.** (b)	**104.** (d)	**105.** (b)	**106.** (a)	**107.** (c)	**108.** (c)
109. (c)	**110.** (c)	**111.** (a)	**112.** (a)	**113.** (a)	**114.** (a)	**115.** (a)	**116.** (a)	**117.** (d)
118. (b)	**119.** (d)	**120.** (a)	**121.** (b)	**122.** (a)	**123.** (a)	**124.** (a)	**125.** (a)	**126.** (c)
127. (b)	**128.** (a)	**129.** (b)	**130.** (b)	**131.** (c)	**132.** (a)	**133.** (a)	**134.** (c)	**135.** (c)
136. (b)	**137.** (c)	**138.** (a)	**139.** (c)	**140.** (b)	**141.** (c)	**142.** (c)	**143.** (a)	**144.** (b)
145. (c)	**146.** (b)	**147.** (b)	**148.** (a)	**149.** (d)	**150.** (c)	**151.** (c)	**152.** (b)	**153.** (b)
154. (b)	**155.** (d)	**156.** (d)	**157.** (a)	**158.** (b)	**159.** (d)	**160.** (b)	**161.** (a)	**162.** (a)
163. (a)	**164.** (a)	**165.** (a)	**166.** (a)	**167.** (b)	**168.** (c)	**169.** (b)	**170.** (b)	**171.** (a)
172. (b)	**173.** (a)	**174.** (b)	**175.** (c)	**176.** (c)	**177.** (a)	**178.** (b)	**179.** (a)	**180.** (d)
181. (a)	**182.** (c)	**183.** (c)	**184.** (d)	**185.** (b)	**186.** (d)	**187.** (c)	**188.** (a)	**189.** (a)
190. (b)	**191.** (c)	**192.** (b)	**193.** (a)	**194.** (a)	**195.** (b)	**196.** (c)	**197.** (d)	**198.** (d)
199. (c)	**200.** (d)	**201.** (b)	**202.** (b)	**203.** (a)	**204.** (c)	**205.** (c)	**206.** (a)	**207.** (d)
208. (c)	**209.** (d)	**210.** (b)	**211.** (b)	**212.** (b)	**213.** (d)	**214.** (d)	**215.** (c)	**216.** (a)
217. (a)	**218.** (d)	**219.** (d)	**220.** (b)	**221.** (a)	**222.** (b)	**223.** **(a)**	**224.** **(b)**	**225.** (b)
226. **(a)**	**227.** (a)	**228.** (d)	**229.** (c)	**230.** (c)	**231.** (b)	**232.** (a)	**233.** (b)	**234.** (d)
235. (a)	**236.** (a)	**237.** (c)	**238.** (d)	**239.** (b)	**240.** (b)	**241.** (c)	**242.** (b)	**243.** (c)
244. (b)	**245.** (c)	**246.** (a)	**247.** (b)	**248.** (b)	**249.** (a)	**250.** (c)	**251.** (b)	**252.** (a)

253. (b)	**254.** (c)	**255.** (d)	**256.** (b)	**257.** (a)	**258.** (a)	**259.** (a)	**260.** (d)	**261.** (a)
262. (a)	**263.** (d)	**264.** (d)	**265.** (b)	**266.** (c)	**267.** (d)	**268.** (a)	**269.** (b)	**270.** (a)
271. (a)	**272.** (c)	**273.** (b)	**274.** (d)	**275.** (a)	**276.** (c)	**277.** (c)	**278.** (d)	**279.** (b)
280. (a)	**281.** (b)	**282.** (c)	**283.** (d)	**284.** (b)	**285.** (a)	**286.** (d)	**287.** (b)	**288.** (c)
289. (c)	**290.** (b)	**291.** (b)	**292.** (b)	**293.** (a)	**294.** (c)	**295.** (b)	**296.** (d)	**297.** (b)
298. (d)	**299.** (a)	**300.** (d)	**301.** (c)	**302.** (c)	**303.** (a)	**304.** (a)	**305.** (d)	**306.** (a)
307. (b)	**308.** (a)	**309.** (d)	**310.** (a)	**311.** (b)	**312.** (c)	**313.** (b)	**314.** (d)	**315.** (a)
316. (b)	**317.** (a)	**318.** (b)	**319.** (a)	**320.** (b)	**321.** (a)	**322.** (c)	**323.** (c)	**324.** (b)
325. (d)	**326.** (d)	**327.** (d)	**328.** (a)	**329.** (d)	**330.** (c)	**331.** (c)	**332.** (c)	**333.** (b)
334. (c)	**335.** (a)	**336.** (b)	**337.** (b)	**338.** (a)	**339.** (c)	**340.** (c)	**341.** (c)	**342.** (a)
343. (a)	**344.** (b)	**345.** (b)	**346.** (a)	**347.** (d)	**348.** (a)	**349.** (c)	**350.** (a)	**351.** (d)
352. (d)	**353.** (b)	**354.** (a)	**355.** (d)	**356.** (c)	**357.** (d)	**358.** (d)	**359.** (c)	**360.** (a)
361. (a)								

Solution & Hints

1. $\sin 43° = \sin(90° - 47°) = \cos 47°$ $[\therefore \sin(90° - \theta) = \cos\theta]$

$\Rightarrow \dfrac{\sin 43°}{\cos 47°} = 1$

2. $\sin\theta + \dfrac{1}{\sin\theta} = 2$

$\sin^2\theta - 2\sin\theta + 1 = 0$

$(\sin\theta - 1)^2 = 0$

$\sin\theta = 1 \quad \Rightarrow \operatorname{cosec}\theta = 1$

$\sin\theta^5 + \operatorname{cosec}^5\theta = (1)^5 + (1)^5 = 2$

3. $\tan\theta + \dfrac{1}{\tan\theta} = 2$

$(\tan\theta - 1)^2 = 0$

$\tan\theta = 1 \quad \Rightarrow \cot\theta = 1$

$\tan^2\theta + \cot^2\theta = (1)^2 + (1)^2 = 2$

4. $\cos\theta + \dfrac{1}{\cos\theta} = 2$

$(\cos\theta - 1)^2 = 0$

$\cos\theta = 1 \quad \Rightarrow \theta = 0°$

$\sin^{100}\theta + \sec^{100}\theta = (0)^{100} + (1)^{100} = 1$

5. $\dfrac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \dfrac{5}{4}$

(Applying C&D rule both the sides)

$\dfrac{\sin\theta + \cos\theta + \sin\theta - \cos\theta}{\sin\theta + \cos\theta - \sin\theta + \cos\theta} = \dfrac{5+4}{5-4}$

$\dfrac{2\sin\theta}{2\cos\theta} = \dfrac{9}{1} \quad \Rightarrow \tan\theta = 9$

$\dfrac{\tan^2\theta + 1}{\tan^2\theta - 1} = \dfrac{(9)^2 + 1}{(9)^2 - 1} = \dfrac{81+1}{81-1} = \dfrac{82}{80} = \dfrac{41}{40}$

6. $\dfrac{\tan\theta}{1 - \cot\theta} + \dfrac{\cot\theta}{1 - \tan\theta}$

$= \dfrac{\tan\theta}{1 - \dfrac{1}{\tan\theta}} + \dfrac{1}{\tan\theta(1 - \tan\theta)}$ $(\therefore \cot\theta = \dfrac{1}{\tan\theta})$

$= \dfrac{\tan^2\theta}{\tan\theta - 1} + \dfrac{1}{\tan\theta(1 - \tan\theta)}$

$= \dfrac{\tan^2\theta}{\tan\theta - 1} - \dfrac{1}{\tan\theta(\tan\theta - 1)}$

$= \dfrac{\tan^3\theta - 1}{\tan\theta(\tan\theta - 1)}$ $[\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)]$

$= \dfrac{(\tan\theta - 1)(\tan^2\theta + \tan\theta + 1)}{\tan\theta(\tan\theta - 1)}$

$\Rightarrow \dfrac{\tan^2\theta + \tan\theta + 1}{\tan\theta}$

$= 1 + \tan\theta + \cot\theta$

7. $\tan\theta + \cot\theta = 2$

$\tan\theta + \dfrac{1}{\tan\theta} = 2$

$\tan^2\theta - 2\tan\theta + 1 = 0$

$(\tan\theta - 1)^2 = 0$

$\tan\theta = 1 \Rightarrow \cot\theta = 1$ ($\therefore \theta$ is in first quadrant, $\tan\theta$ and $\cot\theta$ will always be +ve)

$\tan^n\theta + \cot^n\theta$

$\Rightarrow (1)^n + (1)^n = 2$

8. $\dfrac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta} = \dfrac{5}{3}$

(Applying C&D rule both sides)

$\dfrac{\sec\theta + \tan\theta + \sec\theta - \tan\theta}{\sec\theta + \tan\theta - \sec\theta + \tan\theta} = \dfrac{5+3}{5-3}$

$\dfrac{2\sec\theta}{2\tan\theta} = \dfrac{8}{2} \quad \Rightarrow \dfrac{1}{\cos\theta} \times \dfrac{\cos\theta}{\sin\theta} = 4 \quad \Rightarrow \sin\theta = \dfrac{1}{4}$

9. $\cos^4\theta - \sin^4\theta = \frac{2}{3}$ [$\because a^2 - b^2 = (a+b)(a-b)$]

$(\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta) = \frac{2}{3}$

$\cos^2\theta - \sin^2\theta = \frac{2}{3}$

$(\therefore \cos^2\theta + \sin^2\theta = 1)$

$\Rightarrow 1 - \sin^2\theta - \sin^2\theta = \frac{2}{3}$

$\Rightarrow 1 - 2\sin^2\theta = \frac{2}{3}$

10. $\tan 46° - \cot 44°$

$= \tan 46^o - \cot(90^o - 46^o)$

$(\therefore \cot(90^o - \theta) = \tan\theta)$

$= \tan 46^o - \tan 46^o = 0$

11. $\cos 51^o - \sin 39^o + \sin 37^o - \cos 53^o$

$\cos(90^o - 39^o) - \sin 39^o + \sin 37^o - \cos(90^o - 37^o)$

$\therefore [\cos(90^o - \theta) = \sin\theta]$

$\Rightarrow \sin 39^o - \sin 39^o + \sin 37^o - \sin 37^o = 0$

12. $x + \frac{1}{x} = 2\cos\theta$

$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)$

$= (2\cos\theta)^3 - 3(2\cos\theta)$

$\Rightarrow 8\cos^3\theta - 6\cos\theta$

$= 2(4\cos^3\theta - 3\cos\theta) = 2\cos 3\theta$

13. $\sec\theta + \tan\theta = 3$ ....(1)

Identity: $\sec^2\theta - \tan^2\theta = 1$

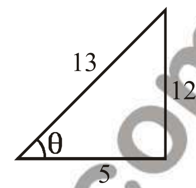
$(\sec + \tan\theta)(\sec - \tan\theta) = 1$

$3(\sec\theta - \tan\theta) = 1 \Rightarrow \sec\theta - \tan\theta = \frac{1}{3}$ ...(2)

Adding eq^n (1) and (2)

$2\sec\theta = 3 + \frac{1}{3} = \frac{10}{3} \Rightarrow \sec\theta = \frac{5}{3}$

14. $\cos\theta = \frac{5}{13} \Rightarrow \tan\theta = \frac{12}{5}, \sec\theta = \frac{13}{5}$



(Putting the values of $\tan\theta$ and $\sec\theta$)

$\tan^2\theta + \sec^2\theta = \frac{144}{25} + \frac{169}{25} = \frac{313}{25}$

15. $\tan\theta = \sqrt{6 + \sqrt{6 + \sqrt{6 + ...\infty}}}$

$\tan\theta = \sqrt{6 + \tan\theta}$ (squaring both sides)

$\tan^2\theta = 6 + \tan\theta \Rightarrow \tan^2\theta - \tan\theta - 6 = 0$

$(\tan\theta + 2)(\tan\theta - 3) = 0 \Rightarrow \tan\theta = 3$

($\therefore \tan\theta = -2$ is not applicable as θ is in 1st quadrant)

now, $\sec^2\theta = \tan^2\theta + 1$

Putting value of $\tan\theta$, we get

$\sec^2\theta = (3)^2 + 1 \Rightarrow 9 + 1 = 10$

16. $\alpha + \beta = 90^o, \alpha = 2\beta \Rightarrow 2\beta + \beta = 90^o$

$3\beta = 90^o \Rightarrow \beta = 30^o$ and $\alpha = 60^o$

$\cos^2 60^o + \sin^2 30^o = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

17. $\dfrac{1 - \tan^2 22\frac{1}{2}^o}{1 + \tan^2 22\frac{1}{2}^o}$

$\left[\cos 2\theta = \frac{1 - \tan^2\theta}{1 + \tan^2\theta}, \text{ here } \theta = 22\frac{1}{2}^o\right]$

$\Rightarrow \cos 45^o = \frac{1}{\sqrt{2}}$

18. $\sin^2 88^o + \cos^2 88^o = 1$

$(\therefore \sin^2\theta + \cos^2\theta = 1$ for any value of $\theta)$

19. $\tan(\theta+\phi) = \dfrac{\tan\theta+\tan\phi}{1-\tan\theta\tan\phi}$

Putting value of $\tan\theta$ and $\tan\phi$

$$\tan(\theta+\phi) = \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}} = \frac{5}{6}\times\frac{6}{5} = 1 = \tan 45^\circ$$

$\Rightarrow \theta + \phi = 45^\circ = \dfrac{\pi}{4}$

20. $(\tan A + \sec A - 1)\cos A$

$\because (\sec^2\theta - \tan^2\theta = 1)$

$= [\tan A + \sec A - (\sec^2 A - \tan^2 A)]\cos A$

$= [\tan A + \sec A - (\sec A + \tan A)(\sec A - \tan A)]\cos A$

taking $(\tan A + \sec A)$ as common

$= (\tan A + \sec A)(1 - \sec A + \tan A)\cos A$

$= \left(\dfrac{\sin A}{\cos A} + \dfrac{1}{\cos A}\right)(\tan A - \sec A + 1)\cos A$

$= \dfrac{(\sin A + 1)}{\cos A}(\tan A - \sec A + 1)\cos A$

$= (1+\sin A)(\tan A - \sec A + 1)$

21. $2\cos\theta = x + \dfrac{1}{x}$

Squaring both the sides

$4\cos^2\theta = x^2 + \dfrac{1}{x^2} + 2$

$2\cos^2\theta = \dfrac{1}{2}(x^2 + \dfrac{1}{x^2}) + 1$

22. $\dfrac{1-\cos A}{1+\cos A}$ (rationalisation)

$= \dfrac{(1-\cos A)}{(1+\cos A)}\times\dfrac{(1-\cos A)}{(1-\cos A)} = \dfrac{(1-\cos A)^2}{(1-\cos^2 A)} = \left(\dfrac{1-\cos A}{\sin A}\right)^2$

23. $\sqrt{\dfrac{(1-\sin\theta)}{(1+\sin\theta)}}$ (rationalisation)

$= \sqrt{\dfrac{(1-\sin\theta)}{(1+\sin\theta)}\times\dfrac{(1-\sin\theta)}{(1-\sin\theta)}} = \sqrt{\dfrac{(1-\sin\theta)^2}{1-\sin^2\theta}}$

$= \sqrt{\dfrac{(1-\sin\theta)^2}{\cos^2\theta}} = \dfrac{1-\sin\theta}{\cos\theta} = \sec\theta - \tan\theta$

24. $\sqrt{\dfrac{1+\cos\theta}{1-\cos\theta}} + \sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}$

$= \dfrac{\sqrt{1+\cos\theta}}{\sqrt{1-\cos\theta}} + \dfrac{\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}}$

$= \dfrac{\left(\sqrt{(1+\cos\theta)}\right)^2 + \left(\sqrt{(1-\cos\theta)}\right)^2}{\sqrt{1-\cos\theta}\sqrt{1+\cos\theta}}$

$= \dfrac{1+\cos\theta+1-\cos\theta}{\sqrt{1-\cos^2\theta}} = \dfrac{2}{\sin\theta} = 2\operatorname{cosec}\theta$

25. $\dfrac{\sqrt{1-\sin\theta}}{\sqrt{1+\sin\theta}} + \dfrac{\sqrt{1+\sin\theta}}{\sqrt{1-\sin\theta}}$

$= \dfrac{1-\sin\theta+1+\sin\theta}{\sqrt{1+\sin\theta}\sqrt{1-\sin\theta}} = \dfrac{2}{\sqrt{1-\sin^2\theta}}$

$= \dfrac{2}{\cos\theta} = 2\sec\theta$

26. $\sec\theta = A \quad \Rightarrow \cos\theta = \dfrac{1}{A}$

$\operatorname{cosec}\theta = B \quad \Rightarrow \sin\theta = \dfrac{1}{B}$

$(\therefore \sin^2\theta + \cos^2\theta = 1)$

$\dfrac{1}{A^2} + \dfrac{1}{B^2} = 1 \quad \Rightarrow \dfrac{A^2+B^2}{A^2B^2} = 1$

$\Rightarrow A^2 + B^2 = A^2B^2$

27. $\text{Sin}^6 A + \cos^6 A$

$\left[\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)\right]$

$= (\sin^2A+\cos^2A)(\sin^4A-\sin^2A.\cos^2A+\cos^4A)$

$= (\sin^2A+\cos^2A)[(\sin^4A+2\sin^2A.\cos^2A+\cos^4A) -3\sin^2A.\cos^2A]$

$\left(\therefore \sin^2 A + \cos^2 A = 1\right)$

$= \left[\left(\sin^2 A + \cos^2 A\right)^2 - 3\sin^2 A\cos^2 A\right]$

$= 1 - 3\sin^2 A\cos^2 A$

28. $\sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - \dfrac{1}{\sec^2\theta}}$

$\left[\because \sin^2\theta + \cos^2\theta = 1\right]$

$= \dfrac{\sqrt{\sec^2\theta - 1}}{\sec\theta}$

29. $\sin\alpha = \dfrac{1}{\sec(30° + \alpha)} = \cos(30° + \alpha)$

$\cos(90° - \alpha) = \cos(30° + \alpha)$

$\left[\because \cos(90° - \theta) = \sin\theta\right]$

$90° - \alpha = 30° + \alpha$

$\alpha = 30° \quad \Rightarrow \sin 30° + \cos 60° = \dfrac{1}{2} + \dfrac{1}{2} = 1$

30. $\sec^2\theta + \cos^2\theta - 2\sec\theta\cos\theta + \text{cosec}^2\theta + \sin^2\theta$

$-2\sin\theta\,\text{cosec}\theta - \tan^2\theta - \cot^2\theta + 2\tan\theta\cot\theta$

$\Rightarrow (\sec^2\theta - \tan^2\theta) + (\cos^2\theta + \sin^2\theta) + (\text{cosec}^2\theta - \cot^2\theta)$

$-2(\sec\theta\cos\theta + \sin\theta\,\text{cosec}\theta - \tan\theta\cot\theta)$

$\Rightarrow 1 + 1 + 1 - 2(1 + 1 - 1) = 3 - 2 = 1$

$$\begin{bmatrix} \sin^2\theta + \cos^2\theta = 1 \\ \sec^2\theta - \tan^2\theta = 1 \\ \text{cosec}^2\theta - \cot^2\theta = 1 \end{bmatrix}$$

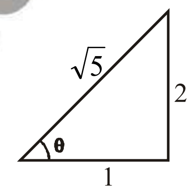
Method 2.

Put $\theta = 45°$

$= \left(\sqrt{2} - \dfrac{1}{\sqrt{2}}\right)^2 + \left(\sqrt{2} - \dfrac{1}{\sqrt{2}}\right)^2 - (1-1)^2$

$= \dfrac{1}{2} + \dfrac{1}{2} = 1$

31. $\dfrac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \dfrac{3}{1}$



$\sin\theta + \cos\theta = 3\sin\theta - 3\cos\theta$

$2\sin\theta = 4\cos\theta$

$\dfrac{\sin\theta}{\cos\theta} = \dfrac{4}{2} = 2$

$\tan\theta = 2$ (From right angle triangle)

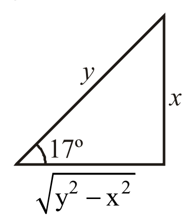
$\sin\theta = \dfrac{2}{\sqrt{5}}, \quad \cos\theta = \dfrac{1}{\sqrt{5}}$

$\Rightarrow \sin^4\theta - \cos^4\theta = (\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$\Rightarrow = \sin^2\theta - \cos^2\theta = \dfrac{4}{5} - \dfrac{1}{5} = \dfrac{3}{5}$

32. $\sin 17° = \dfrac{x}{y}$



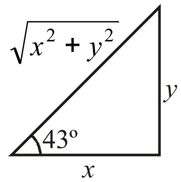
$\sec 17° = \dfrac{y}{\sqrt{y^2 - x^2}}$

$\sin 73° = \cos 17° = \dfrac{\sqrt{y^2 - x^2}}{y}$

$\sec 17° - \sin 73° = \dfrac{y}{\sqrt{y^2 - x^2}} - \dfrac{\sqrt{y^2 - x^2}}{y}$

$= \dfrac{y^2 - y^2 + x^2}{y\sqrt{y^2 - x^2}} = \dfrac{x^2}{y\sqrt{y^2 - x^2}}$

33. $\cos 43^\circ = \dfrac{x}{\sqrt{x^2+y^2}}$



$\tan 47^\circ = \cot 43^\circ = \dfrac{x}{y}$ $\quad [\because \tan(90^\circ - \theta) = \cot\theta]$

34. $\tan\theta = \dfrac{x}{y} = \dfrac{\sin\theta}{\cos\theta}$

here we can put, $\sin\theta = x$

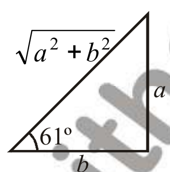
$\cos\theta = y$

$$\frac{x\sin\theta + y\cos\theta}{x\sin\theta - y\cos\theta} = \frac{x.x + y.y}{x.x - y.y} = \frac{x^2+y^2}{x^2-y^2}$$

35. $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta)$

$\sec^2\theta - \tan^2\theta = 1$

36. $\sin 61^\circ = \dfrac{a}{\sqrt{a^2+b^2}}$



$\tan 61^\circ = \dfrac{a}{b} \Rightarrow \tan 29^\circ = \cot 61^\circ = \dfrac{b}{a}$

$(\because \tan(90^\circ - \theta) = \cot\theta)$

$\tan 61^\circ + \tan 29^\circ = \dfrac{a}{b} + \dfrac{b}{a}$

37. $\dfrac{\cos^2\theta}{\dfrac{\cos^2\theta}{\sin^2\theta} - \cos^2\theta} = 3 \Rightarrow \dfrac{\cos^2\theta}{\cos^2\theta\left(\dfrac{1}{\sin^2\theta} - 1\right)} = 3$

$\dfrac{1}{\text{cosec}^2\theta - 1} = 3$ $\quad (\because \text{cosec}^2\theta - 1 = \cot^2\theta)$

$\dfrac{1}{\cot^2\theta} = 3 \Rightarrow \tan^2\theta = 3$

$\tan\theta = \sqrt{3} \Rightarrow \theta = 60^\circ$

38. $\dfrac{2\sin 68^\circ}{\cos 22^\circ} - \dfrac{2\cot 15^\circ}{5\tan 75^\circ} - \dfrac{3\tan 45^\circ \tan 20^\circ \tan 40^\circ \tan 50^\circ \tan 70^\circ}{5}$

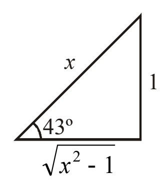
$= \dfrac{2\cos 22^\circ}{\cos 22^\circ} - \dfrac{2\tan 75^\circ}{5\tan 75^\circ} - \dfrac{3}{5}$

$= 2 - \dfrac{2}{5} - \dfrac{3}{5} \Rightarrow 2 - \dfrac{5}{5} = 2 - 1 = 1$

$\left[\begin{array}{l} \because \sin(90^\circ - \theta) = \cos\theta \\ \cot(90^\circ - \theta) = \tan\theta \\ \tan 20^\circ.\tan 70^\circ = 1 \text{ as } (20^\circ + 70^\circ) = 90^\circ \\ \tan 40^\circ.\tan 50^\circ = 1 \text{ as } (40^\circ + 50^\circ) = 90^\circ \end{array}\right]$

39. $\text{cosec } 39^\circ = x \Rightarrow \sin 39^\circ = \dfrac{1}{x}$

$\Rightarrow \cos 39^\circ = \dfrac{\sqrt{x^2-1}}{x} = \sin 51^\circ$



$\Rightarrow \cot 39^\circ = \sqrt{x^2-1}$

$\sin^2 51^\circ + \sin^2 39^\circ + \tan^2 51^\circ - \dfrac{1}{\sin^2 51^\circ.\text{cosec}^2 51^\circ}$

$= \dfrac{x^2-1}{x^2} + \dfrac{1}{x^2} + x^2 - 1 - 1$

$= 1 - \dfrac{1}{x^2} + \dfrac{1}{x^2} + x^2 - 1 - 1 \Rightarrow x^2 - 1$

40. $\dfrac{\sin 39^\circ}{\sin 39^\circ} + 2\tan 11^\circ \tan 79^\circ \tan 31^\circ \tan 59^\circ$

$\tan 45^\circ - 3(\sin^2 21^\circ + \cos^2 21^\circ)$

$\left(\begin{array}{l} \therefore \tan 11^\circ \tan 79^\circ = 1 \\ \tan 31^\circ \tan 59^\circ = 1 \end{array}\right)$

$= 1 + 2 - 3 = 0$

41. $\frac{1}{1+\tan^2\theta}+\frac{1}{1+\frac{1}{\tan^2\theta}} \Rightarrow \frac{1}{1+\tan^2\theta}+\frac{\tan^2\theta}{\tan^2\theta+1}$

$\frac{1+\tan^2\theta}{1+\tan^2\theta}=1$

42. $\frac{\cos^3 20^o-\cos^3 70^o}{\sin^3 70^o-\sin^3 20^o} \Rightarrow \frac{\sin^3 70^o-\sin^3 20^o}{\sin^3 70^o-\sin^3 20^o}=1$

$\left(\begin{matrix}\because \cos 20^o=\sin 70^o \\ \cos 70^o=\sin 20^o\end{matrix}\right)$

43. $\frac{\cos^n 38°-\cot^n 52°}{\sin^n 52°-\tan^n 38°}$

$=\frac{\cos^n(90°-52°)-\cot^n(90°-38°)}{\sin^n 52°-\tan^n 38°}$

$=\frac{\sin^n 52°-\tan^n 38°}{\sin^n 52°-\tan^n 38°}=1$

44. $\frac{\cot^n 29^o-\cot^n 61^o}{\tan^n 61^o-\tan^n 29^o}$

$(\because \cot^n 29^o=\cot^n(90^o-61^o)=\tan^n 61^o)$

similarly, $\cot^n 61^o=\tan^n 29^o$

$=\frac{\tan^n 61^o-\tan^n 29^o}{\tan^n 61^o-\tan^n 29^o}=1$

45. $x=\tan 15^o$

$\tan 15^o=\frac{\sqrt{3}-1}{\sqrt{3}+1}$

$x^2+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)^2-2$

$=\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)^2-2$

$=\left(\frac{(\sqrt{3}-1)^2+(\sqrt{3}+1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)}\right)^2-2$

$=\left(\frac{3+1-2\sqrt{3}+3+1+2\sqrt{3}}{3-1}\right)^2-2$

$=\left(\frac{8}{2}\right)^2-2 \Rightarrow (4)^2-2 \Rightarrow 16-2=14$

46. $x=\cot 75^o=\tan 15^o=\frac{\sqrt{3}-1}{\sqrt{3}+1}$

$\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2=x+\frac{1}{x}+2$

$=\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{\sqrt{3}+1}{\sqrt{3}-1}+2$

$\Rightarrow \frac{(\sqrt{3}-1)^2+(\sqrt{3}+1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)}+2$

$=\frac{3+1-2\sqrt{3}+3+1+2\sqrt{3}}{3-1}+2 \Rightarrow \frac{8}{2}+2=6$

$\therefore \sqrt{x}+\frac{1}{\sqrt{x}}=\sqrt{6}$

47. $\tan(A+B)=\frac{1}{2}$, $\tan(A–B)=\frac{1}{3}$

$\tan 2A=\tan(A+B+A-B)$

$\frac{\tan(A+B)+\tan(A-B)}{1-\tan(A+B)\tan(A-B)}=\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}.\frac{1}{3}}=\frac{5}{6}.\frac{6}{5}=1$

48. Same as 47.

49. $\cos(A-B)=\frac{1}{2}$ & $\sin(A+B)=\frac{1}{2}$

$A-B=\cos^{-1}\frac{1}{2}$, $A+B=\sin^{-1}\frac{1}{2}$

$A-B=60°$ .....(1), $A+B=150°$ ....(2)

Now, adding eq^n (1) & (2)

$\angle A=\frac{60^o+150^o}{2}=105°$

50. $\sin(x+y) = 1$ & $\tan(x-y) = \frac{1}{\sqrt{3}}$

$x + y = \sin^{-1} 1$ & $(x-y) = \tan^{-1}\frac{1}{\sqrt{3}}$

$x + y = 90°$ ... (1) $x - y = 30°$ ... (2)

Now, adding eq^n. (1) and (2)

$2x = 120°$

$x = 60°, y = 30°$

$\sin x + \tan y = \sin 60° + \tan 30° = \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}}$

$= \frac{3+2}{2\sqrt{3}} = \frac{5}{2\sqrt{3}}$

51. $\sin A + \cos A = \frac{3}{5}$

squaring both the sides

$\sin^2 A + \cos^2 A + 2\sin A \cos A = \frac{9}{25}$

$1 + 2\sin A \cos A = \frac{9}{25}$

$2\sin A \cos A = \frac{9}{25} - 1 = \frac{-16}{25}$

Now, finding $(\sin A - \cos A)^2 = \sin^2 A + \cos^2 A - 2\sin A \cos A$

$= 1 - \left(\frac{-16}{25}\right) = 1 + \frac{16}{25} = \frac{41}{25}$

$\sin A - \cos A = \frac{\sqrt{41}}{5}$

52. $\sec^2\theta + \tan^2\theta = \frac{7}{12}$ ... (1)

identity, $\sec^2\theta - \tan^2\theta = 1$ ... (2)

Multiplying eq^n (1) & (2)

$(\sec^2\theta + \tan^2\theta).(\sec^2\theta - \tan^2\theta) = \frac{7}{12}$

$\sec^4\theta - \tan^4\theta = \frac{7}{12}$

53. $\text{cosec}^2\theta + \cot^2\theta = \frac{5}{7}$ ... (1)

identity $\text{cosec}^2\theta - \cot^2\theta = 1$ ... (2)

Multiplying eq^n (1) & (2)

$(\text{cosec}^2\theta + \cot^2\theta).(\text{cosec}^2\theta - \cot^2\theta) = \frac{5}{7}$

$\text{cosec}^4\theta - \cot^4\theta = \frac{5}{7}$

54. $\sec x + \tan x = a$ ... (1)

Identity $\sec^2 x - \tan^2 x = 1$

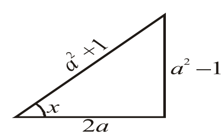
$(\sec x + \tan x)(\sec x - \tan x) = 1$

$\sec x - \tan x = \frac{1}{a}$ ... (2)

Adding eq^n (1) & (2) , We get

$2\sec x = a + \frac{1}{a}$

$\sec x = \frac{a^2+1}{2a}$



$\sin x = \frac{a^2-1}{a^2+1}$

55. $\text{cosec}\, x - \cot x = a$ ...(1)

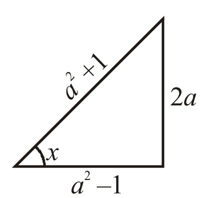
Identity, $\text{cosec}^2 x - \cot^2 x = 1$

$(\text{cosec}\, x + \cot x)(\text{cosec}\, x - \cot x) = 1$

$(\text{cosec}\, x + \cot x) = \frac{1}{a}$ ...(2)

Adding eq^n (1) & (2)

$2\,\text{cosec}\, x = a + \frac{1}{a}$



$\text{cosec}\, x = \frac{a^2+1}{2a}$

$\cos x = \frac{a^2-1}{a^2+1}$

56. $\sin^2 x + \text{cosec}^2 x + 2\sin x\,\text{cosec}\, x + \cos^2 x + \sec^2 x + 2\cos x \sec x - \tan^2 x - \cot^2 x$

$\Rightarrow$ ($\therefore$ $\sin^2 x + \cos^2 x = 1$, $\sec^2 x - \tan^2 x = 1$, $\text{cosec}^2 x - \cot^2 x = 1$)

$\Rightarrow 1+1+1+2+2 = 7$

57. $\sin^2 x + \text{cosec}^2 x + 2\sin x \text{ cosec } x + \cos^2 x + \sec^2 x + 2\cos x \sec x - \tan^2 x - \cot^2 x - 2\tan x \cot x$

$\Rightarrow$ ($\therefore$ $\sin^2 x + \cos^2 x = 1$, $\sec^2 x - \tan^2 x = 1$, $\text{cosec}^2 x - \cot^2 x = 1$)

$\Rightarrow$ $1+1+1+2+2-2 = 5$

58. $\sin^2 x + \text{cosec}^2 x - 2\sin x \text{ cosec } x + \cos^2 x + \sec^2 x - 2\cos x \sec x - \tan^2 x - \cot^2 x + 2\tan x \cot x$

$\Rightarrow$ ($\therefore$ $\sin^2 x + \cos^2 x = 1$, $\sec^2 x - \tan^2 x = 1$, $\text{cosec}^2 x - \cot^2 x = 1$)

$\Rightarrow$ $1+1+1-2-2+2 = 1$

59. $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta)$

$= 2(1 - 3\sin^2\theta\cos^2\theta) - 3(-2\sin^2\theta\cos^2\theta + 1)$

$= 2 - 6\sin^2\theta\cos^2\theta + 6\sin^2\theta\cos^2\theta - 3$

$= -1$

Or As answer is independent of θ

Put any value of θ

Let $\theta = 0°$

$2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta)$

$2(0+1) - 3(0+1)$

$2 - 3 = -1$

60. $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) - (\sin^2\theta + \cos^2\theta)^2$

As answer is independent of θ

Put any value of θ

Let $\theta = 0°$

$= 2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) - (\sin^2\theta + \cos^2\theta)^2$

$= 2 - 3 - 1$

$= -2$

61. $u_n = \cos^n\delta + \sin^n\delta$ $2u_6 - 3u_4 + 2 = 2(\cos^6\delta + \sin^6\delta) - 3(\cos^4\delta + \sin^4\delta) + 2$

Let $\delta = 0°$

$2 - 3 + 2 = 1$

62. $\dfrac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta} = 2\dfrac{51}{79} = \dfrac{209}{79}$

applying component dividend rule

$$\frac{2\sec\theta}{2\tan\theta} = \frac{288}{130}$$

$$\frac{\sec\theta}{\tan\theta} = \frac{144}{65}$$

$$\frac{1}{\cos\theta.\frac{\sin\theta}{\cos\theta}} = \frac{144}{65}$$

$$\frac{1}{\sin\theta} = \frac{144}{65} \Rightarrow \sin\theta = \frac{65}{144}$$

63. $\sin\theta + \sin^2\theta = 1$

$\sin\theta = 1 - \sin^2\theta = \cos^2\theta$

putting, $\cos^2\theta = \sin\theta$

$\cos^2\theta + \cos^4\theta = \sin\theta + \sin^2\theta$

$= 1$

64. $\cos\theta + \cos^2\theta = 1$

$\cos\theta = 1 - \cos^2\theta = \sin^2\theta$

putting, $\sin^2\theta = \cos\theta$

$\sin^4\theta + \sin\theta = \cos^2\theta + \cos\theta$

$= 1$

65. $\cos A + \cos^2 A = 1$

$\cos A = 1 - \cos^2 A = \sin^2 A$

Putting $\sin^2 A = \cos A$

$\sin^8 A + 2\sin^6 A + \sin^4 A = \cos^4 A + 2\cos^3 A + \cos^2 A$

$= (\cos^2 A + \cos A)^2 = 1$

66. $\cos A + \cos^2 A = 1$

$\cos A = 1 - \cos^2 A = \sin^2 A$

$\sin^{12} A + 3\sin^{10} A + 3\sin^8 A + \sin^6 A + \sin^4 A + \sin^2 A$

Putting $\sin^2 A = \cos A$

$= \cos^6 A + 3\cos^5 A + 3\cos^4 A + \cos^3 A + \cos^2 A + \cos A$

$= (\cos^2 A + \cos A)^3 + \cos^2 A + \cos A$

$= 1 + 1$

$= 2$

67. $\sin A + \sin^2 A = 1$

$\sin A = 1 - \sin^2 A = \cos^2 A$

$\cos^{12} A + 3\cos^{10} A + 3\cos^8 A + \cos^6 A + \cos^4 A + \cos^2 A$

Putting $\cos^2 A = \sin A$

$= \sin^6 A + 3\sin^5 A + 3\sin^4 A + \sin^3 A + \sin^2 A + \sin A$

$= (\sin^2 A + \sin A)^3 + \sin^2 A + \sin A$

$= 1 + 1$

$= 2$

68. $\cos^4 x = 1 - \cos^2 x = \sin^2 x$

$\cos^4 x = \sin^2 x$

$$\tan^2 x + \tan^4 x = \frac{\sin^2 x}{\cos^2 x} + \frac{\sin^4 x}{\cos^4 x}$$

$$= \frac{\sin^2 x}{\sin x} + \frac{\sin^4 x}{\sin^2 x}$$

$= \sin x + \sin^2 x$

$= \cos^2 x + \cos^4 x = 1$

69. $3\sin x + 4\cos x = 2 \quad \ldots(1)$

$3\cos x - 4\sin x = t \quad \ldots(2)$

squaring and adding eq^n (1) & (2)

$$= \begin{array}{l} 9\sin^2 x + 16\cos^2 x + 24\sin x\cos x + 9\cos^2 x + \\ 16\sin^2 x - 24\sin x\cos x = 4 + t^2 \end{array}$$

$$= 9\left(\sin^2 x + \cos^2 x\right) + 16\left(\sin^2 x + \cos^2 x\right) = 4 + t^2$$

$9 + 16 = 4 + t^2$

$25 = 4 + t^2$

$\sqrt{21} = t$

70. $\cos\theta - \sin\theta = \sqrt{2}\cos\theta \quad \ldots(1)$

$\cos\theta + \sin\theta = x \quad \ldots(2)$

squaring and adding eq^n (1) & (2)

$1 + 1 = 2\cos^2 q + x^2$

$x^2 = 2\left(1 - \cos^2\theta\right) = 2\sin^2\theta$

$x = \sqrt{2}\sin\theta$

71. $\sin q + \cos q = \sqrt{2} \quad \ldots(1)$

$\sin q - \cos q = x \quad \ldots(2)$

squaring and adding eq^n (1) & (2)

$1 + 1 = 2 + x^2$

$x^2 = 0$

$x = 0$

72. $\sin\theta + \cos\theta = p \quad \& \quad \frac{1}{\sin\theta} + \frac{1}{\cos\theta} = q$

$$\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta} = q$$

$$\sin\theta\cos\theta = \frac{p}{q}$$

$$q(p^2 - 1) = \frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}(\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1)$$

$$= q\left(2\sin\theta\cos\theta + 1 - 1\right) = q\left(2 \times \frac{p}{q}\right) = 2p$$

73. $$\frac{T_3 - T_5}{T_1} = \frac{\sin^3\theta + \cos^3\theta - \sin^5\theta - \cos^5\theta}{\sin\theta + \cos\theta}$$

$$= \frac{\sin^3\theta\left(1 - \sin^2\theta\right) + \cos^3\theta\left(1 - \cos^2\theta\right)}{\sin\theta + \cos\theta}$$

$$= \frac{\sin^3\theta\cos^2\theta + \cos^3\theta\sin^2\theta}{\sin\theta + \cos\theta}$$

$$= \frac{\sin^2\theta\cos^2\theta\left(\sin\theta + \cos\theta\right)}{\sin\theta + \cos\theta}$$

$= \sin^2\theta\cos^2\theta$

74. $\frac{x}{a} = \cos^3\theta, \quad \frac{y}{b} = \sin^3\theta$

$$= \left(\cos^3\theta\right)^{2/3} + \left(\sin^3\theta\right)^{2/3}$$

$= \cos^2\theta + \sin^2\theta = 1$

75. $\left(\frac{x}{a}\right)^{1/n} = \sec\theta, \quad \left(\frac{y}{b}\right)^{1/n} = \tan\theta$

Identity $\sec^2\theta - \tan^2\theta = 1$

$$\left(\frac{x}{a}\right)^{2/n} - \left(\frac{y}{b}\right)^{2/n} = 1$$

76. $\tan^5\theta.\tan^5 5\theta = 1$

$\left(\tan\theta.\tan 5\theta\right)^5 = 1$

$\tan\theta.\tan 5\theta = 1$

$\theta + 5\theta = 90°$

$6\theta = 90°$

$3\theta = 45°$

$\tan^n 45° = 1$

77. $\tan\theta.\tan 2\theta = 1$

$\tan\theta.\tan 2\theta = 1 \Rightarrow \theta = 30°$

$$\sin^2 60° + \tan^2 60° = \frac{3}{4} + 3 = \frac{15}{4} = 3\frac{3}{4}$$

78. $\cos\theta.\operatorname{cosec}23° = 1$

$\theta + 23° = 90°$

$\theta = 67°$

79. $\sin(x + y) = \cos(x - y)$

$\sin(x + y) = \sin\{90° - (x - y)\}$

$x + y = 90° - x + y$

$x = 45°$

$\cos^2 x = \cos^2 45° = \frac{1}{2}$

80. $\sin(2x - 20°) = \sin(90° - 2y - 20°)$

$2x - 20° = 90° - 2y - 20°$ ($\because \sin 90° - \theta = \cos\theta$)

$x + y = 45°$

$\sec(x + y) = \sec 45° = \sqrt{2}$

81. $A + B = 90°$

$\Rightarrow \tan A.\tan B = 1$

$\Rightarrow \cot A.\cot B = 1$

$\Rightarrow \sin A.\sec B = 1$

$B = 90° - A$

$\sin B = \sin(90° - A) = \cos A$

$= \sqrt{\frac{\tan A \tan B + \cot A \cot B}{\sin A.\sec B} - \frac{\cos^2 A}{\cos^2 A}}$

$= \sqrt{\frac{2}{1} - 1}$

$= 1$

82. $A + B = 90°$

$\cos B = \sin A$, $\sin B = \cos A$, $\tan A.\tan B = 1$

$= \sin A.\sin A + \cos A.\cos A + 2\tan A\tan B - (\sec^2 A - \tan^2 A)$

$= \sin^2 A + \cos^2 A + 2\tan A\tan B - (\sec^2 A - \tan^2 A)$

$= 1 + 2 - 1 = 2$

83. $\sin\theta = \cos\theta$

$\therefore \theta = 45°$

$2\tan^2\theta + \sin^2\theta - 1 = 2\tan^2 45° + \sin^2 45° - 1$

$= 2 + \frac{1}{2} - 1 = 1\frac{1}{2}$

84. $\sin(x + y) = \sin(90° - 3x - 3y)$

$x + y = 90° - 3x - 3y$

$4(x + y) = 90°$

$2(x + y) = 45°$

$\tan\{2(x + y)\} = \tan 45° = 1$

85. $\sec(5\theta - 50°) = \sec(90° - \theta - 32°)$

$5\theta - 50° = 90° - \theta - 32°$

$6\theta = 108°$

$\theta = 18°$

86. $\frac{a}{x} = \operatorname{cosec}\theta$, $\frac{b}{y} = \cot\theta$

$\frac{a^2}{x^2} - \frac{b^2}{y^2} = \operatorname{cosec}^2\theta - \cot^2\theta = 1$

87. $\frac{a}{x} = \sec\theta$, $\frac{b}{y} = \tan\theta$

$\frac{a^2}{x^2} - \frac{b^2}{y^2} = \sec^2\theta - \tan^2\theta = 1$

88. $\frac{x}{a} = \sec\theta\cos\theta$, $\frac{y}{b} = \sec\theta\sin\theta$, $\frac{z}{c} = \tan\theta$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = \sec^2\theta\cos^2\theta + \sec^2\theta\sin^2\theta - \tan^2\theta$

$= \sec^2\theta(\cos^2\theta + \sin^2\theta) - \tan^2\theta$

$= \sec^2\theta - \tan^2\theta = 1$

89. $3\sin\theta - 4\sin^3\theta = \sin 3\theta$

$3\sin 20° - 4\sin^3 20° = \sin 60° = \frac{\sqrt{3}}{2}$

90. $-(4\cos q\, 20° - 3\cos q\, 20°) = -\cos 60° = -\frac{1}{2}$

91. $\alpha = 0$, $\beta = 0$

$\tan^3 0° + \sin^5 0° = 0$

92. $\tan 1° \tan 2° \tan 3°$ .... $\tan 88° \tan 89°$

$= 1$ ($\tan A.\tan B = 1$, If $A + B = 90°$)

93. $\tan^n 1° \tan^n 2° \tan^n 3°$ .... $\tan^n 88° \tan^n 89°$

$= 1$ ($\tan A.\tan B = 1$, If $A + B = 90°$)

94. $\cos 1° \cos 2°$ .... $\cos 90° \cos 100° = 0$

($\because \cos 90° = 0$)

95. $\cos 1° \cos 2°$ .... $\cos 90° = 0$ ($\because \cos 90° = 0$)

96. $\sin 1° \sin 2°$ .... $\sin 180° = 0$ ($\because \sin 180° = 0$)

97. $\sin^2 1° + \sin^2 2° + \sin^2 3° + \ldots\ldots\ldots \sin^2 45° + \cos^2 44° + \cos^2 43° + \ldots\ldots\ldots \cos^2 1° + \sin^2 90°$

$= 1 + 1 \ldots\ldots 44 \text{ times} + \frac{1}{2} + 1$

$= 45\frac{1}{2}$

98. $\cos^2 1° + \cos^2 2° + \cos^2 3° + \ldots\ldots + \cos^2 45° + \sin^2 44° + \sin^2 43° + \sin^2 1° + \cos^2 90°$

$= 1 + 1 + \ldots\ldots\ 44 \text{ times} + \frac{1}{2}$

$= 44\frac{1}{2}$

99. $\sin^2 5° + \sin^2 10° + \ldots\ldots + \sin^2 45° + \cos^2 40° + \ldots\ldots\ldots + \cos^2 5° + \sin^2 90°$

$= 1 + 1 + \ldots\ldots\ 8 \text{ times} + \frac{1}{2} + 1$

$= 9\frac{1}{2}$

100. $\sin 10° \sin 50° \sin 70° \sin 30°$

$\left(\because \sin\theta \sin(60° - \theta)\sin(60° + \theta) = \frac{1}{4}\sin 3\theta\right)$

$\therefore \theta = 10°$

$= \frac{1}{4}\sin 30° \sin 30°$

$= \frac{1}{4} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$

101. $\underbrace{\tan 4° . \tan 86°}_{1} . \underbrace{\tan 43° . \tan 47°}_{1} = 1$

$[\because \tan\theta.(\tan(90° - \theta) = 1]$

102. $\cot 10° . \cot 80° . \cot 20° . \cot 70° . \cot 60°$

$= 1 \times \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

$[\because \cot\theta.(\cot(90° - \theta) = 1]$

103. $\tan 10° . \tan 80° . \tan 15° . \tan 75° = 1$

$[\tan\alpha \tan\beta = 1, \text{ if } \alpha + \beta = 90°]$

104. $\tan\frac{\pi}{8} \tan\frac{3\pi}{8} . \tan\frac{\pi}{12} . \tan\frac{5\pi}{12} - \sin^2\frac{\pi}{6}$

$= 1 - \frac{1}{4} = \frac{3}{4}$

$(\therefore \tan\frac{\pi}{8} . \tan\frac{3\pi}{8} = 1, \because \frac{\pi}{8} + \frac{3\pi}{8} = \frac{\pi}{2})$

$(\therefore \tan\frac{\pi}{12} . \tan\frac{5\pi}{12} = 1, \because \frac{\pi}{12} + \frac{5\pi}{12} = \frac{\pi}{2})$

105. $\cot\frac{\pi}{20} \cot\frac{9\pi}{20} . \cot\frac{3\pi}{20} \cot\frac{7\pi}{20} . \cot\frac{5\pi}{20}$

$= 1.1. \cot\frac{\pi}{4} = 1$

106. $\sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2(1 + \cos 16\theta)}}}}$

$= \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2.2\cos^2 8\theta}}}}$

$(\because 2\cos^2\theta = \cos 2\theta + 1)$

$= \sqrt{2 + \sqrt{2 + \sqrt{2 + 2\cos 8\theta}}}$

$= \sqrt{2 + \sqrt{2 + 2\cos 4\theta}}$

$= \sqrt{2 + 2\cos 2\theta} = 2\cos\theta$

107. $\tan 50° = \frac{\tan 70° - \tan 20°}{1 + \tan 70° \tan 20°}$

$\left(\because \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}\right)$

$\tan 50° = \frac{\tan 70° - \tan 20°}{1 + 1}$

$(\because \tan 70° \tan 20° = 1)$

$2\tan 50° + \tan 20° = \tan 70°$

108. $\tan 70° = \frac{\tan 80° - \tan 10°}{1 + \tan 80° \tan 10°}$

$(\because \tan 80° \tan 10° = 1)$

$2\tan 70° + \tan 10° = \tan 80°$

109. $\tan\left(\frac{\pi}{2}-\frac{\theta}{2}\right) = \cot\frac{\theta}{2} = \sqrt{3} \Rightarrow \cot\frac{\theta}{2} = \cot 30^\circ$

$\Rightarrow \frac{\theta}{2} = 30^\circ$

$\Rightarrow \theta = 60^\circ$

$\Rightarrow \cos\theta = \cos 60^\circ = \frac{1}{2}$

110. $A+B+C = 180^\circ$

$\frac{A+B}{2} = \frac{180^\circ - C}{2}$

$\frac{A+B}{2} = 90^\circ - \frac{C}{2}$

$\sin\left(\frac{A+B}{2}\right) = \sin\left(90^\circ - \frac{C}{2}\right) = \cos\frac{C}{2}$

$\cos\left(\frac{A+B}{2}\right) = \cos\left(90^\circ - \frac{C}{2}\right) = \sin\frac{C}{2}$

$\tan\left(\frac{A+B}{2}\right) = \tan\left(90^\circ - \frac{C}{2}\right) = \cot\frac{C}{2}$

$\cot\left(\frac{A+B}{2}\right) = \cot\left(90^\circ - \frac{C}{2}\right) = \tan\frac{C}{2}$

Option C is incorrect.

111. $\text{cosec } A = 2 \Rightarrow A = 30^\circ$

$\frac{1}{\tan A} + \frac{\sin A}{1+\cos A} = \frac{1}{\tan 30^\circ} + \frac{\sin 30^\circ}{1+\cos 30^\circ}$

$= \sqrt{3} + \frac{1}{2\left(1+\frac{\sqrt{3}}{2}\right)} = \sqrt{3} + \frac{1}{2+\sqrt{3}}$

$= \sqrt{3} + \frac{(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})} = \sqrt{3} + 2 - \sqrt{3} = 2$

112. $\tan A = \sqrt{2}-1$

$\sin 2A = \frac{2\tan A}{1+\tan^2 A} = \frac{2\left(\sqrt{2}-1\right)}{1+2+1-2\sqrt{2}} = \frac{2\left(\sqrt{2}-1\right)}{2\left(2-\sqrt{2}\right)}$

$= \frac{\sqrt{2}-1}{\sqrt{2}\left(\sqrt{2}-1\right)} = \frac{1}{\sqrt{2}}$

113. $\frac{5\times\frac{1}{4}+\frac{1}{2}-4\times\frac{1}{3}}{2.\frac{1}{2}.\frac{\sqrt{3}}{2}+1} = \frac{\frac{5}{4}+\frac{1}{2}-\frac{4}{3}}{\frac{\sqrt{3}}{2}+1}$

$\frac{15+6-16}{12} \times \frac{2}{2+\sqrt{3}}$

$= \frac{10}{12\,(2+\sqrt{3})} = \frac{5}{6} \times \frac{2-\sqrt{3}}{(2+\sqrt{3})\,(2-\sqrt{3})}$

$= \frac{5}{6}(2-\sqrt{3})$

114. $\frac{\sin\theta.\text{cosec}\theta.\tan\theta}{\sec\theta.\cos\theta.\tan\theta} = 1$

115. $\tan^2\alpha = \frac{8}{7}$

$\frac{(1+\sin\alpha)\,(1-\sin\alpha)}{(1+\cos\alpha)\,(1-\cos\alpha)} = \frac{1-\sin^2\alpha}{1-\cos^2\alpha} = \frac{\cos^2\alpha}{\sin^2\alpha}$

$= \cot^2\alpha = \frac{1}{\tan^2\alpha} = \frac{7}{8}$

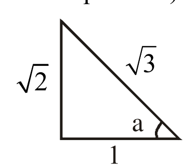
116. $\frac{k\,\text{cosec}^2 30^\circ.\sec^2 45}{8\cos^2 45^\circ.\sin^2 60^\circ} = \tan^2 60^\circ - \tan^2 30^\circ$

$\frac{4.k.2}{8.\frac{1}{2}.\frac{3}{4}} = 3 - \frac{1}{3} \Rightarrow k = 1$

117. $\sec^2\alpha = 3 \Rightarrow \sec\alpha = \pm\sqrt{3}$

But $\sec\alpha = +\sqrt{3}$ ($\because$ α is in Ist quadrant)

$\frac{\tan^2\alpha - \text{cosec}^2\alpha}{\tan^2\alpha + \text{cosec}^2\alpha} = \frac{2-\frac{3}{2}}{2+\frac{3}{2}}$



$= \frac{1}{2} \times \frac{2}{7} = \frac{1}{7}$

118. $5\alpha + 4\alpha = 90^\circ$

$\alpha = 10^\circ$

$2\sin 30^\circ - \sqrt{3}\tan 30^\circ = 2\times\frac{1}{2} - \sqrt{3}\times\frac{1}{\sqrt{3}} = 0$

119.

$$\sin^2\theta+\cos ec^2\theta+2\sin\theta\cos ec\theta+\cos^2\theta+\sec^2\theta+2\cos\theta\sec\theta = K + \tan^2\theta + \cot^2\theta$$

$$\Rightarrow 1+2+2+\cos ec^2\theta-\cot^2\theta+\sec^2\theta-\tan^2\theta = K$$

$$\Rightarrow 5+1+1=K$$

$$K=7$$

120. $\sin^2\alpha+\sec^2\alpha+2\sin\alpha\ \sec\alpha+\cos^2\alpha+\cos ec^2\alpha+2\text{cosa coseca} = K^2+\sec^2\alpha\ \cos ec^2\alpha+2K\ \text{seca coseca}$

$$\Rightarrow 1+\frac{1}{\sin^2\alpha}+\frac{1}{\cos^2\alpha}+\frac{2\sin\alpha}{\cos\alpha}+\frac{2\cos\alpha}{\sin\alpha}=K^2+\sec^2\alpha\ \text{cosec}^2\alpha+2K\sec\alpha\ \text{cosec}\alpha$$

$$\Rightarrow 1+\frac{\sin^2\alpha+\cos^2\alpha}{\sin^2\alpha\cos^2\alpha}+\frac{2(\sin^2\alpha+\cos^2\alpha)}{\sin\alpha\cos\alpha}=K^2+\sec^2\alpha\ \text{cosec}^2\alpha+2K\sec\alpha\ \text{cosec}\alpha$$

$$\Rightarrow 1+\sec^2\alpha\ \cos ec^2\alpha+2\sec\alpha\ \cos ec\alpha=K^2+\sec^2\alpha\cos ec^2\alpha+2K\sec\alpha\ \cos ec\alpha$$

comparing both the sides, we get. $\Rightarrow K = 1$

121. $$\frac{\sin\theta\cot\theta-1-\sin\theta\cot\theta-1}{(\cot\theta+\cos ec\theta)(\cot\theta-\cos ec\theta)}$$

$$=\frac{-2}{\cot^2\theta-\cos ec^2\theta}=\frac{-2}{-1}=2$$

122. $$\frac{\tan A}{1-\frac{1}{\tan A}}+\frac{1}{\tan A(1-\tan A)}=K+\tan A+\cot A$$

$$\Rightarrow -\frac{\tan^2 A}{(1-\tan A)}+\frac{1}{\tan A(1-\tan A)}=K+\tan A+\cot A$$

$$\Rightarrow \frac{1-\tan^3 A}{\tan A(1-\tan A)}=K+\tan A+\cot A$$

$$\Rightarrow \frac{(1-\tan A)(1+\tan^2 A+\tan A)}{\tan A(1-\tan A)}=K+\tan A+\cot A$$

$$\Rightarrow 1+\tan A+\cot A= K+\tan A+\cot A \Rightarrow K=1$$

123. $$\frac{\cos^2\theta}{1-\frac{\sin\theta}{\cos\theta}}+\frac{\sin^3\theta}{\sin\theta-\cos\theta}=K+\sin\theta\ \cos\theta$$

$$\Rightarrow \frac{\cos^3\theta}{\cos\theta-\sin\theta}-\frac{\sin^3\theta}{\cos\theta-\sin\theta}=K+\sin\theta\cos\theta$$

$$\Rightarrow \frac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta}=K+\sin\theta\ \cos\theta$$

$$\frac{(\cos\theta-\sin\theta)(\cos^2\theta+\sin^2\theta+\sin\theta\cos\theta)}{\cos\theta-\sin\theta}=K+\sin\theta\cos\theta$$

$$\Rightarrow 1+\sin\theta\cos\theta=K+\sin\theta\cos\theta$$

Comparing both sides, we get.

$K=1$

124. Put $\theta=0°$

$$=\frac{(1-0+1)^2}{(1+1)(1-0)}=\frac{4}{2}=2$$

125. $(\sec^2\theta-\tan^2\theta)^3=(1)^3=1$

126. $(\cos ec^2\theta-\cot^2\theta)^3=(1)^3=1$

127. $$\frac{\left(\frac{1}{\sin\theta}-\frac{1}{\cos\theta}\right)\left(\frac{\cos\theta}{\sin\theta}-\frac{\sin\theta}{\cos\theta}\right)}{\left(\frac{1}{\sin\theta}+\frac{1}{\cos\theta}\right)\left(\frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta}-2\right)}$$

$$=\frac{(\cos\theta-\sin\theta)(\cos^2\theta-\sin^2\theta)}{(\cos\theta+\sin\theta)\ (1-2\cos\theta\sin\theta)}$$

$$=\frac{(\cos\theta-\sin\theta)\ (\cos\theta+\sin\theta)\ (\cos\theta-\sin\theta)}{(\cos\theta+\sin\theta)\ (\cos^2\theta+\sin^2\theta-2\cos\theta\sin\theta)}$$

$$=\frac{(\cos\theta-\sin\theta)^2}{(\cos\theta-\sin\theta)^2} \Rightarrow \quad = 1$$

128. $\sec^4\alpha-\sec^4\alpha\,.\sin^4\alpha-2\tan^2\alpha$

$=\sec^4\alpha-\tan^4\alpha-2\tan^2\alpha$

$=[(\sec^2\alpha)^2-(\tan^2\alpha)^2]-2\tan^2\alpha$

$=[(\sec^2\alpha+\tan^2\alpha)(\sec^2\alpha-\tan^2\alpha)]-2\tan^2\alpha$

$=\sec^2\alpha+\tan^2\alpha-2\tan^2\alpha$

$(\because \sec^2\alpha-\tan^2\alpha=1)$

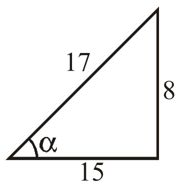
$=\sec^2\alpha-\tan^2\alpha=1$

129. $\sin\theta+\cos\theta=\sqrt{2}\cos\theta$

$\sin\theta=\left(\sqrt{2}-1\right)\cos\theta$

$\cot\theta=\dfrac{1}{\sqrt{2}-1}=\sqrt{2}+1$

130. $\cot\alpha=\dfrac{15}{8}$



$$\frac{(2+2\sin\alpha)(1-\sin\alpha)}{(1+\cos\alpha)(2-2\cos\alpha)}=\frac{\left(2+2\times\frac{8}{17}\right)\left(1-\frac{8}{17}\right)}{\left(1+\frac{15}{17}\right)\left(2-2\times\frac{15}{17}\right)}$$

$$=\frac{\frac{50}{17}\times\frac{9}{17}}{\frac{32}{17}\times\frac{4}{17}}=\frac{50\times 9}{32\times 4}=\frac{225}{64}$$

131. $\dfrac{x^2}{a^2}=\sin^2\alpha,\quad \dfrac{y^2}{b^2}=\cos^2\alpha$

$\sin^2\alpha+\cos^2\alpha=\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$

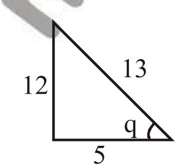
$b^2x^2+a^2y^2=a^2b^2$

132. Put $\theta=45°$

$$=\frac{\cot 45^o}{\cot 45^o-\cot 135^o}+\frac{\tan 45^o}{\tan 45^o-\tan 135^o}$$

$$=\frac{1}{1-(-1)}+\frac{1}{1-(-1)}\Rightarrow\quad=\frac{1}{2}+\frac{1}{2}=1$$

133. $\tan q-\cot q=\dfrac{119}{60}$



$\sin q+\cos q=\dfrac{12}{13}+\dfrac{5}{13}=\dfrac{17}{13}$

134. $16\left(\dfrac{1}{4+4\tan^2\alpha}+\dfrac{1}{4+4\cot^2\alpha}\right)$

$\dfrac{16}{4}\left(\dfrac{1}{\sec^2\alpha}+\dfrac{1}{\operatorname{cosec}^2\alpha}\right)=4\left(\sin^2\alpha+\cos^2\alpha\right)$

$=4$

135. $\cos^2\dfrac{\pi}{16}+\cos^2\dfrac{3\pi}{16}+\cos^2\dfrac{5\pi}{16}+\cos^2\dfrac{7\pi}{16}$

$=\cos^2\dfrac{\pi}{16}+\cos^2\dfrac{3\pi}{16}+\cos^2\left(\dfrac{\pi}{2}-\dfrac{3\pi}{16}\right)+\cos^2\left(\dfrac{\pi}{2}-\dfrac{\pi}{16}\right)$

$=\cos^2\dfrac{\pi}{16}+\cos^2\dfrac{3\pi}{16}+\sin^2\dfrac{3\pi}{16}+\sin^2\dfrac{\pi}{16}$

$=1+1=2$

136. $\cos^2(A-B)+\cos^2B-2\cos(A-B)\cos A\cos B$

Put $A=60°$ & $B=30°$

$=\cos^2 30°+\cos^2 30°-2\cos 30°.\cos 60°.\cos 30°$

$=\dfrac{3}{4}+\dfrac{3}{4}-2\times\dfrac{\sqrt{3}}{2}\times\dfrac{1}{2}\times\dfrac{\sqrt{3}}{2}\Rightarrow\quad=\dfrac{3}{4}+\dfrac{3}{4}-\dfrac{3}{4}$

$=\dfrac{3}{4}=\cos^2 30^o=\sin^2 60^o=\sin^2 A$

137. $\dfrac{\cot^2\frac{\theta}{2}-\tan^2\frac{\theta}{2}}{\cot\theta.\operatorname{cosec}\theta}\Rightarrow$ Put $\theta=60^o$

$=\dfrac{\cot^2 30^o-\tan^2 30^o}{\cot 60^o.\operatorname{cosec}60^o}$

$=\dfrac{3-\frac{1}{3}}{\frac{1}{\sqrt{3}}.\frac{2}{\sqrt{3}}}=\dfrac{8}{3}\times\dfrac{3}{2}=4$

138. Answer is independent of θ. So we can put any value of θ. Put $\theta=0^o$

$=\cos^2\theta+\cos^2(\alpha+\theta)-2\cos\alpha.\cos\theta.\cos(\theta+\alpha)$

$=\cos^2\theta+\cos^2\alpha-2\cos\alpha.\cos\theta.\cos\alpha$

$=1+\cos^2\alpha-2\cos^2\alpha=1-\cos^2\alpha=\sin^2\alpha$

139. $\sin\theta = 3\sin(\theta+2\alpha)$

$$\frac{\sin\theta}{\sin(\theta+2\alpha)} = \frac{3}{1}$$

Apply C & D rule

$$\frac{\sin\theta+\sin(\theta+2\alpha)}{\sin\theta-\sin(\theta+2\alpha)} = \frac{3+1}{3-1} = 2$$

$$\frac{2\sin\left(\frac{\theta+\theta+2\alpha}{2}\right).\cos\left(\frac{\theta-\theta-2\alpha}{2}\right)}{2\cos\left(\frac{\theta+\theta+2\alpha}{2}\right).\sin\left(\frac{\theta-\theta-2\alpha}{2}\right)} = 2$$

$$\frac{\sin(\theta+\alpha).\cos(-\alpha)}{\cos(\theta+\alpha).\sin(-\alpha)} = 2$$

$$\tan(\theta+\alpha) = -2\tan\alpha$$

$$\tan(\theta+\alpha)+2\tan\alpha = 0$$

140. $\tan x + \sec x = -2\tan x$

$\sec x = -3\tan x \Rightarrow \quad \frac{1}{\cos x} = \frac{-3\sin x}{\cos x}$

$\text{cosec } x = -3$

141.

$$\frac{\text{cosec }\theta-\cot\theta}{(\text{cosec }\theta+\cot\theta)(\text{cosec }\theta-\cot\theta)} - \text{cosec }\theta - \tan\theta = 3k\sec\theta\,\text{cosec }\theta$$

$$\Rightarrow \frac{\text{cosec }\theta-\cot\theta}{\text{cosec}^2\theta-\cot^2\theta} - \text{cosec }\theta - \tan\theta = 3k\sec\theta\,\text{cosec }\theta$$

$$\Rightarrow \text{cosec}\theta-\cot\theta-\text{cosec}\theta-\tan\theta = 3k\sec\theta\,\text{cosec }\theta$$

$$\Rightarrow -\left(\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right) = 3k\sec\theta\,\text{cosec }\theta$$

$$\Rightarrow -\frac{(\sin^2\theta+\cos^2\theta)}{\sin\theta\cos\theta} = 3k\sec\theta\,\text{cosec }\theta$$

$$\Rightarrow -\sec\theta\,\text{cosec}\theta = 3k\sec\theta\,\text{cosec }\theta \Rightarrow \quad k = -\frac{1}{3}$$

142. $\tan\theta = \frac{11}{13}$

$$\frac{5\tan\theta-3}{5\tan\theta+2} = \frac{\frac{55}{13}-3}{\frac{55}{13}+2} = \frac{55-39}{55+26} = \frac{16}{81}$$

143. $4\sin^2\theta + 3(\sin^2\theta + \cos^2\theta) = 4$

$4\sin^2\theta = 4-3 \Rightarrow \quad \sin\theta = \frac{1}{2} \quad \Rightarrow \theta = 30^\circ$

$\tan\theta = \tan 30^\circ = \frac{1}{\sqrt{3}}$

144. $(1+\cot\theta-\text{cosec }\theta)(1+\tan\theta+\sec\theta)$

Put $\theta = 45^\circ$

$= \left(1+1-\sqrt{2}\right)\left(1+1+\sqrt{2}\right)$

$= \left(2-\sqrt{2}\right)\left(2+\sqrt{2}\right) = 4-2 = 2$

145. $(\text{cosec}x-\sin x)(\sec x-\cos x)(\tan x+\cot x)$

Put $x = 45^\circ$

$= \left(\sqrt{2}-\frac{1}{\sqrt{2}}\right)\left(\sqrt{2}-\frac{1}{\sqrt{2}}\right)(1+1)$

$= \frac{(2-1)}{\sqrt{2}}\times\frac{(2-1)}{\sqrt{2}}\times 2 = 1$

146.
$$\frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta}{\sin\theta}-\frac{\cos 3\theta}{\sin 3\theta}} + \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}-\frac{\sin 3\theta}{\cos 3\theta}}$$

$$\Rightarrow \frac{\cos\theta}{\sin\theta}\times\frac{\sin\theta\sin 3\theta}{\sin 3\theta\cos\theta - \sin\theta\cos 3\theta} + \frac{\sin\theta}{\cos\theta}\times\frac{\cos\theta\cos 3\theta}{\sin\theta\cos 3\theta - \cos\theta\sin 3\theta}$$

$$\Rightarrow \frac{\cos\theta.\sin 3\theta}{\sin(3\theta-\theta)} + \frac{\sin\theta\cos 3\theta}{-\sin 2\theta}$$

$$\Rightarrow \frac{\cos\theta.\sin 3\theta}{\sin 2\theta} - \frac{\sin\theta\cos 3\theta}{2\sin\theta\cos\theta}$$

$$\Rightarrow \frac{\sin 3\theta}{2\sin\theta} - \frac{\cos 3\theta}{2\cos\theta}$$

$$\Rightarrow \frac{3\sin\theta - 4\sin^3\theta}{2\sin\theta} - \frac{4\cos^3\theta - 3\cos\theta}{2\cos\theta}$$

$$\Rightarrow \frac{3}{2} - 2\sin^2\theta - 2\cos^2\theta + \frac{3}{2}$$

$$\Rightarrow 3 - 2(\sin^2\theta + \cos^2\theta) \Rightarrow \quad 3-2 = 1$$

147. $\dfrac{\cot A + \operatorname{cosec}A - 1}{\cot A - \operatorname{cosec}A + 1}$

$= \dfrac{\cot A + \operatorname{cosec}A - (\operatorname{cosec}^2 A - \cot^2 A)}{\cot A - \operatorname{cosec}A + 1}$

$= \dfrac{(\cot A + \operatorname{cosec}A) - (\cot A + \operatorname{cosec}A)(-\cot A + \operatorname{cosec}A)}{\cot A - \operatorname{cosec}A + 1}$

$= \dfrac{(\cot A + \operatorname{cosec}A)(1 - \operatorname{cosec}A + \cot A)}{(1 - \operatorname{cosec}A + \cot A)}$

$= \cot A + \operatorname{cosec} A = \dfrac{\cos A + 1}{\sin A}$

148. $\dfrac{\sin\theta + 2\sin\theta\cos\theta}{1 + \cos\theta + 2\cos^2\theta - 1}$

$= \dfrac{\sin\theta(1 + 2\cos\theta)}{\cos\theta(1 + 2\cos\theta)} = \tan\theta$

149. $2\operatorname{cosec}2x.\cot x - \cot^2 x = 1$

taking L.H.S.

$= \dfrac{2}{\sin 2x}.\cot x - \cot^2 x$

$= \dfrac{2}{2\sin x\cos x}.\dfrac{\cos x}{\sin x} - \cot^2 x$

$= \dfrac{1}{\sin^2 x} - \cot^2 x$

$= \operatorname{cosec}^2 x - \cot^2 x$

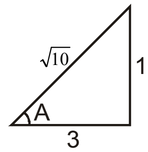
= 1 (from identity).

L.H.S. = R.H.S. for all x.

150. $\sec\theta = \dfrac{1}{2}$ is not possible

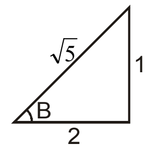
otherwise, $\cos\theta = 2$ which is impossible.

151. $\sin(A + B) = \sin A\cos B + \cos A\sin B$



$= \dfrac{1}{\sqrt{10}} \times \dfrac{2}{\sqrt{5}} + \dfrac{3}{\sqrt{10}} \times \dfrac{1}{\sqrt{5}}$

$= \dfrac{2}{\sqrt{50}} + \dfrac{3}{\sqrt{50}} = \dfrac{5}{\sqrt{50}} = \dfrac{1}{\sqrt{2}}$



$A + B = \sin^{-1}\dfrac{1}{\sqrt{2}} = \dfrac{\pi}{4}$

152. $\sqrt{\dfrac{1 - \sin 2A}{1 + \sin 2A}} = \sqrt{\dfrac{\sin^2 A + \cos^2 A - 2\sin A\cos A}{\sin^2 A + \cos^2 A + 2\sin A\cos A}}$

$= \dfrac{-\sin A + \cos A}{\sin A + \cos A} = \dfrac{-\tan A + 1}{\tan A + 1}$

$= \tan\left(\dfrac{\pi}{4} - A\right)$

153. $\tan\theta = \dfrac{4}{3}$ (θ may be 1st and 3rd quadrant)

$\sin\theta = \dfrac{4}{5}$ (1st quadrant)

$= -\dfrac{4}{5}$ (3rd quadrant)

154. $\dfrac{\tan A + \sec A - (\sec^2 A - \tan^2 A)}{\tan A - \sec A + 1}$

$= \dfrac{\tan A + \sec A - (\sec A - \tan A)(\sec A + \tan A)}{\tan A - \sec A + 1}$

$= \dfrac{(\sec A + \tan A)(1 - \sec A + \tan A)}{(1 - \sec A + \tan A)}$

$= \dfrac{1}{\cos A} + \dfrac{\sin A}{\cos A} = \dfrac{1 + \sin A}{\cos A}$

155. $x = \sec\theta - \tan\theta$, $y = \operatorname{cosec}\theta + \cot\theta$

Put, $\theta = 45^o$

$x = \sqrt{2} - 1$, $\quad y = \sqrt{2} + 1$

$xy = (\sqrt{2} - 1)(\sqrt{2} + 1) = 1$

$y - x = \sqrt{2} + 1 - \sqrt{2} + 1 = 2$

hence, $xy + 1 = y - x$

156. $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$

$\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta = 1$

Squaring and adding both the equations

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1+1=2$

157. $\sec^2\theta = \frac{4xy}{(x+y)^2}$

or, $\cos^2\theta = \frac{(x+y)^2}{4xy}$

$0 \le \cos^2\theta \le 1$

$\frac{(x+y)^2}{4xy} \le 1$

$(x+y)^2 \le 4xy \Rightarrow x^2+y^2+2xy-4xy \le 0$

$(x-y)^2 \le 0$

square at any term can never be less than 0.

So, $(x-y)^2 = 0$

$x = y$

158. $(\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta)$

Put, $\theta = 45^o$

$= \left(\sqrt{2}-\frac{1}{\sqrt{2}}\right)\left(\sqrt{2}-\frac{1}{\sqrt{2}}\right)(1+1)$

$= \frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}}.2 = 1$

159. $\operatorname{cosec}\theta - \cot\theta = q$ ....(i)

$\operatorname{cosec}\theta + \cot\theta = \frac{1}{q}$ ....(ii)

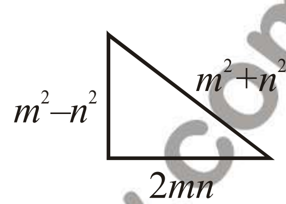
adding both equations

$2\operatorname{cosec}\theta = q + \frac{1}{q}$

$\operatorname{cosec}\theta = \frac{1}{2}\left(q+\frac{1}{q}\right)$

160. $\sin\theta = \frac{m^2-n^2}{m^2+n^2}$

$\tan\theta = \frac{m^2-n^2}{2mn}$



161. $\sec\theta + \tan\theta = p$ ....(i)

$\sec\theta - \tan\theta = \frac{1}{p}$ $\Rightarrow$ ....(ii)

Adding both equations

$2\sec\theta = p + \frac{1}{p} \Rightarrow \quad \sec\theta = \frac{1}{2}\left(p+\frac{1}{p}\right)$

162. $\tan\theta = \frac{p}{q}$

$\Rightarrow \frac{p\sin\theta - q\cos\theta}{p\sin\theta + q\cos\theta} = \frac{p\tan\theta - q}{p\tan\theta + q}$

$= \frac{p.\frac{p}{q} - q}{p.\frac{p}{q} + q} = \frac{\frac{p^2-q^2}{q}}{\frac{p^2+q^2}{q}} = \frac{p^2-q^2}{p^2+q^2}$

163. $\cos 15° = \sqrt{\frac{1+\cos 30°}{2}}$

$(\because \cos 2\theta = 2\cos^2\theta - 1)$

$\therefore \sqrt{\frac{\cos 2\theta + 1}{2}} = \cos\theta$

164. sinA = sinB, cosA = cosB

$\Rightarrow$ A = B

$$\Rightarrow \sin\left(\frac{A-B}{2}\right) = 0$$

165. $\tan 3A = \dfrac{\tan A + \tan 2A}{1 - \tan A \tan 2A}$

tan3A – tan2A – tanA = tan3A tan 2A tanA

166. $$\frac{1 + 2\sin\frac{A}{2}\cos\frac{A}{2} - 1 + 2\sin^2\frac{A}{2}}{1 + 2\sin\frac{A}{2}\cos\frac{A}{2} + 2\cos^2\frac{A}{2} - 1}$$

$$= \frac{\sin\frac{A}{2}\left(\cos\frac{A}{2} + \sin\frac{A}{2}\right)}{\cos\frac{A}{2}\left(\cos\frac{A}{2} + \sin\frac{A}{2}\right)}$$

$$= \frac{\sin\frac{A}{2}}{\cos\frac{A}{2}} = \tan\frac{A}{2}$$

167. $$\frac{1+\cos\theta}{\sin\theta} = \frac{1 + \dfrac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}}{\dfrac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}}$$

$$= \frac{1+\tan^2\frac{\theta}{2} + 1 - \tan^2\frac{\theta}{2}}{2\tan\frac{\theta}{2}}$$

$$= \frac{2}{2\tan\frac{\theta}{2}} = \cot\frac{\theta}{2}$$

168. $\cot x - \tan x = \dfrac{\cos x}{\sin x} - \dfrac{\sin x}{\cos x}$

$$= \frac{\cos^2 x - \sin^2 x}{\sin x \cos x}$$

$$= \frac{2\cos 2x}{2\sin x \cos x}$$

$$= 2 \times \frac{\cos 2x}{\sin 2x} = 2\cot 2x$$

169. $(\sin 50° + \sin 10°) - \sin 70°$

$2\sin 30°.\cos 20° - \sin 70°$

$\cos 20° - \sin 70°$

$\sin 70° - \sin 70° = 0$

170. sin75° = sin(45°+30°)

= sin45°.cos30°+cos45°sin30°

$$= \frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}.\frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

171. $\cos^2 A(3 - 4\cos^2 A)^2 + \sin^2 A(3 - 4\sin^2 A)^2$

$= (3\cos A - 4\cos^3 A)^2 + (3\sin A - 4\sin^3 A)^2$

$= (4\cos^3 A - 3\cos A)^2 + (3\sin A - 4\sin^3 A)^2$

$= (\cos 3A)^2 + (\sin 3A)^2$

$\cos^2 3A + \sin^2 3A = 1$

172. $\tan A = \dfrac{3}{2} \quad \Rightarrow \cot A = \dfrac{2}{3}$

$$\frac{1 + \cot A}{1 - \cot A} = \frac{1 + \frac{2}{3}}{1 - \frac{2}{3}} = \frac{5}{3} \times \frac{3}{1} = 5$$

173. tan75° – cot75° = tan75° – tan15°

$$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} - \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1} - \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$= \frac{3 + 1 + 2\sqrt{3} - 3 - 1 + 2\sqrt{3}}{3 - 1} = \frac{4\sqrt{3}}{2} = 2\sqrt{3}$$

174. $\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$$= \frac{\dfrac{m}{m+1} + \dfrac{1}{2m+1}}{1 - \dfrac{m}{m+1}.\dfrac{1}{2m+1}}$$

$= \dfrac{2m^2 + m + m + 1}{(m+1)(2m+1)} \times \dfrac{(m+1)(2m+1)}{2m^2 + 3m + 1 - m}$

$= \dfrac{(2m^2 + 2m + 1)}{(m+1)(2m+1)} \times \dfrac{(m+1)(2m+1)}{(2m^2 + 2m + 1)} = 1$

$\Rightarrow \alpha + \beta = \tan^{-1} 1 = \dfrac{\pi}{4}$

175. $\tan 20^o \tan 40^o \tan 80^o \tan 60^o$

$\{\because \tan\theta \tan(60^o - \theta)\tan(60^o + \theta) = \tan 3\theta\}$

hence, $\theta = 20°$

$= \tan 60^o . \tan 60^o$

$= \sqrt{3} . \sqrt{3} = 3$

176. $\cos\dfrac{5\pi}{4} - \sin\dfrac{5\pi}{4}$

$= \cos 225^o - \sin 225^o$

$= \cos(180^o + 45^o) - \sin(180^o + 45^o)$

$= -\cos 45° - (-\sin 45°)$

$= -\cos 45° + \sin 45° = -\dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}} = 0$

177. $\tan\theta = \dfrac{b}{a}$

$a\cos 2\theta + b\sin 2\theta = a\left(\dfrac{1-\tan^2\theta}{1+\tan^2\theta}\right) + b\left(\dfrac{2\tan\theta}{1+\tan^2\theta}\right)$

$= \dfrac{a\left(1 - \dfrac{b^2}{a^2}\right) + b \times \dfrac{2b}{a}}{1 + \dfrac{b^2}{a^2}} = \dfrac{\left(\dfrac{a^2 - b^2}{a}\right) + \dfrac{2b^2}{a}}{\dfrac{a^2 + b^2}{a^2}}$

$= \dfrac{(a^2 + b^2)}{a} \times \dfrac{a^2}{(a^2 + b^2)} = a$

178. $\cos A - \cos A + \cos A - \cos A = 0$

$\left(\begin{array}{l} \because \sin(270^o - \theta) = -\cos\theta \\ \cos(180^o + \theta = -\cos\theta \end{array}\right)$

179. $\sin 15^o + \cos 105^o$

$\sin 15^o - \sin 15^o = 0$

$(\because \cos 105^o = \cos(90^o + 15^o) = -\sin 15^o)$

180. $\cos 105° + \sin 105°$

$\cos 105° + \cos 15° = 2\cos 60°.\cos 45° = \dfrac{1}{\sqrt{2}}$

181. $\cos 15^o - \sin 15^o \quad \Rightarrow \quad \sin 75^o - \sin 15^o$

$= 2\cos 45^o \sin 30^o$

$= 2 \times \dfrac{1}{\sqrt{2}} \times \dfrac{1}{2} = \dfrac{1}{\sqrt{2}}$

182. Option is independent of angle q. So, we can put any value of q

Put, q = 0º, Then $x = 1$

$x^2 + (1 + x^2)$ sinq = 1 + 2sin0º = 1

183. $\operatorname{cosec}A.\sin(B + C)$

$= \operatorname{cosec}A.\sin(180^o - A)$

$= \operatorname{cosec}A.\sin A \quad = 1$

184. $\cos\theta = x + \dfrac{1}{x}$ for all real values of x

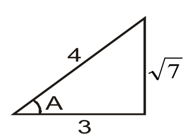
$x + \dfrac{1}{x}$ can never be between –2 and 2.

$x + \dfrac{1}{x} \geq 2$ or $x + \dfrac{1}{x} \leq -2$

but $\cos\theta$ lies in –1 and 1

So, the value of θ is not possible.

185. $\cos A = \dfrac{3}{4} \Rightarrow \sin A = \dfrac{\sqrt{7}}{4}$



$32\sin\dfrac{A}{2}.\cos\dfrac{5A}{2} = 16.2\sin\dfrac{A}{2}.\cos\dfrac{5A}{2}$

$= 16\left(\sin\left(\dfrac{A}{2} + \dfrac{5A}{2}\right) + \sin\left(\dfrac{A - 5A}{2}\right)\right)$

$= 16(\sin 3A - \sin 2A)$

$= 16(3\sin A - 4\sin^3 A - 2\sin A.\cos A)$

$= 16\sin A(3 - 4\sin^2 A - 2\cos A)$

$$= 16 \times \frac{\sqrt{7}}{4}\left(3 - 4 \times \frac{7}{16} - 2 \times \frac{3}{4}\right)$$

$$= 4\sqrt{7}\left(3 - \frac{7}{4} - \frac{6}{4}\right)$$

$$= \sqrt{7}(12 - 7 - 6)$$

$$= -\sqrt{7}$$

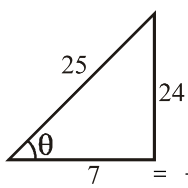
186. $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 3$

$\theta_1 = \theta_2 = \theta_3 = 90°$

$\cos\theta_1 + \cos\theta_2 + \cos\theta_3 = 0$

187. $\sin\theta = \frac{24}{25}$

$\sec\theta + \tan\theta = \frac{25}{-7} + \frac{24}{-7}$



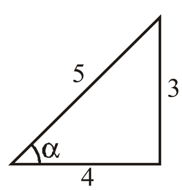
$= \frac{-49}{7} = -7$

188. $\frac{\cos 17° + \sin 17°}{\cos 17° - \sin 17°} = \frac{1 + \tan 17°}{1 - \tan 17°}$

$= \frac{\tan 45° + \tan 17°}{1 - \tan 45° . \tan 17°}$

$= \tan(45° + 17°) = \tan 62°$

189. $\sin\alpha = \frac{-3}{5}$



$\cos\alpha = \frac{-4}{5}$

$2\cos^2\frac{\alpha}{2} - 1 = \cos\alpha$

$2\cos^2\frac{\alpha}{2} = 1 + \cos\alpha = 1 - \frac{4}{5} = \frac{1}{5}$

$\cos^2\frac{\alpha}{2} = \frac{1}{10} \quad \Rightarrow \cos\frac{\alpha}{2} = \pm\frac{1}{\sqrt{10}}$

$\cos\frac{\alpha}{2} = \frac{-1}{\sqrt{10}}$

($\cos\frac{\alpha}{2}$ will be negative because a is in third quadrant so $\frac{\alpha}{2}$ will be in second quadrant.)

190. Greatest value of $\sqrt{3}\sin x + \cos x$

$= \sqrt{\left(\sqrt{3}\right)^2 + (1)^2} = \sqrt{3+1} = 2$

191. $\cos\theta(\tan\theta + 2)(2\tan\theta + 1)$

$= \cos\theta(2\tan^2\theta + \tan\theta + 4\tan\theta + 2)$

$= \cos\theta[2(1 + \tan^2\theta) + 5\tan\theta]$

$= \cos\theta(2\sec^2\theta + 5\tan\theta)$

$= 2\sec^2\theta\cos\theta + \frac{5\sin\theta}{\cos\theta}.\cos\theta$

$= 2\sec\theta + 5\sin\theta$

192. See example in book

193. Answer is independent of θ, so we can put any value of θ as $\theta = 0°$

$\frac{5\cos\theta - 4}{3 - 5\sin\theta} - \frac{3 + 5\sin\theta}{4 + 5\cos\theta} = \frac{1}{3} - \frac{3}{9} = 0$

194. $2\sin\alpha + 15\cos^2\alpha = 7$

$2\sin\alpha + 15(1 - \sin^2\alpha) = 7$

$15\sin^2\alpha - 2\sin\alpha - 8 = 0$

$15\sin^2\alpha - 12\sin\alpha + 10\sin\alpha - 8 = 0$

$5\sin\alpha(3\sin\alpha + 2) - 4(3\sin\alpha + 2) = 0$

$(3\sin\alpha + 2)(5\sin\alpha - 4) = 0$

here, $\sin\alpha = \frac{4}{5}, \frac{-2}{3}$ but, α is acute angle,

so, $\sin\alpha = \frac{4}{5}$ then, $\cot\alpha = \frac{3}{4}$

195. $3\tan\theta + 4 = 0 \Rightarrow \tan\theta = \frac{-4}{3}$

θ is in 2[nd] quadrant,

then, $\cot\theta = \frac{-3}{4}$, $\cos\theta = \frac{-3}{5}$ & $\sin\theta = \frac{4}{5}$

$$2\cot\theta - 5\cos\theta - \sin\theta = 2\left(\frac{-3}{4}\right) - 5\left(\frac{-3}{5}\right) - \frac{4}{5}$$

$$= \frac{-3}{2} + 3 - \frac{4}{5} = \frac{7}{10}$$

196. $\sec^2\theta = 3 \Rightarrow \tan^2\theta = \sec^2\theta - 1 = 2$

$$\sin^2\theta = 1 - \cos^2\theta = 1 - \frac{1}{\sec^2\theta} = 1 - \frac{1}{3} = \frac{2}{3}$$

$$\text{cosec}^2\theta = \frac{3}{2}$$

$$\frac{\tan^2\theta - \text{cosec}^2\theta}{\tan^2\theta + \text{cosec}^2\theta} = \frac{2 - \frac{3}{2}}{2 + \frac{3}{2}} = \frac{1}{7}$$

197. $3\cos\theta - 2\sin\theta = \frac{1}{\sqrt{2}} = 3\times\frac{1}{\sqrt{2}} - 2\times\frac{1}{\sqrt{2}}$

here, $\theta = 45^\circ$ is satisfying

then, $3\sin\theta + 2\cos\theta = \frac{3}{\sqrt{2}} + \frac{2}{\sqrt{2}} = \frac{5}{\sqrt{2}}$

198. $\tan\theta = \cot\theta \Rightarrow \theta = 45^\circ$

then, $\sin\theta - \cos\theta = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 0$

199. $\sin A + \text{cosec} A = 3$

$$\sin A + \frac{1}{\sin A} = 3$$

squaring both sides

$$\sin^2 A + \frac{1}{\sin^2 A} + 2 = 9$$

$$\frac{\sin^4 A + 1}{\sin^2 A} = 9 - 2 = 7$$

200. $\cos\alpha\ \text{cosec}\,\beta = 1$ if $\alpha + \beta = 90^\circ$

$\cos 7^\circ \cos 23^\circ \cos 45^\circ\ \text{cosec}\,83^\circ\ \text{cosec}\,67^\circ$

$= (\cos 7^\circ\ \text{cosec}\,83^\circ)(\cos 23^\circ\ \text{cosec}\,67^\circ)\cos 45^\circ$

$= 1\times 1\times\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

201. $\frac{\tan\theta + \cot\theta}{\tan\theta - \cot\theta} = 2$

$\tan\theta + \cot\theta = 2\tan\theta - 2\cot\theta$

$3\cot\theta = \tan\theta$

$\frac{3}{\tan\theta} = \tan\theta$

$\tan^2\theta = 3 \Rightarrow \tan\theta = \sqrt{3}$ or $\theta = 60^\circ$

$\sin\theta = \sin 60^\circ = \frac{\sqrt{3}}{2}$

202. $\sin x + \cos x = c$

squaring both sides

$\sin^2 x + \cos^2 x + 2\sin x\cos x = c^2$

$\sin x.\cos x = \frac{c^2 - 1}{2}$

we know that,

$\sin^6 x + \cos^6 x = 1 - 3\sin^2 x.\cos^2 x$

$= 1 - 3\left(\frac{c^2-1}{2}\right)^2 = 1 - 3\left(\frac{c^4 + 1 - 2c^2}{4}\right)$

$= \frac{1 + 6c^2 - 3c^4}{4}$

Method-2

put, $x = 0$

then $c = 1$

put, $c = 1$ in all option

option (a) $= \frac{1}{4}$, option (b) $= 1$

option (c) $= \frac{5}{8}$, option (d) $= \frac{5}{2}$

hence, $\sin^6 x + \cos^6 x = 1$ = option (b)

203. $\frac{1 - \sin A.\cos A}{\cos A\,(\sec A - \text{cosec} A)}\cdot\frac{\sin^2 A - \cos^2 A}{\sin^3 A + \cos^3 A}$

$$= \frac{(1 - \sin A\cos A)}{\cos A\left(\frac{\sin A - \cos A}{\sin A\cos A}\right)}\cdot\frac{(\sin A + \cos A)(\sin A - \cos A)}{(\sin A + \cos A)(\sin^2 A + \cos^2 A - \sin A\cos A)}$$

$$= \frac{\sin A\,(1 - \sin A.\cos A)}{(1 - \sin A.\cos A)} = \sin A$$

204. See example 77

205. $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2$

$= \sin^2\theta + \operatorname{cosec}^2\theta + 2 + \cos^2\theta + \sec^2\theta + 2$

$= 5 + \operatorname{cosec}^2\theta + \sec^2\theta$

$= 7 + \tan^2\theta + \cot^2\theta = 7 + 2\sqrt{1 \times 1} = 9$

206. $4\tan^2\theta + 9\cos^2\theta$

$= 4(\sec^2\theta - 1) + 9\cos^2\theta$

$= 9\cos^2\theta + 4\sec^2\theta - 4$

minimum value $= 2\sqrt{9 \times 4} - 4 = 8$

207. maximum and minimum value of $\cos\alpha$ and $\sin\beta$ will be 1 and – 1 (α, β are independent)

maximum value of $7\cos\alpha + 24\sin\beta$

$= 7 \times 1 + 24 \times 1 = 31$

minimum value of $7\cos\alpha + 24\sin\beta$

$= 7 \times (-1) + 24 \times (-1) = -31$

208. $5\sin^2\theta + 10\cos^2\theta + 12\sin\theta.\cos\theta$

$(\sin^2\theta + \cos^2\theta) + [(2\sin\theta)^2 + (3\cos\theta)^2 + 2 \times 2\sin\theta.3\cos\theta]$

$= 1 + (2\sin\theta + 3\cos\theta)^2$

then, minimum value = 1

maximum value $= 1 + \left(\sqrt{4+9}\right)^2 = 14$

209. $A - B = \frac{\pi}{4}$

$\tan(A - B) = \tan\frac{\pi}{4}$

$$\frac{\tan A - \tan B}{1 + \tan A.\tan B} = 1$$

tanA – tanB = 1 + tanA tanB

tanA – tanB – tanA tanB = 1

Adding 1 to both sides

1 + tanA – tanB – tanA tanB = 2

(1 + tanA) – tanB (1 + tanA) = 2

(1 + tanA) (1 – tanB) = 2

210. $A + B = 135°$

$\tan(A + B) = \tan 135° = -1$

$$\frac{\tan A + \tan B}{1 - \tan A.\tan B} = -1$$

tanA + tanB= –1 + tanA.tanB

$$\frac{1}{\cot A} + \frac{1}{\cot B} = \frac{1}{\cot A.\cot B} - 1$$

$$\frac{\cot B + \cot A}{\cot A.\cot B} = \frac{1 - \cot A.\cot B}{\cot A.\cot B}$$

cotA + cotB + cotA.cotB = 1

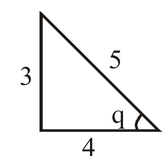
Adding 1 to both sides

(1 + cotA) + cotB (1 + cotA) = 1 + 1

(1 + cotA) (1 + cotB) = 2

211. Put $x = 1$

then, $\sec\theta = 1 + \frac{1}{4} = \frac{5}{4}$



hence, $\sec\theta + \tan\theta = \frac{5}{4} + \frac{3}{4} = 2$

Put $x = 1$ in all options then option (b) is equal to 2 hence option (b) is correct.

Method -2

$\sec\theta = x + \frac{1}{4x}$

$2\sec\theta = 2x + \frac{1}{2x}$ …(1)

$(\because \sec^2\theta - \tan^2\theta = 1)$

$2\tan\theta = 2x - \frac{1}{2x}$ …(2)

Adding both equations

$2(\sec\theta + \tan\theta) = 4x$

$\sec\theta + \tan\theta = 2x$

212. Hint: Same as above Q.211

213. $mn = (\tan\theta + \sin\theta)(\tan\theta - \sin\theta) = \tan^2\theta - \sin^2\theta$

$$= \frac{\sin^2\theta(1 - \cos^2\theta)}{\cos^2\theta}$$

$mn = \tan^2\theta.\sin^2\theta \Rightarrow \sqrt{mn} = \tan\theta.\sin\theta$

$m^2 - n^2 = (m + n)(m - n) = 2\tan\theta.2\sin\theta$

$m^2 - n^2 = 4\tan\theta\sin\theta$

$m^2 - n^2 = 4\tan\theta\sin\theta = 4\sqrt{mn}$

214. $mn = \cot^2\theta - \cos^2\theta$

$= \dfrac{\cos^2\theta\left(1-\sin^2\theta\right)}{\sin^2\theta}$

$m\,n = \cot^2\theta.\cos^2\theta \Rightarrow \sqrt{mn} = \cot\theta\,.\cos\theta$

$m^2 - n^2 = (m+n)(m-n) = 2\cot\theta.2\cos\theta$

$= 4\cot\theta\cos\theta = 4\sqrt{mn}$

215. $mn = (\tan\theta + \sin\theta)(\tan\theta - \sin\theta)$

$= \tan^2\theta - \sin^2\theta = \dfrac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$

$= \dfrac{\sin^2\theta\,(1-\cos^2\theta)}{\cos^2\theta}$

$= \tan^2\theta\sin^2\theta$

$(m^2 - n^2) = (m+n)\,(m-n) = 4\tan\theta\sin\theta$

$\sqrt{mn} = \tan\theta\sin\theta = \dfrac{1}{4}(m^2 - n^2)$

216. $\operatorname{cosec}\theta - \sin\theta = l$ & $\sec\theta - \cos\theta = m$

put $\theta = 45^o$

$\sqrt{2} - \dfrac{1}{\sqrt{2}} = l,\qquad \sqrt{2} - \dfrac{1}{\sqrt{2}} = m$

$\dfrac{1}{\sqrt{2}} = l,\qquad \dfrac{1}{\sqrt{2}} = m$

$\Rightarrow l^2m^2\left(l^2 + m^2 + 3\right) = \dfrac{1}{2}\times\dfrac{1}{2}\left(\dfrac{1}{2} + \dfrac{1}{2} + 3\right) = \dfrac{1}{4}(4) = 1$

217. $\operatorname{cosec}\theta - \sin\theta = m$ & $\sec\theta - \cos\theta = n$

put, $\theta = 45^o$

$m = \dfrac{1}{\sqrt{2}},\ n = \dfrac{1}{\sqrt{2}}$

$(m^2n)^{2/3} + (mn^2)^{2/3} = \left(\dfrac{1}{2\sqrt{2}}\right)^{2/3} + \left(\dfrac{1}{2\sqrt{2}}\right)^{2/3}$

$= \left(\dfrac{1}{8}\right)^{1/3} + \left(\dfrac{1}{8}\right)^{1/3} = \dfrac{1}{2} + \dfrac{1}{2} = 1$

218. $x = \cot\theta + \tan\theta = \dfrac{\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \dfrac{1}{\sin\theta\cos\theta}$

$y = \sec\theta - \cos\theta = \dfrac{1-\cos^2\theta}{\cos\theta} = \dfrac{\sin^2\theta}{\cos\theta}$

$(x^2y)^{2/3} = \left(\dfrac{1}{\sin^2\theta.\cos^2\theta}.\dfrac{\sin^2\theta}{\cos\theta}\right)^{2/3} = \dfrac{1}{\cos^2\theta}$

$(xy^2)^{2/3} = \left(\dfrac{1}{\sin\theta.\cos\theta}.\dfrac{\sin^4\theta}{\cos^2\theta}\right)^{2/3} = \dfrac{\sin^2\theta}{\cos^2\theta}$

$(x^2y)^{2/3} - (xy^2)^{2/3} = \dfrac{1}{\cos^2\theta} - \dfrac{\sin^2\theta}{\cos^2\theta} = \dfrac{\cos^2\theta}{\cos^2\theta} = 1$

Method-2

put $\theta = 45^o$

$x = \cot 45^o + \tan 45^o = 2$

$y = \sec 45^o - \cos 45^o = \dfrac{1}{\sqrt{2}}$

$(x^2y)^{2/3} - (xy^2)^{2/3} = \left(\dfrac{4}{\sqrt{2}}\right)^{2/3} - \left(\dfrac{2}{2}\right)^{2/3}$

$= 2 - 1 = 1$

219. $\sin\theta + \sin^2\theta + \sin^3\theta = 1$

$\sin\theta + \sin^3\theta = 1 - \sin^2\theta$

$\sin\theta + \sin^3\theta = \cos^2\theta$

$\sin\theta\left(1 + \sin^2\theta\right) = \cos^2\theta$

$\sin\theta\left(2 - \cos^2\theta\right) = \cos^2\theta$

Squaring both sides

$\sin^2\theta\left(2 - \cos^2\theta\right)^2 = \cos^4\theta$

$\left(1-\cos^2\theta\right)\left(4 + \cos^4\theta - 4\cos^2\theta\right) = \cos^4\theta$

On solving,

$\cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = 4$

220. $\dfrac{\sin^8\theta - \cos^8\theta}{\cos 2\theta\left(1 + \cos^2 2\theta\right)}$

$$= \frac{(\sin^4\theta+\cos^4\theta)(\sin^2\theta-\cos^2\theta)(\sin^2\theta+\cos^2\theta)}{(\cos^2\theta-\sin^2\theta)(1+(\cos^2\theta-\sin^2\theta)^2)}$$

$$= -\frac{(\sin^4\theta+\cos^4\theta)}{(1+\cos^4\theta+\sin^4\theta-2\sin^2\theta.\cos^2\theta)}$$

$$= -\frac{(1-2\sin^2\theta.\cos^2\theta)}{(1+1-2\sin^2\theta.\cos^2\theta-2\sin^2\theta.\cos^2\theta)}$$

$$= -\frac{1}{2}$$

Method-2

Answer is independent of angle θ, so put $\theta = 90^o$

$$\frac{\sin^8 90^o-\cos^8 90^o}{\cos 180^o(1+\cos^2 180^o)} = \frac{1-0}{-1(1+1)} = -\frac{1}{2}$$

221. $k=(\sec\alpha+\tan\alpha)(\sec\beta+\tan\beta)(\sec\gamma+\tan\gamma)$ ...(i)

$k=(\sec\alpha-\tan\alpha)(\sec\beta-\tan\beta)(\sec\gamma-\tan\gamma)$ ...(ii)

Multiplying equation (i) & (ii)

$k^2=(\sec^2\alpha-\tan^2\alpha)(\sec^2\beta-\tan^2\beta)(\sec^2\gamma-\tan^2\gamma)$

$k^2 = 1 \Rightarrow k=\pm1$

222. $a\sec\theta + b\tan\theta + c = 0$

$p\sec\theta + q\tan\theta + r = 0$

Apply cross-multiplication method to solve these eq^n

$$\frac{\sec\theta}{br-qc} = \frac{\tan\theta}{pc-ar} = \frac{1}{aq-bp}$$

$$\sec\theta = \frac{br-qc}{aq-bp} \;\&\; \tan\theta = \frac{pc-ar}{aq-bp}$$

$$\sec^2\theta - \tan^2\theta = 1$$

$$\left(\frac{br-qc}{aq-bp}\right)^2 - \left(\frac{pc-ar}{aq-bp}\right)^2 = 1$$

then, $(br-qc)^2 - (pc-ar)^2 = (aq-bp)^2$

223. $P = a\cos^3 x + 3a\cos x\sin^2 x$

$Q = a\sin^3 x + 3a\cos^2 x\sin x$

Put $x = 45°$

$P = \frac{a}{2\sqrt{2}} + \frac{3a}{2\sqrt{2}}$, $\quad Q = \frac{a}{2\sqrt{2}} + \frac{3a}{2\sqrt{2}}$

$P = \frac{4a}{2\sqrt{2}} = \sqrt{2}a$, $\quad Q = \sqrt{2}a$

$(P+Q)^{2/3} + (P-Q)^{2/3} = (\sqrt{2}a+\sqrt{2}a)^{2/3} + 0$

$= (2\sqrt{2}a)^{2/3} \quad = 2a^{2/3}$

224. $\sin A + \sin B = -\frac{21}{65}$ …(i)

$\cos A + \cos B = -\frac{27}{65}$ …(ii)

Adding both equation after squaring

$\Rightarrow$ $\sin^2 A + \sin^2 B + 2\sin A.\sin B + \cos^2 A + \cos^2 B +$

$2\cos A.\cos B = \left(-\frac{21}{65}\right)^2 + \left(-\frac{27}{65}\right)^2$

$$= \frac{21^2+27^2}{65^2} = \frac{3^2(7^2+9^2)}{65^2}$$

$$= \frac{9(49+81)}{65\times65} = \frac{9\times130}{65\times65} = \frac{18}{65}$$

$2 + 2(\sin A.\sin B + \cos A.\cos B) = \frac{18}{65}$

$1 + \sin A.\sin B + \cos A.\cos B = \frac{9}{65}$

$\cos(A-B) = -\frac{56}{65}$

$2\cos^2\left(\frac{A-B}{2}\right) - 1 = -\frac{56}{65}$

$2\cos^2\left(\frac{A-B}{2}\right) = -\frac{56}{65} + 1 = \frac{9}{65}$

$\cos^2\left(\frac{A-B}{2}\right) = \frac{9}{130} \Rightarrow \cos\left(\frac{A-B}{2}\right) = \frac{3}{\sqrt{130}}$

225. $8\cos^2\theta + 8\sec^2\theta = 65 \quad \& \quad 0° < \theta < \frac{\pi}{2}$

$$8\cos^2\theta + \frac{8}{\cos^2\theta} = 65$$

$$8\cos^4\theta + 8 = 65\cos^2\theta$$

$$8\cos^4\theta - 64\cos^2\theta - \cos^2\theta + 8 = 0$$

$$8\cos^2\theta\,[\cos^2\theta - 8] - 1\,(\cos^2\theta - 8) = 0$$

$$\cos^2\theta = \frac{1}{8},$$

$\cos^2\theta = 8$ (not possible)

$$\cos 2\theta = 2\times\frac{1}{8} - 1 \quad (\because \cos 2\theta = 2\cos^2\theta - 1)$$

$$= -\frac{3}{4}$$

$$4\cos 2\theta = 4\times\left(-\frac{3}{4}\right) = -3$$

226. $\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8}$

$$\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\left(\pi - \frac{3\pi}{8}\right) + \cos^4\left(\pi - \frac{\pi}{8}\right)$$

$$= \cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{\pi}{8}$$

$$= 2\left(\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8}\right)$$

$$= 2\left(\cos^4\frac{\pi}{8} + \cos^4\left(\frac{\pi}{2} - \frac{\pi}{8}\right)\right)$$

$$= 2\left(\cos^4\frac{\pi}{8} + \sin^4\frac{\pi}{8}\right)$$

$$= 2\left(1 - 2\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right)$$

$$= 2\left(1 - \frac{1}{2}.4\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right)$$

$$= 2\left(1 - \frac{1}{2}\left(\sin\frac{\pi}{4}\right)^2\right)$$

$$= 2\left(1 - \frac{1}{2}.\frac{1}{2}\right) = 2\left(\frac{3}{4}\right) = \frac{3}{2}$$

Sol^n 227. $\cos\frac{\pi}{15}.\cos\frac{2\pi}{15}.\cos\frac{4\pi}{15}.\cos\frac{8\pi}{15}$

$$= \cos 12°\cos 24°\cos 48°\cos 96°$$

$$= \frac{(\cos 12°\cos 48°\cos 72°)\cos 24°\cos 96°}{\cos 72°}$$

$$= \frac{\left(\frac{1}{4}\cos 36°\right)\cos 24°\cos 96°}{\cos 72°}$$

$$= \frac{1}{4}\times\frac{\cos 36°\cos 24°\cos 96°}{\cos 72°}$$

$$= \frac{1}{4}\times\frac{\frac{1}{4}\cos 108°}{\cos 72°} = \frac{1}{16}\times\frac{\cos(180° - 72°)}{\cos 72°}$$

$$= -\frac{1}{16}\times\frac{\cos 72°}{\cos 72°} = -\frac{1}{16}$$

228.

$$\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$$

$$= \left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\left(\pi-\frac{3\pi}{8}\right)\right)\left(1+\left(\pi-\frac{\pi}{8}\right)\right)$$

$$= \left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{\pi}{8}\right)$$

$$= \left(1-\cos^2\frac{\pi}{8}\right)\left(1-\cos^2\frac{3\pi}{8}\right)$$

$$= \left(\sin^2\frac{\pi}{8}\right)\left(\sin^2\frac{3\pi}{8}\right)$$

$$= \frac{1}{4}\left(2\sin\frac{\pi}{8}\sin\frac{3\pi}{8}\right)^2$$

$$=\frac{1}{4}\left(\cos\frac{\pi}{4}-\cos\frac{\pi}{2}\right)^2=\frac{1}{4}\times\frac{1}{2}$$

$$=\frac{1}{8}$$

229. $x = y\cos\frac{2\pi}{3} = z\cos\frac{4\pi}{3}$

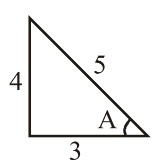
$$\left(\because \cos\frac{2\pi}{3}=\frac{-1}{2}, \cos\frac{4\pi}{3}=\frac{-1}{2}\right)$$

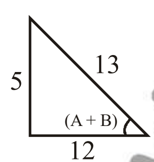
$$x=\frac{-y}{2}=\frac{-z}{2}$$

$$xy+yz+zx=\left(\frac{-y}{2}\right).y+y.y+\left(\frac{-y}{2}\right).y$$

$$=\frac{-y^2}{2}+y^2-\frac{y^2}{2}=0$$

230. $\sin A=\frac{4}{5}$ & $\cos A=\frac{3}{5}$





$$\cos(A+B)=\cos A\cos B-\sin A\sin B=-\frac{12}{13}$$

$$=\frac{3}{5}\cos B-\frac{4}{5}\sin B=-\frac{12}{13}$$

$$=4\sin B-3\cos B=\frac{60}{13} \qquad \ldots(1)$$

$$\sin\ A+B=\sin A\cos B+\cos A\sin B=\frac{5}{13}$$

$$=\frac{4}{5}\cos B+\frac{3}{5}\sin B=\frac{5}{13}$$

$$=3\sin B+4\cos B=\frac{25}{13} \qquad \ldots(2)$$

On solving eq^n (1) and (2) we get

$$\sin B=\frac{63}{65}$$

231. $\cos(\theta-A)=a,\ \cos(\theta-B)=b$

Let $\theta=90°$

$$a=\cos(90^o-A)=\sin A,$$

$$b=\cos(90^o-B)=\sin B$$

$$\cos A=\sqrt{1-a^2}, \qquad \cos B=\sqrt{1-b^2}$$

$$\Rightarrow \sin^2(A-B)+2ab\cos(A-B)$$

$$=(\sin A\cos B-\cos A\sin B)^2+2ab(\cos A\cos B+\sin A\sin B)$$

$$=\left(a\sqrt{1-b^2}-b\sqrt{1-a^2}\right)^2$$

$$+2ab\left(\sqrt{1-a^2}.\sqrt{1-b^2}+ab\right)$$

$$=a^2(1-b^2)+b^2(1-a^2)-2ab\sqrt{1-a^2}.\sqrt{1-b^2}$$

$$+2ab\sqrt{1-a^2}.\sqrt{1-b^2}+2a^2b^2$$

$$=a^2-a^2b^2+b^2-a^2b^2+2a^2b^2$$

$$=a^2+b^2$$

232. $\sin^4\frac{\pi}{8}+\sin^4\frac{3\pi}{8}+\sin^4\frac{5\pi}{8}\sin^4\frac{7\pi}{8}$

$$=2\left(\sin^4\frac{\pi}{8}+\sin^4\frac{3\pi}{8}\right)$$

$$=2\left(\sin^4\frac{\pi}{8}+\cos^4\frac{\pi}{8}\right)$$

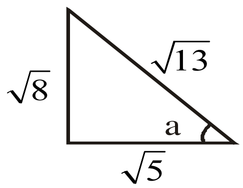
$$\left(\because \sin\frac{3\pi}{8}=\sin\left(\frac{\pi}{2}-\frac{\pi}{8}\right)=\cos\frac{\pi}{8}\right)$$

$$=2\left(\left(\sin^2\frac{\pi}{8}+\cos^2\frac{\pi}{8}\right)^2-2\sin^2\frac{\pi}{8}.\cos^2\frac{\pi}{8}\right)$$

$$=2\left(1-\frac{1}{2}\left(\sin\frac{\pi}{4}\right)^2\right) \quad =2\left(1-\frac{1}{4}\right) \quad =\frac{3}{2}$$

233. $2\sin^2\alpha+15\cos^2\alpha=7$

$$2+13\cos^2\alpha=7$$



$$\cos^2\alpha = \frac{5}{13} \quad \Rightarrow \cos\alpha = \frac{\sqrt{5}}{\sqrt{13}}$$

$$\Rightarrow \cot\alpha = \frac{\sqrt{5}}{2\sqrt{2}}$$

234. $3x\sin\theta + 2y\cos\theta = 4$ …(i)

$2x\sin\theta - 3y\cos\theta = 2$ …(ii)

Adding after multiplying equation (i) by 3 and equation (ii) by 2.

$$9x\sin\theta + 6y\cos\theta + 4x\sin\theta - 6y\cos\theta = 12 + 4$$

$$13x\sin\theta = 16 \quad \Rightarrow 13\sin\theta = \frac{16}{x}$$

Similarly, $13y\cos\theta = 2 \quad \Rightarrow 13\cos\theta = \frac{2}{y}$

Adding after squaring

$$169\sin^2\theta + 169\cos^2\theta = \frac{256}{x^2} + \frac{4}{y^2}$$

$$\Rightarrow \frac{256}{x^2} + \frac{4}{y^2} = 169$$

235. $\sin\theta + \cos\theta = a,\ \sec\theta + \text{cosec}\theta = b$

$$b\left(a^2 - 1\right) = \left(\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}\right)\left(\sin^2 + \cos^2\theta + 2\sin\theta\cos\theta - 1\right)$$

$$= \frac{(\sin\theta + \cos\theta)}{\sin\theta\cos\theta} \times 2\sin\theta\cos\theta = 2a$$

236. $(a\sec\theta + b\tan\theta)(a\sec\theta - b\tan\theta) = 5$

$(a\sec\theta + b\tan\theta) = 1$ …(1)

$(a\sec\theta - b\tan\theta) = 5$ …(2)

From eq^n (1) and (2)

$a\sec\theta = 3, \quad b\tan\theta = -2$

$$a^2b^2 + 4a^2 = \frac{9}{\sec^2\theta} \times \frac{4}{\tan^2\theta} + 4 \times \frac{9}{\sec^2\theta}$$

$$= \frac{36\left(1 + \tan^2\theta\right)}{\sec^2\theta\tan^2\theta} = \frac{36\sec^2\theta}{\sec^2\theta\tan^2\theta}$$

$$= \frac{36}{\tan^2\theta} = \frac{36 \times b^2}{4} = 9b^2$$

237. $\dfrac{\sec^4\alpha}{\sec^2\beta} - \dfrac{\tan^4\alpha}{\tan^2\beta} = 1$

Let $\alpha = \beta = 45°$

$$\frac{\sec^4 45°}{\sec^2 45°} - \frac{\tan^4 45°}{\tan^2 45°} = 1$$

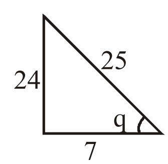
238. $\dfrac{\cos^4\alpha}{\cos^2\beta} + \dfrac{\sin^4\alpha}{\sin^2\beta} = 1$

Let $\alpha = \beta = 45°$

$$\frac{\cos^4 45°}{\cos^2 45°} + \frac{\sin^4 45°}{\sin^2 45°} = 1$$

239. $\dfrac{7}{\sin\theta} + \dfrac{24}{\cos\theta} = \dfrac{25}{\sin\theta\cos\theta}$

$7\cos\theta + 24\sin\theta = 25$



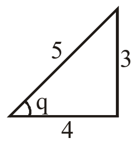
$$\cot\theta = \frac{7}{24}$$

240. $\dfrac{8}{20}\sec\theta + \dfrac{6}{20}\text{cosec}\theta = 1$

$$\frac{2}{5}\sec\theta + \frac{3}{10}\text{cosec}\theta = 1$$

If we consider $\dfrac{2}{5}\sec\theta = \dfrac{3}{10}\text{cosec}\theta = \dfrac{1}{2}$

then, $\sec\theta = \dfrac{5}{4}$ & $\text{cosec}\theta = \dfrac{5}{3}$ which satisfies.

the triangle 

hence, $\cot\theta = \frac{4}{3}$

241. $\left(\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}\right)$

$[2\sin A\cos B = \sin(A+B) + \sin(A-B)]$

$$= \frac{1}{2\sin\frac{\pi}{7}}\left(\sin\frac{3\pi}{7} - \sin\frac{\pi}{7} + \sin\frac{5\pi}{7} - \sin\frac{3\pi}{7} + \sin\frac{7\pi}{7} - \sin\frac{5\pi}{7}\right)$$

$$= \frac{1}{2\sin\frac{\pi}{7}}\left(\sin\pi - \sin\frac{\pi}{7}\right) = -\frac{1}{2}$$

242. $\frac{1}{2}\left(\cos 15°.\cos 7\frac{1}{2}^{\circ}.\cos 82\frac{1}{2}^{\circ}\right)\times 2$

$\frac{1}{2}\cos 15°.\sin 15° = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{8}$

243. $\tan^2\theta = 1 - e^2$

$\tan^2 + 1 = \sec^2\theta = 2 - e^2$

$\sec\theta + \tan^2\theta.\tan\theta\,\mathrm{cosec}\theta = \sec\theta + \tan^2\theta.\sec\theta$

$= \sec\theta\left(1 + \tan^2\theta\right)$

$= \left(2 - e^2\right)^{1/2}.\left(2 - e^2\right)$

$= \left(2 - e^2\right)^{\frac{3}{2}}$

244. $3\tan\theta\tan\phi = 1$

$\theta = \phi = 30°$

$\frac{\cos(30° - 30°)}{\cos(30° + 30°)} = \frac{\cos 0°}{\cos 60°} = 2$

245. $\tan 60° = \frac{\tan 20° + \tan 40°}{1 - \tan 20°\tan 40°}$

$\tan 20° + \tan 40° = \sqrt{3} - \sqrt{3}\tan 20°.\tan 40°$

$\tan 20° + \tan 40° + \sqrt{3}\tan 20°.\tan 40° = \sqrt{3}$

246. $\sin 36°.\sin 72°.\sin 72°.\sin 36°$

$= (\sin 36°.\sin 72°)^2$

$= (\sin 36°.\cos 18°)^2$

$= \left(\frac{1}{4}\sqrt{10 - 2\sqrt{5}}.\frac{1}{4}\sqrt{10 + 2\sqrt{5}}\right)^2$

$= \left(\frac{1}{16}\times\sqrt{80}\right)^2 = \frac{80}{16\times 16} = \frac{5}{16}$

247. $\frac{2\cos 40° - \cos 20°}{\sin 20°} = \frac{\cos 40° - \cos 20° + \cos 40°}{\sin 20°}$

$= \frac{-2\sin 30°.\sin 10° + \cos 40°}{\sin 20°}$

$= \frac{-\sin 10° + \sin 50°}{\sin 20°} = \frac{2\sin 20°.\cos 30°}{\sin 20°}$

$= 2\cos 30° = \sqrt{3}$

248. $2\sqrt{2}\sin 10°\left(\frac{1}{2\cos 5°} + \frac{\cos 40°}{\sin 5°} - 2\sin 35°\right)$

$= 2\sqrt{2}\sin 10°\left(\frac{\sin 5° + 2\cos 5°\cos 40° - 2\sin 35°.2\sin 5°\cos 5°}{2\sin 5°\cos 5°}\right)$

$= 2\sqrt{2}\sin 10°\left(\frac{\sin 5° + \cos 45° + \cos 35° - 2\sin 35°.\sin 10°}{\sin 10°}\right)$

$= 2\sqrt{2}\left(\sin 5° + \cos 45° + \cos 35° - 2\sin 35°.\sin 10°\right)$

$= 2\sqrt{2}\left[\sin 5° + \frac{1}{\sqrt{2}} + \cos 35° - (\cos 25° - \cos 45°)\right]$

$$= 2\sqrt{2}\left[\sin 5° + \frac{1}{\sqrt{2}} + \cos 35° - \cos 25° + \frac{1}{\sqrt{2}}\right]$$

$$= 2\sqrt{2}\left[\sin 5° + \cos 35° - \cos 25° + \frac{2}{\sqrt{2}}\right]$$

$$= 2\sqrt{2}\left[\sin 5° - 2\sin 30°.\sin 5° + \frac{2}{\sqrt{2}}\right]$$

$$= 2\sqrt{2}\left[\sin 5° - \sin 5° + \sqrt{2}\right] = 4$$

249. $\cos x = n \cos y, \qquad \sin x = m \sin y$

$\cos^2 x + \sin^2 x = n^2 \cos^2 y + m^2 \sin^2 y$

$1 = n^2 (1 - \sin^2 y) + m^2 \sin^2 y$

$1 = n^2 + \sin^2 y\,(m^2 - n^2)$

$1 - n^2 = (m^2 - n^2) \sin^2 y$

250. $x\cos\theta + y\sin\theta = 4 \Rightarrow x\cos\theta = y\sin\theta$

$y\sin\theta = 2 = x\cos\theta \Rightarrow \sin\theta = \frac{2}{y}, \cos\theta = \frac{2}{x}$

$\sin^2\theta + \cos^2\theta = \frac{4}{y^2} + \frac{4}{x^2} = 1$

$\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{4}$

251. $(\sec^2\theta - 1) + (\operatorname{cosec}^2\theta - 1) = 14$

$\sec^2\theta + \operatorname{cosec}^2\theta = 16$

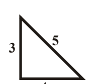
$\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta} = 16$

$\frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta \sin^2\theta} = 16$

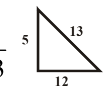
$\frac{1}{\cos^2\theta \sin^2\theta} = 16$

$\sec^2\theta . \operatorname{cosec}^2\theta = 16$

$\sec\theta . \operatorname{cosec}\theta = 4$

252. $\cos(\alpha+\beta) = \frac{4}{5}$ 

$\Rightarrow \tan(\alpha+\beta) = \frac{3}{4}$

$\sin(\alpha-\beta) = \frac{5}{13}$ 

$\Rightarrow \tan(\alpha-\beta) = \frac{5}{12}$

$$\Rightarrow \tan 2\alpha = \tan(\alpha+\beta+\alpha-\beta) = \frac{\tan(\alpha+\beta) + \tan(\alpha-\beta)}{1 - \tan(\alpha+\beta)\tan(\alpha-\beta)}$$

$$= \frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4}.\frac{5}{12}} = \frac{14}{12} \times \frac{48}{48-15} = \frac{56}{33}$$

253. $\tan\theta - \tan\phi = x, \quad \frac{\tan\theta - \tan\phi}{\tan\theta\tan\phi} = y = \frac{x}{\tan\theta\tan\phi}$

$\tan\theta\tan\phi = \frac{x}{y}$

$\cot(\theta-\phi) = \frac{1}{\tan(\theta-\phi)} = \frac{1 + \tan\theta\tan\phi}{\tan\theta - \tan\phi}$

$$= \frac{1 + \frac{x}{y}}{x} = \frac{1}{x} + \frac{1}{y}$$

254. $3\cos\theta = 5\sin\theta$

$\tan\theta = \frac{3}{5} \Rightarrow \sec\theta = \sqrt{1 + \tan^2\theta}$

$$= \sqrt{1 + \frac{9}{25}} = \frac{\sqrt{34}}{5}$$

$\frac{5\tan\theta - 2\sec^4\theta + 2}{5\tan\theta + 2\sec^4\theta - 2} = \frac{5 - 2\sec^4\theta}{1 + 2\sec^4\theta}$

$$= \frac{5 - 2\left(\frac{1156}{625}\right)}{1 + 2\left(\frac{1156}{625}\right)} = \frac{271}{979}$$

255. $\sec\theta + \tan\theta = \sqrt{5} + 2$

$\sec\theta - \tan\theta = \frac{1}{\sec\theta + \tan\theta} = \frac{1}{\sqrt{5}+2} = \sqrt{5} - 2$

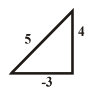
Adding both equation

$2\sec\theta = 2\sqrt{5} \Rightarrow \sec\theta = \sqrt{5} \Rightarrow \cos\theta = \frac{1}{\sqrt{5}}$

$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{1}{5}} = \frac{2}{\sqrt{5}}$

256. $3\tan\theta = -4$

$\tan\theta = \frac{-4}{3}$



$2\left(\frac{-3}{4}\right) - 5\left(\frac{-3}{5}\right) - \left(\frac{4}{5}\right)$

$= \frac{-3}{2} + \frac{15}{5} + \frac{-4}{5} = \frac{-15+30-8}{10} \Rightarrow \quad = \frac{7}{10}$

257. $\frac{1+\sin x}{\cos x} + \frac{\cos x(1-\sin x)}{\cos^2 x} = 4$

$\frac{1+\sin x}{\cos x} + \frac{1-\sin x}{\cos x} = 4 \Rightarrow \frac{2}{\cos x} = 4$

$\sec x = 2 \Rightarrow \quad x = 60°$

258. $\cos\theta.\frac{(\sin\theta + 2\cos\theta)}{\cos\theta}.\frac{(2\sin\theta+\cos\theta)}{\cos\theta}$

$= \frac{2\sin^2\theta + 5\sin\theta\cos\theta + 2\cos^2\theta}{\cos\theta}$

$= \frac{2+5\sin\theta\cos\theta}{\cos\theta} = 2\sec\theta + 5\sin\theta$

259. $\cos\frac{\pi}{7} + \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{5\pi}{7} + \cos\frac{6\pi}{7}$

$= \cos\frac{\pi}{7} + \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} + \cos\left(\pi - \frac{3\pi}{7}\right) +$

$\cos\left(\pi - \frac{2\pi}{7}\right) + \cos\left(\pi - \frac{\pi}{7}\right)$

$= \cos\frac{\pi}{7} + \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} - \cos\frac{3\pi}{7} - \cos\frac{2\pi}{7} - \cos\frac{\pi}{7} = \quad 0$

260. $\tan 9° + \cot 9° - (\tan 27° + \cot 27°)$

$= \frac{\sin 9°}{\cos 9°} + \frac{\cos 9°}{\sin 9°} - \left(\frac{\sin 27°}{\cos 27°} + \frac{\cos 27°}{\sin 27°}\right)$

$= \frac{1}{\sin 9° \cos 9°} - \frac{1}{\sin 27° \cos 27°}$

$= \frac{2}{\sin 18°} - \frac{2}{\sin 54°} = \frac{2(\sin 54° - \sin 18°)}{\sin 18° \sin 54°}$

$= \frac{2(2\sin 18° \cos 36°)}{\sin 18° \cos 36°} = 4$

261. $\sin x + 2\cos x = 1$

Put $x = 90°$

$7\cos x + 6\sin x = 7\cos 90° + 6\sin 90°$

$= 6$

262. put $\theta = 45°$

$\Rightarrow b^3 = \sec 45° - \cos 45° = \sqrt{2} - \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\Rightarrow a^3 = b^3 \Rightarrow a^2b^2(a^2+b^2) = 2a^6 = 2 \times \frac{1}{2} = 1$

263. $\sin A + \sin B = C, \qquad \cos A + \cos B = D$

Put, $A = B = 45°$

$C = \sqrt{2}, \qquad D = \sqrt{2}$

$\sin(A+B) = \sin 90° = 1$

$\frac{2CD}{C^2 + D^2} = \frac{2 \times \sqrt{2} \times \sqrt{2}}{2+2} = 1$

satisfies the equation.

264. $\cos\theta = \frac{3}{5} \quad \& \quad \cos\phi = \frac{4}{5}$

$\Rightarrow 2\cos^2\frac{\theta}{2} - 1 = \frac{3}{5} \;\&\; \Rightarrow 2\cos^2\frac{\phi}{2} - 1 = \frac{4}{5}$

$\Rightarrow \cos\frac{\theta}{2} = \frac{2}{\sqrt{5}} \quad \& \Rightarrow \cos\frac{\phi}{2} = \frac{3}{\sqrt{10}}$

$\Rightarrow \sin\frac{\theta}{2} = \frac{1}{\sqrt{5}} \quad \& \Rightarrow \sin\frac{\phi}{2} = \frac{1}{\sqrt{10}}$

$\Rightarrow \cos\left(\frac{\theta - \phi}{2}\right) = \cos\frac{\theta}{2}\cos\frac{\phi}{2} + \sin\frac{\theta}{2}\sin\frac{\phi}{2}$

$= \frac{6}{\sqrt{50}} + \frac{1}{\sqrt{50}} = \frac{7}{\sqrt{50}} = \frac{7}{2\sqrt{5}}$

265. $\cos^2 48° - \sin^2 12°$

$[\because \cos^2 A - \sin^2 B = \cos(A+B).\cos(A-B)]$

$$\cos 60°.\cos 36° = \frac{1}{2}\left(\frac{\sqrt{5}+1}{4}\right) = \frac{\sqrt{5}+1}{8}$$

266. $$\frac{1}{2}\left[2\sin\left(\frac{\pi}{10}\right)\sin\frac{3\pi}{10}\right]$$

$$= \frac{1}{2}\left(\cos\frac{2\pi}{10} - \cos\frac{7\pi}{10}\right)$$

$$= \frac{1}{2}(\cos 36° - \cos 72°)$$

$$= \frac{1}{2}(\cos 36° - \sin 18°)$$

(By putting the value of cos 36° and sin 18°)

$$= \frac{1}{2}\left(\frac{\sqrt{5}+1}{4} - \frac{\sqrt{5}-1}{4}\right) = \frac{1}{2}\times\frac{1}{2} = \frac{1}{4}$$

267. $$\cot(A - B) = \frac{1}{\tan(A-B)} = \frac{1 + \tan A \tan B}{\tan A - \tan B} = \frac{1+\frac{x}{y}}{x}$$

$$= \frac{1}{x} + \frac{1}{y}$$

$$\left(\because \cot B - \cot A = y \Rightarrow \frac{\tan A - \tan B}{\tan A \tan B} = y \Rightarrow \tan A \tan B = \frac{x}{y}\right)$$

268. $$\tan\left(\frac{\pi}{4}+\theta\right) - \tan\left(\frac{\pi}{4}-\theta\right)$$

$$= \frac{1+\tan\theta}{1-\tan\theta} - \frac{1-\tan\theta}{1+\tan\theta}$$

$$= \frac{1+\tan^2\theta + 2\tan\theta - 1 - \tan^2\theta + 2\tan\theta}{1-\tan^2\theta}$$

$$= \frac{2.2\tan\theta}{1-\tan^2\theta} = 2\tan 2\theta$$

269. $$\frac{1}{\tan\left(\frac{\pi}{4}+\theta\right)}.\frac{1}{\tan\left(\frac{\pi}{4}-\theta\right)}$$

$$\Rightarrow \frac{(1-\tan\theta)(1+\tan\theta)}{(1+\tan\theta)(1-\tan\theta)} = 1$$

270. $\sin(\alpha-\beta) = \cos(\alpha+\beta) = \sin(90° - \alpha - \beta)$

$\alpha - \beta = 90 - \alpha - \beta$

$\alpha = 45°, \quad \beta = 15°$

271. $\alpha + \beta = \pi + \gamma$

$\Rightarrow \sin(\alpha+\beta) = \sin(\pi+\gamma)$

$\Rightarrow \sin\alpha\cos\beta + \cos\alpha\sin\beta = -\sin\gamma$

squaring both the sides

$\Rightarrow \sin^2\alpha(1-\sin^2\beta) + \sin^2\beta(1-\sin^2\alpha) +$
$2\sin\alpha\cos\beta.\cos\alpha\sin\beta = \sin^2\gamma$

$\Rightarrow \sin^2\alpha - \sin^2\alpha\sin^2\beta + \sin^2\beta - \sin^2\alpha\sin^2\beta +$
$2\sin\alpha\cos\beta\,.\cos\alpha\sin\beta = \sin^2\gamma$

$\Rightarrow \sin^2\alpha + \sin^2\beta - \sin^2\gamma$

$= 2\sin\alpha\sin\beta\,(\sin\alpha\sin\beta - \cos\alpha\cos\beta)$

$= -2\sin\alpha\sin\beta\cos(\alpha+\beta)$

$= -2\sin\alpha\sin\beta\cos(180° + \gamma)$

$= 2\sin\alpha\sin\beta\cos\gamma$

272. $1 + \cos 2x + \cos 4x + \cos 6x$

$$1 + 2\cos^2 x - 1 + 2\cos\left(\frac{4x+6x}{2}\right)\cos\left(\frac{6x-4x}{2}\right)$$

$$= 2\cos^2 x + 2\cos 5x.\cos x$$

$$= 2\cos x(\cos x + \cos 5x)$$

$$= 2\cos x\left[2\cos\left(\frac{5x+x}{2}\right)\cos\left(\frac{5x-x}{2}\right)\right]$$

$$= 2\cos x.\ 2\cos 3x.\ \cos 2x$$

$$= 4\cos x\ \cos 2x\ \cos 3x$$

Method – 2

We can put q = 0º in question and all option

$1 + \cos 2x + \cos 4x + \cos 6x$

$= 1 + \cos 0^\circ + \cos 0^\circ + \cos 0^\circ = 4$

Option (a) $2\cos x \cos 2x \cos 3x = 2$

Option (b) $4\sin x \cos 2x \cos 3x = 0$

Option (c) $4\cos x \cos 2x \cos 3x = 4$

Option (d) $\cos x \cos 2x \cos 3x = 1$

hence, Option (c) is correct

273. $\frac{\sin 12^\circ.\sin 48^\circ.\sin 72^\circ.\sin 54^\circ}{\sin 72^\circ}$

$= \frac{1}{4}.\frac{\sin 36^\circ.\sin 54^\circ}{\sin 36^\circ \cos 36^\circ}$

$= \frac{1}{8}.\frac{\sin 54^\circ}{\sin 54^\circ} = \frac{1}{8}$

274. $\frac{1}{2}\left(\frac{\sqrt{3}}{2}\cos 23^\circ - \frac{1}{2}\sin 23^\circ\right)$

$= \frac{1}{2}\left(\cos 30^\circ.\cos 23^\circ - \sin 30^\circ \sin 23^\circ\right)$

$= \frac{1}{2}\left(\cos 53^\circ\right)$

275. Method – 1

$= 2\cos x - (\cos 3x + \cos 5x)$

$= 2\cos x - \left[2\cos\left(\frac{3x+5x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]$

$= 2\cos x - 2\cos 4x \cos x$

$= 2\cos x\,(1 - \cos 4x)$

$= 2\cos x\,(1 - 1 + 2\sin^2 2x)$

$= 4\cos x \sin^2 2x$

$= 4\cos x\,(2\sin x.\cos x)^2$

$= 16\cos^3 x.\sin^2 x$

Method – 2

Put $x = 45^\circ$ in question and all option

$2\cos 45^\circ - \cos 135^\circ - \cos 225^\circ$

$= \frac{2}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{4}{\sqrt{2}}$

When $x = 45^\circ$, $(\sin x = \cos x)$

Option (a) $16\cos^5 x = 16\left(\frac{1}{\sqrt{2}}\right)^5 = \frac{16}{4\sqrt{2}} = \frac{4}{\sqrt{2}}$

Option (b) $\sin^5 x = \left(\frac{1}{\sqrt{2}}\right)^5 = \frac{1}{4\sqrt{2}}$

Option (c) $4\cos^5 x = 4\left(\frac{1}{\sqrt{2}}\right)^5 = \frac{4}{4\sqrt{2}} = \frac{1}{\sqrt{2}}$

Option (d) $4\cos^5 x = 4\left(\frac{1}{\sqrt{2}}\right)^5 = \frac{4}{4\sqrt{2}} = \frac{1}{\sqrt{2}}$

hence, Option (a) is correct.

276. $\frac{\sqrt{3}}{\sin 20^\circ} - \frac{1}{\cos 20^\circ}$

$\Rightarrow \frac{\sqrt{3}\cos 20^\circ - \sin 20^\circ}{\sin 20^\circ.\cos 20^\circ} = \frac{2\left[\frac{\sqrt{3}}{2}\cos 20^\circ - \frac{1}{2}\sin 20^\circ\right]}{\sin 20^\circ.\cos 20^\circ}$

$= \frac{2.2\left[\sin 60^\circ \cos 20^\circ - \cos 60^\circ \sin 20^\circ\right]}{2.\sin 20^\circ.\cos 20^\circ}$

$= \frac{4\sin 40^\circ}{\sin 40^\circ} = 4$

277. $\frac{\sin 78^\circ - \sin 12^\circ}{\sin 78^\circ + \sin 12^\circ} + \frac{\sin 147^\circ}{\cos 147^\circ}$

$\frac{2\cos 45^\circ.\sin 33^\circ}{2\sin 45^\circ.\cos 33^\circ} - \frac{\sin 33^\circ}{\cos 33^\circ} = 0$

278. $\cos 24^\circ + \cos 55^\circ + \cos 125^\circ + \cos 204^\circ + \cos 300^\circ$

$\cos 24^\circ + \cos(90^\circ - 35^\circ) + \cos(90^\circ + 35^\circ) + \cos(180^\circ + 24^\circ) + \cos(360^\circ - 60^\circ)$

$= \cos 24^\circ + \sin 35^\circ - \sin 35^\circ - \cos 24^\circ + \cos 60^\circ$

$= \frac{1}{2}$

279. $\frac{2\sin\theta\tan\theta - 2\sin\theta\tan^2\theta + 2\sin\theta\sec^2\theta}{(1+\tan\theta)^2}$

$= \frac{2\sin\theta\tan\theta + 2\sin\theta\left(\sec^2\theta - \tan^2\theta\right)}{(1+\tan\theta)^2}$

$= \frac{2\sin\theta\tan\theta + 2\sin\theta}{(1+\tan\theta)^2} = \frac{2\sin\theta 1+\tan\theta}{1+\tan\theta^{2}}$

$= \frac{2\sin\theta}{1+\tan\theta}$

280. $m\tan(\theta-30°)=-n\cot(\theta+30°)$

$$\tan(\theta-30°).\tan(\theta+30°)=\frac{-n}{m}$$

$$\frac{\sqrt{3}\tan\theta-1}{\sqrt{3}+\tan\theta}.\frac{\sqrt{3}\tan\theta+1}{\sqrt{3}-\tan\theta}=\frac{-n}{m}$$

$$\frac{3\tan^2\theta-1}{3-\tan^2\theta}=\frac{-n}{m}$$

$$\frac{m}{n}=\frac{3-\tan^2\theta}{1-3\tan^2\theta}$$

$$\frac{m+n}{m-n}=\frac{4-4\tan^2\theta}{2+2\tan^2\theta}$$

$$=\frac{4(1-\tan^2\theta)}{2(1+\tan^2\theta)}=2\cos2\theta$$

281. Method – 1

$$\cos A+2\cos\left(\frac{240°+A+240°-A}{2}\right).\cos\left(\frac{240°+A-240°+A}{2}\right)$$

$=\cos A+2\cos240°.\cos A$

$$=\cos A\left[1+2\left(\frac{-1}{2}\right)\right]=0$$

Method – 2

We can put here A = 240º, then cosA + cos (240º + A) + cos (240º – A)

$=\cos240°+\cos(480°)+\cos0°$

$$=-\frac{1}{2}-\frac{1}{2}+1=0$$

282. $1+\cos56°+\cos58°-\cos66°$

$$1+2\cos^2 28°-1+2\sin\left(\frac{58°+66°}{2}\right)\sin\left(\frac{66°-58°}{2}\right)$$

$=2\cos^2 28°+2\sin62°\sin4°$

$=2\cos^2 28°+2\cos28°\cos86°$

$=2\cos28°(\cos28°+\cos86°)$

$=2\cos28°(2\cos57°\cos29°)$

$=4\cos28°\cos29°\sin33°$

$$\left[\because \cos C+\cos D=2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)\right]$$

283. $\frac{\tan A}{1-\tan A}.\frac{\tan B}{1-\tan B}$

$$=\frac{\frac{1}{\cot A}}{1-\frac{1}{\cot A}}.\frac{\frac{1}{\cot B}}{1-\frac{1}{\cot B}}$$

$$=\frac{1}{(\cot A-1)}.\frac{1}{(\cot B-1)}$$

$$=\frac{1}{2}\quad\left(\begin{array}{l}\because \text{if } A+B=225°\\ \text{then } (\cot A-1).(\cot B-1)=2\end{array}\right)$$

284. $\frac{\sin(B+A)+\cos(B-A)}{\sin(B-A)+\cos(B+A)}$

$$=\frac{\sin B\cos A+\cos B\sin A+\cos A\cos B+\sin A\sin B}{\sin B\cos A-\cos B\sin A+\cos A\cos B-\sin A\sin B}$$

$$=\frac{(\cos A+\sin A)(\cos B+\sin B)}{(\cos A-\sin A)(\cos B+\sin B)}$$

$$=\frac{\cos A+\sin A}{\cos A-\sin A}$$

285. $\tan\theta=\frac{x\sin\phi}{1-\cos\phi}$

$$\Rightarrow\frac{1}{\tan\theta}=\frac{1}{x\sin\phi}-\cot\phi$$

$$\frac{1}{\tan\theta}+\cot\phi=\frac{1}{x\sin\phi}$$

$$\Rightarrow\cot\theta+\cot\phi=\frac{1}{x\sin\phi}\qquad\ldots(i)$$

$$\tan\phi = \frac{y\sin\theta}{1 - y\cos\theta}$$

$$\Rightarrow \frac{1}{\tan\phi} = \frac{1}{y\sin\theta} - \cot\theta$$

$$\frac{1}{\tan\phi} + \cot\theta = \frac{1}{y\sin\theta}$$

$$\Rightarrow \cot\phi + \cot\theta = \frac{1}{y\sin\theta} \quad \ldots(ii)$$

From Eq^n (i) and (ii)

$$\frac{1}{x\sin\phi} = \frac{1}{y\sin\theta} \Rightarrow \frac{x}{y} = \frac{\sin\theta}{\sin\phi}$$

286. $\tan x = \frac{b}{a}$

$$= \sqrt{\frac{1+\frac{b}{a}}{1-\frac{b}{a}}} + \sqrt{\frac{1-\frac{b}{a}}{1+\frac{b}{a}}}$$

$$= \sqrt{\frac{1+\tan x}{1-\tan x}} + \sqrt{\frac{1-\tan x}{1+\tan x}}$$

$$= \frac{(1+\tan x)+(1-\tan x)}{\sqrt{1-\tan^2 x}}$$

$$= \frac{2}{\sqrt{1-\frac{\sin^2 x}{\cos^2 x}}} = \frac{2}{\sqrt{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}}}$$

$$= \frac{2\cos x}{\sqrt{\cos 2x}}$$

287. $\cos(A+B) = \alpha\cos A\cos B + \beta\sin A\sin B$

$\alpha = 1, \qquad \beta = -1$

288. $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + \frac{8(1-\tan^2 4\alpha)}{2\tan 4\alpha}$

$$\left(\because \cot 2\theta = \frac{1-\tan^2\theta}{2\tan\theta}\right)$$

$$= \tan\alpha + 2\tan 2\alpha + \frac{8\tan^2 4\alpha + 8 - 8\tan^2\alpha}{2\tan 4\alpha}$$

$$= \tan\alpha + 2\tan 2\alpha + \frac{4}{\tan 4\alpha}$$

$$= \tan\alpha + 2\tan 2\alpha + \frac{4(1-\tan^2 2\alpha)}{2\tan 2\alpha}$$

$$= \tan\alpha + \frac{2}{\tan 2\alpha} = \tan\alpha + \frac{2(1-\tan^2\alpha)}{2\tan\alpha} = \frac{1}{\tan\alpha}$$

$$= \cot\alpha$$

289 $\left(\cos\left(\frac{\pi}{3}-x\right)+\cos\left(\frac{\pi}{3}+x\right)\right)\left(\cos\left(\frac{\pi}{3}-x\right)-\cos\left(\frac{\pi}{3}+x\right)\right)$

$$\Rightarrow \left[\cos\frac{\pi}{3}\cos x + \sin\frac{\pi}{3}\sin x + \cos\frac{\pi}{3}\cos x - \sin\frac{\pi}{3}\sin x\right] \times$$
$$\left[\cos\frac{\pi}{3}\cos x + \sin\frac{\pi}{3}\sin x - \cos\frac{\pi}{3}\cos x + \sin\frac{\pi}{3}\sin x\right]$$

$$= \left[2\cos\frac{\pi}{3}\cos x\right]\left[2\sin\frac{\pi}{3}\sin x\right]$$

$$= (\cos x)(\sqrt{3}\sin x)$$

$$= \frac{\sqrt{3}}{2}(2\sin x\cos x)$$

$$= \frac{\sqrt{3}}{2}\sin 2x \qquad \text{(Greatest value of } \sin 2x = 1)$$

$$= \frac{\sqrt{3}}{2}$$

290. $\dfrac{\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}{\left(\sqrt{1+\sin x}-\sqrt{1-\sin x}\right)}\times\dfrac{\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}{\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}$

$= \dfrac{1+\sin x+1-\sin x+2\sqrt{1-\sin^2 x}}{1+\sin x-1+\sin x}$

$= \dfrac{2(1-\cos x)}{2\sin x}$, here $\sqrt{\cos^2 x} = +\cos x$

($\because x$ is in IV quadrant)

$= \dfrac{1-1+2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}} = \tan\dfrac{x}{2}$

291. Method 1 :

sinA = n sinB

$\dfrac{\sin A}{\sin B} = \dfrac{n}{1}$

applying component dividend value

$\dfrac{\sin A+\sin B}{\sin A-\sin B} = \dfrac{n+1}{n-1}$

$\dfrac{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)} = \dfrac{n+1}{n-1}$

$\dfrac{\tan\left(\frac{A+B}{2}\right)}{\tan\left(\frac{A-B}{2}\right)} = \dfrac{n+1}{n-1}$

$\left(\dfrac{n-1}{n+1}\right)\tan\left(\dfrac{A+B}{2}\right) = \tan\left(\dfrac{A-B}{2}\right)$

Method 2 :

sinA = n sinB

Put, A = 90° & B = 30°

then, $n = 2$

$\left(\dfrac{n-1}{n+1}\right)\tan 60^\circ = \dfrac{1}{3}\times\sqrt{3} = \dfrac{1}{\sqrt{3}}$

Put these value in options

Option (a) $\sin\left(\dfrac{A-B}{2}\right) = \sin 30^\circ = \dfrac{1}{2}$

Option (b) $\tan\left(\dfrac{A-B}{2}\right) = \tan 30^\circ = \dfrac{1}{\sqrt{3}}$

Option (c) $\cot\left(\dfrac{A-B}{2}\right) = \cot 30^\circ = \dfrac{1}{\sqrt{3}}$

Option (d) $\tan\left(\dfrac{A+B}{2}\right) = \tan 60^\circ = \sqrt{3}$

hence, option (b) is correct.

292. $\dfrac{\sin^2 A - \sin^2 B}{\sin A\cos A - \sin B\cos B}$

$= \dfrac{\sin(A+B).\sin(A-B)}{\frac{1}{2}(2\sin A\cos A - 2\sin B\cos B)}$

$= \dfrac{\sin(A+B).\sin(A-B)}{\frac{1}{2}(2\sin 2A - \sin 2B)}$

$= \dfrac{\sin(A+B).\sin(A-B)}{\frac{1}{2}.2\cos(A+B).\sin(A-B)}$

$= \dfrac{\sin(A+B)\sin(A-B)}{\cos(A+B)\sin(A-B)} = \tan(A+B)$

293. $\cos A = m\cos B$

$\dfrac{\cos A}{\cos B} = \dfrac{m}{1}$

$\dfrac{\cos A + \cos B}{\cos A - \cos B} = \dfrac{m+1}{m-1}$

$\dfrac{2\cos\frac{A+B}{2}.\cos\frac{A-B}{2}}{2\sin\frac{A+B}{2}\sin\frac{B-A}{2}} = \dfrac{m+1}{m-1}$

$\cot\dfrac{A+B}{2}.\cot\dfrac{B-A}{2} = \dfrac{m+1}{m-1}$

$\cot\dfrac{A+B}{2} = \dfrac{m+1}{m-1}\tan\dfrac{B-A}{2}$

294.

$$x\cos\theta = y\cos\left(\theta + \frac{2\pi}{3}\right) = z\cos\left(\theta + \frac{4\pi}{3}\right) = k \text{ (let)}$$

$$\frac{k}{x} + \frac{k}{y} + \frac{k}{z} = \cos\theta + \cos\left(\theta + \frac{2\pi}{3}\right) + \cos\left(\theta + \frac{4\pi}{3}\right)$$

$$= \cos\theta + \cos\theta\cos\left(\frac{2\pi}{3}\right) - \sin\theta\sin\left(\frac{2\pi}{3}\right) + \cos\theta\cos\left(\frac{4\pi}{3}\right) - \sin\theta\sin\left(\frac{4\pi}{3}\right)$$

$$= \cos\theta - \frac{\cos\theta}{2} - \frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta$$

$$= \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$$

295. $2\sin A\cos^3 A - 2\sin^3 A\cos A$

$2\sin A\cos A\left(\cos^2 A - \sin^2 A\right)$

$\frac{1}{2}\left(2\sin 2A.\cos 2A\right) = \frac{1}{2}\sin 4A$

296. $\tan A + \cot A - \tan A - \cot A$

$= \tan A - \cot A$

297. $$\frac{(\sin 3\theta + \sin 5\theta) + (\sin 7\theta + \sin 9\theta)}{(\cos 3\theta + \cos 5\theta) + (\cos 7\theta + \cos 9\theta)}$$

$$= \frac{2\sin 4\theta.\cos\theta + 2\sin 8\theta.\cos\theta}{2\cos 4\theta\cos\theta + 2\cos 8\theta.\cos\theta}$$

$$= \frac{\sin 4\theta + \sin 8\theta}{\cos 4\theta + \cos 8\theta} = \frac{2\sin 6\theta.\cos 2\theta}{2\cos 6\theta.\cos 2\theta}$$

$= \tan 6\theta$

298. A, B, C, D are angles of quadrilateral

$\angle A + \angle C = 180°, \quad \angle B + \angle D = 180°$

$\cos A + \cos B + \cos(180° - A) + \cos(180° - B)$

$\cos A + \cos B - \cos A - \cos B = 0$

299. $$\frac{2\sin\alpha}{1 + \cos\alpha + \sin\alpha} = y$$

Put $\alpha = 45°$

$$\Rightarrow \frac{\frac{2}{\sqrt{2}}}{1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}} = y$$

$$\Rightarrow \quad y = \frac{2}{2 + \sqrt{2}} = 2 - \sqrt{2}$$

$$\Rightarrow \quad \frac{1 - \cos\alpha + \sin\alpha}{1 + \sin\alpha} = \frac{1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}}{1 + \frac{1}{\sqrt{2}}} = \frac{\sqrt{2}}{\sqrt{2} + 1}$$

$$= \frac{\sqrt{2}\left(\sqrt{2} - 1\right)}{1} = 2 - \sqrt{2} = y$$

300. $$1 - \frac{\sin^2 y}{1 + \cos y} + \frac{1 + \cos y}{\sin y} - \frac{\sin y}{1 - \cos y}$$

$$= 1 - \frac{1 - \cos^2 y}{1 + \cos y} + \frac{1 + \cos y}{\sin y} - \frac{\sin y}{1 - \cos y}$$

$$= 1 - \frac{(1 + \cos y)(1 - \cos y)}{1 + \cos y} + \frac{(1 - \cos^2 y) - \sin^2 y}{\sin y(1 - \cos y)}$$

$= 1 - (1 - \cos y) = \cos y$

$$= \frac{\sin^3 y\cos y}{\sin^3 y} = \cos y$$

301. $$\frac{\sin 70° + \cos 40°}{\cos 70° + \sin 40°} = \frac{\cos 20° + \cos 40°}{\sin 20° + \sin 40°}$$

$$= \frac{2\cos 30°.\cos 10°}{2\sin 30°.\cos 10°} = \cot 30° = \sqrt{3}$$

302. $x = \sec\phi - \tan\phi, \qquad y = \operatorname{cosec}\phi + \cot\phi$

$$x = \frac{1 - \sin\phi}{\cos\phi}, \qquad y = \frac{1 + \cos\phi}{\sin\phi}$$

Put $\phi = 45°$

$x = \sqrt{2} - 1, \qquad y = \sqrt{2} + 1$

$$\frac{y - 1}{y + 1} = \frac{\sqrt{2}}{\sqrt{2} + 2} = \frac{\sqrt{2}}{\sqrt{2}\left(\sqrt{2} + 1\right)} = \sqrt{2} - 1$$

hence, option (c) is correct.

303. $\dfrac{\sin^4 A}{a}+\dfrac{\cos^4 A}{b}=\dfrac{1}{a+b}$

$$\left(\frac{a+b}{a}\right)\sin^4 A+\left(\frac{a+b}{b}\right)\cos^4 A=1$$

$$\left(\frac{a+b}{a}\sin^2 A\right)\sin^2 A+\left(\frac{a+b}{b}\cos^2 A\right)\cos^2 A=1$$

$$\left(\because \frac{a+b}{a}\sin^2 A=1 \;\&\; \frac{a+b}{b}\cos^2 A=1\right)$$

$$\sin^2 A=\frac{a}{a+b} \quad \& \quad \cos^2 A=\frac{b}{a+b}$$

$$\frac{\sin^8 A}{a^3}+\frac{\cos^8 A}{b^3}=\frac{a^4}{(a+b)^4 a^3}+\frac{b^4}{(a+b)^4 b^3}$$

$$=\frac{a+b}{(a+b)^4}=\frac{1}{(a+b)^3}$$

304. $2y\cos\theta=x\sin\theta \;\Rightarrow\; 2y\operatorname{cosec}\theta=x\sec\theta$

Put value of $x\sec\theta$ as following

$2x\sec\theta-y\operatorname{cosec}\theta=3$

$2.2y\operatorname{cosec}\theta-y\operatorname{cosec}\theta=3$

$4y\operatorname{cosec}\theta-y\operatorname{cosec}\theta=3$

$3y\operatorname{cosec}\theta=3 \;\Rightarrow\; y\operatorname{cosec}\theta=1$

or $\sin\theta=y$

then, $2y\cos\theta=xy \;\Rightarrow\; 2\cos\theta=x$

hence, $x^2+4y^2=4\cos^2\theta+4\sin^2$

$=4$

Mothod – 2

Put $\theta=45^o$

$2y\cos 45^o=x\sin 45^o \;\Rightarrow\; x=2y$

$2x.\sec 45^o-y\operatorname{cosec}45^o=3$

$\sqrt{2}(2x-y)=3 \;\Rightarrow\; \sqrt{2}(4y-y)=3$

$$y=\frac{1}{\sqrt{2}}$$

$$x=\frac{2}{\sqrt{2}}=\sqrt{2}$$

$$x^2+4y^2=2+4\times\frac{1}{2}=4$$

305. $\tan\theta-\cot\theta=a$ & $\cos\theta+\sin\theta=b$

put, $\theta=45^o$

$\tan 45^o-\cot 45^o=a$,

$\cos 45^o+\sin 45^o=b$

$$a=0, \qquad \frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}=b$$

$$\frac{1+1}{\sqrt{2}}=b \;\Rightarrow\; b=\sqrt{2}$$

$$(b^2-1)^2(a^2+4)=[(\sqrt{2})^2-1]^2(0+4)$$

$$=(2-1)^2(4)$$

$$=4$$

306. $\tan\theta=\dfrac{\sin\alpha-\cos\alpha}{\sin\alpha+\cos\alpha}$

$$\frac{\sin\theta}{\cos\theta}=\frac{\sin\alpha-\cos\alpha}{\sin\alpha+\cos\alpha}$$

let, $\sin\alpha-\cos\alpha=k\sin\theta$

$\sin\alpha+\cos\alpha=k\cos\theta$

adding after squaring.

$\sin^2\alpha+\cos^2\alpha-2\sin\alpha\cos\alpha+\sin^2\alpha+\cos^2\alpha+2\sin\alpha\cos\alpha$

$=k^2(\sin^2\theta+\cos^2\theta)$

$2=k^2$

$k=\sqrt{2}$

hence, $\sin\alpha-\cos\alpha=\sqrt{2}\sin\theta$

$\sin\alpha+\cos\alpha=\sqrt{2}\cos\theta$

307. $\sin\theta+\sin\phi=a$, $\cos\theta+\cos\phi=b$

Put, $\theta=90°$ & $\phi=30°$

$$a=1+\frac{1}{2}=\frac{3}{2}, \qquad b=\frac{\sqrt{3}}{2}$$

$$\tan\left(\frac{\theta-\phi}{2}\right)=\tan 30°=\frac{1}{\sqrt{3}}$$

option (b)

$$\Rightarrow \sqrt{\frac{4-a^2-b^2}{a^2+b^2}} = \sqrt{\frac{4-\frac{9}{4}-\frac{3}{4}}{\frac{9}{4}+\frac{3}{4}}} = \frac{1}{\sqrt{3}}$$

hence, option (b) is correct.

308. Answer is independent of α so put $\alpha = 0^o$

$$\cos^2\alpha + \cos^2(\alpha + 120^o) + \cos^2(\alpha - 120^o)$$

$$= \cos^2 0^o + \cos^2 120^o + \cos^2(-120^o)$$

$$= 1 + \frac{1}{4} + \frac{1}{4} = \frac{3}{2}$$

309. $\sin\frac{3\pi}{14}\ \sin\frac{5\pi}{14}\ \sin\frac{7\pi}{14}\ \sin\frac{9\pi}{14}\ \sin\frac{11\pi}{14}\ \sin\frac{13\pi}{14}$

$$= \sin\frac{\pi}{14}.\sin\frac{3\pi}{14}.\sin\frac{5\pi}{14}.\sin\left(\pi-\frac{5\pi}{14}\right).\sin\left(\pi-\frac{3\pi}{14}\right).\sin\left(\pi-\frac{\pi}{14}\right)$$

$$= \sin\frac{\pi}{14}.\sin\frac{3\pi}{14}.\sin\frac{5\pi}{14}.\sin\frac{5\pi}{14}.\sin\frac{3\pi}{14}.\sin\frac{\pi}{14}$$

$$= \sin^2\frac{\pi}{14}.\sin^2\frac{3\pi}{14}.\sin^2\frac{5\pi}{14}$$

$$= \left(\sin\frac{\pi}{14}.\sin\frac{3\pi}{14}.\sin\frac{5\pi}{14}\right)^2$$

$$= \left(\sin\left(\frac{\pi}{2}-\frac{6\pi}{14}\right).\sin\left(\frac{\pi}{2}-\frac{4\pi}{14}\right).\sin\left(\frac{\pi}{2}-\frac{2\pi}{14}\right)\right)^2$$

$$= \left(\cos\left(\frac{6\pi}{14}\right).\cos\left(\frac{4\pi}{14}\right).\cos\left(\frac{2\pi}{14}\right)\right)^2$$

$$= \left(\cos\left(\frac{3\pi}{7}\right).\cos\left(\frac{2\pi}{7}\right).\cos\left(\frac{\pi}{7}\right)\right)^2$$

$$= \left(\frac{\cos\left(\frac{3\pi}{7}\right).\cos\left(\frac{2\pi}{7}\right).2\sin\left(\frac{\pi}{7}\right).\cos\left(\frac{\pi}{7}\right)}{2\sin\left(\frac{\pi}{7}\right)}\right)^2$$

$$= \left(\frac{\cos\left(\frac{3\pi}{7}\right).\cos\left(\frac{2\pi}{7}\right).2\sin\left(\frac{2\pi}{7}\right)}{2.2\sin\left(\frac{\pi}{7}\right)}\right)^2$$

$$= \left(\frac{\cos\left(\pi-\frac{4\pi}{7}\right).\sin\left(\frac{4\pi}{7}\right)}{2.2\sin\left(\frac{\pi}{7}\right)}\right)^2$$

$$= \left(\frac{-\cos\left(\frac{4\pi}{7}\right).2\sin\left(\frac{4\pi}{7}\right)}{2.2.2\sin\left(\frac{\pi}{7}\right)}\right)^2$$

$$= \left(\frac{-\sin\left(\frac{8\pi}{7}\right)}{8\sin\left(\frac{\pi}{7}\right)}\right)^2$$

$$= \left(\frac{-\sin\left(\pi+\frac{\pi}{7}\right)}{8\sin\left(\frac{\pi}{7}\right)}\right)^2$$

$$= \left(\frac{\sin\left(\frac{\pi}{7}\right)}{8\sin\left(\frac{\pi}{7}\right)}\right)^2 = \left(\frac{1}{8}\right)^2 = \frac{1}{64}$$

310. $\tan A = \frac{1-\cos B}{\sin B} = \frac{1-1+2\sin^2\frac{B}{2}}{2\sin\frac{B}{2}\cos\frac{B}{2}} = \tan\frac{B}{2}$

$$A = \frac{B}{2} \Rightarrow B = 2A$$

$$\tan 2A = \tan B$$

311. $\sin 600° \cos 330° + \cos 120°.\sin 150°$

$\sin(2\times 360^o - 120^o).\cos(360^o - 30^o) + \cos(90^o + 30^o).\sin(180^o - 30^o)$

$(-\sin 120^o).(\cos 30^o) + (-\sin 30^o).(\sin 30^o)$

$-\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2} - \frac{1}{2}.\frac{1}{2} = -\frac{3}{4} - \frac{1}{4} = -\frac{4}{4} = -1$

312.

$\tan 2A = \tan(A + B + A - B) = \frac{\tan(A + B) + \tan(A - B)}{1 - \tan((A + B).\tan(A - B)}$

$= \frac{p + q}{1 - pq}$

313. $\frac{\sec 8A - 1}{\sec 4A - 1} = \frac{1 - \cos 8A}{1 - \cos 4A} \times \frac{\cos 4A}{\cos 8A} = \frac{2\sin^2 4A}{2\sin^2 2A} \times \frac{\cos 4A}{\cos 8A}$

$= \frac{2\sin 4A \cos 4A.\sin 4A}{2\sin^2 2A.\cos 8A}$

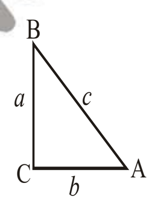
$= \frac{\sin 8A \times 2\sin 2A \cos 2A}{\cos 8A \times 2\sin^2 2A}$

$= \tan 8A.\frac{\cos 2A}{\sin 2A}$

$= \frac{\tan 8A}{\tan 2A}$

314. $\alpha + \beta = \tan A + \tan B = \frac{a}{b} + \frac{b}{a} = \frac{a^2 + b^2}{ab}$

$\alpha\ \beta = \tan A.\tan B = \frac{a}{b}.\frac{b}{a} = 1$



then equation

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$x^2 - \left(\frac{a^2 + b^2}{ab}\right)x + 1 = 0$

$abx^2 - (a^2 + b^2)x + ab = 0$

$abx^2 - c^2x + ab = 0$

$(\because a^2 + b^2 = c^2)$

315. $\sin A + \sin 2A = x$ & $\cos A + \cos 2A = y$

Put, $A = 0$

then, $x = 0$, $y = 2$

$(x^2 + y^2)(x^2 + y^2 - 3) = 4(4 - 3)$

$= 4 = 2y$

316. $\cos(A - B) = \frac{3}{5}$ and $\frac{\sin A \sin B}{\cos A \cos B} = 2$

$\cos A \cos B + \sin A \sin B = \frac{3}{5}$

$3 \cos A \cos B = \frac{3}{5}$

$\cos A \cos B = \frac{1}{5}$

$\sin A \sin B = \frac{2}{5}$

317. $\frac{\cos 9° + \sin 9°}{\cos 9° - \sin 9°} = \frac{\sin 81° + \sin 9°}{\sin 81° - \sin 9°}$

$= \frac{2\sin 45°.\cos 36°}{2\cos 45°.\sin 36°} = \cot 36° = \tan 54°$

318. $\tan\alpha = \frac{1}{7}$, $\tan\beta = \frac{1}{3}$

$\cos 2\alpha = \frac{1 - \tan^2\alpha}{1 + \tan^2\alpha} = \frac{1 - \frac{1}{49}}{1 + \frac{1}{49}} = \frac{48}{50} = \frac{24}{25}$

$\sin 2\beta = \frac{2\tan\beta}{1 + \tan^2\beta} = \frac{\frac{2}{3}}{1 + \frac{1}{9}} = \frac{2}{3} \times \frac{9}{10} = \frac{3}{5}$

$\cos 2\beta = \frac{4}{5}$

$\sin 4\beta = 2\sin 2\beta \cos 2\beta = 2 \times \frac{3}{5} \times \frac{4}{5} = \frac{24}{25}$

hence, $\cos 2\alpha = \sin 4\beta$

319. Given $A = 130°$

$x = \sin A + \cos A$

$x = \sin 130° + \cos 130°$

$= \cos 40° - \sin 40°$

(For $\theta = 0°$ to $45°$ $\quad \cos\theta > \sin\theta$)

$x = +ve$

$x > 0$

320. $\tan A = \frac{1}{2}, \qquad \tan B = \frac{1}{3}$

$$\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\frac{1}{4}}{1+\frac{1}{4}} = \frac{3}{5}$$

$$\sin 2B = \frac{2\tan B}{1+\tan^2 B} = \frac{\frac{2}{3}}{1+\frac{1}{9}} = \frac{2}{3}\times\frac{9}{10} = \frac{3}{5}$$

hence , $\cos 2A = \sin 2B$

321. $\sin(120° - A) = \sin(120° - B)$

$\Rightarrow \quad 120° - A = 120° - B$

$\Rightarrow \quad A = B$

322. Put $\beta = 0°$ such that no option are same

$2\sin^2\beta + 4\cos(\alpha+\beta).\sin\alpha.\sin\beta + \cos 2(\alpha+\beta)$

$= 0 + 0 + \cos 2\alpha$

$= \cos 2\alpha$ option (c) (We can also put $\alpha = 0°$)

323. $\cos 12° + \cos 84° - \cos 24° - \cos 48°$

$= 2\cos\frac{(12°+84°)}{2}\cos\frac{(84°-12°)}{2} - \left[2\cos\frac{(48°+24°)}{2}\cos\frac{(48°-24°)}{2}\right]$

$= 2\cos 48°.\cos 36° - 2\cos 36°\cos 12°$

$= 2\cos 36°(\cos 48° - \cos 12°)$

$= 2\cos 36°(-2\sin 30°.\sin 18°)$

$= -2\cos 36°.\sin 18°$

$$= -2\left(\frac{\sqrt{5}+1}{4}\right)\left(\frac{\sqrt{5}-1}{4}\right)$$

$$= -2\times\frac{4}{4\times 4} = -\frac{1}{2}$$

324. $A + C = B$

$\tan(A + C) = \tan B$

$$\frac{\tan A + \tan C}{1 - \tan A\tan C} = \tan B$$

$\tan A + \tan C = \tan B - \tan A\tan B\tan C$

$\tan A\tan B\tan C = \tan B - \tan C - \tan A$

325. $\tan\theta\sin\left(\frac{\pi}{2}+\theta\right)\cos\left(\frac{\pi}{2}-\theta\right)$

$\tan\theta.\cos\theta.\sin\theta = \sin^2\theta$

326. $x = \dfrac{\cos 10°.\cos 20°.\cos 40°.\cos 80°}{\cos 80°}$

$$x = \frac{1}{4}.\frac{\cos 10°.\cos 60°}{\cos 80°}$$

$$x = \frac{1}{4}.\frac{\sin 80°}{\cos 80°}\times\frac{1}{2}$$

$$x = \frac{1}{8}.\tan 80° = \frac{1}{8}\cot 10°$$

327.

$$\frac{(\sin 12°.\sin 48°.\sin 72°).(\sin 24°.\sin 36°.\sin 84°)}{\sin 72°.\sin 36°}$$

$$= \frac{1}{4}\sin 36°\times\frac{1}{4}\sin 72°\times\frac{1}{\sin 72°\sin 36°}$$

$$= \frac{1}{16}$$

328. $\tan 5x = \dfrac{\tan 3x - \tan 2x}{1 - \tan 3x\tan 2x}$

$= \tan 5x - \tan 5x\tan 3x\tan 2x = \tan 3x + \tan 2x$

$= \tan 5x\tan 3x\tan 2x = \tan 5x - \tan 3x - \tan 2x$

329. $\cos\alpha + \cos\beta = \sin\alpha + \sin\beta$

Squaring both sides

$\Rightarrow \cos^2\alpha + \cos^2\beta + 2\cos\alpha.\cos\beta = \sin^2\alpha + \sin^2\beta + 2\sin\alpha.\sin\beta$

$(\cos^2\alpha - \sin^2\alpha) + (\cos^2\beta - \sin^2\beta) + 2(\cos\alpha.\cos\beta - \sin\alpha.\sin\beta) = 0$

$\cos 2\alpha + \cos 2\beta + 2\cos(\alpha+\beta) = 0$

$\cos 2\alpha + \cos 2\beta = -2\cos(\alpha+\beta)$

330. $A+B+C=180°$

$A+B=180°-C$

$\tan(A+B)=\tan(180°-C)=-\tan C$

$$\frac{\tan A+\tan B}{1-\tan A\tan B}=-\tan C$$

$$\tan A+\tan B=-\tan C+\tan A\tan B\tan C$$

$$\frac{\tan A+\tan B+\tan C}{\tan A\tan B\tan C}=1$$

331. $\cos A = a\cos B \Rightarrow \cos^2 A = a^2\cos^2 B = a^2(1-\sin^2 B)$ ...(i)

$\sin A = b\sin B \Rightarrow \sin^2 A = b^2\sin^2 B$ ...(ii)

Adding both equation

$\cos^2 A+\sin^2 A = a^2 - a^2\sin^2 B + b^2\sin^2 B$

$1 = a^2 + \sin^2 B\,(b^2-a^2)$

$(b^2-a^2)\sin^2 B = 1-a^2$

332. Put $A=90°, B=60°, C=30°$

$(\because A+B+C=\pi)$

then, $\cos 2A+\cos 2B+\cos 2C$

$=\cos 180°+\cos 120°+\cos 60°$

$$=-1-\frac{1}{2}+\frac{1}{2}=-1$$

Put these value in all option

Option (a) $=1+4\cos A.\cos B.\cos C$

$=1$

Option (b) $=-1+4\sin A\sin B.\cos C$

$$=1+4\times1\times\frac{\sqrt3}{2}.\frac{\sqrt3}{2}$$

$=-1+3=2$

Option (c) $=-1-4\cos A\cos B.\cos C$

$=-1-0=-1$

Option (d) $=1+4\sin A\sin B.\sin C$

$$=1+4\times1\times\frac{\sqrt3}{2}.\frac{\sqrt3}{2}$$

$=1+3=4$

hence option (c) is correct

333. Let $A=B=45°$ and $C=90°$

$$\sin^2 A+\sin^2 B+\sin^2 C-2\cos A.\cos B.\cos C$$

$$=\frac{1}{2}+\frac{1}{2}+1-2\times\frac{1}{\sqrt2}\times\frac{1}{\sqrt2}\times0=2$$

334. Method 1 :

$A+B+C=180°$

Put $A=B=45°$ and $C=90°$ in equation and option too.

$\sin 2A+\sin 2B+\sin 2C=\sin 90°+\sin 90°+\sin 180°$ $=1+1+0=2$

Option (a) $=0$

Option (b) $=0$

Option (c) $=4\times\frac{1}{\sqrt2}.\frac{1}{\sqrt2}.1=2$

Option (d) $=8\times\frac{1}{\sqrt2}.\frac{1}{\sqrt2}.1=4$

Option (c) is correct.

Method 2 :

$\sin 2A+\sin 2B+\sin 2C$

$=2\sin(A+B).\cos(A-B)+2\sin C.\cos C$

$=2\sin(180°-C).\cos(A-B)+2\sin C.\cos C$

$=2\sin C[\cos(A-B)+\cos C]$

$=2\sin C[\cos(A-B)+\cos(A+B)]$

$=2\sin C(2\sin A.\sin B)=4\sin A\sin B\sin C$

335. $\cos 52°+\cos 68°+\cos(180°-8°)$

$=2\cos 60°\cos 8°-\cos 8°$

$=\cos 8°-\cos 8°=0$

336. $\cos 2B=\dfrac{\cos(A+C)}{\cos(A-C)}$

$$\frac{\cos^2 B-\sin^2 B}{1}=\frac{\cos A\cos C-\sin A\sin C}{\cos A\cos C+\sin A\sin C}=\frac{\cos^2 B-\sin^2 B}{\cos^2 B+\sin^2 B}$$

(Applying C & D rule)

$$\frac{2\cos A\cos C}{2\sin A\sin C}=\frac{2\cos^2 B}{2\sin^2 B}$$

$$\frac{1}{\tan A.\tan C}=\frac{1}{\tan^2 B}$$

$\tan A.\tan C=\tan^2 B$

$\tan A, \tan B, \tan C$ are in G.P.

337. $(a+b)^2 = 4ab\sin^2\theta$

For, $2a = b$

$(3a)^2 = 4a.2a\sin^2\theta$

$9a^2 = 8a^2\sin^2\theta$

$\sin^2\theta = \frac{9}{8} > 1$ Which is not possible

Given equation can be true if and only it $a = b$

$(2a)^2 = 4a.a\sin^2\theta$

$\sin^2\theta = 1 \quad \Rightarrow a = b$

338. $\frac{\sin A - \sin C}{\cos C - \cos A} = \cot B$

$$\frac{2\cos\frac{A+C}{2}.\sin\frac{A-C}{2}}{2\sin\frac{A+C}{2}.\sin\frac{A-C}{2}} = \cot B$$

$$\cot\left(\frac{A+C}{2}\right) = \cot B$$

$$\frac{A+C}{2} = B$$

then A, B, C are in A.P.

339. $b\sin\alpha = a\sin(\alpha+2\beta)$

$$\frac{\sin\alpha}{\sin(\alpha+2\beta)} = \frac{a}{b}$$

Apply C & D rule

$$\frac{\sin\alpha+\sin(\alpha+2\beta)}{\sin\alpha-\sin(\alpha+2\beta)} = \frac{a+b}{a-b}$$

$$\frac{2\sin(\alpha+\beta).\cos(\beta)}{2\cos(\alpha+\beta).\sin(-\beta)} = \frac{a+b}{a-b}$$

$$\frac{a+b}{a-b} = -\frac{\cot\beta}{\cot(\alpha+\beta)}$$

340. $\tan 10° - \tan 50° + \tan 70°$

$$\tan 10° + \frac{\tan 60° + \tan 10°}{1-\tan 60°.\tan 10°} - \frac{\tan 60° - \tan 10°}{1+\tan 60°.\tan 10°}$$

$$\tan 10° + \frac{\sqrt{3}+\tan 10°}{1-\sqrt{3}\tan 10°} - \frac{\sqrt{3}-\tan 10°}{1+\sqrt{3}\tan 10°}$$

$$\tan 10° + \frac{\sqrt{3}+\tan 10°+3\tan 10°+\sqrt{3}+\tan^2 10°-\sqrt{3}+3\tan 10°+\tan 10°-\sqrt{3}+\tan^2 10°}{1-3\tan^2 10°}$$

$$\tan 10° + \frac{8\tan 10°}{1-3\tan^2 10°}$$

$$= \frac{9\tan 10° - 3\tan^3 10°}{1-3\tan^2 10°}$$

$$= \frac{3(\tan 10° - \tan^3 10°)}{1-3\tan^2 10°} = 3\tan 30° = \sqrt{3}$$

341. Note : If $\alpha = 60°$ or $120°$ or $240°$ or $300°$

then, $\cos^3\theta + \cos^3(\alpha+\theta) + \cos^3(\alpha-\theta) = \frac{3}{4}\cos 3\theta$

then, $\cos^3 10° + \cos^3 110° + \cos^3 130°$

$$= \frac{3}{4}\cos 3\times 10° = \frac{3}{4}\cos 30° = \frac{3}{4}.\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{8}$$

342. Solve it by option

Put, $\alpha = \frac{\pi}{6}$

$$4\cos\frac{\pi}{6} + 3\cos\frac{\pi}{3} - 2\sin\frac{\pi}{2} + \cos\frac{2\pi}{3}$$

$$= 4.\frac{\sqrt{3}}{2} + \frac{3}{2} - 2 - \frac{1}{2}$$

$$= \frac{4\sqrt{3}}{2} - 1 = 2\sqrt{3} - 1$$

hence, option (a) is correct.

343. $(\sin A + \sin B + \sin C)^2 = \sin^2 A + \sin^2 B + \sin^2 C$

$\therefore\ 2(\sin A\sin B + \sin B\sin C + \sin C\sin A) = 0$

$\Rightarrow\ \sin A\sin B + \sin B\sin C + \sin C\sin A = 0$

above condition will be true only when

$A = B = C = 0$

$\therefore\ \sin A + \sin B + \sin C = 0$

344. $\frac{\sin x}{\sin y} = p$ & $\frac{\cos x}{\cos y} = q$

$\frac{\sin x}{p} = \sin y \quad \Rightarrow \frac{\sin^2 x}{p^2} = \sin^2 y$

$\frac{\cos x}{q} = \cos y \quad \Rightarrow \frac{\cos^2 x}{q^2} = \cos^2 y$

Adding both equation

$\frac{\sin^2 x}{p^2} + \frac{\cos^2 x}{q^2} = \sin^2 y + \cos^2 y = 1$

divide by $\cos^2 x$

$\frac{\tan^2 x}{p^2} + \frac{1}{q^2} = \frac{1}{\cos^2 x} = \sec^2 x = 1 + \tan^2 x$

$\tan^2 x\left(\frac{1}{p^2} - 1\right) = 1 - \frac{1}{q^2}$

$\tan^2 x = \frac{\frac{q^2-1}{q^2}}{\frac{1-p^2}{p^2}}$

$\tan x = \frac{p}{q}\sqrt{\frac{q^2-1}{1-p^2}}$

345. If $\cos A = \tan B$, $\cos B = \tan C$ and $\cos C = \tan A$ then $\sin A = \sin B = \sin C = 2\sin 18° =$

$2 \times \frac{\sqrt{5}-1}{4} = \frac{\sqrt{5}-1}{2}$

or after solving above equation we can get A = B = C then

$\cos A = \tan A$

$\Rightarrow \cos A = \frac{\sin A}{\cos A} \Rightarrow \cos^2 A = \sin A$

$\Rightarrow 1 - \sin^2 A = \sin A$

$\Rightarrow \sin^2 A + \sin A - 1 = 0$

then, $\sin A = \frac{-1 \pm \sqrt{5}}{2} = \frac{\sqrt{5}-1}{2}$

346. $\frac{3 - \tan^2 A}{1 - 3\tan^2 A} = k \Rightarrow \tan^2 A = \frac{k-3}{3k-1}$

$\Rightarrow \quad \text{cosec}A\left(3\sin A - 4\sin^3 A\right) = \left(3 - 4\sin^2 A\right)$

$= 3 - \frac{4}{\text{cosec}^2 A} = 3 - \frac{4}{1+\cot^2 A}$

$= 3 - \frac{4}{1 + \frac{3k-1}{k-3}} = \frac{2k}{k-1}$

$\Rightarrow \sin^2 A = \frac{1}{\text{cosec}^2 A} = \frac{1}{1+\cot^2 A} = \frac{k-3}{4(k-1)}$

$0 \le \sin^2 A \le 1$

$0 \le \frac{k-3}{4(k-1)} \le 1 \Rightarrow k \ge \frac{1}{3} \text{ or } k \ \ge \ 3$

347. $m + n = a(\cos\alpha + \sin\alpha)\left(\cos^2\alpha - \cos\alpha\sin\alpha + \sin^2\alpha\right)$

$+3\cos\alpha\sin\alpha(2\cos\alpha + \sin\alpha)$

$= a(\cos\alpha + \sin\alpha)(1 + 2\sin\alpha\cos\alpha)$

$= a(\cos\alpha + \sin\alpha)^3$

$m - n = a(\cos\alpha - \sin\alpha)\left(\cos\alpha + \cos\alpha\sin\alpha + \sin^2\alpha\right)$

$-3\cos\alpha\sin\alpha(\cos\alpha - \sin\alpha)$

$= \ a(\cos\alpha - \sin\alpha)(1 - 2\sin\alpha\cos\alpha)$

$= \ a(\cos\alpha - \sin\alpha)^3$

$(m+n)^{2/3} + (m-n)^{2/3}$

$= \ a^{2/3}(1 + 2\sin\alpha\cos\alpha) + a^{2/3}(1 - 2\sin\alpha\cos\alpha)$

$= \ 2a^{2/3}$

348. $y = a\cos^2 x + 2b\sin x\cos x + c\sin^2 x$

$z = a\sin^2 x - 2b\sin x\cos x + c\cos^2 x$

$y + z = a\left(\sin^2 x + \cos^2 x\right) + c\left(\sin^2 x + \cos^2 x\right)$

$y + z = a + c$

349. $A + B + C = p$

answer is independent of A, B & C

So put $A = B = C = 60°$

$= \dfrac{\cos A}{\sin B\sin C} + \dfrac{\cos B}{\sin C\sin A} + \dfrac{\cos C}{\sin A\sin B}$

$= \dfrac{\cos 60°}{\sin 60°.\sin 60°} + \dfrac{\cos 60°}{\sin 60°.\sin 60°} + \dfrac{\cos 60°}{\sin 60°.\cos 60°}$

$= 3\times\dfrac{\frac{1}{2}}{\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}} = 3.\dfrac{1}{2.\frac{3}{4}} = 2$

350. sin A, cos A & tan A are in G.P.

then, $\cos^2 A = \sin A.\tan A$

$\cos^2 A = \dfrac{\sin A.\sin A}{\cos A}$

$\cos^3 A = \sin^2 A = 1 - \cos^2 A$

$\cos^3 A + \cos^2 A = 1$

351. $\cos(\theta - \alpha)$, $\cos\theta$, $\cos(\theta + \alpha)$

then,

$\cos\theta = \dfrac{2\cos(\theta - \alpha).\cos(\theta + \alpha)}{\cos(\theta - \alpha) + \cos(\theta + \alpha)}$

$\cos\theta = \dfrac{\cos 2\theta + \cos 2\alpha}{2\cos\theta.\cos\alpha}$

$2\cos^2\theta.\cos\alpha = 2\cos^2\theta - 1 + \cos 2\alpha$

$2\cos^2\theta\,(\cos\alpha - 1) = -1 + 1 - 2\sin^2\alpha$

$2\cos^2\theta\left[1 - 2\sin^2\dfrac{\alpha}{2} - 1\right] = -2\sin^2\alpha$

$2\cos^2\theta\left(-2\sin^2\dfrac{\alpha}{2}\right) = -2.4\sin^2\dfrac{\alpha}{2}.\cos^2\dfrac{\alpha}{2}$

$\cos^2\theta = 2\cos^2\dfrac{\alpha}{2}$

$\cos^2\theta.\sec^2\dfrac{\alpha}{2} = 2$

$\cos\theta.\sec\dfrac{\alpha}{2} = \pm\sqrt{2}$

352. $A + B = C$, $\tan A = k\tan B$ and $A - B = \phi$

$\dfrac{\sin A}{\cos A} = \dfrac{k\sin B}{\cos B}$

$\dfrac{\sin A\cos B}{\cos A\sin B} = \dfrac{k}{1}$

Using component & dividend rule

$\dfrac{\sin A\cos B + \cos A\sin B}{\sin A\cos B - \cos A\sin B} = \dfrac{k+1}{k-1}$

$\dfrac{\sin(A + B)}{\sin(A - B)} = \dfrac{k+1}{k-1}$

$\dfrac{\sin C}{\sin\phi} = \dfrac{k+1}{k-1}$

$\sin C = \dfrac{k+1}{k-1}\sin\phi$

353. $\tan\alpha$ & $\tan\beta$ are roots of $x^2 + px + q = 0$

$\tan\alpha + \tan\beta = -p$

$\tan\alpha.\tan\beta = q$

$\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha.\tan\beta} = \dfrac{-p}{1-q} = \dfrac{p}{q-1}$

354. $A + B + C = \dfrac{3\pi}{2}$

Put $A = B = C = \dfrac{\pi}{2}$ in question and all options.

$\cos 2A + \cos 2B + \cos 2C = \cos\pi + \cos\pi + \cos\pi = -3$

Option

(a) $= 1 - 4\sin\dfrac{\pi}{2}.\sin\dfrac{\pi}{2}.\sin\dfrac{\pi}{2} = 1 - 4 = -3$

hence, option (a) is correct.

Method : 2

$(\cos 2A + \cos 2B) + \cos 2C$

$= 2\cos(A+B).\cos(A-B) + 1 - 2\sin^2 C$

$= 2\cos\left(\frac{3\pi}{2} - C\right).\cos(A-B) - 2\sin^2 C + 1$

$= -2\sin C.\cos(A-B) - 2\sin^2 C + 1$

$= 1 - 2\sin C\ [\cos(A-B) + \sin C]$

$= 1 - 2\sin C\ [\cos(A-B) + \cos(A+B)]$

$= 1 - 2\sin C.2\sin A \sin B$

$= 1 - 4\sin A.\sin B \sin C$

355. Maximum value of cot A . cotB . cot C will be possible when cot A = cot B = cot C

$A = B = C = \frac{\pi}{3}$

$k \le \cot\frac{\pi}{3}.\cot\frac{\pi}{3}.\cot\frac{\pi}{3}$

$k \le \frac{1}{3\sqrt{3}}$

356. $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$

$\frac{\sin x \cos y + \cos x \sin y}{\sin x \cos y - \cos x \sin y} = \frac{a+b}{a-b}$

Using component & dividend rule

$\frac{2\sin x.\cos y}{2\cos x.\sin y} = \frac{2a}{2b} \Rightarrow \frac{\tan x}{\tan y} = \frac{a}{b}$

357. $\tan\frac{\pi}{3} = \tan\left(\frac{2\pi}{5} - \frac{\pi}{15}\right)$

$\tan\frac{\pi}{3} = \frac{\tan\frac{2\pi}{5} - \tan\frac{\pi}{15}}{1 + \tan\frac{2\pi}{5}\tan\frac{\pi}{15}}$

$\Rightarrow \tan\frac{\pi}{3} + \tan\frac{\pi}{3}\tan\frac{2\pi}{5}\tan\frac{\pi}{15} = \tan\frac{2\pi}{5} - \tan\frac{\pi}{15}$

$\Rightarrow \tan\frac{2\pi}{5} - \tan\frac{\pi}{15} - \sqrt{3}\tan\frac{2\pi}{5}\tan\frac{\pi}{15} = \sqrt{3}$

358. $\sin A + \cos A = x$

$\sin^2 A + \cos^2 A + 2\sin A.\cos A = x^2$

$2\sin A.\cos A = x^2 - 1$

We know that

$\sin^6 A + \cos^6 A = 1 - 3\sin^2 A.\cos^2 A$

$\frac{1}{4}\left(4 - 3\left(x^2-1\right)^2\right) = 1 - 3\left(\frac{x^2-1}{2}\right)^2$

$1 - \frac{3}{4}\left(x^2-1\right)^2 = 1 - \frac{3}{4}\left(x^2-1\right)^2$

L.H.S. = R.H.S.

Hence this is true of all x.

but, $\sin A + \cos A = x$

So $-\sqrt{2} \le x \le \sqrt{2} \Rightarrow x^2 \le 2$

359. Note : When $\alpha = 60°$ or $120°$ or $240°$ or $300°$

then, $\cos^2\theta + \cos^2(\alpha - \theta) + \cos^2(\alpha + \theta) = \frac{3}{2}$

Hence, $\cos^2 10° + \cos^2 50° + \cos^2 70° = \frac{3}{2}$

360. Answer is independent of A, B, C and A + B + C = 180°.

So put, $A = B = C = 60°$

$\tan^2\frac{A}{2} + \tan^2\frac{B}{2} + \tan^2\frac{C}{2} = \tan^2 30° + \tan^2 30° + \tan^2 30°$

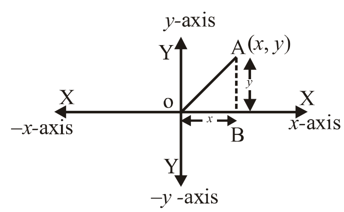
$= \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$

361. $\frac{\cot^2 15° - 1}{\cot^2 15° + 1}$

$= \frac{\cos^2 15° - \sin^2 15°}{\cos^2 15° + \sin^2 15°}$

$= \frac{\cos 30°}{1} = \frac{\sqrt{3}}{2}$

3 Co-ordinate Geometry



XOX' and YOY' is called rectangular co-ordinate axis which intersect at O which is called origin. OB is x co-ordinate of point A and this is called **Abscissa**.

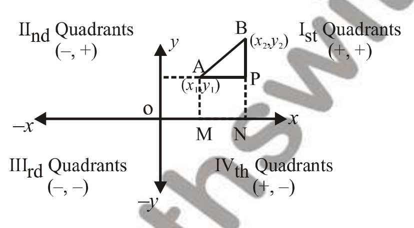
AB is y co-ordinate of point A and this is called **ordinate**. To find the position of any point we take Abscissa positive at right side of O and negative left side of O and we take ordinate positive above O and negative below O.

(a) Ordinate of any point on x –axis is 0.

(b) Absicssa of any point on y–axis is 0.

So cordinate of origin O is (0, 0)

Using Pythagoras in triangle OAB then distance of a point (x, y) from origin is $\sqrt{x^2 + y^2}$



Here ,AM = y_1 and BN = y_2

then BP = $y_2 - y_1$

similarly, AP = $x_2 - x_1$

using Pythagoras in triangle ABP.

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

This is formula to find distance between two point A(x_1, y_1) and B(x_2, y_2)

Ex. 1 What is abcissa of (2, 3) ?

Sol. 2

Ex. 2. In which quadrant(–2, 3) will be ?

Sol. IInd Quadrant

Ex.3 If distance between two point (0, –5) and $(x, 0)$ is equal to 13 unit then find x.

(a) 10 (b) ±10

(c) 12 (d) ±12

Sol. Using Distance formula,

$$13 = \sqrt{(x-0)^2 + (0+5)^2}$$

Squaring both side

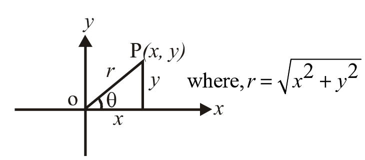
$(13)^2 = x^2 + 25$

$x^2 = 169 - 25 = 144$

$x = \pm\sqrt{144} = \pm 12$

Polar co-ordinate of a point:–

When co-ordinate is written in form of distance from origin and angle then it is called polar form



$$\sin\theta = \frac{y}{r} \text{ and } \cos\theta = \frac{x}{r}$$

$\Rightarrow$ $x = r\cos\theta$ & $y = r\sin\theta$

$$\tan\theta = \frac{y}{x}$$

then polar co-ordinate (x, y) will be $(r\cos\theta, r\sin\theta)$

Ex. 4 Write the polar cordinate of $\left(1, \sqrt{3}\right)$.

Sol. $r = \sqrt{(1)^2 + \left(\sqrt{3}\right)^2} = \sqrt{1+3} = 2$

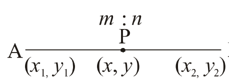
$$\tan\theta = \frac{y}{x} = \frac{\sqrt{3}}{1} = \tan 60^\circ \Rightarrow \theta = 60^\circ$$

then, polar co-ordinate is (2 cos 60º, 2 sin 60º)

Section formula:–

We find the co-ordinate of the point dividing the join of two given points in a given ratio.

(I) Internal Division:



$\left(\frac{PA}{PB} = \frac{m}{n}\right)$

Let P(x, y) is a point which divides the join of two given points A(x_1, y_1) and B(x_2, y_2) in the given ratio $m : n$ internally.

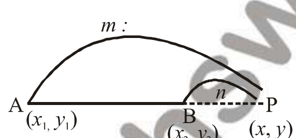
then co-ordinate of P

$$\Rightarrow \quad (x, y) \equiv \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

If P is mid point of AB then $m = n$ and co-ordinate of mid point of AB.

$$\Rightarrow \quad (x, y) \equiv \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

(II) External division:



$\left(\frac{PA}{PB} = \frac{m}{n}\right)$

If the point P(x, y) divides the join of A(x_1, y_1) and B(x_2, y_2) externally in the ratio $m : n$ then co-ordinate of P.

$$\Rightarrow \quad (x, y) \equiv \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$$

Ex. 5 Find the co-ordinate of a point which divides the join of (2, –3) and (–4, 6) internally in the ratio 1 : 2.

(a) (8, 0) (b) (0, 0)
(c) (3, –5) (d) (4, 3)

Sol. Here $x_1 = 2, y_1 = -3, x_2 = -4, y_2 = 6$
$m = 1, n = 2$

$$\Rightarrow \quad (x, y) \equiv \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

$$\equiv \left(\frac{1\times(-4) + 2\times 2}{1+2}, \frac{1\times 6 + 2\times(-3)}{1+2}\right) \equiv (0, 0)$$

Ex. 6 Find the co-ordinate of a point which divides the join of (2, 1) and (3, 5) externally in the ratio 2 : 3.

Sol. $(x, y) \equiv \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$

$$\equiv \left(\frac{2\times 3 - 3\times 2}{2-3}, \frac{2\times 5 - 3\times 1}{2-3}\right) \equiv (0, -7)$$

Ex. 7 In what ratio does the point (6, –6) divide the join of (1, 4) and (9, –12) ?

Sol.

λ	:	1
(1, 4)	(6, –6)	(9, 12)

ratio be $m : n \rightarrow \frac{m}{n} : 1$ (let $\frac{m}{n} \rightarrow \lambda$)

Let $\lambda : 1$ be the ratio internally.

Now, x-co-ordinate $= \left(\frac{mx_2 + nx_1}{m+n}\right)$

here, $m = \lambda$, $n = 1, x_1 = 1, x_2 = 9$

$$\Rightarrow 6 = \frac{9\lambda + 1}{\lambda + 1}$$

$$\Rightarrow 6\lambda + 6 = 9\lambda + 1$$

$$\Rightarrow 3\lambda = 5 \quad \Rightarrow \lambda = \frac{5}{3}$$

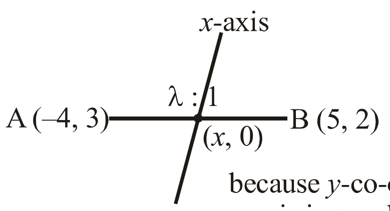
{If we take y-co-ordinate, value of λ will be same}

Hence ratio will be 5 : 3 internally.

Note: If the value of λ is negative we say external division.

Ex. 8 In what ratio x–axis will divide the join of A(–4, 3) and B(5, 2).

Sol.



because y-co-ordinate on x-axis is equal to zero

Let ratio is $\lambda : 1$ internally.

here, y- co-ordinate$=\left(\frac{my_2+ny_1}{m+n}\right)$

$\Rightarrow \quad 0=\frac{2\lambda+1(3)}{\lambda+1}$ (y-co-ordinate on x-axis is 0)

$\Rightarrow \quad 2\lambda+3=0 \qquad \Rightarrow \quad \lambda=-3/2$

Now ratio 3 : 2 but externally. (because λ is negative)

Ex. 9 If the mid point of join (–8, 13) and (k, 7) is (4, 10) then, find value of k.

Sol. here $x_1=-8, x_2=k, x=4$

$\Rightarrow \quad x$-co-ordinate $=\left(\frac{x_1+x_2}{2}\right)$

$\Rightarrow \quad 4=\frac{-8+k}{2} \quad \Rightarrow 8=-8+k$

$\Rightarrow \quad k=16$

Ex. 10 Point (–2, 7) will be in quadrant –

(a) Ist (b) IInd
(c) IIIrd (d) IVth

Ans. (b) IInd

Ex. 11 Find the distance between points (a sinα , a cosα) and (0, 0).

(a) a (b) a^2
(c) $a \sin\alpha$ (d) $a \cos\alpha$

Ans. (a) a

Ex. 12 Distance between (2a, a) and ($-a$, $-3a$) will be

(a) $4a$ (b) $25a$
(c) $5a$ (d) $5\sqrt{a}$

Ans. (c) $5a$

Ex. 13 Distance between (a sinθ, a cosθ) and (a cosθ, $-a$ sin θ) will be

(a) $a\sqrt{2}$ (b) $a\sqrt{3}$
(c) $3a$ (d) $2a$

Ans. (a) $a\sqrt{2}$

Ex. 14 Distance between $\left(at_1^2, 2at_1\right)$ and $\left(at_2^2, 2at_2\right)$ is

(a) $a\left(t_1+t_2\right)\sqrt{\left(t_1+t_2\right)^2-4}$

(b) $a\left(t_2-t_1\right)\sqrt{\left(t_1+t_2\right)^2+4}$

(c) $a\left(t_2-t_1\right)\sqrt{\left(t_1+t_2\right)^2-4}$

(d) none of these

Ans. (b) $a\left(t_2-t_1\right)\sqrt{\left(t_1+t_2\right)^2+4}$

Ex. 15 If distance between points (4, 0) and (0, x) is 5 then x =?

(a) 0 (b) 1
(c) 2 (d) 3

Ans. (d) 3

Ex.16 Point P divides the join of point (8, 9) and (–7, 4) in the ratio 2 : 3 internally then co-ordinate of P is

(a) (–2, 7) (b) (2, 7)
(c) (7, 2) (d) (–7, 2)

Ans. (b) (2, 7)

Ex. 17 In which ratio point P(1, 2) divides the line segment joining (–2, 1) and (7, 4) ?

(a) 1 : 2 (b) 2 : 1
(c) 3 : 2 (d) 2 : 3

Ans. (a) 1 : 2 (Internally)

Ex. 18 In which ratio x–axis divides the line segment joining (2, –3) and (5, 6)?

(a) 1 : 2 externally (b) 2 : 1 externally
(c) 1 : 2 internally (d) 2 : 1 internally

Ans. (c) 1 : 2

Ex. 19 In which ratio y–axis divides the line segment joining (4, 2) and (8, 3)

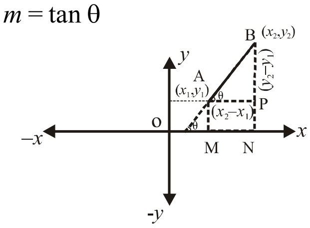
(a) 1 : 2 internally (b) 2 : 1 internally
(c) 1 : 2 externally (d) 2 : 1 externally

Ans. (c) 1 : 2 externally

Line

Slope of line : If a line makes an angle θ with positive x-axis, then tangent of that angle is called slope of line. Slope is denoted by m

$\Rightarrow \quad m=\tan\theta$



$$m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}, \; ax + by + c = 0$$

$$a \text{ line M} = \frac{\text{coestgent of } x}{\text{coestgent of y}}$$

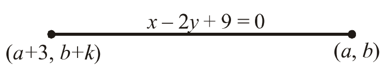
(*a*) Slope of x-axis or a line parallel to x-axis is $m = \tan 0^\circ = 0$

(*b*) Slope of y-axis or a line parallel to y-axis is

$$m = \tan 90^\circ = \infty = \frac{1}{0}$$

Ex.20 Two points $(a+3, b+k)$ & (a, b) are on the line $x - 2y + 9 = 0$. Find the value of k

Sol.



$$x - 2y + 9 = 0 \Rightarrow y = \frac{1}{2}x + \frac{9}{2}$$

then slope of line $= \frac{1}{2}$, we will equate this slope to the slope of join of points $(a+3, b+k)$ & (a, b).

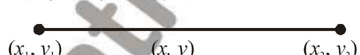
$$\Rightarrow \quad \frac{1}{2} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{b + k - b}{a + 3 - a} = \frac{k}{3}$$

$$\therefore \quad k = \frac{3}{2}$$

Equation of a line

The relation between x & y in which values of x & y satisfies the relation is called equation of line.

(1) Point form of line:



Equate the slope of any two points on the line

$$\Rightarrow \left(\frac{y - y_1}{x - x_1}\right) = \left(\frac{y_2 - y_1}{x_2 - x_1}\right) = m$$

$$\boxed{(y - y_1) = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)} \quad \ldots\text{(i)}$$

Ex.21 Find the equation of line whose ending points are (2, 3) & (5, 4).

$$\Rightarrow \quad (y - 3) = \left(\frac{4 - 3}{5 - 2}\right)(x - 2)$$

$$\Rightarrow \quad 3y - 9 = x - 2$$

Equation of line:- $x - 3y + 7 = 0$

(2) Slope form of a line:

put $\left(\frac{y_2 - y_1}{x_2 - x_1}\right) = m$ in equation (i)

$\Rightarrow \quad y - y_1 = m(x - x_1)$

$\Rightarrow \quad y = mx + y_1 - mx_1$

$\Rightarrow \quad \boxed{y = mx + c}$ (let $c = y_1 - mx_1$ is a constant)

Ex.22 Find the equation of line which makes an angle 45° with positive x-axis & passes through the point (5, 7)

$\therefore$ now equation:- $y = x + 2$

Sol. Slope $m = \tan 45° = 1$

put $m = 1$ in $y = mx + c$

$\Rightarrow \quad y = x + c$ passes through (5, 7)

put $x = 5$ & $y = 7$

$7 = 5 + c \Rightarrow c = 2$

Ex.23 Find the slope of line $3y = 2x + 9$

Sol. $\Rightarrow \quad y = \frac{2}{3}x + \frac{9}{3}$

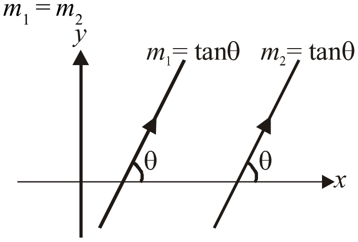
$$y = \frac{2}{3}x + 3 \quad \text{[on comparing with } y = mx + c\text{]}$$

$$\Rightarrow \quad m = \frac{2}{3}$$

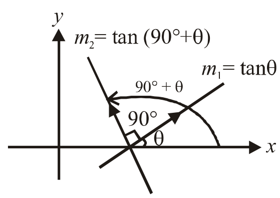
Note:

(a) If two lines are parallel then slopes of lines are equal.

$\Rightarrow \quad m_1 = m_2$



(b) If two lines are perpendicular, then product of their slope is equal to –1.



$m_1 = \tan\theta,\ m_2 = \tan(90° + \theta) = -\cot\theta$

$\Rightarrow \quad m_1 . m_2 = -1$

Ex. 24 If two lines having equation $2y = 3x + 5$ & $4y = kx + 11$ are parallel then find the value of k.

Sol. Here slope of 1st line $m_1 = 3/2$ & slope of 2[nd] line $m_2 = k/4$ because lines are parallel then $k/4 = 3/2$

$\Rightarrow$ ***k* = 6**

Ex. 25 If two lines having equation $y = x + 15$ & $4y = kx + 11$ are perpendicular then find then value of k.

Sol. here slope of 1st line $m_1 = 1$ & slope of 2nd line $m_2 = k/4$ because lines are perpendicular then,

$(k/4).(1) = -1$

$\Rightarrow$ ***k* = –4**

Ex. 26 Find the equation of line which is parallel to $5x + 7y = 199$ & passes through the point (2, 1).

Sol. If two lines are parallel then their slopes will be equal

$$\text{slope} = \frac{\text{coeff. of } x}{\text{coeff. of } y}$$

Slope depend upon the coefficient of x & y so if two lines are parallel then the coefficient of x & y will be equal.

So equation of line parallel to $5x+7y=199$ will be $5x+7y=\lambda$

it passes through (2, 1) so it satisfies equation

$\Rightarrow \quad 5(2) + 7(1) = \lambda$

$\Rightarrow \quad \lambda = 17$

hence, equation $\Rightarrow 5x + 7y = 17$

Ex. 27 Find the equation of line which is perpendicular to $4x + 3y = 111$ and passes through (3, 2).

Sol. Slope depends upon the coefficient of x & y so if two lines are perpendicular then the coefficient of x & y will be interchanged with opposite sign.

So equation of line parallel to $4x + 3y = 111$ is $3x - 4y = \lambda$

it passes through (3, 2) so it satisfies equation

$\Rightarrow 3(3) - 4(2) = \lambda \quad \Rightarrow \lambda = 1$

hence, equation $\Rightarrow \ 3x - 4y = 1$

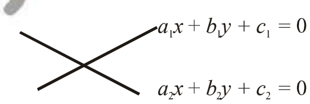
Types of lines: let the two equation

$a_1x + b_1y + c_1 = 0 \quad \ldots$(i)

$a_2x + b_2y + c_2 = 0 \quad \ldots$(ii)

(a) Intersecting lines: it means eq[n] have unique solution and condition is:-

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$



$(x, y) \rightarrow$ unique solution

(b) Parallel lines: it means eq[n] have no solution (parallel lines never intersect) and condition is:-

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

for Ex. $2x + 3y = 11 \quad \ldots$(i)

$4x + 6y = 38 \quad \ldots$(ii)

are parallel lines

because $4x + 6y = 38 \Rightarrow 2x + 3y = 19$

(c) Coincide lines: it means eq[n] have infinite solution and condition is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

for ex. $x + y = 5$

$2x + 2y = 10$

Ex. 28 If $3x + 2y = 11$ & $kx + 4y = 22$ are coincide lines find value of k.

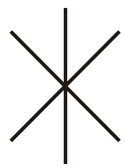
Sol. $\Rightarrow \quad \frac{3}{k} = \frac{2}{4} = \frac{11}{22} = \frac{1}{2}$

$\Rightarrow \quad k = 6$

Ex. 29 If $2x + 3y = 122$ and $4x + ky = 119$ have unique solution.

Sol. $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{2}{4} \neq \frac{3}{k} \Rightarrow k \neq 6$

(D) Concurrent lines: If at least three lines pass through a single point are called concurrent lines.



$a_1x + b_1y + c_1 = 0 \quad$ ...(i)

$a_2x + b_2y + c_2 = 0 \quad$ ...(ii)

$a_3x + b_3y + c_3 = 0 \quad$ ...(iii)

If above three lines are concurrent lines then

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

or

$$a_1(b_2c_3 - c_2b_3) - b_1(a_2c_3 - c_2a_3) + c_1(a_2b_3 - b_2a_3) = 0$$

Ex. 30 If lines $4x + 3y = k$, $2x + 3y = 12$ and $x + y = 5$ are concurrent lines find the value of k.

Sol. We dont use above formula to solve this question even if we can do.We will use another concept.

we find the intersection point of two line and after that we will put that point in third line because third line will pass from the intersection point of two lines.

$2x + 3y = 12 \quad$ ...(i)

$x + y = 5 \Rightarrow 2x + 2y = 10 \quad$ ...(ii)

on solving above two line we find $\Rightarrow x = 3$ & $y = 2$

put these value in $4x + 3y = k$

$\Rightarrow \quad k = 4(3) + 3(2)$

$= 12 + 6 = 18$

(E) Perpendicular lines: $a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

perpendicular lines then $m_1.m_2 = -1$

$\Rightarrow \quad \left(-\frac{a_1}{b_1}\right).\left(-\frac{a_2}{b_2}\right) = -1$

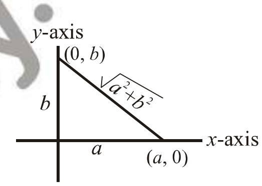
$\Rightarrow \quad \boxed{a_1a_2 + b_1b_2 = 0}$

Ex. 31 If lines $3x + 4y + 9 = 0$ & $kx + 6y + 41 = 0$ are perpendicular then find k.

Sol. $\Rightarrow \quad 3k + 24 = 0$

$\Rightarrow \quad k = -8$

(3) Intercept form of a line:



$\Rightarrow \quad (y - 0) = \frac{0 - b}{a - 0}(x - a)$

$\Rightarrow \quad \frac{-y}{b} = \frac{x}{a} - 1$

$\Rightarrow \quad \boxed{\frac{x}{a} + \frac{y}{b} = 1 \text{ is intercept form of a line.}}$

where, length of intercept on x-axis = a

length of intercept on y-axis = b

length of intercept between both axis $= \sqrt{a^2 + b^2}$

Ex. 32 Find the length of intercept between axis of line

$3x + 4y = 12$

Sol. $\Rightarrow \quad \frac{3x}{12} + \frac{4y}{12} = \frac{12}{12} \Rightarrow \frac{x}{4} + \frac{y}{3} = 1$

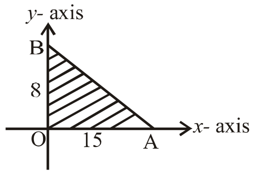
here $a = 4$, $b = 3$ so length of intercept $= \sqrt{4^2 + 3^2} = 5$

Intercept on x–axis $\rightarrow 4$

Intercept on y–axis $\rightarrow 3$

Ex. 33 Find the area of triangle which is formed by three lines $8x + 15y = 120$, x–axis & y–axis.

Sol. $\Rightarrow \frac{8x}{120} + \frac{15y}{120} = 1 \Rightarrow \frac{x}{15} + \frac{y}{8} = 1$

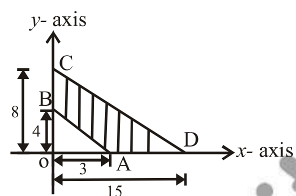


Area of triangle OAB $= \frac{1}{2} \times 15 \times 8 = 60$

Ex. 34 Find the area of quadrilateral formed by four lines $8x + 15y = 120$, $3x + 4y = 12$, x–axis & y–axis.

Sol. $8x + 15y = 120 \Rightarrow \frac{x}{15} + \frac{y}{8} = 1$

$3x + 4y = 12 \Rightarrow \frac{x}{4} + \frac{y}{3} = 1$



Area of quadrilateral ABCD

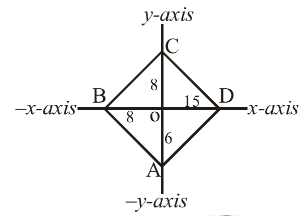
$=$ Area of ΔOCD – Area of ΔOAB

$= \frac{1}{2} \times 15 \times 8 - \frac{1}{2} \times 3 \times 4 = 60 - 6 = 54$

Ex. 35 Find the area of quadrilateral ABCD. AC & BD are diagnols i.e, x–axis, & y–axis respectively. The equation of line AB is $3x + 4y = -24$ & equation of line CD is $8x + 15y = 120$. Find the area of quadrilateral ABCD.

Sol. $3x + 4y = -24 \Rightarrow \frac{x}{-8} + \frac{y}{-6} = 1$

$8x + 15y = 120 \Rightarrow \frac{x}{15} + \frac{y}{8} = 1$



Area of quadraletral ABCD =
Sum of area of triangle OCD, OBC, OAB & OAD

$= \frac{1}{2} \times \left[(8 \times 15) + (8 \times 8) + (8 \times 6) + (15 \times 6) \right]$

$= 161$ unit^2

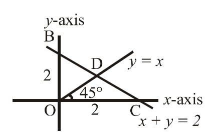
Method : 2 : AC = 8 + 6 = 14, BD = 8 + 15 = 23

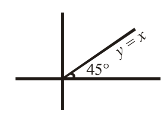
AC $\perp$ BD

then area $= \frac{1}{2}$ AC.BD $= \frac{1}{2} \times 14 \times 23 == 161$ unit^2

Ex. 36 Find the area of triangle formed by three lines. $x + y = 2$, $y = x$ & x–axis

Sol.





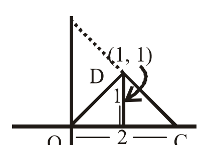
$\frac{x}{2} + \frac{y}{2} = 1$, OD will be the median of triangle OBC so this will divide the triangle OBC into two equal area part.

Area of $\Delta OBC = \frac{1}{2} \times 2 \times 2 = 2$

Area of $\Delta ODC = \frac{1}{2} \times$ Area of $\Delta OBC = 1$

Method 2 : Take intersection point of lines $x = y$ & $x + y = 2$

On solving, that will be (1, 1)



Area of triangle ODC $= \frac{1}{2} \times 2 \times 1 = 1$ sq. unit

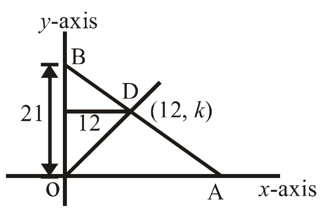
Ex. 37 Find the area of triangle which is formed by three lines $3x + 4y = 84$, $y = x$ & y–axis.

Sol. $\frac{x}{28} + \frac{y}{21} = 1$

We will find only x co-ordinate of intersection point of $3x + 4y = 84$ & $y = x$

(put $x = y$ in first equation) $\Rightarrow 3x + 4x = 84$

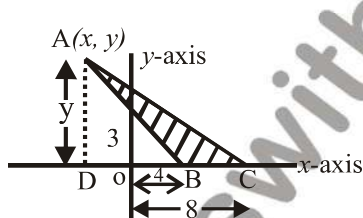
$7x = 84 \Rightarrow x = 12$



Area of Δ ODB $= \frac{1}{2} \times 21 \times 12 = 126$

Ex. 38 Find the area of which is formed by three lines $3x + 4y = 12$, $5x + 8y = 40$ & x–axis.

Sol. $\frac{x}{4} + \frac{y}{3} = 1$ & $\frac{x}{8} + \frac{y}{5} = 1$



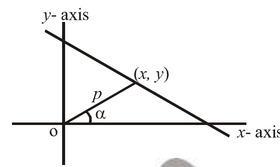
On solving both equations $3x + 4y = 12$ & $5x + 8y = 40$, we will find intersection point of both lines A(–16, 15). The y co-ordinate of this point will be the height of triangle ABC, which is equal to 15.

BC = 8 – 4 = 4, height AD = 15

Area of triangle ABC $= \frac{1}{2} \times$ base $\times$ height

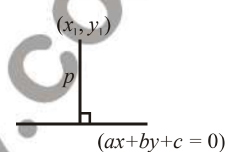
$= \frac{1}{2} \times 4 \times 15 = 30$ sq. unit

(4) Normal form of a line:



$\Rightarrow x \sin \alpha + y \cos \alpha = p$

Perpendicular distance from a point to the straight line:



$$p = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$

Ex. 39 Find the perpendicular distance from a point (2, 3) to the line $3x + 4y + 7 = 0$.

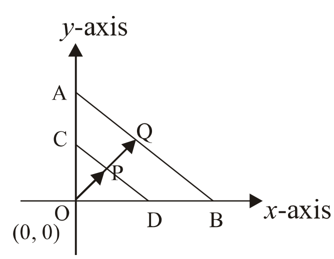
Sol.

$$p = \left| \frac{3(2) + 4(3) + 7}{\sqrt{3^2 + 4^2}} \right|$$

$$= \left| \frac{6 + 12 + 7}{5} \right| = \frac{25}{5} = 5$$

Note-1: Distance of a line $ax + by + c = 0$ from origin (0, 0) will be $\left| \frac{c}{\sqrt{a^2 + b^2}} \right|$.

Note-2 : Distance between two parallel lines (coefficient of x & y in two parallel lines will be equal).



line AB $\cong ax + by + c_1 = 0$

line CD $\cong ax + by + c_2 = 0$

Distance between AB & CD will be the difference of length of perpendicular to both lines from origin.

Let distance between AB & CD is equal to PQ = d

Hence, $d = |OQ - OP|$

$$d = \left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$$

Ex. 40 Find the distance between two parallel lines whose equations are $3x + 4y + 12 = 0$ & $3x + 4y - 13 = 0$.

Sol. $$d = \left|\frac{12-(-13)}{\sqrt{3^2+4^2}}\right| = \frac{25}{5} = 5$$

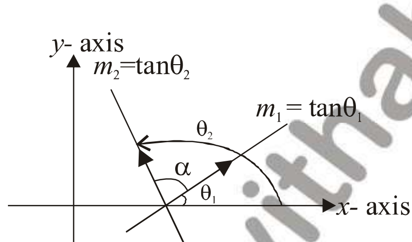
<u>Angle between two lines:</u>

If there are two lines having equations $y = m_1x + c_1$ & $y = m_2x + c_2$.

Let angle between these lines is α. then

$$\tan\alpha = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$$

Proof :



From Fig. $\alpha = \theta_2 - \theta_1$

Taking tan both sides

$$\tan\alpha = \tan(\theta_2 - \theta_1)$$

$$= \frac{\tan\theta_2 - \tan\theta_1}{1 + \tan\theta_2 \tan\theta_1}$$

$$\therefore \quad \tan\alpha = \left|\frac{m_2 - m_1}{1 + m_1m_2}\right|$$

Ex. 41 Find the angle between two lines $x - 3y + 13 = 0$ & $x + 2y - 111 = 0$.

Sol. $3y = x + 13 \Rightarrow y = \frac{x}{3} + \frac{13}{3}$ then $m_1 = \frac{1}{3}$

$2y = -x + 111 \Rightarrow y = \frac{-1}{2}x + \frac{111}{2}$ then

$m_2 = -\frac{1}{2}$

$$\tan\alpha = \left|\frac{\frac{1}{3} - \left(-\frac{1}{2}\right)}{1 + \frac{1}{3}\left(\frac{-1}{2}\right)}\right| = \left|\frac{\frac{5}{6}}{1 - \frac{1}{6}}\right| = 1$$

$\therefore \quad \alpha = 45°$

Ex. 42 Find the angle between two lines $x \sin\alpha + y \cos\alpha = p_1$, & $x \sin\beta + y \cos\beta = p_2$.

Sol. $\Rightarrow \quad x \sin\alpha + y \cos\alpha = p_1$

$$y = \frac{-x\sin\alpha}{\cos\alpha} + \frac{p_1}{\cos\alpha} = -x\tan\alpha + p_1\sec\alpha$$

$\Rightarrow \quad x \sin\beta + y \cos\beta = p_2$

$$y = \frac{-x\sin\beta}{\cos\beta} + \frac{p_2}{\cos\beta} = -x\tan\beta + p_2\sec\beta$$

$m_1 = -\tan\alpha \quad m_2 = -\tan\beta$

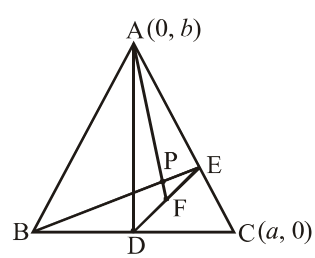
Let angle between both lines is θ

$$\Rightarrow \quad \tan\theta = \left|\frac{-\tan\alpha + \tan\beta}{1 + \tan\alpha\tan\beta}\right| = \tan(\beta - \alpha)$$

$\therefore \quad \theta = (\beta - \alpha)$

Ex.43 ABC is an Isoceles triangle with AB = AC. D is the mid-point of BC. E is the foot of the perpendicular drawn from D to AC & F is the mid-point of DE. Then the angle between AF & BE is.

Sol. Taking D as origin and BC as x-axis, co-ordinates of B and C are of the form $(-a, 0)$ & $(a, 0)$, since ABC is an isoceles triangle, AD, the altitude through A is the y axis of the system.



Let A be $(0, b)$.

Equation of AC is $\frac{x}{a}+\frac{y}{b}=1$ …(i)

Equation of DE is $y=\frac{a}{b}x$ …(ii)

since DE is $\perp$ to AC

Solving (i) and (ii)

Point E will be $\left(\frac{ab^2}{a^2+b^2}, \frac{a^2b}{a^2+b^2}\right)$

Since F is the midpoint of DE,

Point F will be $\left(\frac{ab^2}{2\left(a^2+b^2\right)}, \frac{a^2b}{2\left(a^2+b^2\right)}\right)$

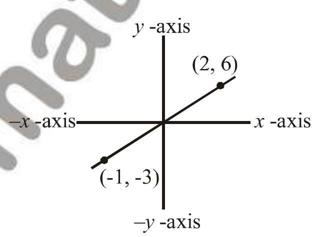
Slope of BE $m_1 = \dfrac{\frac{a^2b}{a^2+b^2}-0}{\frac{ab^2}{a^2+b^2}+a} = \dfrac{ab}{\left(2b^2+a^2\right)}$

Slope of AF $m_2 = \dfrac{\frac{a^2b}{2\left(a^2+b^2\right)}-b}{\frac{ab^2}{2\left(a^2+b^2\right)}} = \dfrac{-\left(2b^2+a^2\right)}{ab}$

Angle between AF and BE $=\frac{\pi}{2}$ $(\because m_1.m_2=-1)$

Question based upon graph (asked in SSC)

Ex. 44 What is the equation of line is passing through origin.

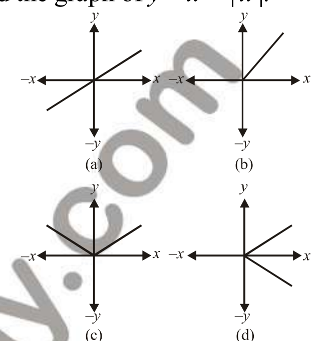


Sol. Slope of this line $=\frac{6-(-3)}{2-(-1)}=\frac{9}{3}=3$

Line passing through origin will be $y = mx$

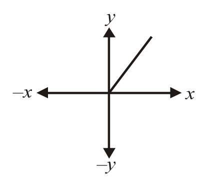
$m=3 \quad \therefore \quad y=3x$

Ex. 45 Find the graph of $y = x + |x|$.



Sol. $y = x + |x|$

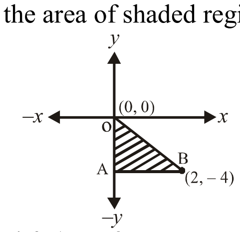
$$y=\begin{cases}2x, & x>0\\ 0 & x<0\end{cases}$$



We know that the definition of $|x|$

$$|x|=\begin{cases}x, & x>0\\ -x & x<0\end{cases}$$

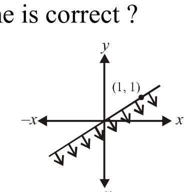
Ex. 46 Find the area of shaded region.



Sol. OA = 4 & AB = 2

then, Area of triangle OAB $=\frac{1}{2}\times 2\times 4=4$

Ex. 47 Which one is correct ?



(a) $y \le x$ (b) $y \ge x$

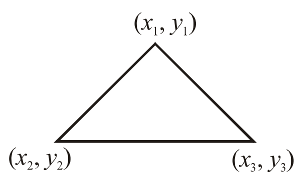
(c) $y \le -x$ (d) $y \ge x$

Sol. Point (1, 1) is on this line so equation of line will be $y = x$ or $\frac{y}{x} = 1 = \tan 45°$. Region below this line angle will be less than 45° so, slope of line will be less than 1.

$$m = \frac{y}{x} \le 1 \Rightarrow y \le x \qquad (\because y = mx)$$

Triangles

Let a triangle ABC whose vertices are $[(x_1, y_1), (x_2, y_2) \& (x_3, y_3)]$



then, Area of triangle = $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$$= \frac{1}{2}\left[x_1(y_2 - y_3) - y_1(x_2 - x_3) + 1(x_2y_3 - y_2x_3)\right]$$

$$= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

Note 1 : The vertices of an equilateral Δ cannot be rational number. For example - If vertices of a triangle are $(2, -5), (6, 7)(5, 4)$ then triangle cannot be equilateral triangle because area of equilateral triangle is in the form of $\sqrt{3}$ and if vertices are rational then area of triangle can never be irritational on using above formula.

Note 2 : If three points $(x_1, y_1), (x_2, y_2) \& (x_3, y_3)$ are collinear, then area of triangle formed by these vertices will be equal to 0.

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Or,

$$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

Ex.48 If $(p, q), (m, n)$ $(p - m, q - n)$ are collinear. find relation between $p, q, m \& n$

(a) $pq = mn$ (b) $mp = nq$

(c) $np = qm$ (d) None of these

Sol. We can find the relation using above formula but calculation will be difficult. So we will use another concept.

Slope of any line is unique so we will equate the slope of line AB & AC.

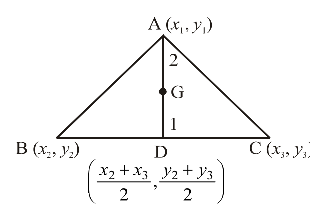
$$\Rightarrow \quad \text{Slope} = \frac{q-n}{p-m} = \frac{q-q+n}{p-p+m}$$

$\Rightarrow \quad np - mn = mq - mn$

$\Rightarrow \quad np = mq$

Centroid of Triangle

Centroid of triangle is the intersection point of medians.



In above triangle ABC, AD is median and G centroid. D will be mid point of DC and G will divide median AD into 2 : 1 internally.

On using section formula on internal division we will find co-ordinate of centroid.

$$G \equiv \left(\frac{x_1 + x_2 + x_3}{3} + \frac{y_1 + y_2 + y_3}{3}\right)$$

Ex. 49 If $x - 2y + k = 0$ is the median of a triangle whose vertices are $(-1, 3), (0, 4), (-5, 2)$, find the value of k.

Sol. Here, we cannot say that median will pass through which vertex but we can say median will pass through controid of the triangle.

Controid of the triangle

$$\equiv\left(\frac{x_1+x_2+x_3}{3}+\frac{y_1+y_2+y_3}{3}\right)$$

$$\equiv\left(\frac{-1+0-5}{3},\frac{3+4+2}{3}\right)\equiv(-2,3)$$

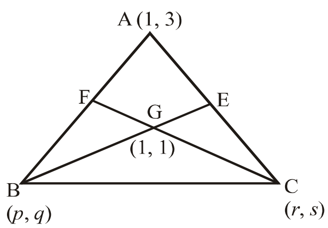
Substituting (–2, 3) in the equation of median

$\Rightarrow \quad (-2)-2(3)+k=0$

$\Rightarrow \quad k=2+6=8$

Ex. 50 The vertex A of triangle ABC is given to be (1, 3) and the medians BE and CF are $x-2y+1=0$ and $y-1=0$. then the equation of all sides of its triangle.

Sol. Solving the two medians the centroid G is (1, 1)



Let the other two vertices be B(p, q) and C (r, s). F lies on $y = 1$ and it is the midpoint of AB.

$\therefore \quad \frac{3+q}{2}=1 \quad \Rightarrow q=-1$

Again y - co-ordinate of G is

$\Rightarrow \quad \frac{3+q+s}{3}=1 \quad \Rightarrow q+s=0$

$\Rightarrow \quad s=-q=1$

E is the mid-point of AC

$\therefore \quad \frac{3+s}{2}=\frac{3+1}{2}=2=y$ - co-ordinate of E

Which lies on BE $x-2y+1=0$ (given)

$\therefore \quad x=3$ as $y=2$

$\frac{1+r}{2}=3$ or $r=5$

$\therefore$ C is (5, 1) B is (p, –1)

$\therefore \quad \frac{1+p+5}{3}=1 \quad \Rightarrow p=-3$

B is (–3, –1)

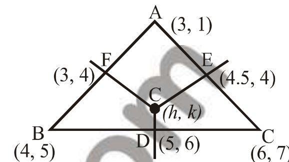
Now the equations of sides of triangle obtained by two point formulae are

$x+2y-7=0,\ x-4y-1=0\ \&\ x-y+2=0$

Circumcentre

Circumcentre of triangle is the intersection point of perpendicular bisectors of sides.

To know the method of finding the co-ordinate of circumcentre we take the following triangle as example:



Here : $DC \perp BC$, $EC \perp AC$ and $FC \perp AB$

$\Rightarrow$ Let co-ordinates of circumcentre C (h, k)

$\therefore$ Slope of line CD $=\left(\frac{k-6}{h-5}\right)$

Slope of line BC $=\left(\frac{7-5}{6-4}\right)=1$

(Slope of line CD) $\times$(Slope of line BC) $=-1$

$\left(\frac{k-6}{h-5}\right)\times 1=-1 \quad \Rightarrow h+k=11 \quad \ldots(1)$

Slope of line CE $=\left(\frac{k-4}{h-4.5}\right)$

Slope of line AC $=\left(\frac{7-1}{6-3}\right)=2$

(Slope of line CE) $\times$(Slope of line AC) $=-1$

$\left(\frac{k-4}{h-4.5}\right)\times 2=-1 \quad \Rightarrow h+2k=12.5\ldots(2)$

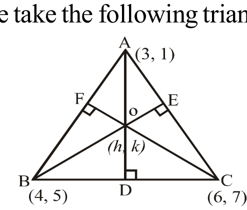
On solving eq^n. (1) & (2),

$(h, k)\equiv(10.5, 1.5)$

Orthocentre

Orthocentre of triangle is the intersection point of altitude on sides.

To know the method of finding the co-ordinate of orthocentre we take the following triangle as example:



Here : $AD \perp BC$, $BE \perp AC$ and $CF \perp AB$

$\Rightarrow$ Let co-ordinates of orthocentre O (h, k)

$\therefore$ Slope of line AD $= \left(\dfrac{k-1}{h-3}\right)$

Slope of line BC $= \left(\dfrac{7-5}{6-4}\right) = 1$

(Slope of line AD) ×(Slope of line BC) = –1

$$\left(\frac{k-1}{h-3}\right) \times 1 = -1 \Rightarrow h + k = 4 \qquad \ldots(1)$$

Slope of line CF $= \left(\dfrac{k-7}{h-6}\right)$

Slope of line AB $= \left(\dfrac{5-1}{4-3}\right) = 4$

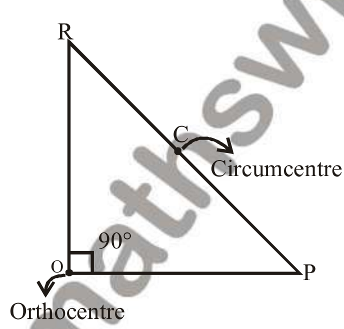
(Slope of line CF) ×(Slope of line AB) = –1

$$\left(\frac{k-7}{h-6}\right) \times 4 = -1 \qquad \Rightarrow 4h + k = 31 \qquad \ldots(2)$$

On solving eq^n. (1) & (2),

$(h, k) \equiv (9, -5)$

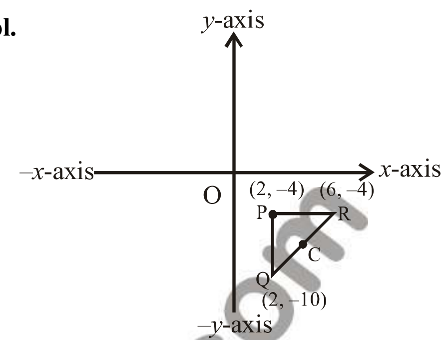
Note : In general triangle co-ordinate of circumcentre is not asked in an exam, only in right angle triangle orthocentre & circumcentre is generally asked.



In Right angle triangle mid-point of hypotenuse is the circumcentre and vertex at 90° is the orthocentre.

Ex. 51 Find the co-ordinates of the circumcentre and orthocentre of a triangle whose vertices are (2, –4), (6 –4) & (2, –10).

Sol.

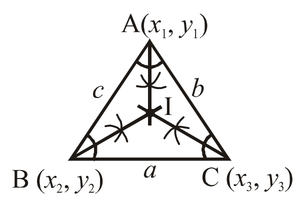


Circumcentre of this triangle PQR will be the mid point of QR $\equiv C \equiv \left(\dfrac{2+6}{2}, \dfrac{-10-4}{2}\right) = (4, -7)$

Orthocentre of this triangle PQR will be the vertex at 90° $\equiv P \equiv (2, -4)$

Incentre

Incentre is the intersection point of angle bisectors of all vertex angle.



Co-ordinates of Incentre :

$$I \equiv \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$$

where a, b, c length of side.

Pair of straight lines (passing through origin)

Standard equation : $ax^2 + 2hxy + by^2 = 0$

If angle between both the lines is θ then

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}$$

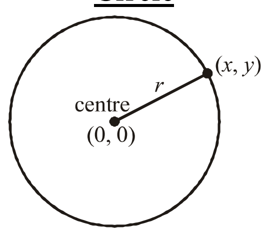
$\Rightarrow$ If $a + b = 0$ then, $\tan\theta = \infty \Rightarrow \theta = 90°$

Ex. 52 What is the angle between pair of straight lines represented by equation $3x^2 - 111xy - 3y^2 = 0$.

Sol. Here, $a + b = 3 - 3 = 0$,

then, angle between lines will be 90°.

Circle



$\Rightarrow \quad r=\sqrt{(x-0)^2+(y-0)^2}$

$\Rightarrow \quad x^2+y^2=r^2$ is called equation of circle.

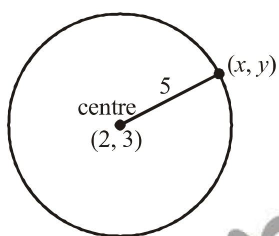
Ex.53 If $x^2+y^2=25$ is equation of a circle, find the radius and its centre.

Sol. centre = (0, 0)

Radius = 5

Ex. 54 Find the equation of circle whose centre is (2, 3) & radius is 5.

Sol.



$\Rightarrow \quad r^2=(x-2)^2+(y-3)^2$

$\Rightarrow \quad 25=(x-2)^2+(y-3)^2$

$\therefore \quad x^2+y^2-4x-6y-12=0$

Standard equation of a circle :

$x^2+y^2+2gx+2fy+c=0$

Where Centre $\equiv(-g,-f)$

Radius $=\sqrt{g^2+f^2-c}$

Ex. 55 If $x^2+y^2+6x+8y+11=0$ is the equation of a circle, find its centre and radius.

Sol. On comparing $x^2+y^2+6x+8y+11=0$ with $x^2+y^2+2gx+2fy+c=0$

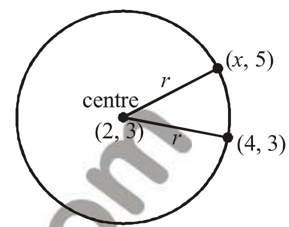
$g=3, f=4$ & $c=11$

then centre $\equiv(-g,-f)\equiv(-3,-4)$

Radius $=\sqrt{(-3)^2+(-4)^2-11}=\sqrt{14}$

Ex. 56 If two points (x, 5) (4, 3) are on the circumference of a circle if the centre of the circle is (2, 3) ,find the value of x.

Sol.



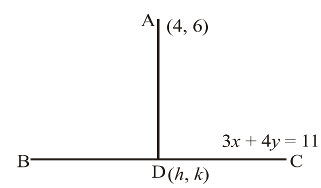
$r=\sqrt{(x-2)^2+(5-3)^2}$

$=\sqrt{(4-2)^2+(3-3)^2}$

$(x-2)^2=0 \Rightarrow x=2$

Foot of the Perpendicular

Ex. 57 Find the foot of the perpendicular in the below figure.



Let (h, k) is the foot of the perpendicular which is drawn to the line $3x+4y=11$ from the point (4, 6). AD is perpendicular to BC so product of slope of both lines will be equal to –1.

$\left(\frac{k-6}{h-4}\right)\left(\frac{-3}{4}\right)=-1 \Rightarrow 4h-3k=-2$ ....(i)

(h, k) is situated on the line $3x+4y=11$ so,

$3h+4k$ ...(ii)

On solving equation (i) & (ii)

$(h, k)\equiv(1, 2)$

Quadrilateral

There is a quadrilateral whose vertices are $A(x_1 y_1) B(x_2 y_2)$, $C(x_3 y_3)$ & $D(x_4 y_4)$ then quadrilateral will be Rhombus, Square, Rectangle & Parallelogram ?

Step I: Find the length of diagonal – There are two cases possible.

(i) case I → diagonal equal → (square or rectangle)

(ii) case II → unequal → (rhombus or parallelogram)

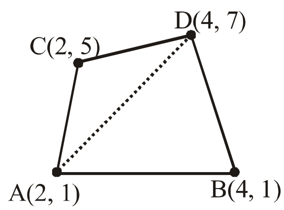
Step II: Find the slopes of diagonal – There are two cases possible.

(i) case I $\rightarrow m_1.m_2 = -1 \rightarrow$ (rhombus or square)

(ii) case II $\rightarrow m_1.m_2 \neq -1 \rightarrow$ (rectangle or parallelogram)

Ex. 58 Find the area quadrilateral formed by joining points (2, 1), (4, 1), (2, 5) and (4, 7)

Sol.



Area of triangle ADC

$$= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

$$= \frac{1}{2}\left[2(7-5) + 4(5-1) + 2(1-7)\right] = 4$$

Area of triangle ABD

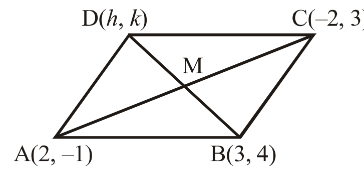
$$= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

$$= \frac{1}{2}\left[2(1-7) + 4(7-1) + 4(1-1)\right] = 6$$

Hence, Area of quadrilateral ABCD = Area of triangle ADC + Area of triangle ABD = 4 + 6 = 10 sq. unit

Ex. 59 If the three vertices of a rhombus are (2, –1), (3, 4), (–2, 3). Find the fourth vertex.

Sol.



Let the co-ordinate of the D is (h, k).

M is the mid-point of AC.

Co-ordinate of M $\equiv \left(\frac{-2+2}{2}, \frac{-1+3}{2}\right) \equiv (0,1)$

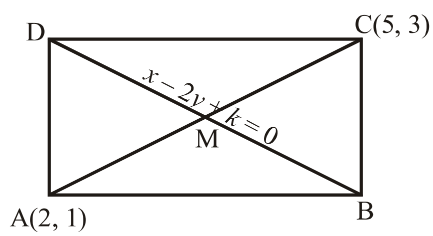
M is also the mid point of BD.

Co-ordinate of M $\equiv \left(\frac{h+3}{2}, \frac{k+4}{2}\right) \equiv (0,1)$

On solving,

D $\equiv (h, k) \equiv (-3, -2)$

Ex. 60 If two opposite vertex of a rectangle are (2, 1) & (5, 3) & the equation of other diagonal is $x - 2y + k = 0$ find k,



M is the mid-point of diagonal AC & BD.

Co-ordinate of M $\equiv \left(\frac{2+5}{2}, \frac{3+1}{2}\right) \equiv (3.5, 2)$

Point M is also situated on line $x - 2y + k = 0$ so,

$3.5 - 2 \times 2 + k = 0$

$$\Rightarrow \quad k = \frac{1}{2}$$

EXERCISE

1. Find the equation of a line x–axis.
(a) $y = 0$ (b) $x = 0$
(c) $x = 1$ (d) $y = 2$

2. Find the equation of a line y–axis.
(a) $y = 0$ (b) $x = 0$
(c) $x = 1$ (d) $y = 2$

3. Find the slope of x–axis.
(a) 0 (b) 1
(c) –1 (d) ∞

4. Find the slope of y–axis.
(a) 0 (b) 1
(c) –1 (d) ∞

5. Which one of the equation passes through origin?
(a) $2x + 3y = 1$ (b) $3x - 5y = -1$
(c) $5x + 7y = 0$ (d) $2x - 5y + 1 = 0$

6. The given equation of the line $2x - 4 = 0$ passes through which quadrant–
(a) I, II (b) II, III
(c) III, IV (d) IV, I

7. The given equation $x^2 + y^2 = 25$ passes through which quadrants–
(a) I, II, III, IV (b) II, III, IV
(c) III, IV (d) IV, I

8. The given equation of the line $2x + 3y = 5$ passes through which quadrant–
(a) I, II, III (b) II, III, IV
(c) III, IV, I (d) I, II, IV

9. The given equation of the line $x - 3y - 9 = 0$ passes through from which quadrant–
(a) I, II, III (b) II, III, IV
(c) III, IV, I (d) I, II, IV

10. Find the intercepts made by the line $3x + 4y - 12 = 0$ on the axis.
(a) 2 & 3 (b) 4 & 3
(c) 3 & 5 (d) None of these

11. The length of the intercept by the straight line $12x - 9y = 108$ between the co-ordinate axis is
(a) 12 unit (b) 18 unit
(c) 15 unit (d) 9 unit

12. Find the points where the straight line $2x - 3y = 12$ cuts x–axis and y–axis.
(a) (6, 0) & (0, 4) (b) (–6, 0) & (0, 4)
(c) (6, 0) & (0, –4) (d) (4, 0) & (6, 0)

13. Find the point where the straight line $3x - 5y = 15$ cuts x–axis and y–axis.
(a) (5, 0) & (0, –3) (b) (–5, 0) & (0, 3)
(c) (–5, 0) & (0, –3) (d) (–3, 0) & (0, 5)

14. Find the distance of point (3, 4) from (i) x–axis (ii) y-axis (iii) origin respectively.
(a) 3, 4, 5 (b) 4, 3, 5
(c) 5, 3, 4 (d) –3, 4, 5

15. Find the distance between the points of a line P(3, 6) and Q(5, 6).
(a) 2 (b) –2
(c) 5 (d) 6

16. Find the distance between the points of a lines P(–2, 6) and Q(7, –1).
(a) $\sqrt{30}$ (b) $\sqrt{24}$
(c) $\sqrt{96}$ (d) $\sqrt{130}$

17. Find the slope of the line passing through the points (1, 4) and (3, 8).
(a) 5 (b) –2
(c) –5 (d) 2

18. Find the slope of the line passing through the points $\left(\frac{1}{2}, 4\right)$ & $\left(3, \frac{4}{3}\right)$.
(a) $\frac{4}{15}$ (b) $\frac{8}{15}$
(c) $\frac{16}{15}$ (d) $-\frac{16}{15}$

19. Find the slope of the line which is perpendicular to the line passing through the points P(2, –1) and Q(–3, –5).
(a) $\frac{4}{5}$ (b) $\frac{2}{3}$
(c) $-\frac{5}{4}$ (d) $-\frac{4}{5}$

20. Find the slope of line whose equation is $2x + 3y = 7$.
(a) $-\frac{3}{2}$ (b) $-\frac{1}{2}$
(c) $\frac{2}{3}$ (d) $-\frac{2}{3}$

21. Find the area of triangle formed by lines $8x - 3y = 24$, x–axis & y–axis.
(a) 12 sq. unit (b) 6 sq. unit
(c) 18 sq. unit (d) 9 sq. unit

22. Find the area of triangle formed by lines $-5x + 3y = 12$, x–axis & y–axis.

(a) $\frac{24}{5}$ sq. unit (b) $\frac{12}{5}$ sq. unit
(c) 24 sq. unit (d) 12 sq. unit

23. Find the area of triangle formed by lines $x - 2y = 5$, $2x + 3y = 10$ and x–axis.

(a) $\frac{165}{12}$ sq. unit (b) $\frac{175}{12}$ sq. unit
(c) 15 sq. unit (d) 12 sq. unit

24. Find the area of triangle formed by lines $2x - 3y + 6 = 0$, $2x + 3y - 18 = 0$ & $y - 1 = 0$.

(a) 27 sq. unit (b) 13.5 sq. unit
(c) 9 sq. unit (d) None of these

25. Find the area of triangle formed by lines $x + y = 4$, $2x - y = 2$ & $x = 0$.

(a) 4 sq. unit (b) 9 sq. unit
(c) 16 sq. unit (d) 6 sq. unit

26. Find the area of quadrilateral intercepted by straight lines $2x + y = 6$ & $4x + 2y = 24$ between the axis.

(a) 18 sq. unit (b) 54 sq. unit
(c) 16 sq. unit (d) 27 sq. unit

27. Find the area of quadrilateral intercepted by straight lines $x + y = 2$ & $3x + 4y = 24$ between the axis.

(a) 22 sq. unit (b) 26 sq. unit
(c) 44 sq. unit (d) 11 sq. unit

28. Find the area of quadrilateral formed by straight lines $x = 1$, $x = 3$, $y = 2$ & $x = y + 3$.

(a) 6 sq. unit (b) 12 sq. unit
(c) 3 sq. unit (d) None of these

29. What is value of k given system of equations $kx + 3y = k - 3$ & $12x + ky = k$ have infinitely many solution?

(a) ± 6 (b) 6
(c) -6 (d) 7.2

30. For what value of k given system of equations $x + 3y = k$ & $2x + 6y = 2k$ are coincident lines.

(a) 1 (b) 2
(c) For all real values of k
(d) For no real values of k

31. For what value of k given system of equations $5x + 20y = 11$ & $2x + ky = 17$ are intersecting lines?

(a) $k \neq 8$ (b) $k \neq 6$
(c) $k \neq 12$ (d) $k \neq 11$

32. For what value of k given pair of equations $5x + 3y = 3$ & $12x + ky = 6$ have no solution?

(a) ± 6 (b) 6
(c) -6 (d) None of these

33. Find the equation of the line passing through the points (2, 7) and slope is 5

(a) $5x - y - 3 = 0$ (b) $5x - 3y = 7$
(c) $x - 5y = 3$ (d) $5x - y = -3$

34. Find the equation of the line passing through the point (–3, 5) and slope is $\frac{2}{3}$.

(a) $2x - 3y = -21$ (b) $2x - 3y = 21$
(c) $3x - 2y = 21$ (d) $2x + 3y = 21$

35. Find the equation of the line passing through the point (–1, 7) and (2, –5)

(a) $4x + y = 7$ (b) $x + 4y = 3$
(c) $4x + y = 3$ (d) $4x - y + 3 = 0$

36. Find the distance between the point P(1, 4) and the line $3x + 4y = 9$.

(a) 1 (b) 5
(c) 4 (d) 2

37. Find the equation of line passing through (2,3) & mid-point of the line whose ending points are (4, 9) and (6, 5).

(a) $4x + 3y = 1$ (b) $4x - 3y = -1$
(c) $3x + 4y = 1$ (d) $3x - 4y = 1$

38. For what value of k the lines $4x + ky = 3$ & $3x + 2y = 7$ are perpendicular to each other?

(a) 6 (b) ± 6
(c) -6 (d) 4

39. For what value of k the lines $(k+1)x + ky = 3$ & $5x - 2y = 7$ are perpendicular to each other?

(a) $\frac{1}{3}$ (b) $-\frac{5}{3}$
(c) $-\frac{1}{3}$ (d) 5

40. The vertices of a triangle ABC be (4, 3) (7, 1) and (9, 3) then triangle will not be –

(a) Scalene (b) Isosceles
(c) Equilateral (d) None of these

41. The vertices of a triangle ABC be A(7, 9), B(3, –7) & C(–3, 3) then the triangle will be –

(a) right angled (b) equilateral
(c) isosceles (d) Both (a) and (c)

42. Three points A(1, – 2), B(3, 4) and C(4,7) form –

(a) Collinear
(b) An equilateral triangle
(c) A right angled triangle
(d) None of these

43. A(–2, –1) B(3, –1) C(4, 3) and D(1, 2) are the four points of a quadrilateral. The quadrilateral will be –

(a) Square (b) Rhombus
(c) Parallelogram (d) None of these

44. Points A(4, –1), B(6, 0), C(7, 2) & D(5, 1) are the vertices of the following quadrilateral which will be –

(a) Square (b) Rectangle
(c) Rhombus (d) None of these

45. The three vertices of a parallelogram taken in a order are (–1, 0) (3, 1) and (2, 2) respectively. Find the coordinates of fourth vertex.

(a) (–1, 2) (b) (–2, 1)
(c) (2, 3) (d) (3, –2)

46. Find the equation of the line which cuts off intercepts 2 and 3 on the axis.

(a) $9x - 7y = 6$ (b) $3x - 2y = 5$
(c) $4x - 3y = 7$ (d) $3x + 2y = 6$

47. Find the equation of the line which passes through the point (3, –4) and makes an angles of 60° with the positive direction of x– axis.

(a) $x\sqrt{2} + y\sqrt{3} = 0$
(b) $x\sqrt{3} - y = 4 + 3\sqrt{3}$
(c) $x\sqrt{3} + y = 3\sqrt{2} + 5$
(d) None of these

48. The triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, –1), (2, 1) and (0, 3). Find the ratio of area of the original triangle to the area of the formed triangle.

(a) 3 : 1 (b) 1 : 3
(c) 4 : 1 (d) 1 : 4

49. Find the co-ordinates of the circumcentre of the triangle whose vertices are (8, 6) (8, –2) and (2, –2).

(a) (2, 5) (b) (5, 2)
(c) (2, 1) (d) (5, 1)

50. Find the co-ordinates of the centroid of a triangle whose vertices are (0, 6), (8, 12) and (8, 0).

(a) $\left(\frac{16}{3}, 6\right)$ (b) $\left(6, \frac{16}{3}\right)$
(c) $(6, 5)$ (d) $(6, 3)$

51. If $x + 4y = 2k$ is a median of the triangle whose vertices are points A(4, 3), B(7, 1) & C(9, 3) find the value of k.

(a) 6 (b) 7
(c) 9 (d) 8

52. The vertices of a triangle are (3, –5) and (–7, 4). If its centroid is (2, – 12), find the third vertex.

(a) (10, –35) (b) (–2, 10)
(c) (10, 35) (d) (–3, 10)

53. The base AB of two equilateral triangles ABC and ABC' with side $2a$ lies along the x– axis such that the mid-points of AB is the origin. Find the co-ordinates of the vertices C and C' of the triangles.

(a) $(0, \sqrt{3}a)$ & $(0, -\sqrt{3}a)$
(b) $(0, \sqrt{4}a)$ & $(0, -\sqrt{4}a)$
(c) $(0, \sqrt{3}a)$ & $(0, -\sqrt{3}a)$
(d) $(\sqrt{4}a, 0)$ & $(-\sqrt{4}a, 0)$

54. Find the area of the quadrilateral ABCD whose vertices are respectively A(1, 1), B(7, – 3), C(12, 2) & D(7, 21).

(a) 132 sq. units (b) 124 sq. units
(c) 136 sq. units (d) 112 sq. units

55. If the points (3, 4), (7, 12) & $(k + 1, k - 2)$ are collinear find value of k.

(a) 4 (b) – 2
(c) – 4 (d) 2

56. The lines $2x + y = 5$ & $x + 2y = 4$ intersect at point.

(a) (1, 2) (b) (2, 1)
(c) (5/2, 0) (d) (0, 2)

57. Area of triangle formed by the graph of the line $2x + 3y + 6 = 0$ along with the co-ordinate axis is –

(a) 3/2 sq. units (b) 3 sq. units
(c) 6 sq. units (d) 1/2 sq. units

58. Area of the trapezium formed by x–axis, y–axis and the lines $3x + 4y = 12$ & $6x + 8y = 60$ will be –

(a) 31.5 sq. units (b) 48 sq. units
(c) 36.5 sq. units (d) 37.5 sq. units

59. If the distance between two points (a, –3) and (3, a) is 6 unit, then a = ?

(a) ±3 (b) ±6
(c) 3 (d) 6

60. The area of the triangle formed by the lines $5x + 7y = 35$, $4x + 3y = 12$ & x–axis is –

(a) 160/13 sq. units (b) 1050/13 sq. units
(c) 140/3 sq. units (d) 10 sq. units

61. The distance between the points (0, 0) and the intersecting point of the graphs of $x = 3$ and $y = 4$ is –

(a) 4 units (b) 3 units
(c) 2 units (d) 5 units

62. In the xy - co-ordinate system, if $(a + 3, b + 1)$ & $(a + 5, b + k)$ are two points on the line defined by the equation $2y = 7x - 9$, then k = ?

(a) 8 (b) 3
(c) 7/3 (d) 1

63. The graph of the equation $2x + 3y = 6$

(a) intersects x axis but not y–axis
(b) intersect y axis but not x – axis
(c) Passes through the origin
(d) intersects both x–axis and y–axis

64. The radius of the circumcirele of the triangle made by x – axis, y axis and $4x + 3y = 12$ is

(a) 2 units (b) 2.5 units
(c) 3 units (d) 4 units

65. Line $x + 4y = 7$ & line $kx + 8y = 18$ are parallel lines. Find the value of k.

(a) 3 (b) 4
(c) 6 (d) 2

66. Find the angle between $2x + 3y = 5$ & $3x - 2y = 1$.

(a) 0° (b) 60°
(c) 90° (d) 120°

67. An equation whose graph passes through the origin, out of the given equations $2x - 3y = 2, 2x - 3y = 3,$ $-2x + 3y = 5$ & $2x + 3y = 0$ is

(a) $-2x + 3y = 5$ (b) $2x - 3y = 3$
(c) $2x + 3y = 0$ (d) $2x - 3y = 2$

68. Line $2x + 5y + k = 0$ passes through quadrants I, II and IV. Find the range of values 'k'can take.

(a) $k < 0$ (b) $k > 0$
(c) $k < 1$ (d) $0 < k < 1$

69. What is the equation of a set of points equidistant from the lines $y = 5$ and $x = -4$?

(a) $x + y = -1$ (b) $x - y = -1$
(c) $x + y = 1$ (d) $-x + y = -1$

70. Rhombus ABCD has A at (2, 0) and B at (4, 4). If one of the sides of the rhombus lies on the x – axis, what is the area of this rhombus ?

(a) 40 sq.unit (b) $8\sqrt{6}$ sq. unit
(c) $4\sqrt{5}$ sq. unit (d) $8\sqrt{5}$ sq. unit

71. Two sides of a triangle are given by equations $2x + 3y = 12$ and $3x - 2y = -6$. If the circumcentre of this triangle lies at (2, 0), find the area of this triangle.

(a) $\frac{48}{13}$ sq unit (b) $\frac{192}{13}$ sq unit
(c) 24 sq unit (d) 96 sq unit

72. What are the coordinates of the incentre of triangle given by (3, 4,), (3, 7) and (7, 4) ?

(a) (5, 5) (b) (5, 4)
(c) (4, 5) (d) (5, 5)

73. Triangle PQR of area 6 sq units has P(0, 0) and Q(4, 0). If the orthocentre, circumcentre and incentre of this triangle lie on the line $x = 2$, find the coordinates of R.

(a) (2, 2) (b) (2, 1)
(c) (2, 4) (d) (2, 3)

74. Point P is given by (0, 3) and point Q is given by (0, 7). Point R exists such that PR + QR = 8. What is the maximum possible value of abscissa of R ?

(a) 9 (b) 3
(c) $3\sqrt{3}$ (d) $2\sqrt{3}$

75. Points P(2, 5), Q(2, 11) and R form a triangle in which the incentre, circumcentre and orthocentre lie on a straight line. Which of the following will be Point R ?

(a) $\left(2-\sqrt{35},\ 10\right)$ (b) (2, 7)
(c) (2, 8) (d) $\left(4-\sqrt{5},\ 1\right)$

76. What is the area enclosed between lines $2x + y = 6$, $x = y + 1$ & y-axis ?

(a) $\frac{49}{6}$ sq. unit (b) $\frac{8}{3}$ sq. unit
(c) 7 sq. unit (d) 8 sq. unit

77. Area of triangle formed by straight lines $2x + 3y = 5$ and $y = 3x - 13$ with x-axis is –

(a) 11 sq. unit (b) 22 sq. unit
(c) $\frac{11}{6}$ sq. unit (d) $\frac{11}{12}$ sq. unit

78. If $b > a$, $d > c$ then area of quadrilateral formed by straight lines $x = a$, $x = b$, $y = c$ and $y = d$ is –

(a) $(b-a)(d-c)$ (b) $\frac{1}{2}(b-a)(d-c)$
(c) $(b+a)(d+c)$ (d) $\frac{1}{2}(b+a)(d+c)$

79. If L_1 and L_2 be perpendicular from the origin upon the straight lines $x\sec\theta + y\operatorname{cosec}\theta = a$ & $x\cos\theta - y\sin\theta = a\cos 2\theta$ respectively, then the value of $4L_1^2 + L_2^2$ is –

(a) a^2 (b) $2a^2$
(c) $4a^2$ (d) $3a^2$

80. The sides AB, BC, CD and DA of a Quadrilateral ABCD have the equations $x + 2y = 3$, $x = 1$, $x - 3y = 4$ & $5x + y + 12 = 0$ respectively, then the angle between the diagonals AC and BD is

(a) 30° (b) 45°
(c) 60° (d) 90°

81. The equation of a line passing through the point (4, 4) and cutting off intercepts on the axis whose sum is 18?

(a) $x + 2y - 12 = 0$ but not $2x + y - 12 = 0$
(b) neither $x + 2y - 12 = 0$ nor $2x + y - 12 = 0$
(c) $2x + y - 12 = 0$ but not $x + 2y - 12 = 0$
(d) $x + 2y - 12 = 0$ or $2x + y - 12 = 0$

82. The area of the triangle formed by the lines $5x + 7y = 35$ and $4x + 3y = 12$ and x-axis will be –

(a) $\frac{160}{13}$ sq. unit (b) $\frac{21}{13}$ sq. unit
(c) 92 sq. unit (d) 21 sq. unit

83. Find the co-ordinate of centre of circle which touch the all sides of a triangles ABC whose vertices are A (– 3, – 2), B(– 2, 3) & C(3, 2) are situated.

(a) (1, 1)
(b) $\left(\frac{2\sqrt{2}}{2-\sqrt{2}}, \frac{3\sqrt{2}}{2-\sqrt{2}}\right)$
(c) $\left(\frac{2\sqrt{2}}{2+\sqrt{2}}, \frac{3\sqrt{2}}{2+\sqrt{2}}\right)$
(d) $\left(\frac{-2\sqrt{2}}{2+\sqrt{2}}, \frac{3\sqrt{2}}{2+\sqrt{2}}\right)$

84. If point (4, 3) is the centroid of the triangle ABC and vertices are A(x, y), B(– 3, 7) & C(9, 7). Find the area of the triangle.

(a) 66 sq. units (b) 55 sq. units
(c) 44 sq. units (d) 72 sq. units

85. The co-ordinates of the point P which divides the join of A(5, –2) and B(9, 6) in the ratio 3 : 1will be –

(a) $(4,-7)$ (b) $\left(\frac{7}{2}, 4\right)$

(c) $(8,4)$ (d) $(12,8)$

86. ABC is a triangle with vertices A(7, –3), B(3, –1) and C(5, 3). If AD is one of its medians, then the length of this median is –

(a) 7 units (b) 5 units

(c) 8 units (d) 6 units

87. A(–3, b) and B(1, b + 4) are two points and the co-ordinates of the middle point of AB are (–1, 1). The value of b is –

(a) 1 (b) –1

(c) 2 (d) 0

88. The area of quadrilateral ABCD with A(1, 1), B(7, –3), C(12, 2) and D(7, 21) as its vertices will be –

(a) 35 sq. units (b) 65 sq. units

(c) 85 sq. units (d) 132 sq. units

89. The number of points in which the circle $x^2 + y^2 = 16$ is intersected by the line $x + y = 16$ is –

(a) 1 (b) 2

(c) more than 2 (d) no point

90. The value of k so that the line joining A(–5, 7) and B(0, –2) and the line joining C(1, –3) and D(4, k) are perpendicular to each other is –

(a) 0 (b) 1

(c) – 4/3 (d) 3

91. The equation of the line passing through the point (1, 1) and perpendicular to the line $3x + 4y - 5 = 0$ is –

(a) $3x+4y-7=0$ (b) $3x+4y+k=0$

(c) $4x-3y+1=0$ (d) $4x-3y-1=0$

92. The angle between the lines represented by the equations $y-\sqrt{3}\,x-5=0$ and $\sqrt{3}\,y-x+6=0$ is –

(a) 30° (b) 45°

(c) 60° (d) 90°

93. If the vertices of a triangle are (4, k) (6, 9) and (h, 4) respectively and co-ordinate of centroid is (3, 6) then h and k will be –

(a) $(2,-3)$ (b) $(2,3)$

(c) $(-1,5)$ (d) $(4,3)$

94. If point (–5,4) divides the line segment between the co-ordinate axis in the ratio 1 : 2, then its equation –

(a) $8x+5y+20=0$ (b) $5x+8y-7=0$

(c) $8x-5y+60=0$ (d) $5x-8y+57=0$

95. If (a, b), (c, d) and ($a - c$, $b - d$) are collinear, then which one of the following is correct?

(a) $bc-ad=0$ (b) $ab-cd=0$

(c) $bc+cd=0$ (d) $ab+cd=0$

96. The middle point of the segment of the straight line joining the points (p, q) and (q, –p) is (r/2, s/2). What is the length of the segment?

(a) $\left[\left(s^2+r^2\right)^{1/2}\right]/2$ (b) $\left[\left(s^2+r^2\right)^{1/2}\right]/4$

(c) $\left(s^2+r^2\right)^{1/2}$ (d) $s+r$

97. The point of intesection of the two lines $x + 3y - 10 = 0$ and $2x + y - 5 = 0$ is at 'd' distance from origin. What is the value of d?

(a) $\sqrt{10}$ (b) $\sqrt{3}$

(c) $\sqrt{5}$ (d) $\sqrt{7}$

98. The line through the points (4, 3) and (2, 5) cuts off intercepts of lenght λ and μ on the axis. Which one of the following is correct?

(a) $\lambda>\mu$ (b) $\lambda<\mu$

(c) $\lambda<-\mu$ (d) $\lambda=\mu$

99. What is the area of the triangle by the lines $y - x = 0$, $y + x = 0$ & $x = c$?

(a) $c/2$ (b) c^2

(c) $2c^2$ (d) $c^2/2$

100. What is the equation to the straight line joining the origin to the point of intersection of the lines $\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{b}+\frac{y}{a}=1$?

(a) $x+y=0$ (b) $x+y+1=0$

(c) $x-y=0$ (d) $x+y+2=0$

101. If line $x\cos\theta+y\sin\theta=2$ is perpendicular to the line $x-y=3$, then what is one of the value of θ?

(a) $\pi/6$ (b) $\pi/4$

(c) $\pi/2$ (d) $\pi/3$

102. What is the product of the perpendicular from the two points $\left(\pm\sqrt{b^2-a^2},0\right)$ to the lines $ax\cos\phi+by\sin\phi=ab$?

(a) a^2 (b) b^2

(c) ab (d) a/b

103. What is the foot of the perpendicular from the point (2, 3) on the line $x+y-11=0$?

(a) (1, 10) (b) (5, 6)

(c) (6, 5) (d) (7, 4)

104. What is the image of the point (1, 2) on the line $3x+4y-1=0$?

(a) $\left(-\frac{7}{5},-\frac{6}{5}\right)$ (b) $\left(\frac{7}{8},\frac{1}{2}\right)$

(c) $\left(\frac{7}{8},-\frac{1}{2}\right)$ (d) $\left(-\frac{7}{5},\frac{1}{2}\right)$

105. If the points $(k, 2-k)$, $(-k+1, 2k)$ & $(-4-k, 6+2k)$ are collinear, then k is equal to –

(a) 1 (b) 2

(c) $\frac{4}{3}$ (d) $\frac{3}{4}$

106. The foot of the perpendicular drawn from the point (2, –1) to a straight line L is (1, 3). The equation of straight line L is –

(a) $x-4y+11=0$ (b) $x+4y+13=0$

(c) $4x-y-1=0$ (d) $4x+y-7=0$

107. Equation of the line through the point (2, 3) and perpendicular to the line joining (–5, 6) and (–6, 5) is given by –

(a) $x+y+5=0$ (b) $x-y+5=0$

(c) $x-y-5=0$ (d) $x+y-5=0$

108. If p be the length of the perpendicular from the origin on the straight line $x+2by=2p$, then what is the value of b = ?

(a) $\frac{1}{p}$ (b) p

(c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$

109. What is the angle between the two straight lines $y=\left(2-\sqrt{3}\right)x+5$ and $y=\left(2+\sqrt{3}\right)x-7$?

(a) 60° (b) 45°

(c) 30° (d) 15°

110. Which one of the following points on the line $2x-3y=5$ is equidistant from (1, 2) and (3, 4)?

(a) (7, 3) (b) (4, 1)

(c) (1, –1) (d) (–2, –3)

111. What is the locus of a moving point equidistant from the straight lines $x+y=0$ and $x-y=0$?

(a) $xy=0$ (b) xy = constant

(c) $x=0$ (d) $y=0$

112. Find the value of k, if the straight line $2x+3y+4+k(6x-y+12)=0$ is perpendicular to a line $7x+5y=4$.

(a) $\frac{29}{37}$ (b) $-\frac{29}{37}$

(c) 1 (d) 4

113. The vertices of a ΔABC are $(\lambda, 2-2\lambda), (-\lambda+1, 2\lambda)$ and $(-4-\lambda, 6-2\lambda)$. If its area be 70 units, then number of integral values of λ is –

(a) 1 (b) 2

(c) 4 (d) 0

114. The area of a triangle is 5 and two of its vertices are $A(2,1), B(3,-2)$. Then, the third vertex which lies on the line $y=x+3$ is –

(a) $\left(\frac{7}{2},\frac{13}{2}\right)$ (b) $\left(\frac{5}{2},\frac{5}{2}\right)$

(c) (5,4) (d) (0, 0)

115. Vertices of a ΔABC are the points (0, 0), (a, 0) and $\left(\frac{a}{2}, \frac{a\sqrt{3}}{2}\right)$. Its incentre is the point

(a) $\left(\frac{a}{3}, \frac{a\sqrt{3}}{2}\right)$ (b) $\left(\frac{a}{2}, \frac{a\sqrt{3}}{6}\right)$

(c) (a, 2a) (d) (0, 0)

116. The orthocentre of the triangle with vertices $\left[2, \frac{(\sqrt{3}-1)}{2}\right], \left(\frac{1}{2}, -\frac{1}{2}\right)$ and $\left(2, -\frac{1}{2}\right)$ is

(a) $\left(\frac{1}{2}, -\frac{1}{2}\right)$ (b) $\left(2, -\frac{1}{2}\right)$

(c) (2, – 1) (d) (0 , 0)

117. If (a, 0), (0, b) and (1, 1) are collinear, what is the ($a + b - ab$) equal to ?

(a) 2 (b) 1

(c) 0 (d) –1

118. Find the equation of the line joining the points of intersection $2x+y=4$ with $x-y+1=0$ and $2x-y-1=0$ with $x+y-8=0$

(a) $2x+3y+6=0$ (b) $3x+2y+12=0$

(c) $3x-2y+1=0$ (d) None of these

119. Find the length of the perpendicular from the point (3, –2) to the straight line $12x-5y+6=0$.

(a) 5 unit (b) 4 unit

(c) 6 unit (d) 8 unit

120. Find the distance between two parallel lines $5x+12y-30=0$ and $5x+12y-4=0$.

(a) 3 unit (b) 7 unit

(c) 5/2 unit (d) 2 unit

121. Find the equation of the line through the point of intersection of $2x-3y+1=0$ and $x+y-2=0$ which is parallel to the y-axis.

(a) $x=1$ (b) $8x=7$

(c) $x+3=0$ (d) $x=6$

122. Find the area of region bounded by straight lines $|x|+|y|=m$.

(a) m^2 (b) $2m^2$

(c) $3m^2$ (d) $4m^2$

123. A traingle is formed by the x-axis and the lines $2x + y = 4$ and $x - y + 1 = 0$ as three sides. Taking the side along x–axis as its base, then corresponding altitude of the triange is –

(a) 2 units (b) 3 units

(c) 5 units (d) 1 unit

124. The triangle formed by the points A(2a, 4a), B(2a, 6a) and C(2a+ $\sqrt{3}a$,5a) is –

(a) right angled (b) isosceles

(c) equilateral (d) None of these

125. What is a locus of a point which is equidistant from the points ($a + b$, $a - b$) and ($b - a$, $a + b$)?

(a) $bx - ay = 0$ (b) $bx + ay = 0$

(c) $-ax + by = 0$ (d) $ax + by = 0$

126. The locus of a point whose sum of distance from point (3, 0) and (– 3, 0) is 4 will be –

(a) $\frac{x^2}{4} - \frac{y^2}{5} = 1$ (b) $\frac{x^2}{5} - \frac{y^2}{4} = 1$

(c) $2x^2 - 3y^2 = 6$ (d) None of these

127. If the lines $3y + 4x = 1$, $y = x + 5$ and $5y + bx = 3$ are concurrent, then what is the value of b?

(a) 1 (b) 3

(c) 6 (d) 0

128. If $ax + by + c = 0$ is a straight line $a \neq 0$, $b \neq 0$, $c = 0$, line will pass through?

(a) (0, 0) (b) (3, 2)

(c) (2, 2) (d) None of these

129. Find the area of triangle which is formed by $y = |x| - 5$ & co-ordinate x-axis.

(a) 10 sq. unit (b) 20 sq. unit

(c) 25 sq. unit (d) 50 sq. unit

130. Find the area of triangle of lines

$\frac{x}{6}+\frac{y}{7}=1,\ -\frac{x}{4}+\frac{y}{7}=1$ & x–axis.

(a) 30 sq. unit (b) 35 sq. unit
(c) 70 sq. unit (d) 60 sq. unit

131. Find the equation of the line which is perpendicular to to $3x-4y-12=0$ & forming a triangle of area 24 with co-ordinate axis.

(a) $3x-4y=\pm 12$ (b) $3x-4y=\pm 24$
(c) $4x+3y=\pm 12$ (d) $4x+3y=\pm 24$

132. Find the circumcentre of the triangle whose equation of sides are $x+y=5, x-y+1=0$ & $y=1$

(a) (2, 1) (b) (–2, 1)
(c) (–2, –1) (d) (1, 2)

133. Find the area of the triangle whose vertices are $(p, q+r), (p, q-r)$ & $(-p, r)$.

(a) $2pr$ (b) $2qr$
(c) $q(p+r)$ (d) $r(p-q)$

134. The number of lines that are parallel to $2x + 6y + 7 = 0$ and have an intercept of length 10 between the co-ordinate axis is –

(a) 2 (b) 1
(c) Infinite (d) 0

135. If the origin gets shifted to (2, 2), then what will be the new co-ordinates of the point (4, –2) ?

(a) (4, 2) (b) (2, 4)
(c) (–2, 4) (d) (2, –4)

136. Two vertices of the triangle are (5, –1) & (–2, 3). If the orthocentre of the triangle is the origin, what will be the co-ordinates of the third vertex ?

(a) (4, 7) (b) (4, –7)
(c) (– 4, 7) (d) (– 4, –7)

137. One side of the rectangle lies along the line $4x + 7y = -5$. Two of its vertices are (–3, 1) & (1, 1). Which of the following will be an equation which represents any of the other three straight lines ?

(a) $y + 1 = 0$ (b) $4x + 7y = 3$
(c) $7x - 4y = 3$ (d) $7x - 4y = -3$

138. What will be the equation of the straight line that passes through the intersection of the straight lines $2x - 3y = -4$ and $3x + 4y = 5$ and is perpendicular to the straight line $3x-4y-5=0$?

(a) $8x+6y=\frac{58}{17}$ (b) $4x+3y=\frac{84}{17}$
(c) $8x+6y=\frac{32}{7}$ (d) $4x+3y=\frac{62}{17}$

139. Four vertices of a parallelogram ABCD are A(–3, –1), B(a, b), C(3, 3) and D(4, 3), then ratio of a to b –

(a) 1 : 3 (b) 3 : 1
(c) 1 : 2 (d) 4 : 1

140. ABC is an isoceles triangle. If the cordinates of B(1, 3) and C(–2, 7), then the cordinates of the vertex A will be

(a) $\left(\frac{1}{3},2\right)$ (b) $\left(\frac{5}{6},6\right)$
(c) (4, 6) (d) (2, 5)

141. If A and B are two points on the line $3x + 4y + 15 = 0$ such that OA = OB = 9 units, where O is the origin. Find the area of the triangle OAB.

(a) $18\sqrt{2}$ sq. units (b) $3\sqrt{2}$ sq. units
(c) $6\sqrt{2}$ sq. units (d) $15\sqrt{2}$ sq. units

142. If the cordinates of the points are P(6, 3), Q(–3, –5), R(4, –2) and S(a, 3a) respectively and if the the ratio of the area of the triangles PQR and SRQ is 2 : 1, then the value of a is

(a) 13 (b) 23
(c) $-\frac{11}{4}$ (d) $-\frac{23}{36}$

143. A(3, 1), B(6, 5) and C(x, y) are three points such that the angle ACB is a right angle and the area of triangle ABC is 7. The number of such points C that are possible is

(a) 4 (b) 3
(c) 1 (d) 2

144. The distance between the orthocentre and circumcentre of the triangle with vertices (1, 0), $\left(-\frac{1}{2},\frac{\sqrt{3}}{2}\right)$ & $\left(\frac{-1}{2},\frac{\sqrt{3}}{2}\right)$ is

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) $\frac{1}{3}$ (d) 0

145. Two sides of a square lie on the lines $x + y - 1 = 0$ and $x + y = -2$. Then area of the square will be

(a) $\frac{9}{2}$ sq. units (b) $\frac{11}{4}$ sq. units

(c) 5 sq. units (d) 4 sq. units

146. If p is the length of perpendicular from the origin to the line $\frac{x}{a}+\frac{y}{b}=1$, then

(a) $\frac{1}{p^2}=\frac{1}{b^2}-\frac{1}{a^2}$ (b) $\frac{1}{p^2}=\frac{1}{a^2}-\frac{1}{b^2}$

(c) $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$ (d) None of these

147. What will be the area of the rhombus formed by lines $ax \pm by \pm c = 0$?

(a) $\frac{c^2}{2ab}$ (b) $\frac{2c^2}{ab}$

(c) $\frac{4c^2}{ab}$ (d) $\frac{c^2}{4ab}$

148. The cordinates of the mid points of the sides of a triangle are (4, 2), (3, 3) and (2, 2). What will be the cordinates of the centroid of the triangle ?

(a) $\left(3,-\frac{7}{3}\right)$ (b) $\left(-3,-\frac{7}{3}\right)$

(c) $\left(-3,\frac{7}{3}\right)$ (d) $\left(3,\frac{7}{3}\right)$

149. If the slope of line is – 6 and the y-intercept is 2. Find the equation of the line.

(a) $6x+y=2$ (b) $6x-y=2$

(c) $6x+y=-2$ (d) $x+6y=2$

150. If a and b are real numbers between 0 and 1 such that the points $(a, 1)$, $(1, b)$ and $(0, 0)$ form an equilateral triangle. Then the value of a is

(a) $-2+\sqrt{3}$ (b) $-1+\sqrt{3}$

(c) $2-\sqrt{3}$ (d) $2+\sqrt{3}$

*****

Answer

1.	(a)	2.	(b)	3.	(a)	4.	(d)	5.	(c)	6.	(d)	7.	(a)	8.	(d)	9.	(c)
10.	(b)	11.	(c)	12.	(c)	13.	(a)	14.	(b)	15.	(a)	16.	(d)	17.	(d)	18.	(d)
19.	(c)	20.	(d)	21.	(a)	22.	(a)	23.	(b)	24.	(b)	25.	(d)	26.	(d)	27.	(a)
28.	(c)	29.	(a)	30.	(c)	31.	(a)	32.	(d)	33.	(a)	34.	(a)	35.	(c)	36.	(d)
37.	(b)	38.	(c)	39.	(b)	40.	(c)	41.	(d)	42.	(a)	43.	(b)	44.	(c)	45.	(b)
46.	(d)	47.	(b)	48.	(c)	49.	(b)	50.	(a)	51.	(d)	52.	(a)	53.	(c)	54.	(a)
55.	(b)	56.	(b)	57.	(b)	58.	(a)	59.	(a)	60.	(a)	61.	(d)	62.	(d)	63.	(d)
64.	(b)	65.	(d)	66.	(c)	67.	(c)	68.	(a)	69.	(c)	70.	(d)	71.	(b)	72.	(c)
73.	(d)	74.	(b)	75.	(c)	76.	(a)	77.	(d)	78.	(a)	79.	(a)	80.	(d)	81.	(d)
82.	(b)	83.	(d)	84.	(d)	85.	(c)	86.	(b)	87.	(b)	88.	(d)	89.	(d)	90.	(c)
91.	(d)	92.	(a)	93.	(c)	94.	(c)	95.	(a)	96.	(c)	97.	(a)	98.	(d)	99.	(b)
100.	(c)	101.	(b)	102.	(a)	103.	(b)	104.	(a)	105.	(c)	106.	(a)	107.	(d)	108.	(d)
109.	(a)	110.	(b)	111.	(a)	112.	(b)	113.	(c)	114.	(a)	115.	(b)	116.	(b)	117.	(c)
118.	(c)	119.	(b)	120.	(d)	121.	(a)	122.	(b)	123.	(d)	124.	(c)	125.	(c)	126.	(a)
127.	(c)	128.	(a)	129.	(c)	130.	(b)	131.	(d)	132.	(a)	133.	(a)	134.	(a)	135.	(d)
136.	(c)	137.	(c)	138.	(d)	139.	(d)	140.	(b)	141.	(a)	142.	(d)	143.	(d)	144.	(b)
145.	(a)	146.	(c)	147.	(b)	148.	(d)	149.	(a)	150.	(d)						

Solution & Hints

Sol^n 1. Equation of x - axis $y = 0$

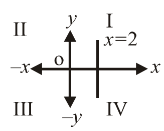
Sol^n 2. Equation of y - axis $x = 0$

Sol^n 3. Slope of x–axis $= 0$ $\left(\therefore\ y = 0x + 0\right)$

Sol^n 4. Slope of y–axis $= \infty = \dfrac{1}{0}$ $\left(\therefore\ 0y = x + 0\right)$

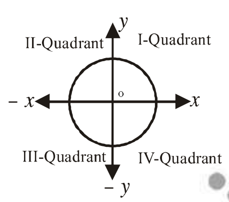
Sol^n 5. In any equation of line passing through origin, constant term will be zero. So $5x + 7y = 0$ will be answer. We can verify all option by putting $x = 0$ & $y = 0$

Sol^n 6. $2x - 4 = 0 \quad \Rightarrow x = 2$

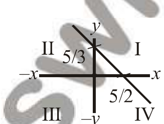


Line will pass through I and IV quadrant

Sol^n 7. Equation $x^2 + y^2 = 25$ is a equation of a circle whose centre is origin (0, 0), so this graph will pass through all four quadrants.



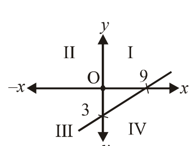
Sol^n 8. $2x + 3y = 5$



$$\frac{x}{5/2} + \frac{y}{5/3} = 1$$

Line will pass through I, II and IV quadrant

Sol^n 9. $x - 3y = 9 \Rightarrow \dfrac{x}{9} + \dfrac{y}{-3} = 1$

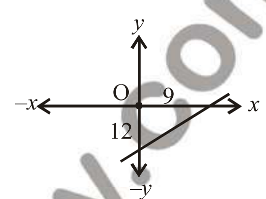


Line will pass through I, III and IV quadrant

Sol^n 10. $3x + 4y = 12 \Rightarrow \dfrac{x}{4} + \dfrac{y}{3} = 1$

Length of intercepts made at x -axis at $x = 4$ & at y -axis at $y = 3$.

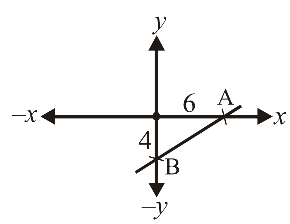
Sol^n 11. $12x - 9y = 108 \Rightarrow \dfrac{x}{9} + \dfrac{y}{-12} = 1$



Length of intercept between axis

$$= \sqrt{(9)^2 + (-12)^2} = 15$$

Sol^n 12. $2x - 3y = 12 \Rightarrow \dfrac{x}{6} + \dfrac{y}{-4} = 1$



at x -axis point A $\equiv$ (6, 0)

at y -axis point B $\equiv$ (0, –4)

Method - 2

line $2x = 3y = 12$ will cut x -axis where $y = 0$

then $2x = 12 \Rightarrow x = 6$ and point (6, 0) line

$2x - 3y = 12$ will cut y-axis where $x = 0$ then

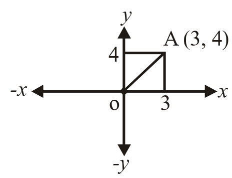
$-3y = 12 \Rightarrow y = -4$ and point (0, – 4)

Sol^n 13. $3x - 5y = 15 \Rightarrow \dfrac{x}{5} + \dfrac{y}{-3} = 1$

at x -axis point $\equiv$ (5, 0)

at y -axis point $\equiv$ (0, -3)

Sol^n 14.



(i) distance from x-axis $= 4$
(ii) distance from y-axis $= 3$
(iii) distance from origin $=$

$OA = \sqrt{(3)^2 + (4)^2} = 5$

Sol^n 15. Distance between the points P and Q

$PQ = \sqrt{(5-3)^2 + (6-6)^2} = \sqrt{(2)^2 + (0)^2} = 2$

Sol^n 16. Distance between the points P and Q

$PQ = \sqrt{(7+2)^2 + (-1-6)^2}$

$= \sqrt{81+49} = \sqrt{130}$

Sol^n 17. $P(x_1, y_1)$————————$Q(x_2, y_2)$

Slope of line $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$P(1, 4)$ & $Q(3, 8)$

then, Slope $= m = \dfrac{8-4}{3-1} = 2$

Sol^n 18. $P\left(\dfrac{1}{2}, 4\right)$ & $Q\left(3, \dfrac{4}{3}\right)$

then, Slope $= m = \dfrac{\frac{4}{3} - 4}{3 - \frac{1}{2}} = \dfrac{-8}{3} \times \dfrac{2}{5} = \dfrac{-16}{15}$

Sol^n 19. $P(2, -1)$ & $Q(-3, -5)$

Slope of line PQ $= m = \dfrac{-5+1}{-3-2} = \dfrac{-4}{-5} = \dfrac{4}{5}$

Slope of line perpendicular to PQ $= \dfrac{-1}{\text{Slope of PQ}}$

$= \dfrac{-1}{4/5} = -\dfrac{5}{4}$

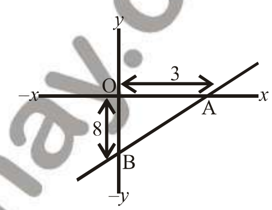
Sol^n 20. If equation of a straight line : $y = mx + c$

where m is the slope of line

$\therefore\ 2x + 3y = 7 \Rightarrow\ y = \dfrac{-2x}{3} + \dfrac{7}{3}$

hence, slope $m = \dfrac{-2}{3}$

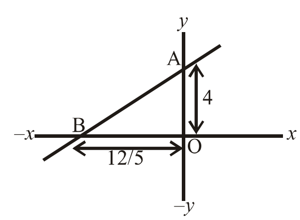
Sol^n 21. $8x - 3y = 24 \Rightarrow \dfrac{x}{3} + \dfrac{y}{-8} = 1$



Area of $\Delta OAB = \dfrac{1}{2} \times OA \times OB = \dfrac{1}{2} \times 8 \times 3$

$= 12$ sq. unit

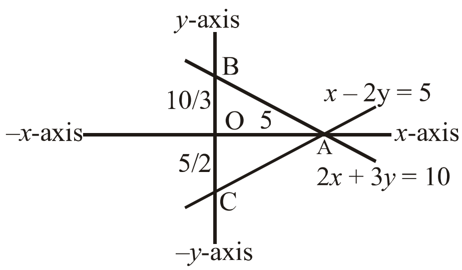
Sol^n 22. $-5x + 3y = 12 \Rightarrow \dfrac{x}{-12/5} + \dfrac{y}{4} = 1$



Area of $\Delta ABC = \dfrac{1}{2} \times 4 \times \dfrac{12}{5} = \dfrac{24}{5}$ sq. unit

Sol^n 23. $x - 2y = 5 \Rightarrow \dfrac{x}{5} + \dfrac{y}{-5/2} = 1$

$2x + 3y = 10 \Rightarrow \dfrac{x}{5} + \dfrac{y}{10/3} = 1$



OA = 5, OB = 10/3 & OC = 5/2

Area of $\Delta ABC = (\text{area of } \Delta OAB) + (\text{area of } \Delta OAC)$

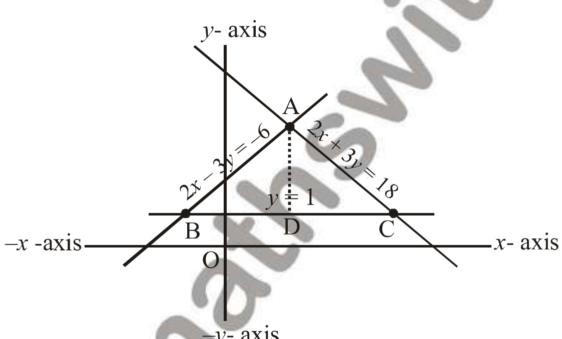
$$= \left(\frac{1}{2}\times 5\times\frac{10}{3}\right)+\left(\frac{1}{2}\times 5\times\frac{5}{2}\right)$$

$$= \frac{25}{3}+\frac{25}{4}$$

$$= \frac{175}{12}\text{sq. unit}$$

Sol^n^ 24. $2x-3y=-6 \quad \Rightarrow \frac{x}{-3}+\frac{y}{2}=1$

$2x+3y=18 \quad \Rightarrow \frac{x}{9}+\frac{y}{6}=1$ and $y = 1$



On solving $2x-3y=-6$ & $y=1$, Point B $\equiv$ (–3/2, 1)

On solving $2x+3y=18$ & $y=1$, Point C $\equiv$ (15/2, 1)

On solving $2x-3y=-6$ & $2x+3y=18$, Point A $\equiv$ (3,4)

Base of this triangle ABC $\Rightarrow$ $BC = \frac{15}{2}+\frac{3}{2}=9$

Height of this triangle ABC $\Rightarrow$ AD = 4 – 1 = 3

Area of this triangle ABC =

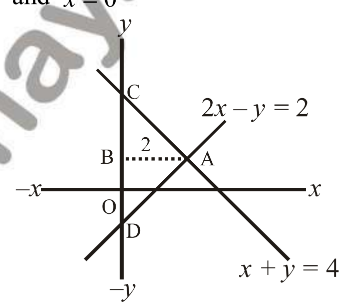
$$\frac{1}{2}\times BC\times AD=\frac{1}{2}\times 9\times 3$$

$$= \frac{27}{2} = 13.5 \text{ sq. unit}$$

Sol^n^ 25. $x+y=4 \quad \Rightarrow \frac{x}{4}+\frac{y}{4}=1$ …(i)

$2x-y=2 \quad \Rightarrow \frac{x}{1}+\frac{y}{-2}=1$ …(ii)

and $x = 0$ …(iii)



On solving $2x-y=2$ & $x+y=4$ $\Rightarrow$ Point A $\equiv$ (2, 2)

Base of this triangle ACD $\Rightarrow$ CD = 4 + 2 = 6

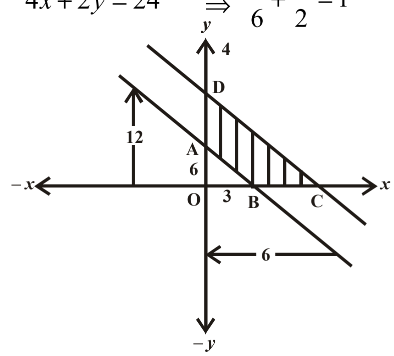
Height of this triangle ACD $\Rightarrow$ AB = 2

Area of triangle ACD =

$$\frac{1}{2}\times CD\times AB=\frac{1}{2}\times 6\times 2 = 6 \text{ sq. unit}$$

Sol^n^ 26. $2x+y=6 \quad \Rightarrow \frac{x}{3}+\frac{y}{6}=1$

$4x+2y=24 \quad \Rightarrow \frac{x}{6}+\frac{y}{2}=1$



Area of quadrilateral = Area of triangle OCD

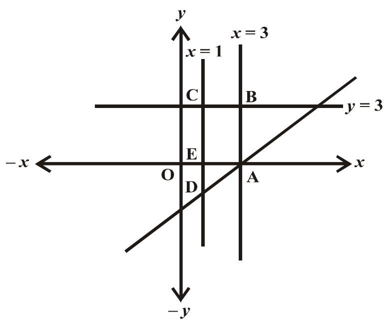
Area of triangle OAB = $\frac{1}{2}\times 6\times 12-\frac{1}{2}\times 3\times 6$

$= 27$ sq. unit

Sol^n 27. Same as above

Sol^n 28. $x=1, x=3, y=2$

and $x=y+3 \quad \Rightarrow \quad \frac{x}{3}+\frac{y}{-3}=1$



Point D will be $(1, -2)$

ABCD is a trapezium.

AB = 2, CD = CE + DE + 2 + 2 = 4

height = AE = 2 – 1 = 1

Area of trapezium ABCD $=\frac{1}{2}\text{AE}(\text{AB}+\text{CD})$

$=\frac{1}{2}\times 1(2+4) \quad = 3$ sq. unit.

Sol^n 29. $kx + 3y = k - 3$

$12x + ky = k$

For infinite many solution

$\Rightarrow \quad \frac{k}{12}=\frac{3}{k}=\frac{k-3}{k}$

$\Rightarrow \quad k^2=12\times 3=36$

$\Rightarrow \quad k=\pm 6$

Sol^n 30. $x+3y=k$

$2x+6y=2k$ are coincident lines.

$\Rightarrow \quad \frac{1}{2}=\frac{3}{6}=\frac{k}{2k}$

here all three ratio are equal for all real values of k.

Sol^n 31. $5x + 20y = 11$ and $2x + ky = 17$ will be intersecting lines or having unique solution

$\Rightarrow \quad \frac{5}{2}\neq\frac{20}{k}\Rightarrow k\neq 8$

Sol^n 32. $5x+3y=3$ and $12x+ky=6$ have no solution.

$\Rightarrow \quad \frac{5}{12}=\frac{3}{k}\Rightarrow 5k=36$

$\Rightarrow \quad k=7.2$

Sol^n 33. Line is passing through point (2,7) and having slope 5 then equation

$y-7=5(x-2)$

$\Rightarrow \quad 5x-y-3=0$

Sol^n 34. Same as above

Sol^n 35. Line is passing through points $(-1, 7)$ & $(2, -5)$ then equation

$$y-7=\frac{-5-7}{2-(-1)}[x-(-1)]$$

$\Rightarrow \quad y-7=-4(x+1)$

$\Rightarrow \quad 4x+y=3$

Sol^n 36. Distance of a point P(1, 4) to line $3x+4y-9=0$

$$d=\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|=\left|\frac{3\times 1+4\times 4-9}{\sqrt{3^2+4^2}}\right|$$

$$=\left|\frac{10}{5}\right|=2$$

Sol^n 37. Mid point of line whose ending point are (4, 9) and (6, 5) will be (5,7)

hence, equation of line passing through (2, 3) and (5, 7) will be

$$(y-3)=\frac{7-3}{5-2}(x-2)$$

$\Rightarrow \quad (y-3)=\frac{4}{3}(x-2)$

$\Rightarrow \quad 3y-9=4x-8$

$\Rightarrow \quad 4x-3y+1=0$

Sol^n 38. $4x+ky=3$ & $3x+2y=7$ are perpendicular to each other.

Condition for perpendicular : $a_1a_2+b_1b_2=0$

$\Rightarrow \quad 4\times3+k\times2=0$

$\Rightarrow \quad k=-6$

Sol^n 39. $(k+1)x+ky=3$ & $5x-2y=7$ are perpendicular to each other

Condition for perpendicular : $a_1a_2+b_1b_2=0$

$\Rightarrow \quad (k+1)\times5+k\times(-2)=0$

$\Rightarrow \quad 5k+5-2k=0$

$\Rightarrow \quad k=\dfrac{-5}{3}$

Sol^n 40. If the vertices of a triangle are rational number then triangle can never be equilateral triangle.

Sol^n 41. A(7, 9), B(3, – 7), and C(– 3, 3)

$$AB=\sqrt{(9+7)^2+(7-3)^2}=\sqrt{272}$$

$$BC=\sqrt{(3+3)^2+(-7-3)^2}=\sqrt{136}$$

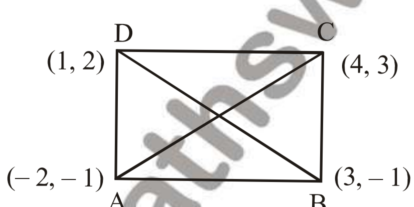
$$CA=\sqrt{(-3-7)^2+(3-9)^2}=\sqrt{136}$$

$\because$ BC = CA $\quad \therefore$ Isoceles triangle

$\because$ $BC^2+CA^2=AB^2$ $\quad \therefore$ right angles triangle

Sol^n 42. **Same as above**

Sol^n 43. A(– 2, – 1), B(3, –1), C(4, 3) & D(1, 2)



Step 1: Find length of diagonal

$$AC=\sqrt{(4+2)^2+(3+1)^2}=\sqrt{52}$$

$$BD=\sqrt{(3-1)^2+(-1-2)^2}=\sqrt{13}$$

$$AC\neq BD$$

then, ABCD will be rhombus or parallelogram…(i)

Step 2 : Find the product of slope of both diagonal.

$$AC\to m_1=\frac{3+1}{4+2}=\frac{2}{3}$$

$$BD\to m_2=\frac{2+1}{1-3}=\frac{-3}{2}$$

here, $\quad m_1\,m_2=-1$

then ABCD will be rhombus or square …(ii)

from eq. (i) & (ii)

ABCD is a rhombus.

Sol^n 44. **Same As above**

Sol^n 45. **See Example 57**

Sol^n 46. $a=2$, $b=3$

$$\frac{x}{a}+\frac{y}{b}=1\Rightarrow\frac{x}{2}+\frac{y}{3}=1$$

$\Rightarrow \; 3x+2y=6$

Sol^n 47. $m=\tan60°=\sqrt{3}$

Points (3, – 4)

$(y+4)=\sqrt{3}(x-3)$

$x\sqrt{3}-y=4+3\sqrt{3}$

Sol^n 48. There is no need to solved we know that the area of triangle formed by joining the mid points of the sides of the triangle is one–forth of original triangle. So required ratio is 4 : 1.

Sol^n 49. **Same as example 49**

Sol^n 50. Centroid of a triangle

$$\equiv\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$$

$$\equiv\left(\frac{0+8+8}{3},\frac{6+12+0}{3}\right)$$

$$\equiv\left(\frac{16}{3},6\right)$$

Sol^n 51. **Same as example 48**

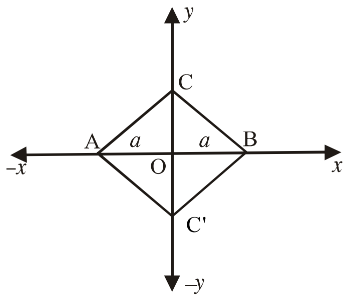
Sol^n 52. Vertices of triangle are (3, – 5) (– 7, 4) & (h, k) and centroid is (2, – 12)

$$(2,-12)\equiv\left(\frac{3-7+h}{3},\frac{-5+4+k}{3}\right)$$

On solving.

$(h,k)\equiv(10,-35)$

Solⁿ 53.



height of both triangle

$= OC = OC' = \frac{\sqrt{3}}{2}(2a)$

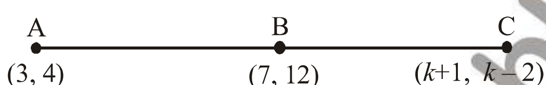
$= \sqrt{3}a$

$C \equiv (0, \sqrt{3}a)$

$C' \equiv (0, -\sqrt{3}a)$

Solⁿ 54. Same as example 56

Solⁿ 55.



We will equate slope of line AB and BC

$\frac{12-4}{7-3} = \frac{k-2-4}{k+1-3} \quad \Rightarrow 2 = \frac{k-6}{k-2}$

$\Rightarrow \quad 2k - 4 = k - 6$

$\Rightarrow \quad k = -2$

Solⁿ 56. $2x + y = 5$ & $x + 2y = 4$

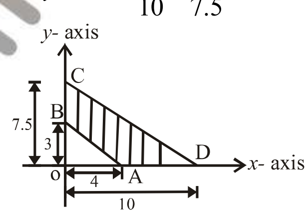
on solving both equation

intersection point $(x, y) \equiv (2, 1)$

Solⁿ 57. See example 33

Solⁿ 58. $3x + 4y = 12 \Rightarrow \frac{x}{4} + \frac{y}{3} = 1$

$6x + 8y = 60 \Rightarrow \frac{x}{10} + \frac{y}{7.5} = 1$



Area of ABCD =

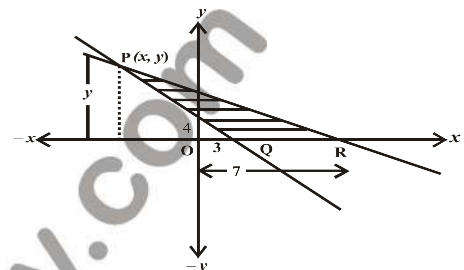
Area of ΔOCD – Area of ΔOAB

$= \frac{1}{2} \times 10 \times 7.5 - \frac{1}{2} \times 4 \times 3$

$= 37.5 - 6 = 31.5$ sq.unit

Solⁿ 59. Hint: Use distance formula.

Solⁿ 60.



$5x + 7y = 35 \quad \Rightarrow \quad \frac{x}{7} + \frac{y}{5} = 1$

$4x + 3y = 12 \quad \Rightarrow \quad \frac{x}{3} + \frac{y}{4} = 1$

both lines intersect at point P (x, y)

we need only y co-ordinate of P (on solving)

$y = \frac{80}{13}$

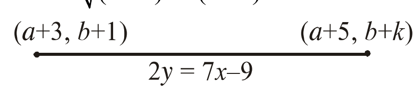
base OR = 7 – 3 = 4

Area of $\Delta PQR = \frac{1}{2} \times 4 \times \frac{80}{13} = \frac{160}{13}$ sq. unit

Solⁿ 61. Then point A will be (3, 4)

$OA = \sqrt{(4-0)^2 + (3-0)^2} = 5$ units

Solⁿ 62.



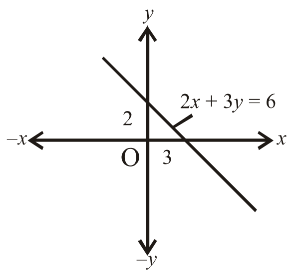
$2y = 7x - 9 \Rightarrow y = \frac{7}{2}x - \frac{9}{2}$

equate the slope of line –

$\frac{b+k-b-1}{a+5-a-3} = \frac{7}{2}$

$\Rightarrow \frac{k-1}{2} = \frac{7}{2} \quad \Rightarrow \quad k = 8$

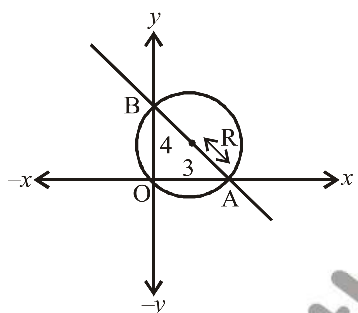
Solⁿ 63. $2x + 3y = 6 \qquad \Rightarrow \frac{x}{3} + \frac{y}{2} = 1$



line will intersect both of x-axis and y-axis

Sol^n 64. $4x+3y=12 \quad \Rightarrow \dfrac{x}{3}+\dfrac{y}{4}=1$

$AB = \sqrt{3^2+4^2} = 5$



Radius of circumcircle

$R = \dfrac{AB}{2} = \dfrac{5}{2} = 2.5$ units

in right angle triangle mid point of hypotenuse is circumcentre

Sol^n 65. $x + 4y = 7$ & $kx + 8y = 14$ are parallel lines

$\Rightarrow \dfrac{1}{k}=\dfrac{4}{8}$

$\Rightarrow \dfrac{1}{k}=\dfrac{1}{2} \quad \Rightarrow k=2$

Sol^n 66. $2x+3y=5 \quad \Rightarrow y=\dfrac{-2}{3}x+\dfrac{5}{3}$

then $m_1=\dfrac{-2}{3}$

$3x-2y=1 \quad \Rightarrow y=\dfrac{3}{2}x-\dfrac{1}{2}$

then $m_2=\dfrac{3}{2}$

If angle between lines is θ then,

$$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

$$\tan\theta=\left|\frac{\frac{-2}{3}-\frac{3}{2}}{1-\frac{2}{3}.\frac{3}{2}}\right|=\left|\frac{\frac{-13}{6}}{0}\right|=\infty=\tan 90^o$$

$\theta=90^o$

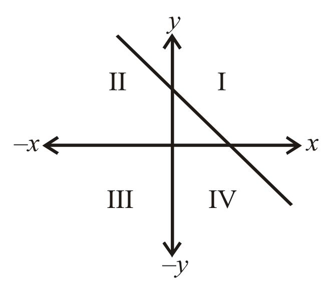
Method 2:

$m_1m_2=\dfrac{-2}{3}.\dfrac{3}{2}=-1$

then lines are perpendicular to each other hence angle is 90^o

Sol^n 67. The lines which pass through origin having no constant term in their equation then, $2x+3y=0$ will pass through origin

Sol^n 68.



$2x+5y+k=0$

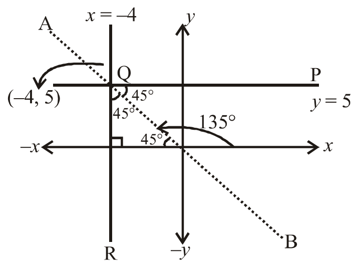
$$\frac{x}{\frac{-k}{2}}+\frac{y}{\frac{-k}{5}}=1$$

If line pass through I, II and IV quadrant intercept on x-axis and y-axis should be positive

$\Rightarrow \dfrac{-k}{2}$ & $\dfrac{-k}{5}$ will be positive when k is negative

so, $k<0$

Sol^n 69.



AB will be the required line which is angle bisector of $\angle PQR$.

slope of this line AB = tan 135° = – 1

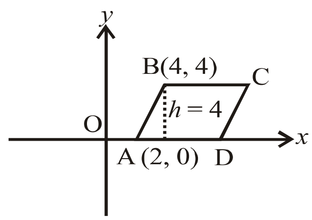
and it will pass through (– 4, 5)

equation will be

$(y-5) = -1\,(x+4)$

$x + y = 1$

Sol^n 70.



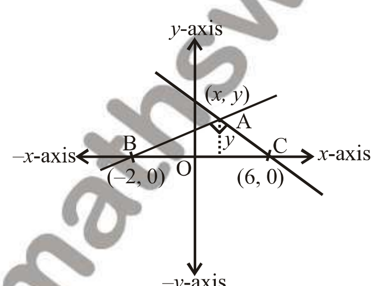
$AD = AB = \sqrt{(4-2)^2 + (4-0)^2}$

$= \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

Area of Rhombus = base × height

$= 2\sqrt{5} \times 4 = 8\sqrt{5}$ sq.unit

Sol^n 71.



$2x + 3y = 12 \Rightarrow \dfrac{x}{6} + \dfrac{y}{4} = 1$

$3x - 2y = -6 \Rightarrow \dfrac{x}{-2} + \dfrac{y}{3} = 1$

both lines are perpendicular

$(\because m_1 m_2 = -1)$

and in a right angle triangle circum centre is on hypotenuse

$\therefore$ Circumcentre (2, 0) will be on x-axis

It means x-axis is hypotenuse

on solving both lines $2x + 3y = 12$ and $3x - 2y = -6$

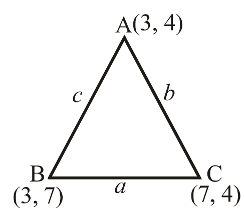
y - co-ordinate =

$$y = \frac{48}{13}$$

Area of $\Delta ABC = \dfrac{1}{2} \times BC \times y$

$= \dfrac{1}{2} \times 8 \times \dfrac{48}{13} \quad = \dfrac{192}{13}$ sq.unit

Sol^n 72.



$a = \sqrt{(7-3)^2 + (4-7)^2} = 5$

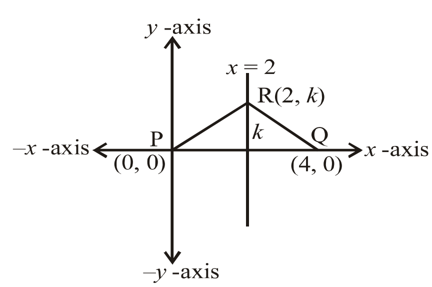
$b = \sqrt{(7-3)^2 + (4-4)^2} = 4$

$c = \sqrt{(3-3)^2 + (7-4)^2} = 3$

Incentre $I \equiv \left(\dfrac{ax_1 + bx_2 + cx_3}{a+b+c}, \dfrac{ay_1 + by_2 + cy_3}{a+b+c} \right)$

$\equiv \left(\dfrac{5\times3 + 4\times3 + 3\times7}{5+4+3}, \dfrac{5\times4 + 4\times7 + 3\times4}{5+4+3} \right) \equiv (4, 5)$

Sol^n 73.

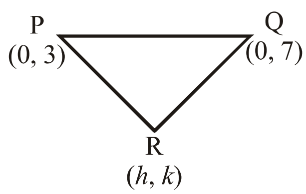


Area of triangle PQR = $\frac{1}{2} \times PQ \times k$

$6 = \frac{1}{2} \times 4 \times k \Rightarrow k = 3$

hence point R will be (2, 3)

Sol^n 74.



$PR + QR = 8$

$$\sqrt{(h-0)^2 + (k-3)^2} + \sqrt{(h-0)^2 + (k-7)^2} = 8$$

Abscissa of R, h will be maximum when $k = 7$ or 3.

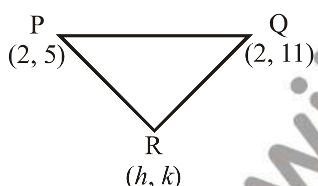
Put $k = 7$

$\sqrt{h^2 + 16} = 8 - \sqrt{h^2 + 0} = 8 - h$

$h^2 + 16 = 64 + h^2 - 16h$

$16h = 48 \Rightarrow h = 3$

Sol^n 75.



Incentre, circumcentre and orthocentre lie on a straight line. So this triangle will be Isoceles triangle.

hence, PR = PR

$$\sqrt{(h-2)^2 + (k-5)^2} = \sqrt{(h-2)^2 + (k-11)^2}$$

After squaring both side

$(k-5)^2 = (k-11)^2$

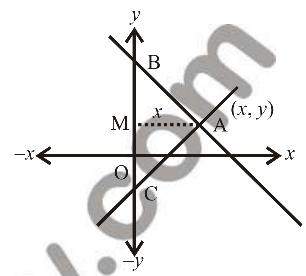
$k^2 + 25 - 10k = k^2 + 121 - 22k$

$12k = 96 \Rightarrow k = 8$

See option and (2, 8) will be the point R.

Sol^n 76. $2x + y = 6 \Rightarrow \frac{x}{3} + \frac{y}{6} = 1$

$x = y + 1 \Rightarrow \frac{x}{1} + \frac{y}{-1} = 1$



If we consider BC is the base of triangle ABC and AM is the height which is x-cordinate of point A.

On solving $2x + y = 6$ & $x = y + 1$

x- co-ordinate = AM = $\frac{7}{3}$

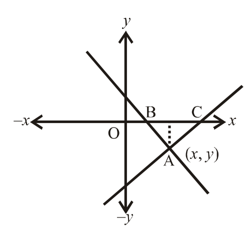
Base BC = 6 + 1 = 7

Hence Area of triangle ABC = $\frac{1}{2} BC \times AM$

$= \frac{1}{2} \times 7 \times \frac{7}{3} = \frac{49}{6}$

Sol^n 77. $2x + 3y = 5 \Rightarrow \frac{x}{5/2} + \frac{y}{5/3} = 1$

$y = 3x - 13 \Rightarrow \frac{x}{13/3} + \frac{y}{-13} = 1$



We have to find area of triangle ABC base

$BC = \frac{13}{3} - \frac{5}{2} = \frac{11}{6}$

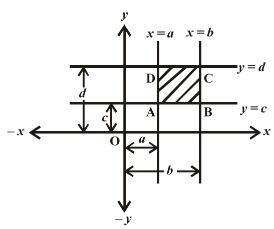
Height of the triangle = y co-ordinate of point A.

On solving $2x+3y=5$ & $y=3x-13 \Rightarrow y=-1$

Hence Area of triangle = $\frac{1}{2}\times$ Base $\times$ height

$$= \frac{1}{2}\times\frac{11}{6}\times 1 = \frac{11}{12} \text{ sq. unit}$$

Sol[n] 78.



AB = $b-a$ = CD
BC = $d-c$ = AD
ABCD will be a rectangle.
Area of ABCD = $(b-a)(d-c)$

Sol[n] 79. $L_1 = \left|\frac{0\times\sec\theta+0\times\text{cosec}\theta-a}{\sqrt{\sec^2\theta+\text{cosec}^2\theta}}\right|$

$$= \frac{a}{\sqrt{\sec^2\theta+\text{cosec}^2\theta}} = \frac{a}{\sqrt{\frac{1}{\cos^2\theta}+\frac{1}{\sin^2\theta}}}$$

$$= \frac{a}{\sqrt{\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta.\cos^2\theta}}} = a\sin\theta.\cos\theta$$

$$L_2 = \left|\frac{0\times\cos\theta+0\times\sin\theta-a\cos 2\theta}{\sqrt{\cos^2\theta+\sin^2\theta}}\right| = a\cos 2\theta$$

$$4L_1^2+L_2^2 = 4a^2.\sin^2\theta.\cos^2\theta+a^2\cos^2 2\theta$$

$$= a^2(2\sin\theta.\cos\theta)^2+a^2\cos^2 2\theta$$

$$= a^2\left(\sin^2 2\theta+\cos^2 2\theta\right) = a^2$$

Method : 2 Put $\theta = 45°$ (because answer is independent of θ)

Line will be $\rightarrow x\sqrt{2}+4\sqrt{2}-9=0$

$$\frac{x}{\sqrt{2}}-\frac{y}{\sqrt{2}}=0 \Rightarrow x-y=0$$

Now, $L_1 = \left|\frac{0+0-a}{\sqrt{\left(\sqrt{2}\right)^2+\left(\sqrt{2}\right)^2}}\right| = \frac{a}{2}$

$2L_1 = a$ …(i)

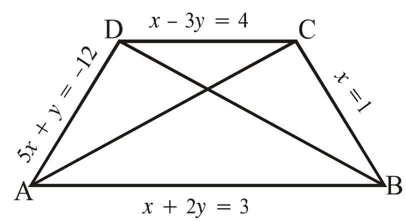
$$L_2 = \left|\frac{0+0-0}{\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2+\left(\frac{-1}{\sqrt{2}}\right)^2}}\right| = 0 \quad \text{…(ii)}$$

Adding equation (i) and (ii) after squaring

$4L_1^2+L_2^2 = a^2$

Sol[n] 80. On solving,

$x+2y=3$ & $5x+y+12=0$, point A $\equiv (-3, 3)$
$x+2y=3$ & $x=1$, point B $\equiv (1, 1)$
$x-3y=4$ & $x=1$, point C $\equiv (1, -1)$
$x-3y=4$ & $5x+y+12=0$, point D $\equiv (-2, 2)$



Slope of AC = $\frac{-1-3}{1+3} = -1 = m_1$

Slope of BD = $\frac{-2-1}{-2-1} = 1 = m_2$

$m_1.m_2 = -1$

Hence angle between both diagonal will be 90º.

Sol[n] 81. Let a line $\frac{x}{a}+\frac{y}{b}=1$

Given, $a+b=18$ …(i)

Passing through (4, 4)

It means $\frac{4}{a}+\frac{4}{b}=1 \Rightarrow ab=4(a+b)=72$

$$(a-b)^2=(a+b)^2-4ab=(18)^2-4\times 72$$
$$=324-288=36$$

$a-b=\pm 6$ …(ii)

On solving eqn. (i) and (ii)

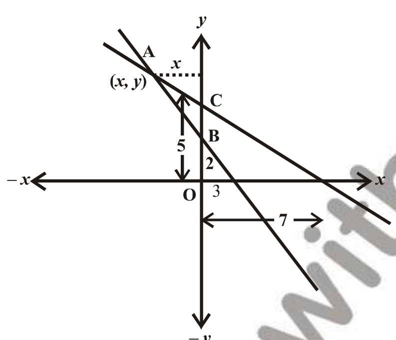
$(a, b) \equiv (12, 6)$ or $(6, 12)$

Line will be $\frac{x}{12}+\frac{y}{6}=1 \Rightarrow x+2y=12$

Or, $\frac{x}{6}+\frac{y}{12}=1 \Rightarrow 2x+y=12$

Sol^n 82. $5x+7y=35 \quad \Rightarrow \quad \frac{x}{7}+\frac{y}{5}=1$

$4x+3y=12 \quad \Rightarrow \quad \frac{x}{3}+\frac{y}{4}=1$



We have to find area of triangle ABC

Base BC = 5 – 3 = 2.

Height of the triangle will be x– co-ordinate intersection point of both lines.

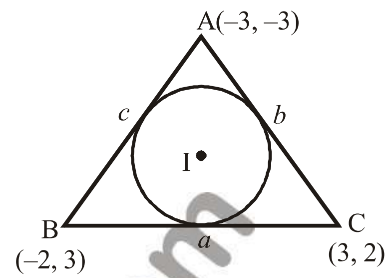
On solving $5x + 7y = 35$ & $4x + 3y = 12$

$$x=\frac{-21}{13}$$

Area of triangle $=\frac{1}{2}\times \text{BC}\times x$

$$=\frac{1}{2}\times 2\times\frac{21}{13}=\frac{21}{13} \text{ sq. unit.}$$

Sol^n 83.



$$\text{BC}=a=\sqrt{(-2-3)^2+(3-2)^2}=\sqrt{26}$$

$$\text{AC}=b=\sqrt{(-3-3)^2+(-2-2)^2}=\sqrt{52}$$

$$\text{AB}=c=\sqrt{(-3+2)^2+(-2-3)^2}=\sqrt{26}$$

$\therefore$ Co-ordinate of incentre

$$\equiv\left(\frac{ax_1+bx_2+cx_3}{a+b+c},\frac{ay_1+by_2+cy_3}{a+b+c}\right)$$

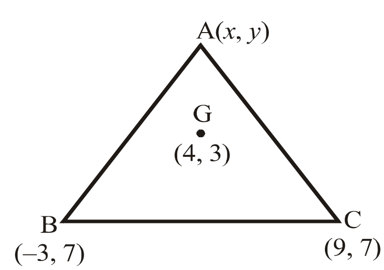
$$\equiv\left(\frac{\sqrt{26}\times(-3)+\sqrt{52}(-2)+\sqrt{26}\times(3)}{\sqrt{26}+\sqrt{52}+\sqrt{26}},\right.$$

$$\left.\frac{\sqrt{26}\times(-2)+\sqrt{52}(3)+\sqrt{26}(2)}{\sqrt{26}+\sqrt{52}+\sqrt{26}}\right)$$

$$\equiv\left(\frac{-2\sqrt{52}}{\sqrt{26}\left(2+\sqrt{2}\right)},\frac{3\sqrt{52}}{\sqrt{26}\left(2+\sqrt{2}\right)}\right)$$

$$\equiv\left(\frac{-2\sqrt{2}}{\left(2+\sqrt{2}\right)},\frac{3\sqrt{2}}{\left(2+\sqrt{2}\right)}\right)$$

Sol^n 84.



$\because \quad G = \left(\dfrac{x_1+x_2+x_3}{3}, \dfrac{y_1+y_2+y_3}{3}\right)$

$\Rightarrow \quad G = \left(\dfrac{x-3+9}{3}, \dfrac{y+7+7}{3}\right)$

$\Rightarrow \; (4, 3) \equiv \left(\dfrac{x-6}{3}, \dfrac{y+14}{3}\right)$

$\therefore \quad \dfrac{x+6}{3} = 4, \quad \Rightarrow \; x = 4\times3-6 = 6$

$\therefore \quad \dfrac{y+14}{3} = 3, \quad \Rightarrow \; y = 3\times3-14 = -5$

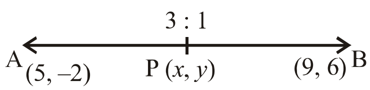
$\therefore \; (x, y) = (6, -5)$

Area of triangle having vertices (x_1, y_1), (x_2, y_2) & (x_3, y_3)

$= \left|\dfrac{1}{2}\left(6(7-7)-3(7+5)+9(-5-7)\right)\right|$

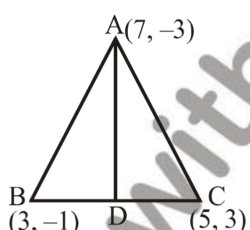
$= 72$sq.unit

Sol^n 85.



$(x, y) \equiv \left(\dfrac{9\times3+1\times5}{3+1}, \dfrac{6\times3+1\times-2}{3+1}\right) \equiv (8, 4)$

Sol^n 86.

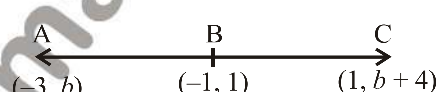


Mid point $\equiv \left(\dfrac{3+5}{2}, \dfrac{-1+3}{2}\right) \equiv (4, 1)$

Length of Median AD $= \sqrt{(7-4)^2+(-3-1)^2}$

$= \sqrt{9+16} = 5$ unit.

Sol^n 87.



y- co-ordinate of Mid point $= 1 = \dfrac{b+b+4}{2}$

$\Rightarrow \quad 1 = \dfrac{2b+4}{2} \quad \Rightarrow b = -1$

Sol^n 88. Same as example 56

Sol^n 89. $x^2 + y^2 = 16$

and $x + y = 16$

On solving both equation $= x^2 + (16-x)^2 = 16$

$\Rightarrow \quad x^2 - 16x + 120 = 0$

here x is imaginary number so number of points of intersection are zero.

Method - 2

Equation of circle

$x^2 + y^2 = 16 = (r)^2$

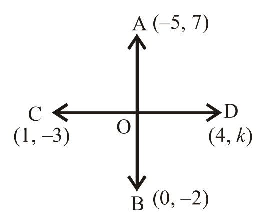
radius $r = 4$ and centre (0, 0)

Distance of line $x + y - 16 = 0$ from centre (0, 0)

$d = \left|\dfrac{0-16}{\sqrt{1^2+1^2}}\right| = \dfrac{16}{\sqrt{2}} = 8\sqrt{2}$

$d > r$ so line will pass outside the circle and line does not intersect circle, so number of points of intersection are zero.

Sol^n 90.



Product of slope of both line AB & CD will be $-1 \quad m_1 . m_2 = -1$

$\Rightarrow \quad \left(\dfrac{7+2}{-5-0}\right).\left(\dfrac{k+3}{4-1}\right) = -1$

$\Rightarrow \quad \left(\dfrac{-9}{5}\right)\left(\dfrac{k+3}{3}\right) = -1$

$\Rightarrow \quad -9k - 27 = -15 \Rightarrow \; -12 = 9k$

$\Rightarrow \quad k = \dfrac{-4}{3}$

Sol^n 91. Same as example 27

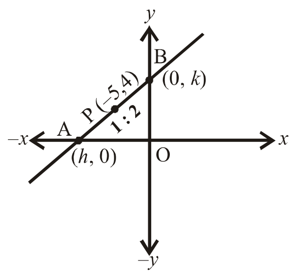
Sol^n 92. Using formula $\tan\theta = \left|\dfrac{m_1-m_2}{1+m_1m_2}\right|$

Where m_1 and m_2 are slope of lines.

Solⁿ 93. Use centroid formula

$$(x, y) \equiv \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

Solⁿ 94.



P(−5, 4) divides AB internally in 1 : 2

$$-5 = \frac{2 \times h + 1 \times 0}{1+2} \Rightarrow h = \frac{-15}{2}$$

Equation of line pass through

$$\left(\frac{-15}{2}, 0\right) \& (-5, 4)$$

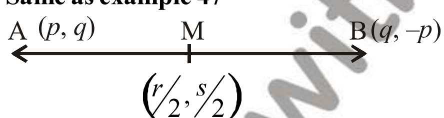
$$(y-4) = \frac{4-0}{-5+\frac{15}{2}}(x+5)$$

$$5(y-4) = 8(x+5)$$

$$8x - 5y + 60 = 0$$

Solⁿ 95. Same as example 47

Solⁿ 96.



$$\frac{r}{2} = \frac{p+q}{2} \Rightarrow p + q = r \quad \ldots\text{(i)}$$

$$\frac{s}{2} = \frac{q-p}{2} \Rightarrow q - p = s \quad \ldots\text{(ii)}$$

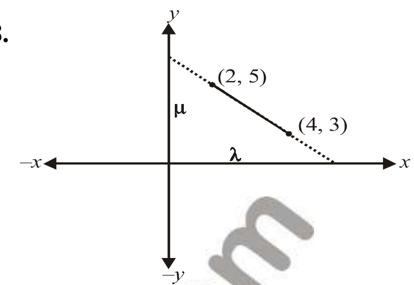
$$AB = \sqrt{(p-q)^2 + (q+p)^2}$$

$$= \sqrt{(s)^2 + (r)^2} = \left(r^2 + s^2\right)^{1/2}$$

Solⁿ 97. On solving point of intersection of line $x + 3y - 10 = 0$ and $2x + y - 5 = 0$ will be (1, 3) Hence distance between point (1, 3) and origin (0, 0)

$$d = \sqrt{(1-0)^2 + (3-0)^2} = \sqrt{10}$$

Solⁿ 98.



Equation of line passing through (2, 5) & (4, 3)

$$y - 3 = \frac{5-3}{2-4}(x-4) = -(x-4)$$

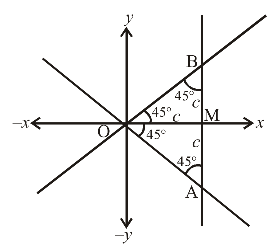
$$\Rightarrow y - 3 = 4 - x$$

$$\Rightarrow x + y = 7$$

$$\Rightarrow \frac{x}{7} + \frac{y}{7} = 1$$

$$\left(\text{Compare with } \frac{x}{\lambda} + \frac{y}{\mu} = 1\right) \lambda = \mu = 7$$

Solⁿ 99.



Line 1 : $y - x = 0 \Rightarrow x = y$
Line 2 : $y + x = 0 \Rightarrow x = -y$
Line 3 : $x = c$

Area of triangle OAB = $\frac{1}{2} \times AB \times OM$

$$= \frac{1}{2} \times 2c \times c = c^2$$

Solⁿ 100. Let equation of line pasing through the intersection point of

$$\frac{x}{a} + \frac{y}{b} = 1 \ \& \ \frac{x}{b} + \frac{y}{a} = 1 \text{ will be}$$

$$\left(\frac{x}{a} + \frac{y}{b} - 1\right) + \lambda\left(\frac{x}{b} + \frac{y}{a} - 1\right) = 0 \quad \ldots(1)$$

$\because$ Line is pasing through origin (0, 0)
so, put $x = y = 0$

$(-1)+\lambda(-1)=0$

$\lambda=-1$

put $\lambda=-1$ in equation (1)

$$\Rightarrow \quad \frac{x}{a}+\frac{y}{b}-1-\frac{x}{b}-\frac{y}{a}+1=0$$

$$\Rightarrow \quad x\left(\frac{1}{a}-\frac{1}{b}\right)+y\left(\frac{1}{b}-\frac{1}{a}\right)=0$$

$$\Rightarrow \quad \left(\frac{1}{a}-\frac{1}{b}\right)(x-y)=0 \Rightarrow x-y=0$$

Sol^n 101. $x\cos\theta+y\sin\theta=2$

$\Rightarrow y\sin\theta=-x\cos\theta+2$

$\Rightarrow y=-x\cot\theta+2\operatorname{cosec}\theta$

$m_1=-\cot\theta$

$x-y=3 \Rightarrow y=x-3$

$m_2=1$

Both lines are perpendiculars to each other so
$m_1 . m_2=-1$
$(-\cot\theta).(1)=-1 \Rightarrow \cot\theta=1$

$\Rightarrow \theta=\pi/4$

Sol^n 102. $(x_1,y_1)\equiv\left(\sqrt{b^2-a^2},0\right)$

$(x_2,y_2)\equiv\left(-\sqrt{b^2-a^2},0\right)$

Product of length of perpendicular from (x_1,y_1) & (x_2,y_2) to
line $ax\cos\phi+by\sin\phi-ab=0$

$$\Rightarrow P_1.P_2=\left|\frac{ax_1\cos\phi+by_1\sin\phi-ab}{\sqrt{a^2\cos^2\phi+b^2\sin^2\phi}}\right|.\left|\frac{ax_2\cos\phi+by_2\sin\phi-ab}{\sqrt{a^2\cos^2\phi+b^2\sin^2\phi}}\right|$$

$$=\left|\frac{a\sqrt{b^2-a^2}\cos\phi-ab}{\sqrt{a^2\cos^2\phi+b^2\sin^2\phi}}\right|.\left|\frac{-a\sqrt{b^2-a^2}\cos\phi-ab}{\sqrt{a^2\cos^2\phi+b^2\sin^2\phi}}\right|$$

$$=\left|\frac{-\left(a\sqrt{b^2-a^2}\cos\phi-ab\right)\left(a\sqrt{b^2-a^2}\cos\phi+ab\right)}{a^2\cos^2\phi+b^2\sin^2\phi}\right|$$

$$=\left|\frac{-\left(a^2(b^2-a^2)\cos^2\phi-a^2b^2\right)}{a^2\cos^2\phi+b^2(1-\cos^2\phi)}\right|$$

$$=\left|\frac{a^2\left\{b^2-(b^2-a^2).\cos^2\phi\right\}}{\left\{b^2-(b^2-a^2).\cos^2\phi\right\}}\right|$$

$= a^2$

Method - 2

we can put $\phi = 0$ because answer is independent of ϕ, then line will be $ax = ab$
$\Rightarrow x - b = 0$

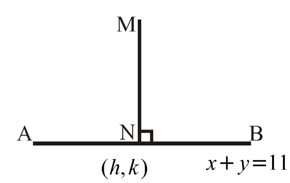
$$P_1.P_2=\left|\frac{x_1-b}{\sqrt{1^2}}\right|\left|\frac{x_2-b}{\sqrt{1^2}}\right|$$

$$=\left|\sqrt{b^2-a^2}-b\right|\left|-\sqrt{b^2-a^2}-b\right|$$

$$=\left|-\left(\sqrt{b^2-a^2}-b\right)\left(\sqrt{b^2-a^2}+b\right)\right|$$

$$=\left|-(b^2-a^2-b^2)\right| \qquad = a^2$$

Sol^n 103.



$x+y=11 \Rightarrow y=-x+11 \quad \Rightarrow m_1=-1$

$MN \perp AB$

Product of slope of both lines = – 1

$m_1.m_2=-1$

$\left(\frac{k-3}{h-2}\right).(-1)=-1$

$k-3=h-2$

$h-k=-1$ …(1)

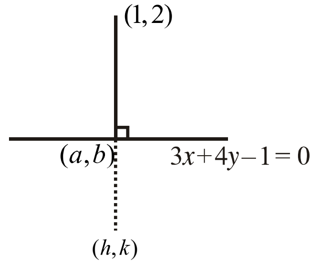
(h, k) is on $x+y=11$

$h+k=11$ …(2)

On solving equation (1) & equation (2)

$(h,k)\equiv(5,6)$

Sol^n^ 104.



We will find foot of the perpendicular (a, b) by previous method. Foot of the perpendicular will be

$$(a,b)\equiv\left(\frac{-1}{5},\frac{2}{5}\right)$$

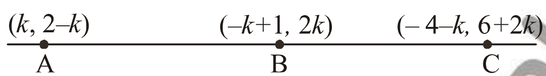
(a, b) will be the mid point of (h, k) & $(1, 2)$

$$\frac{-1}{5}=\frac{h+1}{2}\Rightarrow h=\frac{-7}{5}$$

$$\frac{2}{5}=\frac{k+2}{2}\Rightarrow k=\frac{-6}{5}$$

$$(h,k)\equiv\left(\frac{-7}{5},\frac{-6}{5}\right)$$

Sol^n^ 105.



slope of line AB = slope of line BC

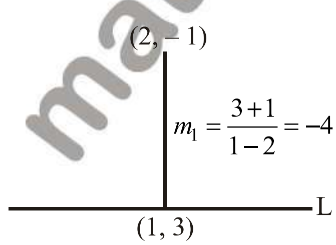
$$\Rightarrow\quad \frac{2k-2+k}{-k+1-k}=\frac{6+2k-2k}{-4-k+k-1}$$

$$\Rightarrow\quad \frac{3k-2}{-2k+1}=\frac{6}{-5}$$

$$\Rightarrow\quad -15k+10=-12k+6$$

$$\Rightarrow\quad 3k-4=0\Rightarrow k=4/3$$

Sol^n^ 106.



$$\text{Slope of line L}=\frac{-1}{m_1}=\frac{-1}{-4}=\frac{1}{4}$$

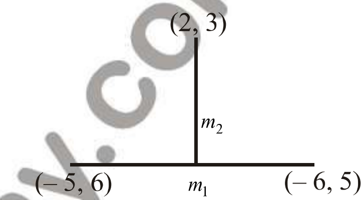
Equation of line L will be

$$(y-3)=\frac{1}{4}(x-1)$$

$$4(y-3)=x-1$$

$$x-4y+11=0$$

Sol^n^ 107.



$$m_1 . m_2=-1$$

$$\left(\frac{6-5}{-5+6}\right)m_2=-1$$

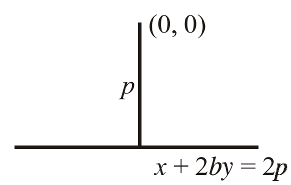
$$m_2=-1$$

Equation of line having slope m_2 and passing through (2, 3) is

$$y-3=-1(x-2)$$

$$x+y-5=0$$

Sol^n^ 108.



Distance from origin (0, 0) to line $x+2by-2p=0$

$$p=\left|\frac{0+0-2p}{\sqrt{(1)^2+(2b)^2}}\right|$$

$$p^2=\frac{4p^2}{1+4b^2}\Rightarrow 1+4b^2=4$$

$$4b^2=4-1=3$$

$$b^2=\frac{3}{4}\quad\Rightarrow b=\frac{\sqrt{3}}{2}$$

Sol^n 109. $y=\left(2-\sqrt{3}\right)x+5 \quad \Rightarrow m_1=2-\sqrt{3}$

$y=\left(2+\sqrt{3}\right)x-7 \quad \Rightarrow m_2=2+\sqrt{3}$

Let angle between lines is θ

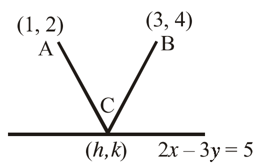
$$\tan\theta=\left|\frac{m_2-m_1}{1+m_1.m_2}\right|$$

$$\tan\theta=\left|\frac{2+\sqrt{3}-2+\sqrt{3}}{1+\left(2-\sqrt{3}\right).\left(2+\sqrt{3}\right)}\right|$$

$$\tan\theta=\sqrt{3}$$

$$\theta=60°$$

Sol^n 110.



AC = AB

$\Rightarrow \quad \sqrt{(k-2)^2+(h-1)^2}=\sqrt{(k-4)^2+(h-3)^2}$

on solving this

$h+k=5$ …(1)

(h, k) is on the line $2x-3y=5$

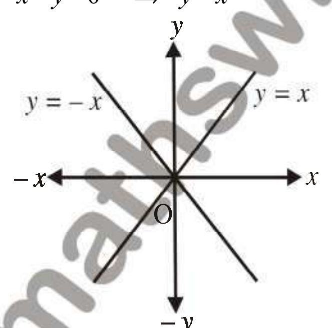
so $2h-3k=5$ …(2)

on solving equation (1) and (2)

$(h,k)\equiv(4,1)$

Sol^n 111. $x+y=0 \quad \Rightarrow y=-x$

$x-y=0 \quad \Rightarrow y=x$



Points which have equal distance from line $y=x$ and $y=-x$ will be on x-axis and y-axis.

so equation of x-axis and y-axis

$y=0$ & $x=0$

so locus will be $xy=0$

Sol^n 112. $2x+2y+4+k(6x-y+12)=0$

$\Rightarrow \quad x(2+6k)+y(3-k)+12k+4=0$

slope of this line $m_1=\dfrac{-(2+6k)}{3-k}$

$7x+5y+4=0 \quad \Rightarrow$ slope $m_2=\dfrac{-7}{5}$

both are perpendicular line.

$m_1.m_2=-1$

$\Rightarrow \quad \left(\dfrac{-(2+6k)}{3-k}\right)\left(\dfrac{-7}{5}\right)=-1$

$14+42k=5k-15$

$37k=-29 \quad \Rightarrow k=\dfrac{-29}{37}$

Sol^n 113. $A(\lambda, 2-2\lambda)\,B(-\lambda+1, 2\lambda)\,\&\,C(-\lambda, 6-2\lambda)$

Area of triangle ABC

$$\Delta=\frac{1}{2}\left(x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right)$$

$\Rightarrow \quad \{\lambda(2\lambda-6+2\lambda)+(-\lambda+1)(6-2\lambda-2+2\lambda)$
$+(-4-\lambda)(2-2\lambda-2\lambda)\}$

$\Rightarrow \quad 140=\left(4\lambda^2-6\lambda+4-4\lambda-8+16\lambda-2\lambda+4\lambda^2\right)$

$\Rightarrow \quad 140=8\lambda^2+4\lambda-4$

$\Rightarrow \quad 2\lambda^2+\lambda-36=0$

$\Rightarrow \quad 2\lambda^2+9\lambda-8\lambda-36=0$

$\Rightarrow \quad \lambda(2\lambda+9)-4(2\lambda+9)=0$

$\Rightarrow \quad (\lambda-4)(2\lambda+9)=0$

$\lambda=4, \dfrac{-9}{2}$

integral value of $\lambda=4$,

Sol^n 114. Let third vertex (h, k) lies on $y=x+3$

hence $k=h+3$

$(h,k)\equiv(h, h+3)$

then, all three vertices are $(2, 1)$ $(3, -2)$ & $(h, h+3)$

Area of triangle $\Delta=5$

$\Rightarrow \quad \frac{1}{2}\{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\}=5$

$\frac{1}{2}\{2(-2-h-3)+3(h+3-1)+h(1+2)\}=5$

$\Rightarrow \quad 2(-5-h)+3(h+2)+h(3)=10$

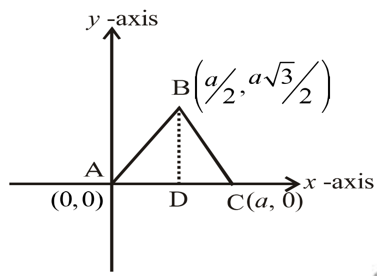
$\Rightarrow \quad -10-2h+3h+6+3h=10$

$\Rightarrow \quad 4h=14$

$\Rightarrow \quad h=\frac{7}{2}$

$\Rightarrow \quad (h,k)\equiv\left(\frac{7}{2},\frac{7}{2}+3\right)\equiv\left(\frac{7}{2},\frac{13}{2}\right)$

Sol^n^ 115.



$BD=\frac{a\sqrt{3}}{2}, \qquad AD=\frac{a}{2}$

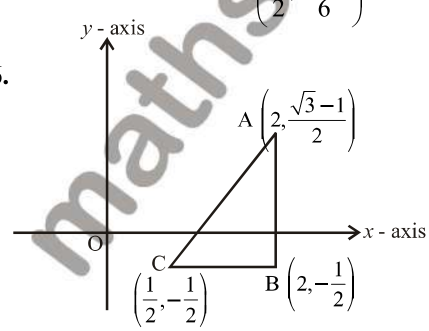
D is mid point and BD is height

So, ABC is equilateral triangle

then, Incentre$\equiv$centroid$\equiv\left(\dfrac{0+a+\frac{a}{2}}{3},\dfrac{0+0+\frac{a\sqrt{3}}{2}}{3}\right)$

$\equiv\left(\frac{a}{2},\frac{a\sqrt{3}}{6}\right)$

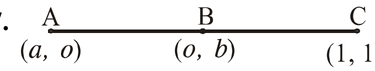
Sol^n^ 116.



ABC is right angle triangle

so point $B\equiv\left(2,-\frac{1}{2}\right)$ will be orthocentre

Sol^n^ 117.



slope of AB & BC will be equal because all three points are collinear (on a line)

Slope of line AB = Slope of line BC

$$\frac{b-0}{0-a}=\frac{1-b}{1-0}$$

$b=-a+ab$

$a+b-ab=0$

Sol^n^ 118. On solving equation of lines.

intersection point of $2x+y=4$ with $x-y+1=0$ will be (1, 2)

Intersection point of $2x-y-1=0$ with $x+y-8=0$ will be (3, 5)

equation of line passing through (1, 2) and (3, 5)

$$y-2=\frac{5-2}{3-1}(x-1)$$

$2y-4=3x-3$

$3x-2y+1=0$

Sol^n^ 119. use formula $p=\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$

Sol^n^ 120. Use formula $d=\left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right|$

Sol^n^ 121. Let line will be

$(2x-3y+1)+\lambda(x+y-2)=0$

$x(2+\lambda)+y(-3+\lambda)+1-2\lambda=0 \qquad \ldots(1)$

line is parallel to y - axis $\Rightarrow x=$ constant

In the equation there is no term of y.

so in equation (1)

coefficient of $y=0$

$-3+\lambda=0 \Rightarrow \lambda=3$

put $\lambda=3$ in equation (1)

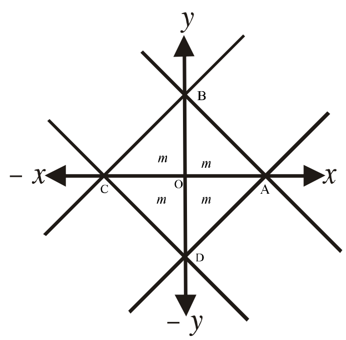
$x(3+2)+y(0)+1-2\times3=0$

$5x-5=0$

$x=1$

Sol^n 122. $|x| + |y| = m$

will represent four lines



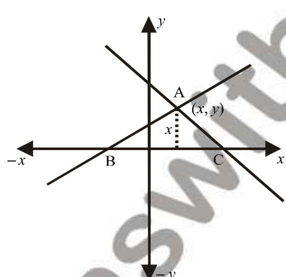
$$x + y = m$$
$$x - y = m$$
$$-x + y = m$$
$$-x - y = m$$

AB = BC = CD = DA = $m\sqrt{2}$

Area of square ABCD = $(AB)^2$

$$= \left(m\sqrt{2}\right)^2 = 2m^2$$

Sol^n 123.



$$2x + y = 4 \Rightarrow \frac{x}{2} + \frac{y}{4} = 1$$

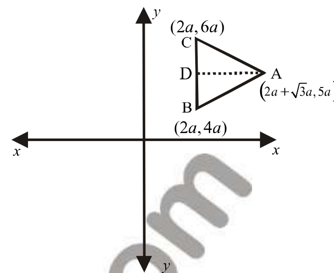
$$x - y + 1 = 0 \Rightarrow \frac{x}{-1} + \frac{y}{1} = 1$$

Altitude of this triangle will be the x - cordinate of point A which is intersection point of line

$2x + y = 4$ and $x - y + 1 = 0$

on solving $x = 1$ unit

Sol^n 124.



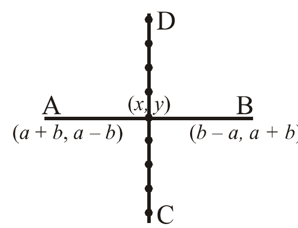
BC = $6a - 4a = 2a$, if ABC is equilateral triangle

then height should be $\frac{\sqrt{3}}{2}(\text{side}) = \frac{\sqrt{3}}{2}(2a) = \sqrt{3}\,a$

and from figure AD = $AD = 2a + \sqrt{3}\,a - 2a = \sqrt{3}\,a$

Height is same so triangle will be equilateral triangle.

Sol^n 125.



All point on the CD will be equidistant to A & B hence CD is perpendicular bisector of AB.

Slope of line $CD = \frac{-1}{\text{slope of AB}}$

$$= \frac{-1}{\dfrac{a+b-a+b}{b-a-a-b}} = \frac{2a}{2b} = \frac{a}{b}$$

$(x, y) \equiv$ (mid point of AB) $\equiv$

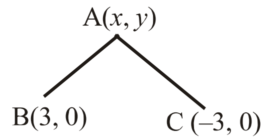
$$\left(\frac{a+b+b-a}{2}, \frac{a-b+a+b}{2}\right) \equiv (b, a)$$

then equation of line CD will be

$$y - a = \frac{a}{b}(x - b)$$

$$by - ab = ax - ab \Rightarrow -ax + by = 0$$

Sol^n 126.



AB – AC = 4

$$\Rightarrow \sqrt{(3-x)^2+(0-y)^2}+\sqrt{(-3-x)^2+(0-y)^2}=4$$

$$\Rightarrow \sqrt{(3-x)^2+y^2}+\sqrt{(-3-x)^2+y^2}=4$$

$$\Rightarrow \sqrt{(3-x)^2+y^2}=4-\sqrt{(3+x)^2+y^2}$$

squaring both side

$$\Rightarrow (3-x)^2+y^2=16+(3+x)^2+y^2-8\sqrt{(3+x)^2+y^2}$$

$$\Rightarrow 9+x^2-6x+y^2=16+9+x^2+6x+y^2-8\sqrt{(3+x)^2+y^2}$$

$$\Rightarrow 8\sqrt{(3+x)^2+y^2}=16+12x$$

$$\Rightarrow 2\sqrt{(3+x)^2+y^2}=4+3x$$

squaring again both side

$$\Rightarrow 4(9+x^2+6x+y^2)=16+9x^2+24x$$

$$\Rightarrow 36+4x^2+24x+4y^2=16+9x^2+24x$$

$$\Rightarrow 5x^2-4y^2=20$$

$$\Rightarrow \frac{x^2}{4}-\frac{y^2}{5}=1$$

Sol^n 127. See example 31

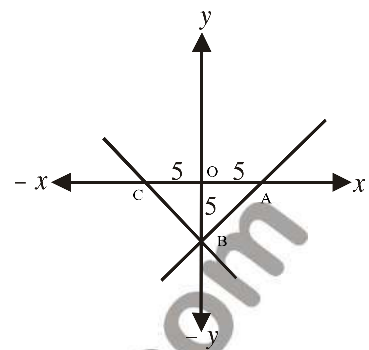
Sol^n 128. $ax + by + c = 0 \qquad c = 0$

constant is zero hence line will pass through origin

Sol^n 129. $y=|x|-5$ will give two lines

$$y=x-5 \quad \Rightarrow \quad \frac{x}{5}+\frac{y}{-5}=1$$

$$y=-x-5 \quad \Rightarrow \quad \frac{x}{-5}+\frac{y}{-5}=1$$



AC = 5 + 5 = 10 unit

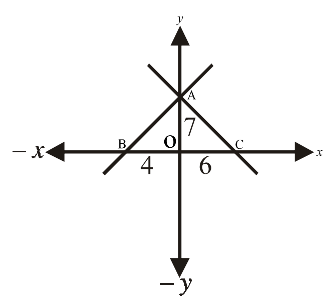
BO = 5 unit

Area of $\Delta ABC = \frac{1}{2}\times AC\times OB$

$= \frac{1}{2}\times 10\times 5$

$=$ 25 sq. unit

Sol^n 130. $\frac{x}{6}+\frac{y}{7}=1,\ \frac{x}{-4}+\frac{y}{7}=1$ & x-axis



BC = 4 + 6 = 10 unit

AO = 7 unit

Area of triangle $= \frac{1}{2}\times BC\times OA$

$= \frac{1}{2}\times 10\times 7$

= 35 unit

Sol^n 131. Equation of line perpendicular to $3x - 4y - 12 = 0$

will be $4x+3y=\lambda \Rightarrow \frac{x}{\frac{\lambda}{4}}+\frac{y}{\frac{\lambda}{3}}=1$

Area of triangle formed by this line with co-ordinate axis = $\frac{1}{2}\times\frac{\lambda}{4}\times\frac{\lambda}{3}$

$\Rightarrow \quad 24 = \frac{\lambda^2}{24}$

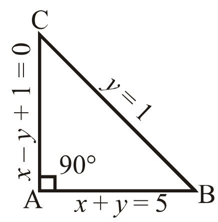
$\Rightarrow \quad \lambda^2 = (24)^2$

$\Rightarrow \quad \lambda = \pm 24$

hence line $\Rightarrow 4x+3y=\pm 24$

Sol^n 132.
$x + y = 5$ …(1)
$x - y + 1 = 0$ …(2)
$y = 1$ …(3)

Equation of line (1) and (2) shows perpendicular lines



mid point of B and C will be circumcentre

Point $B \equiv (4,1)$ and $C \equiv (0,1)$

on solving,

Circumcentre $\equiv \left(\frac{4+0}{2},\frac{1+1}{2}\right) \equiv (2,1)$

Sol^n 133.

$(x_1, y_1)\equiv(p,q+r),(x_2,y_2)\equiv(p,q-r),(x_3,y_3)\equiv(-p,r)$

$$\Delta=\left|\frac{1}{2}\{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\}\right|$$

$$=\left|\frac{1}{2}\{p(q-r-r)+p(r-q-r)-p(q+r-q+r)\}\right|$$

$$=\left|\frac{1}{2}(pq-2pr-pq-2pr)\right|$$

$$=\left|\frac{1}{2}(-4pr)\right|=2pr$$

Sol^n 134. Equation of line parallel to $2x+6y = -7$ will be

$2x+6y=\lambda \quad \Rightarrow \quad \frac{x}{\frac{\lambda}{2}}+\frac{y}{\frac{\lambda}{6}}=1$

length of intercept b/w cordinate axis = $\sqrt{a^2+b^2}$

$$10=\sqrt{\left(\frac{\lambda}{2}\right)^2+\left(\frac{\lambda}{6}\right)^2}=\sqrt{\frac{10\lambda^2}{36}}$$

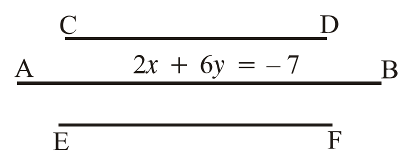
$$100=\frac{10\lambda^2}{36}$$

$\lambda^2 = 360$

$\lambda = \pm\sqrt{360}$

λ have two values so two lines are possible.

Method 2:



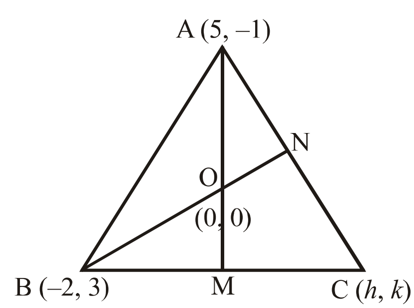
If AB is line then only two lines CD and EF are possible.

Sol^n 135. If origin is shifted to (h, k) then new co-ordinates of (x, y) will be $(x-h, y-k)$

Here origin is shifted to (2, 2)

then new co-ordinates of (4,–2) will be (4 –2, –2–2) $\equiv$ (2, –4)

Sol^n 136.



AM ⊥ BC

Slope of product of AM & BC = –1

$$\left(\frac{0+1}{0-5}\right).\left(\frac{k-3}{h+2}\right)=-1$$

$\Rightarrow \quad 5h-k=-13 \qquad \ldots$(i)

BN ⊥ AC

Slope of product BN & AC = –1

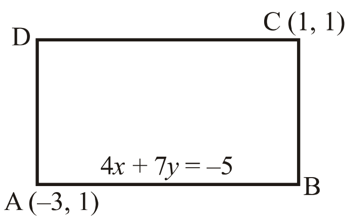
$$\left(\frac{k+1}{h-5}\right).\left(\frac{3-0}{-2-0}\right)=-1$$

$\Rightarrow 2h-3k=13 \qquad \ldots$(ii)

On solving eq^n (i) and (ii)

$(h, k) = (-4, 7)$

Sol^n 137.



We took (–3, 1) on $4x + 7y = -5$ because it is satisfying eq^n of line.

eq^n of BC $\Rightarrow 7x-4y=\lambda$

(it is perpendicular to AB)

it passes through (1, 1)

Put $x = 1, y = 1$

$7\times1-4\times1=\lambda \Rightarrow \lambda=3$

hence, eq^n $\Rightarrow 7x-4y=3$

Sol^n 138. Equation of required line will be

$$(2x-3y+4)+\lambda(3x+4y-5)=0$$

$$x(2+3\lambda)+y(-3+4\lambda)+4-5\lambda=0 \quad \ldots\text{(i)}$$

Slope $m_1 = -\left(\dfrac{2+3\lambda}{-3+4\lambda}\right)$

this line is perpendicular to $3x - 4y = 5 \Rightarrow$

$m_2 = \dfrac{3}{4}$

hence, $m_1 . m_2 = -1$

$$-\left(\frac{2+3\lambda}{-3+4\lambda}\right)\left(\frac{3}{4}\right)=-1$$

$$6+9\lambda=16\lambda-12$$

$$18=\lambda \Rightarrow \lambda=\frac{18}{7}$$

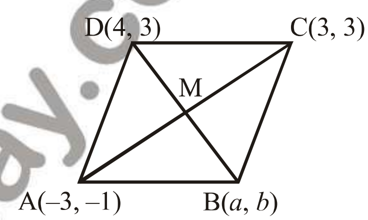
Put λ in equation (i)

$$\frac{68x}{7}+\frac{y51}{7}-\frac{62}{7}=0$$

$$\Rightarrow 68x+51y=62$$

$$\Rightarrow 4x+3y=\frac{62}{17}$$

Sol^n 139.



M will be mid point of both line AC and BD.

$$M \equiv \left(\frac{-3+3}{2},\frac{-1+3}{2}\right)\equiv\left(\frac{a+4}{2},\frac{b+3}{2}\right)$$

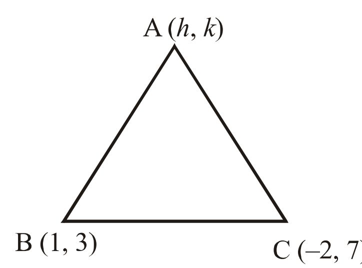
$$(0,1)\equiv\left(\frac{a+4}{2},\frac{b+3}{2}\right)$$

$$\frac{a+4}{2}=0 \Rightarrow a=-4$$

$$\frac{b+3}{2}=1 \Rightarrow b=-1$$

hence, $\dfrac{a}{b}=\dfrac{4}{1}$

Sol^n 140.



Since, AB = AC $\Rightarrow$ AB^2 = AC^2

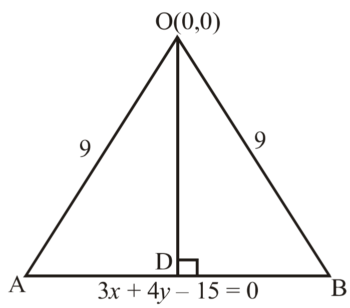
$\Rightarrow (h-1)^2+(k-3)^2=(h+2)^2+(k-7)^2$

$\Rightarrow h^2-2h+1+k^2+9-6k=h^2+4+4h+k^2+99-14k$

$\Rightarrow 6h-8k=-43$

On checking option $\left(\frac{5}{6},6\right)$ is satisfying the equation $A \equiv \left(\frac{5}{6},6\right)$

Sol^n 141.



OD = distance of a point (0, 0) to line $3x + 4y + 5 = 0$

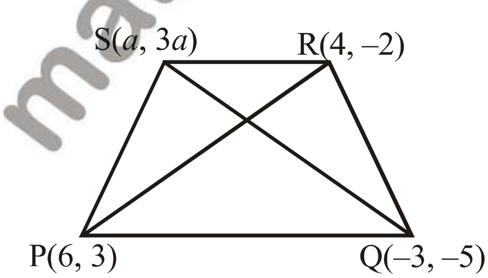
$$OD = \left|\frac{0+0-15}{\sqrt{3^2+4^2}}\right| = 3$$

$BD = \sqrt{OB^2 - DB^2} = \sqrt{9^2-3^2} = \sqrt{72} = 6\sqrt{2}$

$AB = 2 \times BD = 12\sqrt{2}$ (OAB is an isoceles triangle)

Area of triangle OAB $= \frac{1}{2}\times 12\sqrt{2}\times 3 = 18\sqrt{2}$

Sol^n 142.



Area of triangle PQR

$$= \left|\frac{1}{2}\left(6(-5+2)-3(-2-3)+4(3+5)\right)\right| = \frac{29}{2}$$

Area of triangle SRQ

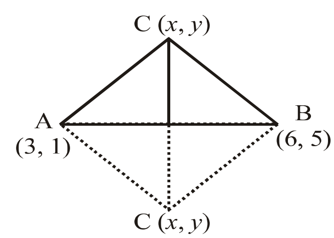
$$= \left|\frac{1}{2}\left(a(-2+5)+4(-5-3a)-3(3a+2)\right)\right| = 9a+13$$

$$\frac{\text{ratio of area of } \Delta PQR}{\text{ratio of area of } \Delta SRQ} = \frac{2}{1}$$

$$\frac{\frac{29}{2}}{9a+13} = \frac{2}{1} \quad \Rightarrow 29 = 36a + 52$$

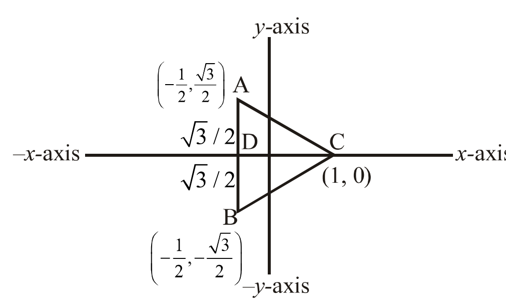
$$36\,a = -23 \quad \Rightarrow a = -\frac{23}{36}$$

Sol^n 143.



Let AB is the base of triangle ACB, only two points C (x, y) will be possible so that perpendicular distance from C to AB is equal in both case. Then area will be equal in both case. So only two points are possible.

Sol^n 144.



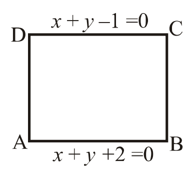
$$AB = \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \sqrt{3}$$

$$DC = 1 + \frac{1}{2} = \frac{3}{2} = \frac{\sqrt{3}}{2}\left(\sqrt{3}\right) = \frac{\sqrt{3}}{2}\left(\text{side AB}\right)$$

Here, DC is the height of triangle in the form of height of equilateral triangle.

Hence, ABC is an equilateral triangle. So all centre will be at a single point and distance between Orthocentre & Circumcentre will be zero.

Sol^n 145.

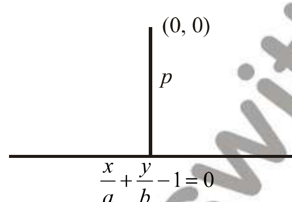


Both are parallel lines.

So distance between AB & CD = $a = \left|\frac{2+1}{\sqrt{1^2+1^2}}\right|$

Area of square ABCD = $a^2 = \frac{9}{2}$

Sol^n 146.



$$p = \left|\frac{0+0-1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}\right| = \frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}$$

$$\sqrt{\frac{1}{a^2}+\frac{1}{b^2}} = \frac{1}{p}$$

$$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

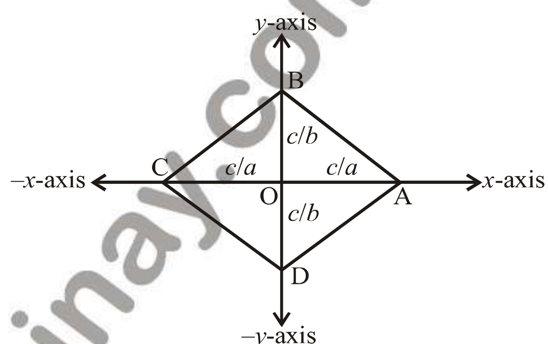
Sol^n 147. $ax \pm by \pm c = 0$ will given four lines.

$$ax + by + c = 0$$

$$ax + by - c = 0$$

$$ax - by + c = 0$$

$$ax - by - c = 0$$

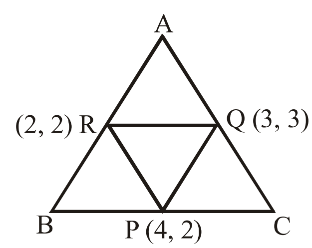


Diagonal AC = $\frac{2c}{a}$ and BD = $\frac{2c}{b}$

Area of rhombus = $\frac{1}{2}d_1 d_2$

$$= \frac{1}{2} \times \frac{2c}{a} \times \frac{2c}{b} = \frac{2c^2}{ab}$$

Sol^n 148. Centroid will be same of triangle made by mid-point to original triangle. So,



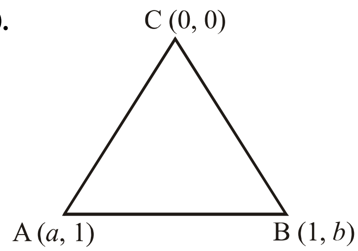
Centroid of triangle

ABC $\equiv$ Centroid of triangle POR

$$\equiv \left|\frac{4+3+2}{3}, \frac{2+3+2}{3}\right| \equiv \left(3, \frac{7}{3}\right)$$

Sol^n 149. Equation of line : $y = mx + c$ where m is slope & c is y - intercept. Here, $m = -6$ & $c = 2$. Then equation of line will be $y = -6x + 2$ or $6x + y = 2$.

Sol^n 150.



$$AB = BC = CA$$

$$\sqrt{(a-1)^2 + (1-b)^2} = \sqrt{(1-0)^2 + (b-0)^2} = \sqrt{(a-0)^2 + (1-0)^2}$$

Squaring all side

$$(a-1)^2 + (1-b)^2 = 1 + b^2 = a^2 + 1$$

taking last two relation $\left(1 + b^2 = a^2 + 1\right)$

$$b = \pm a$$

taking first two relation

$$(a-1)^2 + (1-b)^2 = 1 + b^2$$

$$a^2 + 1 - 2a + b^2 + 1 - 2b = 1 + b^2$$

$$a^2 - 2a - 2b + 1 = 0$$

If we put $b = a$

$$a^2 - 4a + 1 = 0 \Rightarrow a = 2 \pm \sqrt{3}$$

$$a, b \in (0, 1)$$

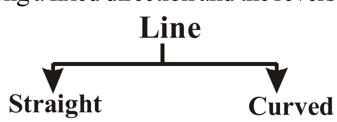
$$a = 2 + \sqrt{3}$$

*****

4 Geometry

Point: A circle of zero radius (.)

Line: A geometric figure formed by a point moving along a fixed direction and the reverse direction.



Straight line: If two points are on shortest distance then, their join is called straight line.

Curved line: If line is not straight (zig-zag)

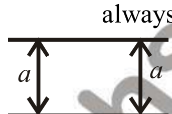
Length of line: A line has infinite length with no ending point.

Line Segment: If we mark starting and ending point on a line at A & B respectively then, 'AB' is called line segment on a line

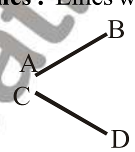
It has finite length with ending point.

Types of lines:

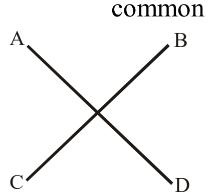
1. **Parallel Lines :** Distance between two lines is always constant.



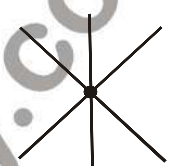
2. **Transversal lines :** Lines which are not parallel.



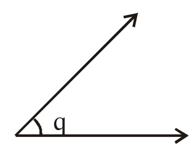
3. **Intersecting lines :** Two lines meeting at a common point.



4. **Concurrent lines:** When more than two lines intersects at a single point are called concurrent lines

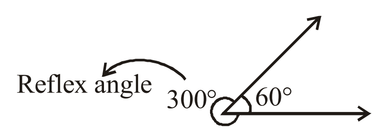


Angle: Slope between two lines is called an angle.



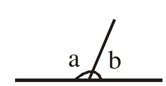
Types of Angle:

1. Acute Angle : $(0 < \theta < 90°)$
2. Right Angle : $(\theta = 90°)$
3. Obtuse Angle : $(90° < \theta < 180°)$
4. Reflex Angle : $(180° < \theta < 360°)$

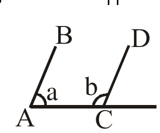


Types of Angles : let α, β are two angles.

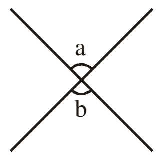
1. **Complementry angles -** If α, β are complementary angles then, $\alpha + \beta = 90°$
2. **Supplementry angles -** If α, β are supplementry angles then, $\alpha + \beta = 180°$
3. **Linear pair angles -** If α, β are linear pair angles then, $\alpha + \beta = 180°$



4. **Adjacent angles** - If AB || CD then, $\alpha + \beta = 180°$



5. **Vertical opposite angles -** α and β are vertical opposite angle then, $\alpha = \beta$.



Ex.1. Find the value of angle which is three times of its supplementary angle?

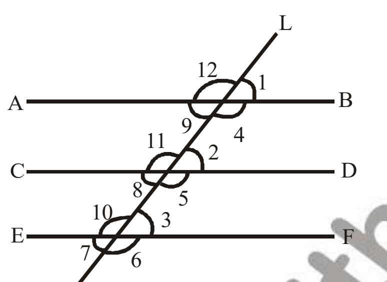
Sol. Let angle be α & 3α

$\Rightarrow \alpha + 3\alpha = 180°$

$\Rightarrow \alpha = 45°$

Required angle $= 3 \times 45° = 135°$

See the below given figure :- Lines AB, CD and EF are parallel lines and a transversal line L is intersecting all three lines.



Corresponding angles : Corresponding angles are always equal.

$\angle 1 = \angle 2 = \angle 3$

$\angle 12 = \angle 10 = \angle 11$

Alternate angles : Alternate angles are always equal.

1. Exterior alternate

   $\angle 1 = \angle 7$

2. Interior alternate

   $\angle 4 = \angle 10$

Note : $\angle 2$ & $\angle 4$ and $\angle 5$ & $\angle 3$ are adjacent angle and lines AB, CD and EF are parallel hence

$$\angle 2 + \angle 4 = \angle 5 + \angle 3 = 180°$$

Unit of Angle :

1. Degree (°) 2. Radian (^{c}) 3. grade (^{g})

Relation between degree, radian and grade :

$180° = \pi$ radian

$$\boxed{1° = \frac{\pi}{180} \text{rad}}$$

$$\boxed{1 \text{radian} = \frac{180°}{\pi} = 57°17'44''}$$

1 deg = 1° = 60 minute = 60'

1 min = 60 sec = 60''

For ex :

sin 1	sin 1°	cos 1	cos 1°
$\Downarrow$	$\Downarrow$	$\Downarrow$	$\Downarrow$
$\approx 57°$	$\approx 0°$	$\approx 57°$	$\approx 0°$
$\approx \sin 60°$	$\approx \sin 0°$	$\approx \cos 60°$	$\approx \cos 0°$
then $\sin 60° > \sin 0°$		then $\cos 60° < \cos 0°$	
$\therefore \boxed{\sin 1 > \sin 1°}$		$\boxed{\cos 1 < \cos 1°}$	

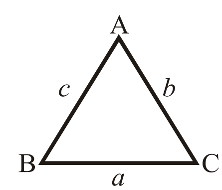
Grade :

1 grade = 100°

$1° = \frac{1}{100}$ grade

360° = 3.6 grade

<u>Triangle</u>



Property - 1. Sum of two sides should be greater than third side.

$a + b > c$

$b + c > a$

$c + a > b$

Property - 2. Difference of two sides should be smaller than third side.

$|b - c| < a$

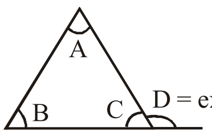
$|c - a| < b$

$|a - b| < c$

Property -3. Sum of angles of a triangle is equal to 180°.

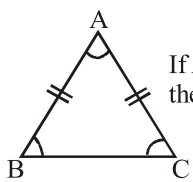
$\angle A + \angle B + \angle C = 180°$

Property - 4. Exterior angle of a triangle is equal to sum of two opposits interior angles.



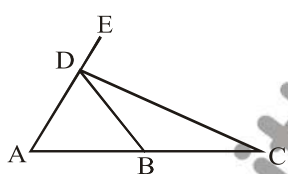
D = exterior angle $= \angle A + \angle B$

Property - 5. If two sides of a triangle are equal then, their opposite angles are equal.

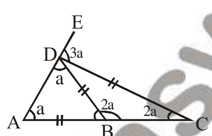


If AB = AC
then $\angle B = \angle C$

Ex.2 In the following figure, AB = BD = CD and $\angle CDE$ = 81°. Find $\angle ACD$.



Sol:



Let $\angle DAB = \alpha$

$\Rightarrow \angle ADB = \alpha \quad \therefore (AB = DB)$

$\Rightarrow \angle DBC = \angle DAB + \angle ADB = 2\alpha$
(exterior angle $\angle DBC$)

$\Rightarrow \angle BCD = 2\alpha \quad \therefore (BD = CD)$

Now, $\angle CDE = \angle DAC + \angle ACD = \alpha + 2\alpha$
(exterior angle $\angle CDE$)

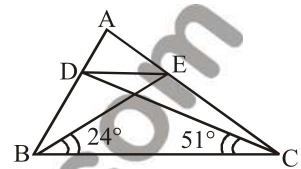
$\angle CDE = 3\alpha \quad \Rightarrow \quad 81° = 3\alpha$

$\Rightarrow \quad \alpha = 27°$

$\angle ACD = 2\alpha = 2 \times 27° = 54°$

Ex.3 In a triangle ABC, $\angle B = \angle C = 78°$. D &E are two points on side AB and AC such that $\angle BCD = 51°$ and $\angle CBE = 24°$ .Find angle $\angle CDE = ?$

Sol.



$\angle B = \angle C = 78°$

$\angle CBE = 24° \& \angle BCD = 51°$

In ΔBCE

$\angle BEC = 180° - 24° - 78° = 78°$

hence ,side BC = BE …(1)

In ΔBCD

$\angle BDC = 180° - 78° - 51° = 51°$

hence, side BC = BD …(2)

from eqn (1) & (2)

BE = BD

hence, $\angle BED = \angle BDE = \alpha$ (Let)

then, ΔBED

$\alpha + \alpha + 54° = 180° \; (\angle DBE = 78° - 24° = 54°)$

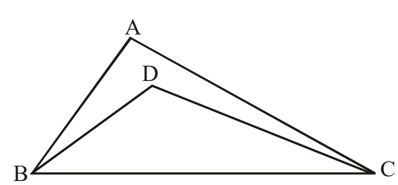
$\alpha = 63° = \angle BDE$

$\angle BDE = 51° + \angle CDE$

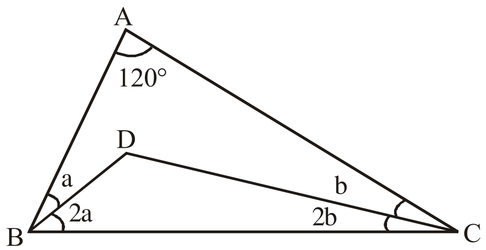
$63° = 51° + \angle CDE$

$\angle CDE = 12°$

Ex 4. In triangle ABC, $\angle A$ is equal to 120°. There is a point D inside the triangle such that $\angle DBC$ = $2\angle ABD$ and $\angle DCB = 2\angle ACD$. What is the measure of $\angle BDC$?



Sol. From the given figure

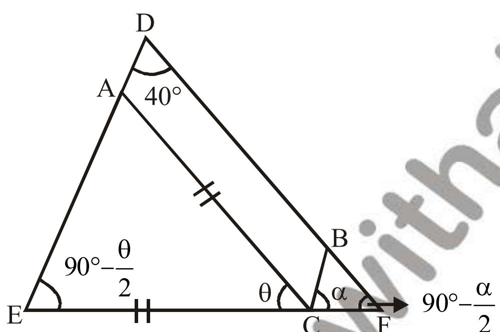


in $\Delta ABC,\ 3\alpha + 3\beta + A = 180° \Rightarrow \alpha + \beta = 20°$

in $\Delta BDC,\ \angle BDC = 180° - 2(\alpha + \beta) = 140°$

Ex.5 In triangle DEF, points A, B, and C are taken on DE, DF and EF respectively such that EC = AC and CF = BC. IF $\angle D = 40^o$ then, $\angle ACB = ?$

Sol.



$\therefore$ Let $\angle ACE = \theta \Rightarrow \angle CEA = 90^o - \frac{\theta}{2}$ and

$\angle BCF = \alpha \Rightarrow \angle BFC = 90^o - \frac{\alpha}{2}$

In Δ DEF

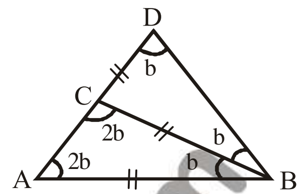
$40^o + 90^o - \frac{\theta}{2} + 90^o - \frac{\alpha}{2} = 180°$

or $\theta + \alpha = 80^o$

$\therefore\ \angle ACB = 180° - (\theta + \alpha) = 180^o - 80^o = 100^o$

Ex.6 In a ΔABD, BC is angle bisector such that $AB = BC = CD$ & $AD = BD$. Find $\angle ADB$.

Sol.



In $\Delta BCD, BC = CD \Rightarrow \angle BDC = \angle CBD = \beta$

Exterior $\angle ACB = 2\beta$

In $\Delta ABC, AB = BC \Rightarrow \angle BAC = \angle ACB = 2\beta$

$\because\ AD = BD \Rightarrow \angle DBA = \angle DAB = 2\beta$

In ΔABD,

$\angle DAB + \angle ABD + \angle BDA = 180°$

$2\beta + \beta + \beta + \beta = 180°$

$5\beta = 180°$

$\beta = 36°$

$\angle ADB = 36°$

Property -6

(a) Perimeter of triangle (*p*) $= a + b + c$

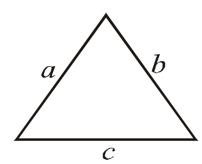
(b) Semi perimeter of triangle (*s*) $= \frac{p}{2} = \frac{a+b+c}{2}$

Property -7

<u>**Area of triangle**</u> (Δ)

(a) Hero's formula :

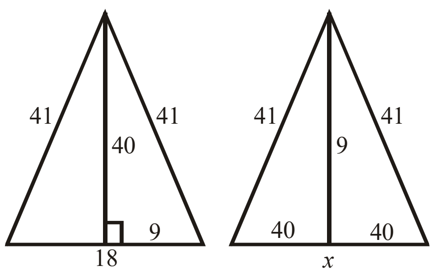
$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$



where, $s = \frac{a+b+c}{2}$

(b) $\Delta = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times a \times h$

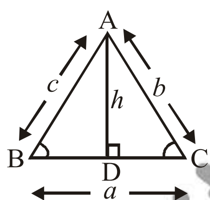
Ex.7 A triangle has sides measuring 41 cm , 41 cm and 18 cm. A second triangle has sides measuring 41cm, 41 cm and and *x* cm, where *x* is a whole number not equat to 18. If the two triangles have equal areas, what is the value of *x* ?



Sol. Area = $\frac{1}{2}$ base × height = $\frac{1}{2}$ × 18 × 40

To keep the area constant we can interchange base and height as shown in the second. figure, Therefore $x = 80$

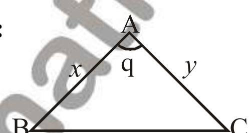
(c) $\Delta = \frac{1}{2}\times$ base × height $= \frac{1}{2}\times a\times h$



$$\Delta = \frac{1}{2}ac\sin B = \frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A$$

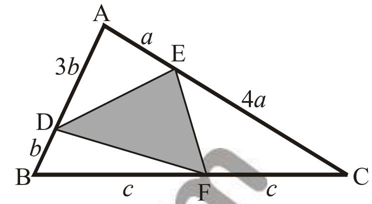
$$\left[\because \sin B = \frac{h}{c} \Rightarrow h = c\sin B\right]$$

Conclusion :



then, area of triangle ABC = $\frac{1}{2}xy\sin\theta$

Ex.8 Points D, E and F divide the sides of triangle ABC in the ratio 1 : 3, 1 : 4 and 1 : 1 as shown in the figure. What fraction of the area of triangle ABC is the area of triangle DEF?



Sol^n

$$\frac{\text{Area of }\Delta ADE}{\text{Area of }\Delta ABC} = \frac{\frac{1}{2}\times a\times 3b\times \sin A}{\frac{1}{2}\times 5a\times 4b\times \sin A} = \frac{3}{20}$$

$$\frac{\text{Area of }\Delta BDF}{\text{Area of }\Delta ABC} = \frac{\frac{1}{2}\times b\times c\times \sin B}{\frac{1}{2}\times 4b\times 2c\times \sin B} = \frac{1}{8}$$

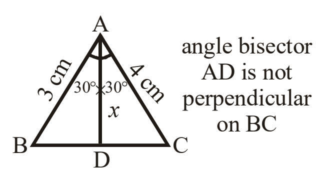
$$\frac{\text{Area of }\Delta CFE}{\text{Area of }\Delta ABC} = \frac{\frac{1}{2}\times 4a\times c\times \sin C}{\frac{1}{2}\times 5a\times 2c\times \sin C} = \frac{2}{5}$$

Therefore,

$$\frac{\text{Area of }\Delta DEF}{\text{Area of }\Delta ABC} = 1 - \left(\frac{3}{20}+\frac{1}{8}+\frac{2}{5}\right) = \frac{13}{40}$$

Ex.9 In a triangle ABC, $\angle A = 60°$, AB = 3 cm & AC = 4 cm. Find the length of AD, if AD is angle bisector.

Sol:



Area of Δ ABD + area of Δ ADC = area of Δ ABC

$$\frac{1}{2}\times 3\times x\times \sin 30° + \frac{1}{2}\times 4\times x\times \sin 30°$$

$$= \frac{1}{2}\times 3\times 4\times \sin 60°$$

$$\frac{1}{2}\left(\frac{3x}{2}+\frac{4x}{2}\right)=\frac{1}{2}\times 12\times\frac{\sqrt{3}}{2}$$

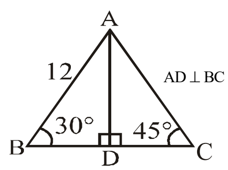
$$\frac{3x}{2}+\frac{4x}{2}=12.\frac{\sqrt{3}}{2}$$

$$7x=12\sqrt{3}$$

$$x=\frac{12\sqrt{3}}{7}\text{ cm}$$

Ex. 10 In a triangle ABC, AB = 12cm, $\angle$ ABC = 30°, $\angle$ ACB = 45° Find the area of triangle ABC?

Sol.



In triangle ABD, $\sin 30° = \frac{AD}{12} \Rightarrow AD = 6$ cm

$$\cos 30° = \frac{BD}{12} \Rightarrow BD = 6\sqrt{3}\text{ cm}$$

In triangle ACD,

$$\tan 45° = \frac{AD}{CD} = \frac{6}{CD} \Rightarrow AD = 6\text{ cm}$$

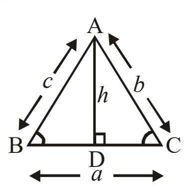
$$BC = BD + CD = 6\left(\sqrt{3}+1\right)$$

$$\text{Area} = \frac{1}{2}\times\text{base}\times\text{height} = \frac{1}{2}\times(BD+CD)\times AD$$

$$=\frac{1}{2}\times\left(6\sqrt{3}+6\right)\times 6 = 18\left(\sqrt{3}+1\right)\text{ cm}^2$$

Property - 8

Sine Rule :



$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$$

$$\text{or }\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

Proof :

In Δ ABD, $\sin B = \frac{h}{c} \Rightarrow h = c\sin B$

and in Δ ADC, $\sin C = \frac{h}{b} \Rightarrow h = b\sin C$

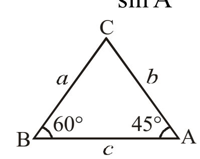
$$\Rightarrow c\sin B = b\sin C$$

$$\frac{\sin B}{b}=\frac{\sin C}{c}$$

similarly, $\frac{\sin B}{b} = \frac{\sin C}{c} = \frac{\sin A}{a}$

Ex. 11 In a triangle ABC, $\angle A = 45°$, $\angle B = 60°$. Find the ratio of the sides ?

Sol. From sine Rule : $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=k$



$$\angle C = 75°$$

$$a = k\sin 45° = \frac{k}{\sqrt{2}}$$

$$b = k\sin 60° = k\frac{\sqrt{3}}{2}$$

$$c = k\sin 75° = k.\left(\frac{\sqrt{3}+1}{2.\sqrt{2}}\right)$$

$$a : b : c = 2 : \sqrt{6} : \left(\sqrt{3}+1\right)$$

Ex. 12 In a triangle ABC, AD divides BC in the ratio 2 : 3.

If $\angle B = 60°$ & $\angle C = 45°$ then, find the $\frac{\sin\angle BAD}{\sin\angle CAD}$.

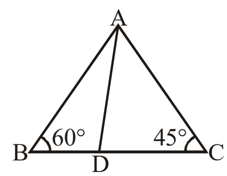
Sol: Apply sine rule in ΔABD & ΔACD.

$$\frac{\sin\angle BAD}{2}=\frac{\sin 60°}{AD}\Rightarrow AD.\sin\angle BAD=\sqrt{3}\ \ldots(1)$$

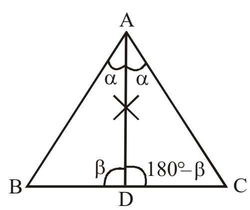
$$\frac{\sin\angle CAD}{3}=\frac{\sin 45°}{AD}\Rightarrow AD.\sin\angle CAD=\frac{3}{\sqrt{2}}\ \ldots(2)$$

Divide Eqn. (1) by Eqn. (2)

$$\frac{\sin\angle BAD}{\sin\angle CAD}=\frac{\sqrt{2}}{\sqrt{3}}$$



Property - 9 (a) Interior Angle Bisector Theorem



If AD is angle bisector of $\angle A$ then,,

$$\boxed{\frac{AB}{AC}=\frac{BD}{CD}}$$

Proof : By sine rule,

In triangle ABD, $\frac{\sin\alpha}{BD}=\frac{\sin\beta}{AB}\Rightarrow\frac{\sin\alpha}{\sin\beta}=\frac{BD}{AB}\ \ldots(i)$

Similarly ,in triangle ADC

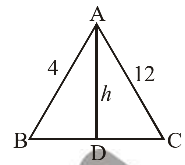
$$\frac{\sin\alpha}{CD}=\frac{\sin(180°-\beta)}{AC}=\frac{\sin\beta}{AC}\Rightarrow\frac{\sin\alpha}{\sin\beta}=\frac{CD}{AC}\ \ldots(ii)$$

From eqn. (i) & (ii),

$$\frac{BD}{AB}=\frac{CD}{AC}\Rightarrow\boxed{\frac{AB}{AC}=\frac{BD}{CD}}$$

Ex.13 In a triangle ABC, AB = 4 cm, AC = 12 cm, if the area of triangle ABD = 40 cm^2. Find the area of triangle ABC if AD is angle bisector.

Sol. $\frac{AB}{AC}=\frac{BD}{DC}\Rightarrow\frac{4}{12}=\frac{BD}{DC}$



$$\Rightarrow\frac{BD}{DC}=\frac{1}{3}$$

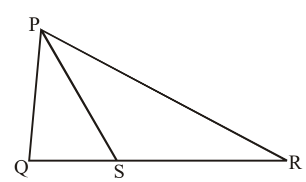
$$\frac{\text{Area of }\Delta ABD}{\text{Area of }\Delta ADC}=\frac{\frac{1}{2}\times BD\times h}{\frac{1}{2}\times DC\times h}=\frac{1}{3}$$

$$\frac{40}{\text{Area of }\Delta ADC}=\frac{1}{3}$$

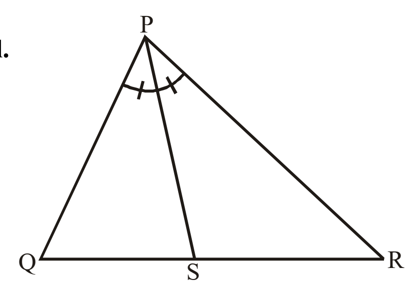
Area of ADC = 40× 3 = 120cm^2

Area of ΔABC = 40 + 120 = 160 cm^2

Ex.14 In the following figure, PS bisect $\angle QPR$. The area of the ΔPQS = 40 sq. cm. and PR is 2.5 times of PQ. Find the area of ΔPQR.



Sol.



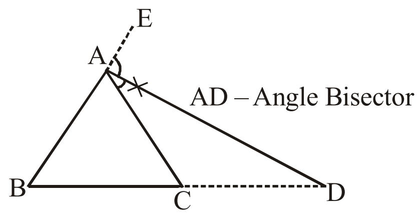
PS is the angle bisector of $\angle QPR$

$$\frac{PQ}{PR}=\frac{QS}{SR}\Rightarrow SR=(2.5)QS$$

Area of Δ PSR = 2.5× Area Δ PQS = 2.5×40 = 100

Area of Δ PQR = $100+40=140$ sq. cm

(b) Exterior Angle Bisector Theorem

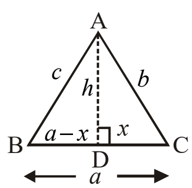


AD is angle Bisecter of exterior angle $\angle CAE$

then,, $\boxed{\dfrac{AB}{AC}=\dfrac{BD}{CD}}$

Property - 10

Cosine Rule :



Proof :

Let CD = x

$(AD \perp BC)$

$\cos C = \dfrac{x}{b}$

$x = b\cos C$

Using Pythagoras theorem in ΔABD & ΔACD

$h^2 = c^2-(a-x)^2 = b^2-x^2$

$c^2-a^2-x^2+2ax = b^2-x^2$

$2ax = b^2+a^2-c^2$ (put $x = b\cos C$)

$2ab\cos C = b^2+a^2-c^2$

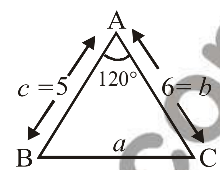
$\cos C = \dfrac{b^2+a^2-c^2}{2ab}$

$\boxed{\cos A = \dfrac{b^2+c^2-a^2}{2bc}}$

$\boxed{\cos B = \dfrac{c^2+a^2-b^2}{2ca}}$ $\boxed{\cos C = \dfrac{a^2+b^2-c^2}{2ab}}$

Ex.15 In a triangle ABC, $\angle A = 120°$. Find the length of side BC, if AB = 5cm & AC = 6 cm

Sol:



$\cos A = \dfrac{b^2+c^2-a^2}{2bc}$

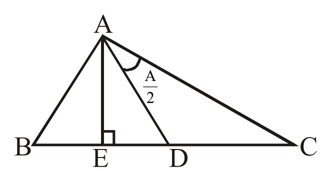
$\cos 120° = \dfrac{(6)^2+(5)^2-(a)^2}{2\times 6\times 5}$

$\dfrac{-1}{2}\times 60 = 36+25-a^2$

$\left(\begin{aligned}&\cos 120° = \cos(180°-60°)\\ &= -\cos 60° = -\frac{1}{2}\end{aligned}\right)$

$a^2 = 61+30 \Rightarrow BC = a = \sqrt{91}$ cm

Property - 11 In ΔABC, AE $\perp$ BD and AD is angle bisector of $\angle A$ then, $\angle EAD = \dfrac{1}{2}|\angle B - \angle C|$



Proof :

In ΔADC

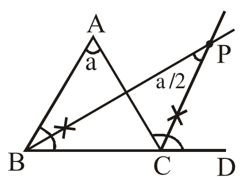
$\angle ADE$ is exterior angle $\left(\angle ADE = \dfrac{\angle A}{2}+\angle C\right)$

$\angle EAD = 90° - \left(\dfrac{\angle A}{2}+\angle C\right)$

$\angle EAD = 90° - \left(\angle C + \dfrac{180°-\angle B-\angle C}{2}\right)$

$\angle EAD = \left|\dfrac{\angle B - \angle C}{2}\right|$

Property - 12 In a ΔABC, angle bisector of interior $\angle B$ & exterior $\angle C$ meet at P, then, $\angle BPC = \dfrac{\angle A}{2}$



Proof : exterior $\angle ACD = \angle A + \angle B$

$$\angle PCD = \frac{\angle A + \angle B}{2}$$

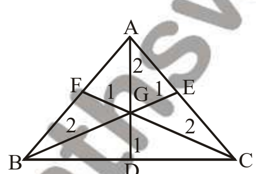
$$\angle PBC + \angle BPC = \angle PCD$$

$$\frac{\angle B}{2} + \angle BPC = \frac{\angle A}{2} + \frac{\angle B}{2}$$

$$\angle BPC = \frac{\angle A}{2}$$

Property - 13

Centroid : It is the intersection point of the all medians. A median of a triangle is a line segment joining a vertex to the midpoint of the opposite side. Every triangle has exactly three medians, one from each vertex, and they all intersect each other at the triangle's centroid.



Point I - G divides all the median in the ratio of 2 : 1.

Point II - Sum of two sides is always greater than 3rd side.

$BG + GC > BC$

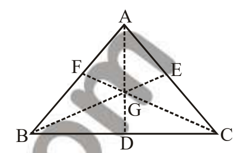
$BG + GA > AB$

$CG + GA > AC$

Adding above relation

$2(BG + CG + AG) > AB + BC + CA$

$$\left[\because BG = \frac{2}{3}BE, CG = \frac{2}{3}CF \text{ \& } AG = \frac{2}{3}AD\right]$$



$$2.\frac{2}{3}[AD + BE + CF] > AB + BC + CA$$

$$\frac{4}{3}[AD + BE + CF] > AB + BC + CA$$

$$3(AB + BC + CA) < 4(AD + BE + CF)$$

Point III - Length of Medians

$$AD = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$$

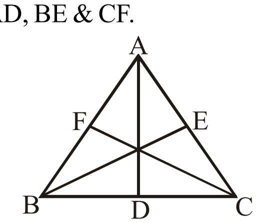
$$BE = \frac{1}{2}\sqrt{2a^2 + 2c^2 - b^2}$$

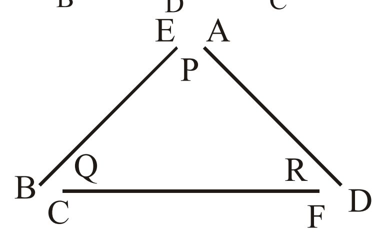
$$CF = \frac{1}{2}\sqrt{2b^2 + 2a^2 - c^2}$$

Point IV -

$$AB^2 + BC^2 + CA^2 = \frac{4}{3}\left(AD^2 + BE^2 + CF^2\right)$$

Point V - If we form a new triangle after taking sides as medians, AD, BE & CF.





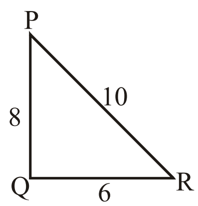
$\left[\text{Area of triangle} = \frac{4}{3} \times \text{area of triangle PQR formed by median}\right]$

Area of Δ PQR formed by median

$= \frac{3}{4}$ [Area of Δ ABC formed by sides]

Ex.16 Find the area of triangle whose length of the median, 6, 8, 10.

Sol:



Area of triangle PQR $= \frac{1}{2} \times 6 \times 8$

Area of triangle $= \frac{4}{3}$ [area of median triangle]

$\frac{4}{3}\left[\frac{1}{2} \times 6 \times 8\right] = 32$ unit^2

Ex.17 If the sides of a triangle ABC are 18, 24, 30. Find the area of triangle made by the medians of triangle ABC.

Sol. $\frac{1}{2} \times 18 \times 24 = \frac{4}{3}(\Delta)$

$\Delta = 162$ unit^2

Ex.18 Find the area of triangle whose medians are 6, 5 & 5.

Sol. $s = \frac{a+b+c}{2}$

$s = \frac{6+5+5}{2} = 8$

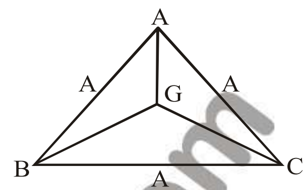
area of triangle formed by median $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{8(2)(3)(3)} = 12$

Area of triangle $= \frac{4}{3}(\Delta$ formed by medians$)$

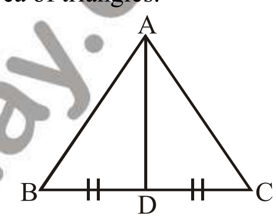
$= \frac{4}{3} \times 12 = 16$ unit^2

Point - VI Centroid G divides area of triangle into three equal parts.



area of ΔBGC = area of ΔAGC = area of Δ ABG

Point - VII Median divides the area of triangle into two equal area of triangles.

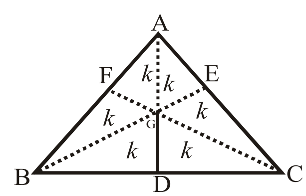


area of ΔABD = area of ΔACD

Hence, We draw all medians in a triangle then, 6 triangles will be formed and all have same area.

Ex.19 In a ΔABC, G is centroid and AD, BE, CF are medians. Find the ratio of area of quadrilateral BDGF and area of ΔABC.

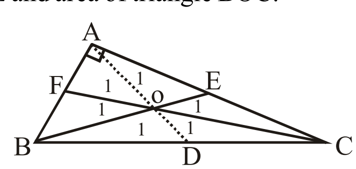
Sol.



$\frac{\text{area of quadrilateral BDGF}}{\text{area of } \Delta\text{ABC}} = \frac{2k}{6k} = \frac{1}{3}$

Ex.20 In ΔABC $\angle A = 90°$, median BE & CF intersect at O. Find the ratio of area of quadrilateral AFOE and area of triangle BOC.

Sol.

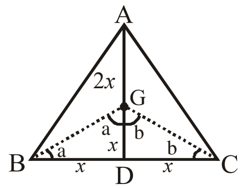


$\frac{\text{area of quadrilateral AFOE}}{\text{area of } \Delta\text{BOC}} = \frac{2}{2} = 1$

Ex. 21 In a triangle ABC, G is centroid, AG = BC. Find $\angle BGC$ =?

Sol. AG = BC

let, AG = $2x$ = BC



Centroid divides median in 2 : 1

GD = x = BD = CD, then,

$\angle DBG = \angle BGD = \alpha \ \& \ \angle DCG = \angle CGD = \beta$

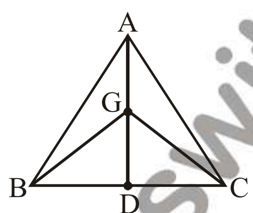
In Δ BGC

$\alpha + \alpha + \beta + \beta = 180°$

$\angle BGC = \alpha + \beta = 90°$

Ex.22 In a triangle ABC, AD is median and G is mid point of AD. If Area of Δ AGC = 11 cm^2 then, find area of Δ BGC.

Sol.



In Δ ADC, G is mid point of AD

Area of Δ AGC = Area of Δ GDC = 11

Similarly,

in Δ ABC, D is mid point of BC

Area of Δ BGD = Area of Δ GDC = 11

Now,

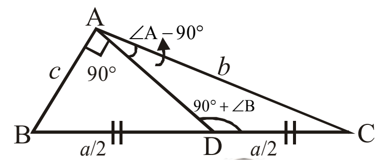
Area of Δ BGC = Area of Δ BGD + Area of Δ GDC

= 11 + 11

= 22 cm^2

Ex.23 In a ΔABC, median AD is perpendicular to side AB. Find the value of tan A + 2tanB.

Sol:



In ΔABC, applying sine rule

$$\frac{\sin A}{a} = \frac{\sin B}{b} \Rightarrow \frac{\sin A}{\sin B} = \frac{a}{b} \qquad ...(i)$$

In ΔADC,

$$\frac{\sin(A - 90°)}{a/2} = \frac{\sin(90° + B)}{b}$$

$$\frac{-2\cos A}{a} = \frac{\cos B}{b} \Rightarrow \frac{-2\cos A}{\cos B} = \frac{a}{b} \qquad \ldots(ii)$$

From Eqn. (i) and (ii)

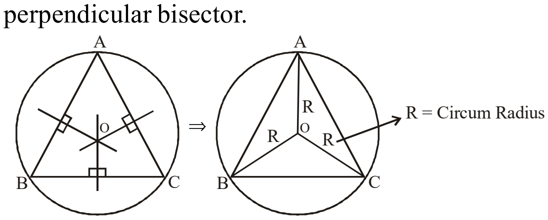
$$\frac{\sin A}{\sin B} = \frac{-2\cos A}{\cos B} \Rightarrow \frac{\sin A}{\cos A} = \frac{-2\sin B}{\cos B}$$

$\tan A = -2\tan B$

$\boxed{\dfrac{\tan A}{\tan B} = -2}$ or $\boxed{\tan A + 2\tan B = 0}$

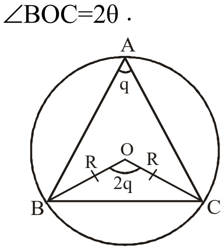
Property-14

Circumcentre : It is the intersection point of perpendicular bisector.

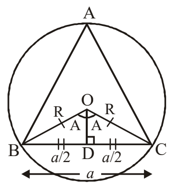


Circumcumcentre is a point in the triangle which is equidistant from each vertices.

Point I. In a ΔABC, O is the circumcentre. If $\angle A = \theta$ then, $\angle BOC = 2\theta$.



Point II.



In a ΔOBD, $\sin A = \dfrac{a/2}{R} = \dfrac{a}{2R}$

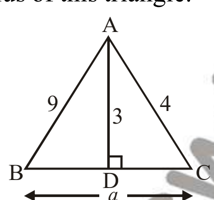
$$\boxed{R = \frac{a}{2\sin A} = \frac{b}{2\sin B} = \frac{c}{2\sin C}}$$

$$\boxed{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R \text{ (sine rule)}}$$

$$\Delta(\text{Area}) = \frac{1}{2}bc\sin A$$

$$\sin A = \frac{2\Delta}{bc} \quad \Rightarrow R = \frac{a}{2.\frac{2\Delta}{bc}} = \quad \boxed{R = \frac{abc}{4\Delta}}$$

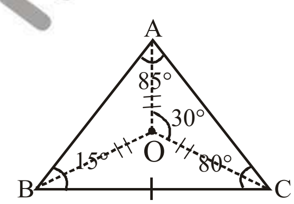
Ex.24 In a triangle ABC, AB = 9 cm, AC = 4cm, AD is perpendicular on BC and AD = 3 cm. Find the circum radius of this triangle.



$$R = \frac{abc}{4\Delta} = \frac{a \times 4 \times 9}{4 \times \frac{1}{2} \times a \times 3} = 3 \times 2 = 6 \text{ cm}$$

Ex.25 In a triangle ABC, $\angle A = 85°$ & $\angle C = 80°$ where O is circumcentre. Find $\angle OAC$.

Sol.



then,, $\angle B = 180° - 85° - 80° = 15°$

then, , $\angle AOC = 30°$

In triangle OAC,

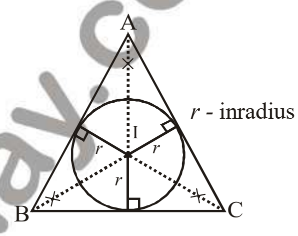
$$\angle AOC + \angle OAC + \angle ACO = 180°$$

$30° + 2\angle OAC = 180°$ $\quad (\because OA = OC)$

hence, $\angle OAC = 75°$

Property-15

Incentre : It is the intersection point of angle bisector.



Incentre is a point in a triangle which has equal distance from each side.

Point-1 Area of triangle ABC will be the sum of area of triangle BIC, AIB & AIC. $\Rightarrow$

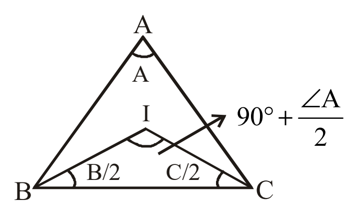
$$\frac{1}{2}r \times a + \frac{1}{2} \times r \times b + \frac{1}{2} \times r \times c = \Delta$$

$$r\left(\frac{a+b+c}{2}\right) = \Delta$$

$$r.s = \Delta$$

$$\boxed{r = \frac{\Delta}{s}}$$

Point-2



$$\angle BIC = 180° - \left(\frac{\angle B + \angle C}{2}\right) = 180° - \left(\frac{180° - \angle A}{2}\right)$$

$$\angle BIC = 180° - 90° + \frac{\angle A}{2}$$

$$\boxed{\angle BIC = 90° + \frac{\angle A}{2}}$$

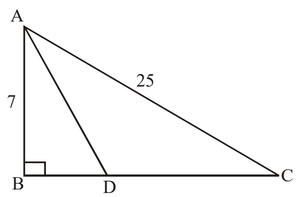
Ex.26 In a triangle ABC, I is incentre $\angle BIC = 116°$. Find $\angle A$.

Sol. $\angle BIC = 90° + \frac{\angle A}{2}$

$$116° = 90° + \frac{\angle A}{2} \Rightarrow 116° - 90° = \frac{\angle A}{2}$$

$\angle A = 52°$

Ex.27 AC = 25 and D is a point on BC such that AD is the bisector of angle A, as shown in the figure. What is the length of AD? ($\angle B = 90°$)



⇒

Sol. $BC = \sqrt{25^2 - 7^2} = 24$

AD is the angle bisector

$$\frac{BD}{DC} = \frac{AB}{AC} = \frac{7}{25}$$

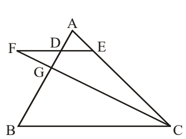
$$BD = BC \times \frac{7}{25+7} = 24 \times \frac{7}{32} = \frac{21}{4}$$

$\& AB = BD \Rightarrow BD = 7$

$$\Rightarrow \quad AD = \sqrt{AB^2 + BD^2} = \sqrt{49 + \frac{441}{16}}$$

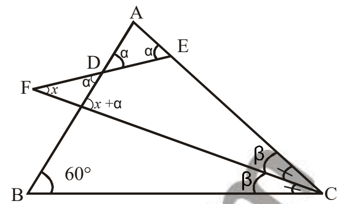
$= 8.75$

Ex.28 In triangle ABC, D and E are any points on AB and AC such AD = AE. The bisector of $\angle C$ meets DE at F. It is known that $\angle B = 60°$.



What is the degree measure of angle EFC?

Sol.



ΔEFC में

$\alpha = \beta + x$ (1)

ΔBGC में

$x + \alpha = 60° + \beta$ (2)

$x + \beta + x = 60° + \beta$

$2x = 60$

$x = 30°$

Ex.29 In a triangle ABC, perimeter is 24 cm and inradius is 7cm. Find area of Δ ABC.

Sol. $\Delta = r.s$

$p = 24$ cm $\Rightarrow$ then, $s = 12$, $r = 7$

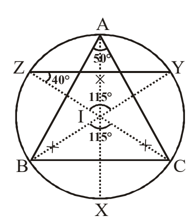
$\Delta = 12 \times 7 = 84 \text{ cm}^2$

Ex.30 In a triangle ABC, angle bisector of $\angle A, \angle B$ & $\angle C$ cuts circumcircle at X, Y, Z respectively. If $\angle CZY$ = 40° & $\angle A = 50°$, then, find $\angle BYZ$.

Sol. Because I is the intersection point of all angle bisectors so I will be incentre, hence

$$\angle BIC = 90° + \frac{\angle A}{2} = 115°$$

$\angle ZIY = \angle BIC = 115°$ (Vertical opposite angles)



In ΔZIY,

$\angle BYZ = 180° - 115° - 40° = 25°$

Ex.31 If the ratio of sides of a triangle is 5 : 7 : 8. find the ratio of circumradius (R) to inradius (r).

Sol. $R = \frac{abc}{4\Delta}$ & $r = \frac{\Delta}{s}$

$$\because \quad s = \frac{a+b+c}{2} = \frac{5+7+8}{2} = 10$$

$$\frac{R}{r} = \frac{abc}{4\Delta} \times \frac{s}{\Delta} = \frac{abc.s}{4\Delta^2}$$

$$\Rightarrow \frac{R}{r} = \frac{abc.s}{4s(s-a)(s-b)(s-c)} = \frac{abc}{4(s-a)(s-b)(s-c)}$$

$$= \frac{5\times7\times8}{4\times5\times3\times2} = \frac{7}{3}$$

$R : r = 7 : 3$

Property- 16

Orthocentre : It is the intersection point of all altitudes.

In ΔCBE,

$\angle CBE = 90° - \angle C$

In ΔBCF,

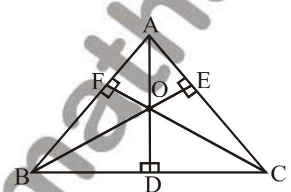
$\angle BCF = 90° - \angle B$

In ΔBOC,

$\angle BOC = 180° - (90° - \angle B) - (90° - \angle C)$

$\angle BOC = \angle B + \angle C = 180° - \angle A$

$\angle BOC + \angle A = 180°$



Point (I) $\angle BOC = 180° - \angle A$

Point (II)

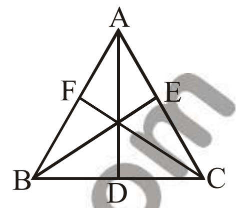
Sum of Sides > Sum of altitudes

$AB + BC + CA > AD + BE + CF$

Proof – In triangle ABD $\Rightarrow AB > AD$

In triangle ACF $\Rightarrow AC > CF$

In triangle BCE $\Rightarrow BC > BE$



Ex.32 In a triangle ABC, $\angle BOC = 130°$, if O is orthocentre. find $\angle A$.

Sol. $\angle BOC + \angle A = 180°$

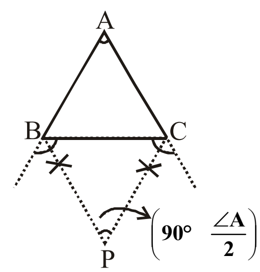
$\angle A = 180° - 130°$

$\angle A = 50°$

Property -17

Ex-centre : It is the intersection point of bisector angles.

$$\angle BPC = 90° - \frac{\angle A}{2}$$



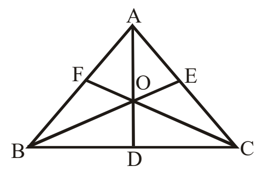
Ex.33 In a triangle ABC, angle bisector of exterior angle B and C intersect at P. If $\angle BPC = 40°$. Find $\angle A$.

Sol. $\angle BPC = 90° - \frac{\angle A}{2}$

$40° = 90° - \frac{\angle A}{2}$ $\quad \frac{\angle A}{2} = 90° - 40° = 50°$

$\angle A = 100°$

Ex.34 If O is the orthocentre of Δ ABC then, A will be the orthocentre of which triangle ?



(a) ΔBOC (b) ΔCOA

(c) ΔBOA (d) ΔBFC

Sol. $BF \perp CF \Rightarrow BA \perp CF$

$CE \perp BE \Rightarrow CA \perp BE$

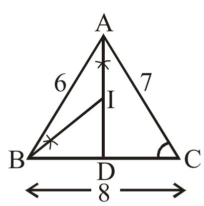
$OD \perp BC \Rightarrow AD \perp BC$

All perpendiculars of ΔBOC intersect at A, hence 'A' will be the orthocentre of ΔBOC.

Ex.35 In a ΔABC, AD is angle bisector of, $\angle A$, AB = 6 cm, AC = 7cm & BC = 8 cm. If I is the incentre, find the ratio AI to ID.

Sol. AD is angle bisector of $\angle A$, then, $\frac{AB}{AC} = \frac{BD}{CD}$

$$\Rightarrow \frac{6}{7} = \frac{BD}{CD}$$



BD & CD will be divided in the ratio 6 : 7.

then,, $BD = \frac{8}{13} \times 6 = \frac{48}{13}$

I is incentre so BI is the angle bisector of $\angle B$ in ΔABD.

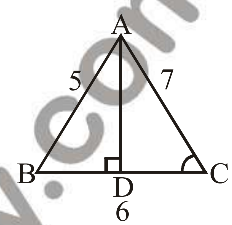
So, BI will divide AD in the ratio AB : BD.

$$\frac{AI}{ID} = \frac{AB}{BD} = \frac{6 \times 13}{48} = \frac{13}{8}$$

Ex.36 In a ΔABC, AB = 5 cm, BC = 6 cm & AC = 7 cm. If AD is perpendicular to BC. Find the length of BD.

Sol. In ΔABC, applying cosine rule,

$$\cos B = \frac{BD}{5} = \frac{5^2 + 6^2 - 7^2}{2 \times 5 \times 6}$$



$$\frac{BD}{5} = \frac{25 + 36 - 49}{60} = \frac{12}{60} = \frac{1}{5}$$

$BD = 1$

Property-18

1. Distance between circumcentre and incentre in any triangle is equal to $\sqrt{R^2 - 2Rr}$

where, R = circumradius & r = inradius

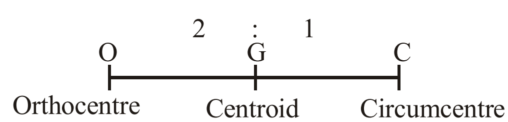
Ex.37 If the circumradius of a triangle is 6cm and inradius is 2 cm. Find the distance between circumcentre and incentre.

Sol. Distance between circumcentre and incentre in any triangle $= \sqrt{R^2 - 2Rr}$

$$= \sqrt{(6)^2 - 2 \times 6 \times 2} = \sqrt{12} = 2\sqrt{3} \text{ cm}$$

Property -19

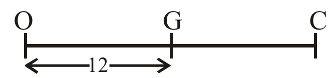
In any triangle orthocentre, centroid and circumcentre are colinear and centroid divides the join of orthocentre and circumcentre in the ratio 2 : 1.



$$\frac{OG}{GC} = \frac{2}{1}$$

Ex.38 In a triangle the distance between centroid and orthocentre is 12 cm. Find the distance between orthocentre and circumcentre.

Sol:



$$\frac{OG}{GC}=\frac{2}{1}=\frac{12}{6}$$

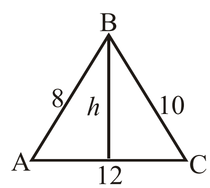
$OC = OG + GC = 12 + 6 = 18$ cm

Ex.39 In a ΔABC, AB = 8 cm, BC = 10 cm and AC = 12 cm. Find the length of smallest altitude.

Sol:

Note : Smallest altitude will be on the largest side and largest altitude will be on smallest side

hence, smallest altitude will be on side of length 12 cm.



$$s=\frac{8+10+12}{2}=\frac{30}{2}=15$$

$$\text{Area}(\Delta)=\sqrt{s(s-a)(s-b)(s-c)}=\frac{1}{2}\times\text{base}\times\text{height}$$

$$\text{Area}(\Delta)=\sqrt{15(5)(3)(7)}=\frac{1}{2}\times 12\times h$$

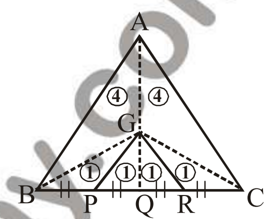
$$15\sqrt{7}=\frac{1}{2}\times h\times 12$$

$$h=\frac{5\sqrt{7}}{2}\text{cm}$$

Ex. 40 In a triangle ABC, there are three points P, Q & R on side BC, such that BP = PQ = QR = RC. If G is the centroid, then, find the ratio of area of ΔPGR to area of ΔABC.

Sol. Height and base of ΔGBP, ΔGPQ, ΔGQR & ΔGRC are equal, so area of ΔGBP, ΔGPQ, ΔGQR & ΔGRC will be equal. Let area of these triangle is 1, then, area of ΔBGC is equal to 4. Area of triangle ΔAGB, & ΔAGC are equal.

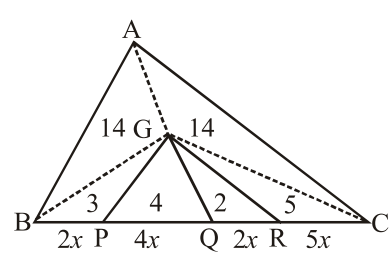
Hence, area of triangle ABC will be equal to 12.



$$\frac{\text{area of }\Delta PGR}{\text{area of }\Delta ABC}=\frac{1+1}{4+4+4}=\frac{2}{12}=\frac{1}{6}$$

Ex.41 In a ΔABC, P, Q & R are three points on side BC, such that $BP = 3x$, $QR = 2x$, $PQ = 4x$ and $RC = 5x$. If G is centroid then, find the ratio of area of ΔPGR to area of ΔABC.

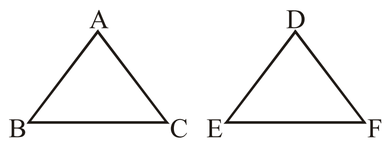
Sol. Height of ΔGBP, ΔGPQ, ΔGQR & ΔGRC are equal, so area of ΔGBP, ΔGPQ, ΔGQR & ΔGRC will be divided in the ratio of their base. Hence area of ΔGBP, ΔGPQ, ΔGQR & ΔGRC will be in the ratio 3 : 4 : 2 : 5. Let area of these triangle ΔGBP, ΔGPQ, ΔGQR & ΔGRC is 3, 4, 2 and 5 respectively, then, area of ΔBGC is equal to 14. Area of triangle ΔAGB, ΔBGC & ΔAGC are equal. Hence, area of triangle ABC will be equal to 42.



$$\frac{\text{area of }\Delta PGR}{\text{area of }\Delta ABC}=\frac{6}{42}=\frac{1}{7}$$

Property -20

Congruency of triangles : If two triangles ABC and DEF are congruent then, every corresponding part of triangle will be equal.



$\Delta ABC \cong \Delta DEF$

$AB = DE, \quad BC = EF \;\&\; AC = DF$

$\angle A = \angle D, \angle B = \angle E \;\&\; \angle C = \angle F$

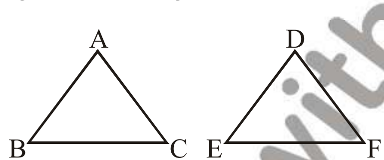
$r_1 = r_2, \quad h_1 = h_2 \;\&\; R_1 = R_2$

$\Delta_1 = \Delta_2, \quad m_1 = m_2 \;\&\; P_1 = P_2$

Condition for congruency :

1. AAS
2. SAS
3. SSS

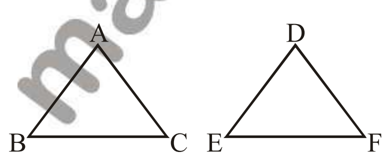
AAS – If two corresponding angles and their connecting side of two triangles are equal then, triangle will be congruent.



If, $\angle B = \angle E$, $\angle C = \angle F$ & $BC = EF$

Then, $\Delta ABC \cong \Delta DEF$

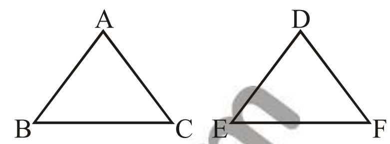
SAS – If two corresponding sides and angle between them of two triangles are equal then, triangles will be congruent.



$AB = DE, \quad BC = EF \;\&\; \angle B = \angle E$

Then, $\Delta ABC \cong \Delta DEF$

SSS – If all three corresponding sides of two triangles are equal then, triangle will be congruent.

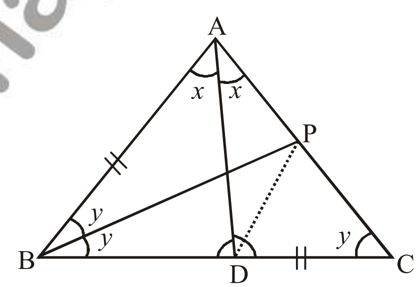


$AB = DE, \quad BC = EF \;\&\; AC = DF$

Then, $\Delta ABC \cong \Delta DEF$

Ex.42 In a triangle ABC, $\angle B = 2\angle C$. AD & BE are bisectors of angle BAC and angle ABC. If AB = CD, then, find of angle ABC.

Sol.



In ΔABC,

$\angle B = 2\angle C$ or, $\angle B = 2y$ where, $\angle C = y$

AD is the bisector of $\angle BAC$.

$\angle BAD = \angle CAD = x$ (let)

Let BP be the bisector of $\angle ABC$. Join PD.

In ΔBPC,

$\angle CBP = \angle BCP = y$

$\Rightarrow BP = PC$

In ΔABP and ΔDCP,

$\angle ABP = \angle DCP = y$

$AB = DC$ (given)

and, $BP = PC$

by SAS congruency

$\Delta ABP \cong \Delta DCP$

$\Rightarrow \angle BAP = \angle CDP$ and $AP = DP$

$\Rightarrow \angle CDP = 2x$ and $\angle ADP = DAP = x$

$[\therefore \angle A = 2x]$

In ΔABD,

$\Delta ADC = \Delta ABD + \angle BAD$

$\Rightarrow x + 2x = 2y + x \Rightarrow x = y$

In ΔABC,

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow 2x + 2y + y = 180°$

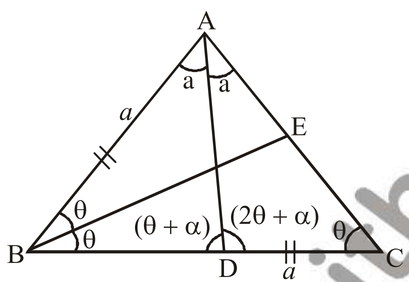
$\Rightarrow 5x = 180°$

$\Rightarrow x = 36°$

$\angle BAC = 2x = 72°$

Method 2.

Applying sine rule,



Let, AD = x

In Δ ABD,

$$\frac{\sin(\theta+\alpha)}{a} = \frac{\sin 2\theta}{x}$$

$$\Rightarrow \frac{\sin(\theta+\alpha)}{\sin 2\theta} = \frac{a}{x} \quad \ldots(1)$$

In Δ ADC

$$\frac{\sin\theta}{x} = \frac{\sin\alpha}{a}$$

$$\Rightarrow \frac{a}{x} = \frac{\sin\alpha}{\sin\theta} \quad \ldots(2)$$

from Eqn. (1) and (2)

$$\Rightarrow \frac{\sin(\theta+\alpha)}{\sin 2\theta} = \frac{\sin\alpha}{\sin\theta}$$

$$\Rightarrow \frac{\sin\theta\cos\alpha + \cos\theta\sin\alpha}{2\sin\theta\cos\theta} = \frac{\sin\alpha}{\sin\theta}$$

$\Rightarrow \sin\theta\cos\alpha + \cos\theta\sin\alpha = 2\cos\theta\sin\alpha$

$\Rightarrow \sin\theta\cos\alpha - \cos\theta\sin\alpha = 0$

$\Rightarrow \sin(\theta-\alpha) = 0 = \sin 0°$

$\Rightarrow \theta = \alpha$

In triangle ΔABC,

$\angle ABC + \angle BAC + BCA = 180°$

$2\alpha + 2\alpha + \alpha = 180°$

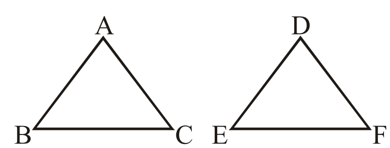
$\alpha = 36°$

$\angle ABC = 2\alpha = 2 \times 36$

$= 72°$

Property-20

Similarity of triangle : If two triangles are similar then, the ratio of corresponding organ of triangles will be equal.

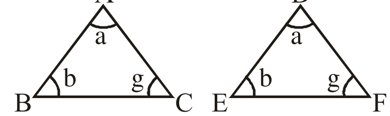


$\Delta ABC \sim \Delta DEF$

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{p_1}{p_2} = \frac{m_1}{m_2} = \frac{h_1}{h_2} = \frac{r_1}{r_2} =$$

$$\frac{R_1}{R_2} = \sqrt{\frac{\Delta_1}{\Delta_2}}$$

Proof of Similarity: - Similarity is nothing but sine rule. Two triangles will be similar when their corresponding angles are equal.



$$\frac{\sin\alpha}{BC} = \frac{\sin\beta}{AC} \quad \ldots(1)$$

$$\frac{\sin\alpha}{EF} = \frac{\sin\beta}{DF} \quad \ldots(2)$$

Divide Eqn. (1) by Eqn. (2)

$$\frac{EF}{BC} = \frac{DF}{AC}, \quad \text{hence Proved.}$$

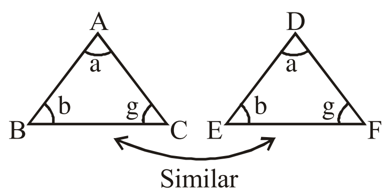
Condition of similarity of triangles

1. AAA

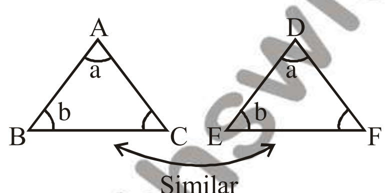
2. AA

3. SSS (every congruent triangle is similar triangle.)

<u>AAA</u> – If all three corresponding angles of two triangles are equal then, triangle will be similar.

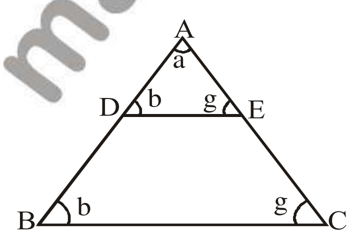


<u>AA</u> – If two corresponding angles of two triangles are equal then, triangle will be similar. Because third angle will also be equal.



Thales theorem:

1.



DE||BC

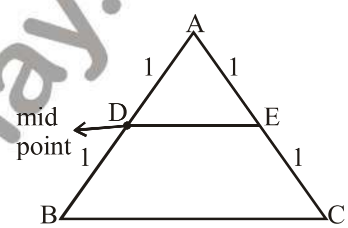
$\Delta ADE \sim \Delta ABC$

$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$$

$$\Rightarrow \frac{AD}{BD} = \frac{AE}{EC}$$

2. When D and E will be mid point of AB & AC, then, the ratio of AD & DB and ratio of AE & AC are equal to 1.

Then triangle ADE & ABC will be similar, hence DE || BC.

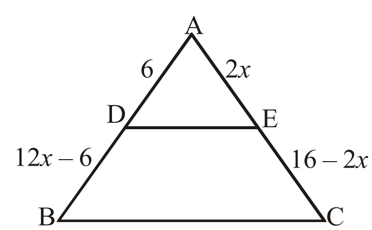


$$\frac{AD}{AB} = \frac{1}{2} = \frac{DE}{BC} \Rightarrow DE = \frac{1}{2}BC$$

$$\frac{\text{Area of ADE}}{\text{Area of ABC}} = \left(\frac{DE}{BC}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\text{Area of ADE} = \frac{1}{4} \text{ of area of ABC}$$

Ex.43 In a triangle ABC, D & E are two point on AB & AC and $DE||BC$. If $AD = 6$ cm, $BD = (12x - 6)$ cm, $AE = 2x$ cm $CE = (16 - 2x)$ cm, then, find the value of x.



$AB = 12x - 6 + 6 = 12x$ and $AC = 2x + 16 - 2x = 16$

If DE||BC then, ΔADE & ΔABC will be similar.

$\Delta ADE \sim \Delta ABC$

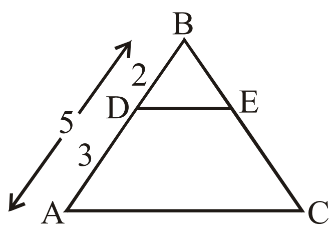
$$\frac{AD}{AB} = \frac{AE}{AC}$$

$$\frac{6}{12x} = \frac{2x}{16}$$

$x^2 = 4 \Rightarrow x = 2$ cm

Ex.44 In a triangle ABC, D & E are two points on sides AB and BC such that DE || AC and AD : DB = 3 : 2. Find the ratio of area of trapezium ACED and area of Δ BED.

Sol.



In Δ BDE and Δ BAC

$\angle B = \angle B$ (common)

$\angle BDE = \angle BAC$ $(\because DE \parallel AC)$

$\angle BED = \angle BCA$ $(\because DE \parallel AC)$

$\therefore \Delta BDE \sim \Delta BAC$

In similar triangles

ratio of area = (ratio of sides)^2

$$\frac{\text{area of } \Delta BDE}{\text{area of } \Delta BAC} = \left(\frac{BD}{BA}\right)^2 = \left(\frac{2}{5}\right)^2 = \frac{4}{25}$$

let area of Δ BDE = 4

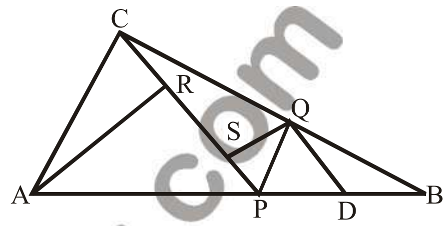
& area of Δ BAC = 25

Area of trapezium ACED = area of Δ BAC – area of Δ BDE

$= 25 - 4 = 21$

$$\frac{\text{area of trapezium ACED}}{\text{area of } \Delta BED} = \frac{21}{4}$$

EX. 45 In the figure, P is an point on AB such that AP : PB = 4 : 3, PQ is parallel to AC and QD is parallel to CP. In Δ ARC, $\angle ARC = 90°$ and in Δ PQS, $\angle PSQ = 90°$. The length of QS is 6 cm. What is ratio AP : PD?



Sol: PQ is parallel to AC $\Rightarrow \frac{AP}{PB} = \frac{CQ}{QB} = \frac{4}{3}$

Let AP = $4x$ and PB = $3x$.

QD is parallel CP $\Rightarrow \frac{PD}{DB} = \frac{CQ}{QB} = \frac{4}{3}$

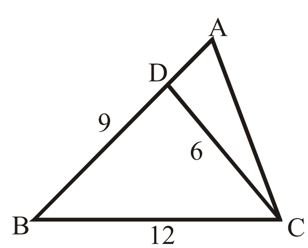
$$\frac{PD}{PD + DB} = \frac{4}{7}$$

$$\frac{PD}{PB} = \frac{4}{7}$$

$$PD = \frac{4PB}{7} = \frac{12x}{7}$$

$$AP : PD = 4x : \frac{12x}{7} = 7 : 3$$

Ex.46 Consider the triangle ABC shown in the following figure where BC = 12 cm, DB = 9 cm, CD = 6 cm and $\angle BCD = \angle BAC$. What is the ratio of the perimeter of the triangle ADC to the area of the triangle BDC?



Sol: In ΔBAC and ΔBCD, $\angle BCD = \angle BAC$, $\angle B$

is common $\Rightarrow \angle BDC = \angle BCA$. Therefore, the two triangles are similar.

$$\frac{AB}{BC} = \frac{AC}{CD} = \frac{BC}{BD} \Rightarrow AB = \frac{BC^2}{BD} = 16$$

AD = 7,

$$\text{Similarly, } AC = \frac{BC \times CD}{BD} = 8$$

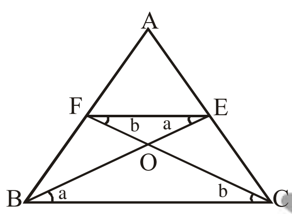
Perimeter of $\Delta ADC = 7 + 6 + 8 = 21$, Perimeter of $\Delta BDC = 27$,

$$\text{Therefore, ratio} = \frac{7}{9}$$

Ex.47 In a triangle ABC, median BE and CF intersect at O. Find the ratio of area of triangle OEF and ABC.

Sol. E & F are mid point of AC and AB then, EF || BC.

$\Delta OFE \sim \Delta OBC$

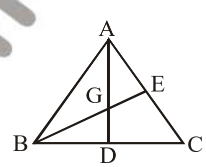


$$\frac{\text{Area of OFE}}{\text{Area of OBC}} = \left(\frac{FE}{BC}\right)^2 = \left(\frac{FE}{2FE}\right)^2 = \frac{1}{4}$$

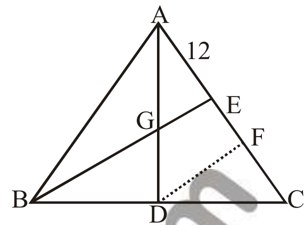
Let the area of ΔOBC is 4 then, area of ΔABC will be 12.

$$\frac{\text{Area of } \Delta OFE}{\text{Area of } \Delta ABC} = \frac{1}{12}$$

Ex.48 In the following figure, if AG : GD = 3 : 4 and BD : DC = 4 : 7 and AE = 12 cm, then, find the length of EC (in cm).



Sol. We draw a line DF parallel to BE.



In Δ ADF, GE || DF then,

$$\frac{AG}{GD} = \frac{AE}{EF}$$

$$\frac{3}{4} = \frac{12}{EF} \Rightarrow EF = 16\text{cm}$$

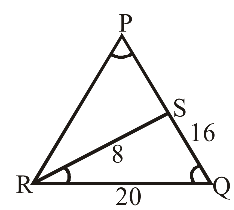
$\therefore$ EF = 16 cm

Now, In Δ BEC, DF || BE

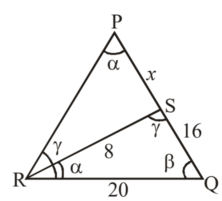
$$\frac{BD}{DC} = \frac{EF}{FC} \Rightarrow \frac{4}{7} = \frac{16}{FC} \Rightarrow FC = 28\text{cm}$$

EC = EF + FC = 16 + 28 = 44 cm

Ex.49 Consider the triangle PQR shown in the figure, where, S is a point on PQ such that QR = 20 cm, SQ = 16 cm, RS = 8 cm and $\angle QRS = \angle QPR$. What is the perimeter of the triangle PRS?



Sol.



Note : When we use similarity then, confusion is that which angle is equal to angle of other angle.

So, we will take the ratio of opposite side of equal angle in both triangle.

e.g. here, In ΔRSQ & ΔPQR ,

$\angle QRS = \angle QPR = a$ and $\angle Q = \beta$ (common in both traingle).

In ΔRSQ ,16 is opposite side of $\angle\alpha$ and 8 is opposite side of $\angle\beta$.

In ΔPQR ,20 is opposite side of $\angle\alpha$ and PR is opposite side of $\angle\beta$.

Hence, the ratio of these side will be equal

$$\frac{16}{20} = \frac{8}{PR} \Rightarrow PR = 10\,cm$$

Similarly, in the case of $\angle\gamma$ & $\angle\alpha$,

$$\frac{20}{x+16} = \frac{16}{20} \Rightarrow x = 9\,cm$$

Perimeter of $\Delta RSQ = PR + RS + PS$

$= 10 + 8 + 9 = 27$

II Rule:-

$\Delta PQR \sim \Delta RSQ$

$$\frac{P_1}{P_2} = \frac{20}{16}$$

$$\frac{P_1}{44} = \frac{5}{4}$$

$P_1 = 55$

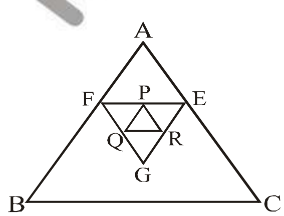
Perimeter of $\Delta PQR = 55$

Side $PR + PS = 55 - 16 - 20 = 19$

Perimeter of $\Delta PRS = 19 + 8 = 27$

Ex.50 In a ΔABC, F and E are the mid point of AB and AC respectively. G is the centroid of ΔABC. If P, Q and R are the mid points of EF, FG & GE respectively. Find the ratio of area of ΔPQR to area of ΔABC.

Sol.



From Example (2)

$$\frac{\text{area of }\Delta EFG}{\text{area of }\Delta ABC} = \frac{1}{12}$$

P, Q, R are mid-points of sides of triangle EFG then,

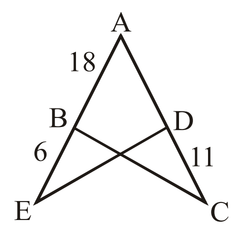
$$\frac{\text{area of }\Delta PQR}{\text{area of }\Delta EFG} = \frac{1}{4}$$

$$\text{area of }\Delta PQR = \frac{1}{4} \times \text{area of }\Delta EFG$$

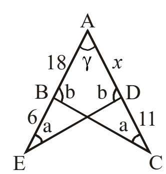
$$= \frac{1}{4} \times \frac{1}{12} \times \text{area of }\Delta ABC$$

$$\frac{\text{area of }\Delta PQR}{\text{area of }\Delta ABC} = \frac{1}{48}$$

Ex.51 In the below figure, AB = 18 cm, BE = 6 cm and CD = 11 cm. If $\angle AED = \angle ACB$, then, find the length of side AD (in cm).



Sol.



In ΔAED & ΔACB,

$\angle AED = \angle ACB = \alpha$

$\angle A = \angle A$ (common in both triangle)

then,, remaining third angle will be equal.

$\angle ADE = \angle ABC = \beta$

then, both triangle are similar hence ratio of opposite side of same corresponding angle will be equal.

$$\Rightarrow \frac{x}{18} = \frac{24}{x+11}$$

$x^2 + 11x = 18 \times 24$

$x^2 + 11x - 18 \times 24 = 0$
$x^2 + 27x - 16x - 18 \times 24 = 0$
$x(x+27) - 16(x+27) = 0$
$(x+27)(x-16) = 0$
$x = 16$ cm

Types of triangle

Based upon sides	Based upon angles
1. Scelene triangle	1. Acute angle triangle
2. Isosceles triangle	2. Obtuse angle triangle
3. Equilateral triangle	3. Right angle triangle

Property-22

Scelene triangle : no sides and no angles are equal.

Ex.52 ABC is a triangle, D is the point on AB. Such that AD : DB = 1 : 3, E is point on AC such that AE : EC = 3: 1, O is the midpoint of DE. Find the area of triangle BOC if area of triangle ABC = 1

Sol. In ΔOAC, ratio of base of two triangle OCE and OAE is 1 : 3 then, area will be above in this ratio 1 : 3.

let area of $\Delta OCE = K(\text{let})$

then, area of $\Delta OAE = 3K$

O is the mid point of DE, it means AO is median of ΔADE then, area of ΔADO = area of $\Delta AOE = 3K$

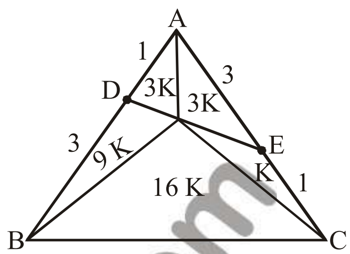
AD : DB = 1 : 3 So area of ΔAOD and ΔBOD will be in the ratio of 1 : 3

area of $\Delta BOD = 3 \times 3K = 9K$

$$\frac{\text{area of } \Delta ADE}{\text{area of } \Delta ABC} = \frac{\frac{1}{2} \times 1 \times 3 \times \sin A}{\frac{1}{2} \times 4 \times 4 \times \sin A} = \frac{3}{16} = \frac{6K}{32K}$$

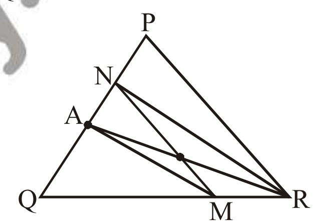
then,, area of ΔBOC = area of ΔABC – area of ΔAOB – area of $\Delta AOC = 32K - (9K + 3K) - (3K + K) = 16K$

it means area of ΔBOC is half of area of ΔABC.



Ex.53 In a triangle PQR, A is the mid-point of side PQ. There is any point M on side QR. A line RN is drawn parallel to AM which intersects PQ at N. If area of Δ PQR is 2 square unit then, find the area of Δ NQM.

Sol.



$\because$ A is the mid-point of PQ, then, AR is the median of ΔPQR.

$\therefore$ Area of Δ AQR = Area of Δ APR = $\frac{1}{2}$ area of Δ PQR ...(i)

$\because$ AM || RN

$\therefore$ Area of Δ AMR = Area of Δ AMN

Area of Δ AQR = Area of Δ AQM + Area of Δ AMR

$\Rightarrow$ Area of Δ AQM + Area of Δ AMN

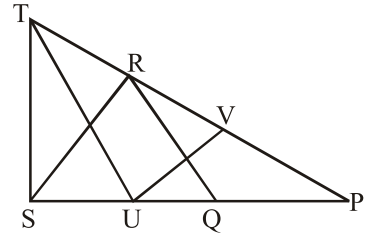
$\Rightarrow$ Area of Δ AQR = Area of Δ NQM

From Eqn. (i)

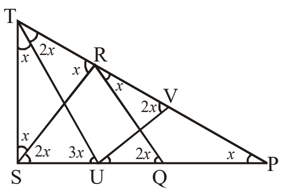
$\frac{1}{2}$ area of Δ PQR = Area of Δ NQM

Area of Δ NQM = $\frac{1}{2} \times 2 = 1$

Ex.54 In the below figure, PQ = QR = RS = ST = TU = UV = VP then $\angle SPT$ is approximately.



Sol :



$PQ = QR = RS = ST = TU = UV = VP$

Let $\angle QPR = x$

$\Rightarrow QRP = x [\because PQ = QR]$

and $\angle VUP = x [\because UV = VP]$

$\angle TVU = \angle VUP + \angle VPU = x + x = 2x$ ...(1)

$RQS = \angle QRP + \angle QPR = x + x = 2x$ ...(2)

(Since the exterior angle of a triangle at a vertex is equal to the sum of the two opposite interior angles)

$\Rightarrow \angle UTV = \angle UVT = 2x [\therefore TU = UV]$ ...(3)

$\Rightarrow \angle RSQ = \angle RQS = 2x [\therefore QR = RS]$ ...(4)

$\angle TUS = \angle UPT + \angle UTP = x + 2x = 3x$ ...(5)

(Since the exterior angle of a triangle at a vertex is equal to the sum of the two opposite interior angles)

$\angle TSU = \angle TUS = 3x [\therefore ST = TU]$ ...(6)

$\angle TRS = \angle RSP + \angle RPS = 2x + x = 3x$

But, $\angle TRS = \angle RTS$, because SR = TS.

$\therefore \angle RTS = 3x$ ...(7)

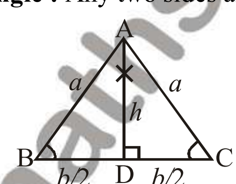
Hence, $\angle PTS + \angle TPS + \angle PST = 3x + 2x + 3x = 7x = 180°$

$3x + 2x + 3x = 180°$

$\therefore x = \frac{180°}{7} = 25.7° \simeq 26°$

Property-23

Isosceles triangle : Any two sides are equal.



AB = AC

then,, $\angle C = \angle B$

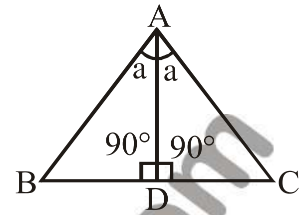
$AD \perp BC$

then, BD = CD (AD = angle bisector)

$$AD = \sqrt{a^2 - \frac{b^2}{4}} = \sqrt{\frac{4a^2 - b^2}{4}}$$

$$\text{Area} = \frac{1}{2} \times b \times \frac{\sqrt{4a^2 - b^2}}{2} = \frac{b}{4}\sqrt{4a^2 - b^2}$$

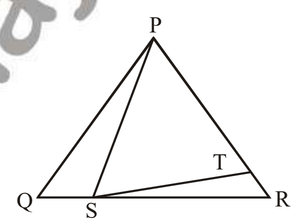
In Isosceles Triangle



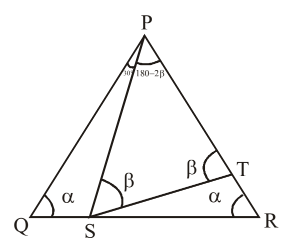
$ABD \cong ADC$

then,, BD = DC

EX. 55 In the figure shown, PQR is an isosceles triangle with PQ = PR, S is a point on QR such that PS = PT, Also, $\angle QPS = 30°$. Find $\angle RST$



Sol.



Let $\angle PST = \angle PTS = \beta$ & $\angle Q = \angle R = \alpha$

$\angle P = 30° + 180° - 2\beta$

in ΔPQR,

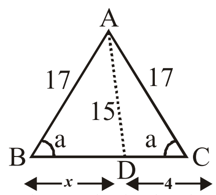
$(30° + 180° - 2\beta) + 2\alpha = 180°$

$\beta - \alpha = 15°$,

As $\angle STP$ is the external angle of $\Delta RST \Rightarrow \beta - \alpha = \angle RST = 15°$

Ex.56 In a triangle ABC, AB = AC. D is the any point on BC. Find the length of BD If AB = 17 cm, AD = 15 cm, CD = 4 cm.

Sol:



Applying cosine rule in both triangle ABD & triangle ACD.

$$\cos\alpha = \frac{17^2 + 4^2 - 15^2}{2 \times 17 \times 4} = \frac{17^2 + x^2 - 15^2}{2 \times 17 \times x}$$

$$\frac{80}{4} = \frac{64 + x^2}{x}$$

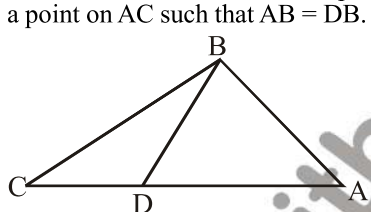
$$x^2 - 20x + 64 = 0$$

$$(x - 16)(x - 4) = 0$$

$$BD = x = 16 \text{ cm}$$

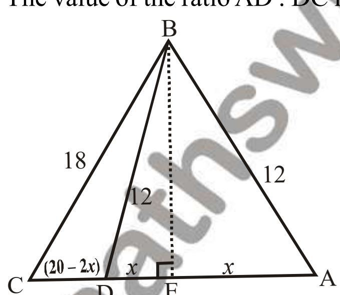
$$(x \neq 4)$$

Ex.57 In triangle ABC the length of the sides AB, BC, and AC are 12, 18 and 20 units, respectively. D is a point on AC such that AB = DB.



The value of the ratio AD : DC is

Sol.



Let $BE \perp AC$

As ABD is isosceles $\triangle$

$\therefore$ DE = EA = x (say)

In $\triangle$ BDE $\rightarrow$ $BE^2 = 144 - x^2$

In $\triangle$ BCE $\rightarrow$ $BE^2 = 324 - (20 - 2x + x)^2$

Comparing both the equation's

$144 - x^2 = 324 - (20 - x)^2$

$\Rightarrow \quad x = 5.5$

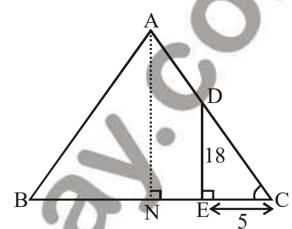
So AD : DC = 11 : 9

Ex.58 In a ΔABC, D and E are two points on AC & BC such that DE is perpendicular to BC and DE = 18 cm, CE = 5 cm & $\tan\angle B = 3.6$. Find the ratio AC to CD.

(a) $\frac{BC}{2CE}$ (b) $\frac{CE}{2BC}$

(c) $\frac{2CE}{BC}$ (d) $\frac{2BC}{CE}$

Sol.



$\tan\angle B = 3.6$

In ΔCDE

$$\tan\angle C = \frac{18}{5} = 3.6 = \tan\angle B$$

Hence, $\angle B = \angle C$

then, ABC will be an isosceles triangle. We draw a perpendicular AN to side BC. In an isosceles triangle perpendicular will divide side BC in two equal parts.

$$\left(\therefore BN = NC = \frac{BC}{2}\right)$$

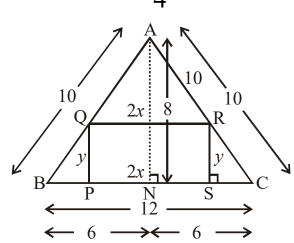
ΔANC & ΔCDE are similar.

$$\frac{AC}{CD} = \frac{NC}{CE} = \frac{BC}{2CE}$$

$$\frac{AC}{CD} = \frac{BC}{2CE}$$

Ex.59 In an isosceles triangle with base BC = 12 cm and AB = AC = 10 cm. There is a rectangle PQRS inside the triangle whose base PS lies on BC such that PQ = SR = y and QR = PS = $2x$. Find the value of $x + \frac{3y}{4}$.

Sol.



ABC is an isosceles triangle and $AN \perp BC$, then, $BN = NC = 6$ cm

$$AN = \sqrt{AC^2 - NC^2} = \sqrt{100-36} = 8$$
$$PN = NS = x$$

$$SC = BP = \frac{12-2x}{2} = 6 - x$$

$AN \perp BC$ & $RS \perp BC$ hence, ΔANC & ΔRSC are similar triangles.

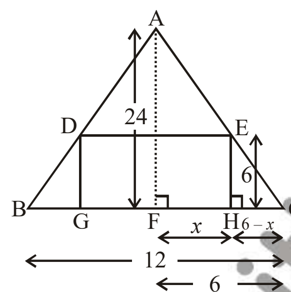
In ΔANC & ΔRSC,

$$\frac{AN}{RS} = \frac{NC}{SC} \Rightarrow \frac{8}{y} = \frac{6}{6-x}$$

$$6 - x = \frac{3y}{4} \Rightarrow x + \frac{3y}{4} = 6$$

Ex.60 ABC is a triangle with base $BC = 12$. There is a rectangle GHED inside the triangle such that $HE = 6$ cm, F is the midpoint of BC. If $AF = 24$ cm, then, find area of rectangle.

Sol.



ΔAFC & ΔEHC are similar.

$$\frac{AF}{EH} = \frac{FC}{HC} \Rightarrow \frac{24}{6} = \frac{6}{6-x}$$

$$\Rightarrow 6 - x = \frac{3}{2}$$
$$\Rightarrow x = 4.5$$

$GH = 2 \times x = 2 \times 4.5 = 9$ and $HE = 6$

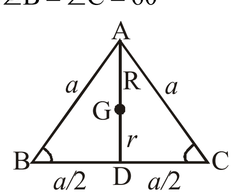
Area $= GH \times HE = 9 \times 6 = 54$

Property-24

Equilateral triangle : All sides and angles are equal.

$\Rightarrow AB = BC = CA = a$

$\Rightarrow \angle A = \angle B = \angle C = 60°$



$$AD = h = \sqrt{a^2 - \frac{a^2}{4}} = \frac{\sqrt{3}}{2}a$$

(i) all centre (orthocentre, circumecentre, incentre & centroid) lies on a same point.

(ii) all medians = all altitudes = all perpendicular bisector = all angle bisector $= \frac{\sqrt{3}}{2}a$

(iii) Circumradius $R = \frac{a}{2\sin A} = \frac{a}{2\sin 60°} \Rightarrow \boxed{R = \frac{a}{\sqrt{3}}}$

(iv) Inradius $r = \frac{\Delta}{s} \Rightarrow \boxed{r = \frac{a}{2\sqrt{3}}}$

(v) $\frac{R}{r} = \frac{2}{1}$

(vi) $\frac{\pi R^2}{\pi r^2} = \frac{\text{area of circumcircle}}{\text{area of incircle}} = \frac{4}{1}$

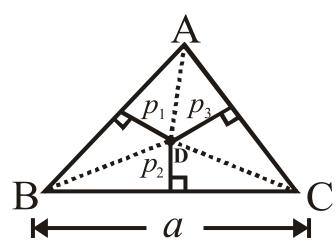
(vii) Perimeter $p = 3a$ then, semi-perimeter $s = \frac{3a}{2}$

(viii) $\boxed{\text{area of } \Delta = \frac{\sqrt{3}}{4}a^2}$ $\left(\therefore \Delta = \frac{1}{2}a \times a \times \sin 60°\right)$

(ix) D is a point inside the equilateral triangle ABC. Three perpendiculars of length p_1, p_2 & p_3 are drawn on sides AB, BC & AC respectively. Hence, area of triangle ABC will be equal to the sum of all three triangles ADB, BDC & ADC.

$$\Delta = \frac{\sqrt{3}}{4}a^2 = \frac{1}{2}ap_1 + \frac{1}{2}ap_2 + \frac{1}{2}ap_3$$

$$\frac{\sqrt{3}}{4}a^2 = \frac{1}{2}a\left[p_1 + p_2 + p_3\right]$$



$$\boxed{a = \frac{2}{\sqrt{3}}\left[p_1 + p_2 + p_3\right]}$$

Ex.61 In an equilateral triangle, there is a point inside the triangle whose perpendicular distance from each side is $2\sqrt{3}$, $3\sqrt{3}$ & $5\sqrt{3}$ respectively. Find the area of triangle.

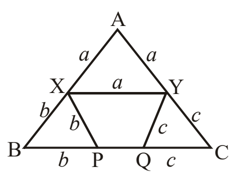
Sol. $a = \frac{2}{\sqrt{3}}\left(2\sqrt{3} + 3\sqrt{3} + 5\sqrt{3}\right) = \frac{2}{\sqrt{3}} \times 10\sqrt{3}$

$= 20$

Area $= \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4} \times 20 \times 20 = 100\sqrt{3}$

Ex. 62 ABC is an equilateral triangle of side 30cm. XY is parallel to BC, XP is parallel to AC and YQ is parallel to AB. If XY + XP + YQ = 40 cm, then, find the length of PQ.

Sol.



$\angle A = \angle B = \angle C = 60°$

$XY \parallel BC \Rightarrow \angle AXY = \angle ABC = 60°$

$\Rightarrow \angle AYX = \angle ABC = 60°$

Hence ,AXY is an equilateral triangle.

$\Rightarrow$ AX = XY = AY = a (Let)

Similarly, BXP & CYQ are equilateral triangle.

$\Rightarrow$ BX = BP = XP = b (Let)

$\Rightarrow$ CY = YQ = QC = c (Let)

Perimeter of triangle = 3 × 30 = 90

$\Rightarrow$ AB + BC + CA = 90

$\Rightarrow (a + b) + (b + PQ + c) + (a + c) = 90$

$\Rightarrow 2(a + b + c) + PQ = 90$ ($\because$ XY + XP + YQ = 40)

$\Rightarrow 2 \times 40 + PQ = 90$ ($\because a + b + c = 40$)

PQ = 10 cm

Property-25

Acute angle triangle: all angles < 90°

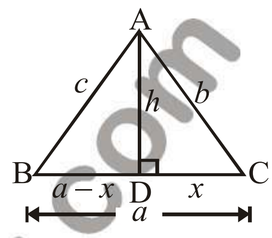
In ΔABD & ΔACD, applying Pythagoras theorem.

$$h^2 = c^2 - (a - x)^2 = b^2 - x^2$$

$$c^2 - a^2 - x^2 + 2ax = b^2 - x^2$$

$$c^2 = a^2 + b^2 - 2ax$$

Hence, we can say c^2 will be less than sum of a^2 and b^2. In any acute angle triangle square of largest side is always less than sum of two smaller sides.



Point 1. $c^2 < a^2 + b^2$ or $b^2 < a^2 + c^2$ or $a^2 < b^2 + c^2$

Point 2. In acute angle triangle all centres (centroid, orthocentre, circumcentre and incentre) lies inside the triangle.

e.g. What will be the types of triangle if sides of the triangle are 6, 7, 8 ?

Sol. $(8^2) < (6^2) + (7^2)$

$64 < 36 + 49$

this is acute angle triangle.

Property-26

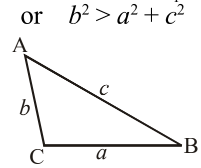
Obtuse angle triangle : One angle > 90°

(Only one angle is greater than 90°)

In any obtuse angle triangle square of largest side is always greater than sum of two smaller sides.

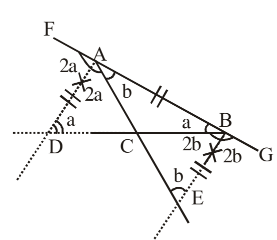
$(\text{larger side})^2 > (\text{smaller side}_1)^2 + (\text{smaller side}_2)^2$

$c^2 > a^2 + b^2$ or $b^2 > a^2 + c^2$ or $a^2 > b^2 + c^2$



Ex.63 In ΔABC, $\angle$C in an obtuse angle. The bisectors of the exterior angles at A and B meet BC and AC produced at D and E respectively. If AB = AD = BE, then, $\angle$ACB = ?

Sol:



AB = AD $\Rightarrow \angle BDA = \angle ABD = \alpha$(let)

AB = BE $\Rightarrow \angle BAC = \angle AEB = \beta$(let)

$\angle DAF = 2\alpha$ & $\angle EBG = 2\beta$ (both are exterior angle of triangle ADB & ABE respectively)

$\angle DAC = 2\alpha$ & $\angle EBC = 2\beta$

(AD and BE is angle bisector)

$\angle ACB$ is exterior angle of ΔADC and ΔBCE.

then, $\angle ACB = 3\alpha = 3\beta \Rightarrow \alpha = \beta$

now in ΔACB

$$3\alpha + \alpha + \alpha = 180°$$

$$\alpha = 180°$$

$$\alpha = 36°$$

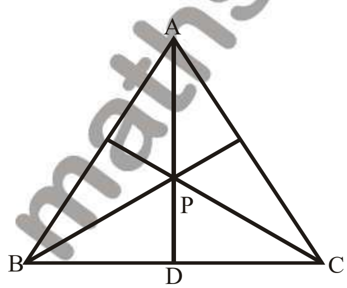
$$\angle ACB = 3\alpha = 108°$$

Ex.64 Let ABC be an equilateral triangle with sides x. Let P be the point of intersection of the three angle bisectors. What is the length of AP?

(a) $\frac{4x\sqrt{3}}{3}$ (b) $\frac{x\sqrt{3}}{6}$

(c) $\frac{5\sqrt{3}}{6}$ (d) $\frac{2x\sqrt{3}}{6}$

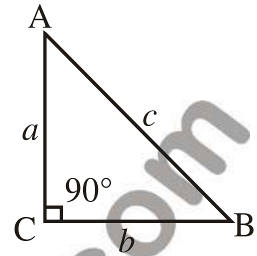
Sol. In an equilateral triangle , medians, angle bisectors, and altitudes are the something.



$$PA = \frac{2}{3}\, AD = \frac{2}{3} \times \frac{\sqrt{3}}{2} x = \frac{2\sqrt{3}x}{6}$$

Property-27

Right angle triangle : One angle is equal to 90°

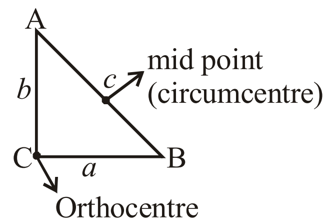


$\angle C = 90°$ then, $AB^2 = AC^2 + BC^2$

$c^2 = a^2 + b^2$ (This is Pythagoras theorem)

(i) $\Delta = \text{Area} = \frac{1}{2}ab$

(ii) $R = \frac{abc}{4\Delta} = \frac{abc}{4.\frac{1}{2}ab} = \frac{c}{2} \Rightarrow \boxed{R = \frac{c}{2}}$



(iii) $r = \frac{\Delta}{s} = \frac{\frac{1}{2}ab}{\frac{a+b+c}{2}} = \boxed{\frac{ab}{a+b+c}}$

$$r = \frac{ab}{(a+b)+c} \times \frac{a+b-c}{(a+b)-c} = \frac{ab(a+b-c)}{(a+b)^2 - c^2}$$

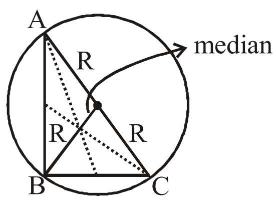
$$= \frac{ab(a+b-c)}{a^2+b^2+2ab-c^2} = \frac{ab(a+b-c)}{2ab} = \frac{a+b-c}{2}$$

(iv) $\boxed{r = \frac{a+b-c}{2}}$

$$r = \frac{a+b}{2} - \frac{c}{2} = \frac{a+b}{2} - R$$

$$r = \frac{a+b}{2} - R$$

(v) $\Rightarrow$ $\boxed{2(r+R)=a+b}$



In right angle triangle shortest median = R (Circumradius)
Area of triangle ABC

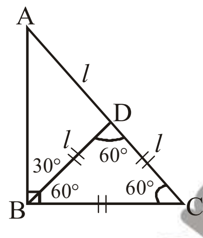
$$\Delta = r.s = \left(\frac{a+b-c}{2}\right).s = \left(\frac{a+b+c-2c}{2}\right).s$$

$$= \left(\frac{a+b+c}{2} - c\right).s = (s-c).s$$

(vi) $\boxed{\Delta = (s-c).s}$ where c is hypotenuse or twice of shortest median in the triangle.

Ex.65 In a right angle triangle ABC (right angled at B), median BD of length l divides angle B in the ratio of 2 : 1. Find area of Δ ABC.

Sol.



$BD = CD = AD = l$

then, $\angle DBC = \angle DCB = 60° \Rightarrow (\angle BDC = 60°)$

area of equilateral $\Delta BDC = \frac{\sqrt{3}}{4} \times l^2$

Area of $\Delta ABC = 2 \times$ Area of BDC

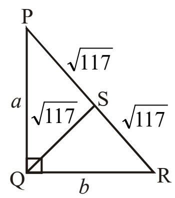
$= \frac{\sqrt{3}}{4} l^2 \times 2 = \frac{\sqrt{3}}{2} l^2$

(BD is the median and will divide the triangle in two equal area parts.)

वक्त बदलता है,
फिर बदलेगा, सारा जहां अपना होगा।

Ex.66 In a triangle PQR, $\angle Q = 90°$, S is the midpoint of PR, $QS = \sqrt{117}$. Sum of sides of PQ & QR is 30 cm. Find the area of triangle PQR.

Sol.



$QS = \sqrt{117}$ then, $PS = QR = \sqrt{117}$

$PQ + QR = 30$

$a + b = 30$

$a^2 + b^2 = \left(2\sqrt{117}\right)^2 = 468$

$(a+b)^2 = a^2 + b^2 + 2ab$

$900 = 468 + 2ab \Rightarrow 2ab = 432$

$ab = 216$

$\text{Area} = \frac{1}{2}ab = \frac{1}{2} \times 216 = 108 \text{ cm}^2$

Method 2 : Here

$$s = \frac{PQ + QR + PR}{2} = \frac{30 + 2\sqrt{117}}{2} = 15 + \sqrt{117}$$

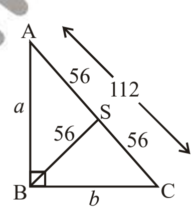
$$r = \frac{PQ + QR - PR}{2} = \frac{30 - 2\sqrt{117}}{2} = 15 - \sqrt{117}$$

$$\Delta = r.s = \left(15 + \sqrt{117}\right).\left(15 - \sqrt{117}\right) = 15^2 - \left(\sqrt{117}\right)^2$$

$= 225 - 117 = 108 \text{ cm}^2$

Ex. 67 If the semiperimeter of a right angle triangle is 120 cm and length of smallest median is 56 cm then, find the area of triangle.

Sol.



$a^2 + b^2 = (112)^2$

$s = \frac{a+b+c}{2}$

$120 = \frac{a+b+112}{2} \Rightarrow a + b + 112 = 240$

$a + b = 128$

$(a+b)^2 = a^2 + b^2 + 2ab \Rightarrow (128)^2 = (112)^2 + 2ab$

$2ab = (128)^2 - (112)^2$

$ab = \frac{240 \times 16}{2} = 1920$

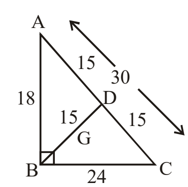
$\text{Area} = \frac{1}{2}ab = \frac{1}{2} \times 1920 = 960 \text{ cm}^2$

Method 2 : Here, $s = 120$ and $c = 112$ cm

$\Delta = s(s-c) = 120(120 - 112) = 960 \text{ cm}^2$

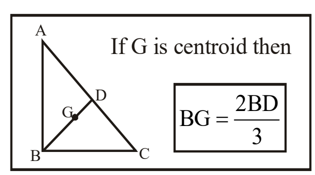
Ex.68 In a triangle ABC, right angled at B. AB = 18 cm, BC = 24 cm. Find the length of BG, if G is the centroid.

Sol.



$AC^2 = (18)^2 + (24)^2$

$AC = 30$ cm



then, $AD = CD = 15$ cm   then, $BD = 15$ cm

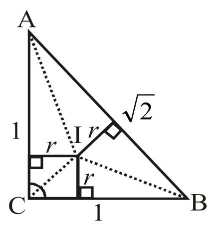
centroid divide median into 2 : 1

then, $BG : GD = 10 : 5$

$BG = 10$ cm

Ex.69 In an isosceles right angle triangle ABC, I is incenter of triangle then, find the ratio area of ΔAIB, ΔBIC and ΔAIC

Sol.



Area of $\Delta AIB = \frac{1}{2}\times 1\times r = \frac{1}{2}r$

Area of $\Delta BIC = \frac{1}{2}\times 1\times r = \frac{1}{2}r$

Area of $\Delta AIC = \frac{1}{2}\times\sqrt{2}\times r = \frac{\sqrt{2}}{2}r$

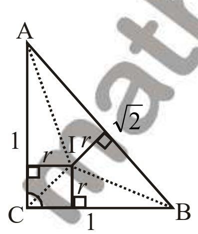
$\Delta AIB : \Delta BIC : \Delta AIC = \frac{1}{2}r : \frac{1}{2}r : \frac{\sqrt{2}}{2}r$

$= 1:1:\sqrt{2}$

We can see here height of all three triangles are same and equal to radius then, area will be divided into the ratio of their base which is $1:1:\sqrt{2}$

Ex.70 In an isosceles right angle triangle ABC right angled at C, I is the incetre then, find the ratio of area of ΔAIB to ΔABC.

Sol.



area of $\Delta AIB = \frac{1}{2}\times AB\times r = \frac{1}{2}\times\sqrt{2}\times r$

area of $\Delta ABC = \frac{1}{2}\times AB\times AC = \frac{1}{2}\times 1\times 1$

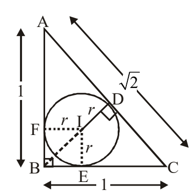
$r = \frac{a+b-c}{2} = \frac{1+1-\sqrt{2}}{2} = \frac{2-\sqrt{2}}{2}$

$\frac{\text{area of }\Delta AIB}{\text{area of }\Delta ABC} = \frac{\frac{1}{2}\times\sqrt{2}\times r}{\frac{1}{2}\times 1\times 1} = \frac{\sqrt{2}\left(2-\sqrt{2}\right)}{2}$

$= \frac{2-\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}-1}{1}$

Ex.71 ABC is a right angle triangle, (right angled at B) incircle touches the sides AB, BC & AC at F, E & D respectively. If BD is perpendicular to AC then, find the ratio of AF to FB.

Sol. I is incentre then, BI will be angle bisector and ID will be perpendiculatr to AC, but it is given that $BD \perp AC$. So, line BID is a straight line and this is angle bisector and perpendiculr at also to opposite side. Hence, ABC will be Isosceles right angle triangle.



$IF \perp AB$ & $IE \perp BC$ then, $IF = IE = BF = r$

$r = \frac{a+b-c}{2} = \frac{1+1-\sqrt{2}}{2} = \frac{2-\sqrt{2}}{2}$

$\frac{AF}{FB} = \frac{1-r}{r}$

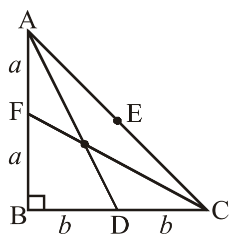
$\frac{AF}{FB} = \frac{1-\frac{2-\sqrt{2}}{2}}{\frac{2-\sqrt{2}}{2}} = \frac{\sqrt{2}}{2-\sqrt{2}} = \frac{1}{\sqrt{2}-1}$

$$\frac{AF}{FB} = \frac{\sqrt{2}+1}{1}$$

Property-28

If the triangle formed by the medians of a right angle triangle is also proof a right angle triangle then, the ratio of the sides of original triangle will be in the ratio $1 : \sqrt{2} : \sqrt{3}$. kkmll,,,

Property -29



$AD^2 = b^2 + 4a^2$

$CF^2 = a^2 + 4b^2$

then, $AD^2 + CF^2 = 5(a^2 + b^2)$

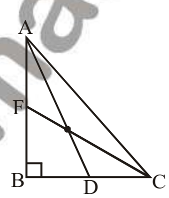
$$= \frac{5}{4}\left(4a^2 + 4b^2\right) = \frac{5}{4}AC^2$$

$$\left[\because\ 4a^2 + 4b^2 = AC^2\right]$$

$$\boxed{4\left(AD^2 + CF^2\right) = 5AC^2}$$

Ex.72 In a triangle ABC, right angle at B, AC = 5 cm, median AD = $\frac{3\sqrt{5}}{2}$. Find the length of median CF.

Sol.



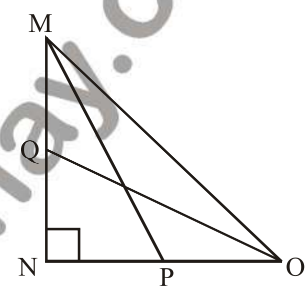
$$4\left[\left(\frac{3\sqrt{5}}{2}\right)^2 + CF^2\right] = 5\times(5)^2$$

$$4\times\frac{45}{4} + 4CF^2 = 125$$

$4CF^2 = 125 - 45$

$$CF^2 = \frac{80}{4} = 20 \quad \Rightarrow \quad CF = 2\sqrt{5}$$

Ex.73 Find the sum of squares of the medians MP and OQ drawn from the two acute angled verticles of a right angled triangle MNO. The longest side of ΔMNO is 20 cm.

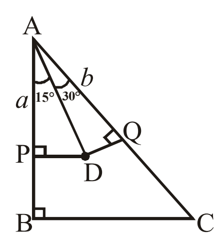


Sol. $MP^2 + QO^2 = \frac{5}{4}\left(MO^2\right) = \frac{5}{4}\times 20\times 20 = 500$

Ex.74 In an isosceles right angle triangle ABC (right angled at B), perpendiculars are drawn from a point D inside the triangle on side AB and AC meet at P & Q respectively. If AP = a unit, AQ = b unit and ∠ PAD = 15°, then, find the value of sin75°.

(a) $\frac{2a}{\sqrt{3}b}$ (b) $\frac{\sqrt{3}a}{2b}$

(c) $\frac{a\sqrt{3}}{b}$ (d) $\frac{b}{2a\sqrt{3}}$



Sol: $AB = BC \Rightarrow \angle BAC = \angle BCA = 45°$

$\angle PAD = 15° \Rightarrow \angle DAQ = 30°$ & $\angle PDA = 75°$

In ΔDAQ,

$$\cos 30° = \frac{b}{AD} \Rightarrow AD = \frac{2b}{\sqrt{3}}$$

In ΔPAD,

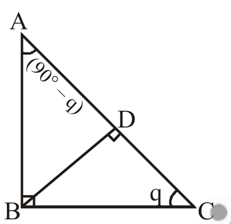
$$\sin 75° = \frac{a}{AD} = \frac{a}{\frac{2b}{\sqrt{3}}} = \frac{\sqrt{3}a}{2b}$$

Propery-30

Similarity in right angle triangle

We will never use similarity in right angle triangle, it creates confusion.

We will use trigonometric ratio in right triangle. Let a right angle triangle (right angled at B), a perpendicular BD is drawn to hypotenuse AC.



(1). $BD^2 = CD.AD$

Proof –

$\angle B = 90°$ & $BD \perp AC$

In ΔBDC,

$$\tan\theta = \frac{BD}{CD} \quad \ldots(i)$$

In ΔADB,

$$\tan(90° - \theta) = \cot\theta = \frac{BD}{AD} \quad \ldots(ii)$$

Multiply (i) and (ii) :-

$$\tan\theta.\cot\theta = 1 = \frac{BD}{CD}.\frac{BD}{AD}$$

$$BD^2 = AD.CD$$

(2). $BD = \frac{AB.BC}{AC}$

Proof –

$\angle B = 90°$ & $BD \perp AC$

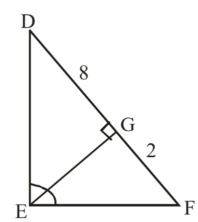
We equate area of triangle to take BC & AC as base and AB & BD as perpendicular respectively.

$$\text{Area of } \Delta ABC = \frac{1}{2}.AB.BC = \frac{1}{2}.BD.AC$$

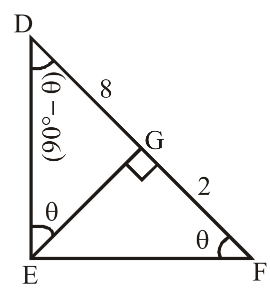
$$\Rightarrow BD = \frac{AB.BC}{AC}$$

So, we can find length of any side after taking cos, sin & tan ratio.

Ex.75 Δ DEF is right-angled triangles (right angled at E). $EG \perp DF$. If DG = 8 cm and GF = 2cm, then, find the ratio of DE : EF.



Sol.



$$\frac{\text{Area of } \Delta DEG}{\text{Area of } \Delta EGF} = \frac{DG}{EF} = \frac{8}{2} = 4$$

Δ DEG and Δ EFG are similar

$$\frac{\text{Area of } \Delta DEG}{\text{Area of } \Delta EFQ} = \frac{DE^2}{EF^2} = 4 \Rightarrow \frac{DE}{EF} = 2$$

Method-2

In ΔEGF, $\tan\theta = \frac{EG}{2} \Rightarrow EG = 2\tan\theta$ ...(i)

In ΔDGE, $\tan(90^\circ - \theta) = \cot\theta =$

$\frac{EG}{8} \Rightarrow EG = 8\cot\theta$ ...(ii)

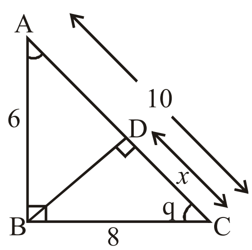
from equation (i) & (ii)

$2\tan\theta = 8\cot\theta \Rightarrow \tan^2\theta = 4$

$\tan\theta = 2 = \frac{DE}{EF}$ (in ΔDEF)

Ex.76 In a triangle ABC, $\angle B = 90°$, AB = 6 cm, BC = 8 cm. BD is perpendicular to AC, then, find the length of CD, AD and BD.

Sol.



$AC = \sqrt{AB^2 + BC^2} = 10$ cm

In ΔBDC, $\cos\theta = \frac{x}{8}$

In ΔABC, $\cos\theta = \frac{8}{10}$

Equate $\cos\theta \Rightarrow \frac{x}{8} = \frac{8}{10} \Rightarrow x = 6.4$ cm

CD = x = 6.4 cm

AD = 10 – 6.4 = 3.6 cm

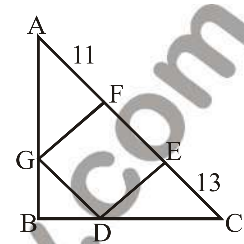
Area of Δ ABC = $\frac{1}{2} \times AB \times BC = \frac{1}{2} \times BD \times AC$

$AB \times BC = BD \times AC$

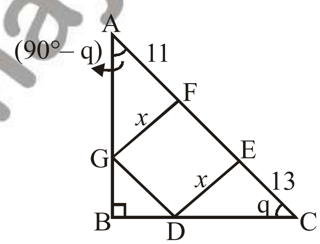
$6 \times 8 = BD \times 10$

$\frac{6\times8}{10} = BD \Rightarrow BD = \frac{6\times8}{10} = 4.8$ cm

Ex.77 In a right angle triangle ABC, (right angled at B) there are two point on side AC such that AF = 11 cm and EC = 13 cm. There are two points G and D on sides AB and BC respectively. Find the area of square DEFG.



Sol.



In ΔDEC, $\Rightarrow \tan\theta = \frac{x}{13}$ ...(i)

In ΔAGF, $\Rightarrow \tan(90° - \theta) = \cot\theta = \frac{x}{11}$ ...(ii)

Multiply Eqn. (i) & (ii)

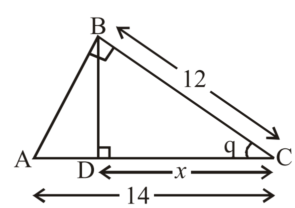
$\tan\theta \cdot \cot\theta = 1 = \frac{x}{13} \times \frac{x}{11}$

Area of square DEFG

$x^2 = 13 \times 11 = 143$ sq. cm

Ex.78 ΔABC is a right angle triangle (right angled at B), BD is perpendicular to AC. If AC = 14 cm, BC = 12cm, find the length of CD.

Sol.



In ΔBDC, $\cos\theta = \frac{x}{12}$

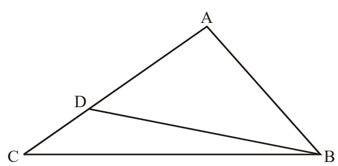
$$\text{In } \Delta ABC, \cos\theta = \frac{12}{14}$$

$$\text{Equate } \cos\theta \Rightarrow \frac{x}{12} = \frac{12}{14}$$

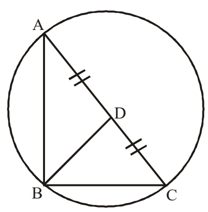
$$x = \frac{12\times12}{14} = \frac{72}{7}$$

$$CD = x = 10\frac{2}{7}\text{ cm}$$

Ex.79 In ΔABC, $\angle B$ is a right angle, AC = 6 cm, and D is the midpoint of AC. The length of BD is .



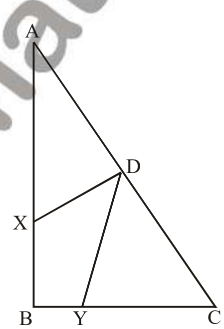
Sol.



If we form a circle, taking AC is diameter. The circle will pass through vertex B and AD, DC & BD will become radius of the circle as shown in the figure.

$$\therefore\ BD = AD = DC = \frac{1}{2}\times 6 = 3\text{ cm.}$$

Ex.80 In right triangle ABC, AX = AD and CY = CD, as shown in the figure below. what is the measure of $\angle XDY$?



Sol. Let $\angle ADX = \angle AXD = \alpha$

$\Rightarrow \quad \angle XAD = 180° - 2\alpha$

Let $\angle CYD = \angle CDY = \beta$

$\Rightarrow \quad \angle DCY = 180° - 2\beta$

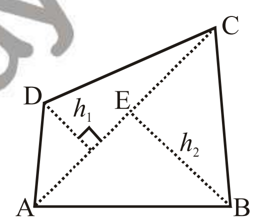
$\angle XAD + \angle DCY = 90°$

$180° - 2\alpha + 180° - 2\beta = 90°$

$\Rightarrow \quad \alpha + \beta = 135°$

$\Rightarrow \quad \angle XDY = 180° - (\alpha + \beta) = 45°$

Quadrilateral



Property-31

$\angle A + \angle B + \angle C + \angle D = 360°$

Property-32

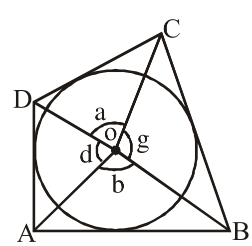
$$\text{Area of Quadrilateral} = \frac{1}{2}(h_1 + h_2).\ AC$$

Or,

Area of ABCD = Area of ACD + Area of ABC
(using Hero's Formula)

Property-33

If a circle touches all the sides of a quadrilateral ABCD

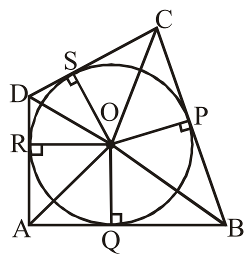


(i) AB + CD = AD + BC

(ii) If O is the centre of circle then,

$\alpha + \beta = 180°$ and $\gamma + \delta = 180°$

Proof-



I. Length of a tangent drawn from external point to the circle are equal.

$SC = PC$ …(i)

$QB = PB$ …(ii)

$QA = AR$ …(iii)

$DS = RD$ …(iv)

Adding all four equation,

$SC + QB + QA + DS = PC + PB + AR + RD$

$(QA + QB) + (SC + DS) = (PB + PC) + (AR + RD)$

$\Rightarrow AB + CD = AD + BC$

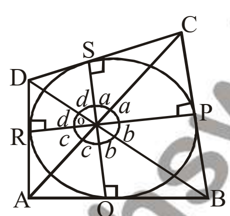
II. In ΔOCS & ΔOPC,

$OC = OC$ (Common)

$\angle OSC = \angle OPC = 90°$ (CP & CS are tangent).

$CS = CP$

ΔOCS & ΔOPC are congruent.



hence, $\angle SOC = \angle POC = a$ (let)

Similarly, $\angle POB = \angle QOB = b$ (let)

$\angle QOA = \angle ROA = c$ (let)

$\angle ROD = \angle SOD = d$ (let)

$a + a + b + b + c + c + d + d = 360°$

$2(a + b + c + d) = 360°$

$a + b + c + d = 180°$

$(a + d) + (b + c) = 180° \Rightarrow \alpha + \beta = 180°$

$(a + b) + (c + d) = 180° \Rightarrow \gamma + \delta = 180°$

Ex.81 If a circle touches all the sides of a quadrilateral ABCD, if AB = 7 cm, CD = 3 cm, BC = 8.5 cm. Find the length of AD.

Sol. $AB + CD = AD + BC$

$7 + 3 = AD + 8.5$

$AD = 1.5$ cm

Ex.82 A circle of centre O inscribed in a quadrilateral ABCD which touches all the sides of a quadrilateral. If $\angle AOB = 115°$. Find $\angle COD$.

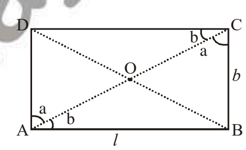
Sol. $\angle AOB + \angle COD = 180°$

$\angle COD = 180° - 115° = 65°$

Types of Quadrilateral :

Propety-34

(i) Rectangle -



I. Perimeter (P) = $2(l + b)$

II. Area = $l \times b$

III. $AB = CD = l$ & $AD = BC = b$

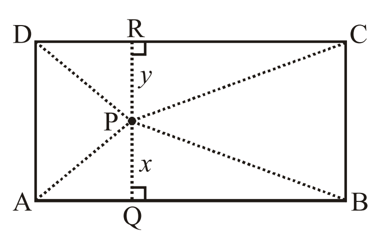
IV. $\angle A = \angle B = \angle C = \angle D = 90°$

V. Diagonals are equal and bisects each other but do not intersect at 90°. Diagonals do not bisect any vertex angle. $(\alpha \neq \beta)$. $AC = BD = \sqrt{l^2 + b^2}$

VI. Area of all four triangles $\Delta OAB, \Delta OBC, \Delta OCD$ & ΔOAD are equal.

VII. If P is a any point inside the rectangle then,

$AP^2 + PC^2 = BP^2 + PD^2$



Proof – In ΔAPB,

$PQ^2 = AP^2 - AQ^2 = PB^2 - QB^2$ …(i)

In ΔDPC,

$PR^2 = PC^2 - RC^2 = PD^2 - DR^2 \quad \ldots(ii)$

Adding Eqn. (i) & (ii)

$$AP^2 + \underbrace{PC^2 - RC^2}_{} - AQ^2 = PB^2 + \underbrace{PD^2 - DR^2}_{} - QB^2$$

$$\Downarrow \qquad \Downarrow$$

$$QB^2 \qquad DR^2$$

$$\boxed{AP^2 + PC^2 = PB^2 + PD^2}$$

Ex.83 There is a point P in a rectangle ABCD, such that PA = 4, PD = 5, PB = 8, find PC.

Sol: $AP^2 + PC^2 = PD^2 + PB^2$

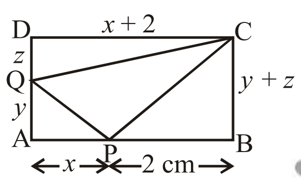
$16 + PC^2 = 25 + 64$

$PC^2 = 73$

$PC = \sqrt{73}$

Ex.84 ABCD is a rectangle. There are two points P & Q on side AB and AD such that area of triangles PAQ, CDQ & PBC are equal. If the length of BP is 2cm, find the length of AP.

Sol. Let, AP = x, AQ = y & QD = z, so BC = $y + z$ & CD = $x + 2$



Area of ΔPAQ = Area of ΔPBC = Area of ΔCDQ

$$\frac{1}{2}xy = \frac{1}{2}.2(y+z) = \frac{1}{2}z(x+2)$$

$xy = 2y + 2z = zx + 2z$

From last two relation ($2y + 2z = zx + 2z$)

$$\Rightarrow z = \frac{2y}{x}$$

From first & last relation $\{xy = z(x+2)\}$

Put the value of z in above relation

$$xy = \frac{2y}{x}(x+2) \Rightarrow x^2 = 2(x+2)$$

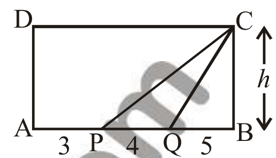
$x^2 - 2x - 4 = 0$

$$x = \frac{2 \pm \sqrt{4+16}}{2} = 1 + \sqrt{5}$$

$AP = 1 + \sqrt{5}$

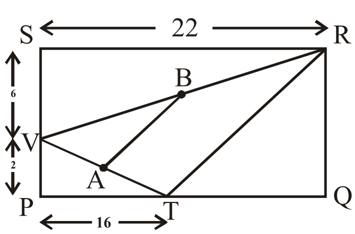
Ex. 85 ABCD is a rectangle. P & Q are two points on AB, such that AP : PQ : QB = 3 : 4 : 5. Find the ratio of area ΔPQC and rectangle ABCD.

Sol.



$$\frac{\text{Area of } \Delta PQC}{\text{Area of } \square ABCD} = \frac{\frac{1}{2} \times PQ \times h}{AB \times h} = \frac{PQ}{2AB} = \frac{4}{2 \times 12} = \frac{1}{6}$$

Ex.86 In the given figure, PQRS is a rectangle and VTR is a triangle whose vertices lie on the sides of PQRS. RS = 22, SV = 6, PT = 16 and VP = 2. Find the length of the line joining the mid points (A & B) to the side VT and VR.



Sol. TQ = 22 – 16 = 6 and RQ = 8

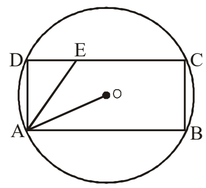
$TR^2 = 8^2 + 6^2$

TR = 10

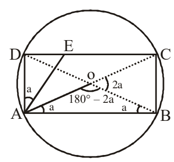
A & B are mid points of VT and VR.

As, TR = 10 then, AB = $\frac{1}{2} \times TR = 5$

Ex.87 In the below figure, rectangle ABCD is inscribed in the circle with centre at O. The length of side AB is greater than that of side BC. The ratio of the area of the circle to the area of the rectangle ABCD is $\pi : \sqrt{3}$. The line segment AE intersects CD at E such that $\angle OAB = \angle EAD$. What is the ratio AE : AD?



Sol.



$\angle EAD = \angle OAB = \alpha$

The area of (ΔAOD, ΔAOB, ΔOBC & ΔDOC) are equal.

Area of OAB $= \frac{1}{2}.r.r.\sin(180° - 2\alpha) = \frac{1}{2}r^2 \sin 2\alpha$

Area of rectangle $= \frac{1}{2}r^2 \sin 2\alpha \times 4$

$= 2r^2 \sin 2\alpha$

$\frac{\text{Area of circle}}{\text{Area of rectangle}} = \frac{\pi}{\sqrt{3}}$

$\frac{\pi r^2}{2 \times r^2 \times \sin 2\alpha} = \frac{\pi}{\sqrt{3}}$

$\sin 2\alpha = \frac{\sqrt{3}}{2} = \sin 60°$

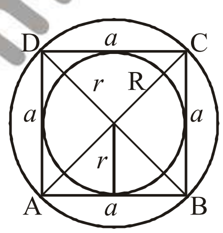
$2\alpha = 60° \quad \Rightarrow \quad \alpha = 30°$

$\cos 30° = \frac{AD}{AE} \quad \Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{AD}{AE}$ Sec30°

$\frac{AE}{AD} = \frac{2}{\sqrt{3}}$

Propety-35

(ii) Square :



I. $AB = BC = CD = DA = a$

II. $\angle A = \angle B = \angle C = \angle D = 90°$

III. Diagonals are equal and bisects each other at 90°

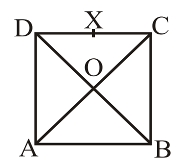
Length of diagonals $AC = BD = a\sqrt{2}$

IV. Inradius $r = \frac{a}{2}$

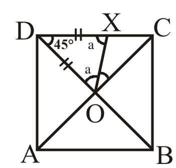
V. Circumradius $R = \frac{a\sqrt{2}}{2} = \frac{a}{\sqrt{2}}$

VI. Ratio of area of incircle to circumcircle $= \frac{\pi r^2}{\pi R^2} = \frac{1}{2}$

Ex.88 If ABCD is a square. X is a point on CD, such that DX = DO. Find $\frac{\angle DOX}{\angle XOC}$. O is the intersection point of diegonals.



Sol.



In ΔDXO $\quad 2\alpha + 45° = 180°$

$2\alpha = 135°$

$\alpha = 67.5$

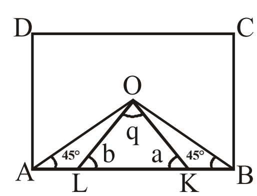
$\angle XOC = 90° - \alpha = 90° - 67.5° = 22.5°$

$\frac{\angle DOX}{\angle XOC} = \frac{67.5}{22.5} = \frac{3}{1}$

Ex.89 ABCD is square. K and L are two points on AB such that AO = AK and BO = BL and $\angle LOK = \theta$.

Find the value of $\tan\theta$.

Sol.



In ΔAOK, $45° + 2\alpha = 180°$

$2\alpha = 135° \Rightarrow \alpha = 67.5°$

In ΔBOL, $45° + 2\beta = 180°$

$2\beta = 135° \Rightarrow \beta = 67.5°$

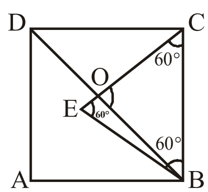
In ΔLOK, $\Rightarrow \beta + \alpha + \theta = 180°$

$135° + \theta = 180° \Rightarrow \theta = 45°$

hence, $\tan\theta = \tan 45° = 1$

Ex.90 ABCD is a square. BEC is an equilateral triangle inside the square. If CE & BD intersects at O. Find $\angle BOC = ?$

Sol:



BEC is an equilateral triangle

$\angle BEC = \angle BCE = \angle CBE = 60°$.

diagonals of square bisects the vertex angle

$\angle CBD = 45°$

In ΔBOC

$\angle BOC = 180° - \angle OCB - \angle CBO$

$= 180° - 60° - 45° = 75°$

OR,

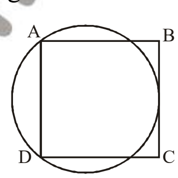
$\angle CBD = 45°$

then,, $OBE = 60° - 45° = 15°$

exterior angle $\angle BOC = \angle CEB + \angle OBE$

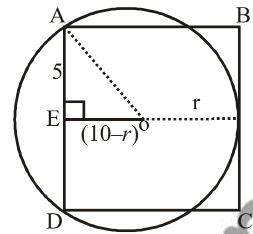
$= 60° + 15° = 75°$

Ex.91 ABCD is a square with side length 10. A circle is drawn through A and D so that it is tangent to BC.



What is the radius of circle?

Sol.



Let O is the center of circle $OE \perp AD$

$AE = ED = 5$

Let r be the radius of circle.

Hence, $OE = (10 - r)$

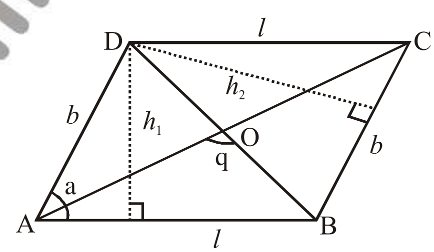
In $\Delta AEO \Rightarrow AE^2 + EO^2 = AO^2$

$5^2 + (10 - r)^2 = r^2$

on solving, $r = 6.25$

Property-36

(iii) Parallelogram :



I. $AB || CD$ & $BC || AD$

II. $AB = CD$ & $BC = AD$

III. $\angle A = \angle C$ & $\angle B = \angle D$

IV. $\angle A + \angle B = \angle B + \angle C = \angle C + \angle D = \angle D + \angle A = 180°$

V. Diagonals are not equal but bisect each other but not at 90º. Diagonals do not bisect any vertex angle.

VI. Area = Base × height $= l \times h_1 = b \times h_2$

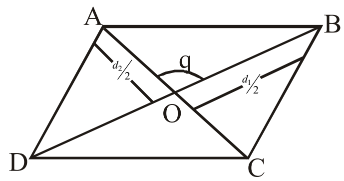
VII. Diagonal divides parallelogram in two equal area parts.

$$\Delta ABD = \Delta BCD = \Delta ABC = \Delta ACD = \frac{1}{2}(\square ABCD)$$

$$\text{Area} = \frac{1}{2} lb\sin\alpha \times 2 = lb\sin\alpha$$

where, α = angle between two common side.

VIII.



The area of four part

$(\Delta AOB, \Delta BOC, \Delta COD \ \& \ \Delta DOA)$

will be equal.

$$\text{Area} = \frac{1}{2} \times \frac{d_1}{2} \times \frac{d_2}{2} \sin\theta \times 4$$

$$= \frac{1}{2} d_1 \times d_2 \sin\theta$$

where, θ = angle between diagonal

IX. $(AC^2 + BD^2) = 2(AB^2 + BC^2)$

$$d_1^2 + d_2^2 = 2(l^2 + b^2)$$

Ex.92 Area of parallelogram 1000 cm^2 and ratio of two adjacent sides 3 : 2. If the distance between larger sides is 20cm, then, find the distance between smaller sides?

Sol. Area = $l \times h_1 = b \times h_2$

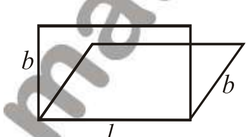
$$\frac{h_1}{h_2} = \frac{b}{l} \Rightarrow \frac{20}{h_2} = \frac{2}{3}$$

h_2 = 30 cm

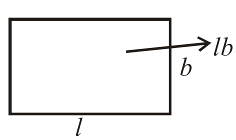
Ex.93 If perimeter of rectangle & Parallelogram is equal with same base. Find the ratio of area of rectangle and area of parallelogram.

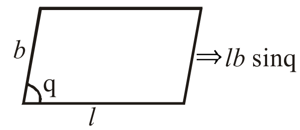
(a) = 1 (b) > 1
(c) < 1
(d) None of these

Sol.



Area of rectangle > Area of Parallelogram





If θ is less than 90° than value of $\sin\theta$ will be less than one .

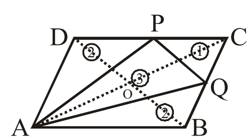
$$\frac{\text{area of rectangle}}{\text{area of parallelogram}} = \frac{lb}{lb\sin\theta} = K$$

$\sin\theta = \frac{1}{K} < 1$ (value of $\sin\theta$ will always be less than 1)

$1 < K$
$K > 1$

Property :37

If ABCD is a parallelogram. P & Q are mid points of CD & BC.



Let the area of parallelogram be 8 unit

Diagonal AC will divide the parallelogram in two equal parts.

As AQ is median of ΔABC,

Area of $\Delta ABQ = \frac{1}{2}(\Delta ABC) = 2$

similarly, AP is median of ΔADC,

Area of $\Delta ADP = \frac{1}{2}(\Delta ADC) = 2$

Now in ΔBCD, P and Q are mid point of sides DC and BC respectively.

Area of $\Delta PCQ = \frac{1}{4}$ of $\Delta BCD = \frac{1}{4} \times (4) = 1$

Now area of ΔAPQ = (area of parallelogram) – [(area of ΔADP) + (area of ΔABQ) + (area of ΔPCQ)]

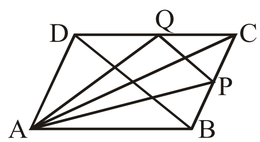
$= 8 - (2 + 2 + 1) = 3$

hence,

area of APQ : area of ABCD = 3 : 8

Ex.94 ABCD is a parallelogram. P and Q are mid points of BC & CD. Find the area of ΔAPQ if area of ΔABC is 12.

Sol.

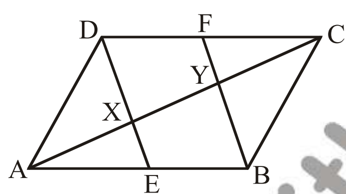


As the area of Δ ABC = 12 so area of parallelogram ABCD will be 24.

$$\text{Area of } \Delta APQ = \frac{3}{8} \text{ (area of parallelogram)}$$

$$= \frac{3}{8} \times 24 = 9$$

Ex.95 ABCD is a parallelogram. E and F are mid-point of side AB and CD respectively. DE and BF intersect diagonal AC at X & Y respectively. If AC = 15 cm, find XY.



Sol. EB = DF $\Rightarrow XE \| BY \,\&\, FY \| DX$

In ΔAYB, E is the mid point of side AB

$AX = XY = a$ (Let)

similarly in ΔCXD, F is mid point of side CD

$CY = XY = a$ (Let)

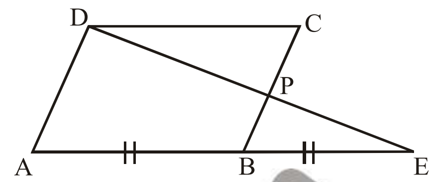
Now $AC = AX + XY + YC = 3a$

$15 = 3a$

$a = XY = 5\text{cm}$

$$\boxed{XY = \frac{1}{3}AC}$$

Ex.96 ABCD is a parallolegram. E is a point on extended side AB such that AB = BE . By joining D to E it intersects side BC at P. Find length of PC if BC = 15 cm .



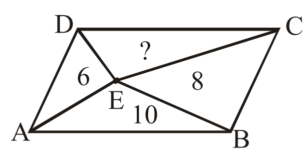
Sol. B is the mid-point of AE and PB $\|$ AD. So in ΔAED, two triangles $\Delta EPB \,\&\, \Delta EDA$ are similar.

$BP = \frac{1}{2}AD = \frac{1}{2}BC$ $(\because BC = AD)$

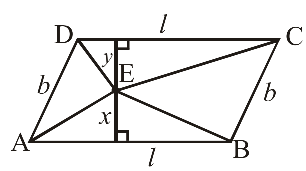
$AD = BC = 15$

$BP = PC = \frac{15}{2} = 7.5$ (P will be mid point of DE)

Ex.97 In a parallelogram ABCD, there is a point E inside the parallelogram such that area of Δ ADE = 6 sq. unit, area of Δ AEB = 10 sq. unit and area of Δ BEC = 8 sq. unit. Find the area of Δ DEC.



Sol.



$h = x + y$

Area of $\square$ABCD $= l(x + y)$

$$\text{Area of } \Delta \text{ ABE} + \text{Area of } \Delta \text{ DEC} = \frac{1}{2} \times l \times x + \frac{1}{2} \times l \times y$$

$$= \frac{1}{2}\, l(x + y)$$

$\text{Area of } \Delta \text{ ABE} + \text{Area of } \Delta \text{ DEC} = \frac{1}{2}(\text{Area of } \square \text{ABCD})$

Similarly,

$\text{Area of } \Delta \text{ BCE} + \text{Area of } \Delta \text{ ADE} = \frac{1}{2}(\text{Area of } \square \text{ABCD})$

Hence,

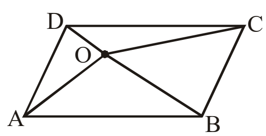
Property-38

Area of (ΔABE + ΔDEC) = Area of (ΔBCE + ΔADE)

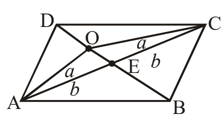
10 + Area of Δ CDE = 8 + 6

Area of Δ CDE = 14 – 10 = 4 sq. unit.

Ex.98 In the below figure, ABCD is a parallelogram. If area of Δ OAB = 19cm^2 then, find the area of Δ OBC.



Sol.



Line joining A and C cuts BD at E .

∴ As diagonals of parallelogram bisects each other,

⇒ OE and BE will be median of ΔOAC and ΔABC respectively,

Area of Δ OAE = Area of Δ OEC = *a* (let)

Area of Δ AEB = Area of Δ EBC = *b* (let)

Area of Δ OAB = Area of Δ OAE + Area of Δ AEB = (*a* + *b*)

Similarly,

Area of Δ OBC = Area of Δ OEC + Area of Δ EBC = (*a* + *b*)

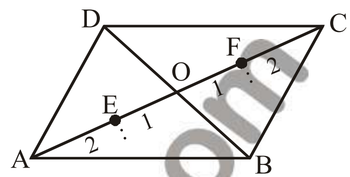
Hence, If O is any point on diagonal BD in parallelogram ABCD then,,

area of ΔOAB = area of ΔOBC

Area of ΔOBC = 19 cm^2

Ex.99 ABCD is a parallelogram . E & F are centroid of triangles ABD and BDC respectively. Find the length of EF if length of diagonal AC = 12 cm.

Sol.



Centroid divides the median in the ratio of 2 : 1

AC = 12 cm, then, AO = OC = 6 cm

In Δ ABD

AO is a median

$$\frac{AE}{EO} = \frac{2}{1}$$

$$EO = \frac{1}{3} \times AO = \frac{1}{3} \times 6 = 2 \text{ cm}$$

In Δ BCD,

CO is a median

$$\frac{CF}{OF} = \frac{2}{1}$$

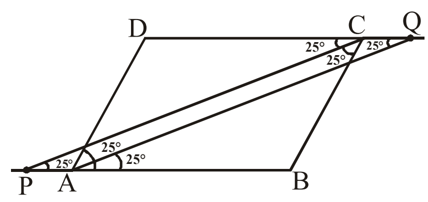
$$OF = \frac{1}{3} \times OC = \frac{1}{3} \times 6 = 2\text{cm}$$

EF = EO + OF = 2 + 2 = 4 cm

$$\text{or } \left(EF = \frac{AC}{3}\right) \Rightarrow EF = \frac{12}{3} = 4 \text{ cm}$$

Ex.100 ABCD is a parallelogram. Angle bisector of ∠A and angle bisector of ∠C cuts extended side DC and AB at Q and P respectively. If ∠A = 50°, find ∠P + ∠Q.

Sol.



∠A = ∠C = 50°

as AQ and CP are angle bisector

$\Rightarrow \angle DAQ = \angle QAB = 25°$

$\Rightarrow \angle DCP = \angle BCP = 25°$

$(\because DC \parallel AB)$

$\Rightarrow \angle CPB = \angle PCD = 25°$ (alternate interior angle)

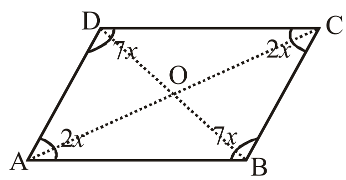
$\Rightarrow \angle DQA = \angle QAB = 25°$ (alternate interior angle)

$\angle P + \angle Q = 25° + 25° = 50°$

Ex.101 If the ratio of angles of a quadrilateral is 2 : 7 : 2 : 7 then, quadrilateral will be .

(a) Rectangle (b) Square
(c) Parallelogram (d) Rhombus

Sol.



Let the angles of quadrilateral are $2x$, $7x$, $2x$ & $7x$

$\therefore 2x + 7x + 2x + 7x = 180°$

$x = 20°$

then, angles of quadrilateral are 40°, 140°, 40° & 140°

As two opposite angles are same and sum of two adjacent angles is 180°. Then figure will be either parallelogram or rhombus.

According to figure in ΔAOB

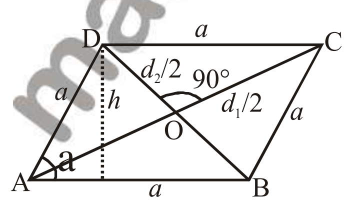
It is not necessary that AC is angle bisector of vertex angle A.So $\angle AOB$ is 90° or not.

It is necessary that Diagonals bisect each other at 90°

$\therefore$ Quadrilateral will be a parallelogram.

Property-39

(iv) Rhombus:-



I. $AB \parallel CD$ & $BC \parallel AD$

II. $AB = BC = CD = DA = a$

III. $\angle A = \angle C$ & $\angle B = \angle D$

IV. $\angle A + \angle B = \angle B + \angle C = \angle C + \angle D = \angle D + \angle A = 180°$

V. Diagonals are not equal but bisect each other at 90°. Diagonals bisect each vertex angle.

VI. Perimeter $= 4a$

VII. Area of rhombus $= a \times h = a^2 \sin\alpha = \frac{1}{2} \cdot d_1 d_2$

VIII. In ΔDOC, $(OD)^2 + (OC)^2 = (DC)^2$

$$\left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2 = a^2$$

$$\boxed{(d_1)^2 + (d_2)^2 = 4a^2}$$

Ex.102 If the perimeter of rhombus is 150 cm and length of one diagonal is 50 cm. Then find the length of second diagonal and area of rhombus.

Sol. perimeter $= 4a = 150 \Rightarrow 2a = 75$ cm

$4a^2 = d_1^2 + d_2^2 \Rightarrow (2a)^2 = (d_1)^2 + (d_2)^2$

$(75)^2 = (50)^2 + d_2^2$

$(75)^2 - (50)^2 = d_2^2$

$d_2^2 = (75 + 50)(75 - 50)$

$= 125 \times 25$

$= 25 \times 25 \times 5$

$d_2 = 25\sqrt{5}$

Area $= \frac{1}{2} d_1 d_2 = \frac{1}{2} \times 50 \times 25\sqrt{5}$

$= 625\sqrt{5}$ cm^2

Ex.103 If the sum of length of diagonal of a rhombus is $\sec\theta$. and perimeter is $2\tan\theta$. Find the area of rhombus.

Sol. Perimeter $= 4a = 2\tan\theta \Rightarrow 2a = \tan\theta$

$4a^2 = d_1^2 + d_2^2$

$(2a)^2 = d_1^2 + d_2^2$

$\tan^2\theta = d_1^2 + d_2^2$ …(i)

$d_1 + d_2 = \sec\theta$ (squaring in both sides)

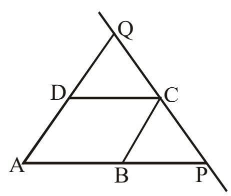
$d_1^2 + d_2^2 + 2d_1 d_2 = \sec^2\theta$

$2d_1 d_2 = \sec^2\theta - \tan^2\theta = 1$

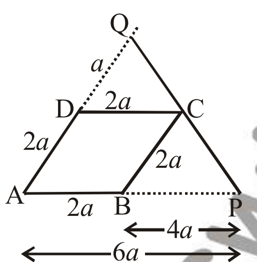
$d_1 d_2 = \frac{1}{2}$

$\text{Area} = \frac{1}{2} d_1 d_2 = \frac{1}{4}$

Ex.104 ABCD is a rhombus. A line passing through C cuts extended line AD and AB at Q and P respectively. If QD = $\frac{1}{2}$ AB . Then find the ratio AB to PB.



Sol.



$\frac{QD}{AQ} = \frac{DC}{AP}$

$\left(\frac{QD}{AQ} = \frac{a}{3a} = \frac{1}{3}\right) = \frac{DC}{AP}$

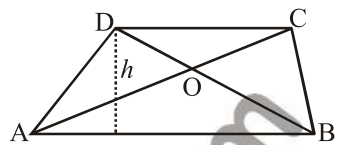
$AP = 3DC = 3 \times 2a = 6a$

then,, $BP = 6a - 2a = 4a$

$\frac{AB}{PB} = \frac{2a}{4a} = \frac{1}{2}$

Property-40

(v) Trapezium –



I. AB||CD

II. $\text{Area} = \frac{1}{2} \times h\,(AB + CD)$

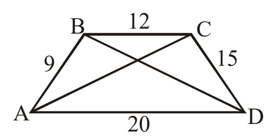
III. $\frac{OA}{OC} = \frac{OB}{OD}$

IV. Sum of square of its diagonal :–

$\Rightarrow AC^2 + BD^2 = AD^2 + BC^2 + 2 \times AB \times CD$

Ex.105 ABCD is a trapezium of sides in which BC || AD & AB = 9 cm, BC = 12 cm, CD = 15 cm & DA = 20 cm. Find the sum of square of its diagonal.

Sol.



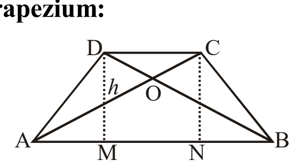
$AC^2 + BD^2 = AB^2 + CD^2 + 2 \times BC \times AD$

$= 81 + 225 + 2 \times 12 \times 20$

$= 786 \text{ cm}^2$

Property-41

Isosceles Trapezium:



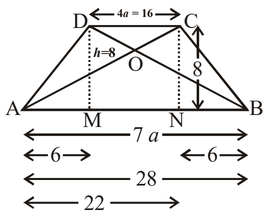
I. AB||CD & AD = BC

II. AM = NB & MN = CD

III. Diagonals are equal $\Rightarrow$ AC = BD

Ex. 106 Area of isosceles trapezium is 176 cm^2. Ratio of its parallel sides is 7 : 4. Height is equal to $\frac{2}{11}$ of sum of parallel sides. Find the length of diagonal.

Sol.



$$h = \frac{2}{11} \times (7a + 4a) = \frac{2}{11} \times 11a = 2a$$

$$\text{Area of trapezium} = \frac{1}{2} \times h\,(AB + CD)$$

$$176 = \frac{1}{2} \times 2a\,(11a)$$

$$\Rightarrow 176 = 11a^2$$

$$a^2 = 16 \;\Rightarrow\; a = 4$$

then, height $h = 2a = 2 \times 4 = 8$

Side $AB = 7a = 28$ & $CD = 4a = 16$

$MN = 16$ then, $AM = NB = 6$

hence $AN = 6 + 16 = 22$

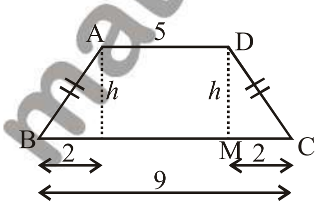
$AC^2 = CN^2 + AN^2 = 8^2 + 22^2 = 2^2(4^2 + 11^2)$

$AC = 2\sqrt{4^2 + 11^2}$

$AC = BD = 2\sqrt{137}$

Ex.107 ABCD is an isosceles trapezium in which AB = CD and $AD \parallel BC$. If AD = 5 cm, BC = 9cm and area of trapezium is 35 cm^2, then, find the length of side CD.

Sol.



$$\text{Area of trapezium} = \frac{1}{2} h\,(AD + BC)$$

$$35 = \frac{1}{2} \times h \times 14$$

$$h = 5$$

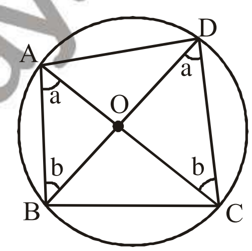
In ΔCMD,

$$CD^2 = h^2 + 2^2 = 5^2 + 2^2$$
$$CD^2 = 29$$
$$CD = \sqrt{29}$$

Proerty-42

Cyclic quadrilateral :

I. All vertices of quadrilateral are on circle then, quadrilateral is cyclic quadrilateral.



O is not the centre of the circle, it is the intersection point of the diagonals.

II. $\angle A + \angle C = 180°$ & $\angle B + \angle D = 180°$

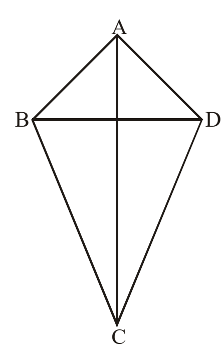
III. $AB \times CD + BC \times AD = AC \times BD$

IV. $\Delta AOB \sim \Delta DOC$ (similar triangles)

$$\frac{OB}{OC} = \frac{OA}{OD} \Rightarrow OB.\,OD = OA.\,OC$$

Property-43

Kite:



I. $AC \perp BD$

II. $AB = AD$ & $BC = CD$

III. Area $= \frac{1}{2} \times AC \times BD$ (In any quadrilateral, if diagonals intersect at 90º then, area will be equal to $\frac{1}{2} d_1 d_2$)

Important notes:-

Figure:-	**Figure formed by joining mid points of sides**
Quadrialateral	Parallelogram
Parallelogram	Parallelogram
Rhombus	Rectangle
Rectangle	Rhombus
Square	Square

Property-44

Polygon:-

I. Sum of all interior angles $= (n-2) \times 180°$
where, n = no. of side

II. Sum of all exterior angles $= 360°$

Property-45

Regular Polygon – All sides & all angles are equal. If n is no. of sides and a is length of side then,

I. Each interior angle $= \frac{(n-2)\times 180°}{n}$

II. Each exterior angle $= \frac{360°}{n}$

III. Area of polygon $= \frac{na^2}{4} \cot\left(\frac{\pi}{n}\right)$

E.g. - An equilateral triangle is a regular polygon.

$$\text{Area} = \frac{3a^2}{4} \cot\left(\frac{180°}{3}\right) = \frac{3a^2}{4} \times \frac{1}{\sqrt{3}} = \frac{\sqrt{3}\,a^2}{4}$$

IV. Circumradius $R = \frac{a}{2} \text{cosec}\left(\frac{\pi}{n}\right)$

V. Inradius $r = \frac{a}{2} \cot\left(\frac{\pi}{n}\right)$

VI. No. of diagonals $= \frac{n(n-3)}{2}$

Ex.108 In a polygon, five interier angles are 172° each , and all remaining interior angles are 160º each. Find the no. of sides and no. of diagonals.

Sol. Let the no. of sides be n

$(n-2) \times 180° = (5 \times 172°) + (n-5) \times 160°$

$180n - 360° = 860° + 160n - 800°$

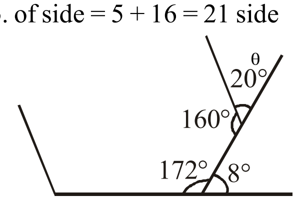
$n = 21$

Method 2 : (5 exterior angles will be 8° each and remaining exterior angles will be 20° each. As the summation of all exterior angles is 360° then, summation of remaining exterior angles will be 360° – 5 × 8° = 320°. As each remaining exterior angle is 20° then, no. of remaining angles will be 320°/20° = 16) then, total angles or sides = 16 + 5 = 21

No. of angles	(5)	x
interior	172°	160°
exterior	8°	20°
Sum =	40°	360° – 40° = 320°

then,, $x = \frac{320}{20} = 16$

no. of side = 5 + 16 = 21 side



No. of diagonals $= \frac{n(n-3)}{2} = \frac{21\times 18}{2} = 189$

Ex.109 In a polygon 6 interior angles are equal to 170º each and all remaining angles are 150º each. Find the no. of sides and no. of diagonals.

Sol.

No. of angles	6	x
interior	170°	150°
exterior	10°	30°
Sum =	6 × 10° = 60°	360° – 60° = 300°

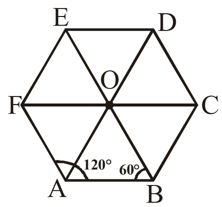
then,, $x = \frac{300°}{10°} = 30$

No. of sides = 6 + 10 = 16

No. of diagonals $= \frac{16\times13}{2} = 104$

Property-46

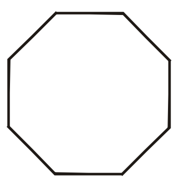
Regular Hexagon: No. of sides $n = 6$, There are all six equilateral triangle in a Hexagon. OA = OB = OC = OD = OE = OF = *a.*



Area $= 6\times\frac{\sqrt{3}\,a^2}{4} = \frac{3\sqrt{3}\,a^2}{2}$

Property-47

Octagon : No. of sides $n = 8$ and Length of side = a



Area of Octagon $= 2\left(\sqrt{2}+1\right)a^2$

Each interior angle $= \frac{(8-2)\times180°}{8} = 135°$

Ex.110 If area of octagon is $18\left(\sqrt{2}+1\right)$. Find the length of each side.

Sol.

$\because$ Area of octagon $= 2\left(\sqrt{2}+1\right)a^2$

$18\left(\sqrt{2}+1\right) = 2\left(\sqrt{2}+1\right)a^2$

$a^2 = 9$

$a = 3$

Ex.111 Ratio of no. of sides of two regular polygon is 1 : 2. and ratio of their each interior angle is 3 : 4 . Find the no. of sides and diagonals.

Sol. Each interior angle $= \frac{(n-2)\times180°}{n}$

Let the No. of sides in first polygon is n then, no. of sides of second polygon will be $2n$.

$$\frac{\frac{(n-2)\times180°}{n}}{\frac{(2n-2)\times180°}{2n}} = \frac{3}{4}$$

$$\frac{(n-2)\times2}{(2n-2)} = \frac{3}{4}$$

$8n - 16 = 6n - 6$

$2n = 10$

$n = 5$

No. of sides $= n,\ 2n = 5, 10$

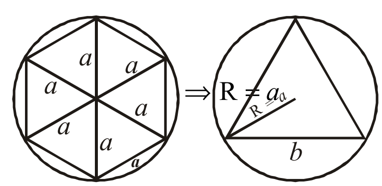
No. of diagonal $= \frac{n(n-3)}{2}$ and $\frac{2n(2n-3)}{2}$

$= \frac{5(5-3)}{2}$ and $\frac{10(10-3)}{2}$

$= 5$ and 35

Ex.112 There is a regular hexagon and an equilateral triangle inside a circle . If sides of the hexagon is a and side of triangle is b then, find the relation between a and b.

Sol.



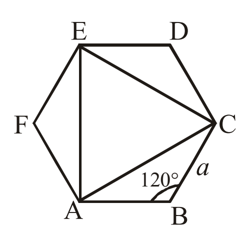
$\Rightarrow R = \frac{b}{\sqrt{3}}$

then,, $R = a = \frac{b}{\sqrt{3}} \Rightarrow b = \sqrt{3}a$

$\Rightarrow b^2 = 3a^2$

Ex.113 ABCDEF is a regular hexagon. Find the ratio of area of triangle ACE and hexagon ABCDEF.

Sol.



$$\text{Area of ABCDEF} = 6 \times \frac{\sqrt{3}\,a^2}{4} = 6\,A$$

Area of Δ ABC = Area of Δ DCE = Area of Δ AFE

$$= \frac{1}{2} \times a \times a \times \sin 120°$$

$$= \frac{\sqrt{3}\,a^2}{4} = A$$

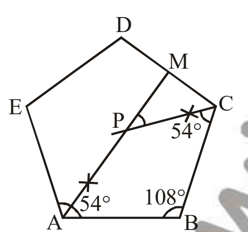
$$\frac{\text{Area of triangle ACE}}{\text{Area of hexagon ABCDEF}} = \frac{6A - 3A}{6A} = \frac{1}{2}$$

Ex.114 ABCDE is a regular pentagon. Angle bisector of $\angle BAE$ meets CD at M. Angle bisector of $\angle BCD$ meets AM at P. Find the $\angle CPM$.

Sol. Each angle of pentagon $= \frac{(n-2)\times 180}{n}$

$$= \frac{(5-2)\times 180°}{5}$$

$$= \frac{3\times 180}{5} = 108°$$



$\angle$ ABC = 108° & $\angle$ BAF = 108°

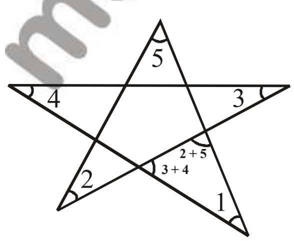
then,, $\angle$ BAP = 54° and $\angle$ BCP = 54°

$\angle$ APC = 360° – 108° – 54° – 54° = 144°

hence, $\angle$ CPM = 180° – 144 = 36°

Property-48

Star:



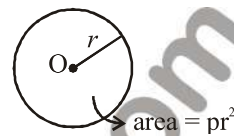
$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 = 180°$

If there are m nodes in a star.

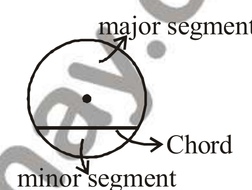
then, sum of all interior angle = $(m - 4) \times 180°$

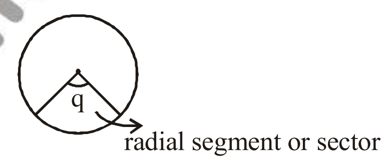
Property-49

Circle :



r = radius, circumference = $2\pi r$





Property-50

length of arc $= \frac{2\pi r}{360} \times \theta$ (θ in degree)

$l = r\theta$ (θ in radian) ($\because 2\pi = 360°$)

Property-51

Area of sector $= \frac{\pi r^2}{360} \times \theta$ (in degree)

$= \frac{1}{2} r^2 \theta$ (θ is in radian) ($2\pi = 360°$)

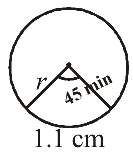
$= \frac{1}{2} lr$ ($\because l = r\theta$)

Ex.115 There are many points on the circumference of a circle such that distance between each consecutive point is 1.1cm and time taken to move from one point to next point is 45 minutes. Find the radius of that circle and number of points located on circumference.

Sol.

$\theta = 45$ min

$45 \text{ min} = \frac{45°}{60} = \frac{3}{4}^{\circ}$



length of arc $(l) = \frac{2\pi r}{360°} \times \theta$

$1.1 = 2 \times \frac{22}{7} \times \frac{r}{360°} \times \frac{3}{4}$

$r = 84$ cm

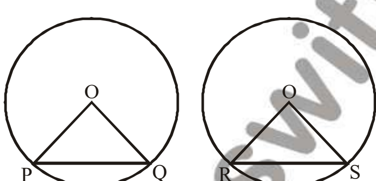
No. of points $= \frac{2\pi r}{1.1} = 2 \times \frac{22}{7} \times 84 \times \frac{1}{1.1}$

$\frac{360°}{314} = 480$

Property-52

In a circle (or congruent circles) equal chords are made by equal arcs.

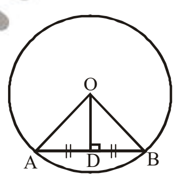
$PQ = RS \quad \therefore \quad \overparen{PQ} = \overparen{RS}$



Property-53

The perpendicular from the centre of a circle to a chord bisects the chord i.e., $OD \perp AB$

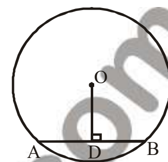
$\therefore AB = 2AD = 2BD$



Property-54

The line joining the centre of a circle to the mid-point of a chord is perpendicular to the chord.

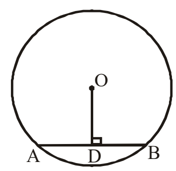
$AD = DB \quad OD \perp AB.$



Property-55

Perpendicular bisector of a chord passes through the centre i.e., $OD \perp AB$ and $AD = DB$

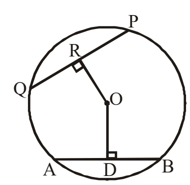
$\therefore$ O is the centre of the circle



Property-56

Equal chords of a circle (or of congruent circles) are equidistant from the centre

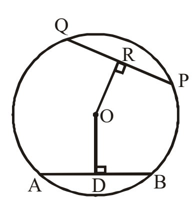
$\therefore AB = PQ \ \& \ \therefore OD = OR$



Property-57

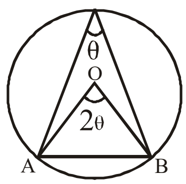
Chords of a circle (or of congruent circles) are equi-distant from the centre

$\because OD = OR \ \& \ \therefore AB = PQ$



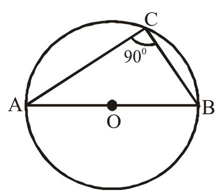
Property-58

The angle subtended by an arc (the degree measure of the arc) at the centre of a circle is twice the angle subtended by the arc at any point on the remaining part of the circle, m$\angle AOB = 2\angle ACB$.



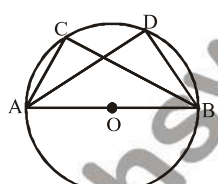
Property-59

- Angle in an semicircle is a right angle.



Property-60

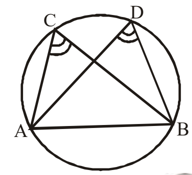
- Angles in the same segment of a circle are equal i.e., $\angle ACB = \angle ADB$



Property-61

. If a line segment joining two points subtends equal angle at two other points lying on the same side of the line containing the segment, then, the four points lie on the same circle. $\because \angle ACB = \angle ADB$

$\therefore$ Points A, C, D, B are concyclic i.e., lie on the circle



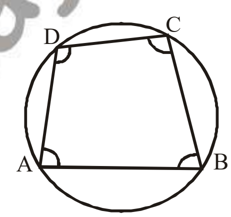
Property-62

The sum of pair of opposite angles of a cyclic quadrilateral is 180°

$\angle DAB + \angle BCD = 180°$

and $\angle ABC + \angle CDA = 180°$

(Inverse of this theorem is also true)

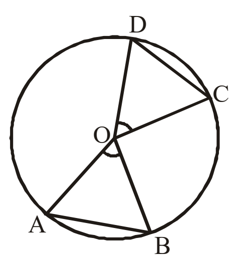


Property-63

Equal chords (or equal arcs) of a circle (or congruent circles) subtended equal angles at the centre.

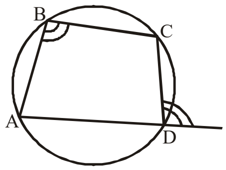
$AB = CD$ (or $\overset{\frown}{AB} = \overset{\frown}{CD}$) $\angle AOB = \angle COD$

(Inverse of this theorem is also true)



Property-64

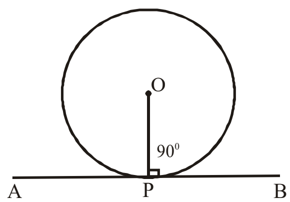
- If a side of a cyclic quadrilateral is produced, then, the exterior angle is equal to the interior opposite angle. m$\angle CDE = \angle ABC$



Property-65

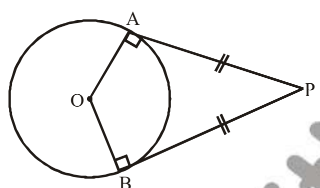
A tangent at any point of a circle is perpendicular to the radius through the point of contact.

(Inverse of this theorem is also true.)



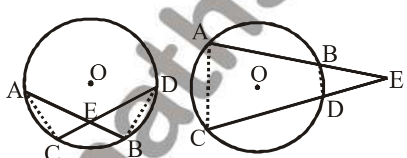
Property-66

The lengths of two tangents drawn from an external point to a circle are equal i.e., AP=BP



Property-67

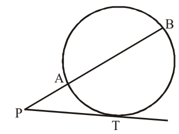
If two chords AB and CD of a circle, intersect inside a circle (outside the circle when produced at a point E), then, AE×BE=CE×DE



Property-68

If PB be a secant which intersects the circle at A and B and PT be a tangent at T then,

$$PA \times PB = (PT)^2$$

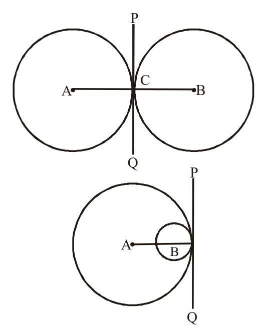


Property-69

The point of contact of two tangents lies on the straight line joining the two centres.

(a) When two circles touch externally then, the distance between their centres is equal to sum of the radii, i.e., $AB = AC + BC$

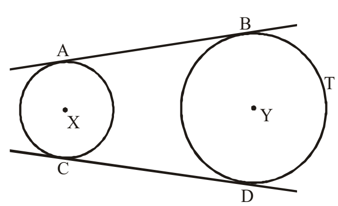
(b) When two circles touch internally the distance between their centres is equal to the difference between their radii, i.e., $AB = AC - BC$



Property-70

For the two circles with centre X and Y and radii r_1 and r_2 AB and CD are two direct common tangent, then, length of AB = CD =

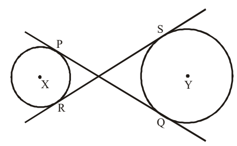
$$\sqrt{(\text{Distance between centres})^2 - (r_1 - r_2)^2}$$



Property-71

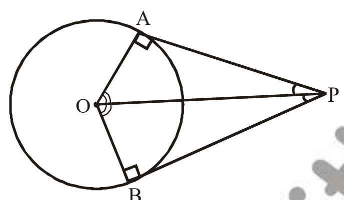
For the two circles with centre X and Y and radii r_1 and r_2 PQ and RS are two transverse common tangent, then, length of PQ = RS =

$$\sqrt{(\text{Distance between centres})^2 - (r_1 + r_2)^2}$$



Property-72

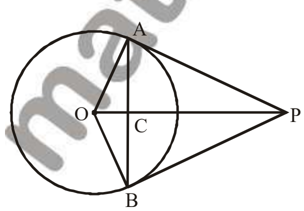
From an external point from which the tangents are drawn to the circle with centre O, then, (a) they subtend equal angles at the centre (b) they are equally inclined to the line segment joining the centre of that point $\angle AOP = \angle BOP$ and $\angle APO = \angle BPO$



Property-73

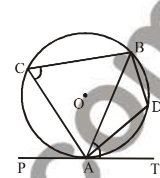
If P is an external point from which the tangents to the circle with centre O touch it at A and B then, OP is the perpendicular bisector of AB.

$OP \perp AB$ and $AC = BC$

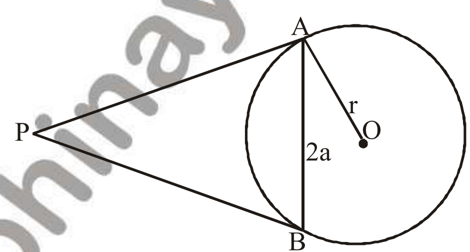


Property-74

If from the point of contact of a tangent, a chord is drawn then, the angles which the chord makes with the tangent line are equal respectively to the angles formed in the corresponding alternate segments. In the adjoining diagram. $\angle BAT = \angle BCA$ and $\angle BAP = \angle BDA$



Property-75



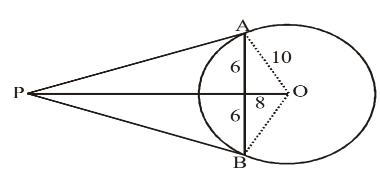
If AB = $2a$
radius = r
then, length of tangent

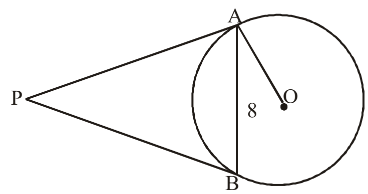
$$PA = PB = \frac{ar}{\sqrt{r^2 - a^2}} \text{ (if chord AB = } 2a\text{)}$$

Ex.116 There is a chord AB in a circle of radius 10 cm. Length of chord is 12 cm. Two tangents are drawn from an external point P to A&B . Find the

(i) Length of PA & PB.
(ii) Area of triangle PAB
(iii) Area of quadrilateral OAPB
(O is centre of circle)

Sol.





$$PA = \frac{ar}{\sqrt{r^2 - a^2}} \quad \text{(If chord AB = 2}a\text{)}$$

$$= \frac{6\times10}{\sqrt{10^2-6^2}} = \frac{60}{8} = 7.5 = PB$$

Area of quadrilateral OAPB = 2 × area of triangle OAP

$$= 2 \times \frac{1}{2} \times AO \times AP$$

$$= 2 \times \frac{1}{2} \times 10 \times 7.5 = 75 \text{ sq. unit.}$$

$$\text{Area of triangle OAB} = \frac{1}{2} \times AB \times 8 = \frac{1}{2} \times 12 \times 8$$

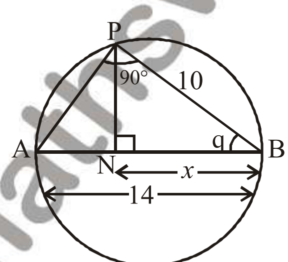
$$= 48 \text{ sq. unit}$$

then, area of ΔPAB = area of ▭OAPB – area of ΔOAB

$$= 75 - 48 = 27 \text{ sq. unit.}$$

Ex.117 There is a circle of diameter is 14 cm. P is any point on the circumference of circle such that PN ⊥ AB. Length of chord PB = 10 cm. Find length of BN.

Sol.



In ΔPNB,

$$\cos\theta = \frac{x}{10} \quad \ldots\text{(i)}$$

In Δ PAB,

$$\cos\theta = \frac{10}{14} \quad \ldots\text{(ii)}$$

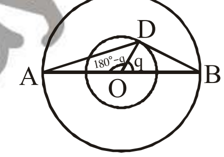
From (i) and (ii) -

$$\frac{x}{10} = \frac{10}{14} \Rightarrow x = \frac{50}{7} = 7\frac{1}{7} \text{ cm}$$

$$BN = 7\frac{1}{7} \text{ cm}$$

Ex.118 Radius of two concentric circles are 8 cm and 13 cm. If AB is the diameter of bigger circle and BD is the tangent on smaller circle, find the length of AD.

Sol. Method 1.



In ΔODB,

$$\cos\theta = \frac{OD}{OB} = \frac{8}{13} \quad \ldots(1)$$

In ΔAOD,

$$\cos(180° - \theta) = \frac{8^2 + 13^2 - x^2}{2\times8\times13}$$

$$-\cos\theta = \frac{64+169-x^2}{2\times8\times13} \quad \ldots(2)$$

From (1) and (2),

$$\frac{-8}{13} = \frac{64+169-x^2}{2\times8\times13}$$

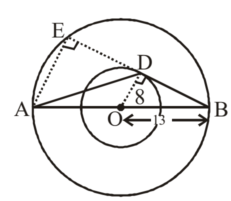
$-128 = 233 - x^2$

$x^2 = 361$

$x = 19$

AD = 19 cm

Method 2 :



OD ⊥ BD ⇒ OD || AE

$\angle AED = 90°$

$\Delta ODB \sim \Delta AEB$

O is the mid-point of AB, then,

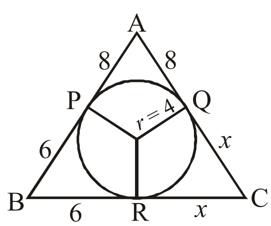
$$BD = DE = \sqrt{13^2 - 8^2} = \sqrt{105}$$

$$AE = 2 \times OD = 16$$

then, $AD = \sqrt{16^2 + \left(\sqrt{105}\right)^2} = 19$ cm

Ex.119 The radius of the incircle of a triangle is 4 cm and the segments into which one side is divided by the point of contact are 6 cm and 8 cm. Determine the perimeter of the triangle.

Sol.



$$s = \frac{8+6+6+x+x+8}{2} = 14 + x$$

$$r = \frac{\Delta}{s}$$

$\Rightarrow \quad 4 = \dfrac{\sqrt{(14+x)x \times 6 \times 8}}{14+x}$ (squaring both sides)

$\Rightarrow \quad 16 = \dfrac{(14+x)x \times 6 \times 8}{(14+x)^2} = \dfrac{x \times 6 \times 8}{14+x}$

$\Rightarrow \quad 3x = 14 + x \quad \Rightarrow x = 7$

$\Rightarrow \quad 42 = 13x \Rightarrow x = \dfrac{42}{13}$

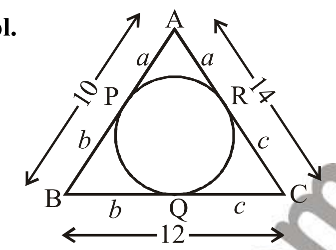
$\Rightarrow \quad BC = 6 + x = 6 + 7 = 13$

$\Rightarrow \quad AC = 8 + x = 8 + 7 = 15$

hence, perimeter $= AB + BC + CA = 14 + 13 + 15 = 42$cm

Ex.120 A circle is inscribed in a Δ ABC having sides 10 cm, 12 cm and 14 cm cuts the sides of triangle AB, BC & AC at P, Q & R respectively. Find length of AP.

Sol.



Tangents drawn from an external point on the circle are equal

$a + b = 10$ …(i)

$b + c = 12$ …(ii)

$c + a = 14$ …(iii)

Adding all three equations

$2(a + b + c) = 36$

$a + b + c = 18$

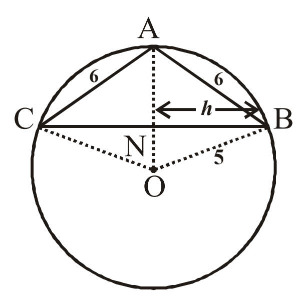
$AP = a = 18 - (b + c)$

$= 18 - 12$

hence, AP = 6 cm

Q.121 There is a circle of radius 5 cm. AB and AC are two equal chords such that AB = AC = 6 cm. Find the length of chord BC.

Sol.



$\because \quad s = \dfrac{a+b+c}{2} = \dfrac{6+5+5}{2} = 8$

$\therefore$ Area of Δ OAB $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{8 \times 2 \times 3 \times 3} = 3 \times 4 = 12$ …(i)

Area of Δ OAB $= \dfrac{1}{2} \times BN \times AO$ …(ii)

From Eqn.(i) & (ii)

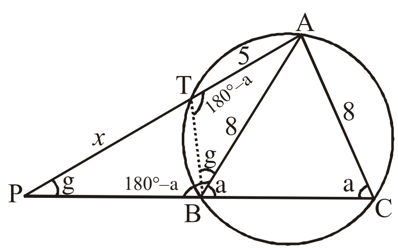
$$12 = \frac{1}{2} \times h \times 5 \Rightarrow h = \frac{24}{5}$$

$$h = 4.8$$

$$BC = 2 \times 4.8 = 9.6 \text{ cm}$$

Q.122 There are two chords AB and AC of equal length 8cm . CB is produced to P. AP cuts circle at T such that AT = 5 cm. Find length of PT.

Sol.



$\because AB = AC$

$\Rightarrow \angle ABC = \angle ACB = \alpha$ (Let)

$\angle ABP = 180° - \angle ABC = 180° - \alpha$

$\therefore$ ATBC is a cylic quadrilateral

$\Rightarrow \angle ATB + \angle ACB = 180°$

$\Rightarrow \angle ATB = 180° - \alpha$

Now, In Δ ABP and Δ BTA

$\angle ABP = \angle BTA = 180° - \alpha$

$\angle BAP = \angle BAT$ (common)

$\angle BPA = \angle TBA = \gamma$ (Let)

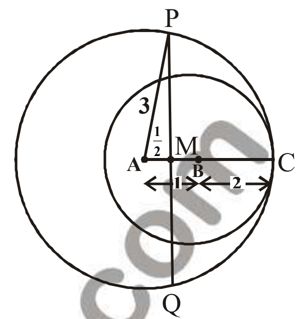
$\therefore \Delta ABP \sim \Delta BTA$

$$\frac{AP}{AB} = \frac{AB}{AT} \Rightarrow \frac{x+5}{8} = \frac{8}{5}$$

$$\Rightarrow PT = x = \frac{39}{5}$$

Ex.123 In the given figure, two circles with centres A and B and of radii 3 cm and 2 touch each other internally. If the perpendicular bisector of segment AB meets the bigger circle at P and Q, find the length of PQ.

Sol.



AB = 3 – 2 = 1 cm

$$AM = \frac{AB}{2} = \frac{1}{2} \text{ cm}$$

AP = 3 cm

$$PM = \sqrt{PA^2 - AM^2}$$

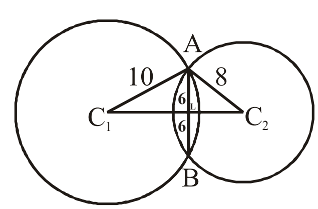
$$= \sqrt{9 - \frac{1}{4}} = \frac{1}{2}\sqrt{35}$$

then,, $PQ = 2 \times PM = 2 \times \frac{1}{2}\sqrt{35}$

$$= \sqrt{35}$$

Ex.124 Two circles of radii 10 cm and 8 cm intersect and the length of the common chord is 12 cm. Find the distance between their centres.

Sol:



Let C_1 and C_2 be the centres of the circles of radii 10 cm and 8 cm respectively and let AB be their common chord.

$C_1A = 10$ cm, $C_2A = 8$ cm and AB = 12 cm.

$\therefore \quad AL = \frac{1}{2} AB = 6$ cm

In right angle triangle C_1AL,

$C_1A^2 = C_1L^2 + AL^2$

$\Rightarrow C_1L = \sqrt{C_1A^2 - AL^2} = \sqrt{10^2 - 6^2} = \sqrt{64} = 8$ cm

In right angle triangle C_2LA,

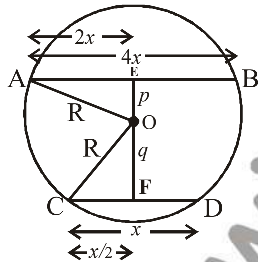
$C_2A^2 = C_2L^2 + LA^2$

$\Rightarrow C_2L = \sqrt{C_2A^2 - LA^2} = \sqrt{8^2 - 6^2} = \sqrt{28} = 5.29$ cm

$\therefore \quad C_1C_2 = C_1L + C_2L = (8 + 5.29)$ cm $= 13.29$ cm

Ex.125 There is a circle of centre O and radius R. There are two parallel chords AB &CD opposite side of centre. The distance from the centre to AB and CD is p and q respectively. Chords AB is 4 times of chord CD. If CD is x unit. Find the value of x in terms of p and q.

Sol:



In right angle Δ AEO,

$R^2 = p^2 + (2x)^2 \qquad \ldots$(i)

In right angle ΔOCF,

$R^2 = q^2 + \left(\frac{x}{2}\right)^2 \qquad \ldots$(ii)

from (i) and (ii) we have,

$R^2 = p^2 + (2x)^2 = q^2 + \left(\frac{x}{2}\right)^2$

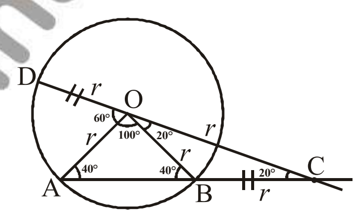
$\Rightarrow p^2 + 4x^2 = q^2 + \frac{x^2}{4}$

$4x^2 - \frac{x^2}{4} = q^2 - p^2$

$15x^2 = 4(q^2 - p^2) \Rightarrow x = \frac{2}{\sqrt{15}}\sqrt{(q^2 - p^2)}$

Ex.126 AB is the chord of circle with centre O and DOC is a line segment originating from a point D on the circle and intersecting produced AB at C such that BC = OD if $\angle BCD = 20°$. Find angle AOD .

Sol.



$\therefore$ BC = OD (given) and OD = OB (radius of circle)

BC = OB

In Δ OBC

$\angle BCO = \angle BOC = 20°$ (isosceles triangle)

Now,

$\angle OBA = 20° + 20°$ ($\therefore$ Exterior angle of Δ OBC)

$\therefore \quad$ OA = OB (radius of circle)

$\angle OAB = \angle OBA = 40°$

In Δ AOB

$\angle OAB + \angle AOB + \angle OBA = 180°$

$\angle AOB = 180° - 40° - 40° = 100°$

Now,

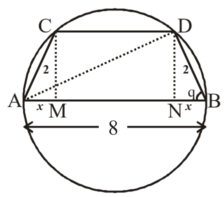
$\angle AOD + \angle AOB + \angle BOC = 180°$

($\therefore$ DOC is a straight line)

$\angle AOD + 100° + 20° = 180°$

$\angle AOD = 60°$

Ex.127 AB is diameter of circle with length 8cm and ABCD is a trapezium in which AC = 2 cm and BD = 2 cm then, find the length of CD.



Sol. Two perpendicular CM and DN are drawn on diameter AB.

CD = MN

AM = NB = x (Let)

In right angle ΔDBN,

$$\cos\theta = \frac{x}{2} \quad \ldots(i)$$

and In right angle ΔADB,

$$\cos\theta = \frac{2}{8} \quad \ldots(ii)$$

from (i) and (ii)

$$\frac{x}{2} = \frac{2}{8} \Rightarrow x = \frac{1}{2}$$

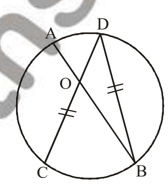
Now,

CD = MN = AB – AM – NB

$$= 8 - \frac{1}{2} - \frac{1}{2} = 7 \text{ cm}$$

CD = 7 cm

Ex.128



In the above figure, AB is a diameter of the circle and C and D are such points that CD = BD. AB and CD intersect at O. If angle AOD = 45°, Find angle ADC.

Sol. Draw AC and CB.

CD = BD $\Rightarrow \angle DCB = \angle DBC = \angle\theta$ (let)

$\angle ACB = 90^o \Rightarrow \angle ACD = 90^o - \theta$

$\angle ABD = \angle ACD = 90^o - \theta$

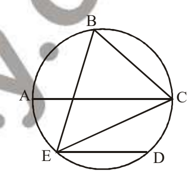
$\Rightarrow \angle ABC = \theta - (90^o - \theta) = 2\theta - 90^o$

In ΔOBC,

$$45^o + 2\theta - 90^o + \theta = 180^o \Rightarrow 3\theta = 225^o \Rightarrow \theta = 75^o$$

$\angle ADC = \angle ABC = 2\theta - 90^o = 60^o$

Ex.129 In the figure, chord ED is paralled to the diameter AC of the circle. If angle CBE = 65°, then, what is the value of angle DEC.

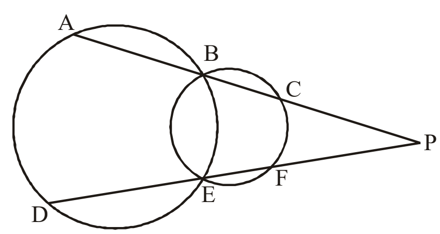


Sol. $\angle ABC = 90° \Rightarrow \angle ABE = 90 - \angle EBC = 25°$,

$\angle ABE = \angle ACE = 25°$,

$\angle ACE = \angle CED = 25°$ (alternate angles)

Ex.130 In the figure, PC = 9, PB = 12, PA = 18 and PF = 8, Then, find the length of DE.



Sol: In the smaller circle PC × PB = PF × PE

$$\Rightarrow PE = 12 \times \frac{9}{8} = \frac{27}{2}$$

In the larger circle, PB × PA = PE × PD

$$\Rightarrow 12 \times 18 = PD \times \frac{27}{2} \Rightarrow PD = 16$$

Therefore, DE = PD – PE = 16 – 13.5 = 2.5

Ex.131 A circle is inscribed in a triangle with sides measuring 4 cm, 6 cm and 8 cm. What is the area of the circle in square centimeters ?

(a) $\frac{7\pi}{6}$ (b) $\frac{3\pi}{2}$

(c) $\frac{5\pi}{3}$ (d) $\frac{7\pi}{4}$

Sol. If r is the radius of the circle.

$\Rightarrow \Delta = r \times s,$ $s = \frac{4+6+8}{2} = 9$ cm,

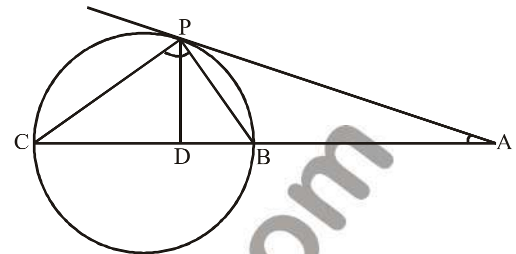
$\Delta = \sqrt{9.5.3.1} = 3\sqrt{15}$

$3\sqrt{15} = r \times 9 = r = \sqrt{\frac{5}{3}}$ cms

Area of the circle = $\frac{5\pi}{3}$

Ex.132 In the adjoining figure, AP is tangent to the circle at P, ABC is a secant and PD is the bisector of $\angle BPC$. Also, $\angle BPD$ = 25° and ratio of $\angle ABP$ to $\angle APB$ is 5:3. Find $\angle APB$.

Sol. $\angle APB = \angle PCB$



Now $\frac{\angle ABP}{\angle APB} = \frac{\angle ABP}{\angle PCB} = \frac{5}{3}$

Or $\angle ABP = 5x$ $\angle PBC = 180° - 5x$

And $\angle PCB = 3x$

PD is angle bisector so $\angle CPB = 2 \times 25° = 50°$

In Δ PBC

$\angle CPB + \angle PCB + \angle PBC = 180°$

$50° + 3x + 180° - 5x = 180°$

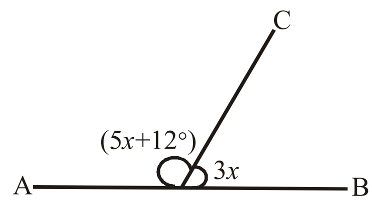
$x = 25°$

$\angle APB = \angle PCB = 3x = 75°$

*****

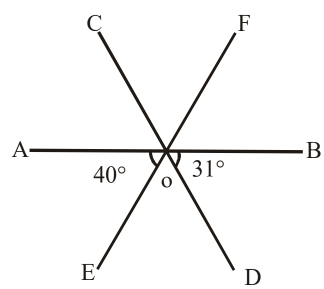
Exercise-Lines & Angles

1. What is the value of x in figure?



(a) 18° (b) 20°
(c) 21° (d) 24°

2. In the figure find the value of $\angle BOC$:



(a) 101° (b) 149°
(c) 71° (d) 80°

3. If $(2x + 17°)$, $(x + 4°)$ are complementary, find x:

(a) 63° (b) 53°
(c) 35° (d) 23°

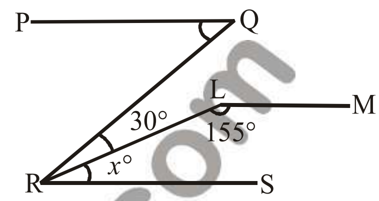
4. If $(5y + 62°)$, $(22° + y)$ are supplementary, find y:

(a) 16° (b) 32°
(c) 8° (d) 21°

5. If two supplementary angles are in the ratio 13 : 5 find the greater angle:

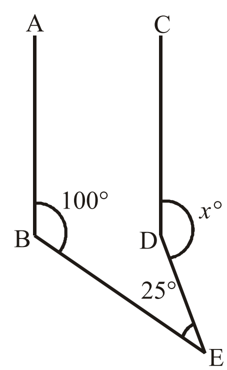
(a) 130° (b) 65°
(c) 230° (d) 21°

6. In the figure PQ||LM||RS. Find the value of $\angle LRS$:



(a) 25° (b) 55°
(c) 65° (d) 75°

7. In the figure AB||CD, $\angle ABE = 100^o$. Find $\angle CDE$:



(a) 125° (b) 55°
(c) 65° (d) 75°

8. Find the angle which is one fifth of its supplementry angle?

(a) 15° (b) 30°
(c) 75° (d) 150°

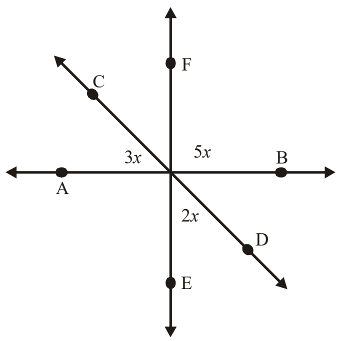
9. Find the measure of an angle which is complement of itself.

(a) 30° (b) 45°
(c) 60° (d) 90°

10. Find the measure of an angle which forms a pair of supplementary angles with itself.

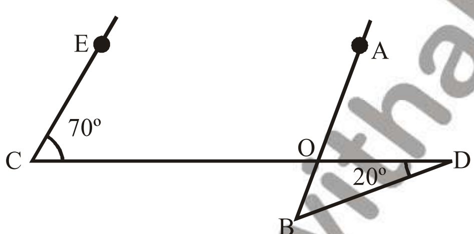
(a) 60° (b) 90°
(c) 120° (d) 150°

11. In fig. Find the value of x.



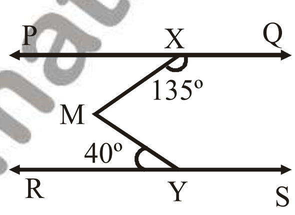
(a) 18° (b) 20°
(c) 10º (d) 30º

12. In the given figure if EC $\parallel$ AB, $\angle$ECD = 70º. $\angle$BDO = 20º, then $\angle$OBD is equal to :



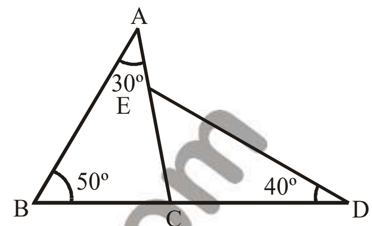
(a) 70º (b) 60º
(c) 50º (d) 20º

13. In the following figure, if PQ $\parallel$ RS, $\angle$MXQ= 135º and $\angle$MYR = 40º. Find $\angle$XMY.



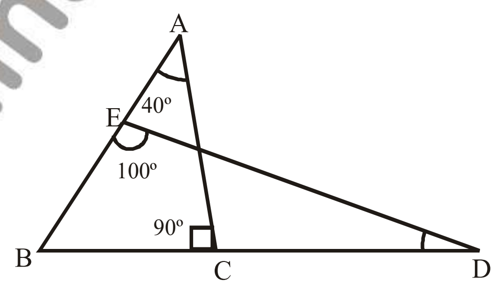
(a) 85º (b) 75º
(c) 65º (d) 80º

14. In the given figure, $\angle$BAC = 30º, $\angle$ABC = 50º and $\angle$CDE = 40º. Then $\angle$AED = ?



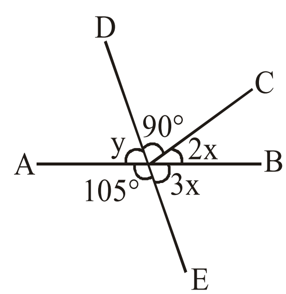
(a) 120º (b) 100º
(c) 80º (d) 110º

15. In the given figure, $\angle$BAC = 40º, $\angle$ACB = 90º and $\angle$BED = 100º. Then $\angle$BDE = ?



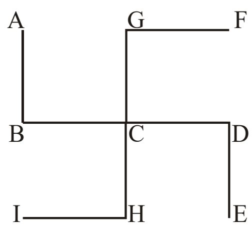
(a) 50º (b) 30º
(c) 40º (d) 25º

16. In a figure, AB is a straight line. Find (x + y):



(a) 55° (b) 65°
(c) 75° (d) 80°

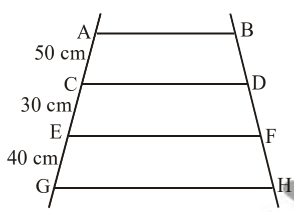
17. In a figure AB || GH || DE and GF || BD || HI, ∠FGC = 80°. Find the value of ∠CHI



(a) 80° (b) 120°

(c) 100° (d) 160°

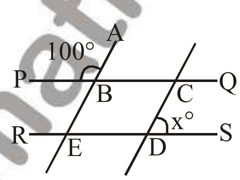
18. In a figure AB || CD || EF || GH and BH = 100 cm. Find the value of DF.



(a) 26 cm. (b) 40 cm.

(c) 25 cm. (d) 24 cm.

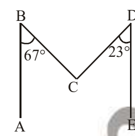
19. In a figure AE || CD and BC || ED, then find x:



(a) 60° (b) 80°

(c) 90° (d) 75°

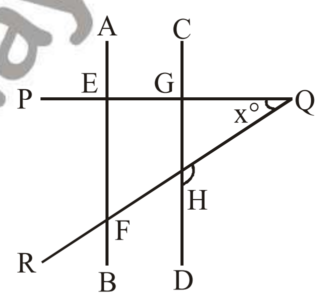
20. In a figure AB || DE, ∠ABC = 67° and ∠EDC= 23°. Find ∠BCD :



(a) 90° (b) 44°

(c) 46° (d) 67°

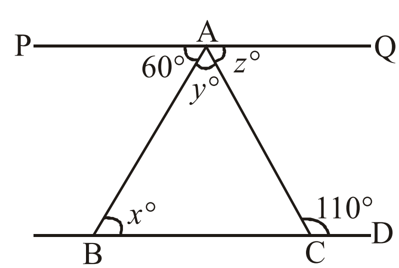
21. In a Figure AB || CD, ∠PEB = 80° and ∠DHQ = 120°. Find x°:



(a) 40° (b) 20°

(c) 100° (d) 30°

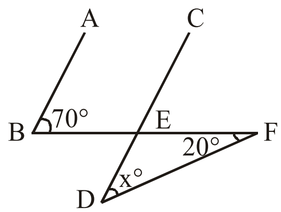
22. In a figure PQ || BC, ∠PAB = 60° and ∠ACD = 110°. Find y° :



(a) 50° (b) 60°

(c) 70° (d) 80°

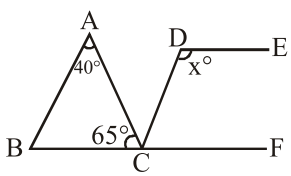
23. In a figure AB || CD, find x° :



(a) 50° (b) 60°

(c) 70° (d) 80°

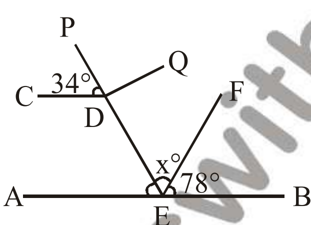
24. In the figure AB || DC and DE || BF. Find the value of x°:



(a) 140° (b) 155°

(c) 105° (d) 115°

25. In the figure AB || CD and EF || DQ. Find the value of x°:



(a) 68° (b) 78°

(c) 34° (d) 39°

26. Two supplementary angles differ by 34°. Find the greater angles.

(a) 107° (b) 90°

(c) 115° (d) 70°

27. An angle is equal to five times its complement. Determine its measure.

(a) 75 (b) 65

(c) 55 (d) 45

28. An angle is equal to one-third of its supplementry. Find its measure.

(a) 60 (b) 45

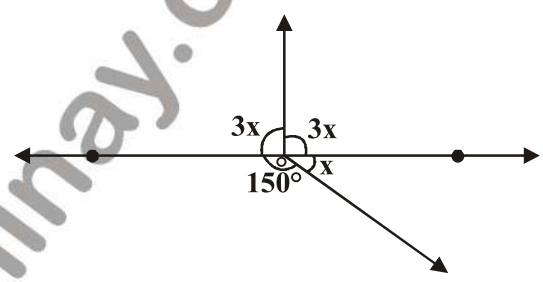
(c) 75 (d) 50

29. Two supplementary angles are in the ratio 2:3. Find the smaller angles.

(a) 72° (b) 40°

(c) 60° (d) 55°

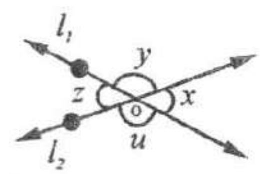
30. In the given Fig. determine the value of x.



(a) 30° (b) 40°

(c) 50° (d) 60°

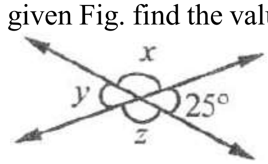
31. In the given Fig. if $x = 45$, find the values of y



(a) 145° (b) 135°

(c) 185° (d) 175°

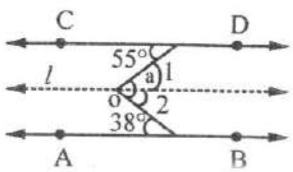
32. In the given Fig. find the values of y.



(a) 25° (b) 26°

(c) 45° (d) 55°

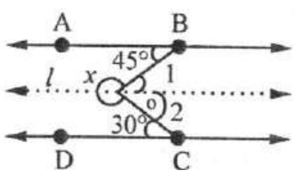
33. In the given Fig. AB || CD, Determine $\angle 1$.



(a) 55° (b) 60°

(c) 70° (d) 48°

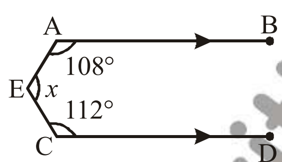
34. In the given Fig. AB || CD. Determine x



(a) 285° (b) 260°

(c) 240° (d) 210°

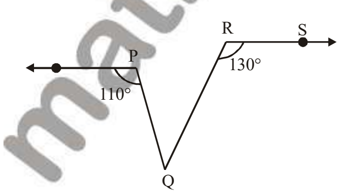
35. In the given Fig. AB || CD. Find the value of x.



(a) 72° (b) 140°

(c) 108° (d) 112°

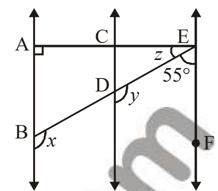
36. In the given Fig. OP || RS. Determine $\angle PQR$.



(a) 70° (b) 65°

(c) 60° (d) 45°

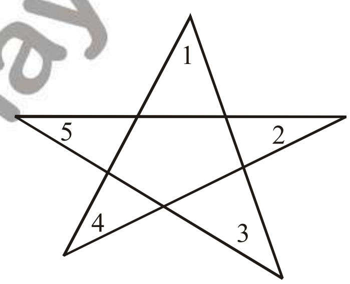
37. In the given Fig. AB||CD and CD||EF. Also, EA $\perp$ AB. If $\angle BEF = 55°$, find the value of z.



(a) 35° (b) 45°

(c) 125° (d) 55°

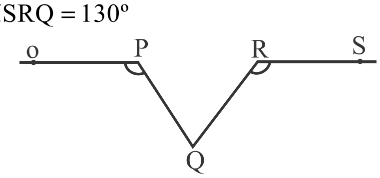
38. In the given Fig. find $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5$?



(a) 180° (b) 270°

(c) 360° (d) 540°

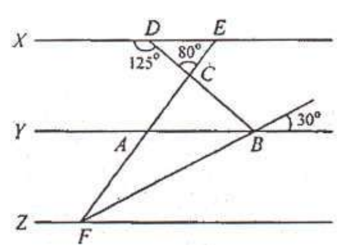
39. In the given figure OP || RS . Determing $\angle PQR$ if $\angle OPQ = 110°$ $\angle SRQ = 130°$



(a) 45° (b) 60°

(c) 50° (d) 40°

40.



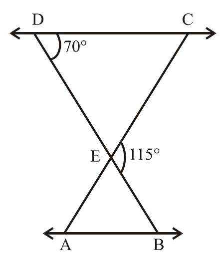
Three straight lines X, Y and Z are parallel and the angles are as shown in the figure above. $\angle AFB$ equal to?

(a) 20^0 (b) 15^0
(c) 30^0 (d) 10^0

41. The line segments AB and CD intersect at O. OF is the internal bisector of obtuse $\angle$ BOC and OE is the internal bisector of acute $\angle$ AOC. If $\angle BOC = 130^o$, what is the measure of $\angle FOE$?

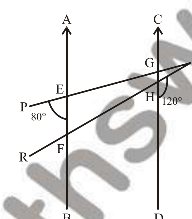
(a) 90^o (b) 110^o
(c) 115^o (d) 120^o

42. In the fig. If $\angle EDC = \angle EBA$, $\angle BEC = 115°$ and $\angle EDC = 70°$. Find the $\angle DCE$ and $\angle AEB$.



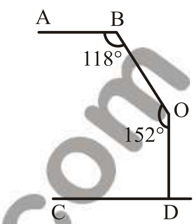
(a) 25°, 45° (b) 45°, 55°
(c) 15°, 25° (d) 45°, 65°

43. In the figure, AB ||CD and PQ, QR intersect AB and CD both at E, F, G and H respectively. Given that $\angle PEB = 80°$, $\angle QHD = 120°$ and $\angle PQR = x$, then find the value of x.



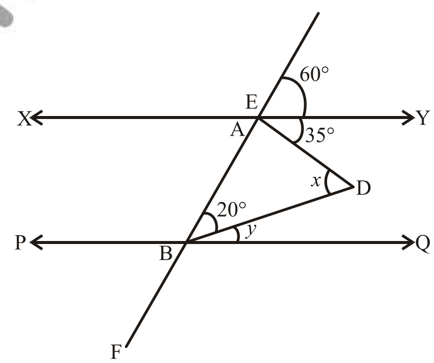
(a) 40° (b) 20°
(c) 100° (d) 120°

44. In the Fig. If AB||CD, $\angle ABO = 118°$ and $\angle BOD = 152°$ then find the value of $\angle ODC$.



(a) 70° (b) 80°
(c) 90° (d) 34°

45. In the given figure, if XY || PQ, then find the value of $\angle x$ and $\angle y$.



(a) 75°, 40° (b) 45°, 60°
(c) 75°, 45° (d) 60°, 45°

Answer

1.	(c)	2.	(b)	3.	(d)	4.	(a)	5.	(a)	6.	(a)	7.	(a)	8.	(b)	9.	(b)
10.	(b)	11.	(a)	12.	(c)	13.	(a)	14.	(a)	15.	(b)	16.	(b)	17.	(a)	18.	(c)
19.	(b)	20.	(a)	21.	(b)	22.	(a)	23.	(a)	24.	(c)	25.	(a)	26.	(a)	27.	(a)
28.	(b)	29.	(a)	30.	(a)	31.	(b)	32.	(a)	33.	(a)	34.	(a)	35.	(b)	36.	(c)
37.	(a)	38.	(a)	39.	(b)	40.	(b)	41.	(a)	42.	(d)	43.	(b)	44.	(c)	45.	(a)

Solution & Hints

Sol^n 1. Sum of linear pair angle $= 180^o$

so, $5x + 12^o + 3x = 180^o$

$8x = 168^o$

$x = 21^o$

Sol^n 2. **In Ques. fig.**

$\Rightarrow \angle AOE + \angle EOD + \angle DOB = 180^o$

$\Rightarrow 40^o + \angle EOD + 31^o = 180^o$

$\Rightarrow \angle EOD = 109^o$

$\Rightarrow \angle COF = \angle EOD = 109^o$

$\Rightarrow \angle FOB = \angle AOE = 40^o$

$\Rightarrow \angle BOC = \angle BOF + \angle COF = 40^o + 109^o = 149^o$

Sol^n 3. For complementary Angle

$2x + 17^o + x + 4^o = 90^o$

$3x = 90^o - 21^o$

$x = 23^o$

Sol^n 4. $5y + 62^o + 22^o + y = 180^o$

$6y = 180^o - 84^o$

$y = 16^o$

Sol^n 5. For supplementery Angle

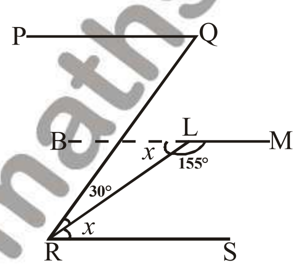
$13x + 5x = 180^o$

$18x = 180^o$

$x = 10^o$

greater angle $= 13 \times 10 = 130^o$

Sol^n 6.

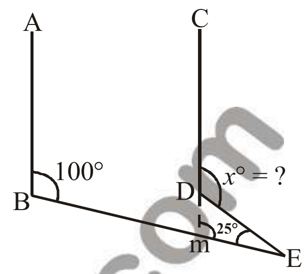


$\angle BLR = \angle SRL$

$\angle BLR = 180^o - 155^o = 25^o$

$\angle SRL = x = 25^o$

Sol^n 7.



$\angle ABE = \angle CME = 100^o$

and $\angle CDE = (\angle CME + \angle DEM)$

($\because$ Exterior Angle property)

$\angle CDE = 100^o + 25^o = 125^o$

Sol^n 8. Let angle be x

then,

$$x = \frac{1}{5}(180^o - x)$$

$$x = 30^o$$

Sol^n 9. Let angle be x

then,

$$x + x = 90^o$$

$$x = 45^o$$

Sol^n 10. Let angle be x

then,

$\Rightarrow x + x = 180^o$

$\Rightarrow x = 90^o$

Sol^n 11. $\angle COF = \angle DOE = 2x$

$\angle FOB = \angle AOE = 5x$

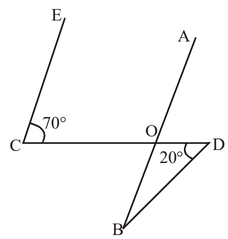
$\angle COA = \angle DOB = 3x$

then,

$\Rightarrow 2(2x + 3x + 5x) = 360^o$

$10x = 180^o \quad x = 18^o$

Sol^n 12.



EC || AB

$\Rightarrow$ $\angle ECO \parallel \angle AOD = 70^o$

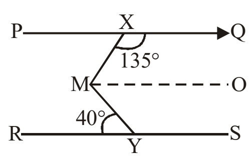
$\angle ECO \parallel \angle AOD$ (Correspondence angle)

$\angle AOD = \angle OBD + ODB$

$\angle OBD = 70^o - 20^o = 50^o$

Sol^n 13. $PQ \parallel RS$, $\angle MXQ = 135^o$

$\angle MYR = 40^o$



$\angle OMY = \angle RYM = 40^o$

$\angle MXQ + \angle MXP = 180^o$

then,

$\angle MXP = 180^o - 135^o = 45^o$

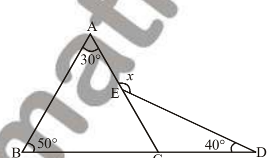
&

$\angle MXP = \angle XMO = 45^o$ and

$\angle XMY = \angle XMO + \angle OMY = 45^o + 40^o$

$= 85^o$

Sol^n 14.



$\angle ACD = 30^o + 50^o = 80^o$

$\angle AED = \angle ACD + \angle EDC$

$= 80^o + 40^o = 120^o$

Sol^n 15. $\angle ABC = 180^o - 90^o - 40^o = 50^o$

$\angle EDB = 180^o - 100^o - 50^o = 30^o$

Sol^n 16. $y + 90^o + 2x + 3x + 105^o = 360^o$

$5x + y = 360^o - 195^o$

$5x + y = 165^o$ ....(i)

and $2x + y + 90^o = 180^o$

$2x + y = 90^o$ ....(ii)

From equation (i) & (ii)

$x = 25^o$

$y = 40^o$

$x + y = 25^o + 40^o = 65^o$

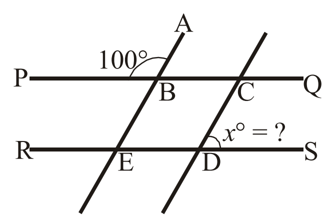
Sol^n 17. $\angle FGC = \angle CHI = 80^o$

Sol^n 18. AB|| CD|| EF ||GH

$$\frac{CE}{AG} = \frac{DF}{BH}$$

$$\Rightarrow DF = \frac{30}{120} \times 100 = 25\,cm$$

Sol^n 19.

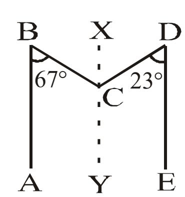


$\angle ABP = \angle CBE = \angle CDE = 100^o$

$\angle CDS = 180^o - \angle CDE$

$= 180^o - 100^o = 80^o$

Sol^n 20.



Draw a parallel line XY || AB|| DE

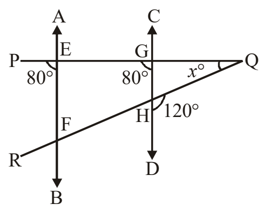
$\angle XCB = \angle ABC = 67^\circ$

$\angle XCD = \angle CDE = 23^\circ$

$\angle BCD = \angle BCX + \angle DCX$

$= 67^\circ + 23^\circ = 90^\circ$

Sol^n 21.



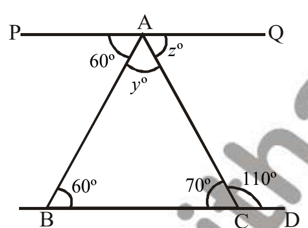
$\angle PEF = \angle PGD = 80^\circ$

$\angle HGQ = 180^\circ - 80^\circ = 100^\circ$

$\angle GQH = 120^\circ - 100^\circ = 20^\circ$

(Exterior Angle property)

Sol^n 22.

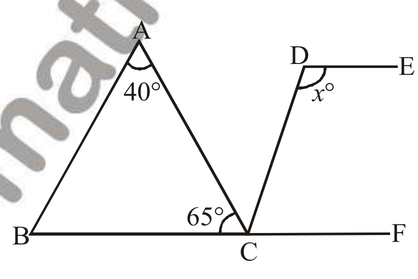


$\angle BAC = 180^\circ - 60^\circ - 70^\circ = 50^\circ$

Sol^n 23. $\angle ABE = \angle CEF = 70^\circ$

$\angle EDF = 70^\circ - 20^\circ = 50^\circ$

Sol^n 24.



AB ||CD and DE||BF

$\angle ABC = \angle DCF = 180^\circ - 40^\circ - 65^\circ = 75^\circ$

$\angle EDC + \angle DCF = 180^\circ$

$\angle EDC = 180^\circ - 75^\circ = 105^\circ$

Sol^n 25. $\angle PEA = \angle POC = 34^\circ$

$\therefore 34^\circ + x + 78^\circ = 180^\circ$

$x = 68^\circ$

Sol^n 26. $x + y = 180^\circ$ …(i)

$x - y = 34^\circ$ …(ii)

From equation (i) & (ii) we get

$x = 107^\circ$

$y = 73^\circ$

Sol^n 27. Let angle = x°

According to Question

$x = 5(90^\circ - x)$

$6x = 5 \times 90^\circ$

$x = 75^\circ$

Sol^n 28. Let Angle = x°

According to Question

$x = \frac{1}{3}(180^\circ - x)$

$x = 45^\circ$

Sol^n 29. $2x + 3x = 180^\circ$

$x = 36^\circ$

smaller Angle = 72°

Sol^n 30. $3x + 3x + x + 150^\circ = 368^\circ$

$x = 30^\circ$

Sol^n 31. $x^\circ = z^\circ = 45^\circ$

$y^\circ = 180^\circ - x^\circ = 135^\circ$

Sol^n 32. $y = 25^\circ$

(opposite vertical angle)

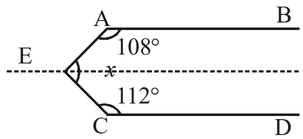
Sol^n 33. $\angle 1 = 55^\circ$

(alternate angle)

Sol^n 34. $x = 360^\circ - (45^\circ + 30^\circ)$

$x = 285^\circ$

Sol^n 35.

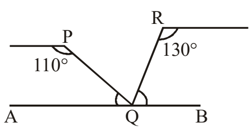


$108^\circ + x^\circ + 112^\circ = 360^\circ$

$x = 360^\circ - 220^\circ$

$= 140^\circ$

Sol^n 36.



$OP \| RS$

$\angle PQA = 70^\circ$

$\angle RQB = 50^\circ$

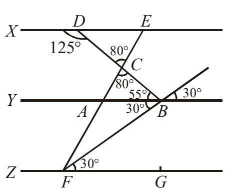
$\angle PQR = 180^\circ - 120^\circ = 60^\circ$

Sol^n 37. $\angle z = 90^\circ - 55^\circ = 35^\circ$

Sol^n 38. $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 = 180^\circ$

Sol^n 39. Same Q. 36

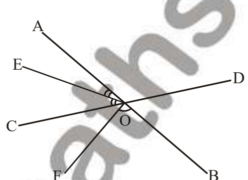
Sol^n 40.



$\angle DBY = 180^\circ - 125^\circ = 55^\circ$

$\angle AFB = 180^\circ - 80^\circ - 55^\circ - 30^\circ = 15^\circ$

Sol^n 41.



$\angle BOC = 130^\circ$

$\angle COF = \dfrac{130^\circ}{2} = 65^\circ$

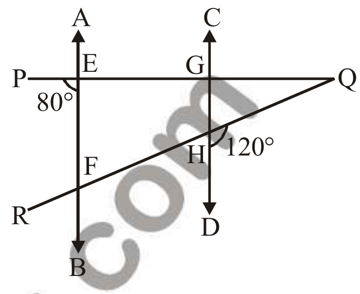
$\angle FOC = \dfrac{50^\circ}{2} = 25^\circ$

$\angle FOE = 65^\circ + 25^\circ = 90^\circ$

Sol^n 42. $\angle DCE = 115^\circ - 70^\circ = 45^\circ$

and $\angle AEB = 180^\circ - 115^\circ = 65^\circ$

Sol^n 43.



$\angle PEF = \angle EGH = 80^\circ$

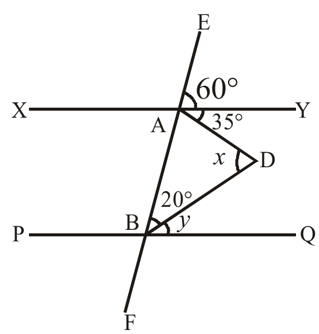
$\angle GHQ = 180^\circ - 120^\circ = 60^\circ$

$\angle PQR = 80^\circ - 60^\circ = 20^\circ$

Sol^n 44. $\angle ABO + \angle BOD + \angle ODC = 360^\circ$

$\angle ODC = 360^\circ - 118^\circ - 152^\circ = 90^\circ$

Sol^n 45.



$XY \| PQ$

$\angle BAD = 180^\circ - 60^\circ - 35^\circ = 85^\circ$

In ΔADB

$\angle BAD + \angle ADB + \angle ABD = 180^\circ$

$85^\circ + x^\circ + 20^\circ = 180^\circ$

$x = 75^\circ$

$\angle EAY = \angle EBQ = 60^\circ$

$\angle EBQ = 20^\circ + y^\circ = 60^\circ$

$y^\circ = 40^\circ$

Exercise-Triangle

1. If the medians of a triangle are equal, then the triangle is:
(a) Right angled (b) Isosceles
(c) Equilateral (d) Scalene

2. The Incentre of a triangle is determined by the:
(a) Medians (b) Angle bisector
(c) Sides Perpendicular bisector
(d) Altitudes (height)

3. The circum-centre of a triangle is determined by the:
(a) Medians (b) Angle bisector
(c) Perpendicular bisector
(d) Altitudes (height)

4. The points of intersection of the angle bisector of a triangle is:
(a) Ortho centre (b) Centroid
(c) In centre (d) Circum centre

5. In $\Delta ABC, AC=5cm$. Calculate the length of AE where DE||BC. Given that AD = 3cm and BD = 7 cm.
(a) 2 cm (b) 1 cm
(c) 1.5 cm (d) 2.5 cm

6. If O is the circum-centre of ΔABC and $\angle OBC = 35^o$. Find $\angle BAC$
(a) 55° (b) 110°
(c) 70° (d) 35°

7. If G is the centroid and AD, BE, CF are three medians of ΔABC with area 72cm^2, then the area of ΔBDG is:
(a) 12 cm^2 (b) 16 cm^2
(c) 24 cm^2 (d) 8 cm^2

8. Three medians AD, BE and CF of ΔABC intesect at point G. If the area of ΔABC is 60sq. cm, then the area of the quadrilateral BDGF is:
(a) 10 cm^2 (b) 15 cm^2
(c) 20 cm^2 (d) 30 cm^2

9. In ΔABC, AD is the median and $AD = \frac{1}{2} BC$. If $\angle BAD = 30°$, then the measure of $\angle ACB$ is:
(a) 30° (b) 60°
(c) 90° (d) 45°

10. D and E, are the mid-points of AB and AC of ΔABC, BC is produced to any point P, DE, DP and EP are joined, then,
(a) $\Delta PED = \Delta BEC$ (b) $\Delta ADE = \Delta BEC$
(c) $\Delta BDE = \Delta BEC$ (d) $\Delta PED = \frac{1}{4} \Delta ABC$

11. In the right angle ABC. BD divides the triangle ABC into two triangles of equal perimeters. Find the length of BD, given that AC = 100, BC = 80. $\angle B = 90^o$
(a) 25 (b) $24\sqrt{5}$
(c) $20\sqrt{5}$ (d) None of these

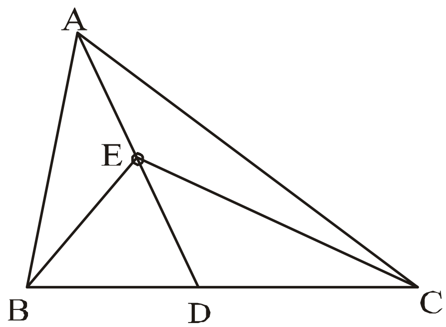
12. If the ratio of areas of two similar triangles is 9 : 16, then the ratio of their corresponding sides is:
(a) 3 : 5 (b) 3 : 4
(c) 4 : 5 (d) 4 : 3

13. Two medians AD and BE of ΔABC intersect at G at right angles. If AD = 9 cm and BE = 6 cm. Then the length of BD in cm is:
(a) 10 (b) 6
(c) 5 (d) 3

14. In ΔABC, PQ is parallel to BC. If AP:PB = 1:2 and AQ = 3 cm AC is equal to :
(a) 6 cm (b) 9 cm
(c) 12 cm (d) 8 cm

15. If O be the in-centre of a triangle ABC and D be a point on the side BC of ΔABC, such that $OD \perp BC$. If $\angle BOD = 15°$ then $\angle ABC$ = ?
(a) 75° (b) 45°
(c) 150° (d) 90°

16. The vertex A of ΔABC is joined to a point D on BC. If E is the midpoint of AD then ar $(\Delta BEC) = ?$



(a) $\frac{1}{2}$ area (ΔABC) (b) $\frac{1}{3}$ area (ΔABC)

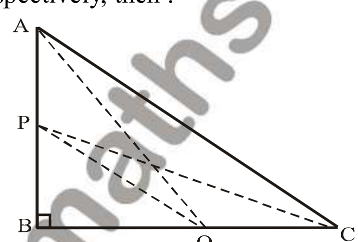
(c) $\frac{1}{4}$ area (ΔABC) (d) $\frac{1}{6}$ area (ΔABC)

17. A triangle PQR is drawn to circumscribe a circle of radius 8 cm such that the segments QT and TR, into which QR is divided by the point of contact T, are of lengths 14 cm and 16 cm respectively. If the area of ΔPQR is 336 cm^2, find the sides PQ and PR.

(a) 26, 28 (b) 18, 26

(c) 24, 26 (d) 20, 22

18. In a right-angled ΔABC, right-angled at B, if P and Q are points on the sides AB and BC respectively, then :



(a) $AQ^2 + CP^2 = (AC^2 + PQ^2)$

(b) $2(AQ^2 + + CP^2) = AC^2 + PQ^2$

(c) $AQ^2 + CP^2 = AC^2 + PQ^2$

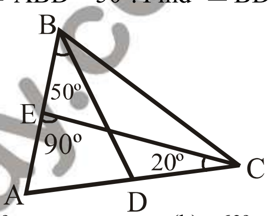
(d) $AQ + CP = \frac{1}{2}(AC + PQ)$

19. In a triangle ABC, the sum of the exterior angles at B and C is equal to :

(a) $180° + \angle BAC$ (b) $180° - \frac{1}{2}\angle BAC$

(c) $180° + \frac{1}{2}\angle BAC$ (d) $180° + 2\angle BAC$

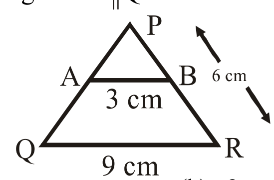
20. In the given figure $CE \perp AB$, $\frac{1}{2}\angle ACE = 20°$ and $\angle ABD = 50°$. Find $\angle BDA$:



(a) 50º (b) 60º

(c) 70º (d) 80º

21. In the figure $AB \parallel QR$. Find the length of PB:



(a) 2 cm (b) 3 cm

(c) 2.5 cm (d) 4 cm

22. Area of $\Delta ABC = 30$ cm^2. D and E are the midpoint of BC and AB. Find Area (ΔBDE) :

(a) 10 cm^2 (b) 7.5 cm^2

(c) 15 cm^2 (d) 30 cm^2

23. In ΔABC, P and Q are the mid points of the sides AB and AC. R is a point on the segment PQ such that PR : RQ = 1 : 2. If PR = 2 cm, then BC = ?

(a) 4 cm (b) 2 cm

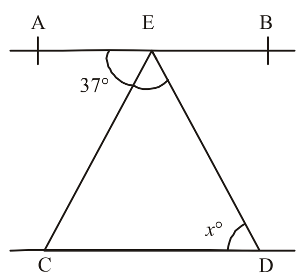
(c) 12 cm (d) 6 cm

24. The in-radius of an equilateral triangle is of length 3 cm. Then the length of each of its medians is :

(a) 12 cm (b) $\frac{9}{2}$ cm.

(c) 4 cm. (d) 9 cm.

25. If AB || CD and CE $\perp$ ED, then find $x°$.



(a) 53° (b) 63°
(c) 37° (d) 45°

26. Side AB of rectangle ABCD is divided into four equal parts by points X, Y, Z the ratio of the.

$\frac{\text{Area of } \Delta XYD}{\text{Area of Rectangle ABCD}}$ is

(a) 1/7 (b) 1/6
(c) 1/9 (d) 1/8

27. Two angles of a triangle are 1/2 radian and 1/3 radian. The measure of the third angle in degrees $\left(\pi = \frac{22}{7}\right)$.

(a) $132\frac{1}{11}°$ (b) $32\frac{2}{11}°$

(c) $132\frac{3}{11}°$ (d) 132°

28. Consider ΔABD such that $\angle ADB = 20°$ and C is a point on BD such that AB = AC and CD = CA. Then the measure of $\angle ABC$ is

(a) 40° (b) 45°

(c) 60° (d) 30°

29. Two sides of a triangle are of length 4 cm and 10 cm. If the length of the third side is a cm. then,

(a) $a < 6$ (b) $6 < a < 14$

(c) $a > 5$ (d) $6 \leq a \leq 12$

30. The side BC of Δ ABC is produce to D. If $\angle ACD = 120°$ and $\angle B = \frac{1}{2} \angle A$, then $\angle A$ is

(a) 108° (b) 80°

(c) 36° (d) 72°

31. O is the circumcentre of the triangle ABC with circumradius 13 cm. Let BC = 24 cm and OD is perpendicular to BC. Then the length of OD is

(a) 3 (b) 4

(c) 5 (d) 7

32. By decreasing 15° of each angle of a triangle, the ratio of their angles are 2 : 3 : 5. The radian measure greatest angle is :

(a) $\frac{7\pi}{12}$ (b) $\frac{\pi}{24}$

(c) $\frac{5\pi}{24}$ (d) $\frac{11\pi}{24}$

33. In a right angled triangle ABC, $\angle B$ is right angle and AC = $2\sqrt{5}$ cm. If AB – BC = 2 cm, then the value of $(\cos^2 A - \cos^2 C)$ is

(a) $\frac{3}{5}$ (b) $\frac{6}{5}$

(c) $\frac{3}{10}$ (d) $\frac{2}{5}$

34. The perimeter of two similar triangles ΔABC and ΔPQR are 36 cm and 24 cm respectively. If PQ = 10. Then AB is

(a) 10 (b) 15
(c) 20 (d) 25

35. ΔABC and ΔDEF are similar and their areas are respectively 64 cm^2 and 121 cm^2. If EF = 15.4 cm, then BC is

(a) 11.2 (b) 12.1
(c) 11.0 (d) 12.3

36. In ΔABC, $\angle A$ is a right angle and AD is perpendicular to BC. If AD = 4 cm BC = 12, then the value of (cot B + cot C) is

(a) 4 (b) $\frac{3}{2}$
(c) 6 (d) 3

37. In ΔABC, AD is drawn perpendicular from A on BC. If AD^2 = BD. CD, then $\angle BAC$ is
(a) 60° (b) 90°
(c) 30° (d) 45°

38. In a right angled triangle ABC, AB = 2.5 cm, cos B = 0.5, $\angle ACB = 90°$. The length of side AC, in cm is:

(a) $\frac{5}{4}\sqrt{3}$ (b) $\frac{5}{16}\sqrt{3}$
(c) $5\sqrt{3}$ (d) $\frac{5}{2}\sqrt{3}$

39. In ΔABC, D, E, F are mid-points of AB, BC, CA respectively and $\angle B = 90°$, AB = 6 cm, BC = 8 cm. Then area of ΔDEF (in sq. cm) is:
(a) 6 (b) 12
(c) 24 (d) 28

40. If in a triangle ABC, the angles at B and C are 1.5 and 2.5 times of the angle at A respectively, then angle at B is :
(a) 48° (b) 72°
(c) 36° (d) 54°

41. In triangle ABC, AB = 12 cm, $\angle B = 60°$, then perpendicular from A to BC meets it at D. The bisector of $\angle ABC$ meets AD at E. Then E divides AD in the ratio.
(a) 1 : 1 (b) 2 : 1
(c) 3 :1 (d) 6 : 1

42. If P denotes are perimeter and S denotes the sum of the distances of a point within a triangle from its angular points, then:
(a) S < P (b) $S \leq P$
(c) P < S (d) P < 2S

43. The length of the circum radius of a triangle having sides of length 12 cm, 16cm and 20 cm is:
(a) 15 cm (b) 10 cm
(c) 18 cm (d) 16 cm

44. If BE and CF be the two medians of a Δ ABC and G be their intersection. Also let EF cut AG at O. Then AO : OG is
(a) 1 : 1 (b) 1 : 2
(c) 2 : 1 (d) 3 : 1

45. O and C are respectively the orthocentre and circum-centre of an acute-angled triangle PQR. The points P and O joined and produced to meet the sides QR at S. If $\angle PQS = 60°$ and $\angle QCR = 130°$ then $\angle RPS = ?$
(a) 30°- (b) 35°
(c) 100° (d) 60°

46. D is any point on side AC of ΔABC. If P, Q, X, Y are the mid points of AB, BC, AD and DC respectively, then the ratio of PX and QY is :
(a) 1 : 2 (b) 1 : 1
(c) 2 : 1 (d) 2 : 3

47. In Δ ABC, DE||BC and DE : BC = 3 : 5 then, Area (Δ ADE) : Area (trapezium BDEC) is :
(a) 3 : 2 (b) 3 : 5
(c) 9 : 25 (d) 9 : 16

48. ABCD is a square. BCE and ACF are equilateral triangles described on side BC and diagonal AC respectively then, the area (ΔBCE) : area (ΔACF) is :
(a) 1 : 2 (b) 2 : 1
(c) 4 : 1 (d) 1 : 4

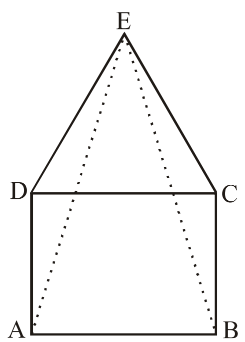
49. One angle of a triangle is 108°; the angle included between the internal bisectors of the two acute angles of the triangle is
(a) 144° (b) 54°
(c) 72° (d) 136°

50. ΔABC is right angled at A. AB = 3 units. AC = 4 units and AD is perpendicular to BC. What is the area of the ΔADB

(a) $\frac{9}{25}$ sq unit (b) $\frac{54}{25}$ sq unit

(c) $\frac{72}{25}$ sq unit (d) $\frac{96}{25}$ sq unit

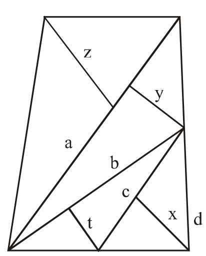
51. If ABCD is a square and DCE is an equilateral triangle in the given figure, then $\angle$ DAE is equal to



(a) 45° (b) 30°

(c) 15° (d) $22\frac{1}{2}$

52. A surveyor in his field book has drawn the plot as shown in the given figure. The area of the plot is:



(a) $\frac{1}{2}(az + by + ct + dx)$

(b) $\frac{1}{2}(bt + cx + ay + az)$

(c) $\frac{1}{2}(cx + bt + by + az)$

(d) $\frac{1}{2}(b + t)(c + x) + \frac{1}{2}(a + b)(y + z)$

53. In a triangle ABC, points D, E and F are mid-points of sides AB, BC and CA. If $\angle BAC = 70^o$ then find $\angle DEF$?

(a) 130º (b) 70º

(c) 60º (d) 50º

54. In ΔABC, D and E are points on AB and AC respectively such that $DE \parallel BC$ and DE divides the ΔABC into two parts of equal areas. Then ratio of AD and BD is:

(a) $1:\sqrt{2}$ (b) $1:\sqrt{2}+1$

(c) $1:1$ (d) $1:\sqrt{2}-1$

55. O is any point inside a ΔABC. The bisectors of $\angle AOB, \angle BOC$ and $\angle COA$ meet the sides AB, BC and CA in points D, E, F respectively. $AD \times BE \times CF$ is equal to

(a) DB.EC.FA (b) DB.AC.FA

(c) AB.EC.FA (d) DB.EC.AC

56. In a ΔABC, AD is a median. The bisectors of $\angle ADB$ and $\angle ADC$ meet AB and AC at E and F respectively. If the ratio of $AE:BE = 3:4$, then find the ratio of $EF:BC$.

(a) $3:4$ (b) $4:3$

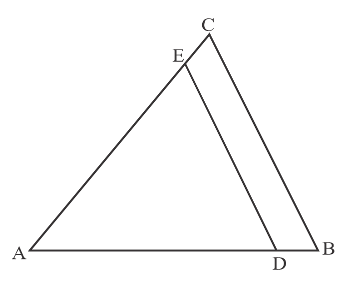
(c) $7:3$ (d) $3:7$

57. In an equilateral ABC, if AD $\perp$ BC then which of the following would be correct?

(a) $2AB^2 = 3AD^2$ (b) $3AB^2 = 4AD^2$

(c) $5AB^2 = 6AD^2$ (d) $4AB^2 = 5AD^2$

58.



In the figure given above, DE||BC, AD = x, AE = x+2, DB = x–2 and EC = x–1

What is the value of x?

(a) 3 (b) 4

(c) –3 (d) – 4

59. ABC is an equilateral traingle. P and Q are two points on $\overline{AB}$ and $\overline{AC}$ respectively such that $\overline{PQ} \parallel \overline{BC}$. If $\overline{PQ} = 5cm$, then area of ΔAPQ is-

(a) 25 cm^2 (b) $\frac{25\sqrt{3}}{4}$ cm^2

(c) $\frac{25}{\sqrt{3}}$ cm^2 (d) $25\sqrt{3}$ cm^2

60. ΔPQR is right angled triangle at Q, PR = 5 cm and QR = 4 cm. If the lengths of sides of another ΔABC are 3 cm, 4 cm and 5 cm, than which one of the following is correct?

(a) Area of ΔPQR is double that of ΔABC

(b) Area of ΔABC is double that of ΔPQR

(c) $\angle B = \frac{\angle Q}{2}$

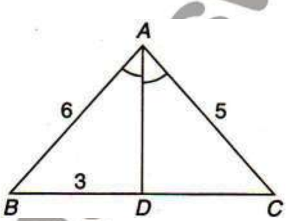
(d) Both triangles are congruent

61. In ΔABC and ΔDEF, it is given that AB = 5 cm, BC = 4 cm and CA = 4.2 cm, DE = 10 cm, EF = 8 cm and FD = 8.4 cm. If AL is perpendicular to BC and DM is perpendicular to EF, then what is the ratio of AL to DM?

(a) 1/2 (b) 1/3

(c) 1/4 (d) 1

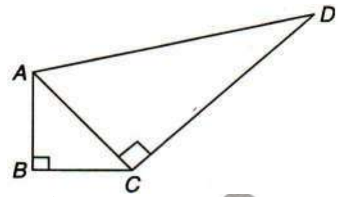
62. In the above figure, AD is the bisector of $\angle BAC$, AB = 6 cm, AC = 5cm and BD = 3 cm. Find DC.



(a) 11.3 cm (b) 2.5 cm

(c) 3.5 cm (d) 4 cm

63. In the given figure, ΔABC and ΔACD are right angle triangles and AB = x cm, BC = y cm, CD = $z$cm and $x.y = z$ and x, y and z has minimum integral value. Find the area of ABCD



(a) 36 cm^2 (b) 64 cm^2

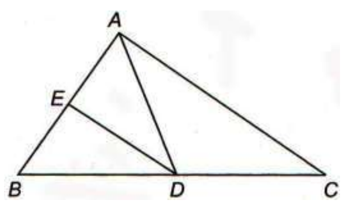
(c) 24 cm^2 (d) 25 cm^2

64. The points D and E are taken on the sides AB and AC of ΔABC such that $AD = \frac{1}{3}AB$, $AE = \frac{1}{3}AC$. If the length of BC is 15 cm, then the length of DE is

(a) 10 cm (b) 8 cm

(c) 6 cm (d) 5 cm

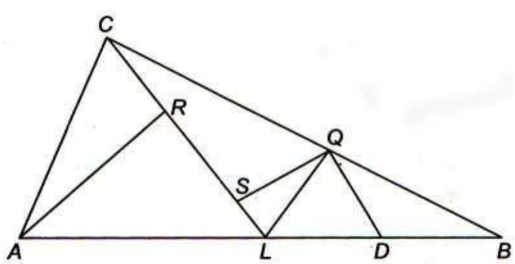
65. In the given figure, ABC is a triangle in which AD and DE are medians to BC and AB respectively, the ratio of the area of ΔBED to that of ΔABC is



(a) 1 : 4 (b) 1 : 16

(c) Date inadequate (d) None of these

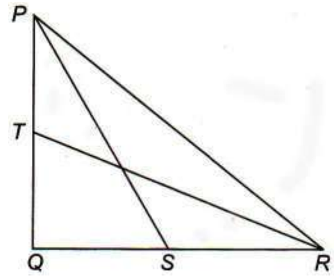
66. In the figure (not drawn to scale) given below, L is a point on AB such that AL : LB = 4 : 3. LQ is parallel to AC and QD is parallel to CL. In $\Delta ARC, \angle ARC = 90^o$ What is ratio $AL : LD$?



(a) 3 : 7 (b) 4 : 3

(c) 7 : 3 (d) 8 : 3

67. In the figure given below PS & RT are the medians each measuring 4 cm. triangle PQR is right angled at Q. what is the area of the triangle PQR?



(a) 5.2 (b) 6.4
(c) 6.2 (d) 7.2

68. ΔABC is a right angled triangle AB = 6 cm BC= 8 cm and AC = 10 cm then the in radius of the incircle is:
(a) 1 cm (b) 2 cm
(c) 3 cm (d) 4 cm

69. A right triangle has hypotenuse p cm and one side of length q cm. If $p - q = 1$, find the length of the third side of the triangle.
(a) $\sqrt{2q+1}\,cm$ (b) $\sqrt{2p+1}$
(c) $\sqrt{p^2+q^2}$ (d) None of these.

70. ABC is an isosceles triangle right-angled at B. Similar triangles ACD and ABE are constructed on sides AC and AB. Find the ratio between the areas of ΔABE and ΔACD.
(a) 1 : 2 (b) 2 : 1
(c) 1 : 4 (d) 4 : 1

71. If ABC is a triangle right angled at C and having u units, v units, w units as the lengths of its sides opposite to the vertices A, B, C respectively, then what is the tangent of the angle at A + tangent of the angle at B equal to ?
(a) $\frac{u^2}{(vw)}$ (b) 1
(c) $u + v$ (d) $\frac{w^2}{(uv)}$

72. In ΔABC, $\angle B = 90^o$, AB = 5 cm, BC = 12 cm find BQ If Q is centroid.
(a) $3\frac{1}{2}$ cm (b) $4\frac{1}{3}$ cm
(c) $4\frac{1}{2}$ cm (d) $5\frac{1}{2}$ cm

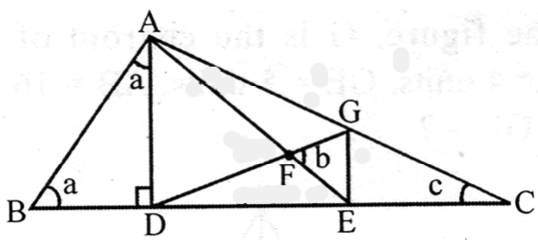
73. In a triangle ABC, the lengths of the sides AB and AC equal to 17.5 cm an 9 cm respecively. Let D be a point on the line segment BC such that AD is perpendicular to BC. If AD = 3 cm, the what is the radius (in cm) of the circle circumscribing the triangle ABC?
(a) 27.85 (b) 32.25
(c) 26.25 (d) 22.45

74. In ΔPQR, S and T are points on sides PR and PQ respectively such that $\angle PQR = \angle PST$. If PT = 5 cm, PS = 3 cm and TQ = 3 cm, then length of SR is:
(a) 5 cm (b) 6 cm
(c) $\frac{31}{3}$ cm (d) $\frac{41}{3}$ cm

75. The sides of a triangle are in geometric progression with common ratio $r < 1$. If the triangle is a right-angled triangle, then r^2 is given by
(a) $\frac{\sqrt{5}+1}{2}$ (b) $\frac{\sqrt{5}-1}{2}$
(c) $\frac{\sqrt{3}+1}{2}$ (d) $\frac{\sqrt{3}-1}{2}$

76. ABC is a triangle in which AB = AC. Let BC is produced to D. From a point E on the line AC let EF be a straight line such that EF is parallel to AB. Consider the quadrilateral ECDF thus formed. If $\angle ABC = 65^0$ and $\angle EFD = 50^0$, then what is $\angle FDC$ equal to ?
(a) 43^0 (b) 41^0
(c) 37^0 (d) 65^0

77. In ΔABC (not drawn to scale). $AD = AG = GC$ and $FD = FG$. Find $a + b + c$.



(a) 210° (b) 165°
(c) 135° (d) 175°

78. In triangle PQR, PS is the median to QR. Find the length of PS if $PQ = PR = 5$ cm and $QR = 4$ cm

(a) $\sqrt{21}$ cm (b) $\sqrt{19}$ cm
(c) $\sqrt{30}$ cm (d) None of these

79. In a triangle ABC, angle bisector of $\angle BAC$ cut the side BC at D and meet the circumcircle of ΔABC at E, then

$AB.AC + DE.AE =$

(a) AD^2 (b) AE^2
(c) CE^2 (d) CD^2

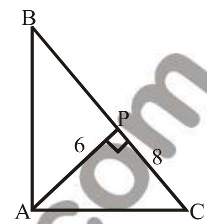
80. The length of each side of an equilateral triangle is $14\sqrt{3}$ cm. The area of the incircle $\left(\text{in cm}^2\right)$ is:

(a) 450 (b) 308
(c) 154 (d) 77

81. A triangular region ABC whose height AD is $20\sqrt{2}$ cm is to be divided into two regions of equal areas by drawing a straight line L Parallel to the straight line BC. What is the perpendicular distance of L from the point A?

(a) 10 cm (b) $10\sqrt{2}\,cm$
(c) 15 cm (d) 20 cm

82. In the given figure $\angle BAC = 90^o$, AP = 6 cm, PC = 8 cm and $\angle APC = 90^o$, find the area of ΔABC. (where P is point on, BC)



(a) 75 cm^2 (b) 240 cm^2
(c) $\frac{75}{2}$ cm^2 (d) 150 cm^2

83. In a ΔABC, AB = BC and $\angle BAC = 15^\circ$, AB = 10 cm. find the area of ΔABC :

(a) 50 cm^2 (b) 40 cm^2
(c) 25 cm^2 (d) 32 cm^2

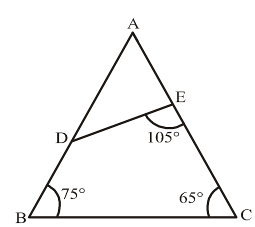
84. In ΔABC, G is the centroid, AB = 15 cm, BC= 18 cm, and AC = 25 cm. Find GD, Where D is the mid point of BC :

(a) $\frac{1}{2}\sqrt{86}$ cm (b) $\frac{1}{3}\sqrt{86}$ cm
(c) $\frac{7}{3}\sqrt{86}$ cm (d) $\frac{2}{3}\sqrt{86}$ cm

85. ΔABC, the bisector of $\angle B$ meets AC at D. A line PQ||AC meets AB, BC and BD at P, Q and R respectively. Then AB $\times$ CQ = ?

(a) $AP \times BC$ (b) $BC \times AB$
(c) $CQ \times AB$ (d) $RQ \times BA$

86. In the given figure, if $\frac{DE}{BC} = \frac{2}{3}$ and AE = 10 cm, then find the value of AB.



(a) 16 cm (b) 12 cm
(c) 15 cm (d) 18 cm

87. ABC is an isosceles triangle in which AB = AC and $\angle A = 2\angle B$, AB = 4cm. What is the ratio of inradius to the circumradius.

(a) 1 : 2 (b) $(\sqrt{2}-1):1$
(c) $1:(2\sqrt{2}+1)$ (d) None of these

88. In triangle ABC a straight line parallel to BC intersects AB and AC at D and E respectively. IF AB = 2AD then DE : BC is

(a) 2 : 3 (b) 1 : 2
(c) 3 : 1 (d) 1 : 3

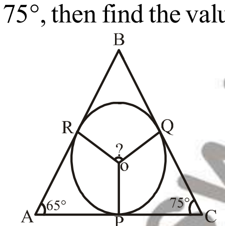
89. ABC is an isosceles triangle such that AB = AC and AD is the median to the base BC with $\angle ABC = 35°$. Then $\angle BAD$ is

(a) 35° (b) 55°
(c) 70° (d) 110°

90. If D, E and F are the mid-points of sides BC, CA and AB respectively of a ΔABC, then DEF is congruent of triangle(s)-

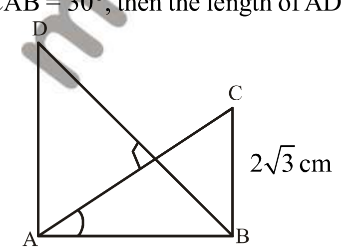
(a) ABC (b) AEF
(c) BFD, CDE (d) AFE, BFD, CDE

91. In a ΔABC, O is the incentre, $\angle BAC = 65°$ and $\angle BCA = 75°$, then find the value of $\angle ROQ$.



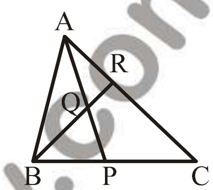
(a) 80° (b) 120°
(c) 140° (d) Can't be determined

92. In the figure given below, ABC is right angled at B and ΔABD is right angled at A. If BD is perpendicular to AC and BC = $2\sqrt{3}$ cm with $\angle CAB = 30°$, then the length of AD is –



(a) $5\sqrt{3}$ cm (b) $4\sqrt{3}$ cm
(c) $7\sqrt{3}$ cm (d) $6\sqrt{3}$ cm

93. In the figure, P is the mid-point of BC and Q is the mid-point of AP. If BQ when produced meets AC at R, then $\frac{RA}{CA} = ?$



(a) $\frac{1}{3}$ (b) $\frac{1}{2}$
(c) 1 (d) $\frac{3}{4}$

94. In a triangle ABC, angle bisector of $\angle BAC$ cut the side BC at D and meet the circumcircle of ΔABC at E. If AC = 4 cm, AD = 5 cm, DE = 3 cm. Find the length of AB.

(a) 10 (b) 4
(c) 15 (d) 8

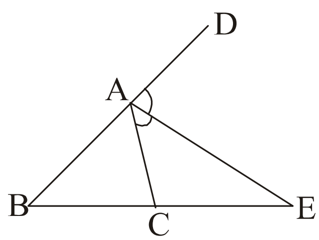
95. In a triangle ABC, AB + BC = 12 cm, BC + CA = 14 cm and CA + AB = 18 cm. Find the radius of the circle (in cm) which has the same perimeter as the triangle.

(a) $\frac{5}{2}$ (b) $\frac{7}{2}$
(c) $\frac{9}{2}$ (d) $\frac{11}{2}$

96. In a right-angled triangle XYZ, right-angled at Y, if XY = $2\sqrt{6}$ and XZ – YZ = 2, then sec X + tan X is

(a) $\frac{1}{\sqrt{6}}$ (b) $\sqrt{6}$
(c) $2\sqrt{6}$ (d) $\frac{\sqrt{6}}{2}$

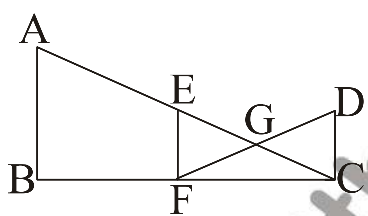
97. In the adjoining figure, AE is the bisector of $\angle CAD$ meeting BC produced at E. If AB = 10 cm, AC = 6 cm and BC = 12 cm, then CE is equal to :



(a) 6 cm (b) 12 cm

(c) 18cm (d) 20 cm

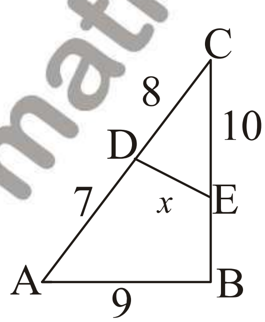
98. In the adjoining figure, AB, EF and CD are parallel lines. Given that EG = 5 cm, GC = 10 cm and DC = 18 cm, then EF is equal to :



(a) 11 cm (b) 5 cm

(c) 6 cm (d) 9 cm

99. In the figure $\angle A = \angle CED$, then find the value of x



(a) 5 (b) 6

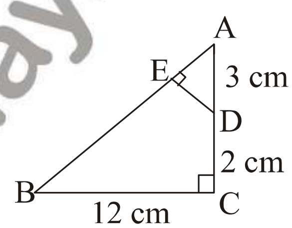
(c) 7 (d) 8

100. In a triangle ABC, the lengths of the sides AB, AC and BC are 3, 5 and 6 cm respectively. If a point D on BC is drawn such that the line AD bisects the $\angle A$ internally, then what is the length of BD?

(a) 2 cm (b) 2.25 cm

(c) 2.5 cm (d) 3 cm

101. In the given figure, ΔABC is right angled at C and $DE \perp AB$. Find the lengths of AE.



(a) $AE = \frac{15}{13}$ cm

(b) $AE = \frac{13}{15}$ cm

(c) $AE = \frac{11}{13}$ cm

(d) $AE = \frac{11}{15}$ cm

102. D is a point on the side BC of a triangle ABC such that $AD \perp BC$. E is a point on AD such that AE : ED = 5 : 1. If $\angle BAD = 30^o$ and $\tan(\angle ACB) = 6\tan(\angle DBE)$,

then $\angle ACB = ?$

(a) 30° (b) 45°

(c) 60° (d) 15°

103. Find the radii of the inscribed and circumscribed circles to a triangle whose sides measure 21 cm, 72 cm and 75 cm respectively.

(a) 9 cm, 37. 5 cm (b) 8 cm, 13 cm

(c) 19 cm, 12 cm (d) 6 cm and 15 cm

Answer

1.	(c)	2.	(b)	3.	(c)	4.	(c)	5.	(c)	6.	(a)	7.	(a)	8.	(c)	9.	(b)
10.	(d)	11.	(b)	12.	(b)	13.	(c)	14.	(b)	15.	(c)	16.	(a)	17.	(a)	18.	(c)
19.	(a)	20.	(b)	21.	(a)	22.	(b)	23.	(c)	24.	(d)	25.	(a)	26.	(d)	27.	(c)
28.	(a)	29.	(b)	30.	(b)	31.	(c)	32.	(a)	33.	(a)	34.	(b)	35.	(a)	36.	(d)
37.	(b)	38.	(a)	39.	(a)	40.	(d)	41.	(b)	42.	(d)	43.	(b)	44.	(d)	45.	(b)
46.	(b)	47.	(d)	48.	(a)	49.	(a)	50.	(b)	51.	(c)	52.	(b)	53.	(b)	54.	(d)
55.	(a)	56.	(d)	57.	(b)	58.	(b)	59.	(b)	60.	(d)	61.	(a)	62.	(b)	63.	(a)
64.	(d)	65.	(a)	66.	(c)	67.	(b)	68.	(b)	69.	(a)	70.	(a)	71.	(b)	72.	(b)
73.	(c)	74.	(c)	75.	(b)	76.	(d)	77.	(b)	78.	(a)	79.	(b)	80.	(c)	81.	(d)
82.	(c)	83.	(c)	84.	(d)	85.	(a)	86.	(c)	87.	(b)	88.	(b)	89.	(b)	90.	(d)
91.	(c)	92.	(d)	93.	(a)	94.	(a)	95.	(b)	96.	(b)	97.	(c)	98.	(d)	99.	(b)
100.	(b)	101.	(a)	102.	(c)	103.	(a)										

Solution & Hints

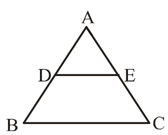
Sol.1 If the medians are equal then sides will be equal. It is equilateral triangle.

Sol.2 Incentre is a intersection point of angle bisector. Incentre is a point in the triangle . Which is equi distance from sides.

Sol.3 Circumcentre is a intersection point of perpendicular bisector. Circumcentre is a point in the triangle which is equi distance from vertexes.

Sol.4 Incentre

Sol.5

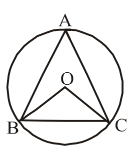


DE || BC

$$\frac{AD}{BD} = \frac{AE}{EC} \text{ or } \frac{AD}{AB} = \frac{AE}{AC}$$

$$\Rightarrow \frac{3}{10} = \frac{AE}{5} \Rightarrow AE = \frac{3}{2} = 1.5\text{cm}$$

Sol.6



In Δ BOC ,

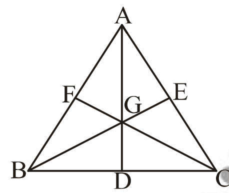
$\angle OBC = \angle OCB = 35^\circ$

$\angle BOC = 180^\circ - 70^\circ = 110^\circ$

In a circumcentre,

$$\angle BAC = \frac{1}{2} \angle BOC = \frac{110^\circ}{2} = 55^\circ$$

Sol.7



ΔABC,

AD, BE, CF are three medians

and they divide the triangle into six equal area part in the figure.

$$\because \text{ Area of } \Delta BDG = \frac{1}{6} \times \text{Area of } \Delta ABC$$

$$\Delta BDG = \frac{1}{6} \times 72 = 12 \text{ cm}^2$$

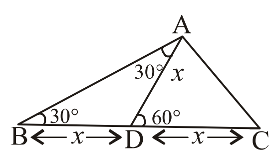
Sol.8 In fig. sol. 7

area of quadrilateral BDGF $= 2 \times$ area ΔBDG

$$= 2 \times \frac{1}{6} \times \text{area of } \Delta ABC$$

$$= 2 \times \frac{1}{6} \times 60 = 20\text{cm}^2$$

Sol.9



AD is median. D is mid point of BC.

AD = BD = DC = x

$\angle DAB = \angle ABD = 30^\circ$

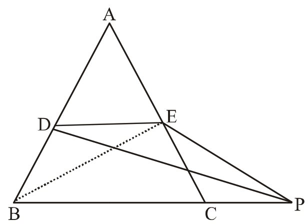
$\angle ADC = 30^\circ + 30^\circ = 60^\circ$ (exterior angle ΔABD)

$\because$ AD = DC

$\therefore \angle DAC = \angle DCA = 60°$

then, $\angle ACB = 60°$

Sol.10



$DE || BC$

$DE = \frac{1}{2}BC \Rightarrow \frac{DE}{BC} = \frac{1}{2}$

area of $\Delta ADE = \frac{1}{4}$ of area of ΔABC

= area of ΔDEB

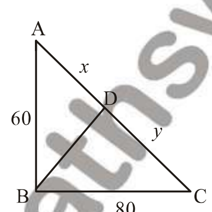
In ΔDEB and ΔDEP

$\because$ DE base are same

hence area of ΔDEB = area of ΔDEP

$$\boxed{\text{area of } \Delta DEP = \frac{1}{4} \text{ area of } \Delta ABC}$$

Sol.11



$AC = 100$

Perimeter of ΔABD = perimeter of ΔBDC

$60 + x + BD = 80 + y + BD$

$x - y = 20$

and $x + y = 100$

hence, $x = 60$ and $y = 40$

Now,

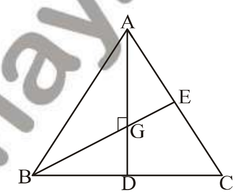
$$\cos\theta = \frac{60^2 + 60^2 - BD^2}{2 \times 60 \times 60} = \frac{60}{100}$$

$\Rightarrow \quad BD = 24\sqrt{5}$

Sol.12 Area of two similar triangle = 9 : 16

Ratio of sides = $\sqrt{9} : \sqrt{16}$ = 3 : 4

Sol.13



In a triangle median intersect at centroid G

Which divides the median into 2 : 1

AD = 9 cm. BE = 6 cm

3 unit = 9 cm (AD)

1 unit = 3

GD = 1 × 3 = 3

BC = 6 cm

3 unit = 6 cm

1 unit = 2 cm

BG = 2 × 2 = 4 cm

Since $\angle BGC = 90°$

Hence, ΔBGC is a right angle triangle

$BD^2 = BG^2 + DG^2$

$BD^2 = (4)^2 + (3)^2$

$16 + 9 = 25$

BD = 5 cm

Sol.14 In a ΔABC , PQ is parallel to BC

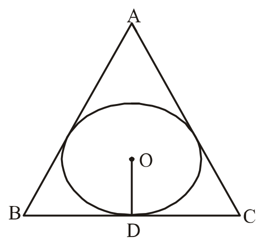
$$\frac{AP}{BP} = \frac{1}{2} \Rightarrow \frac{AQ}{QC} = \frac{1}{2}$$

$\because$ AQ = 3 cm

$\therefore$ QC = 2 × AQ = 2 × 3 = 6 cm

then, AC = 3 + 6 = 9 cm

Sol.15



O is an incentre,

$OD \perp BC$, $\angle D = 90°$, $\angle BOD = 15°$

In ΔBOD, $\angle OBD = 180° - (90° + 15°) = 75°$

$\because$ OB is angle bisector of $\angle ABC$

then, $\angle ABC = 2 \times 75° = 150°$

$\angle ABC = 150°$

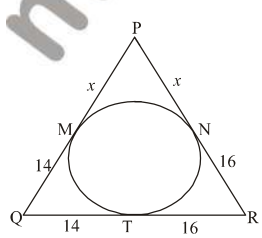
Sol.16 E is the mid point of AD

hance, in ΔABD

BE is median which divide ABD in two equal parts.

$$\text{Area } \Delta BEC = \frac{1}{2} \text{Area } \Delta ABC$$

Sol.17



$\Delta = 336$

$r = 8$

$$S = \frac{\Delta}{r} = \frac{336}{8} = 42 \text{ cm}$$

P = 2 × 42 = 84 cm

PQ + QR + PR = 84

$2x + 60 = 84 \Rightarrow x = 12$

PQ = 26, PR = 28

Sol.18 In ΔABC

$\angle ABC = 90°$

Hence, ΔABC is a right angle triangle

In ΔABQ,

$AQ^2 = AB^2 + BQ^2$ …(i)

In ΔPBC

$PC^2 = PB^2 + BC^2$ …(ii)

Add both equation

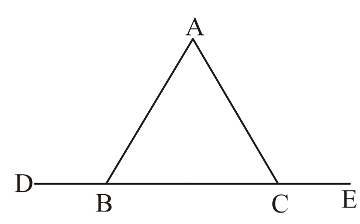
$AQ^2 + CP^2 = AB^2 + BC^2 + BQ^2 + PB^2$

$(AC^2 + AB^2 + BC^2)$

$(PC^2 + BP^2 + BQ^2)$ an equilateral

$AQ^2 + CP^2 = AC^2 + PQ^2$

Sol.19



In ΔABC ,

Exterior angle is summation of two interior angles

$\angle ACE = \angle CAB + \angle ABC$ ....(i) and $\angle ABD = \angle BAC + \angle ACB$ .....(ii)

Adding (i) and (ii)

$\angle ACE + \angle ABD = \angle CAB + \angle ABC + \angle BAC + \angle ACB$

$= 180° + \angle BAC$

Sol.20 ΔAEC is an equilateral triangle

$\angle EAC = 180° - (90° + 20°) = 70°$

In ΔABD, $\angle BDA = 180° - (50° + 70°)$

$= 180° - 120°$

$= \angle BDA = 60°$

Sol.21 AB || QR

$$\frac{AB}{QR} = \frac{PB}{PR}$$

$$\frac{3}{9} = \frac{PB}{6}$$

PB = 2 cm

Sol.22 DE || BC

$$DE = \frac{1}{2} AC$$

Area of $\Delta BDE = \frac{30}{4} = 7.5\ cm^2$

Sol.23 In ΔABC

PQ || BC

$$PQ = \frac{1}{2} BC \qquad \left(\frac{PR}{RQ} = \frac{1}{2}\right)$$

If PR = 2 cm

then, RQ = 4 cm

So, PQ = PR + RQ = 2 + 4 = 6 cm

$\because$ $PQ = \frac{1}{2} BC$

$\therefore$ BC = 2PQ

BC = 6 × 2 = 12

BC = 12 cm

Sol.24 In any triangle median is divided by centroid in ratio 2 : 1

and, centroid is meeting point of medians,

if radius = 3

Then, median = 3 × 3 = 9 cm

Sol.25 AB || CD

AB is a straight line.

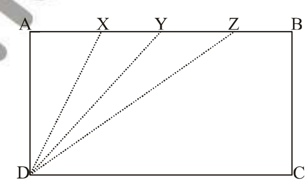
$\angle DEB = 180° - (37° + 90°)$

$\angle DEB = 180° - 127° = 53°$

$\angle DEB = \angle CDE$ (Alternate angle)

$\angle x = 53°$

Sol.26



Let four equal parts = 1 unit

Let BC = x

$$\frac{\text{Area of } \Delta XYD}{\text{Area of rectangle ABCD}} = \frac{\frac{1}{2} \times 1 \times x}{4 \times x} = \frac{1}{8}$$

Sol.27 Sum of all angle = πradian

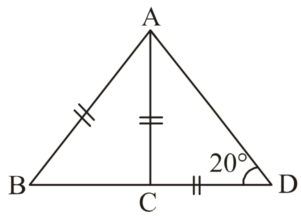
let third angle is α

$$\alpha + \frac{1}{2} + \frac{1}{3} = \pi = \frac{22}{7}$$

$$\alpha = \frac{22}{7} - \frac{5}{6} = \frac{132 - 35}{42} = \frac{97}{42} \text{radian}$$

$$= \frac{97}{42} \times \frac{180°}{\pi} = \frac{97}{42} \times \frac{180°}{22} \times 7 = 132\frac{3°}{11}$$

Sol.28



CA = CD

$\angle$ CDA = $\angle$ CAB = 20°

$\angle$ ACB = 40° [External angle]

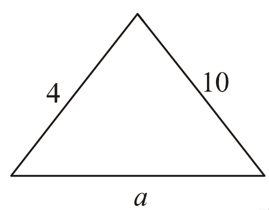
In ΔABC

AB = AC

$\angle$ ACB = $\angle$ ABC = 40°

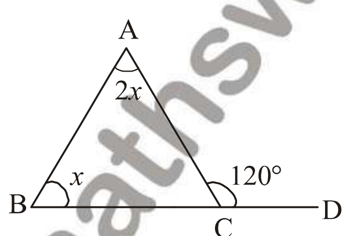
$\angle$ ABC = 40°

Sol.29



Length of Third side = $6 < a < 14$

Sol.30

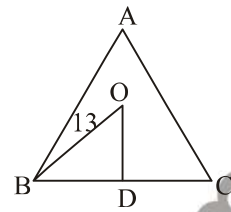


$x + 2x = 120°$

$x = 40°$

$\angle A = 2x = 80°$

Sol.31



BD = 24/2 cm = 12 cm

$OB^2 = OD^2 + BD^2$

$(13)^2 = (OD)^2 + (12)^2$

OD = 5 cm

Sol^n. 32 $\angle A + \angle B + \angle C = 180°$

After decreasing 15° by each angle sum will be 180° – 45° = 135°

Divide 135° in 2 : 3 : 5

Greatest part will be $= 135° \times \frac{5}{10}$

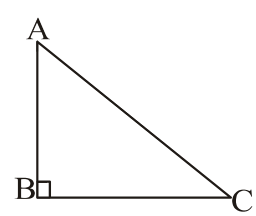
$= \frac{135°}{2}$

Greatest angle will be $= \frac{135°}{2} + 15°$

$= \frac{165°}{2} \times \frac{\pi}{180}$ Radian

$= \frac{11\pi}{24}$

Sol.33



$\therefore AB = BC + 2$

$AC^2 = AB^2 + BC^2$ .... **(Pythagoras theorem)**

$\Rightarrow \left(2\sqrt{5}\right)^2 = (BC + 2)^2 + BC^2$

$\Rightarrow 20 = 2BC^2 + 4 + 4BC$

$\Rightarrow BC^2 + 2BC - 8 = 0$

$\Rightarrow BC = 2$ cm

and AB = 4 cm

$$\cos^2 A - \cos^2 C = \left(\frac{4}{2\sqrt{5}}\right)^2 \left(\frac{2}{2\sqrt{5}}\right)^2 = \frac{12}{20} = \frac{3}{5}$$

Sol.34 If two triangle are similar then ratio of sides is equal to ratio of perimeter.

$$\frac{P_1}{P_2} = \frac{AB}{PQ} \Rightarrow \frac{36}{24} = \frac{AB}{10}$$

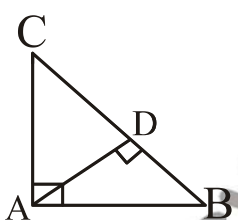
$\Rightarrow AB = 15$ cm

Sol.35 If two triangle are similar then ratio of its sides is equal to under root of ratio of area of triangle

$$\frac{EF}{BC} = \sqrt{\frac{A_1}{A_2}} \Rightarrow \frac{15.4}{BC} = \sqrt{\frac{121}{64}} = \frac{11}{8}$$

$\Rightarrow BC = 11.2$ cm

Sol.36



In Δ ABD and Δ ADC

$$\text{CotB} + \text{cotC} = \frac{BD}{AD} + \frac{CD}{AD}$$

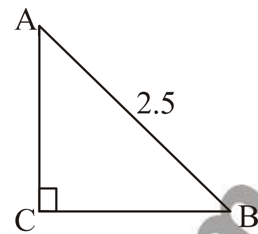
$$= \frac{BD + CD}{AD} = \frac{12}{4} = 3$$

Sol.37 In a right angle triangle

'Perpendicular AD on BC $\Rightarrow AD^2 = BD.CN$

So $\angle BAC$ will be 90°

Sol.38



$$\text{CosB} = 0.5 = \frac{5}{10} = \frac{1}{2} = \cos 60°$$

B = 60°

$$\text{SinB} = \text{Sin}60° = \frac{\sqrt{3}}{2} = \frac{AC}{2.5}$$

$$AC = \frac{\sqrt{3} \times 2.5}{2 \times 10} = \frac{5}{4}\sqrt{3}$$

Sol.39 Area of $\Delta DEF = \frac{1}{4}(\text{Area of } \Delta ABC)$

$$= \frac{1}{4} \times \frac{1}{2} \times 6 \times 8 = 6 \text{ cm}^2$$

Sol.40 Let $\angle A = x$

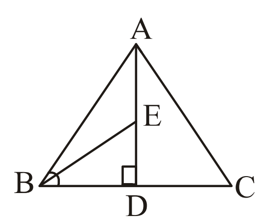
$\angle A + \angle B + \angle C = 180°$

$x + 1.5x + 2.5x = 180°$

$5x = 180° \Rightarrow x = 36°$

$\angle B = 36 \times 1.5 = 54°$

Sol.41

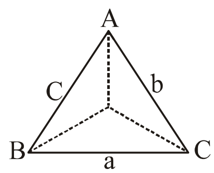


$$\frac{AE}{ED} = \frac{AB}{BD}$$

In $\Delta ABD = \cos 60° = \frac{BD}{AB} = \frac{1}{2}$

$\Rightarrow \frac{AE}{ED} = \frac{1}{2}$

Sol.42



$P = a + b + c$, $S = S_1 + S_2 + S_3$

$S_1 + S_2 > c$
$S_2 + S_3 > a$
$S_1 + S_3 > b$

$2(S_1 + S_2 + S_3) > a + b + c$ adding
$2S > P \Rightarrow P < 2S$

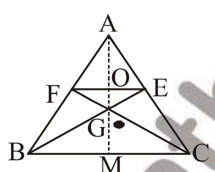
Sol.43 $\therefore (20)^2 = (16)^2 + (12)^2$

Hence, ΔABC is a right angle triangle

with circumradius $= \frac{1}{2} \times (\text{Hypotenuse})$

$= \frac{1}{2} \times (20) = 10$ cm

Sol.44 BGC = GFE (Similar)



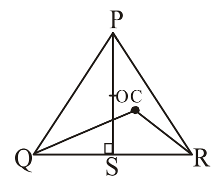
$\frac{GM}{OG} = \frac{BC}{FE} = \frac{2}{1}$

If GM = 2, then AG = 4

AO = AG – OG = 4 – 1 = 3

$\frac{AO}{OG} = \frac{3}{1}$

Sol.45



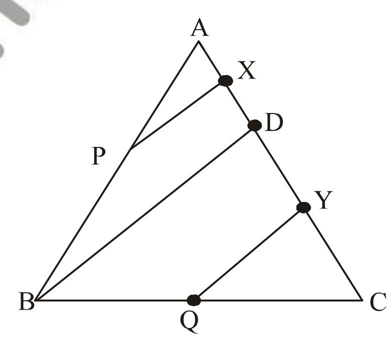
C is circumcentre

$\angle QPR = \frac{\angle OCR}{2} = \frac{130°}{2} = 65°$

$\angle QPS = 180° - \angle PQS - \angle PSQ = 180° - 60° - 90°$
$= 30°$

$\angle RPS = 65° - 30 = 35°$

Sol.46



In ΔABD,

$PX = \frac{1}{2} BD$ ....(i)

In ΔCBD

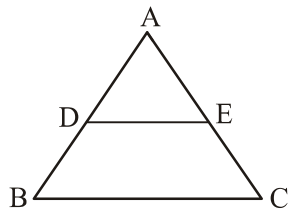
$QY = \frac{1}{2} BD$ ....(ii)

From equation (i) and equation (ii)

PX = QY

PX : QY = 1 : 1

Sol.47



$\because$ DE || BC

$$\frac{\text{area of } \Delta ADE}{\text{area of } \Delta ABC} = \left(\frac{DE}{BC}\right)^2 = \left(\frac{3x}{5x}\right)^2 = \left(\frac{9x}{25x}\right)$$

Hence,

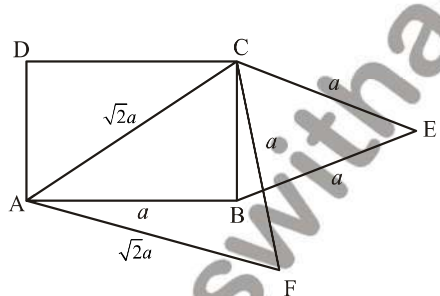
area of trapezium DEBC

$= \text{area of } \Delta\, ABC - \text{aera of } \Delta\, ADE$

$= 25x - 9x = 16x$

and, $\dfrac{\text{Area } (\Delta ADE)}{\text{Area}(\text{Trapezum BDEC})} = \dfrac{9x}{16x} = 9{:}16$

Sol.48



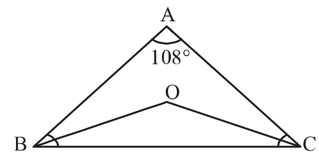
Let the side of square be a

$\therefore$ BC $= a$

and diagonal AC $= \sqrt{2}a$

$$\frac{\text{area of } \Delta BCE}{\text{area of } \Delta ACF} = \frac{\frac{\sqrt{3}}{4}a^2}{\frac{\sqrt{3}}{4}\left(\sqrt{2}a\right)^2} = 1:2$$

Sol.49



In ΔABC,

$\therefore \quad \angle A + \angle B + \angle C = 180°$

$\angle B + \angle C = 180° - 108° = 72°$

and $\dfrac{\angle B}{2} + \dfrac{\angle C}{2} = 36°$

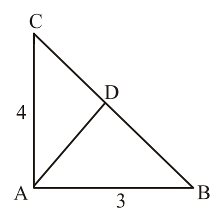
In ΔOBC,

$\therefore \quad \angle OBC + \angle OCB + \angle BOC = 180°$

$\Rightarrow \quad \dfrac{\angle B}{2} + \dfrac{\angle C}{2} + \angle BOC = 180°$

$\Rightarrow \quad \angle BOC = 144°$

Sol.50



BC $= 5$

ΔADB and ΔACB are Similar

$$\frac{\text{area of } \Delta ADB}{\text{area of } \Delta ACB} = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$$

Area of Δ ADB $= \dfrac{9}{25} \times \dfrac{1}{2} \times 3 \times 4 = \dfrac{54}{25}$ cm²

Sol.51 In the fig of question.

$\angle ADE = 90° + 60° = 150°$

In Δ DAE,

$AD = DE$ (Sides)

$\angle DAE + \angle DEA = 180 + 150° = 30°$

$2\angle DAE = 30°$

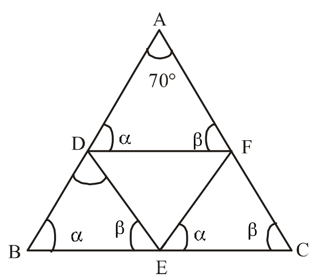
$\angle DAE = 15°$

Sol.52 Area of plot = area of all triangles

$= \frac{1}{2}az + \frac{1}{2}ay + \frac{1}{2}bt + \frac{1}{2}cx$

$= \frac{1}{2}(bt + cx + ay + az)$

Sol.53



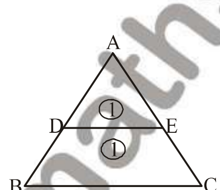
$\angle ABC = \angle ADF = \alpha$

$\angle ACB = \angle AFD = \beta$

$\angle DEF = 180° - (\alpha + \beta) = 180° – 110° = 70°$

$[\because (\alpha + \beta = 180° - 70° = 110°)]$

Sol.54



$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta ABC} = \frac{1}{2} = \left(\frac{AD}{AB}\right)^2$

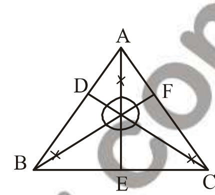
$\frac{AD}{AB} = \frac{1}{\sqrt{2}}$

$AB = \sqrt{2}\,AD$

And

$\frac{AD}{BD} = \frac{AD}{AB - AD} = \frac{AD}{AD(\sqrt{2}-1)} = \frac{1}{\sqrt{2}-1}$

Sol.55



In all three triangles areangles bisector theorem

$\frac{OA}{OB} = \frac{AD}{BD}$ ....(i)

$\frac{OB}{OC} = \frac{BE}{EC}$ . ...(ii)

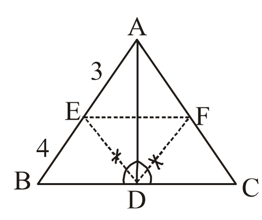
$\frac{OC}{OA} = \frac{CF}{AF}$ ....(iii)

Multiply all three equation

$\frac{OA}{OB} \times \frac{OB}{OC} \times \frac{OC}{OA} = \frac{AD \times BE \times CF}{BD \times EC \times AF}$

$\Rightarrow AD \times BE \times CF = BD \times EC \times AF$

Sol.56



In ΔABD

$\frac{AD}{BD} = \frac{AE}{EB} = \frac{3}{4}$ (angle bisector theorem)

$\therefore$ BA = DC

So $\frac{AD}{DC} = \frac{3}{4}$

In ΔADC

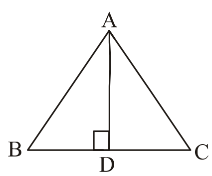
$\frac{AF}{AC} = \frac{AD}{DC} = \frac{3}{4}$ (angle bisector theorem)

Hence, $\frac{AE}{EB} = \frac{AF}{FC}$

$\Delta AEF = \Delta ABC$ [similar]

$\frac{EF}{BC} = \frac{AE}{AB} = \frac{3}{3+4} = \frac{3}{7}$

Sol.57



height of equilateral triangle $= \frac{\sqrt{3}}{2}a$

$AD = \frac{\sqrt{3}}{2}AB$

$2AD = \sqrt{3}AB \Rightarrow 4AD^2 = 3AB^2$

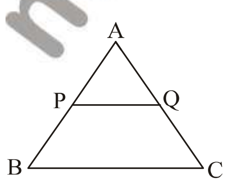
Sol.58 $\Delta ADE \sim \Delta ABC$ [similar]

$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{x}{x-2} = \frac{x+2}{x-1}$

$x^2 - x = x^2 - 4$

$\Rightarrow x = 4$

Sol.59



PO || BC

$\angle APQ = \angle ABC = 60°$

$\angle AQP = \angle ACB = 60°$

Hence, APQ is an equilateral triangle.

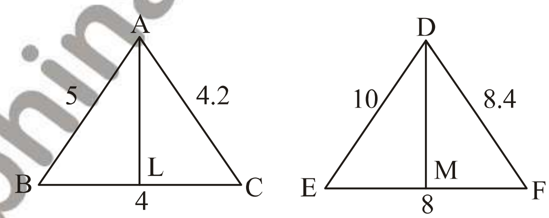
Then, area $= \frac{\sqrt{3}}{4}(PQ)^2 = \frac{\sqrt{3}}{4} \times 25 = \frac{25\sqrt{3}}{4}$ cm²

Sol.60 All sides are equal

hence,

Both triangles are congruent

Sol.61



Here, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{1}{2}$

Hence, both triangle are similar

Then, $\frac{AL}{DM} = \frac{1}{2}$

Sol.62 From angle bisector theorem

$\frac{AB}{AC} = \frac{BD}{DC} \Rightarrow \frac{6}{5} = \frac{3}{DC}$

$\Rightarrow$ DC = 2.5 cm

Sol.63 X,Y, Z has minimum interval valeus

least triplet is 3,4, 5

Let AB = ? BC = 4 then, AC = 5

CD = Z = XY = 12

In ΔACD,

If AC = 5

CD = 12

Then, AD = 13

Area of ABCD = area of Δ ABC + area of Δ ACD

$$= \frac{1}{2}\times 3\times 4+\frac{1}{2}\times 5\times 12$$

$$= 36 \text{ cm}^2$$

Sol.64 $AD = \frac{1}{3}AB$, $AE = \frac{1}{3}AC$

Then,

$DE = \frac{1}{3}BC = \frac{1}{3}\times 15 = 5cm$ (Due to similarly)

Sol.65 Area of ΔABC = 2 × Area of triangle ABD

$$= 2 \times \left(2\times \text{area of } \Delta BED\right)$$

$$\frac{\text{area of } \Delta BED}{\text{area of } \Delta ABC} = \frac{1}{4}$$

Sol.66 $\frac{AL}{LB} = \frac{4}{3}$

In ΔABC

LQ | | AC

Then, $\frac{CQ}{QB} = \frac{AL}{LB} = \frac{4}{3} = \frac{4\times 7}{3\times 7} = \frac{28}{21}$

In ΔBCL

QD| |CL

$$\frac{LD}{DB} = \frac{CD}{QB} = \frac{4}{3}$$

Let LB = 21 and AL = 28

$\therefore$ LD : DB = 4 : 3

Hence, $LD = \frac{21}{7}\times 4 = 12$

Hence, AL : LD = 28 : 12 = 7 : 3

Sol.67 We know that,

$$4\left(PS^2 + QR^2\right) = 5PR^2$$

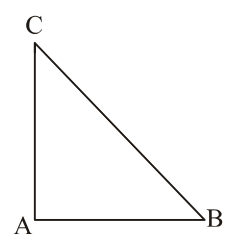
$$4\left(16+16\right) = 5PR^2$$

$PR^2 = 25.6$ $\left(\because PS = RT = 4\right)$

$\therefore$ Both medians are equal then PQ = QR and triangle will be isoscles triangle

$\text{area} = \frac{1}{4}PR^2 = 6.4\text{cm}^2$

Sol.68



$$r = \frac{a+b-c}{2} = \frac{6+8-10}{2} = 2\text{cm}$$

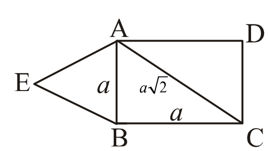
Sol.69 $p - q = 1 \Rightarrow p = q + 1$

$AC^2 = p^2 - q^2 = (p + q)(p - q)$

$= p + q = 2q + 1$

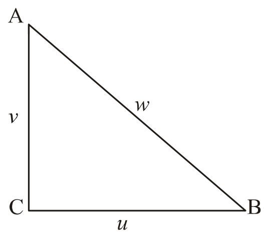
$AC = \sqrt{2q+1}$

Sol.70



$$\frac{\text{area of } \Delta ABE}{\text{area of } \Delta ACD} = \left(\frac{AB}{AC}\right)^2 = \left(\frac{a}{a\sqrt{2}}\right)^2 = \frac{1}{2}$$

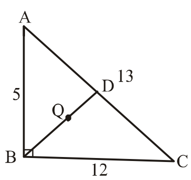
Sol.71



$$\tan A + \tan B = \frac{u}{v} + \frac{v}{u}$$

$$= \frac{u^2 + v^2}{uv} = \frac{w^2}{uv} \qquad (\because v^2 + u^2 = w^2)$$

Sol.72

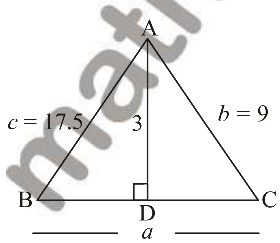


BD is median and will be half of AC

$$BD = \frac{13}{2}$$

$$BQ = \frac{2}{3}BD = \frac{2}{3} \times \frac{13}{2} = 4\frac{1}{3}$$

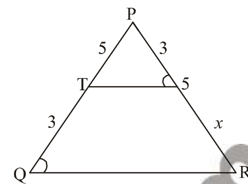
Sol.73



$$R = \frac{abc}{4\Delta} = \frac{a \times 9 \times 17.5}{4 \times \frac{1}{2} \times a \times 3} = \frac{9 \times 17.5}{2 \times 3}$$

$= 26.25$ cm

Sol.74



$\angle PST = \angle PQR$

$\angle P = \angle P$

Then, $\angle PTS = \angle PRS$

Hence, $\Delta PST = \Delta PQR$ (Similar triangle)

$$\frac{PS}{PQ} = \frac{PT}{PR} \Rightarrow \frac{3}{8} = \frac{5}{x+3}$$

$3(x + 3) = 40$

$$x + 3 = \frac{40}{3} \Rightarrow x = \frac{31}{3}$$

Sol.75 Let side are a, ar, ar^2

hence, a is biggest side (hypotenuse)

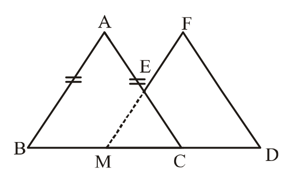
$a^2 = (ar)^2 + (ar^2)^2$

$1 = r^2 + r^4$

$r^4 + r^2 - 1 = 0$

$$r^2 = \frac{-1 + \sqrt{1+4}}{2} = \frac{\sqrt{5} - 1}{2}$$

Sol.76



EF || AB $\Rightarrow$ AB||MF

$\angle ABC = \angle FMC = 65°$

$\angle FED = 50°$

In ΔMFD

$\angle FDC = 180° - \angle EFD - \angle FMC$

$= 180° - 65° - 50°$

$= 65°$

Sol.77 In ΔABD

$2a = 90° \Rightarrow a = 45°$

In ΔADG

AD = AG and DF = FG

F is mid point

$\Rightarrow AF \perp DG$

b = 90°

In ΔADC,

$$\text{Sin C} = \frac{AD}{AC} = \frac{x}{2x} = \frac{1}{2}$$

C = 30°

$a + b + c = 45° + 90° + 30° = 165°$

Sol.78 $\Delta PQS \sim \Delta PRS$

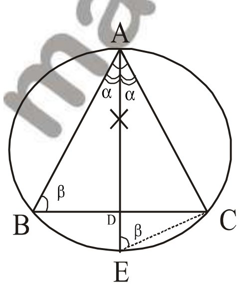
$$\frac{PQ}{QS} - = \frac{PR}{SR}$$

$$QS = SR = \frac{1}{2} \times QR = 2 \text{ cm}$$

then,

$$PS = \sqrt{(PQ)^2 - (QS)^2} = \sqrt{25-4} = \sqrt{21} \text{ cm}$$

Sol.79



$AB \times AC + DE \times AE$

$= AB \times AC + (AE - AD) AE$ [$\therefore DE : AE - AD$]

$= AB \times AC + AE^2 - AD \times AE$

$= AD \times AE + AE^2 - AD \times AE = AE^2$

$\therefore AB \times AC = AD \times AE$

($\therefore$ ΔABD and ΔACE are similar)

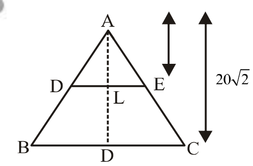
Sol.80 Side of triangle $a = 14\sqrt{3}$

$$\text{radius of incircle } (r) = \frac{a}{2\sqrt{3}} = \frac{14\sqrt{3}}{2\sqrt{3}} = 7 \text{ cm}$$

$$\text{area of incircle} = \pi r^2 = \frac{22}{7} \times 7 \times 7$$

$= 154 \text{ cm}^2$

Sol.81



$$\therefore \frac{\text{area } (\Delta ADE)}{\text{area } (\Delta ABC)} = \left(\frac{AL}{AD}\right)^2$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{AL}{AD} = \frac{AL}{20\sqrt{2}}$$

$\Rightarrow AL = 20$ cm

Sol^n. 82 In ΔAPC,

$$\therefore AC = \sqrt{(AP)^2 + (PC)^2}$$

$$\Rightarrow AC = \sqrt{(6)^2 + (8)^2} = 10 \text{ cm}$$

Now, In ΔABC

$$PC = \frac{AC^2}{BC} \quad \Rightarrow \quad 8 = \frac{100}{BC}$$

$$BC = \frac{25}{4}$$

$AB^2 = BC^2 - AC^2$

$AB^2 = \sqrt{\frac{625}{4} - 100} = \sqrt{\frac{225}{4}}$

$AB = \frac{15}{2}$

Area of Triangle ABC $= \frac{1}{2} \times \frac{15}{2} \times 10$

$= \frac{75}{2}\, cm^2$

Sol.83 $\therefore \angle BAC = \angle ACB = 15°$

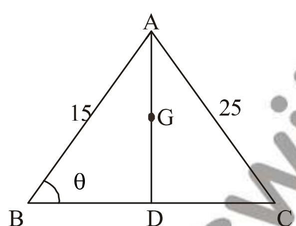
Then, $\angle ABC = 150°$

Area of $\Delta ABC = \frac{1}{2} \times AB \times BC \times \sin \angle ABC$

$= \frac{1}{2} \times 10 \times 10 \sin 150°$

$= 25\ cm^2$

Sol.84



In ΔABC and ΔACQ

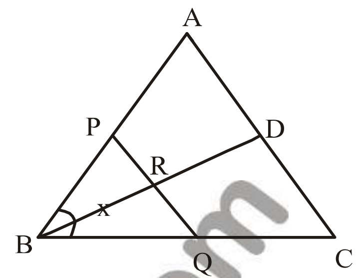
$\cos\theta = \frac{(25)^2 + (18)^2 - (15)^2}{2 \times 25 \times 18}$

$\cos\theta = \frac{(25)^2 + (9)^2 - (AD)^2}{2 \times 25 \times 9}$

$\Rightarrow AD = 2\sqrt{86}$

$GD = \frac{1}{3} \times AD = \frac{2}{3}\sqrt{86}$

Sol.85



$\therefore PQ || AC$

$\Delta PBQ = \Delta ABC$

$\frac{PB}{AB} = \frac{BQ}{BC} \Rightarrow \frac{AB}{PB} = \frac{BC}{BQ}$ .....(i)

Subtracting both the sides, we get

$\frac{AP}{PB} = \frac{OC}{BQ}$ ....(ii)

divide eq^n (ii) by (i)

$\frac{AB}{AP} = \frac{BC}{QC} \Rightarrow AB \times CQ = AP \times BC$

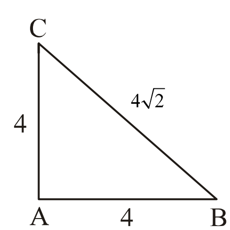
Sol.86 $\angle AED = 75°$

$\therefore \Delta AED \sim \Delta ABC$

$\frac{ED}{BC} = \frac{AE}{AB}$

$\frac{2}{3} = \frac{10}{AB} \Rightarrow AB = 15\,cm$

Sol.87



$BC = 4\sqrt{2}$

Circumradius $(R) = \dfrac{BC}{2} = 2\sqrt{2}$

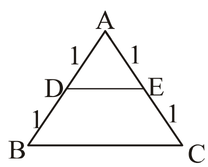
inradius $(r) = \dfrac{1}{2}(AB + AC - BC)$

$= 4 - 2\sqrt{2}$

$= \dfrac{r}{R} = \dfrac{4-2\sqrt{2}}{2\sqrt{2}}$

$= (\sqrt{2}-1):1$

Sol. 88

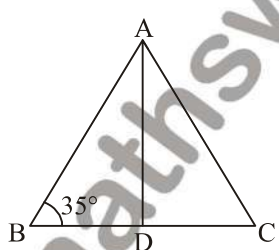


$\therefore$ $DE \,||\, BC$

$\Delta ADE = \Delta ABC$

$\dfrac{AD}{AB} = \dfrac{DE}{BC} = \dfrac{1}{2}$

Sol.89



$\angle B = \angle C = 35°$

$\angle BAC = 180° - 35° - 35° = 110°$

and $\Delta BAD = \Delta DAC$

$\therefore$ $\angle BAD = \dfrac{1}{2}\angle BAC = \dfrac{1}{2} \times 110° = 55°$

Sol.90 Hint :- all triangles will be congruent.

Sol.91 $\angle B = 40°$

$\therefore$ $\angle BRO + \angle ROQ + \angle OQB + \angle B = 360°$

$\Rightarrow$ $90° + \angle ROQ + 90° + 40° = 360°$

$\Rightarrow$ $\angle ROQ = 140°$

Sol.92 $\tan 30° = \dfrac{2\sqrt{3}}{AB}$

$AB = 6$ cm

In ΔAEB

$\angle DEA = \angle EAB + \angle EBA$
[exterior angle]

$90° = 30° + \angle EBA$

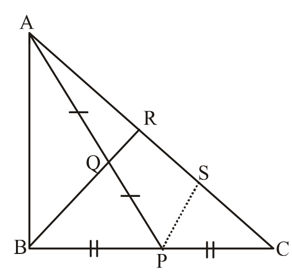
$\Rightarrow$ $\angle EBA = 60°$

In ΔADB,

$\tan 60° = \dfrac{AD}{AB}$

$AD = AB\sqrt{3} = 6\sqrt{3}$ cm

Sol.93



A line PS is drawn parallel to BR

In ΔAPS, $\dfrac{AQ}{QP} = \dfrac{AR}{RS} = 1$

$\Rightarrow$ $AR = RS$

In ΔBRC, $\frac{BP}{PC} = \frac{RS}{SC} = 1$

$\Rightarrow RS = SC$

Hence,

$$\frac{RS}{CA} = \frac{RA}{AR + RS + SC} = \frac{1}{3}$$

Sol.94 Use the concept of Q. 122

Sol.95 $AB + BC = 12$ cm ......(i)

$BC + CA = 14$ cm ....(ii)

$CA + AB = 18$ cm ......(iii)

Adding all above equations

$2(AB + BC + CA) = 44$

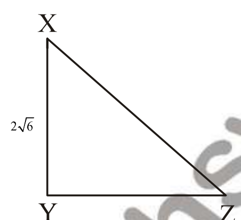
$AB + BC + CA = 22$ cm

Perimeter of triangle = Perimeter of circle = 22

$2\pi r = 22$

$\Rightarrow r = \frac{7}{2}$ cm

Sol.96



$(XY)^2 = (XZ)^2 - (YZ)^2$

$\left(2\sqrt{6}\right)^2 = (XZ - YZ)(XZ + YZ)$

$24 = 2(XZ + YZ)$

$XZ + YZ = 12$ cm

and $XZ - YZ = 2$

Hence, $XZ = 7$ cm and $YZ = 5$ cm

$$SecX + \tan X = \frac{7}{2\sqrt{6}} + \frac{5}{2\sqrt{6}}$$

$$= \frac{12}{2\sqrt{6}} = \sqrt{6}$$

Sol.97 $\therefore \frac{AB}{AC} = \frac{BE}{CE} = \frac{BC + CE}{CE}$

$$\frac{10}{6} = \frac{BC}{CE} + 1$$

$$\frac{BC}{CE} = \frac{2}{3}$$

$$CE = \frac{3}{2} \times BC = \frac{3}{2} \times 12 = 18 \text{ cm}$$

Sol.98 In ΔABC,

$EF || AB || CD$

$\Delta EFG \sim \Delta DGC$ (Similar)

$$\frac{EG}{GC} = \frac{EF}{DC}$$

$$\frac{5}{10} = \frac{EF}{18}$$

$EF = 9$ cm

Sol.99 In ΔCAE and ΔCBA

$\angle A = \angle CFD$

$\angle C = \angle C$

then, $\angle EDC = \angle ABC$

$\Delta CDE \sim \Delta CBA$

$$\frac{DE}{AB} = \frac{CE}{AC}$$

$\frac{x}{9}=\frac{10}{15}$

$x=6$

Sol.100 $\frac{BD}{CD}=\frac{AB}{AC}=\frac{3}{5}$ (Angle bisector property)

$BD=\frac{3}{3+5}\times 6=\frac{15}{8}=2.25$

Sol.101 In given triangle

$AB=\sqrt{12^2+5^2}=13$

In ΔABC,

$CosA=\frac{AE}{AD}=\frac{AE}{B}$ .....(i)

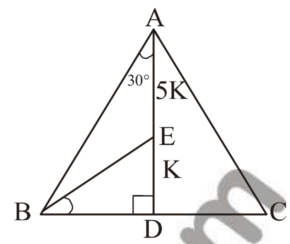
In ΔABC,

$CosA=\frac{AC}{AB}=\frac{5}{13}$ ....(ii)

from equation (i) and equation (ii)

$\frac{AE}{3}=\frac{5}{13}\Rightarrow AE=\frac{15}{13}$

Sol.102



In ΔABD

$\angle B=60°$

$\tan B=\frac{AD}{BD}$

$BD=\frac{6K}{13}$

$\tan(\angle ACB)=6\tan(\angle DBE)$

$6\times\frac{K}{BD}=6\times\frac{K}{6K}\times\sqrt{3}$

$\sqrt{3}=\tan 60°$

$\angle ACB=60°$

Sol.103 21cm, 72 cm and 75 cm

sides are in the ratio (7 : 24 : 25) hence

this is right angle triangle

7, 24, 25 is a triplet

Circumradius (R) $=\frac{\text{Hypotenuse}}{2}=\frac{75}{2}$

inradius $(r)=\frac{a+b-c}{2}=\frac{21+72-75}{2}=9$

☆☆☆☆☆

Exercise-(Quadrilateral)

1. ABCD is a cyclic trapezium whose side AD and BC are parallel to each other. If $\angle ABC = 72°$ then the measure of the $\angle BCD$ is:

(a) 162° (b) 18°
(c) 108° (d) 72°

2. The measures of the angles of a quadrilateral taken in order are proportional to 1 : 2 : 3 : 4, then the quadrilateral is:

(a) Parallelogram (b) Trapezium
(c) Rectangle (d) Rhombus

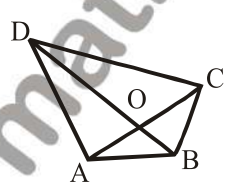
3. Diagonals of a parallelogram are 8m and 6m. If one of side is 5m, then the area of parallelogram is:

(a) 18 m^2 (b) 30 m^2
(c) 24 m^2 (d) 48 m^2

4. The parallel sides of a trapezium are p and q respectively. The line joining the mid-points of its non-parallel sides will be

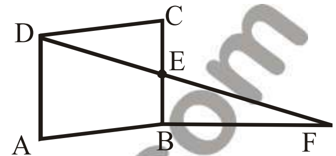
(a) $\sqrt{pq}$ (b) $\frac{2pq}{p+q}$
(c) $\frac{(p+q)}{2}$ (d) $\frac{1}{2}$ $(p-q)$

5. If ABCD is a quadrilateral whose diagonals AC and BD intersect at O, then :



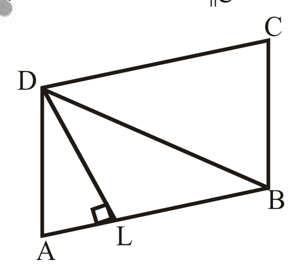
(a) (AB + BC + CD + AD) < (AC + BD)
(b) (AB + BC + CD + DA) > 2 (AC + BD)
(c) (AB + BC + CD + DA) > (AC + BD)
(d) AB + BC + CD + DA = 2 (AC + BD)

6. In the given figure, ABCD is a ∥ gm and E is the mid-point of BC. Also, DE and AB when produced meet at F. Then.



(a) $AF = \frac{3}{2}AB$ (b) AF = 2AB
(c) AF = 3 AB (d) $AF^2 = 2AB^2$

7. In the given figure, ABCD is a ∥ gm in which $DL \perp AB$. If AB = 10 cm and DL = 4 cm, then the area of ∥gm ABCD = ?



(a) 40 cm^2 (b) 80 cm^2
(c) 20 cm^2 (d) 196 cm^2

8. In a quadrilateral ABCD, with unequal sides if the diagonals AC and BD intersect at right angles, then

(a) $AB^2 + BC^2 = CD^2 + DA^2$
(b) $AB^2 + CD^2 = BC^2 + DA^2$
(c) $AB^2 + AD^2 = BC^2 + CD^2$
(d) $AB^2 + BC^2 = 2 (CD^2 + DA^2)$

9. If the length of the side PQ of the rhombus PQRS is 6 cm and $\angle PQR = 120°$, then the length of QS, in cm is :

(a) 4 (b) 6
(c) 3 (d) 5

10. ABCD is a cyclic quadrilateral whose vertices are equidistant from the point O (centre of the circle). If $\angle COD = 120°$ and $\angle BAC = 30°$, then the measure of $\angle BCD$ is :

(a) 180° (b) 150°
(c) 60° (d) 90°

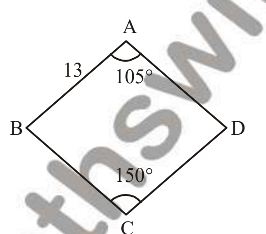
11. The area of a trapezium is 105 sq. m and the lengths of its parallel sides are 9 m and 12 m respectively then the height of the trapezium is:

(a) 15 m (b) 12 m
(c) 5 m (d) 10 m

12. ABCD is a trapezium, such that AB = CD and
AD || BC. AD = 5cm, BC = 9 cm. If area of ABCD is 35 sq.cm then CD is :

(a) $\sqrt{29}$ cm (b) 5cm
(c) 6 cm (d) $\sqrt{21}$ cm

13. In given fig.
$\angle A = 105°$, $\angle C = 150°$ & AB=13 cm
Find the length of AC.



(a) 12 cm (b) 17 cm
(c) 13 cm (d) Can't determined

14. If P, R, T are the area of a Parallelogram, a rhombus and a triangle standing on the same base and between the same parallels, which of the following is true?

(a) R < P < T (b) P > R > T
(c) R = P = T (d) R = P = 2T

15. ABCD is a square. M is the mid-point of AB and N is the mid-point of BC. DM and AN are joined and they meet at O. then which of the following is correct?

(a) OA : OM = 1 : 2
(b) AN = MD
(c) $\angle ADM = \angle ANB$
(d) $\angle AMD = \angle BAN$

16. If an exterior angle of a cyclic quadrilateral be 50°, then the opposite interior angle is :

(a) 130° (b) 40°
(c) 50° (d) 90°

17. A parallelogram ABCD has sides AB = 24 cm and AD = 16 cm. The distance between the sides AB and DC is 10 cm. Find the distance between the sides AD and BC.

(a) 16 cm (b) 18 cm
(c) 15 cm (d) 26 cm

18. The ratio of the angle $\angle A$ and $\angle B$ of a non-square rhombus ABCD is 4 : 5, then the value of $\angle C$ is:

(a) 50° (b) 45°
(c) 80° (d) 95°

19. ABCD is a cyclic trapezium such that AD || BC. If $\angle ABC$ = 70°, then the value of $\angle BCD$ is :

(a) 60° (b) 70°
(c) 40° (d) 80°

20. ABCD is a quadrilateral such that $\angle D = 90°$. A circle C(o, r) touches the sides AB, BC, CD and DA at P, Q, R and S respectively. If BC = 38 cm, CD = 25 cm and BP = 27 cm. Find r

(a) 7 cm (b) 14 cm
(c) 13 cm (d) 11 cm

21. A circle touches the sides of a quadrilateral ABCD at P, Q, R and S respectively. Find the angles subtended at the centre by a pair of opposite sides.

(a) 180° (b) 270°
(c) 225° (d) None of these

22. The difference between two parallel sides of a trapezium is 4 cm. The perpendicular distance between them is 19 cm. If the area of the trapezium is 475 cm^2. Find the lengths (cm) of the parallel sides:

(a) 27, 23 (b) 27, 24
(c) 27, 31 (d) 29, 25

23. The area of a field in the shape of trapezium measures 1440m^2. The perpendicular distance between its parallel sides is 24 m. If the ratio of the parallel sides is 5 : 3, the length of the longer parallel side is :

(a) 45 m (b) 60 m
(c) 75 m (d) 120 m

24. An equilateral triangle, a square and a circle have equal perimeters. If T denotes the area of triangle, S, the area of square and C, the area of the circle, then:

(a) $S < T < C$ (b) $T < C < S$
(c) $T < S < C$ (d) $C < S < T$

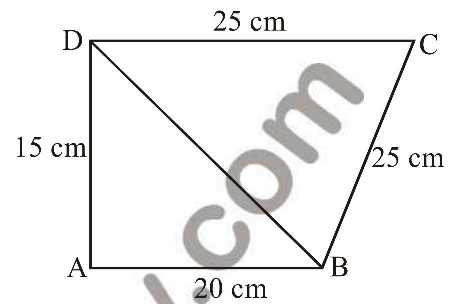
25. ABCD is a parallelogram, $\angle DAB = 30°$, BC = 20 cm and AB=40cm. Find the area of parallelogram:

(a) 150 cm^2 (b) 200 cm^2
(c) 400 cm^2 (d) 260 cm^2

26. ABCD is a parallelogram, BD is diagonal. $\angle BAD = 65°$ and $\angle DBC = 45°$, then $\angle BDC$ is :

(a) 65° (b) 70°
(c) 20° (d) None of these

27. In the given figure AD = 15 cm, AB = 20 cm and
BC = CD = 25 cm. Find the area of ABCD.



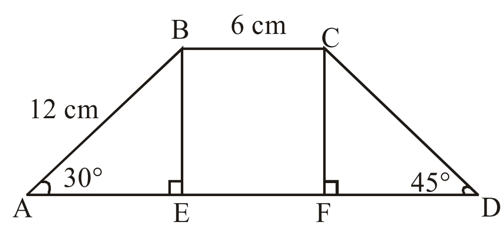
(a) $\frac{25}{4}\left(24+25\sqrt{3}\right)cm^2$

(b) $24\left(25+24\sqrt{3}\right)cm^2$

(c) $\frac{25}{2}\left(24+25\sqrt{3}\right)cm^2$

(d) None of these

28. In a trapezium ABCD, $\angle BAC = 30°$, $\angle CDF = 45°$, BC = 6 cm and AB = 12 cm. Find the area of ABCD.



(a) $18(3+\sqrt{3})cm^2$

(b) $36\sqrt{3}\ cm^2$

(c) $12(3+2\sqrt{3})cm^2$

(d) None of these

29. A square and a rhombus have the same base and rhombus is inclined at 30°. What is the ratio of area of the square to the area of the rhombus.

(a) $\sqrt{2}$: 1 (b) 2 : 1

(c) 1 : 1 (d) 2 : $\sqrt{3}$

30. ABCD is a quadrilateral in which diagonal BD = 64 cm. AL $\perp$ BD and CM $\perp$ BD, such that AL = 13.2 cm and CM = 16.8cm. The area of the quadrilateral ABCD is square centimeters is :

(a) 422.4 (b) 690.0

(c) 537.6 (d) 960.0

31. If the sides of a quadrilateral ABCD touch a circle and AB = 6cm, CD = 5cm, BC = 7cm, then the length of AD in cm is:

(a) 8 (b) 9

(c) 4 (d) 6

32. ABCD is a parallelogram in which diagonals AC and BD interest at O. If E, F, G and H are the mid points of AO, DO, CO and BO respectively, then the ratio of the perimeter of the quadrilateral EFGH to the perimeter of parallelogram ABCD is:

(a) $1:4$ (b) $2:3$

(c) $1:2$ (d) $1:3$

33. ABCD is a parallelogram AB is divided at P and CD at Q so that $AP:PB = 3:2$ and $CQ:QD = 4:1$ if PQ meets AC at R then AR =

(a) $\frac{2}{7}AC$ (b) $\frac{3}{7}AC$

(c) $\frac{4}{7}AC$ (d) $\frac{5}{7}AC$

34. In a quadrilateral ABCD, OA and OB are the angle bisectors of $\angle DAB$ and $\angle CBA$. If $\angle ADC = 70^o$ and $\angle BCD = 80^o$ then find $\angle AOB$?

(a) 70º (b) 80º

(c) 90º (d) 75º

35. ABCD is a cyclic quadrilateral in which AB is a diameter, BC = CD and $\angle ABD = 50°$ then $\angle DBC$ is:

(a) 25º (b) 20º

(c) 45º (d) 30º

36. ABCD is a cyclic quadrilateral. The tangents at A and C meet at a point P. If $\angle ABC = 100^o$ then $\angle APC$ will be equal to:

(a) 10º (b) 20º

(c) 40º (d) 80º

37. Find the area of a trapezium ABCD in which $AB \parallel DC$, AB = 26cm, BC = 25cm, CD= 40 cm and DA = 25 cm.

(a) 648cm^2 (b) 792cm^2

(c) 660cm^2 (d) 798cm^2

38. OABC is a rhombus whose three vertices A, B and C lie on a circle of radius 10cm and with centre O. Find the area of the rhombus.

(a) $30\sqrt{3}cm^2$ (b) $50\sqrt{3}cm^2$

(c) $10\sqrt{3}cm^2$ (d) $100\sqrt{3}cm^2$

39. ABCD is a cylclic trapezium whose side AD and BC are parallel to each other. If $\angle ABC = 62^o$, then the measure of the $\angle BCD$ is:

(a) 120º (b) 118º

(c) 62º (d) 28º

40. In a parallelograom ABCD, the bisector of $\angle A$ also bisects BC at E, find the AD.

(a) AB (b) 2 AB

(c) 3AB (d) $\frac{1}{2}$ AB

41. The ratio of the length of the diagonal of a rhombus is 2:5. Then, the ratio of the area of the rhombus to the square of the shorter diagonal:

(a) $5:4$ (b) $5:2$

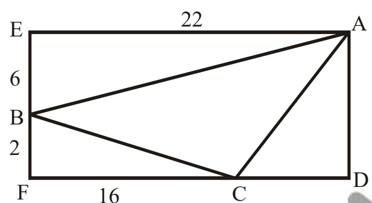
(c) $2:5$ (d) None of these

42. If the sum of the lengths of the diagonals of a rhombus is 10 m and if its area is $9m^2$, then what is the sum of the square of the diagonals?

(a) $36\,m^2$ (b) $64\,m^2$

(c) $80\,m^2$ (d) $100\,m^2$

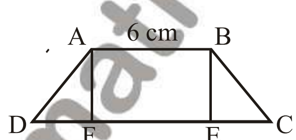
43. In the given figure. EADF is a rectangle and ABC is a triangle whose vertices lie on the sides of EADF. $AE = 22\,cm, BE = 6\,cm, CF = 16\,cm$ and BF = 2 cm. Find the length of the line joining the mid- points to the side AB and BC



(a) $4\sqrt{2}$ cm (b) 5 cm

(c) 3.5 cm (d) None of these

44. ABCD is a trapezium in which AB is parallel to DC, $AD = BC, AB = 6$ cm, $AB = EF$ and $DF = EC$. If two lines AF and BE are drawn so that area of ABEF is half of ABCD. Find DF/CD



(a) $\frac{1}{4}$ (b) $\frac{1}{3}$

(c) $\frac{2}{5}$ (d) $\frac{1}{6}$

45. One of the diagonal of a rhombus is double the other diagonal. The area is 25 sq.cm. The sum of the diagonals is:

(a) 10 cm (b) 12 cm

(c) 15 cm (d) 16 cm

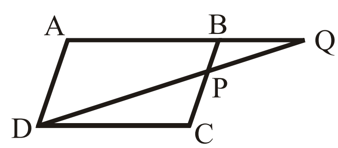
46. Let WXYZ be a square. Let P,Q, R, be the midpoints of WX, XY and ZW respectively and K, L be the midpoints of PQ and PR respectively. What is the value of

$$\frac{\text{Area of triangle PKL}}{\text{Area of square WXYZ}} = ?$$

(a) $\frac{1}{32}$ (b) $\frac{1}{16}$

(c) $\frac{1}{8}$ (d) $\frac{1}{64}$

47.



In the figure given above, ABCD is a parallelogram. P is a point on BC such that PB : PC =1 : 2. DP produced meets AB produced at Q. If the area of the triangle BPQ is 20 square units, what is the area of the triangle DCP?

(a) 20 square units (b) 30 square units

(c) 40 square units

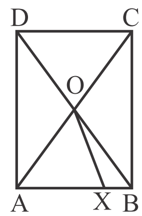
(d) None of the above

48. ABCD is a trapezium, in which AD and BC are parallel. If the four sides AB, BC, CD and DA are respectively 9 cm, 12 cm, 15 cm and 20 cm then find the magnitude of the sum of the squares of the two diagonals.

(a) 638 (b) 786

(c) 838 (d) 814

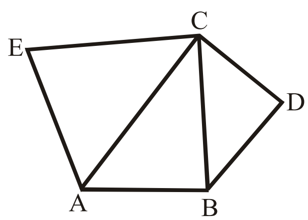
49.



In the given figure, ABCD is a square in which AO = AX, then $\angle XOB$ equals to ?

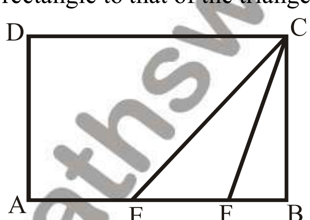
(a) 22.5º (b) 25º
(c) 30º (d) 45º

50. ABC is an isosceles traingle right angled at B. Two equilateral triangles are constructed with side BC and AC as shown in figure. Find the ratio of $ar(\Delta BCD)$ and $ar(\Delta ACE)$.



(a) 2 : 1 (b) 1 : 4
(c) 4 : 1 (d) 1 : 2

51. In the below diagram, ABCD is a rectangle with $AE = 2EF = 3FB$. What is the ratio of the area of the rectangle to that of the triangel CEF?

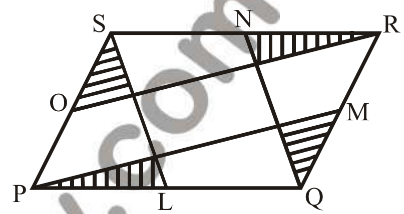


(a) 11 : 3 (b) 22 : 3
(c) 11 : 6 (d) None of these

52. ABCD is a square, P is the mid point of side AD and Q is the intersecting point of its diagonals. Then the ratio of Area (□ABQP) : Area (□ABCD) is

(a) 5 : 8 (b) 3 : 8
(c) 1 : 4 (d) None of these

53. In the parallelogram PQRS, L, M, N and O are mid points of sides PQ, QR, RS and SP respectively. PM, QN, RO and SL are joined. Find the ratio of the area of the shaded region to the area the parallelogram PQRS.



(a) 1/5 (b) 1/4
(c) 4/15 (d) 1/6

54. In a parallelogram ABCD, the mid-point of AB is H. The line parallel to DH and passing through B meets extended AD at K. If BC= 6 cm, then DK is–

(a) 10 cm. (b) 4 cm.
(c) 8 cm. (d) 6 cm.

55. ABCD is a trapezium whose parallel sides AD and BC are in the ratio of 3 : 2. The shortest distance between them is 10 cm. AB and DC are extended to meet at O. If the area of ABCD be 100 sq cm, then the area of ΔOBC is-

(a) 60 sq. cm (b) 80 sq. cm
(c) 90 sq. cm (d) 120 sq. cm

56. If the perimeter of a rectangle is P unit and its diagonal is *d* unit, then the difference between the length and width of the rectangle is-

(a) $\sqrt{\frac{8d^2-P^2}{4}}$ unit

(b) $\sqrt{\frac{8d^2-P^2}{2}}$ unit

(c) $\sqrt{\frac{8d^2+P^2}{2}}$ unit

(d) $\sqrt{\frac{8d^2+P^2}{4}}$ unit

57. Perimeter of a rhombus is $2p$ unit and sum of length of diagonals is m unit, then area of the rhombus is–

(a) $\frac{1}{4}m^2 p$ sq. unit

(b) $\frac{1}{4}mp^2$ sq. unit

(c) $\frac{1}{4}\left(m^2 - p^2\right)$ sq. unit

(d) $\frac{1}{4}\left(p^2 - m^2\right)$ sq. unit

58. If ABCD be a cyclic quadrilateral in which $\angle A = 4x$,

$\angle B = 7x$, $\angle C = 5y$, $\angle D = y$ then $x : y$ is-

(a) 3 %4 (b) 4 %3

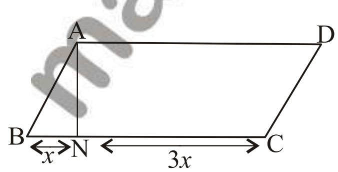
(c) 5 %4 (d) 4 %5

59. ABCD is a trapezium with AD and BC parallel sides. E is a point on AD. The ratio of the area of ABCD to that of BED is–

(a) $\frac{AD}{BC}$ (b) $\frac{BE}{EC}$

(d) $\frac{AD + BE}{AD + CE}$ (d) $\frac{AD+BC}{AD}$

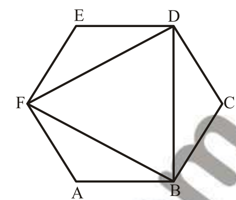
60. In the given figure, the ratio of the areas of the parallelogram ABCD to that of traingle ABN is:



(a) 6 : 1 (b) 5 : 1

(c) 4 : 1 (d) 8 : 1

61. ABCDEF is a regular hexagon of side 6 cm. what is the area of triangle BDF?



(a) $32\sqrt{3}$ cm^2 (b) $27\sqrt{3}$ cm^2

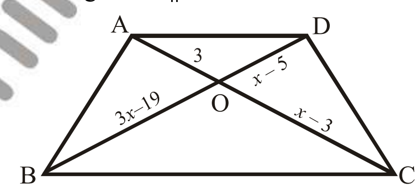
(c) 24 cm^2 (d) None of these

62. Area of a quadrilateral ACDE is 36cm^2. If B is the mid point of AC. Find the area of ΔABE if AC || DE and BE||CD

(a) 10 cm^2 (b) 9 cm^2

(c) 12 cm^2 (d) Can't be determined

63. In the figure BC||AD. Find the value of x.



(a) 9, 10 (b) 7, 8

(c) 10, 12 (d) 8, 9

64. The area of a trapezium is 180 cm^2 and its height to 9 cm. If one of the parallel sides is longer than the other by 6 cm, find the two parallel sides.

(a) 15 cm., 21 cm (b) 14 cm., 20 cm.

(c) 17 cm., 23 cm. (d) 12 cm., 18 cm

65. If sides of a parallelogram are 12 cm and 8 cm. It's one diagonal is 10 cm, then other diagonal is–

(a) 17.8 cm (b) 16.2 cm

(c) 16.9 cm

(d) Can't be determined

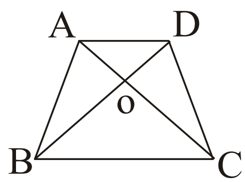
66. If l, b and p be the length, breadth and perimeter of a rectangle and b, l and p are in GP (in order)

then $\frac{l}{b}$ is–

(a) 2 : 1 (b) $\left(\sqrt{3}-1\right):1$

(c) $\left(\sqrt{3}+1\right):1$ (d) $2:\sqrt{3}$

67. In the adjoining figure, ABCD is a trapezium in which BC || AD and its diagonals intersect at O. If AO = $(3x - 1)$. OC = $(5x - 3)$, BO = $(2x + 1)$ and OD = $(6x - 5)$ then x is equal to.



(a) 1 (b) 3
(c) 2 (d) 4

68. ABCD is a trapezium in which AB || CD and the diagonals intersect at O. If AB = 6 cm and DC = 3 cm then the ratio of the areas of ΔAOB and ΔCOD is

(a) 4 : 1 (b) 1 : 2
(c) 2 : 1 (d) 1 : 4

69. The side of a rectangle ABCD, AB = 16 cm and AD = 8 cm. M, N, P are the mid points of side AB , AD and DC respectively. The area of quadrilateral formed by joining B, N, M, P is

(a) 256 (b) 48
(c) 64 (d) 128

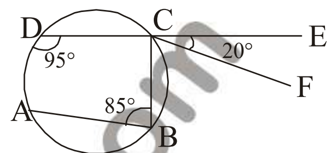
70. The difference of areas of two squares drawn on two line segements of different lengths is 32 sq. cm. Find the length of the greater line segment if one is longer than the other by 2 cm.

(a) 7 cm (b) 9 cm
(c) 11 cm (d) 16 cm

71. The area of a square is equal to that of a rectangle. The length of rectangle is 5cm more than the side of square and width is 3cm less than the side of square. The perimeter of the rectangle is

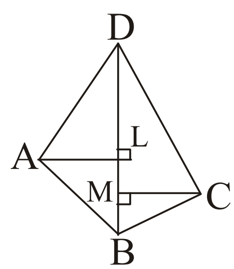
(a) 17 cm (b) 26 cm
(c) 30 cm (d) 34 cm

72. In the given figure, ABCD is a cyclic quadrilateral in which DC is produced to E and CF is drawn parallel to AB such that $\angle ADC = 95°$ and $\angle ECF = 20°$. Then, $\angle BAD = ?$



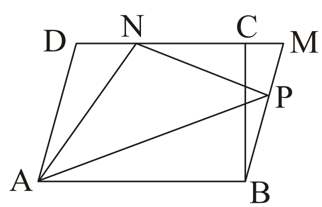
(a) 95° (b) 85°
(c) 105° (d) 75°

73. In a quadrilateral ABCD, it is given that BD=16 cm. If $AL \perp BD$ and $CM \perp BD$ such that AL = 9 cm and CM = 7cm, then ar (quad. ABCD) =



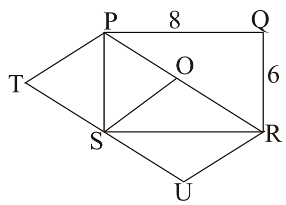
(a) 256 cm^2 (b) 128 cm^2
(c) 64 cm^2 (d) 96 cm^2

74. Parallelograms ABCD and ABMN are on the base AB, where AB || DM. If the area of || gm ABMN is 80sq. unit, what will be the area of ΔAPN ?



(a) 20 sq. unit
(b) 30 sq. unit
(c) 40 sq. unit
(d) 160 sq. unit

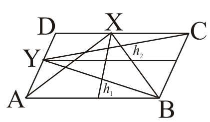
75. PQRS is a rectangle of dimensions 8 units and 6 units. PTUR is a rectangle drawn in such a way that diagonal PR of the first rectangle is one side and side opposite to it is touching the first rectangle at S as shown in the given figure. What is the ratio of the area of rectangle PQRS to that of PTUR?



(a) 2 (b) $\frac{3}{2}$

(c) 1 (d) $\frac{8}{9}$

76. Two points X and Y are on the sides DC and AD of the parallelogram ABCD. The $ar(\Delta ABX)$ is -



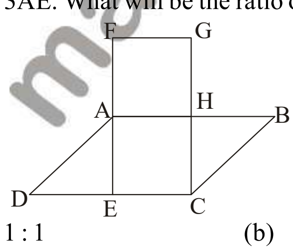
(a) $\frac{1}{2}$ of $ar(\Delta BYC)$

(b) equal to $ar(\Delta BYC)$

(c) $\frac{1}{3}$ of $ar(\Delta BYC)$

(d) Twice the $ar(\Delta BYC)$

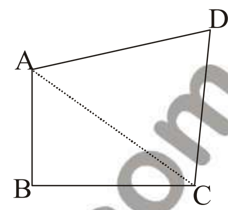
77. In the given figure, ABCD is a parallelogram whose sides AD = a units, DC = 2 a units and DE : EC = 1 : 2, CEFG is a rectangle whose side EF = 3AE. What will be the ratio of the area?



(a) 1 : 1 (b) 1 : 2

(c) 2 : 1 (d) 2 : 3

78. In the quadrilateral ABCD, $\angle B = 90°$ and $AD^2 = AB^2 + BC^2 + CD^2$, then find the measure of $\angle ACD$.



(a) 45° (b) 60°

(c) 90° (d) 30°

79. In trapezium ABCD, AB || DC and DC = 2 AB. EF drawn parallel to AB cuts AD at F and BC at E such that $\frac{BE}{EC} = \frac{3}{4}$. Diagonal DB intersect EF at G. Find $\frac{AB}{FE}$.

(a) $\frac{10}{7}$ (b) $\frac{4}{7}$

(c) $\frac{3}{7}$ (d) $\frac{7}{10}$

80. Each interior angle of a regular polygon is 120°. The number of sides is:

(a) 10 (b) 8

(c) 6 (d) 9

81. Each interior angle of a regular polygon is 140°. The number of diagonals is:

(a) 21 (b) 24

(c) 27 (d) 18

82. The sum of the interior angles of a polygon is 900°. The number of sides of the polygon is:

(a) 6 (b) 7

(c) 8 (d) 9

83. If each interior angle of a regular polygon is 4 times its exterior angle, the number of sides of the polygon is:

(a) 6 (b) 8

(c) 10 (d) 12

84. Difference the interior and exterior angles of regular polygon is 60°. The number of sides in the polygon is:

(a) 5 (b) 6
(c) 8 (d) 9

85. The sum of all the interior angles of a regular polygon is two times the sum of its exterior angles. The polygon is:

(a) Hexagon (b) Triangle
(c) Quadrilateral (d) Pentagon

86. The ratio between the number of sides of regular polygon 1 : 2 and the ratio between their interior angle is 2 : 3. The number of sides of these polygons are respectively.

(a) 3, 6 (b) 5, 10
(c) 6, 9 (d) 4, 8

87. The ratio of an angle of a regular hexagon to its exterior angle is :

(a) 1 : 2 (b) 2 : 1
(c) 1 : 3 (d) 3 : 1

88. If one of the interior angles of a regular polygon is equal to 3/4 times of one the interior angles of a regular hexagon, then the number of sides of the polygon is:

(a) 3 (b) 4
(c) 6 (d) 8

89. In a polygon five interior angles is equal to 172° and all remaining interior angles is equal to 160°. Find the number of sides of the polygon.

(a) 20 (b) 21
(c) 22 (d) 23

90. ABCDEF is a regular hexagon of side 2 feet. The area in square feet of the rectangle BCEF is :

(a) 8 (b) $4+4\sqrt{3}$
(c) 4 (d) $4\sqrt{3}$

91. The ratio of between the number of sides of regular polygon 1 : 2 and the ratio between their interior angle is 1 : 2. The number of sides of these polygons are respectively.

(a) 3, 6 (b) 5, 10
(c) 6, 9 (d) 4, 8

92. In a polygon five interior angles is equal to 162° and all remaining interior angles is equal to 150°. Find the number fo sides of the polygon.

(a) 12 (b) 14
(c) 16 (d) 18

93. The ratio of the number of sides of two regular polygons is 1 : 2. If each interior angle of the first polygon is 120°, then the measure of each interior angle of the second polygon is

(a) 150° (b) 160°
(c) 140° (d) 135°

94. If the interier angles of a polygon are in A.P. with common difference 5º and the smallest angle 120º, then the number of sides of the polygon is:

(a) 9 or 16 (b) 9
(c) 13 (d) 3 or 16

95. There are two regular polygons with number of sides equal to (n–1) and (n+2). Their exterior angle differ by 6°. Then value of n is–

(a) 14 (b) 12
(c) 13 (d) 11

96. Two regular polygone are such that the ratio between their number of sides is 1 : 2 and the ratio of measures of their interior angles is 3 : 4. Find the number of sides of each polygon.

(a) 4, 8 (b) 5, 10
(c) 6, 12 (d) 3, 6

97. ABCDE is a regular pentagon and bisector of $\angle BAE$ meets CD at M. If bisector of $\angle BCD$ meets AM at P, find $\angle CPM$.

(a) 36° (b) 54°
(c) 48° (d) 68°

98. ABCDE is a regular pentagon. The bisector of $\angle A$ of the pentagon meets the side CD in M. Find $\angle AMC$.
(a) 80° (b) 90°
(c) 75° (d) 60°

99. If one of the interior angles of a regular polygon is found to be equal to (9/8) times of one of the interior angles of a regular hexagon, then the number of sides of the polygon is.
(a) 8 (b) 7
(c) 9 (d) 14

100. If one of the interior angles of a regular polygon is equal to $\frac{7}{6}$ times one of the interior angles of a regular hexagon. Then what is the number of sides of the polygon?
(a) 8 (b) 9
(c) 11 (d) 14

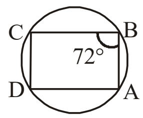
101. A polygon has 27 diagonals. The number of sides in the polygon is:
(a) 7 (b) 6
(c) 9 (d) 12

102. A regular polygon is inscribed in a circle. If a side subtends an angle of 72° at the centre, then the number of sides of the polygon is:
(a) 5 (b) 7
(c) 6 (d) 8

Answer

1. (d)	2. (b)	3. (c)	4. (c)	5. (c)	6. (b)	7. (a)	8. (b)	9. (b)
10. (d)	11. (d)	12. (a)	13. (c)	14. (d)	15. (b)	16. (c)	17. (c)	18. (c)
19. (b)	20. (b)	21. (a)	22. (a)	23. (c)	24. (c)	25. (c)	26. (c)	27. (a)
28. (a)	29. (b)	30. (d)	31. (c)	32. (c)	33. (b)	34. (d)	35. (b)	36. (b)
37. (b)	38. (b)	39. (c)	40. (b)	41. (a)	42. (b)	43. (b)	44. (b)	45. (c)
46. (b)	47. (d)	48. (d)	49. (a)	50. (d)	51. (b)	52. (b)	53. (a)	54. (d)
55. (b)	56. (a)	57. (c)	58. (b)	59. (d)	60. (d)	61. (b)	62. (c)	63. (d)
64. (c)	65. (a)	66. (c)	67. (b)	68. (a)	69. (c)	70. (b)	71. (c)	72. (c)
73. (b)	74. (b)	75. (c)	76. (b)	77. (b)	78. (c)	79. (a)	80. (c)	81. (c)
82. (b)	83. (c)	84. (b)	85. (a)	86. (d)	87. (b)	88. (b)	89. (b)	90. (d)
91. (a)	92. (b)	93. (a)	94. (a)	95. (c)	96. (b)	97. (a)	98. (b)	99. (a)
100.(b)	101. (c)	102. (a)						

Solution & Hints

1. 

$\therefore$ AD$\parallel$BC

Then, $\angle A + 72° = 180°$

$\angle A = 180°$

In cyclic quadrilateral ABCD

$\therefore \angle A + \angle C = 180°$

$\Rightarrow \angle C = 180° - 108° = 72°$

2. The measures of the angles of quadrilateral taken in order are proportion to 1: 2 : 3 : 4

summation of four angles = 360º

$\therefore x + 2x + 3x + 4x = 360º$

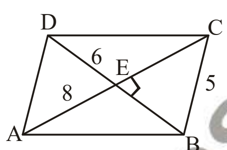
$\Rightarrow 10x = 360º$

$\Rightarrow x = 36°$

Now, angles will be :- 36º, 72º, 108º, 144º

Hence, It will be trapezium

3. Diagonals of a parallelogram are 8 m and 6 m. In parallelogram.



$AC^2 + BD^2 = 2(AB^2 + BC^2)$

$(8)^2 + (6)^2 = 2[(5)^2 + (BC)^2]$

$\Rightarrow \frac{100}{2} = 25 + BC^2$

$\Rightarrow 50 - 25 = BC^2$

$BC = \sqrt{25} = 5$ cm.

sides are equal diagonals meet at 90º

In $\triangle$BEC

DB = 6cm, BE = 3 cm.

AC = 8 cm, EC = 4 cm.

And BC = 5 cm.

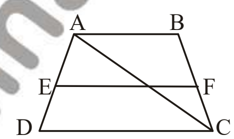
So, In right angle triangle DEC area of triangle =

$\frac{1}{2}$ Base × height

$= \frac{1}{2} \times 4 \times 3$

$= 6 \text{ cm}^2$

So, the area of parallelogram = 4 × 6 = 24 cm^2

4. 

In a trapezium ABCD

AB$\parallel$CD

E and F are mid point of its non parallel sides

a line is drawn from A to C, which intersects EF at G

In $\triangle$ADC

EF$\parallel$DC

$EG = \frac{1}{2} DC$ ......(i)

Similarly, In $\triangle$ABC,

AB$\parallel$GF

$GF = \frac{1}{2} AB$ .....(ii)

Adding eq. (i) and (ii) we get,

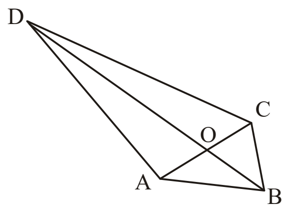
$EG + GF = \frac{1}{2} DC + \frac{1}{2} AB$

$\Rightarrow \quad EF = \frac{1}{2}DC + \frac{1}{2}AB$

$\Rightarrow \quad EF = \frac{1}{2}(p+q)$

$\begin{bmatrix} \because DC = p \\ AB = q \end{bmatrix}$

5.



In any triangle sum of two sides is always greater than third side.

In $\Delta ABD \Rightarrow AB + AD > BD$ .... (i)

In $\Delta BCD \Rightarrow BC + CD > BD$ .... (ii)

In $\Delta ABC \Rightarrow AB + BC > AC$ .... (iii)

In $\Delta ACD \Rightarrow AD + CD > AC$ .... (iv)

Adding Eq. (i), (ii), (iii) and (iv) we get,

$2(AB + BC + CD + DA) > 2(AC + BD)$

$\Rightarrow AB + BC + CD + DA > AC + BD$

6. ABCD is a parallelogram E is mid point of BC

$\therefore \quad BE = CE$

Now, In triangle ΔDCE and ΔEBF

$\angle DBC = \angle EBF$

$\angle CDE = \angle EFB$

$CD = BE$

by SAS, Both triangle are congurrent

Hence, $CD = BF \Rightarrow CD = AB$

$BF = AB$

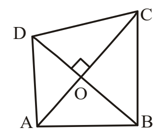
Hence, $AF = AB + BF = 2AB$

7. Area of parallelogram ABCD = height × base

$= DL \times AB$

$= 4 \times 10 = 40 cm^2$

8.



In right angle triangles BOC, OCD, OAD, OAB

$OB^2 + OC^2 = BC^2$

$OC^2 + OD^2 = CD^2$

$OD^2 + OA^2 = AD^2$

$OA^2 + OB^2 = AB^2$

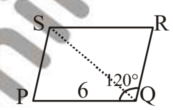
On adding all above equations we get,

$2(OB^2 + OA^2 + OD^2 + OC^2) = AB^2 + BC^2 + CD^2 + DA^2$

$\Rightarrow 2(AB^2 + CD^2) = AB^2 + BC^2 + CD^2 + DA^2$

$\Rightarrow AB^2 + CD^2 = BC^2 + DA^2$

9.



$\therefore$ diagonal of rhombus bisects vertex angle

$\Rightarrow \angle PQS = \frac{1}{2}\angle PQR = 60°$

$\Rightarrow \angle PQS = \angle PSQ = 60°$

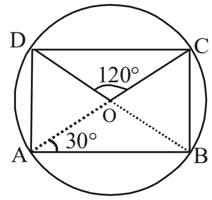
and $\angle SPQ + \angle PQR = 180°$

$\angle SPQ = 180° - 120° = 60°$

$\therefore \Delta SPQ$ is an equilateral triangle

$SP = PQ = QS = 6$ cm

10.



$OD = OC$ (radius)

In ΔODC,

$\angle ODC + \angle DOC + \angle OCD = 180°$

$\Rightarrow \angle ODC = \angle OCD = 30°\ (\because OD = OC)$

and $\angle DBC = \frac{1}{2}\angle DOC$

($\because$ angle made by chord on circle is half of angle made by same chord at central)

$\Rightarrow \angle DBC = 60°$

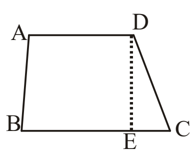
In ΔDBC

$\angle DBC + \angle BDC + \angle BCD = 180°$

$\Rightarrow 60° + 30° + \angle BCD = 180°$

$\Rightarrow \angle BCD = 90°$

11.



In trapezium ABCD

$\text{Area} = \frac{1}{2} \times (\text{sum of parallel sides}) \times \text{Height}$

$= \frac{1}{2}(AD + BC) \times DE$

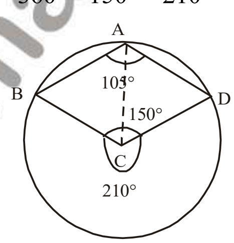
$\Rightarrow 105 = \frac{1}{2}(9 + 12) \times DE$

$\Rightarrow DE = 10\text{ cm}$

12. See example 107.

13 . Reflex angle

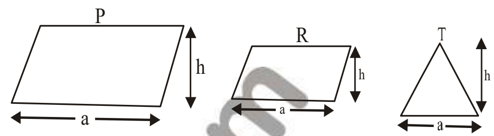
$\angle BCD = 360° - 150° = 210°$



210° is double of 105° hence A,B,D are on the circle and C is centre then

radius = AC = BC = CD = 13 cm

14.



Let base side be a and height be h

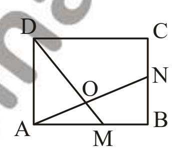
area of parallelogram (P) $= ah$ **............. (i)**

area of rhombus (R) $= ah$ **............(ii)**

area of triangle (T) $= \frac{1}{2}ah$

$\Rightarrow R = P = 2T$

15.



ABCD is a square and M, N are mid Points of AB and BC respectively

If $AB = 2x$

then, $BN = x$

$\therefore AN = \sqrt{AB^2 + BN^2}$

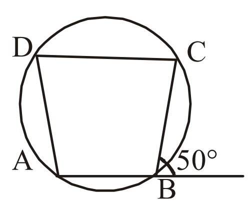
$= \sqrt{4x^2 + x^2} = \sqrt{5}\,x$

Similarly : $MD = \sqrt{AD^2 + AM^2}$

$= \sqrt{4x^2 + x^2} = \sqrt{5}\,x$

Hence, AN = MD

16.



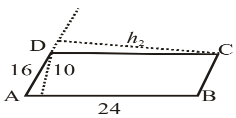
$\angle ABC = 180° - 50° = 130°$

In cyclic quadrilateral

$\therefore \angle ABC + \angle ADC = 180°$

$\Rightarrow \angle ADC = 180° - 130° = 50°$

17.



$h_1 = 10$ cm, $l = 24$ cm, $b = 16$ cm

$h_2 = ?$

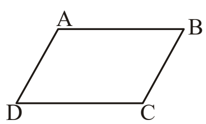
area of parallelogram

$lh_1 = bh_2$

$\Rightarrow 24 \times 10 = 16 \times h_2$

$\Rightarrow h_2 = 15$ cm

18.



Let $\angle A$ and $\angle B$ be $4x$ and $5x$ respetively.

In non– square rhombus

$\angle A + \angle B = 180°$

$4x + 5x = 180^o$

$\Rightarrow x = 20^o$

$\Rightarrow \angle A = 4x = 80^o$

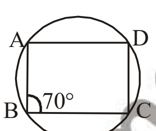
$\Rightarrow \angle B = 5x = 100^o$

and, $\angle B + \angle C = 180^o$

$\Rightarrow \angle C = 180^o - 100^o$

Hence, $\angle C = 80^o$

19.



$AD \parallel BC$

$\therefore \angle ABC + \angle BAD = 180^o$ (adjacent angles)

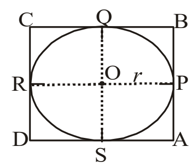
$\Rightarrow \angle BAD = 180^o - 70^o = 110^o$

ABCD is cyclic trapezium

$\therefore \angle BAD + \angle BCD = 180°$

$\angle BCD = 180° - 110° = 70°$

20.



(Two tangents drawn from an external point to a circle are equal)

$\therefore$ BP = BQ = 27cm

$\Rightarrow$ CQ = BC – BQ = 38 – 27 = 11 cm

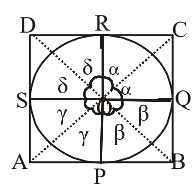
$\Rightarrow$ CQ = CR = 11cm

$\Rightarrow$ DR = CD – CR = 25 – 11 = 14 cm

$\therefore$ OR $\perp$ CD and OS $\perp$ AD (line drawn from centre to tangent will be perpendicular to tangent)

$\Rightarrow$ radius (r) = DR = 14 cm

21.



In triangle OQC and ORC

$\therefore$ OR = OD (radius)

and RC = QC

(tangent from an external point are equal length)

$\angle ORC = \angle OQC = 90°$

$\Delta OQC \cong \Delta ORC$ (congruent triangle)

$\Rightarrow \angle ROC = \angle QOC = \alpha$ (let)

and

$\angle QOB = \angle POB = \beta$ (let)

$\angle POA = \angle SOA = \gamma$ *(let)*

$\angle SOD = \angle ROD = \delta$ (let)

angle on a centre of circle = 360^o

$2\alpha + 2\beta + 2\gamma + 2\delta = 360^\circ$

$\alpha + \beta + \gamma + \delta = 180^\circ$

$(\alpha + \delta) + (\beta + \gamma) = 180^\circ$

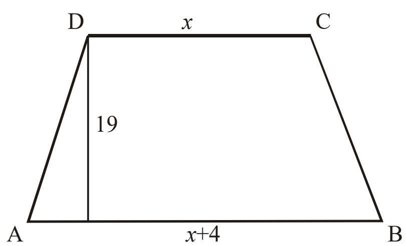
$\angle COD + \angle AOB = 180^\circ$

Similarly,

$\angle BOC = \angle AOD$

hence, angle subtended at the centre by a pair of opposite sides is equal to 180º

22.



ABCD is a trapezium

$AB = x + 4$, and $CD = x$

$\therefore$ Area of trapezium $= \dfrac{1}{2}(AB + CD) \times CE$

$\Rightarrow \ 475 = \dfrac{1}{2}(x + 4 + x) \times 19$

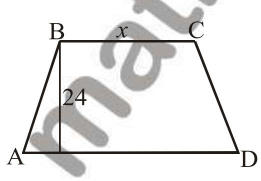
$\Rightarrow 2x + 4 = 50$

$\Rightarrow x = 23$

$\Rightarrow$ AB = 27 cm

$\Rightarrow$ CD = 23 cm

23.



ABCD is a trapezium

Let the side $AD = 5x$ and $BC = 3x$

$\therefore \quad$ Area $= \dfrac{1}{2}(AD + BC) \times$ height

$\Rightarrow \ 1440 = \dfrac{1}{2}(8x) \times 24$

$\Rightarrow \ x = \dfrac{360}{24} = 15m.$

$\Rightarrow BC = 5x = 75\ m$

$\Rightarrow AD = 3x = 45\ m$

24. Let side of equilateral triangle be a and of square be b

and radius of circle be r.

$\therefore$ Perimeter are equal

$3a = 4b = 2\pi r$

$\Rightarrow b = 3a/4$

$\Rightarrow r = 3a/2\pi$

$\Rightarrow$ area of triangle $P = \dfrac{\sqrt{3}}{4}a^2$ ..... (i)

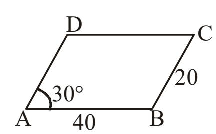
$\Rightarrow$ area of square $S = b^2 = \left(\dfrac{3a}{4}\right)^2 = \dfrac{9a^2}{16}$ .....(ii)

$\Rightarrow$ area of circle $C = \pi r^2 = \dfrac{9a^2}{4\pi}$ .... (iii)

From (i), (ii) and (iii)

$T < S < C$

25.



$\therefore AD \parallel BC$

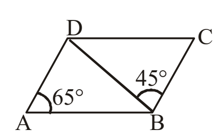
$\Rightarrow \angle ABC = 180^\circ - 30^\circ = 150^\circ$

$\therefore$ area of parallelogram = AB. BC sin 150º

$= 40 \times 20 \times \sin 150^\circ$

$= 400\ cm^2$

26.



In Parallelogram ABCD

$\therefore \quad \angle A + \angle C = 180°$

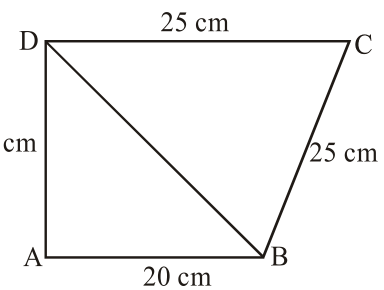
$\angle C = 180° - 65° = 115°$

$\therefore$ In ΔBDC

$\angle BDC + \angle DCB + \angle DBC = 180°$

$\Rightarrow \angle BDC = 180° - 115° - 45° = 20°$

27.



In right angle triangle DAB,

$BD^2 = AD^2 + AB^2$

(Using Pythagoras theorem)

$(BD)^2 = (15)^2 + (20)^2$

$BD = 25$ cm

$\therefore$

Now, ΔBCD is an equilateral triangle

$\text{Area} = \frac{\sqrt{3}}{4}(\text{side})^2$

$= \frac{\sqrt{3}}{4}(25)^2$

Area of ABCD $= \Delta DAB + \Delta BCD$

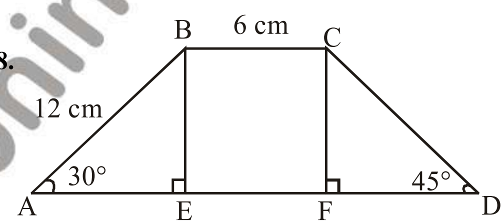
$= \frac{1}{2} \times \text{Base} \times \text{height} + \frac{\sqrt{3}}{4} \times 25 \times 25$

$= \frac{1}{2} \times 15 \times 20 + \frac{\sqrt{3}}{4} \times 25 \times 25$

$= 150 + \frac{\sqrt{3}}{4} \times 25 \times 25$

$= \frac{25}{4}(24 + 25\sqrt{3})$

28.



In Trapezium ABCD,

In ΔABE,

$\cos 30° = \frac{AE}{AB}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AE}{12}$

$\Rightarrow AE = 6\sqrt{3}$

In ΔCFE,

$\tan 45° = \frac{CF}{FD}$

$\Rightarrow 1 = \frac{6}{FD}$

$\Rightarrow FD = 6$ cm

$\because BC = EF = 6$ cm.

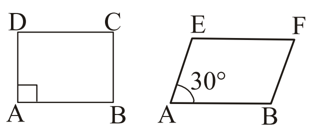
Now the area of trapezium = $\frac{1}{2}$ ×(Sum of parallel sides)×height

$= \frac{1}{2} \times (BC + AD) \times BE$

($\therefore$ AD=AE+EF+FD= $6\sqrt{3}+6+6$)

$= \frac{1}{2}(18 + 6\sqrt{3}) \times 6$

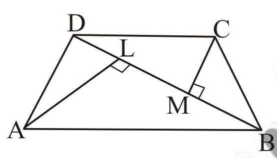
Area = $18(3+\sqrt{3})$ cm^2

29. 

Area of square ABCD = a^2
area of rhombus ABEF = $a^2 \sin 30°$

$= \frac{a^2}{2}$

$\therefore \frac{\text{area of square}}{\text{area of rhombus}} = 2 : 1$

30. 

Area of Δ ABD $= \frac{1}{2} \times AL \times BD$

$= \frac{1}{2} \times 13.2 \times 64$

$= 422.4$ cm^2

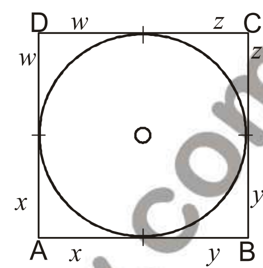
Area of Δ BCD $= \frac{1}{2} \times CM \times BD$

$= \frac{1}{2} \times 16.8 \times 64 = 537.6$ cm^2

$\Rightarrow$ area of quadrilateral ABCD = area of Δ ABD + area of ΔBCD

$= 422.4 + 537.6$

$= 960$ cm^2

31. 

($\therefore$ Tangents from an external point to a circle are equal)

AB = $x + y = 6$ cm ....(i)

BC = $y + z = 7$ cm ....(ii)

CD = $w + z = 5$ cm ....(iii)

AD = $w + x = ?$

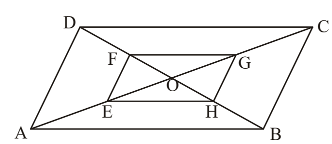
from (i) and (ii)

$x - z = -1$ ....(iv)

adding (iii) and (iv) we get

$x + w = 4$ cm

$\therefore$ AD = 4 cm

32. 

$\therefore$ E and H are mid points of OA and OB respectively

$\Rightarrow$ EH = $\frac{1}{2}$ AB

similarly, GH = $\frac{1}{2}$ BC, FG = $\frac{1}{2}$ CD, EF = $\frac{1}{2}$ AQ

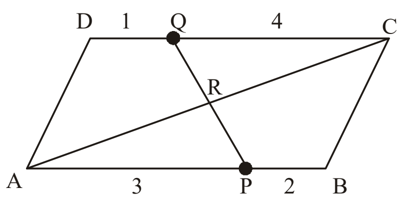
Perimeter of parallelogram = (AB + BC + CD + DA)

Perimeter of quadrilateral EFGH = EH + HG + FG + EF

$= \frac{1}{2}(AB+BC+CD+DA)$

$\therefore \quad \frac{\text{Perimeter of EFGH}}{\text{Perimeter of ABCD}} = 1:2$

33.



$AB \parallel CD$

In ΔARP and ΔQRC

$\Rightarrow \angle RAP = \angle RCQ$ (alternate interior angles)

$\Rightarrow \angle RPA = \angle RQC$

$\Delta ARP \sim \Delta RCQ$ (similar triangle)

$\Rightarrow \frac{RC}{AR} = \frac{QC}{AP} = \frac{4}{3}$

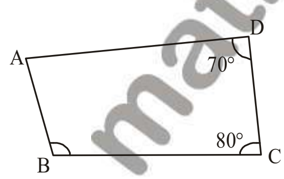
adding 1 both the sides

$\Rightarrow \frac{RC}{AR} + 1 = \frac{4}{3} + 1$

$\Rightarrow \frac{RC+AR}{AR} = \frac{7}{3}$

$\Rightarrow AR = \frac{3}{7} AC$

34.



ABCD is a quadrilateral

$\because \angle A + \angle C = 180^\circ$

$\Rightarrow \angle A = 180^\circ - 80^\circ = 100^\circ$

$\because \angle B + \angle D = 180^\circ$

$\Rightarrow \angle B = 180^\circ - 70^\circ = 110^\circ$

$\angle OAB = \frac{1}{2} \angle A \Rightarrow \frac{100}{2} = 50^\circ$

$\angle OBC = \frac{1}{2} \angle B \Rightarrow \frac{110}{2} = 55^\circ$

In ΔAOB

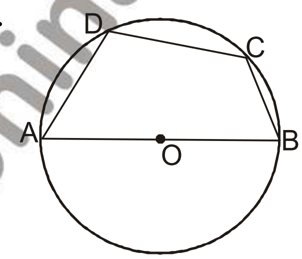
$\angle AOB + \angle OAB + \angle ABO = 180^\circ$

$\Rightarrow \angle AOB + 50^\circ + 55^\circ = 180^\circ$

$\Rightarrow \angle AOB = 180^\circ - 105^\circ$

$\angle AOB = 75^\circ$

35.



AB is diameter, $\angle ADB = 90^\circ$

In ΔADB

$\therefore \angle ADB + \angle ABD + \angle DAB = 180^\circ$

$\Rightarrow \angle DAB + 50^\circ + 90^\circ = 180^\circ$

$\Rightarrow \angle DAB = 40^\circ$

$\therefore \quad \angle DAB + \angle DCB = 180^\circ$

($\therefore$ opposite angle of cyclic quadrilateral)

$\angle DCB = 180^\circ - 40^\circ = 140^\circ$

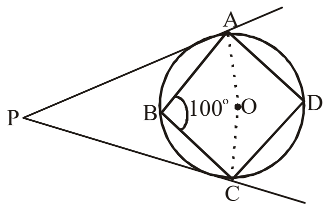
Now, In ΔDCB,

$\therefore CD = BC \Rightarrow \angle CDB = \angle DBC$

$\angle CDB + \angle DBC + \angle DCB = 180^\circ$

$\therefore \angle DBC = \frac{1}{2}(180^\circ - 140^\circ) = 20^\circ$

36.



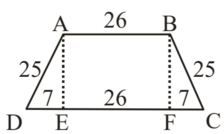
$\angle ADC = 180° - 100° = 80°$

$\Rightarrow \angle AOC = 2 \times \angle ADC = 2 \times 80° = 160°$

$\therefore \angle APC + \angle AOC = 180°$

$\Rightarrow \angle APC = 180° - 160° = 20°$

37.



Let height of parallelogram be h

Two perpendicular AE and EF are drawn on CD

$\therefore AB = EF = 26$ cm

$\Rightarrow DE = FC = \frac{1}{2}(40 - 26) = 7$ cm

In $\triangle AED$,

$\therefore (AD)^2 = (DE)^2 + (AE)^2$ **(Pythagoras)**

$\Rightarrow (25)^2 = (7)^2 + h^2$

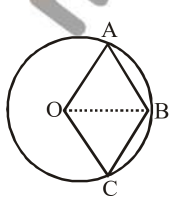
$\Rightarrow h = 24$ cm

$\therefore$ area of trapezium $= \frac{1}{2} \times h \times$ (sum of parellel sides)

$= \frac{1}{2} \times 24 \times (26 + 40)$

$= 792\ cm^2$

38.



$OA = AB = BC = OC = 10cm = a$ (side of rhombus)

$\therefore OB = d_1 = 10$ cm **..........(radius)**

$\Rightarrow d_1^2 + d_2^2 = 4a^2$

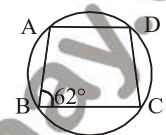
$\Rightarrow d_2^2 = 400 - 100 = 300$

$AC = d_2 = 10\sqrt{3}$ cm

$\therefore$ area of rhombus $= \frac{1}{2} d_1 d_2$

$= \frac{1}{2} \times 10 \times 10\sqrt{3} = 50\sqrt{3}\ cm^2$

39.



$\therefore$ ABCD is a cyclic trapezium

$\Rightarrow \angle ABC + \angle CDA = 180°$

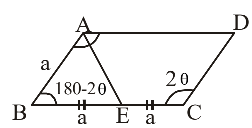
$\Rightarrow \angle CDA = 180° - 62° = 118°$

$\therefore AD \parallel BC$

$\Rightarrow \angle CDA + \angle BCD = 180°$ **(adjacent angles)**

$\Rightarrow \angle BCD = 180° - 118° = 62°$

40.



Let $\angle A = 2\theta$

$\Rightarrow \angle BAE = \angle EAB = \theta$ [$\therefore$ AE is angle bisector of $\angle A$]

$\Rightarrow \angle A + \angle B = 180°$

$\Rightarrow \angle B = 180° - 2\theta$

In $\triangle ABE$,

$\therefore BAE + \angle AEB + \angle ABE = 180°$

$\Rightarrow \theta + \angle AEB + 180° - 2\theta = 180°$

$\Rightarrow \angle AEB = \theta$

$\therefore \angle BAE = \angle AEB = \theta$

$\Rightarrow AB = BE = a$

and, $AQ = BC = 2a$

Hence, $AD = 2AB$

41. Let the two diagonals be $2d$ and $5d$

$$\text{area} = \frac{1}{2}d_1d_2 = \frac{1}{2} \times 2d \times 5d = 5d^2$$

$$\therefore \quad \frac{\text{area}}{\text{square of shorter diagonal}} = \frac{5d^2}{4d^2} = 5:4$$

42. let d_1 and d_2 are the diagonal of rhombus

$\therefore\ d_1 + d_2 = 10$ m ....(i)

$$\text{area of rhombus} = \frac{1}{2}\ d_1d_2 = 9\ \text{m}^2$$

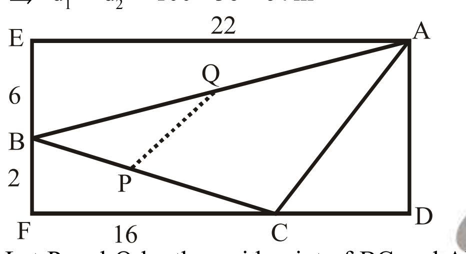
$\Rightarrow\ d_1d_2 = 18$

squaring eq. (i) we get.

$\therefore\ \ d_1^{\,2} + d_2^{\,2} + 2d_1d_2 = 100$

$\Rightarrow\ d_1^{\,2} + d_2^{\,2} = 100 - 36 = 64\ \text{m}^2$

43.



Let P and Q be the mid point of BC and AB respectively.

$$\therefore \quad PQ = \frac{1}{2}\ AC$$

$\Rightarrow\ CD = FD - FC = 22 - 16 = 6$ cm

$\Rightarrow\ AD = 8$ cm

In Δ ACD $\Rightarrow\ AC^2 = CD^2 + AD^2$ **(Pythagoras)**

$$\Rightarrow AC = \sqrt{(6)^2 + (8)^2} = 10\ \text{cm}$$

$$PQ = \frac{1}{2} \times AC = \frac{1}{2} \times 10 = 5\ \text{cm}$$

44. $\therefore$ area of square ABEF $= \frac{1}{2} \times$ (area of trapezium ABCD)

$$AB \times AF = \frac{1}{2} \times \left[\frac{1}{2} \times AF \times (AB + CD)\right]$$

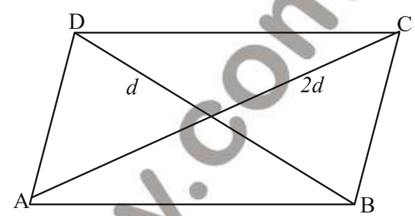
$$\Rightarrow \quad 6 \times AF = \frac{1}{2} \times \left[\frac{1}{2} \times AF \times (6 + CD)\right]$$

$\Rightarrow\ CD = 18$ cm

$$\Rightarrow DF = EC = \frac{1}{2}\ (DC - EF) = \frac{1}{2}\ (18 - 6) = 6\ \text{cm}$$

$$\Rightarrow \frac{DF}{CD} = \frac{6}{18} = \frac{1}{3}$$

45.



ABCD is a rhombus

AC = 2 × BD **......... (given)**

If BD = d then AC = $2d$

$$\therefore \text{Area} = \frac{1}{2}\ (AC \times BD)$$

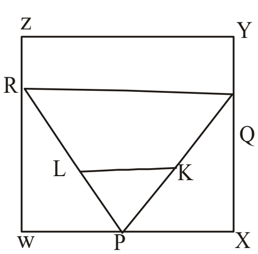
$$\Rightarrow 25 = \frac{1}{2}\ (2d \times d)$$

$\Rightarrow 25 = d$

$\Rightarrow d = 5$, and $2d = 10$

Sum of diagonals = / 15 cm.

46.



Let the side of square be $2a$

$\therefore$ area of square $= (2a)^2 = 4a^2$

Now,

In Δ RQP

$\Rightarrow$ RQ = 2a

$\Rightarrow$ RP = QP $= \frac{1}{2}\sqrt{(2a)^2 + (2a)^2} = \sqrt{2}a$

LK = $\frac{1}{2}$ RQ.

area of Δ PLK = 1 Unit (let)

Then, area of Δ RPQ =4 Units

so area of Quadrilateral WXYZ=16

$$\frac{\text{area of triangle PKL}}{\text{area of square WXYZ}} = \frac{1}{16}$$

47. AB || CD

In Δ DCP and Δ QBP

$\angle$ DCP = $\angle$ QBP (alternate interior angles)

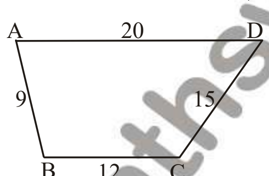
$\angle$ PDC = $\angle$ QBP (alternate interior angle)

hence, $\Delta DCP \sim \Delta QBP$

$$\frac{\text{area of } \Delta DCP}{\text{area of } \Delta OBP} = \left(\frac{PC}{PB}\right)^2$$

$\Rightarrow$ area of Δ DCP $= 20 \times \left(\frac{2\times2}{1\times1}\right) = 80$ square unit

48.



$AC^2 + BD^2 = AD^2 + BC^2 + 2AB.CD$

$= (20)^2 + (12)^2 + 2.(9).(15)$

$= 400 + 144 + 270 = 814$

49. AO = AX

$\angle$ AOX = $\angle$ AXO

In Δ AOX

$\therefore$ $\angle$ OAX + $\angle$ AOX + $\angle$ AXO = 180º

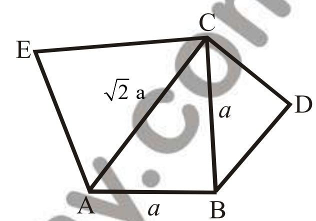
$\Rightarrow$ $\angle$ AOX + $\angle$ AXD = 180º – 45º = 135º

$\Rightarrow$ $\angle$ AOX = $\angle$ AXD = $\frac{1}{2}$ × 135º = 67.5º

$\therefore$ $\angle$ AOX + $\angle$ XOB = 90º **(diagonal bisects at 90º)**

$\Rightarrow$ $\angle$ XOB = 90º – 67.5º = 22.5º

50.



Let AB = BC = a

$\therefore$ AC = $\sqrt{AB^2 + BC^2} = \sqrt{a^2 + a^2} = \sqrt{2}a$

.... (pythagoras theorem)

$$\frac{\text{area }(\Delta BCD)}{\text{area }(\Delta ACE)} = \frac{\frac{\sqrt{3}}{4}a^2}{\frac{\sqrt{3}}{4}(\sqrt{2}a)^2} = 1:2$$

51. $\therefore$ AE = 2EF = 3 FB

AE : EF : FB = 6 : 3 : 2

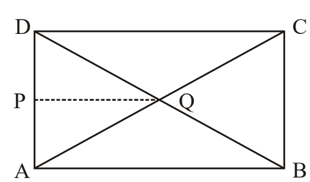
Let AE, EF and FB are $6x$, $3x$ and $2x$ respectively.

$\Rightarrow$ AB = AE + EF + EF + FB = $11x$

$$\frac{\text{area (rectangle ABCD)}}{\text{area }(\Delta CEF)} = \frac{AB \times CB}{\frac{1}{2} \times EF \times CB}$$

$$= \frac{11x}{\frac{3x}{2}} = 22:3$$

52.



$\therefore$ area of trapezium ABPQ $= \frac{1}{2} \times$ (Sum of parallel sides)×Height

$= \frac{1}{2} \quad (PQ+AB)\times AP \quad = \frac{3AB}{4} \times AP$

$(\because PQ = \frac{1}{2} AB)$

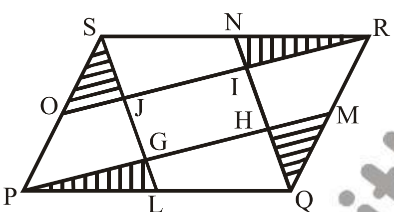
$\therefore$ area of square ABCD = AB×AD = AB×(2AP)

$(\because AP = \frac{1}{2} AD)$

$$\Rightarrow \frac{\text{area of trapezium ABPQ}}{\text{area of square ABCD}} = \frac{\frac{3}{4} AB\times AP}{AB\times(2AP)}$$

$= 3:8$

53.



In ΔSRO, IN || SJ & L is Mid point of PQ

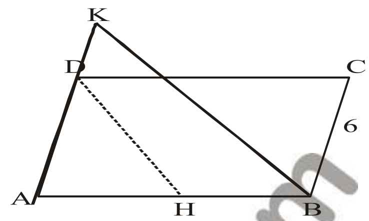
$$\frac{\text{Area of } \Delta NIR}{\text{Area of } \Delta SJR} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$\Rightarrow$ Area of $\Delta NIR = 1$ unit then area of trapezium NIJS = 3 unit

Similarly, IN ΔSPG, ΔPHQ, and ΔQRI

$$\frac{\text{Area of dashed region}}{\text{Area of PQRS}} = \frac{1+1+1+1}{5\times 4} = \frac{1}{5}$$

54.



$\therefore$ H is mid point of AB

$AH = \frac{1}{2} AB = HB$

In Δ ADH and Δ ABK

$\angle A = \angle A$ (common)

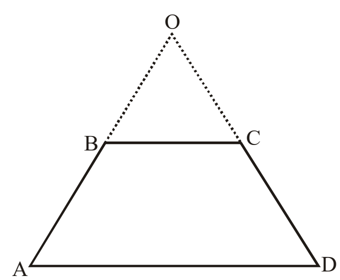
$\angle AHD = \angle ABK$ (DH || BK)

$\therefore \Delta AHD \sim \Delta ABK$

$$\Rightarrow \frac{AH}{HB} = \frac{AD}{DK} = 1$$

AD = DK = 6 cm

55.



Let AD and BC are 3*k* and 2*k* repectively

area of ABCD $= \frac{1}{2} \times h \times (AD + BC)$

$\Rightarrow 100 = \frac{1}{2} \times 10\,(3k + 2k)$

$\Rightarrow k = 4$

$\Rightarrow AD = 4 \times 3 = 12$ cm and $BC = 4 \times 2 = 8$ cm

Now, In ΔOBC and ΔOAD

$\therefore$ BC || AD

$\angle OBC = \angle OAD$ (corresponding angle)

$\angle OCB = \angle ODA$ (corresponding angle)

$\Delta OBC \sim \Delta OAD$

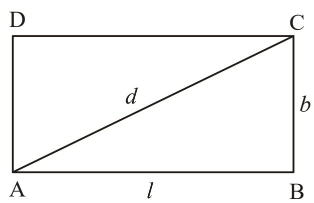
$$\therefore \quad \frac{\text{area of } \Delta OBC}{\text{area of } \Delta OAD} = \left(\frac{BC}{AD}\right)^2 = \frac{4}{9}$$

$$\Rightarrow \quad \frac{\text{area of } \Delta OBC}{\text{area of } \Delta OBC + \text{area of } ABCD} = \frac{4}{9}$$

$$\Rightarrow \quad \frac{\text{area of } \Delta OBC}{\text{area of } \Delta OBC + 100} = \frac{4}{9}$$

$\Rightarrow$ area of Δ OBC = 80 cm^2

56.



Perimeter (P) = $2(l + b)$

diagonal (d) = $\sqrt{l^2 + b^2}$

$\Rightarrow d^2 = l^2 + b^2$ ....(i)

$\Rightarrow \frac{P^2}{4} = l^2 + b^2 + 2lb$ ....(ii)

from eq. [(ii) – (i)] we get

$$\frac{P^2}{4} - d^2 = 2lb$$

Now, putting value of P and d

$(l - b)^2 = l^2 + b^2 - 2lb$

$$\therefore \quad (l - b)^2 = d^2 - \frac{P^2}{4} + d^2 = \frac{8d^2 - P^2}{4}$$

$$\Rightarrow \quad l - b = \sqrt{\frac{8d^2 - P^2}{4}} \text{ unit}$$

57. Sum of diagonal length = m **..........(given)**

Perimieter = $2p$ **........(given)**

area of Rhombus = ?

$4a^2 = d_1^2 + d_2^2$

$\because$ $4a = 2P$ (perimeter)

$$\Rightarrow a = \frac{P}{2}$$

and $d_1 + d_2 = m$

Squaring both sides

$\Rightarrow d_1^2 + d_2^2 + 2d_1d_2 = m^2$

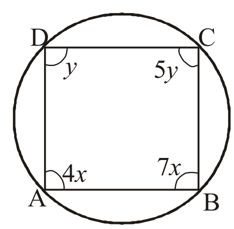
$\Rightarrow 4a^2 + 2d_1d_2 = m^2$

$\Rightarrow 2d_1d_2 = m^2 - 4a^2$

$\Rightarrow 2d_1d_2 = m^2 - 4\left(\frac{P}{2}\right)^2$ (Put $a = P/2$)

$$\Rightarrow \frac{1}{2}d_1d_2 = \frac{m^2 - P^2}{4}$$

58.



ABCD is a cyclic quadrilateral

$\therefore \angle A + \angle C = 180^\circ$

$\Rightarrow 4x + 5y = 180^\circ$ .....(i)

$\angle B + \angle D = 180^\circ$

$\Rightarrow 7x + y = 180^\circ$ ......(ii)

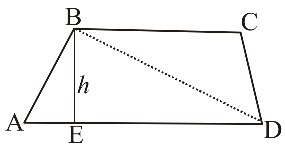
from equation [(ii) × 5 – (i)]

$31x = 720$

$$x = \frac{720}{31} \text{ and } y = \frac{540}{31}$$

$\Rightarrow x : y = 4 : 3$

59.



Height of trapezium = height of Δ BED $= h$

$$\frac{\text{area of ABCD}}{\text{area of }\Delta\text{BED}} = \frac{\frac{1}{2}\times h(\text{BC}+\text{AD})}{\frac{1}{2}\times h(\text{AD})} = \frac{\text{AD}+\text{BC}}{\text{AD}}$$

60. $\therefore$ Area of parallelogram (ABCD) = Base × Height

$= \text{BC}\times\text{AN}$

$= 4x\times\text{AN}$

$\therefore$ Area of triangle (ABN) $= \frac{1}{2}\times$ Base × Height

$= \frac{1}{2}\times\text{BN}\times\text{AN}$

$= \frac{1}{2}\times x\times\text{AN}$

$$\Rightarrow \frac{\text{area of parallelogram ABCD}}{\text{area of traingle ABN}} = \frac{4x\times\text{AN}}{\frac{1}{2}x\times\text{AN}}$$

$= 8 : 1$

61. Let each side of hexagon be a

$a = 6\ cm$

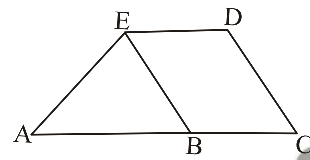
height of hexagon = $\sqrt{3}a$

$\therefore$ BD = FB = FD = $6\sqrt{3}$ cm

area of Δ BDF = $\frac{\sqrt{3}a^2}{4} = \frac{\sqrt{3}}{4}\left(6\sqrt{3}\right)^2$

$= 27\sqrt{3}\ \text{cm}^2$

62.



$\text{AB} = \text{BC} = \frac{1}{2}\text{AC}$

BCDE is a parallelogram

$\therefore\ \text{ED} = \frac{1}{2}\text{AC}$

Let height of quadrilateral be h.

area of quadrilateral = 36cm^2

$\Rightarrow \frac{1}{2}\times h\times(\text{ED}+\text{AC}) = 36$

$\Rightarrow h\times(3\text{ED}) = 72$

$\Rightarrow h\times\text{ED} = 24$

area of Δ ABE = $\frac{1}{2}\times h\times\text{AB} = \frac{1}{2}\times 24 = 12\ \text{cm}^2$

$(\because \text{AB} = \text{ED})$

63. $\therefore\ \frac{\text{OA}}{\text{OC}} = \frac{\text{OD}}{\text{OB}}$

$\Rightarrow \frac{3}{x-3} = \frac{x-5}{3x-19}$

$\Rightarrow 9x - 57 = x^2 - 8x + 15$

$\Rightarrow x^2 - 17x + 72 = 0$

$\Rightarrow (x-9)(x-8) = 0$

$x = 8, 9$

64. Let one side be l then another side will be $l + 6$

$\therefore$ area = $\frac{1}{2}$ × height × (sum of parallel sides)

$\Rightarrow 180 = \frac{1}{2}\times 9\,(l + l + 6)$

$\Rightarrow l = 17$ cm

$l + 6 = 23$ cm

two sides will be 17 cm and 23 cm.

65. Let two diagonals be d_1 and d_2

$d_1 = 10$ cm **..............(given)**

$d_2 = ?$

$\therefore \quad d_1^2 + d_2^2 = 2(l^2 + b^2)$

$\Rightarrow \quad 100 + d_2^2 = 2(144 + 64)$

$\Rightarrow \quad d_2 = 17.8$ cm

66. $P = 2(l + b)$

$\therefore \quad b, l$ and P are in G.P.

$\Rightarrow \quad l^2 = bP$

$\Rightarrow \quad l^2 = b \times 2(l + b)$

$$\Rightarrow \quad \frac{l^2}{b^2} = 2\left(\frac{l}{b} + 1\right)$$

$$\Rightarrow \quad \left(\frac{l}{b}\right)^2 - 2\left(\frac{l}{b}\right) - 2 = 0$$

$$\frac{l}{b} = \frac{2 \pm \sqrt{4+8}}{2} = \frac{2 + 2\sqrt{3}}{2} = \left(\sqrt{3}+1\right) : 1$$

(Negative factor can not consider other wise value of l/b will be negative which is not possible.)

67. $\therefore \quad \frac{OA}{OC} = \frac{OD}{OB}$

$$\Rightarrow \quad \frac{3x-1}{5x-3} = \frac{6x-5}{2x+1}$$

$\Rightarrow \quad 6x^2 + x - 1 = 30x^2 - 43x + 15$

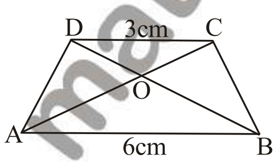
$\Rightarrow \quad 24x^2 - 44x + 16 = 0$

$\Rightarrow \quad 6x^2 - 11x + 4 = 0$

$\Rightarrow \quad (x - 3)(x - 8) = 0$

$x = 3, 8$

68.



AB || CD

In Δ DOC and Δ AOB

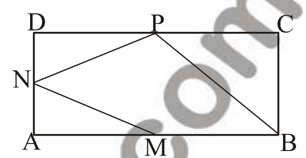
$\therefore \quad \angle OAB = \angle OCD$ (alternate interior angles)

$\therefore \quad \angle OBA = \angle ODC$

$\Delta DOC \approx \Delta BOA$

$$\therefore \quad \frac{\text{area of } \Delta AOB}{\text{area of } \Delta COD} = \left(\frac{AB}{CD}\right)^2 = \left(\frac{6}{3}\right)^2 = 4 : 1$$

69.



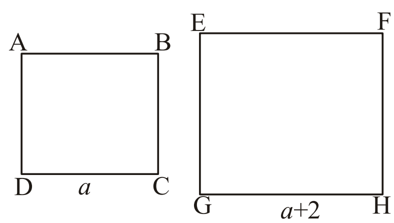
Area of quadrilateral BMNP

= (area of rectangle ABCD) – (area Δ ANM) – (area of Δ DNP) – (area Δ PCB)

$$= (16 \times 8) - \left(\frac{1}{2} \times 4 \times 8\right) - \left(\frac{1}{2} \times 4 \times 8\right) - \left(\frac{1}{2} \times 8 \times 8\right)$$

$= 64$

70.



Let length of one segment of square ABCD be a, then another segment of EFGH will be $a + 2$

$\Rightarrow \quad (a + 2)^2 - a^2 = 32$

$\Rightarrow \quad a^2 + 4 + 4a - a^2 = 32$

$a = 7$

greater segment = $a + 2 = 9$ cm

71. Let side of square be a,

then,

length of rectangle (l) = $a + 5$

Breadth of rectangle (b) = $(a - 3)$

$\therefore \quad$ area of square = area of rectangle

$\Rightarrow\ a^2 = (a+5)(a-3)$

$\Rightarrow\ a^2 = a^2 + 2a - 15$

$\Rightarrow\ a = 7.5$

$\therefore$ Perimeter of square $= 4a = 4 \times 7.5 = 30$ cm

72. AB || CF

$\angle ABC = \angle BCF = 85^\circ$ (alternate interior angle)

$\angle DCB = 180^\circ - 85^\circ - 20^\circ = 75^\circ$

$\therefore\ \angle DCB + \angle BAD = 180^\circ$

$\angle BAD = 180^\circ - 75^\circ = 105^\circ$

73. Area of quadrilateral ABCD = (area of Δ ABD) + (area of Δ BCD)

$$= \frac{1}{2}\,AL \times BD + \left(\frac{1}{2} \times CM \times BD\right)$$

$$= \left(\frac{1}{2} \times 9 \times 16\right) + \left(\frac{1}{2} \times 7 \times 16\right)$$

$$= 72 + 56 = 128\ cm^2$$

74. Parallelogram ABCD and ABMN are on the base AB where,

AB || DM. N and P are the midpoint of CD and BC. Both parallelogram are on same base and between same parallel lines.

So, area of ABCD = area of ABMN

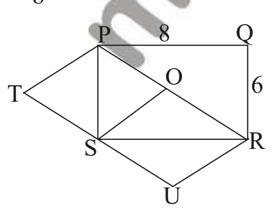
= 80

We know that

Area of $\Delta APN = \frac{3}{8} \times$ area of parallelogram ABCD

$= \frac{3}{8} \times 80 = 30$ sq. unit

75.



In ΔPQR,

$PR^2 = \sqrt{PQ^2 + QR^2}$ **($\therefore$ Pythagoras theorem)**

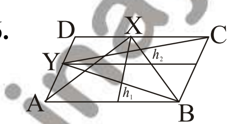
$= \sqrt{8^2 + 6^2} = 10$ cm

$PT = OS = \frac{PS \times SR}{PR}$

$= \frac{6 \times 8}{10} = 4.8$ cm

$\frac{\text{area of rectangle PQRS}}{\text{area of rectangle PTUR}} = \frac{6 \times 8}{10 \times 4.8} = 1$

76.



area of Δ AXB $= \frac{1}{2} \times AB \times h_1$

$= \frac{1}{2} \times$ (area of parallelogram ABCD)

area of BYC $= \frac{1}{2} \times BC \times h_2$

$= \frac{1}{2} \times$ (area of parallelogram ABCD)

$\Rightarrow$ area of Δ AXB = area of Δ BYC

77. $\therefore$ DE : EC = 1 : 2 and DC = DE + EC

$DE = \frac{1}{3} \times DC = \frac{2a}{3}$ cm

Similarly, $EC = \frac{2}{3} \times DC = \frac{4a}{3}$ cm.

$AE = \sqrt{AD^2 - DE^2}$

$AE = \sqrt{a^2 - \left(\frac{2a}{3}\right)^2} = \frac{\sqrt{5}}{3}a$

area of parallelogram = DC × AE

$$= 2a \times \frac{\sqrt{5}}{3} a = \frac{2\sqrt{5}}{3} a^2$$

$$EF = 3\,AE = 3 \times \frac{\sqrt{5}}{3} a = \sqrt{5}\, a$$

$$\text{area of rectangle} = EF \times EC = \sqrt{5}\, a \times \frac{4a}{3} = \frac{4\sqrt{5}}{3} a^2$$

$$\therefore \frac{\text{area of parallelogram}}{\text{area of rectangle}} = \frac{\frac{2\sqrt{5}}{3} a^2}{\frac{4\sqrt{5}}{3} a^2} = 1 : 2$$

78. $\therefore$ $AB^2 + BC^2 = AC^2$ .... (i) **(Pythagoras)**

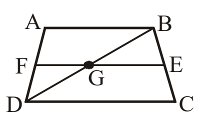
$AD^2 = AB^2 + BC^2 + CD^2$ .... (ii) **(given)**

From eq. (i) and (ii)

$\Rightarrow$ $AD^2 = AC^2 + CD^2$

$\angle ACD = 90^o$ (As third side is sum of square of two sides. Hence, Δ ACD is a right angle triangle)

79.



$\therefore EF \parallel CD$

In Δ BGE and Δ BDC,

$\angle$ BGE = $\angle$ BDC (corresponding angle)

$\angle$ B = $\angle$ B (common)

Hence, $\Delta BGE \sim \Delta BDC$ (similar triangle)

then, $\frac{GE}{DC} = \frac{BE}{BC} = \frac{3}{7}$

$\Rightarrow \frac{GE}{AB} = \frac{6}{7}$ ....(i)

($\because DC = 2AB$)

similarly,

In Δ FDG and Δ ADB,

$\angle$ D = $\angle$ D (common)

$\angle$ DAB = $\angle$ DFG (corresponding angles)

$\Delta FDG \sim \Delta ADB$

$\frac{FG}{AB} = \frac{3}{4}$ ......(i)

$\frac{DF}{AD} = \frac{4}{7}$ ......(ii)

Adding (i) and (ii) we get,

$\Rightarrow \frac{GE}{AB} + \frac{FG}{AB} = \frac{6}{7} + \frac{4}{7}$

$\Rightarrow \frac{FE}{AB} = \frac{10}{7}$

80. Each interior angle $= \frac{(n-2)\times 180^o}{n}$

($\because$ n is no. of side.)

$\Rightarrow 120^o = \frac{(n-2)\times 180^o}{n}$

$\Rightarrow 60^o n = 360^o$

$\Rightarrow n = 6$

81. Each interior angle $= \frac{(n-2)\times 180^o}{n}$

$\Rightarrow 140° = \frac{(n-2)\times 180^o}{n}$

$\Rightarrow 40^o \times n = 360^o$

$n = 9$

no. of diagonals $= \frac{n(n-3)}{2}$

$= \frac{9\times 6}{2} = 27$

82. Sum of interior angles $= (n-2) \times 180^o$

$900^o = (n-2) \times 180^o$

$\Rightarrow n = 7$

83. Each interior angle $= \frac{(n-2)}{n} \times 180^\circ$

$\therefore$ Each exterior angle $= \frac{360^\circ}{n}$

$\Rightarrow \frac{(n-2)}{n} \times 180^\circ = \frac{4\times 360^\circ}{n}$

$\Rightarrow n = 10$

84. According to question.

$\frac{(n-2)\times 180^\circ}{n} - \frac{360^\circ}{n} = 60^\circ$

$\Rightarrow 180^\circ n = 720^\circ + 60^\circ n$

$\Rightarrow 120^\circ n = 720^\circ$

$\Rightarrow n = 6$

85. Sum of all interior angles

$= 2 \times$ (sum of its exterior angle) .... **(given)**

$\therefore (n-2)\ 180^\circ = 2 \times 360^\circ$ (where n is no. of sides)

$\Rightarrow n = 6$

Hence, Polygon is hexagon.

86. Let the no. of sides of two polygon be n and $2n$.

$\therefore \frac{\frac{(n-2)\times 180^\circ}{n}}{\frac{(2n-2)\times 180^\circ}{2n}} = \frac{2}{3}$

$\Rightarrow \frac{2(n-2)}{2(n-1)} = \frac{2}{3}$

$\Rightarrow 3n - 6 = 2n - 2$

$\Rightarrow n = 4$

No. of sides of first polygon $= n = 4$

No. of sides of second polygon $= 2n = 8$

87. No of sides of hexagon $(n) = 6$

$\therefore$ each interior angle $= \frac{(n-2)180^\circ}{n}$

$= \frac{(6-2)\times 180^\circ}{6} = 120^\circ$

$\therefore$ Each exterior angle $= \frac{360^\circ}{n} = \frac{360^\circ}{6} = 60^\circ$

$\Rightarrow \frac{\text{Interior angle}}{\text{Exterior angle}} = \frac{120^\circ}{60^\circ} = 2 : 1$

88. One interior angle of hexagon $= 120^\circ$

One interior angle of polygon $= \frac{3}{4} \times 120^\circ = 90^\circ$

Each interior angle $= \frac{(n-2)\times 180^\circ}{n}$

[$\because n$ = no. of side]

$\Rightarrow 90^\circ = \frac{(n-2)\times 180^\circ}{n}$

$\Rightarrow n = 2n - 4$

$\Rightarrow n = 4$

89. Interior angle $= 172^\circ$

Exterior angle $= 180^\circ - 172^\circ = 8^\circ$

remaining exterior angle $= 180^\circ - 160^\circ = 20^\circ$

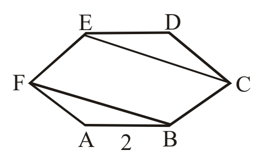
$\therefore 5 \times 8^\circ + n \times 20^\circ = 360^\circ$

(n = no. of remaining sides)

$n = 16$

Hence, total no of sides $= 16 + 5 = 21$

90.



Let each side of hexagon be $a = 2$ feet

Height = FB = $\sqrt{3}a$

Area of rectangle BCEF = FB×EF

$= \sqrt{3}a^2 = 4\sqrt{3}$

91. Let the no of sides of the polygon be n and $2n$

$\therefore$ ratio of each interior angles $= \frac{\frac{(n-2)\times 180^\circ}{n}}{\frac{(2n-2)\times 180^\circ}{2n}} = \frac{1}{2}$

$\Rightarrow \dfrac{2(n-2)}{2(n-1)} = \dfrac{1}{2}$

$\Rightarrow 2n - 4 = n - 1$

$\Rightarrow n = 3$

no of sides of first polygon = $n = 3$

no of sides of second polygon = $2x = 6$

92. Five exterior angles = $180^o - 162^o = 18^o$

sum of all exterior angle = 360^o

$\therefore 5 \times 18^o + n \times 30^o = 360^o$

$\Rightarrow n = 9$

Hence, total no of sides = 9 + 5 = 14

93. Let the no. of sides of two regular polygon are n and $2n$. each interior angle of first polygon is 120^o then each exterior angle of this polygon is $(180^o - 120^o) = 60^o$

then, no. of side of first polygon

$$n = \frac{360^o}{60^o} = 6$$

Hence, side of second polygon is $2n = 12$.

each interior angle of second polygon

$$= \frac{(12-2)\times 180^o}{12} = 150^o$$

94. Summation of A.P = $\dfrac{n}{2}\left[2a + (n-1)d\right]$

(Where a = smallest number, d = common difference)

Summation of interior angles of polygon = $(n\text{-}2)\ 180^\circ$

$\Rightarrow \dfrac{n}{2}\left[2\times 120^o + (n-1)\times 5^o\right] = (n-2)\times 180^\circ$

$\Rightarrow \dfrac{n}{2}\times 5\left[48^o + n - 1\right] = (n-2)\times 180^\circ$

$\Rightarrow n^2 - 25n + 144 = 0$

$\Rightarrow (n-16)(n-9) = 0$

$\Rightarrow n = 9$ or 16

95. $\therefore$ Each interior angle of regular polygon = $\dfrac{360^o}{n}$

(Where n no of side) According to question,

$\therefore \dfrac{360^o}{n-1} - \dfrac{360^o}{n+2} = 6$

$\Rightarrow \dfrac{n+2-n+1}{(n-1)(n+2)} = \dfrac{6}{360} = \dfrac{1}{60}$

$\Rightarrow n^2 + n - 182 = 0$

$\Rightarrow n^2 + 14n - 13n - 182 = 0$

$\therefore n+14 = 0 \Rightarrow n = -14$ (not possible)

$\therefore (n-13) = 0 \Rightarrow n = 13$

Hence, no. of sides = 13

96. Let number of sides of two polygon be n and $2n$

$$\frac{\dfrac{(n-2)\times 180^o}{n}}{\dfrac{(2n-2)\times 180^o}{2n}} = \frac{3}{4}$$

$\Rightarrow \dfrac{2(n-2)}{2(n-1)} = \dfrac{3}{4}$

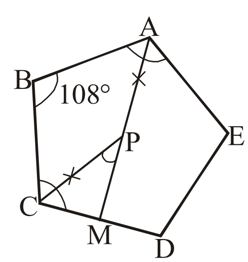
$\Rightarrow 4n - 8 = 3n - 3$

$\Rightarrow n = 5$

Hence, No. of sides of first polygon = $n = 5$

No. of sides of second polygon = $2n = 10$

97.



We know that the measure of each interior angle of a regular pentagon is 108°

$\therefore \angle BAM = \frac{1}{2}(108°) = 54°$

Since the sum of the angles of a quadrilateral is 360°. Therefore, in quadrilateral ABCM, we have

$\angle BAM + \angle ABC + \angle BCM + \angle CMA = 360°$

$\Rightarrow 54° + 108° + 108° + \angle CMA = 360°$

$\Rightarrow \angle CMA = 90°$

Since, CP is the bisector of $\angle BCD$

$\therefore \angle PCM = 54°$

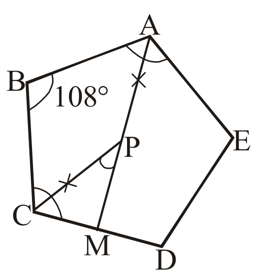
Now, in ΔCPM, we have

$\angle PCM + \angle CMP + \angle CPM = 180°$

$\Rightarrow 54° + 90° + \angle CPM = 180°$

$\Rightarrow \angle CPM = 36°$

98.



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Since the sum of the angles of a quadrilateral is 360°. Therefor, in quadrilateral ABCM, we have

$\angle BAM + \angle ABC + \angle BCM + \angle AMC = 360°$

$\Rightarrow 54° + 108° + 108° + \angle AMC = 360°$

$\Rightarrow \angle AMC = 90°$

99. $\therefore$ Interior angle of regular hexagon = 120°

Interior angle of regular polygon

$= \frac{9}{8} \times 120° = 135°$

$\therefore$ Each interion angle $= \frac{(n-2)180°}{n}$

(Where n is no of sides)

$\Rightarrow \frac{(n-2)\times 180°}{n} = 135°$

$\Rightarrow 180° n - 360° = 135° \times n$

$\Rightarrow$ n = 8

100. Each exterior angle of hexagon $= \frac{360°}{6} = 60°$

$\therefore$ Each exterior angle of hexagon = 180° – 60°=120°

$\therefore$ Each exterior angle of regular polygon

$= 120° \times \frac{7}{6} = 140°$

$\therefore$ Each exterior angle = 180° – 140° = 40°

$\therefore$ Total number of sides $= \frac{360°}{40°} = 9$

101. Number of diagonal $= \frac{n(n-3)}{2} = 27$

$\Rightarrow n^2 - 3n - 54 = 0$

$\Rightarrow (n+6)(n-9) = 0$

$\Rightarrow n = -6, 9$

Number of sides in the polygon = 9

102. Each side of regular polygon subtends equal angle on centre.

Let the number of sides be n

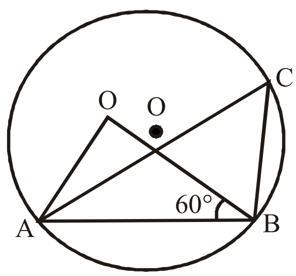
angle subtends by each side is 72°

$\Rightarrow n \times 72° = 360°$

$\Rightarrow n = \frac{360°}{72°} = 5$

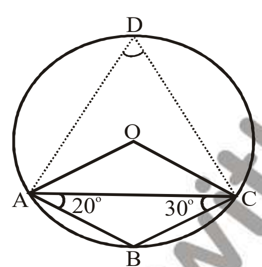
Exercise - Circle

1. In the figure, $\angle ABO = 60^o$. Find $\angle ACB$



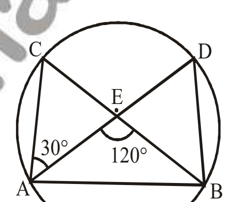
(a) 40° (b) 60°
(c) 50° (d) 30°

2. In the figure, O is centre of a circle. If $\angle BAC = 20^o$ and $\angle ACB = 30°$. Find $\angle AOC$



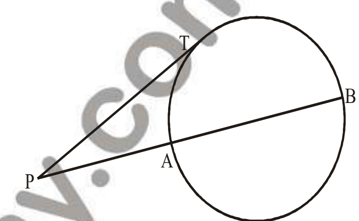
(a) 70° (b) 110°
(c) 100° (d) 80°

3. In the figure, $\angle CAE = 30°$ and $\angle AEB = 120°$. Find $\angle ADB$



(a) 60° (b) 80°
(c) 90° (d) 120°

4. In the figure, PT= 5 cm, PA = 4 cm. Find AB



(a) $\frac{7}{4}$ cm (b) $\frac{11}{4}$ cm

(c) $\frac{9}{4}$ cm (d) $\frac{13}{4}$ cm

5. ABC is a right angled triangle AB=3cm, BC = 5cm and AC = 4cm, then the inradius of the incircle is:

(a) 1 cm (b) 1.25 cm
(c) 1.5 cm (d) 2 cm

6. Three circles touch each other externally. The distance between their centre is 5 cm, 6 cm and 7 cm. Find the radius of the circles.

(a) 2 cm, 3 cm, 4 cm
(b) 3 cm, 4 cm, 1 cm
(c) 1 cm, 2.5 cm, 3.5 cm
(d) 1 cm, 2 cm, 4 cm

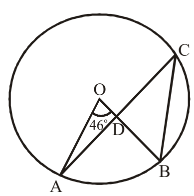
7. Two circles of radius 13 cm and 5 cm touch internally each other. Find the distance between their centres:

(a) 18 cm (b) 12 cm
(c) 9 cm (d) 8 cm

8. A point P is 25 cm from the centre of the circle. The length of the tangent drawn from P to the circle is 24 cm. Find the radius of the circle.

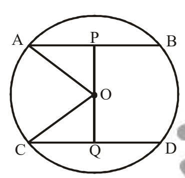
(a) 5 cm (b) 7 cm
(c) 6 cm (d) 8 cm

9. In the figure, $\angle AOB = 46°$; AC and OB intersect at right angle $\angle OBC$ equals to (o is the centre of circle)?



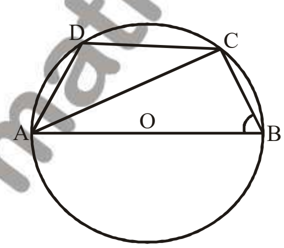
(a) 44° (b) 46°
(c) 67° (d) 78.5°

10. In the figure, find PQ, if AB = 8 cm, CD = 6 cm, If Radius of circle = 5 cm



(a) 6 cm (b) 8 cm
(c) 9 cm (d) 7 cm

11. In the figure, $\angle ADC = 140°$ and AOB is the diameter of circle the $\angle BAC$ will be:



(a) 4° (b) 50°
(c) 70° (d) 75°

12. AB is a diameter of the circle, CD is a chord equal to the radius of the circle. AC and BD when extended intersect at a point E. Find the $\angle AEB$.

(a) 30° (b) 60°
(c) 90° (d) 45°

13. C is a point on the minor arc AB a circle with centre O. If $\angle AOB = 100°$, what is $\angle ACB$?

(a) 80° (b) 90°
(c) 100° (d) 130°

14. Two concentric circles with centre O have A, B, C, D as the points of intersection with the line PQ. The line PQ intersect at A, D outer circles and B, C inner circles. If AD = 12 cm and BC= 8 cm, find the lengths of AB.

(a) 2 cm (b) 4 cm
(c) 6 cm (d) 3 cm

15. AB and CD are two parallel chords of a circle such that AB = 10 cm and CD = 24 cm. If the chords are on the opposite sides of centre and the distance between them is 17 cm, find the radius of the circle.

(a) 12 cm (b) 13 cm
(c) 14 cm (d) 16 cm

16. Two circles of radii 8 cm and 2 cm respectively touch each other externally at the point A. PQ is the direct common tangent of these two circles of centres O_1 and O_2 respectively. The length of PQ is equal to:

(a) 4 cm (b) 8 cm
(c) 2 cm (d) 3 cm

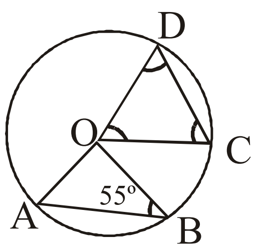
17. The radius of a circle is 13 cm and XY is a chord which is at a distance of 12 cm from the centre. The length of the chord is:

(a) 12 cm (b) 10 cm
(c) 20 cm (d) 15 cm

18. Two tangents are drawn from a point P to a circle at A and B. O is the centre of the circle. If ∠AOP = 60° then ∠APB is:

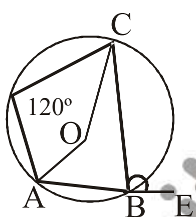
(a) 60° (b) 30°
(c) 120° (d) 90°

19. In the given figure, chords AB and CD are equal. If ∠OBA = 55º then ∠COD is :



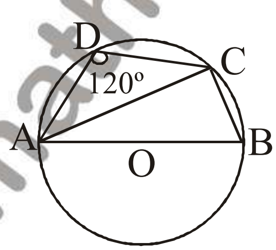
(a) 65º (b) 55º
(c) 70º (d) 50º

20. In the given figure, ∠AOC = 120º. Find ∠CBE, where O is the centre :



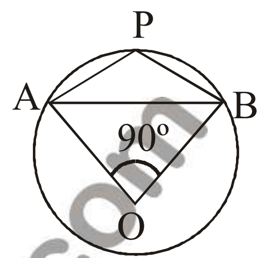
(a) 60º (b) 100º
(c) 120º (d) 150º

21. In the given figure, AB is the diameter of the circle. ∠ADC = 120º. Find ∠CAB :



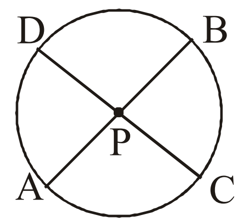
(a) 20º (b) 30º
(c) 40º (d) can't be determined

22. In the given figure, O is the centre of the circle, ∠AOB = 90º. Find ∠APB :



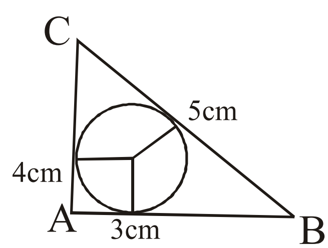
(a) 130º (b) 150º
(c) 135º
(d) can't bedetermined

23. In the given figure, AP = 2 cm, BP = 6 cm and CP = 3 cm. Find DP :



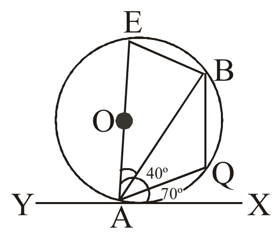
(a) 6 cm (b) 4 cm
(c) 2 cm (d) 3 cm

24. ABC is a right angled triangle AB = 3 cm, BC = 5 cm and AC = 4 cm, then the inradius of the circle is :



(a) 1 cm (b) 1.25
(c) 1.5 cm (d) None of these

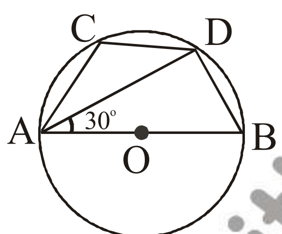
25. In the figure XY is a tangent to the circle with centre O at A. If $\angle BAX = 70^\circ$, $\angle BAQ = 40^\circ$ then $\angle ABQ$ is equal to :



(a) 20º (b) 30º

(c) 35º (d) 40º

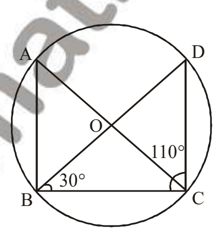
26. In the given figure, AOB is a diameter of a circle and $CD \parallel AB$. If $\angle BAD = 30^\circ$, then $\angle CAD$ =?



(a) 30º (b) 60º

(c) 45º (d) 50º

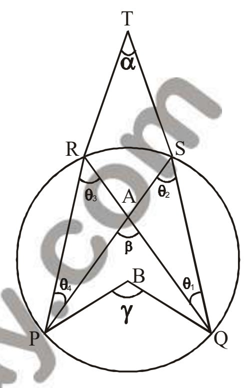
27. In this figure $\angle DBC = 30^\circ$ and $\angle BCD = 110^\circ$. Find $\angle BAC$



(a) 35° (b) 40°

(c) 55° (d) 60°

28. In this figure B is centre, if $\alpha = 50^\circ$, $\beta = 80^\circ$. Find the value of γ.



(a) 50° (b) 80°

(c) 130° (d) 65°

29. The distance between the centres of equal circles each of radius 3 cm is 10 cm. The length of a transversal tangent is :

(a) 4 cm (b) 6 cm

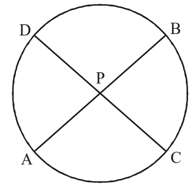
(c) 8 cm (d) 10 cm

30. O and O^1 are the centres of two circles which touch each other externally at P. AB is a common tangent at P. Find $\angle APO$

(a) 90° (b) 120°

(c) 60° (d) None of these

31. In the Figure AP = 2 cm, BP = 6 cm and CP = 3 cm. Find DP

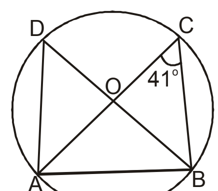


(a) 6 cm (b) 4 cm
(c) 2 cm (d) 3 cm

32. ΔABC is an isosceles triangle and AB = AC. If all the sides AB, BC and CA of a ΔABC touch a circle at D, E and F respectively. Then BE is equal to:

(a) $\frac{1}{2}$BC (b) $\frac{1}{3}$BC
(c) $\frac{1}{4}$BC (d) $\frac{2}{3}$BC

33. In the given figure, BD is the diameter of the circle and $\angle BCA = 41°$, then find the value of $\angle ABD$.



(a) 41° (b) 49°
(c) 22.5° (d) 20.5°

34. The in circle of Δ ABC touches the sides BC, CA and AB at D, E and F respectively. Then AF + BD + CE is equal to :

(a) $\frac{1}{2}$ (Perimeter of ΔABC)

(b) $\frac{1}{3}$ (Perimeter of ΔABC)

(c) $\frac{1}{4}$(Perimeter of ΔABC)

(d) Perimeter of ΔABC

35. A circle touches all the four sides of a quadrilateral ABCD. Then :

(a) BC + DA = AB + CD
(b) ABCD is a rectangle
(c) ABCD is square
(d) None of these

36. A circle is inscribed in Δ ABC having sides 8 cm, 10 cm and 12 cm. A circle touches the sides AB, BC and CA at D, E and F. Find AD

(a) 5 cm (b) 6 cm
(c) 4 cm (d) 10 cm

37. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Then $\angle PTQ$ is equal to:

(a) $\angle OPQ$ (b) $2\angle OPQ$
(c) $\frac{1}{2}\angle OPQ$ (d) $\frac{1}{3}\angle OPQ$

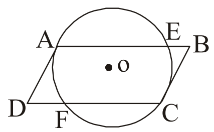
38. PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length TP.

(a) $\frac{10}{3}$ (b) $\frac{20}{3}$
(c) $\frac{23}{3}$ (d) $\frac{13}{3}$

39. In a circle, PQ and RS are two parallel tangents at A and B. The tangent at C makes an intercept DE between PQ and RS. Then $\angle DOE$ is equal to : (where O is centre).

(a) 90° (b) 120°
(c) 60° (d) 45°

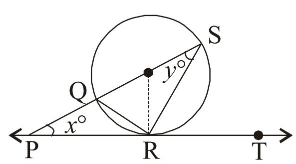
40. In the given figure 'O' is the centre of the circle. If AE = 4 cm, EB = 2 cm, also AE = FC and BC = AD, then DF = ?



(a) 1 cm (b) 2 cm

(c) 4 cm (d) 6 cm

41. In the given figure, PT is the tangent of a circle with centre O at point R. If diameter SQ is increased, it meets with PT at point P. iF $\angle SPR = x°$ and $\angle QSR = y°$. What is the value of $x° + 2y°$?



(a) 90° (b) 105°

(c) 135° (d) 180°

42. O is the centre of the circle. PA and PB are tangent segments. Then the quadrilateral AOBP is:

(a) Rectangle (b) Square

(c) Cyclic (d) None of these

43. Circles C(o, r) and C $\left(O^1, \frac{r}{2}\right)$ touch internally at a point A and AB is a chord of the circle intersecting C$\left(O^1, \frac{r}{2}\right)$ at P. Then AP is equal to:

(a) PB (b) OA

(c) OB (d) $\frac{1}{3}$ AB

44. In two Concentric circles, AB and CD are two chords of the outer circle which touch the inner circle at E and F. Then:

(a) AB = CD (b) AB = $\frac{1}{2}$ CD

(c) AB $\neq$ CD (d) None of these

45. Two circles with centers A and B of radii 3 cm and 4 cm respectively intersect at two points C and D such that AC and BC are two tangents to the two circles. Find the length of the common chord CD.

(a) 5 cm (b) 4.8 cm

(c) 5.8 cm (d) 3.8 cm

46. The radii of two circles are 5 cm and 12 cm. The area of a third circle is equal to the sum of the areas of the two circles. The radius of the third circle is:

(a) 13 cm (b) 21 cm

(c) 30 cm (d) 17 cm

47. O is the centre of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects the circle at E. If AB is the tangent to the circle at E. Find the length of AB.

(a) $\frac{10}{3}$ (b) $\frac{20}{3}$

(c) $\frac{16}{3}$

(d) Cannot be determined

48. PQ is tangent at a point R of the circle with centre O. If ST is a diameter of the circle and $\angle$TRQ = 30° find $\angle$PRS

(a) 30° (b) 60°

(c) 45° (d) 120°

49. Two circles touch externally at a point P from a point T on the tangent at P, tangents TQ and TR are drawn to the circles with points of contact Q are R respectively. Then TQ is equal to:

(a) TR (b) $\frac{1}{2}$TP

(c) $\frac{1}{2}$TR (d) None of these

50. There are two concentric circles with centre O of radii 5 cm and 3 cm. From an external point P, tangents PA and PB are drawn to these circle respectively. If AP = 12 cm find the length of BP.

(a) 12 cm (b) $12\sqrt{3}$

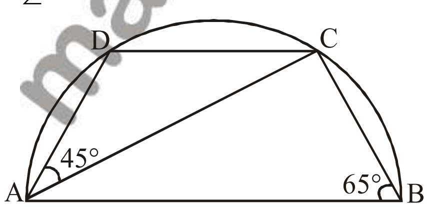
(c) $4\sqrt{10}$ (d) $10\sqrt{4}$

51. AB is a chord of length 16 cm of a circle of radius 10 cm. The tangents at A and B intersect at a point P. Find the length of PA.

(a) $\frac{20}{3}$ (b) $\frac{40}{3}$

(c) $\frac{50}{3}$ (d) $\frac{10}{3}$

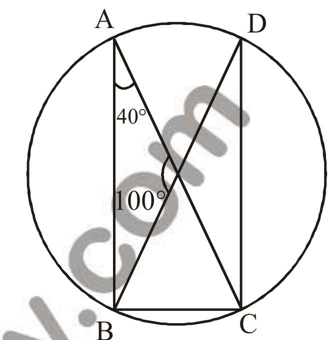
52. In the given figure, AB is diameter of the circle, C and D lie on the semicircle. $\angle$ABC = 65° and $\angle$CAD = 45°, find $\angle$DCA.



(a) 45° (b) 25°

(c) 20° (d) None of these

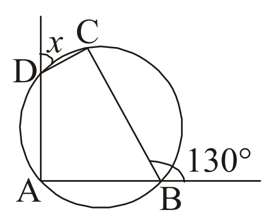
53. In the figure given, what is the measure of $\angle$ACD? (where A, B, C or D are the points on the circle.)



(a) 40° (b) 50°

(c) 80° (d) cannot be determined

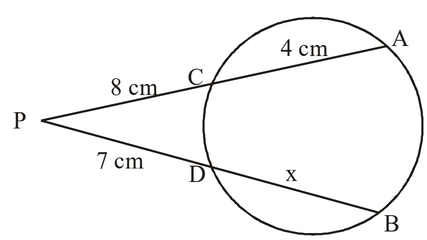
54. In the adjoining figure A, B, C, D are the concyclic points. The value of '*x*' is:



(a) 50° (b) 60°

(c) 70° (d) 90°

55. Find the value of x in the given figure-



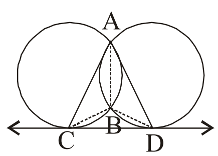
(a) 6 cm (b) 7 cm

(c) 6.7 cm (d) 7.7 cm

56. X and Y are centres of circles of a radii 9 cm and 2 cm respectively, XY = 17 cm. Z is the centre of a circle of radius r cm which touches of above circles externally. Given that $\angle XZY = 90°$, the value of r is

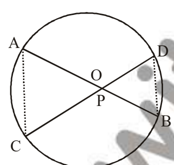
(a) 13 cm (b) 6 cm
(c) 9 cm (d) 8 cm

57. CD is direct common tangent to two circles intersecting each other at A and B. The $\angle CAD + \angle CBD$ is equal to



(a) 180° (b) 90°
(c) 360° (d) 120°

58. In the given figure O is the centre of the circle. If AB = 16 cm, CP = 6 cm, PD = 8 cm and AP > PB then what will be the value of AP.



(a) 12 cm (b) 24 cm
(c) 8 cm (d) 6 cm

59. In a triangle ABC, AB + BC = 12 cm, BC + CA = 14 cm and CA + AB = 18 cm. Find the radius of the circle (in cm) which has the same perimeter as the triangle.

(a) $\frac{9}{2}$ (b) $\frac{11}{2}$
(c) $\frac{5}{2}$ (d) $\frac{7}{2}$

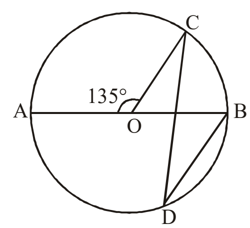
60. A circle (with centre at O) is touching two intersecting lines AX and BY. The two points of contact A and B subtend an angle of 65° at any point C on the circumference of the circle. If P is the point of intersection for the two lines, then the measure of $\angle APO$ is :

(a) 25° (b) 65°
(c) 90° (d) 40°

61. P and Q are centres of two circles with radii 9 cm and 2 cm respectively, where PQ = 17 cm, R is the centre of another circle of radius ×cm which touches each of the above two circles externally. If $\angle PRQ = 90°$, then the value of x is–

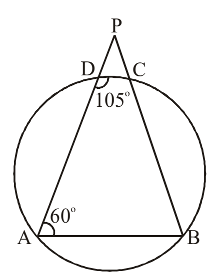
(a) 4 cm (b) 6 cm
(c) 7 cm (d) 8 cm

62. In the figure, AB is the diameter of the circle. If $\angle AOC = 135°$ them find the $\angle CDB$.



(a) $67\frac{1}{2}°$ (b) $22\frac{1}{2}°$
(c) 45° (d) 90°

63. In the figure below, if $\angle BAD = 60°$, $\angle ADC = 105°$, then what is $\angle DPC$ equal to:

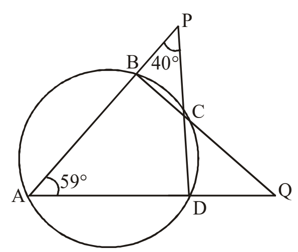


(a) 40° (b) 45°
(c) 50° (d) 60°

64. C is a point on the minor arc AB a circle with centre O. If ∠AOB = 100°, what is ∠ACB?

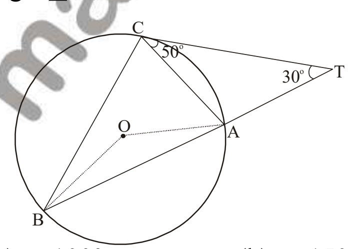
(a) 80° (b) 90°
(c) 100° (d) 130°

65. In the given figure if ∠PAQ = 59°, ∠APD = 40°, then what is ∠AQB ?



(a) 19° (b) 20°
(c) 22° (d) 27°

66. In the figure given below (not drawn to scale), A , B and C are three points on a circle with centre O. The chord BA is extended to a point T such that CT becomes a tangent to the circle at point C. If ∠ATC = 30° and ∠ACT = 50°, then the angle ∠BOA is:



(a) 100° (b) 150°
(c) 80°
(d) not possible to determine

67. C is a point on the minor arc AB of the circle with centre O. Given ∠ACB = x° and ∠AOB = y°. Calculate x, if ACBO is a parallelogram.

(a) 120° (b) 180°
(c) 140° (d) 110°

68. PQ and RS are two parallel chords of a circle whose centre is O and radius is 10 cm. If PQ = 16 cm and RS = 12 cm, find the distance between PQ and RS, if they lie:

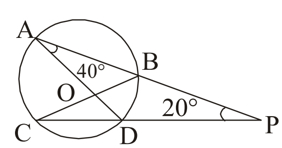
(i) On the same side of the centre O.
(ii) On opposite side of the centre O.

(a) 2] 14 cm. (b) 4] 14 cm.
(c) 2] 12 cm. (d) 4] 12 cm.

69. In a circle with centre O, chords AB and CD intersect inside the circumference at E. Find ∠AEC. If ∠AOC = 40° and ∠BOD = 50°.

(a) 90° (b) 45°
(c) 30° (d) 60°

70. PBA and PDC are two secants. AD is the diameter of the circle with centre at O. ∠A = 40°, ∠P = 20°. Find the measure of ∠DBC.

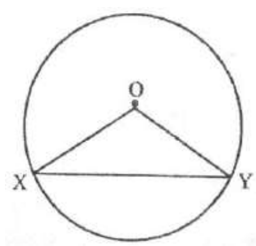


(a) 45° (b) 30°
(c) 60° (d) 75°

71. If a circle with radius of 10 cm and two parallel chords 16 cm and 12 cm and they are on the same side of the centre of the circle, then the distance between the two parallel chords is.

(a) 5 cm. (b) 8 cm.
(c) 2 cm. (d) 3 cm.

72. If the following figure, O is the centre of the circle and XO is perpendicular to OY. If the area of the triangle XOY is 32, then the area of the circle is :



(a) 64π (b) 256π
(c) 16π (d) 32π

73. Two circles of radii 4 cm and 9 cm respectively touch each other externally at a point and a common tangent touches them at the points P and Q respectively. Then the area of a square with one side PC, is:

(a) 97 sq. cm. (b) 194 sq. cm.
(c) 72 sq. cm. (d) 144 sq. cm.

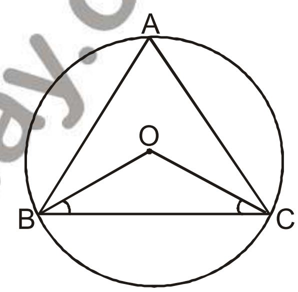
74. Two tangents are drawn from a point P to a circle at A and B. O is the centre of the circle. If $\angle AOP = 60°$ then $\angle APB$ is

(a) 60° (b) 30°
(c) 120° (d) 90°

75. PQ is a direct common tangent of two circles of radii r_1 and r_2 touching each other externally at A. Then the value of PQ^2 is:

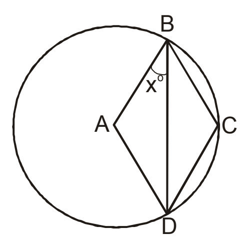
(a) r_1r_2 (b) $2r_1r_2$
(c) $3r_1r_2$ (d) $4r_1r_2$

76. BC is a chord of a circle with centre O. A is a point on major arc BC as shown in the below figure. What is the value of $\angle BAC + \angle OBC$?



(a) 120° (b) 60°
(c) 90° (d) 180°

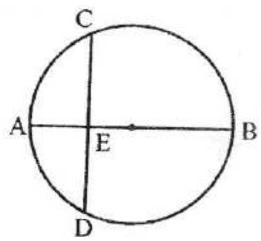
77.



In the figure given above, A is the centre of the circle and AB = BC = CD. What is the value of x?

(a) 20° (b) $22\frac{1}{2}°$
(c) 25° (d) 30°

78. In the given figure, AB is a diameter of a circle and CD is perpendicular to AB, if AB = 10 cm and AE = 2 cm, then what is the length of ED ?



(a) 5 cm. (b) $\sqrt{20}$ cm.
(c) $\sqrt{10}$ cm. (d) 4 cm.

79. Two parallel chords AB and CD are drawn on the same side of centre 'O' of a circle. If the radius of the circle is 65m and the lengths of the chords AB and CD are 126 m and 112 m respectively, then find the area of the quadrilateral ABCD.
(a) 2033 m^2 (b) 2023 m^2
(c) 1023 m^2 (d) 1078 m^2

80. Two circles of radius 4 units and 3 units are at some distance such that the length of the transverse common tangent and the length of their direct common tangent are in the ratio 1:2. What is the distance between the centres of those circles.
(a) $\sqrt{50}$ (b) $\sqrt{65}$
(c) 8 (d) Cannot be determined

81. O is the centre of a circle. AC and BD are two chords of the circle intersecting each other at P. If $\angle AOB = 15^o$ and $\angle APB = 30^o$, then $\tan^2 \angle APB + \cot^2 \angle COD$ is equal to:
(a) $\frac{4}{3}$ (b) $\frac{10}{3}$
(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

82. The tangents at two points A and B on the circle with centre O intersect at P. If in quadrilateral PAOB, $\angle AOB : \angle APB = 5 : 1$, then measure of $\angle APB$ is:
(a) 30^o (b) 60^o
(c) 45^o (d) 15^o

83. The length of a chord of a circle is equal to the radius of the circle. The angle which this chord subtends in the major segment of the circle is equal to:
(a) 30^o (b) 45^o
(c) 60^o (d) 90^o

84. In a circle if AD and BC are two chords such that they meet each other at P when they are produced to p in the external part of the circle, if PA = 10cm, PB = 8cm, PC = 5cm, AC = 6cm then the length of PD is:
(a) 5 cm (b) 4 cm
(c) 6 cm (d) 4.5 cm

85. Find the length of the common chord of the two circles of radii 6 cm and 8 cm with their centres 10 cm apart:
(a) 10 cm (b) 4.5 cm
(c) 9.6 cm (d) 12 cm

86. In ΔABC, O is the circum centre of the circle.If $\angle OBC = 37°$ then $\angle BAC$ is equal to:
(a) 74^o (b) 106^o
(c) 53 (d) 37^o

87. A circle and a square have the same perimeter. Which one of the following is correct?
(a) The area of the circle is equal to that of square.
(b) The area of the circle is larger than that of square.
(c) The area of the circle is less than that of square.
(d) No conclusion can be drawn.

88. Consider the following statements in respect of any triangle

I. The three medians of a triangle divide it into six triangles of equal area.

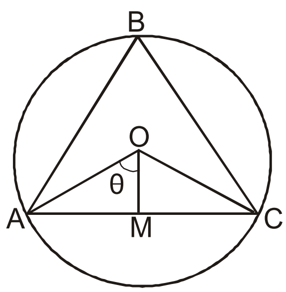
II. The perimeter of a triangle is greater than the sum of the lengths of its three medians.

Which of the statements given above is/ are correct?

(a) I only (b) II only

(c) Both I and II (d) Neither I nor II

89.



In the figure given above, O is the centre of the circle, OA = 3cm, AC = 3 cm and OM is perpendicular to AC. What is $\angle$ ABC equal to?

(a) 60° (b) 45°

(c) 30° (d) 90°

90. The diameter of a circle is 80 cm. The radii (in cme) of their concentric circles drawn in such a manner that the whole area is divided into four equal parts.

(a) $20\sqrt{2}$] $20\sqrt{3}$] 20

(b) $\frac{10\sqrt{3}}{3}$] $\frac{10\sqrt{2}}{3}$] $\frac{10}{3}$

(c) $10\sqrt{3}$] $10\sqrt{2}$] 10

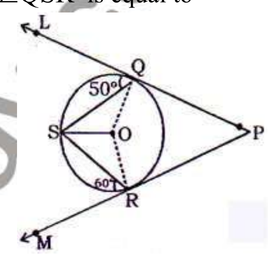
(d) 17] 14] 9

91- Two chords AB and CD of a circle with centre O, intersect each other at P. If $\angle AOC = 100°$ and $\angle BOD = 70°$ then the value of $\angle APC$ is-

(a) 80° (b) 75°

(c) 85° (d) 95°

92. In the given figure, PQL and PRM are tangents to the circle with centre 'O' at the points Q and R, respectively and S is a point on the cirlce such that $\angle SQL = 50°$ and $\angle SRM = 60°$. Then $\angle QSR$ is equal to



(a) 110º (b) 60º

(c) 70º (d) 90º

93. Two chords AB and CD of circle which centre O meet at point P and $\angle$ AOC = 50º $\angle$ BOD = 40º. Then the value of $\angle$ BPD is:

(a) 60º (b) 40º

(c) 45º (d) 75º

94. Two chords of a circle, of lengths 2a and 2b are mutually perpendicular. If the distance of the point, at which the chords interect, from the centre of the circle is c (c < radius of the circle), then the radius of the circle is:

(a) $\sqrt{\frac{a^2+b^2+c^2}{2}}$ (b) $\frac{\sqrt{ab}}{c}$

(c) $a+b-c$ (d) $\frac{\sqrt{a^2+b^2-c^2}}{2}$

95. Two circles with radii 5 cm and 8 cm touch each other externally at a point A. If a straight line through the point A cuts the circles at points P and Q respectively, then AP : AQ is:

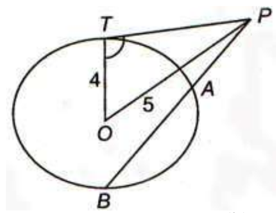
(a) 8 : 5 (b) 5 : 8

(c) 3 : 4 (d) 4 : 5

96. ABCD is a cyclic quadrilateral. Sides AB and DC, when produced meet at the point P and sides AD and BC, when produced meet at the point Q. If $\angle ADC = 85°$ and $\angle BPC = 40°$, then $\angle CQD$ is equal to:

(a) 55º (b) 85º
(c) 30º (d) 40º

97. O is the centre of the circle. $OP = 5$ and $OT = 4$, and $AB = 8$. The line PT is a tangent to the circle. Find PB

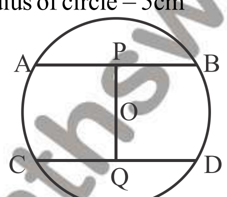


(a) 9 cm (b) 10 cm
(c) 7 cm (d) 8 cm

98. C_1 and C_2 are two concentric circles with centres at O. Their radii are 12 cm and 3 cm respectively. B and C are the points of contact of two tangents draw to C_2 from a point A lying on the circle C_1. Then the area of the quadrilateral ABOC is–

(a) $\frac{9\sqrt{15}}{2}$ cm² (b) $12\sqrt{15}$ cm²
(c) $9\sqrt{15}$ cm² (d) $6\sqrt{15}$ cm²

99. In the given figure find PQ? if AB = 8 cm, CD = 6 cm, if Radius of circle = 5cm

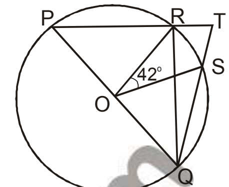


(a) 6 cm (b) 8 cm
(c) 9 cm (d) 7 cm

100. Two concentric circles having common centre 'O' and chord AB of the outer circle intersect the inner circle at points C and D. If the distance of chord from the centre is 3 cm, outer radius is 13cm and inner radius is 7 cm, then length of AC is cm is -

(a) $8\sqrt{10}$ (b) $6\sqrt{10}$
(c) $4\sqrt{10}$ (d) $2\sqrt{10}$

101.



In the figure given above, PQ is a diameter of the circle whose centre is at O. If $\angle ROS = 42°$, then what is the value of $\angle RTS$?

(a) 46º (b) 64º
(c) 69º (d) None of the above

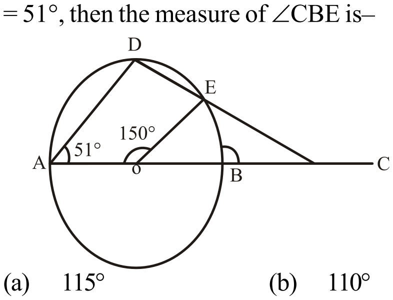
102. Three circles touch each other externally. The distance between their centre is 3 cm, 4 cm and 5 cm. Find the radii of the circles:

(a) 1 cm, 2 cm, 3 cm
(b) $\frac{1}{2}$ cm, $\frac{3}{2}$ cm, $\frac{5}{2}$ cm
(c) 1 cm, 2.5 cm, 3.5 cm,
(d) None of these

103. PT is a tangent to a circle of radius 6 cm. If P is at a distance of 10 cm from the centre O. PBC is secant line and PB = 5 cm, then what is the length of the chord BC ?

(a) 7.8 cm (b) 8.0 cm
(c) 8.4 cm (d) 9.0 cm

104. In the following figures, AB be diameter of a circle whose centre is O. If $\angle AOE = 150°$, $\angle DAO = 51°$, then the measure of $\angle CBE$ is–



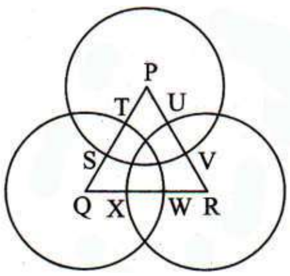
(a) 115° (b) 110°
(c) 105° (d) 120°

105. The sum of the circumferences of two circles which touch each other externally is 176 cm. What is the ratio of the radius of the larger circle to that of the smaller circle, if the sum of squares of the radii of the circles is 400 cm?

(a) $7:8$ (b) $8:7$

(c) $3:4$ (d) $4:3$

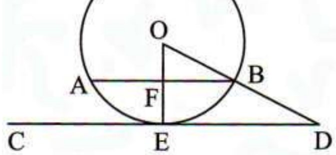
106. There are three circles with centres P, Q and R, each having a radius 24 cm. The three circles intersect each other as shown in the figure below. If ST = 4 cm, UV = 7 cm and WX = 10 cm, find the perimeter of the triangle formed by joining the centres of the three circles.



(a) 123 cm (b) 144 cm

(c) 125 cm (d) 136 cm

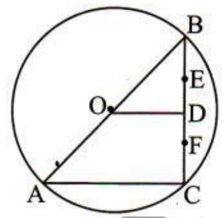
107. In the figure above, O is the center of the circle CD is a tangent to the circle at E. OE bisects the chord AB at F. AB = 16 cm and EF = 2 cm. Find the length of DE (in cm).



(a) $\frac{108}{15}$ (b) $\frac{118}{15}$

(c) $\frac{126}{15}$ (d) $\frac{136}{15}$

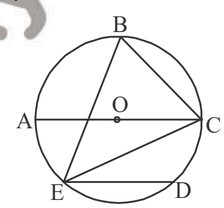
108. In the above figure, AB is the diameter of the circle with centre O. AB = 24 cm. OD is perpendicular to BC. OE bisects $\angle BOD$ and $BE:ED = 2:1$. F is the mid-point of DC. Find the length of AF (in cm).



(a) $\sqrt{171}$ (b) $\sqrt{181}$

(c) $\sqrt{161}$ (d) $\sqrt{211}$

109. In figure chord ED is parallel to the diameter. AC of the circle. If $\angle CBE = 65^o$ then $\angle DEC = ?$



(a) 35º (b) 55º

(c) 45º (d) 25º

110. PQ and RS are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersect PQ at A and RS at B. the find $\angle$ AOB

(a) 60º (b) 120º

(c) 90º (d) 180º

111. In a circle two chords AB and CD intersect each other at external point P (out side the circle), if PA = 8cm, PD=4cm, CD=3cm then AB is equal to:

(a) 4 cm (b) 3 cm

(c) 3.5 cm (d) 4.5 cm

112. Two circles with centres P and Q intersect at B and C. A, D are points on the circles with centres P and Q respectively such that A, C, D are colliner. If $\angle APB = 130^o$, and $\angle BQD = x°$, then the value of x is:

(a) 65 (b) 130

(c) 195 (d) 135

113. C_1 and C_2 are two concentric circles with centres at O. Their radii are 12 cm and 3 cm respectively. B is the point of contact of two tangents drawn to C_2 from a point A lying on the circle C_1. Then the area of the triangle OAB is:

(a) $\frac{9\sqrt{15}}{2}$ sq. cm (b) $12\sqrt{15}$ sq. cm

(c) $9\sqrt{15}$ sq. cm (d) $6\sqrt{15}$ sq. cm

114. Two chords AB, CD of a circle with centre O interest each other at P. $\angle ADP = 23°$ and $\angle APC = 70°$, then the $\angle BCD$ is:

(a) 45º (b) 47º

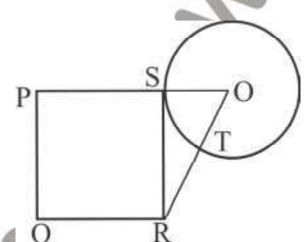
(c) 57º (d) 67º

115. Chords AB and CD of a circle interest at E and are perpendicular to each other. Segments AE, EB and ED are of lengths 2 cm, 6 cm and 3 cm respectively. Then the length of the diameter of the circle in cm is:

(a) $\sqrt{65}$ (b) $\frac{1}{2}\sqrt{65}$

(c) 65 (d) $\frac{65}{2}$

116- PQRS is a square. SR is a tangent (at point S) to the circle with centre O and TR = OS. Then, the ratio of area of the square to the area of the circle is:



(a) $\frac{\pi}{3}$ (b) $\frac{11}{7}$

(c) $\frac{3}{\pi}$ (d) $\frac{7}{11}$

117. A circle is inscribed in an equilateral triangle and a square is inscribed in the circle. The ratio of the area of the triangle to the square is:

(a) $\sqrt{3}:\sqrt{2}$ (b) $3\sqrt{3}:2$

(c) $3:\sqrt{2}$ (d) $\sqrt{2}:1$

118. AB is diameter of circle with length 8 cm and $CD \parallel AB$. If $AC = BD = 2$ cm then find CD.

(a) 7 (b) $7\sqrt{3}$

(c) $7\sqrt{2}$ (d) $2\sqrt{15}$

119. AB and CD are two chords of a circle such that $AB = 8cm, CD = 10cm$ and $AB \parallel CD$. If the perpendicular distance between AB and CD is 2cm , then what is the radius of the circle equal to:

(a) $\frac{5\sqrt{17}}{4}$ cm (b) $\frac{4\sqrt{17}}{5}$ cm

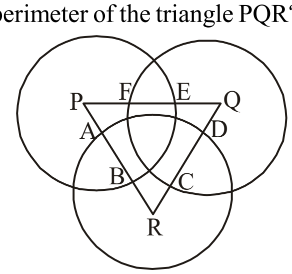
(c) $\frac{3\sqrt{17}}{5}$ cm (d) $\sqrt{17}$ cm

120. The difference between the area of a square and that of an equilateral triangle on the same base is 1/4 cm^2. What is the length of side of triangle ?

(a) $\left(4-\sqrt{3}\right)^{1/2}$ cm (b) $\left(4+\sqrt{3}\right)^{1/2}$ cm

(c) $\left(4-\sqrt{3}\right)^{-1/2}$ cm (d) $\left(4+\sqrt{3}\right)^{-1/2}$ cm

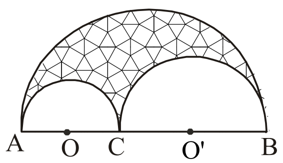
121. The figure below shows three circles, each of radius 25 and centres at P, Q and R respectively. Further $AB = 6, CD = 12$ and $EF = 15$. What is the perimeter of the triangle PQR?



(a) 117 (b) 116

(c) 113 (d) 121

122. Find the area of shaded portion given that the circles with centers O and O' are 6 cm and 18 cm in diameter respectively and ACB is a semi circle.



(a) $54\pi cm^2$ (b) $27\pi cm^2$

(c) $36\pi cm^2$ (d) $18\pi cm^2$

123. A, B, C, D are points on a circle, such that ABD is an equilateral triangle and AC is diameter of the circle. What is the raito of the perimeter of the quadrilateral ABCD to the perimeter of the circle?

(a) $\sqrt{2}+1:\pi$ (b) $3+\sqrt{2}:2\pi$

(c) $\sqrt{3}+1:\pi$ (d) $4+\sqrt{3}:3\pi$

124. Two circles with centers A and B and radius 2 units touch each other externally at 'C' and radius '2' units meets other two at D and E. Then the area of the quadrilateral ABDE is:

(a) $2\sqrt{2}$ sq. unit (b) $3\sqrt{3}$ sq. unit

(c) $3\sqrt{2}$ sq. unit (d) $2\sqrt{3}$ sq. unit

125. AB is a chord of a circle (centre o) and DOC is a line segment originating from a point D on the Circle and intersecting AB on Producing at C such that BC=OD. If $\angle BCD = 20^0$, then $\angle AOD$:-

(a) 30^0 (b) 40^0

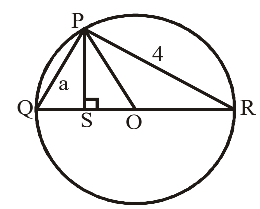
(c) 10^0 (d) 60^0

126. Two tangents PA and PB are drawn from a point P to the circle. If the radius of the circle is 5 cm and AB = 6cm and O is the centre of the circle. OP cuts AB at C and OC = 4cm, then OP:-

(a) $\frac{25}{4}$ cm (b) 25cm

(c) 13cm (d) None of these

127. If in the given figure, PQ = a, PR = 4 cm, while O is the centre of the circle and S is a point between O and Q such that PS ⊥ QR. Find the length of OP.



(a) $\frac{\sqrt{16+a^2}}{2}$ (b) $\frac{16-a^2}{2\sqrt{a^2+16}}$

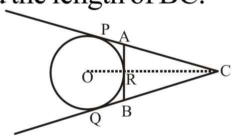
(c) $\frac{4a-16}{16a-a^2}$ (d) $\frac{2\sqrt{a^2-16}}{16+a^2}$

128. P and Q are the middle points of two chords (not diameters) AB and AC respectively of a circle with centre at a point O. The lines OP and OQ are produced to meet the circle respectively at the points R and S. T is any point on the major arc between the points R and S of the circle. If $\angle BAC = 32^0$, $\angle RTS$ = ?

(a) 32^0 (b) 74^0

(c) 106^0 (d) 64^0

129. In figure, CP and CQ are tangents from an external point C to a circle with centre O. AB is another tangent which touches the circle at R. If CP = 11 cm and BR = 4 cm. find the length of BC.



(a) 7 cm (b) 8 cm
(c) 5 cm (d) 10 cm

130. Two circles with centres A and B of radii 3 cm and 4 cm respectively intersect at two points C and D such that AC and BC are tangents to the two circles. Find the length of the common chord CD.

(a) 4.2 cm (b) 8.4 cm
(c) 2.4 cm (d) 4.8 cm

131. PR is a tangent at a point Q on a circle, in which Q is the centre and its radius is 8 cm. If OR = 16 cm and OP = 10 cm., then the length of PR is–

(a) 18 cm. (b) 19 cm.
(c) 19.8 cm. (d) 21.86 cm.

132. Two concentric circles with centre P have radii 6.5 cm and 3.3 cm. Through a point A of the larger circle, a tangent is drawn to the smaller circle touching it at B and the larger circle at C. Find AC.

(a) 5.6 cm. (b) 11.2 cm.
(c) 11.8 cm. (d) 6.5 cm.

Answer

1.	(d)	2.	(c)	3.	(c)	4.	(c)	5.	(a)	6.	(a)	7.	(d)	8.	(b)	9.	(c)
10.	(d)	11.	(b)	12.	(b)	13.	(d)	14.	(a)	15.	(b)	16.	(b)	17.	(b)	18.	(a)
19.	(c)	20.	(c)	21.	(b)	22.	(c)	23.	(b)	24.	(a)	25.	(b)	26.	(a)	27.	(b)
28.	(c)	29.	(c)	30.	(a)	31.	(b)	32.	(a)	33.	(b)	34.	(a)	35.	(a)	36.	(a)
37.	(b)	38.	(b)	39.	(a)	40.	(b)	41.	(a)	42.	(c)	43.	(a)	44.	(a)	45.	(b)
46.	(a)	47.	(d)	48.	(b)	49.	(a)	50.	(c)	51.	(b)	52.	(c)	53.	(a)	54.	(a)
55.	(c)	56.	(b)	57.	(a)	58.	(a)	59.	(d)	60.	(a)	61.	(b)	62.	(b)	63.	(b)
64.	(d)	65.	(c)	66.	(a)	67.	(a)	68.	(a)	69.	(b)	70.	(b)	71.	(c)	72.	(a)
73.	(d)	74.	(a)	75.	(d)	76.	(c)	77.	(d)	78.	(d)	79.	(b)	80.	(b)	81.	(a)
82.	(a)	83.	(a)	84.	(b)	85.	(c)	86.	(c)	87.	(b)	88.	(c)	89.	(c)	90.	(a)
91.	(c)	92.	(c)	93.	(c)	94.	(a)	95.	(b)	96.	(c)	97.	(a)	98.	(c)	99.	(d)
100.	(d)	101.	(c)	102.	(a)	103.	(a)	104.	(c)	105.	(d)	106.	(a)	107.	(d)	108.	(a)
109.	(d)	110.	(c)	111.	(d)	112.	(b)	113.	(a)	114.	(b)	115.	(a)	116.	(c)	117.	(b)
118.	(a)	119.	(a)	120.	(c)	121.	(a)	122.	(b)	123.	(c)	124.	(b)	125.	(d)	126.	(a)
127.	(a)	128.	(b)	129.	(a)	130.	(d)	131.	(c)	132.	(b)						

Solution & Hints

1. AO & BO are the radii of circle

$\angle ABO = \angle BAO = 60^o$ $(\therefore AO = BO)$

In ΔAOB,

$\Rightarrow \angle AOB + \angle ABO + \angle BAO = 180^O$

$\Rightarrow \angle AOB = 180^o - (60^o + 60^o)$

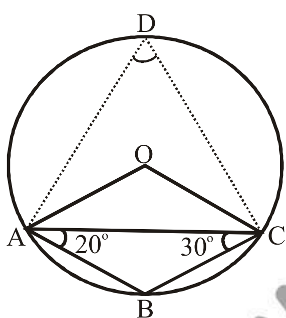
$= 60^o$

We know that,

$\Rightarrow \frac{1}{2} \angle AOB = \angle ACB$

$\therefore \angle ACB = 30^o$

2.



In ΔABC

$\Rightarrow \angle ABC = 180^o - \angle BAC - \angle ACB$

$= 180^o - 30^o - 20^o$

$\Rightarrow \angle ABC = 130^o$

$\Rightarrow \angle ADC = 180^o - 130^o$

$= 50^o$

$\therefore \angle AOC = 2 \times \angle ADC$

$= 2 \times 50 = 100^o$

3. $\angle AEB = 120^o$

$\angle AEC = 180^o – 120^o = 60^o$ [$\therefore$ Linear pair angle]

In ΔAEC,

$\angle AEC = 180^o - \angle CAE - \angle AEC$

$= 180^o - 30^o - 60^o = 90^o$

$\Rightarrow \angle ACE = 90^o$

$\Rightarrow \angle ACB = \angle ADB$ ($\therefore$ Angle formed on the same chord)

$\therefore \angle ADB = 90^o$

4. $PT^2 = PA \times PB$

Let $AB = x$

$\therefore 25 = 4 \times (4 + x)$

$25 = 16 + 4x$

$4x = 9$

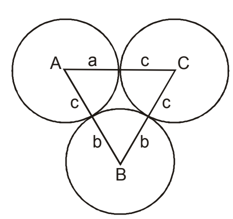
$AB = x = \frac{9}{4}$ cm

5. In right angle Δ

Inradius $r = \frac{AB + AC - BC}{2}$

$\therefore r = \frac{4 + 3 - 5}{2} = \frac{2}{2} = 1$ cm

6.



$\Rightarrow$ Distance between AB (a+b) = 5cm

$\Rightarrow$ Distance between BC (b+c) = 6cm

$\Rightarrow$ Distance between CA (c+a) = 7cm

$\Rightarrow 2a + 2b + 2c = 18$

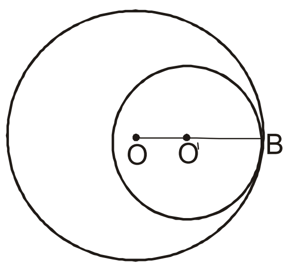
$\therefore a + b + c = 9$

$\Rightarrow$ Radius $a = 9 - 6 = 3$ cm

$\Rightarrow$ Radius $b = 9 - 7 = 2$ cm

$\Rightarrow$ Radius $c = 9 - 5 = 4$ cm

7.



Radius of bigger circle OB = 13 cm

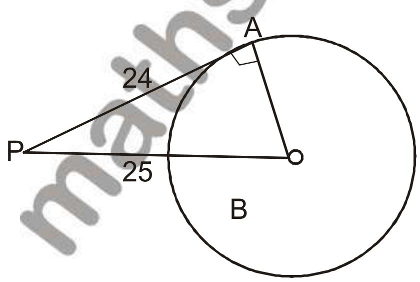
Radius of smaller circle O'B = 5 cm

$\therefore$ Distance between their centers OO' = OB – O'B

= 13 – 5

= 8 cm

8.



OP = 25 cm

AP = 24 cm

$\Rightarrow AO = \sqrt{OP^2 - AP^2}$

[$\therefore$ By pythagoras theorm]

$= \sqrt{25^2 - 24^2} = 7$

$\therefore$ AO = 7 cm

9. $\angle ACB = \frac{1}{2} \angle AOB$

$\angle ACB = \frac{1}{2} \times 46 = 23^o$

Now, In Δ BDC

$23° + \angle B + 90° = 180°$

$\therefore \angle B = 67^o$

10. AB = 8 cm

CD = 6 cm

$\Rightarrow PO = \sqrt{5^2 - 4^2} = 4$

PO = 3 cm

$\Rightarrow QO = \sqrt{5^2 - 3^2} = 4$

QO = 4

PQ = PQ + QO = 4 + 3

$\therefore$ PQ = 7 cm

11. $\angle ABC + \angle ADC = 180^o$

[$\therefore$ Sum of two opposite angle of cyclic quadrilateral]

$\angle ABC + 140^o = 180^o$

$\angle ABC = 40^o$

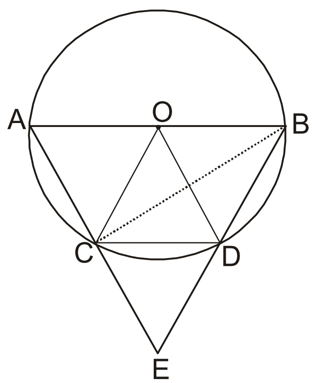
$\angle ACB = 90^o$ [$\therefore$ Angle formed in semicircle]

In Δ ABC,

$\Rightarrow \angle BAC = 180^o - 90^o - 40^o$

$\therefore \angle BAC = 50^o$

12.



Chord CD = r

In Δ OCD,

OC = OD = CD = r (Radius)

So, Δ OCD is equilateral triangle

$\Rightarrow \angle COD = 60^o$

$\Rightarrow \angle CBD = \frac{1}{2} \angle COD$

$\therefore \angle CBD = 30^o$

$\angle ACB = 90^o$ [$\therefore$ Angle formed in semicircle]

$\angle BCE = 180^o - 90^o = 90^o$

In Δ BCE,

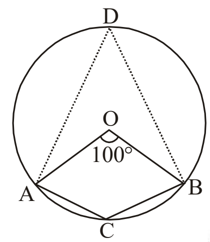
$\angle BEC = 180^o - \angle BCE - \angle CBE$

$= 180^o - 90^o - 30^o$

$\angle BEC = 60^o$

$\therefore \angle AEB = 60°$

13.



$\angle ADB = \frac{1}{2} \angle AOB$

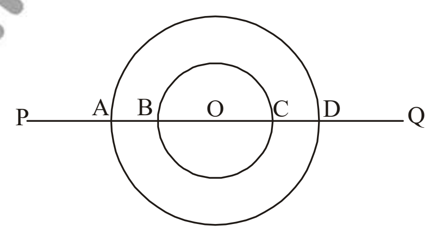
$\therefore \angle ADB = 50^o$

$\Rightarrow \angle ADB + \angle ACB = 180°$

$\Rightarrow \angle ACB = 180° - 50°$

$\therefore \angle ACB = 130°$

14.

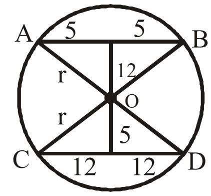


AD = 12 $\Rightarrow$ OA = $\frac{12}{2}$ = 6 cm

BC = 8 cm $\Rightarrow$ OB = $\frac{8}{2}$ = 4 cm

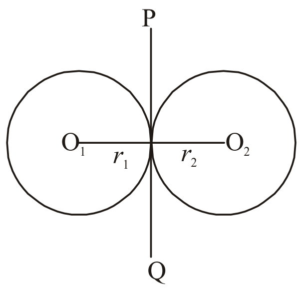
$\therefore$ AB = OA – OB = 6 – 4 = 2 cm

15.



Radius (r) = $\sqrt{(12)^2 + (5)^2} = \sqrt{169} = 13$ cm

16.



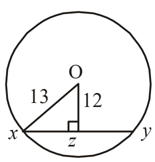
$PQ = \sqrt{4r_1 r_2}$

$= \sqrt{4 \times 8 \times 2}$

$= \sqrt{64}$

$= 8$ cm

17.



In $\triangle$ XOZ

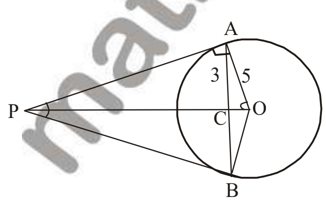
$XZ = \sqrt{13^2 - 12^2}$

$= \sqrt{25}$

$\therefore$ XZ = 5

$\therefore$ Chord XY = 2 × 5 = 10 cm

18.



In $\triangle$ PAO and $\triangle$ PBO

AP = PB [Equal Tangents]

OA = OB [Radius]

OP = OP [Common]

$\therefore$ Δ OAP $\cong$ Δ OPB

So, $\angle$ APO = $\angle$ BPO

In Δ AOP ,

$\Rightarrow$ $\angle APO = 180° - 90° - 60°$

$\Rightarrow$ $\angle APO = 30°$

$\therefore$ $\angle APB = 2(\angle APO)$

$= 2 \times 30° = 60°$

19. In Δ AOB

$\Rightarrow$ $\angle OBA = \angle OAB = 55^o$ [$\because$ OB = OA]

$\Rightarrow$ $\angle AOB = 180° - (55° + 55°)$

$\Rightarrow$ $\angle AOB = 70^o$

$\Rightarrow$ $\angle AOB = \angle COD$ [AB = CD]

$\therefore$ $\angle COD = 70^o$

20. $\angle CBA = \frac{1}{2} \angle COA$

$= \frac{1}{2} \times 120°$

$\therefore$ $\angle CBA = 60^o$

$\Rightarrow$ $\angle CBE = 180° - \angle CBA$

$= 180° - 60°$

$\therefore$ $\angle CBE = 120^o$

21. $\angle ACB = 90°$ ($\therefore$ Angle formed in semicircle)

$\angle ADC + \angle ABC = 180°$

($\therefore$ Sum of two opposite angles of quadrilateral is 180°)

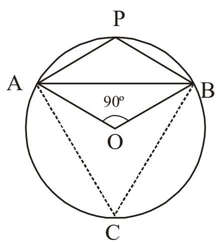
$\Rightarrow$ $\angle ABC = 180° - 120°$

$\therefore$ $\angle ABC = 60^o$

$\Rightarrow \angle CAB = 180° - 90° - 60°$

$\therefore \angle CAB = 30^o$

22.



$\Rightarrow \quad \angle ACB = \frac{1}{2} \angle AOB$

$\Rightarrow \quad \angle ACB = \frac{1}{2} \times 90 = 45^o$

Since, $\angle APB + \angle ACB = 180^o$

$\Rightarrow \quad \angle APB = 180° - 45°$

$\therefore \quad \angle APB = 135^o$

23. Let $PD = x$ cm

$\Rightarrow AP \times PB = CP \times PD$

$\Rightarrow 2 \times 6 = 3 \times x$

$\Rightarrow x = 4$ cm

$\Rightarrow \quad PD = 4$ cm

24. Inradius of circle = $\frac{AB + AC - BC}{2}$

$= \frac{4+3-5}{2} = \frac{2}{2} = 1$ cm

25. $\angle QAX = \angle BAX - \angle BAQ$

$\angle QAX = 70° - 40° = 30^o$

Now, $\angle BAY = 180° - \angle BAX$

$= 180° - 70°$

$\angle BAY = 110°$

$\angle EBA = 90^o$

$\therefore \quad \angle BAY = \angle AQB = 110^o$

Now, In $\triangle ABQ$

$\angle ABQ = 180 - \angle BAQ - \angle AQB$

$= 180° - 40° - 110°$

$\therefore \quad \angle ABQ = 30°$

26. Now, $\angle ADB = 90°$ [Angle in Semicircle]

$\angle ADC = \angle DAB = 30^o$ [Alternate angle]

Now in $\triangle ABD$

$\angle DAB + \angle ABD + \angle ADB = 180^o$

$30^o + \angle ABD + 90^o = 180^o$

$\angle ABD = 60^o$

$\Rightarrow \angle ACD = 180 - \angle ABD = 180\text{-}60^o = 120^o$

Now in ΔCAD

$\angle ACD + \angle CAD + \angle ADC = 180^o$

$120^o + \angle CAD + 30^o = 180^o$

$\therefore \quad \angle CAD = 180^o \text{-} 150^o = 30^o$

27. In ΔBDC

$\angle BDC = 180° - 110° - 30°$

$\angle BDC = 40°$

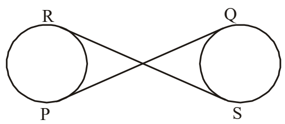
$\angle BAC = \angle BDC = 40°$

($\therefore$ Angle in same segment)

28. $\alpha + \gamma = 180°$

$50 + \gamma = 180°$

$\gamma = 180° - 50° = 130°$

29. 

$PQ = RS$

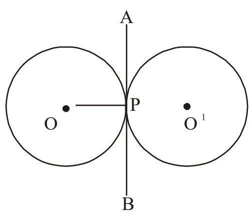
$= \sqrt{(\text{distance between center})^2 - (r_1 + r_2)^2}$

$= \sqrt{(10)^2 - (3+3)^2}$

$= \sqrt{100 - 36}$

$= \sqrt{64}$

$= 8$ cm

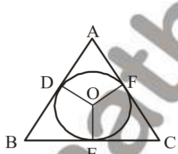
30. 

$\angle APO = 90°$ (Radius is perpendicular to the tangent)

31. $AP \times BP = DP \times CP$

$2 \times 6 = DP \times 3$

$DP = 4$ cm.

32. 

Draw angle bisector OE, OF and OD on BC, CA and AB respectively

$\Rightarrow OB = AC$

AE divides the line BC in equal parts

$\Rightarrow BE = \frac{1}{2} BC$

33. $\angle BCA = \angle ADC$

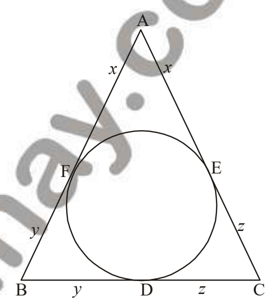
(angle made by same chord)

In ΔABD,

$\angle ABD + \angle BDA + \angle DAB = 180°$

$\angle ABD + 41° + 90° = 180°$

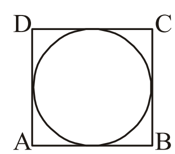
$\angle ABD = 49°$

34. 

$2(x+y+z) =$ perimeter

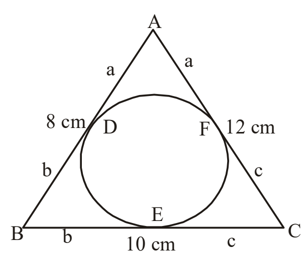
$\Rightarrow (x+y+z) = \frac{1}{2} \times$ perimeter of ΔABC

Hence, $AF + BD + CE = \frac{1}{2}$ (perimeter of ΔABC)

35. 

If Circle touch all the sides of a quadrilateral then sum of two opposite sides is equal.

$\therefore BC + DA = AB + CD$

36. 

Perimeter of $\Delta ABC = 30$

$\Rightarrow 2(a+b+c) = 30$

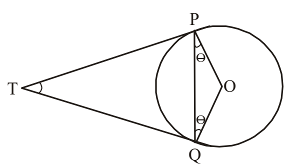
$\Rightarrow a+b+c = 15$

$\therefore AD = a+b+c-(b+c)$

$= 15-10$

$= 5cm$

37.



$\angle OPQ = \angle OQP = \theta$

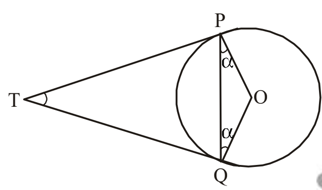
In Δ OPQ,

$\Rightarrow \angle POQ = 180° - 2\theta$

$\Rightarrow \angle PTQ = 180° - (180° - 2\theta) = 2\theta$

$\therefore \angle PTQ = 2\angle OPQ$

38.



$OM = \sqrt{(5)^2 - (3)^2} = 3$ cm.

Now, in Δ PMO

$\Rightarrow \tan\alpha = \frac{4}{3}$ ....(i)

In Δ OPT

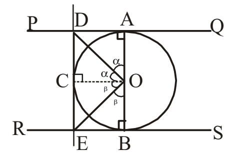
$\Rightarrow \tan\alpha = \frac{TP}{5}$ ....(ii)

From eq. (i) and (ii)

$\frac{4}{3} = \frac{TP}{5}$

$\therefore TP = \frac{20}{3}$ cm.

39.



In Δ DCO and Δ DAO

$\angle C = \angle A = 90°$

DO = DO [Common]

DA = DC = [Tangent]

$\therefore \Delta DCO \cong \Delta DAO$

Now, $\angle AOD = \angle DOC = \gamma$

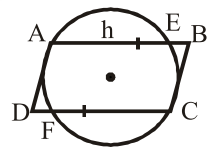
Similarly, $\angle COE = \angle BOE = \beta$

$\Rightarrow \alpha + \alpha + \beta + \beta = 180^o$ [Linear Pair angle]

$\Rightarrow 2(\alpha + \beta) = 180^o$

$\therefore \angle DOE = (\alpha + \beta) = 90^o$

40.



$\Rightarrow BE \times AB = BC^2$

$2 \times 6 = BC^2$

$\Rightarrow BC = \sqrt{12} = 2\sqrt{3}$

$\Rightarrow BC = AD = 2\sqrt{3}$

$\Rightarrow AD^2 = DC \times DF$

$\Rightarrow \left(2\sqrt{3}\right)^2 = (DF + FC) \times DF$

$\Rightarrow 12 = (4 + x)x$

$\Rightarrow x^2 + 4x - 12 = 0$

$\Rightarrow (x+6)(x-2) = 0$

$\Rightarrow x = 2, x = -6$

$\therefore x = 2$

41. $\angle QRS = 90°$

$\Rightarrow \angle QRP = y°$

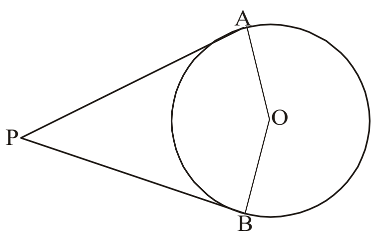
$\Rightarrow \angle SQR = x° + y°$ ($\therefore$ External angle)

In ΔPRQ,

$\Rightarrow x° + y° + y° + 90° = 180°$

$\therefore x° + 2y° = 90°$

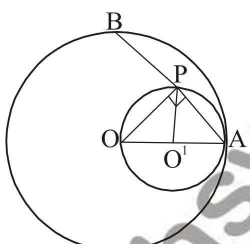
42.



Angle OAP & angle OBP are 90º then sum of remaining two angle of Quadrilateral (angle P & O) will be 180º.Hence,

$\therefore$ AOBP is a cyclic quadilateral.

43.

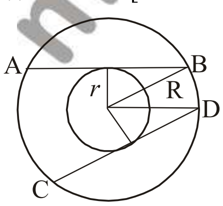


$\angle OPA = 90°$ [AB is a chord of bigger circle]

$OP \perp AB$

$\therefore AP = PB$ [AB is a chord of bigger circle]

44.

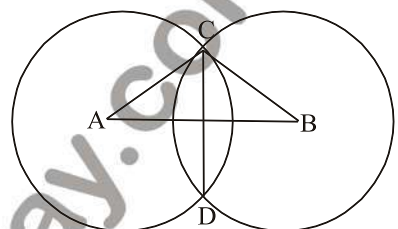


$AB = 2\left(R^2 - r^2\right)$

$CD = 2\left(R^2 - r^2\right)$

$\therefore AB = CD$

45.



$\Rightarrow AB^2 = AC^2 + BC^2$

$\Rightarrow AB = \sqrt{3^2 + 4^2} = 5$

$\Rightarrow AB = 5$ cm

$$CD = \frac{2s_1s_2}{\sqrt{s_1^2 + s_2^2}}$$

$$= \frac{2\times3\times4}{\sqrt{3^2+4^2}} = \frac{24}{5} = 4.8$$

$\Rightarrow CD = 4.8cm$

46. Area of III circle = Area of I + Area of II

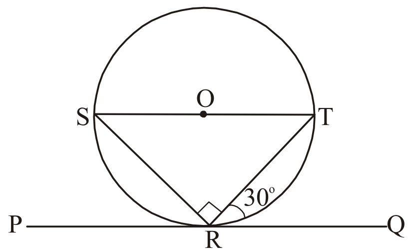
$\pi R_3^2 = \pi R_1^2 + \pi R_2^2$

$R_3^2 = (5)^2 + (12)^2$

$\therefore$ $R_3 = 13$ cm

Sol^n 47. The length of a line is not finite so it cannot be determined.

48.



$\angle TRQ = 30^o$ (given)

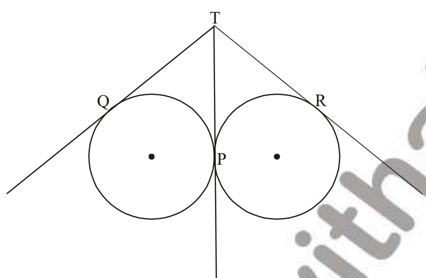
$\angle SRT = 90^o$ ($\therefore$ Angle formed on semicircle)

$\Rightarrow \angle PRS + \angle SRT + \angle TRQ = 180^o$ ($\therefore$ Linear Angle)

$\angle PRS + 90^o + 30^o = 180^o$

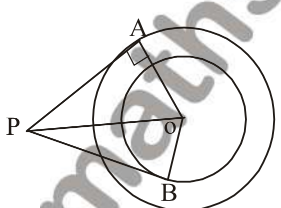
$\therefore \angle PRS = 60^o$

49.



$TQ = TP$
$TP = TR$
$\Rightarrow TQ = TR$

50.



$AO = 5\text{ cm}$

$BO = 3\text{ cm}$

$AP = 12\text{ cm}$

$\Rightarrow OP = \sqrt{AP^2 + OA^2} = \sqrt{12^2 + 5^2} = \sqrt{169}$

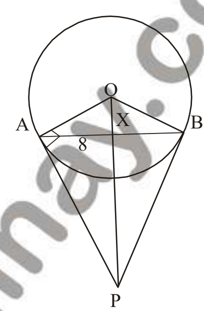
$\Rightarrow OP = 13\text{ cm}$

$\Rightarrow BP = \sqrt{OP^2 - OB^2}$

$= \sqrt{(13)^2 - (3)^2} = \sqrt{160}$

$\therefore BP = 4\sqrt{10}$

51.



$OA = OB = 10\text{ cm}$

$\Rightarrow AB = 16\text{ cm}$

$AX = XB = 8\text{ cm}$

$\Rightarrow XO = \sqrt{AO^2 - AX^2} = \sqrt{100 - 64}$

$\Rightarrow XO = \sqrt{36} = 6\text{ cm}$

Now, $\dfrac{AP}{AO} = \dfrac{AX}{XO} = \dfrac{AP}{10} = \dfrac{8}{6}$

$AP = \dfrac{80}{6} = \dfrac{40}{3}$

$\therefore AP = \dfrac{40}{3}\text{ cm}$

52. $\angle B = 65°$

$\Rightarrow \angle D = 180° - 65° = 115°$

$\Rightarrow \angle DCA = 180^o - 115^o - 45^o$

$\Rightarrow \angle DCA = 20^o$

53. $\angle BAC = \angle BDC = 40^o$

[Angle formed by the same chord]

$\Rightarrow \angle AOB = \angle COD = 100^o$

[Vertically opp. angle]

$\Rightarrow \angle COD + \angle ODC + \angle OCD = 180^o$

[Sum of all angle of Δ is 180°]

$\Rightarrow 100^o + 40^o + \angle ODC = 180^o$

$\Rightarrow \angle COD = 40^o$

$\therefore \angle ACD = 40^o$

54. $\angle CBA = 180° - 130°$

$\angle CDA + \angle CBA = 180°$

$\angle CDA = 180° - 50° = 130°$

$x + 130° = 180°$

$x = 50^o$

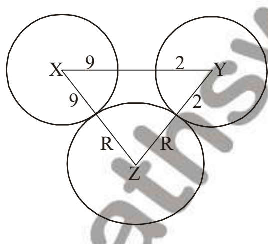
55. $AP \times PC = BP \,.\, DP$

$12 \times 8 = (7 + x) \times 7$

$x = \frac{96}{7} - 7$

$\therefore x = 6.7$ cm

56.



$YZ = 2 + x$

$XY = 17$

$XZ = 9 + x$

$\Rightarrow XY^2 = YZ^2 + XZ^2$

$\Rightarrow 17^2 = (2 + r)^2 + (9 + r)^2$

$\Rightarrow 289 = 4 + 4r + r^2 + 81 + 18r + r^2$

$\Rightarrow 2r^2 + 22r + 85 - 289 = 0$

$\Rightarrow r^2 + 11r - 102 = 0$

$\Rightarrow (r - 6)(r + 17) = 0$

$r = 6, r = -17$

$\therefore r = 6$ cm

57. $\angle BCD = \angle CAB$ ....(i)

$\angle BDC = \angle DAB$ ....(ii)

Add these equations

$\Rightarrow \angle CAB + \angle DAB = \angle BCD + \angle BDC$

$\Rightarrow 180° - \angle CBD = \angle CAD$

$\therefore \angle CAD + \angle CBD = 180°$

58. $AP > PB$

$AB = 16$

$AP = x$

$BP = 16 - x$

$\Rightarrow AP. PB = CP.DP$

$\Rightarrow x(16 - x) = 6 \times 8$

$\Rightarrow x^2 - 16x + 48 = 0$

$\Rightarrow x^2 - 12x - 4x + 48 = 0$

$\Rightarrow x(x - 12) - 4(x - 12) = 6$

$x = 12$

$x = 4$

$\Rightarrow (AD > PB)$

$\therefore AB = 12$ cm

59. $AB + BC = 12$

$BC + CA = 14$

$CA + AB = 18$

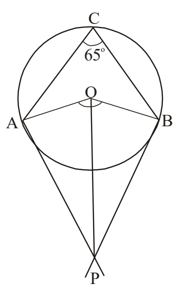
$\text{Perimeter} = \frac{12 + 14 + 18}{2} = 22$

$\Rightarrow 2\pi r = 22$

$\Rightarrow r = \dfrac{22 \times 7}{22 \times 2}$

$\therefore r = \dfrac{7}{2}$ cm

60.



$\Rightarrow \angle ACB = 65°$

$\Rightarrow \angle AOB = 2 \times 65 = 130^o$

$\Rightarrow \angle OAP = 90^o$

$\Rightarrow \angle AOP = 65^o$

$\Rightarrow \angle APO = 180° - 90° - 65°$

$\therefore \angle APO = 25°$

61. Same as Q.56

62. $\angle AOC = 135°$

$\Rightarrow \angle COB = 180° - 35°$

$\Rightarrow \angle COB = 45°$

$\angle CDB = 22\frac{1}{2}^o$

[CB is a common chord]

63. $\angle PDC = 180° - 105°$

$\Rightarrow \angle PDC = 75°$

$\angle DCB = 180° - 60°$

$\Rightarrow \angle DCB = 120°$

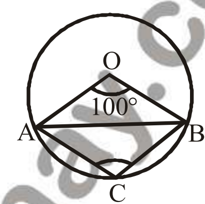
$\angle PCB = 180° - 120° = 60°$

$\Rightarrow \angle PCD = 60^o$

$\angle DPC = 180° - 75° - 60°$

$\therefore \angle DPC = 45^o$

64.



$\angle AOB = 100°$

External angle of $\angle AOB = 360° - 100° = 260°$

$\Rightarrow \angle ACB = \dfrac{1}{2} \angle AOB$

$\Rightarrow \angle ACB = \dfrac{1}{2} \times 260° = 130^o$

$\therefore \angle ACB = 130^o$

65. In ΔADP,

$\angle ADP = 180° - 59° - 40°$

$\Rightarrow \angle ADP = 81^o$

$\Rightarrow \angle ABC = 180° - 81° = 99^o$

$\Rightarrow \angle CDQ = \angle ABC = 99^o$

$\Rightarrow \angle QCB = \angle BAP = 59^o$

Now, In ΔCQD,

$\Rightarrow \angle CQD = 180 - 59° - 99° = 22°$

$\therefore \angle CQD = 22^o$

$\therefore \angle AQB = 22^{o}$

66. $\angle CAB = 50° + 30° = 80°$

$\angle OCT = 90°$

$\angle OCA = 90° - 50° = 40°$

$\therefore OC = OA$

So, $\angle OAC = 40°$

$\angle OAB = 80° - 40° = 40°$

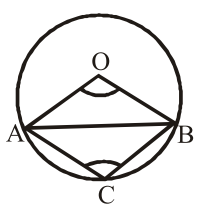
$\therefore OB = OA$

So, $\angle OBA = 40°$

In ΔBOA

$\therefore \angle BOA = 180° - 40° - 40° = 100°$

67.



ACBO is Parallelogram

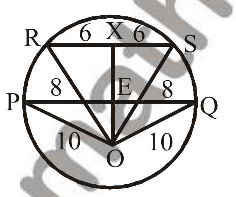
$\therefore x = y$

which is possible

When

$x = y = 120^{o}$

68.



$OE = \sqrt{(10)^2 - (8)^2} = \sqrt{36} = 6$

$OX = \sqrt{(10)^2 - (6)^2} = \sqrt{64} = 8$

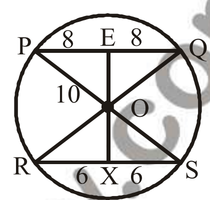
$OX = 8$ cm

$\Rightarrow EX = OX - OE$

$\Rightarrow EX = 8 - 6$

$\Rightarrow EX = 2$ cm

Triplet (6, 8, 10)



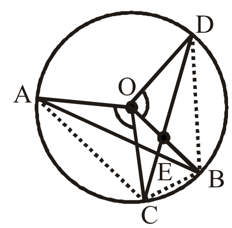
$OE = 6$ cm

$OX = 8$ cm

$EX = 8 + 6 = 14$ cm

$\therefore 2$ cm, 14 cm

69.



$\angle AOC = 40^{o}$

$\angle DOB = 50^{o}$

$\Rightarrow \angle ABC = \dfrac{40}{2} = 20°$

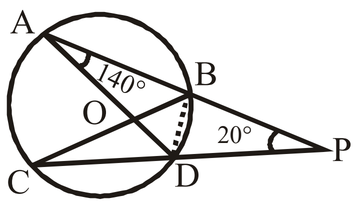
(Angle formed by the same chord)

$\Rightarrow \angle DCB = \dfrac{50}{2} = 25°$

$\Rightarrow \angle AEC = 20° + 25°$

$\therefore \angle AEC = 45°$

70.



$\angle ABD = 90^\circ$

$\angle BCD = \angle DAB = 40^\circ$

$\Rightarrow \angle ABC = \angle BCP + \angle CPB = 40^\circ + 20^\circ$

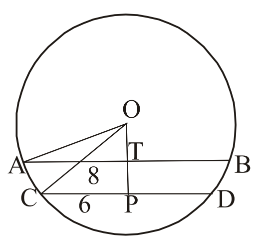
[$\therefore$ external angle]

$\Rightarrow \angle ABC = 60^\circ$

$\Rightarrow \angle DBC = 90^\circ - \angle ABC = 90^\circ - 60^\circ$

$\therefore \angle DBC = 30^\circ$

71.



In Δ OTA

$OA^2 = OT^2 + AT^2$

$\Rightarrow (10)^2 = (OT)^2 + (8)^2$

$\Rightarrow OT = \sqrt{36} = 6$ cm

Now, In Δ OCP

$\Rightarrow OC^2 = OP^2 + CP^2$

$\Rightarrow (10)^2 = OP^2 + CP^2$

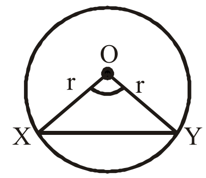
$\Rightarrow OP = \sqrt{64} = 8$cm.

Distance between two paralled chords TP

$\therefore TP = OP - OT$

$\therefore TP = 8\text{-}6 = 2$cm

72.



Area of $\Delta = 32$

$\Rightarrow \frac{1}{2} r \times r = 32$

$\Rightarrow r^2 = 64$

$\Rightarrow r = 8$

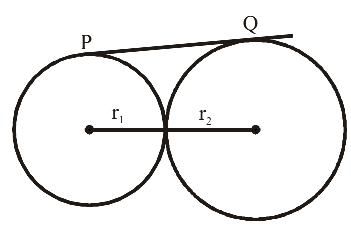
Since, XO = OY

XO = 8

r = OY = 8

$\therefore$ Area of circle = $\pi r^2 = \pi (8)^2 = 64\pi$

73.



$r_1 + r_2 = 13$ cm

$r_2 - r_1 = 9 - 4 = 5$ cm

$PQ = \sqrt{(\text{distance between center})^2 - (r_2 - r_1)^2}$

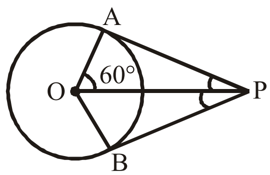
$\Rightarrow \quad PQ = \sqrt{(13)^2 - (5)^2}$

$\Rightarrow \quad PQ = 12$ cm

$\therefore \quad$ Area of square = a^2

$= 12 \times 12 = 144$ cm^2

74.



AP = PB

OA = OB

OP = OP

$\therefore \Delta OAP \cong \Delta OPB$

So, $\angle AOP = \angle POB$

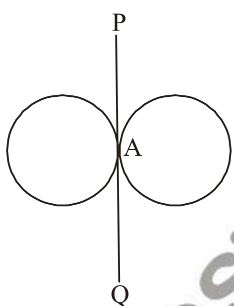
Now, $\angle APO = 180° - 90° - 60°$

$\therefore \angle APO = 30°$

$\Rightarrow \angle APB = 2 \times \angle APO$

$\Rightarrow \angle APB = 2 \times 30° = 60°$

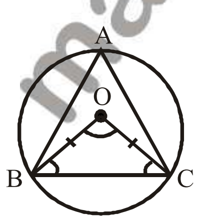
75.



$(PQ)^2 = (r_1 + r_2)^2 - (r_1 - r_2)^2$

$(PQ)^2 = 4 r_1 r_2$ $[\therefore (a+b)^2 - (a-b)^2 = 4ab]$

76.



$\Rightarrow \angle BOC = 2\angle BAC$

$\therefore$ OB = OC

$\Rightarrow \angle OBC = \angle OCB$

$\Rightarrow \angle OBC = 90° - \dfrac{\angle \frac{1}{2} BOC}{2}$

$(\therefore \angle BOC = 2\angle BAC)$

$\Rightarrow \angle OBC = 90° - \angle BAC$

$\therefore \angle OBC + \angle BAC = 90°$

77. AB = AD = BC = CD

If we join AC, then

$\Rightarrow$ In ΔABC,

AB = AC = BC

So, ABC is equilateral Δ

$\Rightarrow \angle ABC = 60°$

BD bisect the $\angle ABC$]

$\therefore \angle ABD = x° = 30°$

78. Diameter AB = 10 cm.

Radius OA = 5cm.

$\Rightarrow$ AE = 2cm.

$\Rightarrow$ OE = 5–2 = 3cm.

$\therefore$ OD is also a radius.

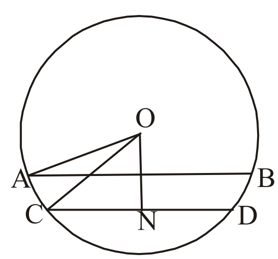
OD = 5 cm

Now, In Δ OED = $OD^2 = OE^2 + ED^2$

$(5)^2 = (3)^2 + ED^2$

$\therefore ED = \sqrt{16} = 4cm$

79.



Perpendicular drwan from centre of a circle to a chord, bisects the chord

In ΔOMA ,

$\therefore$ $(OA)^2 = (AM)^2 + (OM)^2$

$\Rightarrow$ $(65)^2 = (63)^2 + (OM)^2$

$\Rightarrow$ OM = 16 m

Now, In OCN

$\therefore$ $(OC)^2 = (CN)^2 + (ON)^2$ ($\therefore$ by pythagoras)

$\Rightarrow$ $(63)^2 = (56)^2 + (ON)^2$

$\Rightarrow$ ON = 33 m

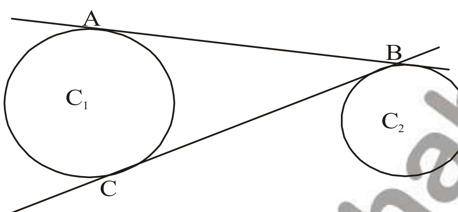
Height of Quadrilateral ABCD (MN) = 33–16 = 17

Area of quadrilateral = $\frac{1}{2}$ ×Height ×(Sum of sides)

$= \frac{1}{2} \times 17 \times (126+112)$

$= 1023\ m^2$

80.



Length of AB = $\sqrt{(c_1 c_2)^2 - (r_1 - r_2)^2}$

Lenght of BC = $\sqrt{(c_1 c_2)^2 - (r_1 + r_2)^2}$

($\therefore$ where c_1c_2 is distance between circles $r_1 = 4$, $r_2 = 3$)

$\Rightarrow \frac{\text{Length of BC}}{\text{Length of AB}} = \frac{1}{2}$

$\Rightarrow \frac{\sqrt{(c_1 c_2)^2 - (r_1 + r_2)^2}}{\sqrt{(c_1 c_2)^2 - (r_1 - r_2)^2}} = \frac{1}{2}$

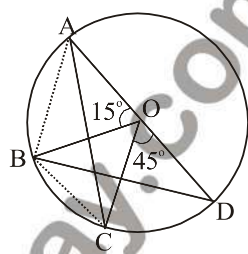
$\Rightarrow \frac{(c_1 c_2)^2 - (4+3)^2}{(c_1 c_2)^2 - (4-3)^2} = \frac{1}{4}$

$\Rightarrow \frac{(c_1 c_2)^2 - 49}{(c_1 c_2)^2 - 1} = \frac{1}{4}$

$\Rightarrow$ $3\,(c_1c_2)^2 = 195$

$\therefore c_1c_2 = \sqrt{65}$

81.



$\angle$ APB = 30°

$\angle$ AOB = 15°

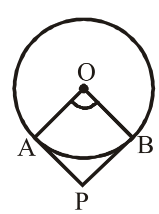
So, $\angle$ COD = 45°

$\Rightarrow$ $\tan^2 \angle APB + \cot^2 \angle COD$

$= \tan^2 30° + \cot^2 45°$

$= \left(\frac{1}{\sqrt{3}}\right)^2 + 1 = \frac{4}{3}$

82.



$\Rightarrow$ $\angle$ OAP = $\angle$ OBP = 90°

[Perpendicular to tangent]

$\Rightarrow$ $\angle$ AOB + $\angle$ APB = 180°

$\Rightarrow \frac{\angle AOB}{\angle APB} = \frac{5}{1}$

$\Rightarrow \angle AOB = 5 \angle APB$

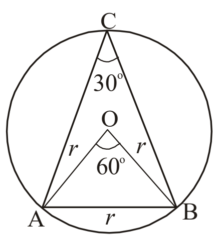
Put value of $\angle AOB$

$\Rightarrow 5\angle APB + \angle APB = 180°$

$\Rightarrow 6\angle APB = 180°$

$\therefore \angle APB = 30°$

83.



Radius = r

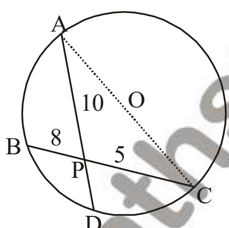
Length of Chord = r

$\therefore \Delta AOB$ is equilateral triangle.

Now, $\angle AOB = 60°$

Hence, $\angle ACB = \frac{1}{2} \times \angle AOB = 30°$

84.

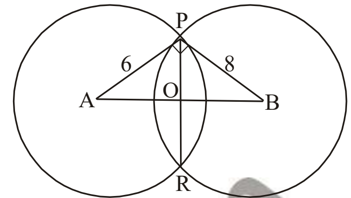


$\Rightarrow \frac{AP}{BP} = \frac{PC}{PD}$

$\Rightarrow \frac{10}{8} = \frac{5}{PD}$

PD = 4cm

85.



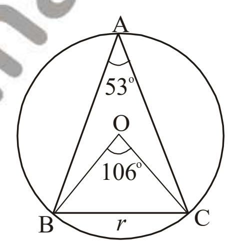
In ΔAPO and ΔAPB

$\Rightarrow \frac{8}{10} = \frac{PO}{6}$

$\Rightarrow PO = 4.8$

$\therefore PR = 9.6$ cm

86.



BO = CO

$\Rightarrow \angle OBC = \angle OCB = 37°$

Now, $\angle BOC = 180° - 37° - 37° = 106°$

$\Rightarrow \angle BAC = \frac{106°}{2} = 53°$

87. Perimeter of square = Perimeter of circle

$4a = 2\pi r$

$a = \frac{\pi r}{2}$

$\frac{\text{Area of square}}{\text{Area of circle}} = \frac{\left(\frac{\pi r}{2}\right)^2}{\pi r^2} = \frac{22}{28} \quad \left(\pi = \frac{22}{7}\right)$

$\therefore$ Area of circle > Area of square

88. Both I and II

89. In ΔAMO,

$$\Rightarrow \text{Sin}\theta = \frac{\frac{3}{2}}{3} = \frac{1}{2}$$

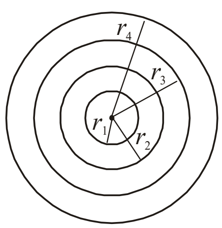
$\Rightarrow \theta = 30°$

$\Rightarrow \angle AOC = 60°$

$$\Rightarrow \angle ABC = \frac{1}{2} \times \angle AOC$$

$= 30°$

90.



If area of is equal of four parts, then

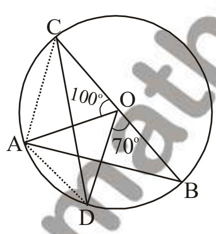
$r_1 : r_2 : r_3 : r_4$

$\sqrt{1} : \sqrt{2} : \sqrt{3} : \sqrt{4}$

Here given, $2 \rightarrow 40$ $1 \rightarrow 20$

$\therefore$ So, area = $20, 20\sqrt{3}, 20\sqrt{2}$

91.



As, $\angle AOC = 100°$

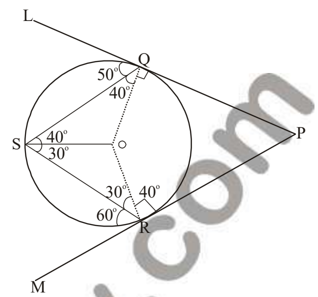
So, $\angle CDA = 50°$

As, $\angle BOD = 70°$

So, $\angle BAD = 35°$

$\therefore \angle APC = 50° + 35° = 85°$[exterior angle]

92.



$\angle PQO = \angle PRO = 90°$

$\Rightarrow \angle SQO = 180° - (50°+90°) = 40°$

$\Rightarrow \angle SRO = 180° - (60°+90°) = 30°$

Now, $OQ = OS$ [Radius]

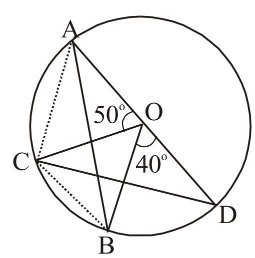
$\angle OQS = \angle SOQ = 40°$

$OR = OS$ [Radius]

$\angle ORS = \angle OSR = 30°$

$\therefore \angle QSR = 40°+30° = 70°$

93.



$\Rightarrow \angle AOC + \angle BOD$

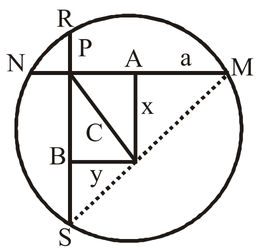
$= 2\angle ABC + \angle BCD$

$\Rightarrow 2\angle ABC + \angle BCD = 2\angle BPD$

$$\Rightarrow \angle BPD = \frac{1}{2}(50° + 40°) = 45°$$

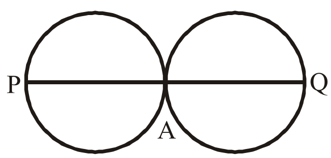
$\therefore \angle BPD = 45°$

94.



$$\therefore r = \sqrt{\frac{a^2+b^2+c^2}{2}}$$

95.



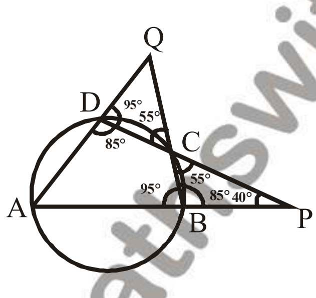
$\Rightarrow$ AP = 5×2 = 10 cm

$\Rightarrow$ AQ = 8×2 = 16 cm

$\Rightarrow$ AP : AQ = 10 : 16

$\therefore$ AP : AQ = 5 : 8

96.



$\angle$ QDC = 180° – 85° = 95°

$\Rightarrow \angle$ QDC = 95º

$\Rightarrow \angle$ ADC + $\angle$ ABC = 180°

$\Rightarrow \angle$ ABC = 180° – 85° = 95°

$\Rightarrow \angle$ CBP = 180° – 95° = 85º

Now, In Δ BCP

$\Rightarrow \angle$ PCB = 180° – 85° – 40°

$\Rightarrow \angle$ PCB = 55°

$\therefore \angle$ PCB = $\angle$ QCD

$\therefore \angle$ QCD = 55°

In Δ QCD,

$\angle$ CQD = 180° – 95° – 55°

= 30º

$\therefore \angle$ CQD = 30º

97. PT = $\sqrt{(5)^2-(4)^2}$

= 3cm

PA = x, AB = 8

Now, PT^2 = PA×PB

$(3)^2 = x \times (x+8)$

PT = $\sqrt{(5)^2-(4)^2}$

$\Rightarrow x^2 + 8x - 9 = 0$

$\Rightarrow x^2 + 8x - 9 = 0$

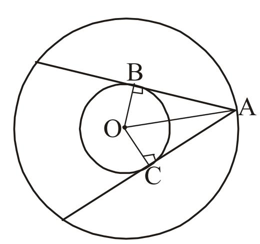
$\Rightarrow x^2 + 9x - x - 9 = 0$

$(x+9)(x-1) = 0$

$x = 1$

$\Rightarrow$ PB = 8+1 = 9cm

98.



OA = 12 cm

OB = 3 cm

$\Rightarrow AB = \sqrt{12^2 - 3^2}$

$\Rightarrow AB = \sqrt{15 \times 9}$

$= 3\sqrt{15}$

$\Rightarrow$ Area of ABOC $= 2 \times \Delta ABO$

$= 2 \times \frac{1}{2} \times OB \times AB$

$= 2 \times \frac{1}{2} \times 3 \times 3\sqrt{15}$

$= 9\sqrt{15}$ cm^2

99. $OP \perp AB$

$AP = PB = 4$cm.

$OQ \perp CD$

$CQ = QD = 3$cm.

In Δ OPA,

$AO^2 = AP^2 + PO^2$

$(5)^2 = (4)^2 + (PO)^2$

$\therefore PO = 3$ cm.

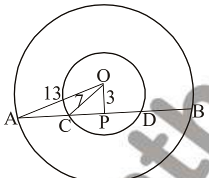
In ΔOCQ,

$(OC)^2 = (CQ)^2 + (OQ)^2$

$OQ = 4$cm

$\therefore\ PQ = 3+4 = 7$cm

100.



In Δ OCP,

$(OC)^2 = (CP)^2 + (PO)^2$

$(7)^2 = (CP)^2 + (3)^2$

$CP = \sqrt{49-9}$

$= 2\sqrt{10}$

Now, In ΔOAP,

$(AO)^2 = (AP)^2 + (OP)^2$

$(13)^2 = (AP)^2 + (3)^2$

$AP = \sqrt{169-9} = \sqrt{160}$

$= 4\sqrt{10}$ cm

$\therefore AC = AP - CP = 4\sqrt{10} - 2\sqrt{10} = 2\sqrt{10}$ cm

101. $\angle PRQ = 90°$ [Angle is semicircle]

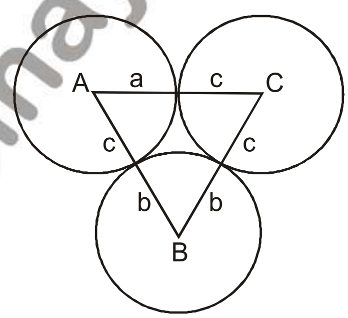
$\Rightarrow \angle QRT = 180° - 90° = 90°$

$\Rightarrow \angle RQS = 21°$

Now, In ΔRTQ,

$\therefore \angle RTS = 180° - (90° + 21°) = 69°$

102.



$a+c = 3$

$b+c = 4$

$c+a = 5$

$\Rightarrow 2(a+b+c) = 12$

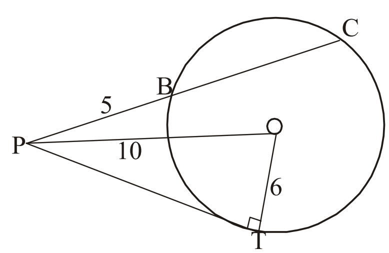
$\Rightarrow a+b+c = 6$cm

$\therefore$ Radius of circle A $= 6 - 4 = 2$cm

$\therefore$ Radius of Circle B $= 6 - 3 = 3$cm

$\therefore$ Radius of Circle C $= 6 - 5 = 1$ cm

103.



$\Rightarrow PT^2 = PO^2 - OT^2$

$= 10^2 - 6^2$

$\therefore PT = 8$ cm.

$PT^2 = PB \times PC$

$\Rightarrow (8)^2 = 5 \times PC$

$\Rightarrow PC = \frac{64}{5}$

$\therefore BC = \frac{64}{5} - 5 = \frac{64-25}{5} = \frac{39}{5} = 7.8$ cm

104. $\angle EOB = 180° - 150°$

$\Rightarrow \angle EOB = 30°$

$OE = OB =$ radius

$\Rightarrow \angle OEB + \angle OBE + \angle EOB = 180°$

$\Rightarrow \angle OEB = \frac{180° - 30°}{2} = \frac{150°}{2} = 75°$

$\Rightarrow \angle OBE = 75°$

$\therefore \angle CBE = 180° - 75° = 105°$

105. R_1 = Radius of larger Circle

R_2 = Radius of Smaller Circle

$\Rightarrow 2\pi R_1 + 2\pi R_2 = 176$

$\Rightarrow 2\pi (R_1 + R_2) = 176$

$\Rightarrow R_1 + R_2 = 28$ .... (1)

$\Rightarrow R_1^2 + R_2^2 = 400$

$\Rightarrow R_1^2 + R_2^2 = (20)^2$

Squaring eq. 1

$\Rightarrow R_1^2 + R_2^2 + 2R_1R_2 = (28)^2$

$\Rightarrow (20)^2 + 2R_1R_2 = (28)^2$

$\Rightarrow 2R_1R_2 = (28)^2 - (20)^2$

$\Rightarrow R_1R_2 = \frac{8 \times 40}{2} = 192$

$\Rightarrow R_2 = \frac{192}{R_1}$

Put the value of R_2 in eq 1

$\Rightarrow R_1 + \frac{192}{R_1} = 28$

$\Rightarrow R_1^2 - 28R_1 + 192 = 0$

$\Rightarrow R_1^2 - 16R_1 - 12R_1 + 192 = 0$

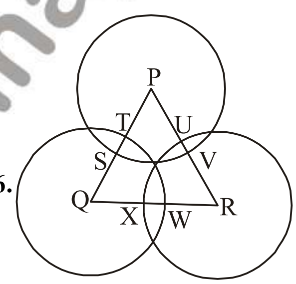
$\Rightarrow R_1(R_1 - 16) - 12(R_1 - 16) = 0$

$\Rightarrow R_1 = 16, 12$

$\Rightarrow R_2 = \frac{192}{12} = 16$

$\therefore \frac{R_1}{R_2} = \frac{16}{12} = \frac{4}{3}$

106.



Radius PS = 24 cm.

ST = 4 cm.

$\Rightarrow PT = 24 - 4 = 20$ cm.

$\therefore$ Similarly, QS = 20 cm

Radius QW = 24 cm.

QX = QW – XW

= 24 – 10

= 14cm

$\therefore$ Similarly, RW = 14 cm

Radius RV = 24 cm.

RV = RU – UV = 24 – 7 = 17 cm

$\therefore$ Similarly PU = 17 cm.

$\therefore$ Perimeter = PT = TS + SQ + QX + XW+ WR+ RV+ VV+ UP

= 20 + 4 + 20 + 14 + 10 + 14 + 17 + 7 + 17 = 123 cm.

107. OE bisect the chord AB

AF = FB = 8 cm

Now, In ΔOFB,

$\Rightarrow r^2 = (r-2)^2 + (8)^2$

$\Rightarrow r = 17$

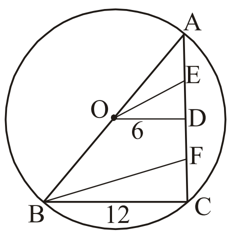
In ΔOFB and in ΔOED

$\Rightarrow \frac{r-2}{8} = \frac{r}{x}$

$\Rightarrow \frac{17-2}{8} = \frac{17}{x}$

$\therefore x = \frac{136}{15}$ cm.

108.



$\frac{AE}{ED} = \frac{OA}{OD} = \frac{2}{1}$

$\Rightarrow OD = 6$

$\frac{OA}{OP} = \frac{OA}{AC}$

$\Rightarrow AC = 12$

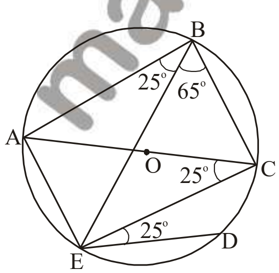
Now, In ΔAOD ,

$\Rightarrow AD = \sqrt{144-36} = \sqrt{108}$

$\Rightarrow FC = \frac{\sqrt{108}}{2}$

$\therefore AF = \sqrt{144 + \frac{108}{4}} = \sqrt{171}$

109.



AC is diameter

So, $\angle ABC = 90°$

$\Rightarrow \angle EBA = 90° - 65° = 25°$

AE is chord so,

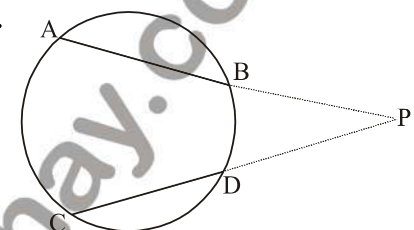
$\Rightarrow \angle ABE = \angle ACE = 25°$

AC || ED

$\therefore$ So, $\angle ACE = \angle CED = 25°$

110. Same as questions 39

111.



PA = 8 cm

PD = 4 cm

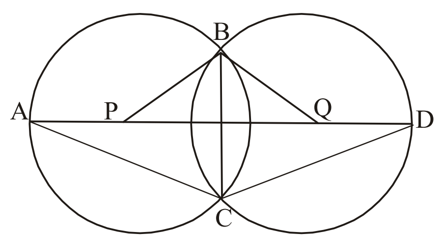
$\Rightarrow PB \times 8 = 4 \times (4+3)$

$\Rightarrow PB = \frac{28}{8} = 3.5$

$\therefore AB = PA - PB = 8 - 3.5$

$= 4.5$

112.



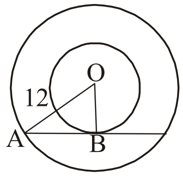
$\Rightarrow \angle BCA = \frac{1}{2}\angle APB = \frac{1}{2}\times 130° = 65°$

$\Rightarrow \angle BCD = 180° - 65° = 115°$

$\Rightarrow \angle BQD = 2 \times \angle BCD = 2 \times 115 = 230$

$\therefore \angle BQD = 360° - 230° = 130°$

113.



$OB = OC = 3$ cm

$OA = 12$

$\angle ABO = 90°$

$\Rightarrow AB = \sqrt{12^2 - 3^2}$

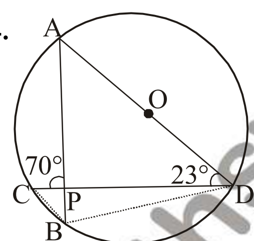
$\Rightarrow AB = \sqrt{135} = 3\sqrt{15}$

$\Rightarrow$ Area of OAB $= \frac{1}{2} \times OB \times AB$

$= \frac{1}{2} \times 3 \times 3\sqrt{15}$

$\therefore$ Area of $\Delta OAB = \frac{9\sqrt{15}}{2}$ cm^2

114.



$\angle APC = 70°$

$\Rightarrow \angle APC = \angle DPB$

$\Rightarrow \angle APD = 130° - 70°$

$\Rightarrow \angle APD = 110°$

$\Rightarrow \angle APD = \angle BPC = 110°$

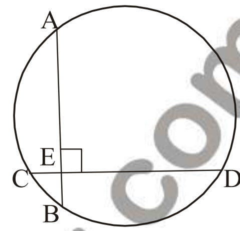
$\Rightarrow \angle ADC = \angle ABC = 23°$

Now, In ΔBPC,

$\Rightarrow \angle BCP = 180° - 23° - 110° = 47°$

$\therefore \angle BCD = 47°$

115.



$AE \times EB = DE \times EC$

$\Rightarrow EC = \frac{D \times 6}{3} = 4$

$\Rightarrow$ Diameter $= \sqrt{7^2 + 4^2}$

$= \sqrt{49 + 16}$

$= \sqrt{65}$ cm

116. Let $OS = TR = r$

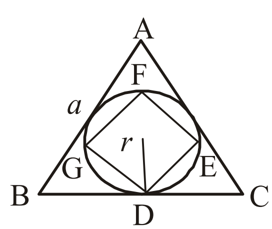
$OS = OT = TR = r$

Now, In Δ OSR,

$\Rightarrow SR = \Delta\sqrt{(2r)^2 - r^2} = \sqrt{3}r$

$\therefore \frac{\text{Area of square}}{\text{Area of Circle}} = \frac{3r^2}{\pi r^2} = \frac{3}{\pi}$

117.



$\Rightarrow$ Area of equilateral $\Delta = \dfrac{\sqrt{3}a^2}{4}$

$\Rightarrow$ Inradius $r = \dfrac{a}{2\sqrt{3}}$

$\Rightarrow$ ED is diagonal of square = $2 \times \dfrac{a}{2\sqrt{3}} = \dfrac{a}{\sqrt{3}}$

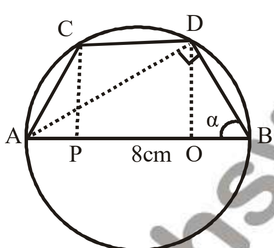
$\Rightarrow$ Diagonal $\sqrt{2}x = \dfrac{a}{\sqrt{3}}$

$\Rightarrow x = \dfrac{a}{\sqrt{6}}$

$\Rightarrow$ Area of square = $= \left(\dfrac{a}{\sqrt{6}}\right)^2 = \dfrac{a^2}{6}$

$\therefore \dfrac{\text{Area of Triangle}}{\text{Area of Square}} = \dfrac{\frac{\sqrt{3}a^2}{4}}{\frac{a^2}{6}} = \dfrac{3\sqrt{3}}{2}$

118.



In ΔODB,

$\Rightarrow \text{Cos}\alpha = \dfrac{OB}{2}$ .... (1)

Now, In ΔADB ,

$\Rightarrow \text{Cos}\alpha = \dfrac{2}{8}$ .... (2)

From eq. 1 & 2

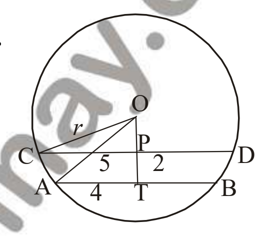
$\Rightarrow \dfrac{OB}{2} = \dfrac{2}{8}$

$\Rightarrow OB = \dfrac{1}{2}$

$\Rightarrow OB = AP = \dfrac{1}{2}$

$\Rightarrow PO = CD = 8 - \dfrac{1}{2} - \dfrac{1}{2} = 7$ cm

119.



$\sqrt{r^2 - 16}$ - $\sqrt{r^2 - 25} = (2)^2$

$\sqrt{r^2 - 16} = \sqrt{r^2 - 25} + 4$

Squaring on both side

$\Rightarrow r^2 - 16 = r^2 - 25 + 16 + \sqrt{r^2 - 25}$

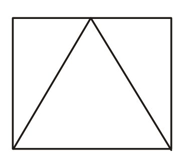
$\Rightarrow 5 = 4\sqrt{r^2 - 25}$

$\Rightarrow r^2 - 25 = \dfrac{25}{16}$

$\Rightarrow r^2 = \dfrac{25}{16} + 25 = \dfrac{425}{16}$

$\Rightarrow r = \dfrac{5\sqrt{17}}{4}$ cm

120.



$\Rightarrow$ Area of square – Area eq, $\Delta = \frac{1}{4}\text{cm}^2$

$\Rightarrow a^2 - \frac{\sqrt{3}a^2}{4} = \frac{1}{4}$

$\Rightarrow \frac{4a^2 - \sqrt{3}a^2}{4} = \frac{1}{4}$

$\Rightarrow a^2 = \frac{1}{4-\sqrt{3}}$

$\therefore a = \left(4-\sqrt{3}\right)^{-1/2}$

121. Radius PB = 25

$\Rightarrow$ PA = PB – AB = 25-6 = 19 cm

Similarly RB = 19 cm

Radius RD = 25

$\Rightarrow$ RC = RD – CD = 25 – 12 = 13cm

Similarly QD = 13cm

Radius QF = 25 cm.

$\Rightarrow$ QE = QF – EF = 25 – 15 = 10cm

Similarly PF = 10cm

$\therefore$ Perimeter of Δ =

19+6+19+13+12+13+10+15+10

= 117cm

122. Diameter AB = 6+18 = 24 cm.

Radius R = 12cm.

Area of semicircle = $\frac{1}{2}\pi R^2 = \frac{1}{2}\pi(12)^2 = 72\pi$

$\Rightarrow$ Radius OA = r_1 = 3cm

$\Rightarrow$ Area = $\frac{1}{2}\pi r_1{}^2 = \frac{1}{2}\pi r(3)^2 = \frac{9}{2}\pi$

$\Rightarrow$ Radius O^1C (r_2) = 9cm

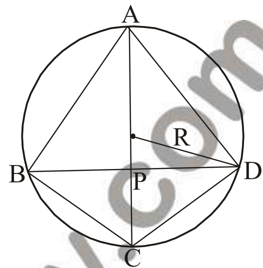
$\Rightarrow$ Area = $\frac{1}{2}\pi r_2{}^2 = \frac{1}{2}\pi(9)^2 = \frac{81}{2}\pi$

$\therefore$ Area of shaded portion = $72\pi - \frac{9}{2}\pi - \frac{81}{2}\pi$

$= \frac{144\pi - 90\pi}{2} = \frac{54\pi}{2}$

$= 27\pi$ cm²

123.



AB = BD = AD = $a = \sqrt{3}R$

$\Rightarrow$ PD = $\frac{a}{2}$

$\Rightarrow$ PC = R-r = $\frac{a}{\sqrt{3}}$

$\Rightarrow$ CD = $\sqrt{\frac{a^2}{4} + \frac{a^2}{12}} = \frac{a}{\sqrt{3}}$

$\Rightarrow$ CD = BC

Perimeter of ABCD

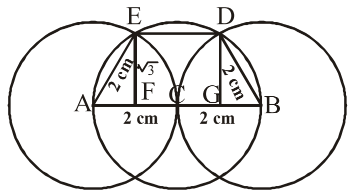
$= 2a + \frac{2a}{\sqrt{3}}$

$= 2\sqrt{3}R + \frac{2\sqrt{3}R}{\sqrt{3}} = 2(\sqrt{3}+1)R$

$\therefore \frac{\text{Perimeter of ABCD}}{\text{Perimeter of Circle}} = \frac{2\left(\sqrt{3}+1\right)R}{2\pi R}$

$\therefore \sqrt{3}+1 : \pi$

124.



ABDE is a trapezium

$DE = \frac{1}{2} AB = 2\,cm$

$EA = DB = 2\,cm$

$FG = 2\,cm$

$\therefore AF = GB = 1cm$

In ΔEAF,

$EA^2 = EF^2 + AF^2$

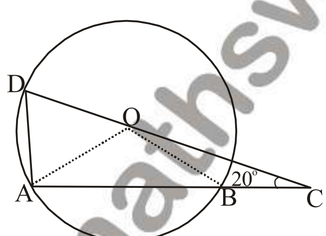
$4 = EF^2 + 1$

$EF = \sqrt{3}$

Area of trapezium ABDE $= \frac{1}{2} \times (2+4) \times \sqrt{3}$

$= 3\sqrt{3}\ c\,m^2$

125.



Construt line OA & OB.

$\Rightarrow OD = OB$ [Radius]

$\Rightarrow OD = BC$ [given]

SO, $OB = BC$

$\Rightarrow \angle BOC = \angle BCO = 20°$

$\Rightarrow \angle OBA = 40°$ [$\therefore$ Sum of two opposite internal angle is equal to exterior angle]

$\therefore OB = OA$ [Radius]

$\angle OAB = \angle OBA = 40°$

In ΔOAB,

$\Rightarrow \angle AOB + \angle OAB + \angle OBA = 180°$

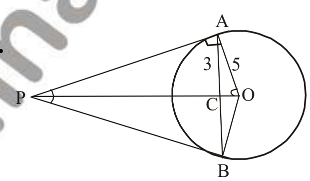
$\Rightarrow \angle AOB = 180° - 40° - 40° = 100°$

Now, $\angle AOD + \angle AOB + \angle BOC = 180°$ [Linear pair]

$\Rightarrow \angle AOD + 100° + 20° = 180°$

$\therefore \angle AOD = 60°$

126.



In ΔOCA,

$\Rightarrow Cos\alpha = \frac{OC}{OA} = \frac{4}{5}$ .... (1)

In ΔOAP,

$\Rightarrow Cos\alpha = \frac{OA}{OP} = \frac{5}{OP}$ .... (2)

From 1 & 2 eq.

$\Rightarrow \frac{4}{5} = \frac{5}{OP}$

$\therefore OP = \frac{25}{4}\,cm$

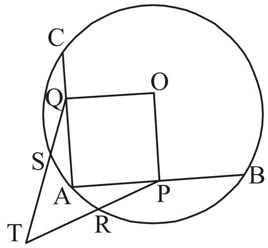
127. In ΔQPR,

$\angle QPR = 90°$

$\Rightarrow QR = \sqrt{a^2 + 4^2}$

$\Rightarrow OP = \frac{QR}{2} = \frac{\sqrt{16+a^2}}{2}$

128.



$\Rightarrow \angle QOP + \angle QAP = 180°$

$\Rightarrow \angle QOP = 180° - 32° = 148°$

$\Rightarrow \angle QOP = \angle SOR = 2\angle STR$

$\Rightarrow \angle STR = \frac{1}{2}\angle QOP = \frac{1}{2} \times 148° = 74°$

$\Rightarrow \angle STR = 74°$

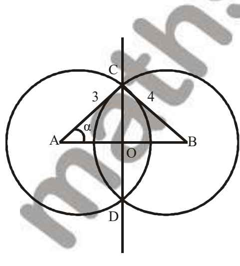
$\therefore \angle RTS = 74°$

129. BR = BQ = 4 cm (equal tangents of circle)

CP = CQ = 11cm

BC = CQ – BQ = 11 – 4 = 7 cm

130.



$AB = \sqrt{3^2+4^2} = 5$ cm

In ΔACB,

$\Rightarrow \sin\alpha = \frac{4}{5}$ ...... (1)

In ΔAOC,

$\Rightarrow \sin\alpha = \frac{OC}{3}$ ....(2)

$\frac{4}{5} = \frac{OC}{3} \Rightarrow OC = \frac{12}{5}$

$\therefore CD = \frac{24}{5} = 4.8$ cm.

Method – 2

AC and BC are tangent.

Angle C = 90°

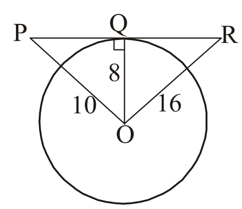
$AB = \sqrt{3^2+4^2} = 5$ cm

Area of $\Delta ABC = \frac{1}{2} \times AC \times BC = \frac{1}{2} \times CO \times AB$

$3 \times 4 = CO \times 5 \Rightarrow CO = 2.4$ cm

$CD = 2 \times 2.4 = 4.8$ cm

131



In ΔOQP,

$\Rightarrow PQ = \sqrt{(10)^2 - (8)^2} = 6$

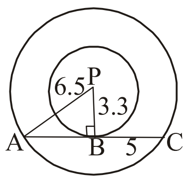
In ΔOQR,

$\Rightarrow QR = \sqrt{(16)^2 - (8)^2} = 8\sqrt{3} = 13.856$

$\left[\sqrt{3} = 1.732\right]$

$\therefore PR = 6 + 13.8 = 19.8$ cm

132.



$AP = 6.5$ cm $\qquad PB = 3.3$ cm

$AB = \sqrt{(6.5)^2 - (3.3)^2}$

$= \sqrt{(6.5 + 3.3)(6.5 - 3.3)}$

$= \sqrt{9.8 \times 3.2} = 5.6$cm

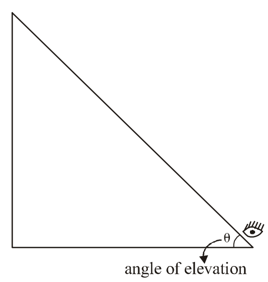
$\therefore AC = 2 \times AB = 2 \times 5.6 = 11.2$ cm

☆☆☆☆☆

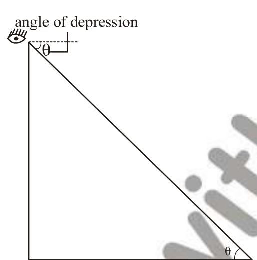
5 Height and Distance

Basic concepts–

Point (i) When we see above then the angle with horizontal is called angle of elevation.

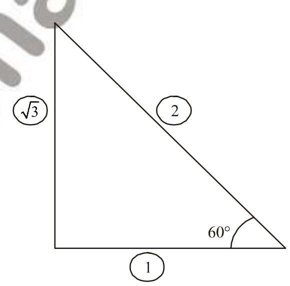


Point (ii) When we see below then the angle with horizontal is called angle of depression.

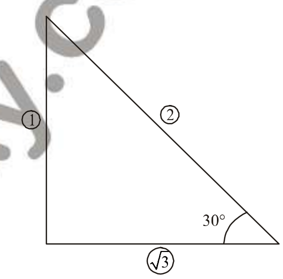


Point (iii) In maximum questions, angles are 30°, 45° and 60° so we will remember ratio of sides in the case of these angles.

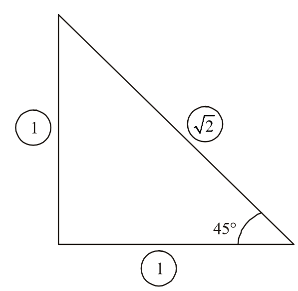
(A) If angle is 60° then we will take perpendicular as $\sqrt{3}$ unit, base as 1 unit and hypotenuse as 2 unit.



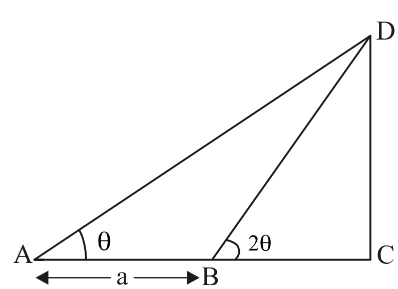
(B) If angle is 30° then we will take prependicular as 1 unit, base as $\sqrt{3}$ unit and hypotenuse as 2 unit.



(C) If angle is 45° then we will take prependicular as 1 unit, base as 1 unit and hypotenuse as $\sqrt{2}$ unit.



Point (iv) If on moving distance *a,* angle becomes double then



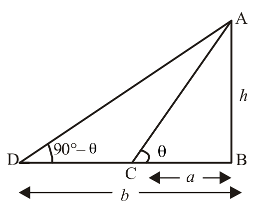
$\angle ADB = 2\theta - \theta = \theta$

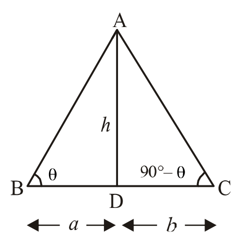
(2θ is exterior angle of ΔADB)

In ΔABD,

AB = BD = a (sides of opposite equal angles) then we can find height DC after using only one triangle BCD ($\because$ BD $= a$ is known)

Point (v) The angles of elevation of the top of a tower from the points C and D at distances of 'a' and 'b' respectively from the base of the tower and in the same straight line with it are complementary. The height of the tower is





$$\tan\theta = \frac{h}{b} \quad \text{...(i)}$$

$$\tan(90 - \theta) = \cot\theta = \frac{h}{a} \quad \text{...(ii)}$$

multiply both equation

$$\tan\theta . \cot\theta = \frac{h^2}{ab}$$

$$h^2 = ab \Rightarrow h = \sqrt{ab}$$

Exercise -1

1. The upper part of a tree broken by the wind makes an angle of 30° with the ground and the distance from the root to the point where the top of the tree touches the ground is 10m. What was the height of the tree?

(a) $10\sqrt{3}$ m (b) $10/\sqrt{3}$ m

(c) $20\sqrt{3}$ m (d) None of these

2. A tower stands at the end of a straight road. The angles of elevation of the top of the tower from two points on the road 500m apart are 45° and 60° respectively. Find the height of the tower.

(a) $\frac{500\sqrt{3}}{\sqrt{3}-1}$ m (b) $5000\sqrt{3}$ m

(c) $\frac{500\sqrt{3}}{\sqrt{3}+1}$ m. (d) None of these

3. The shadow of a tower standing on a level plane is found to be 50 m longer when sun's altitude is 30° than when it is 60°. Find the height of the tower.

(a) $20\sqrt{2}$ m (b) $25\sqrt{2}$ m

(c) $25\sqrt{3}$ m (d) $20\sqrt{3}$ m

4. The height of a tower is 100 m. When the angle of elevation of the sun changes from 30° to 45°, the shadow of the tower becomes x mertres less. The value of x is :

(a) 100 m (b) $100(\sqrt{3}-1)$m

(c) $\frac{100\sqrt{3}-1}{\sqrt{3}}$ m (d) $\frac{100}{\sqrt{3}}$ m

5. A electric pole 20 m high stands upright on the ground with the help of steel wire to its top and affixed on the ground. If the steel wire makes 60° with the horizontal ground, find the length of the steel wire.

(a) $20\sqrt{3}$ m (b) $40\sqrt{3}$ m

(c) $\frac{20}{\sqrt{3}}$ m (d) $\frac{40}{\sqrt{3}}$ m

6. From the top of a cliff 25 m high. The angle of elevation of a tower is found to be equal to the angle of depression of the foot of the tower. Find the height of the tower.

(a) 40 m (b) 48 m

(c) 50 m (d) 52 m

7. Angle of depression from the top of a light house of two boats are 45° and 30° due east which are 60 m apart. The height of the light house is:

(a) $60\sqrt{3}$ (b) $30\left(\sqrt{3}-1\right)$

(c) $30\left(\sqrt{3}+1\right)$ (d) None of these

8. The angle of elevation of the top of a hill from each of the vertices A, B, C of a horizontal triangle is α. The height of the hill is:

(a) $b \tan\alpha.\text{cosecB}$

(b) $\frac{a}{2}\tan\alpha.\text{cosecA}$

(c) $c\tan\alpha.\text{cosecC}$

(d) None of these

9. The angle of elevation of the top of a TV tower from three points A, B, C in a straight line through the foot of the tower are $\alpha, 2\alpha \,\&\, 3\alpha$ respectively. If AB = a the height of the tower is:

(a) $a\tan\alpha$ (b) $a\sin\alpha$

(c) $a\sin 2\alpha$ (d) $a\sin 3\alpha$

10. The angle of elevation of the top of an unfinished tower at a point distance 120 m from its base is 45°. If the elevation of the top at the same point is to 60°, the tower must be raised to a height.

(a) $120\left(\sqrt{3}+1\right)$m

(b) $120\left(\sqrt{3}-1\right)$m

(c) $10\left(\sqrt{3}+1\right)$m

(d) None of these

11. A person walking along a straight road towards a hill observes at two points, distance $\sqrt{3}$ km, the angles of elevation of the hill to be 30° and 60°. The height of the hill is:

(a) $\frac{3}{2}$km (b) $\sqrt{\frac{2}{3}}$ km

(c) $\frac{\sqrt{3}+1}{2}$km (d) $\sqrt{3}$ km

12. The tops of two poles of height 20 m and 14 m are connected by a wire. If the wire makes an angle 30° with the horizontal, then the length of the wire is :

(a) 12 m (b) 10 m

(c) 8 m (d) None of these

13. A person observe from a tower that a car is moving towards tower. If car takes 36 minute to change the angle of the deprerssion 30º to 60º. In how many minutes will the car reach at tower ?

(a) 18 min (b) 12 min

(c) 36 min (d) None of these

14. The top of a 15 m high tower makes an angle of elevation of 60° with the bottom of an electric pole and an angle of elevation of 30° with the top of the pole. What is the height of the electric pole?

(a) 5 m (b) 8 m

(c) 10 m (d) 20 m

15. A man is watching from the top of a tower a boat speeding away from the tower. The boat makes an angle of depression of 45° with the man's eye when at a distance of 60 m from the tower. After 5 sec. the angle of depression becomes 30°. What is the speed of the boat, assuming it is running in still water?

(a) $12\left(\sqrt{3}-1\right)$ m/s

(b) $40\left(\sqrt{3}-1\right)$ m/s

(c) $60\left(\sqrt{3}-1\right)$ m/s

(d) $10\left(\sqrt{3}-1\right)$ m/s

16. From the bridge on the river, 15 m high, the angle of depression of a boat is 30°. If the speed of the boat be 6 km/hr, then the time taken by the boat to reach just below the bridge will be:

(a) $9\sqrt{3}$ sec. (b) $19\sqrt{3}$ sec.

(c) $3\sqrt{3}$ sec. (d) None of these

17. The angle of elevation of the top of a tower from the top and bottom of a building of height '*a*' are 30° and 45°, respectively. If the tower and the building stand at the same level, the height of the tower is:

(a) $\sqrt{3}\,a$ (b) $\left(\sqrt{3}-1\right)a$

(c) $\frac{\left(3+\sqrt{3}\right)}{2}a$ (d) $\left(\sqrt{3}+1\right)a$

18. From the top of a house 32 m high, the angle of elevation of the top of a tower is 45° and the angle of depression of the foot of the tower is 30°. The distance of the tower from the house is:

(a) $30\sqrt{3}$ m (b) $32\sqrt{3}$ m

(c) $35\sqrt{3}$ m (d) None of these

19. Length of the shadow of a person is x when the angle of elevation of the sun is 45°. If the length of the shadow increases by $\left(\sqrt{3}-1\right)x$, then the angle of elevation of the sun should become:

(a) 30° (b) 60°

(c) 45° (d) 75°

20. The angle of elevation of an aeroplane from a point on the ground is 45°. After 15 seconds flight, the elevation changes to 30°. If the aeroplane is flying at a height of 3000 m, the speed of the plane in metre per second is :

(a) $200\left(\sqrt{3}-1\right)$ m/s

(b) $100\left(\sqrt{3}-1\right)$ m/s

(c) $400\left(\sqrt{3}-1\right)$ m/s

(d) $50\left(\sqrt{3}-1\right)$ m/s

21. A balloon of radius r makes an angle α at the eye of an observer and the angle of elevation β. The height of its centre from the ground level is given by:

(a) $r\cos\frac{\beta}{2}.\sec\alpha$ (b) $r\cos\beta.\sec\frac{\alpha}{2}$

(c) $r\sin\frac{\beta}{2}.\operatorname{cosec}\alpha$ (d) $r\sin\beta.\operatorname{cosec}\frac{\alpha}{2}$

22. A landmark on a river bank is observed from two points X and Y on the opposite bank of the river. The lines of sight make equal angles with the bank of the river. If XY = 1 km, then the width of the river is :

(a) 3/2 km (b) 1/2 km

(c) $\frac{3\sqrt{2}}{2}$ km (d) $\frac{2\sqrt{3}}{2}$ km

23. Two men are on the opposite side of a tower. They measure the angle of elevation of the top of the tower as 30° and 60° respectively. If the height of the tower is 30 m, find the distance between the men.

(a) $20\sqrt{3}$ m (b) $20/\sqrt{3}$ m

(c) $40\sqrt{3}$ m (d) $40/\sqrt{3}$ m

24. An artist climbs on a rops stretched from the top of a pole and fixed at the ground. The height of the pole is 10 m and the angle made by the rope with the ground is 30°. Calculate the length of the rope.

(a) 10 m (b) 20 m

(c) 30 m (d) 40 m

25. The length of a string between a kite and a point on the ground is 90 m. The string makes an angle of 60° with the level ground. Assuming that there is no slack in the string, then the height of the kite is:

(a) $45\sqrt{3}$ m (b) $5\sqrt{3}$ m

(c) $50\sqrt{3}$ m (d) $50/\sqrt{3}$ m

26. The angles of elevation of an aeroplane flying vertically above the ground as observed from two consecutive stones 1 km apart are 45° and 60°. The height of the aeroplane above the ground in km is:

(a) $\frac{\sqrt{3}+1}{2}$ (b) $\frac{3+\sqrt{3}}{2}$

(c) $3+\sqrt{3}$ (d) $\sqrt{3}+1$

27. From the top of a light house, the angle of depression of two ships on the opposite side of it are observed to be $\alpha \& \beta$. If the height of the light house be h m and the line joining the ships passes through the foot of the light house, the distance between the ships is:

(a) $\frac{h(\cot\alpha+\cot\beta)}{\cot\alpha\cot\beta}$

(b) $\frac{h(\tan\alpha+\tan\beta)}{\tan\alpha\tan\beta}$

(c) $h(\tan\alpha+\tan\beta)$

(d) $\frac{h\tan\alpha+\tan\beta}{\tan\alpha+\tan\beta}$

28. The angles of elevotion of the top of a tower standing on a horizontal plane from two points on a line passing through the foot of the tower at a distance 9 ft. and 16 ft. respectively are opposite complementary angles. The height of the tower is :

(a) 9 fit (b) 01 fit

(c) 16 fit (d) 033 fit

29. A boy standing in the middle of a field, observes a flying bird in the north at an angles of elevation of 30° and after 2 minutes, he observes the same bird in the south at an angle of elevation of 60°. If the bird flies all along in a straight line at a height of $50\sqrt{3}$ m, then its speed in km/h is:

(a) 4.5 (b) 3

(c) 9 (d) 6

30. The angles of elevation of the top of a tower from two points A and B lying on the horizontal through the foot of the tower are respectively 15° and 30°. If A and B are on the same side of the tower and AB = 48 metre, then the height of the tower is ($\tan 15° = 2-\sqrt{3}$)

(a) $24\sqrt{3}$ m (b) 24 m

(c) $28\sqrt{3}$ m (d) 96 m

31. Two poles of equal heights are standing opposite to each other on either side of a road which is 100 m wide. From a point between then on road, angles of elevation of their tops are 30° and 60°. The height of each pole (in metre) is:

(a) $25\sqrt{3}$ (b) $20\sqrt{3}$

(c) $28\sqrt{3}$ (d) $30\sqrt{3}$

32. The angles of elevation of the top of building and top of the chimney on the roof of building from a point on the ground are x and 45° respectively. The height of building is h metre. Then the height of the chimney (in m) is:

(a) $h\cot x+h$ (b) $h\cot x-h$

(c) $h\tan x-h$ (d) $h\tan x+h$

33. At a point on a horizontal line through the base of a monument, the angle of elevation of the top of the monument is found to be such that its tangent is $\frac{1}{5}$. On walking 138 metres towards the monument the secant of the angle of elevation is found to be $\frac{\sqrt{193}}{12}$. The height of the monument (in metre) is

(a) 35 (b) 49

(c) 42 (d) 56

34. The angles of elevation of the top of a building from the top and bottom of a tree are x and y respectively. If the height of the tree is h metre, then, in metre, the height of the building is

(a) $\dfrac{h\cot x}{\cot x+\cot y}$ (b) $\dfrac{h\cot y}{\cot x+\cot y}$

(c) $\dfrac{h\cot x}{\cot x-\cot y}$ (d) $\dfrac{h\cot y}{\cot x-\cot y}$

35. A telegraph post is bent at a point above the ground due to storm. Its top just meets the ground at a distance of $8\sqrt{3}$ metres from its foot and makes an angle of 30º, then the height of the post is :

(a) 16 metres (b) 26 metres

(c) 24 metres (d) 10 metres

36. Two posts are x metres apart and the height of one is double that of the other. If from the mid-point of the line joining their feet, an observer finds the angular elevations of their tops to be complementary, then the height (in metres) of the shorter post is

(a) $\dfrac{x}{2\sqrt{2}}$ (b) $\dfrac{x}{4}$

(c) $x\sqrt{2}$ (d) $\dfrac{x}{\sqrt{2}}$

37. A man standing at a point B is watching the top of a tower, which makes an angle of elevation of 30º. The man walks some distance towards the tower and then his angle of elevation of the top of the tower is 60º. If the height of the tower is 30 m, then the distance he moves is

(a) 22m (b) $22\sqrt{3}$ m

(c) 20m (d) $20\sqrt{3}$ m

38. There are two temples, one on each bank of a river, just opposite to each other. One temple is 54 m high. From the top of this temple, the angles of depression of the top and the foot of the other temple are 30º and 60º respectively. The length of the temple is :

(a) 18m (b) 36m

(c) $36\sqrt{3}$ m (d) $18\sqrt{3}$ m

39. From a tower 125 metres high, the angles of depression of two objects, which are in horizontal line through the base of the tower, are 45º and 30º and they are on the same side of the tower. The distance (in metres) between the objects is

(a) $125\sqrt{3}$ (b) $125\left(\sqrt{3}-1\right)$

(c) $125\left(\sqrt{3}-1\right)$ (d) $125\left(\sqrt{3}+1\right)$

40. The distance between two vertical poles is 60 m. The height of one of the poles is double the height of the other. The angles of elevation of the top of the poles from the middle point of the line segment joining their feet are complementary to each other.Then height of both the pole

(a) 10 m and 20 m

(b) 20 m and 40 m

(c) 20.9 m and 41.8 m

(d) $15\sqrt{2}$ m and $30\sqrt{2}$ m

41. At the foot of a mountain the elevation of its summit is 45º. After ascending 2 km towards the mountain, upon an incline of 30º, the elevation changes to 60º. Find the height of the mountain.

(a) $\sqrt{3}+1$ (b) $\sqrt{3}+2$

(c) $2\sqrt{3}+2$ (d) $\sqrt{3}$

42. During the super cyclone in Orissa when wind blew a coconut tree touched the top of a pole 9 ft. high. The distance of the pole from the tree 12 ft. After some time the tree brokedown and touched the bottom of another pole which is $5\sqrt{3}$ ft. from the tree. Find the angle from by the broken part of tree with the ground.

(a) 30º (b) 20º
(c) 60º (d) 45º

43. The angle of elevation of the top of a TV tower from three points A, B, C in a straight line through the foot of the tower are α, 2α, 3α, respectively. If AB = a and BC = b then height of the tower is

(a) $\frac{a}{2b}\sqrt{(a-b)(3b+a)}$

(b) $\frac{3a}{2b}\sqrt{(a-b)(3b-a)}$

(c) $\frac{a}{2b}\sqrt{(a+b)(3b-a)}$

(d) None of these

44 There are two part of a vertical pole having height more than 100m. Lower part of the pole is one-third of the whole pole. Upper part makes an angle α at a point 40 m far away from the foot of the pole where

$\tan\alpha = \frac{1}{2}$ .Find the height of the tower.

(a) 40 (b) 150
(c) 120 (d) None of these

45. From the top of a hill 200 m high, the angle of depression of the top and the bottom of a tower are observed to be 30º and 60º. The height of the tower is (in m) :

(a) $\frac{400\sqrt{3}}{3}$ (b) $166\frac{2}{3}$

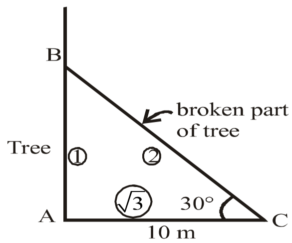
(c) $133\frac{1}{3}$ (d) $200\sqrt{3}$

Answer

1.	(a)	2.	(a)	3.	(c)	4.	(b)	5.	(d)	6.	(c)	7.	(c)	8.	(b)	9.	(c)
10.	(b)	11.	(a)	12.	(a)	13.	(a)	14.	(d)	15.	(a)	16.	(a)	17.	(c)	18.	(b)
19.	(a)	20.	(a)	21.	(d)	22.	(b)	23.	(c)	24.	(b)	25.	(a)	26.	(b)	27.	(b)
28.	(b)	29.	(d)	30.	(b)	31.	(a)	32.	(b)	33.	(c)	34.	(c)	35.	(a)	36.	(a)
37.	(d)	38.	(b)	39.	(b)	40.	(d)	41.	(a)	42.	(a)	43.	(c)	44.	(c)	45.	(c)

Solution & Hints

Sol.1



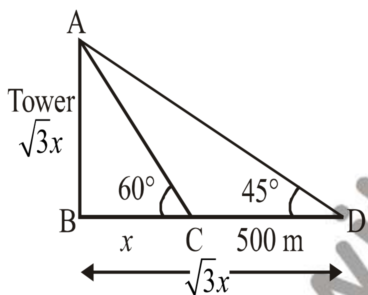
Total length of tree = 1+2 = 3unit

Value of $\sqrt{3}$ unit → 10m

Value of 1 unit → $\dfrac{10}{\sqrt{3}}$m

Then value of 3 unit → $\dfrac{30}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} = 10\sqrt{3}$m

Sol.2



$AB = BD$ ($\angle A = \angle D = 45°$)

Value of $(\sqrt{3}x - x) \rightarrow 500$m

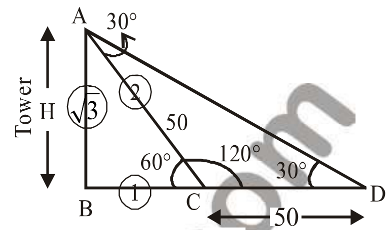
$x(\sqrt{3}-1) = 500$

$x = \dfrac{500}{(\sqrt{3}-1)}$

Then, height of the tower = value of

$\sqrt{3}x = \dfrac{500\sqrt{3}}{\sqrt{3}-1}$ m

Sol.3



In Δ ACD

$\angle ACD = 120°$

[$180° - 60° = 120°$]

So,

$\angle CAD = 180° - 120° - 30° = 30°$

$\angle CAD = CDA = 30°$

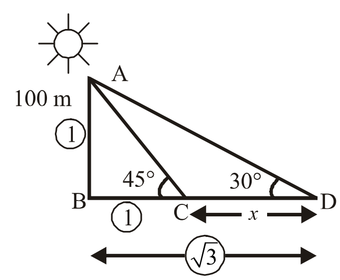
So $DC = AC = 50$m

Value of 2unit $\rightarrow$ 50m

Value of 1unit $\rightarrow$ 25m

Value of $\sqrt{3}$ unit $\rightarrow 25\sqrt{3}$ = Height of Tower

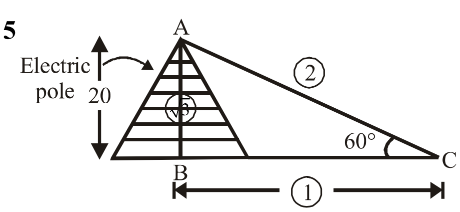
Sol.4



Value of 1 unit → 100m

Value of $(\sqrt{3}-1)$unit → $100(\sqrt{3}-1) = x$

Sol. 5

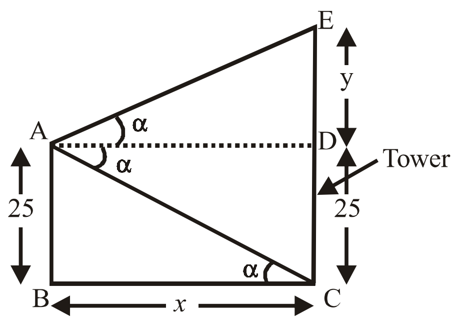


Value of $\sqrt{3}$ unit $\rightarrow$ 20m

Value of 1 unit $\rightarrow \dfrac{20}{\sqrt{3}}$

Value of 2 unit $\rightarrow \dfrac{20}{\sqrt{3}} \times 2 = \dfrac{40}{\sqrt{3}}$ = length of wire

Sol.6



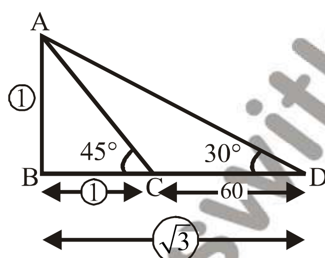
In ΔADE and ΔACD

AD is angle bisector and $AD \perp EC$ so ACE is an isoceles triangle

hence, CD = DE = 25m

Height of tower = 25 + 25 = 50m

Sol.7



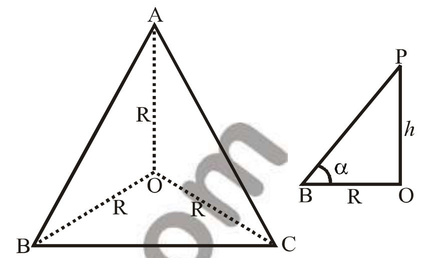
Value of $\left(\sqrt{3}-1\right)$ unit $\rightarrow$ 60m

Value of 1 unit $\rightarrow \dfrac{60}{\sqrt{3}-1}$ (Rationalization)

Height $= \dfrac{60}{\sqrt{3}-1} \times \dfrac{\sqrt{3}+1}{\sqrt{3}+1} = \dfrac{60(\sqrt{3}+1)}{2}$

height of tower $= 30\left(\sqrt{3}+1\right)$ m.

Sol.8



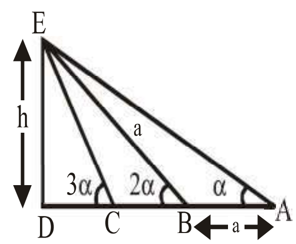
tower will be on circum centre

In triangle BOP,

$\tan\alpha = \dfrac{h}{R} \Rightarrow h = R\tan\alpha = \dfrac{a}{2}\operatorname{cosec}A \tan\alpha$

$\left(\because R = \dfrac{a}{2\text{SinA}}\right)$

Sol.9



In Δ ABE

$\angle A = \alpha$

$\angle EBC = 2\alpha = \angle BEA + \angle BAE$

$2\alpha = \angle BEA + \angle BAE$

$\alpha = \angle BEA$

In Δ ABE

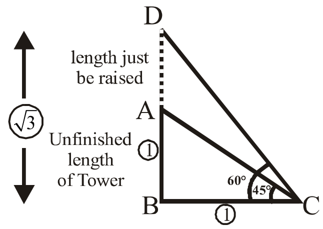
$\angle A = \angle E$ then, AB = BE = a

In Δ EDB

$\sin 2\alpha = \dfrac{h}{a}$

$h = a \sin 2\alpha$

Sol.10

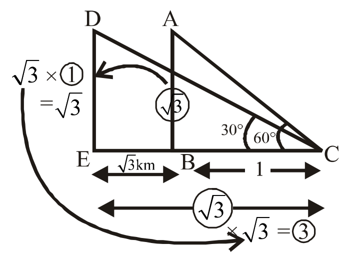


DA = DB – AB

Value of 1 unit → 120m.

Value of $\left(\sqrt{3}-1\right)$ unit → DA → $120\left(\sqrt{3}-1\right)$m

Sol.11



AB = DE

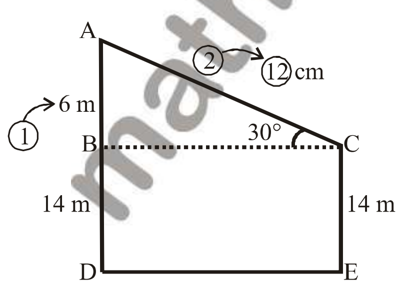
Value of 3–1 = 2 unit = EB ⟶ $\sqrt{3}$ km

Value of 1 unit ⟶ $\frac{\sqrt{3}}{2}$ km

hence,

height of hill = DE = Value of $\sqrt{3}$ unit ⟶ $\frac{3}{2}$ km

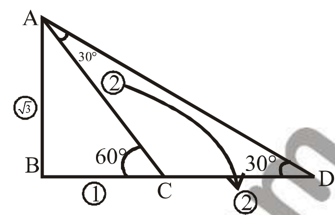
Sol.12



Value of 1 unit → 6m

Value of 2 unit → 12 m = AC

Sol.13



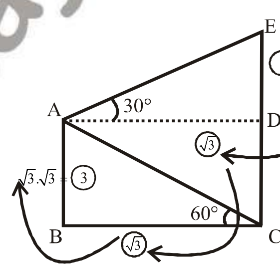
$\angle CAD = 60^\circ - 30^\circ = 30^\circ$

In ΔACD,

CD = AD = 2 unit

If car moves 2 unit (C to D) in 36 minute then Car will take 18 minute to move 1 unit D to

Sol.14



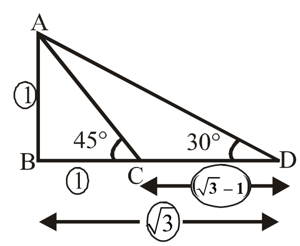
Value of 3 unit → 15m

Value of 1 unit → 5

height of electric pole ED + DC = 1 + 3 = 4m

Value of 4 unit → 4×5 = 20m

Sol.15



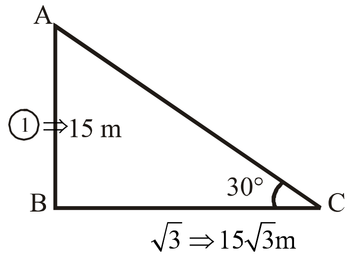
Value of 1 unit → 60m.

Value of $\left(\sqrt{3}-1\right)$ unit → $60\left(\sqrt{3}-1\right)$

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{time}} = \frac{60\left(\sqrt{3}-1\right)}{5}$$

$= 12\left(\sqrt{3}-1\right)$m/s

Sol.16



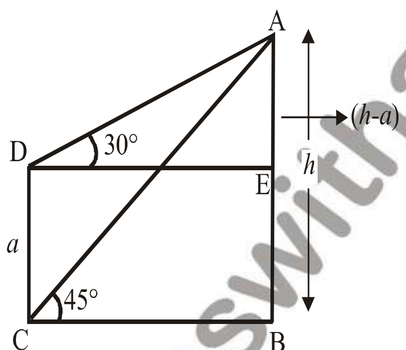
Value of 1 unit → 15m

Value of $\sqrt{3}$ unit → $15\sqrt{3}$m

$$\text{Speed} = 6\ km/h = 6\times\frac{5}{18} = \frac{5}{3}\text{ m/s}$$

$$\text{time taken} = \frac{\text{Distance}}{\text{Speed}} = \frac{15\sqrt{3}}{\frac{5}{3}} = 9\sqrt{3}\text{ sec}$$

Sol.17



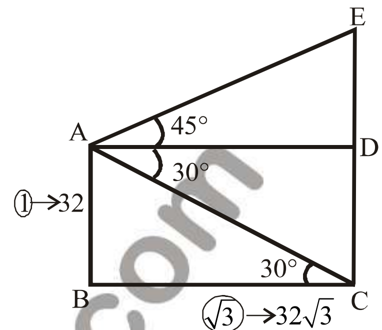
In Δ ABC

$AB = BC = h = DE\ (\angle C = \angle A = 45°)$

In ΔADE , $\tan 30 = \dfrac{1}{\sqrt{3}} = \dfrac{h-a}{h}$

$$h = \frac{a\sqrt{3}}{\sqrt{3}-1} = \frac{a}{2}\left(3+\sqrt{3}\right)$$

Sol.18



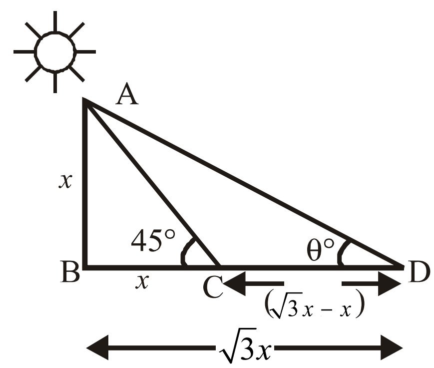
In Δ ABC

Value of 1 unit → 32m

Value of $\sqrt{3}$ unit → $32\sqrt{3}$m

$\Rightarrow BC = AD = 32\sqrt{3}$ m

Sol. 19



In Δ ABC

$AB = BC = x$

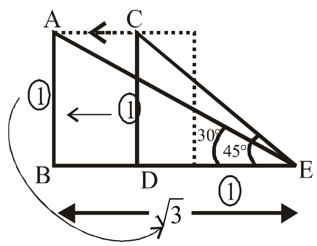
$CD = \sqrt{3}x - x$ **(given)**

then $BD = \sqrt{3}x$

in ΔABD

$\tan\theta = \dfrac{x}{\sqrt{3}x} = \dfrac{1}{\sqrt{3}} \quad \Rightarrow \theta = 30°$

Sol.20



Value of 1 unit $\rightarrow$ 3000 m

from fig. BD $\rightarrow \left(\sqrt{3}-1\right)$unit

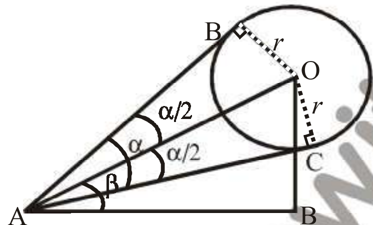
Value of $\left(\sqrt{3}-1\right)$unit $\rightarrow 3000\left(\sqrt{3}-1\right)$

hence Distance travelled by aeroplene = $3000\left(\sqrt{3}-1\right)$m

$$\text{Speed} = \frac{\text{Dis tan ce}}{\text{time}} = \frac{3000\left(\sqrt{3}-1\right)}{15}$$

$= 200\left(\sqrt{3}-1\right)$m / s

Sol.21



In ΔOAC

$$\text{Sin}\frac{\alpha}{2} = \frac{OC}{OA} = \frac{r}{OA}$$

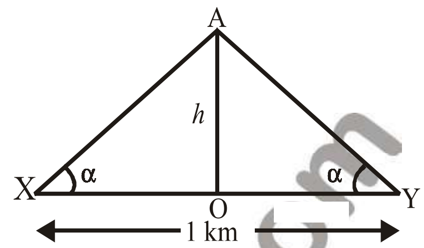
$$OA = r\ \text{cosec}\ \frac{\alpha}{2}$$

In ΔOAB ,

$$\sin\beta = \frac{OB}{OA}$$

height of the balloon = OB = OA $\sin\beta = r\text{cosec}\ \frac{\alpha}{2}\sin\beta$

Sol.22



AO is land mark

Δ XOA $\cong$ Δ AOY

(i) $\angle$X = $\angle$Y given

(ii) So AX = AY

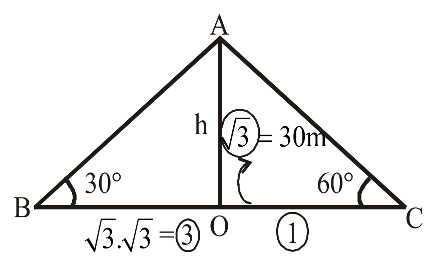
(iii) AO is common in both triangle

From SAS. Both triangle are cogurent.

$\Rightarrow$ O is mid point of XY

so width of river (OY) = $\frac{1}{2}$km.

Sol.23



AO is a tower

In Δ AOB

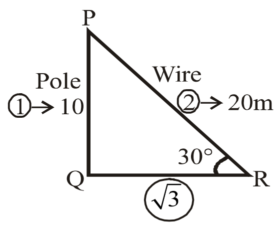
value of $\sqrt{3}$unit $\rightarrow$ 30

value of unit $\rightarrow \frac{30}{\sqrt{3}} = 10\sqrt{3}$

BC = 3+1 = 4 unit

BC = Value of 4unit $\rightarrow 40\sqrt{3}$m

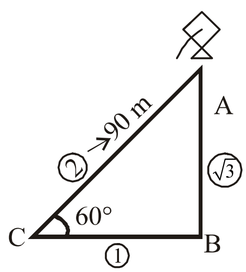
Sol.24



Value of 1unit → 10m

PR = length of wire → Value of 2 unit → 20m

Sol.25.



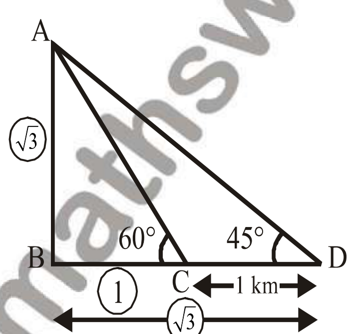
Value of 2 unit ⟶ 90m

value of 1 unit ⟶ 45m

Value $\sqrt{3}$ unit ⟶ $45\sqrt{3}$ m

Heigh of kite from ground = $45\sqrt{3}$ m

Sol.26.



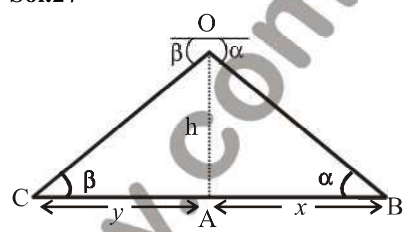
Value of $\left(\sqrt{3}-1\right)$ unit → 1km.

Value of 1unit → $\frac{1}{\sqrt{3}-1}=\frac{\sqrt{3}+1}{2}$ km

Value of $\sqrt{3}$ unit = $\frac{\sqrt{3}\left(\sqrt{3}+1\right)}{2}$

$=\frac{3+\sqrt{3}}{2}=$ height of plane

Sol.27



In $\tan\alpha=\frac{h}{x}\Rightarrow x=\frac{h}{\tan\alpha}$ …(i)

In ΔOAC,

$\tan\beta=\frac{h}{y}\Rightarrow y=\frac{h}{\tan\beta}$ …(ii)

Distance between ships = $x + y$

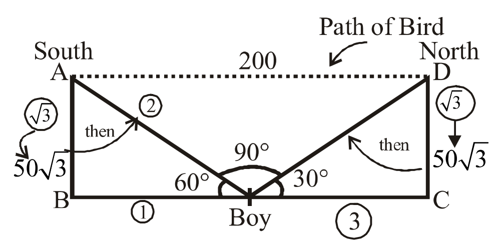
$=h\left(\frac{\tan\beta+\tan\alpha}{\tan\alpha.\tan\beta}\right)$

Sol.28 In this case

height = $h=\sqrt{ab}$

$=\sqrt{9\times16}=12$ ft

Sol.29



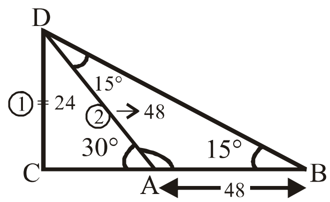
Value of $\sqrt{3}$ unit → $50\sqrt{3}$m

value of 1 unit → 50m

BC → Value of 4 unit = 200 m

Speed $=\frac{200}{2\times60}=\frac{5}{3}$ m/s $=\frac{5}{3}\times\frac{18}{5}=6$ km/h

Sol.30

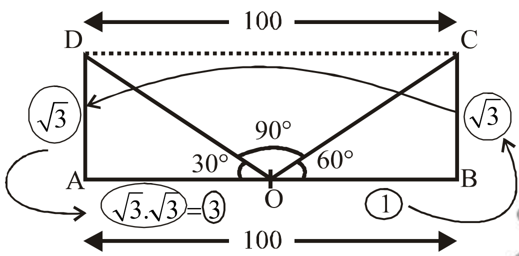


In ΔDCA,

$\angle DAC$ is an exterior angle of ΔDAB

Value of 2 unit → 48m
then value of 1 unit → 24m
hence, Height of tower = 24m.

Sol.31

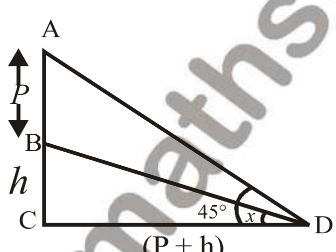


AB → 3+1 = 4 unit
Value of 4 unit → 100m
Value of 1 unit → 25 m

In Δ ADO

length of tower = value of $\sqrt{3}$ unit → $25\sqrt{3}$ m

Sol.32



In Δ ACD

$$\tan 45^\circ = \frac{AC}{CD} = \frac{1}{1} = \frac{P+h}{1}$$

AC = CD = $P + h$

In ΔBCD,

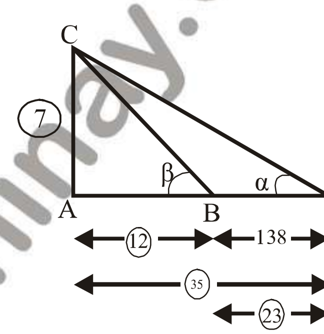
$$\tan x = \frac{h}{P+h}$$

$P \tan x + h \tan x = h$
$P \tan x = h - h \tan x$

$$P = \frac{h - h\tan x}{\tan x} = h\cot x - h$$

AB = Height of chimney = P = h cot$x$ – h

Sol.33



correction in figure

$$\tan\alpha = \frac{1}{5} = \frac{7}{35},$$

$$\tan\beta = \sqrt{\sec^2\beta - 1} = \sqrt{\frac{193}{144} - 1} = \frac{7}{12}$$

hence, in ΔACD,

$\tan\alpha = \dfrac{7}{35}$ then AD = 35 unit, height = 7 unit

In ΔABC,

$\tan\beta = \dfrac{7}{12}$ then AB = 12 unit

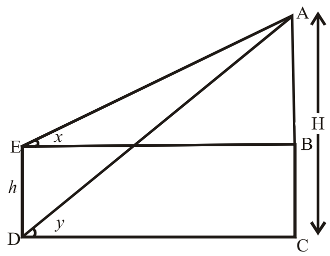
hence, BD = 35 – 12 = 23 unit

value of 23 unit → 138m

Value of 1unit → 6m

Value of 7 unit → 7×6 = 42m = height of tower

Sol.34



In ΔACD ,

$$\tan y = \frac{H}{CD}$$

$CD = H\cot y = BE$

In ΔABE ,

$$\tan x = \frac{AB}{BE}$$

$AB = BE\tan x = H\cot y \,.\, \tan x$

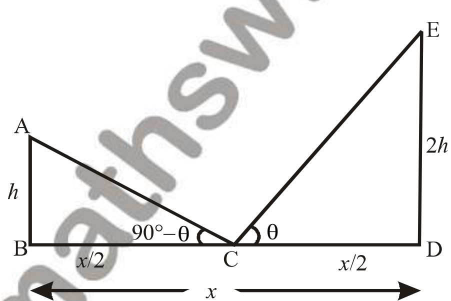
$H - h = H \cot y.\tan x$

$H(1 - \cot y.\tan x) = h$

Height $H = \dfrac{h\cot x}{\cot x - \cot y}$

Sol. 35 based on angle ratio property.

Sol. 36



In ΔABC

$$\tan\theta = \frac{x}{2h} \qquad \ldots(i)$$

In ΔEDC

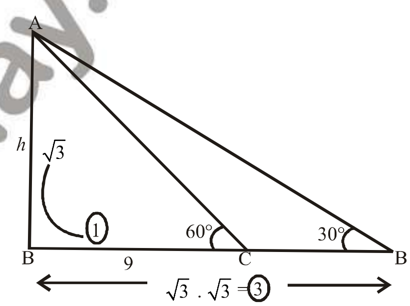
$$\tan\theta = \frac{2h}{\frac{x}{2}} = \frac{4h}{x} \qquad \textbf{......... (ii)}$$

From (i) and (ii)

$$\frac{x}{2h} = \frac{4h}{x} \Rightarrow x^2 = 8h^2$$

$$h = \frac{x}{2\sqrt{2}}$$

Sol.37



Value of $\sqrt{3}$unit $\rightarrow$ 30m

Value of 1 unit $\rightarrow \dfrac{30}{\sqrt{3}}$ m

$=10\sqrt{3}$m

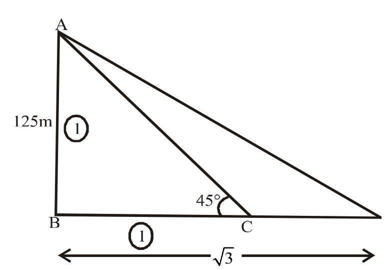
Moving distance = CD = Value of (3-1) unit

= Value of 2 unit

$= =2\times10\sqrt{3} = 20\sqrt{3}$ m

Sol.38 based on angle ratio property.

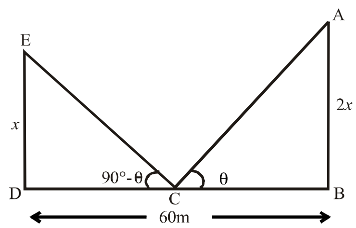
Sol.39



Value of 1 unit → 125 m
Distance between object = CD

= Value of $\left(\sqrt{3}-1\right)$ unit $\Rightarrow 125\left(\sqrt{3}-1\right)$m

Sol.40



C is the mid point of BD.
CD = BC = 30 m.

$$\tan\left(90°-\theta\right)=\frac{x}{30}$$

$$=\cot\theta=\frac{x}{30} \qquad \ldots(1)$$

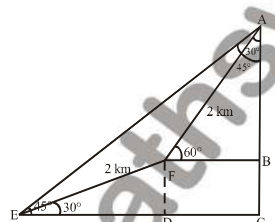
$$=\tan\theta=\frac{2x}{30} \qquad \ldots(2)$$

Multiply (1) and (2)

$$1=\frac{2x^2}{30\times 30}\Rightarrow x=15\sqrt{2}$$

DE = $15\sqrt{2}$ & AB = $30\sqrt{2}$

Sol. 41



In ΔAFE
$\angle$AEF = 15° = $\angle$FAE
then EF = AF = 2km

In ΔFDE ,

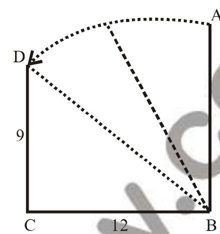
$$\text{Sin}30° = \frac{1}{2}=\frac{FD}{2}\Rightarrow FD = 1\text{ km}$$

In ΔABF ,

$$\text{Sin }60° = \frac{\sqrt{3}}{2}=\frac{AB}{AF}=\frac{AB}{2}\Rightarrow AB=\sqrt{3}\text{ km}$$

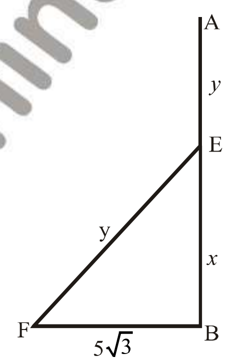
height of montain = AB + BC = $\left(\sqrt{3}+1\right)$km

Sol. 42



hence length of tree = AB = BD $=\sqrt{9^2+12^2}=15$m

$x+y=15 \qquad \ldots$(i)



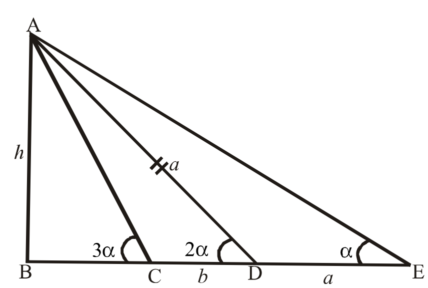
$$y^2-x^2=\left(5\sqrt{3}\right)^2=75$$

$(y+x)(y-x)=75 \Rightarrow y-x=5 \qquad \ldots$(ii)

from eq^n (i) & (ii)
$x=5, y=10$

$$\Rightarrow \text{Sin}\alpha=\frac{x}{4}=\frac{5}{10}=\frac{1}{2}\Rightarrow\alpha=30°$$

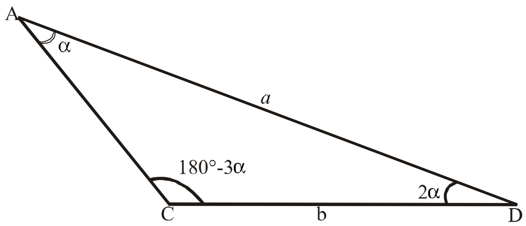
Sol.43



$\angle DAE = 2\alpha - \alpha = \alpha$

then in ΔADE,
$AD = DE = a$

$\angle CAD = 3\alpha - 2\alpha = \alpha$



apply sine rule in above triangle

$$\frac{\sin(180-3\alpha)}{a} = \frac{\text{Sin}\alpha}{b}$$

$$\Rightarrow \frac{\sin 3\alpha}{a} = \frac{\text{Sin}\alpha}{b}$$

$$\Rightarrow \frac{3\sin\alpha - 4\text{Sin}^3\alpha}{a} = \frac{\text{Sin}\alpha}{b}$$

$$\Rightarrow 3 - 4\sin^2\alpha = \frac{a}{b}$$

$$\text{Sin}^2\alpha = \frac{3b-a}{4b} \Rightarrow \cos^2\alpha = \frac{a+b}{4b}$$

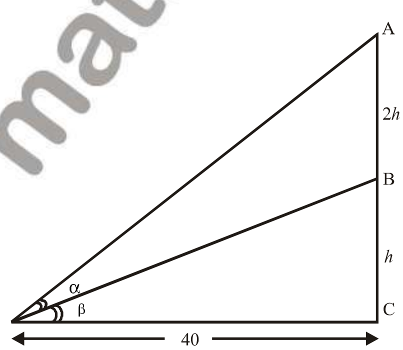
In ΔADE,

$$\sin 2\alpha = \frac{h}{a} \Rightarrow h = a\text{Sin}2\alpha = 2a\sin\alpha.\cos\alpha$$

$$= 2a\sqrt{\frac{(3b-a)}{4b}.\frac{(a+b)}{4b}}$$

$$= \frac{a}{2b}\sqrt{(3b-a)(a+b)}$$

Sol.44



let height $AC = 3h$

lower part $= \frac{1}{3}$ of $3h = h$

In ΔBCD,

$$\tan\beta = \frac{h}{40}, \tan\alpha = \frac{1}{2}$$

$$\tan(\alpha+\beta) = \frac{3h}{40} \Rightarrow \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha.\tan\beta} = \frac{3h}{40}$$

$$\Rightarrow \frac{\frac{1}{2} + \frac{h}{40}}{1 - \frac{1}{2}.\frac{h}{40}} = \frac{3h}{40} \Rightarrow \frac{40+2h}{80-h} = \frac{3h}{40}$$

$1600 + 80h = 240h - 3h^2$

$3h^2 - 160h + 1600 = 0$

$3h^2 - 120h - 40h + 1600 = 0$

$3h(h-40) - 40(h-40) = 0$

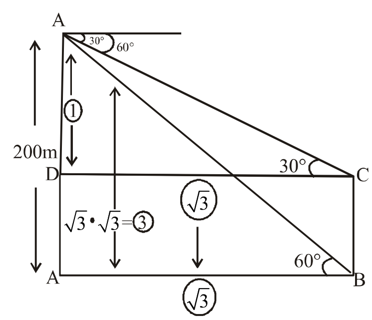
$(3h-40)(h-40)$

$(3h \neq 40$ but $3h > 100)$

then $h = 40$

hence, height of the tower = 120m

Sol.45



Value of 3 → unit 200 m

Value of 1 unit → $\frac{200}{3}$ m

height of the tower BC = value of (3-1) unit

= Value 2.unit → $\frac{400}{3}$ m. = $133\frac{1}{3}$ m

6 Mensuration Theory

Cuboid

1. To find volume of a cuboid if its length, breadth and height are given.

Volume of a cuboid = length × breadth × height

Ex. Find the volume of a cuboid 24 m long, 18 broad and 16 m high.

Sol. Volume of the cuboid = 24 × 18 × 16

= 6912 cubic metres.

2. To find volume of a cuboid if its area of base or top, area of side face, and area of other side face are given.

Volume of the cuboid = $\sqrt{A_1 \times A_2 \times A_3}$

$= \sqrt{\text{area of base or top} \times \text{area of one face} \times \text{area of the other face}}$

Where, A_1 = area of base or top,

A_2 = area of one side face and

A_3 = area of other side face.

Ex. Area of the base of a cuboid is 9 sq metres, area of side face and area of other side face is 16 sq metres and 25 sq metres respectively. Find the volume of the cuboid.

Sol: The required answer = $\sqrt{9 \times 16 \times 25}$

= $\sqrt{3600}$ = 60 cu. metres.

Ex. Find the volume of a cuboid whose area of base and two adjactent faces area is 180 sq cm, 96 sq cm and 120 sq cm respectively.

Sol: We have, volume of a cuboid

$= \sqrt{\text{area of base} \times \text{area of one face} \times \text{area of the other face}}$

$= \sqrt{180 \times 96 \times 120}$ = 1440 cu. cm.

3. To find the whole surface area of cuboid if its length, breadth and height are given.

Whole surface area of the cuboid = $2(lb + bh + lh)$

Where, l = lenght, b = breadth and h = height of the cuboid.

Ex. Find the surface area of a slab of stone measuring 4 metres in length, 2 metres in width and $\frac{1}{4}$ metre in thickness.

Sol: surface area = $2\left(4 \times 2 + 4 \times \frac{1}{4} + 2 \times \frac{1}{4}\right)$ = 19 sq m

4. To find the diagonal of a cuboid if its length, breadth and height are given.

Diagonal of cuboid = $\sqrt{l^2 + b^2 + h^2}$; where l = length, b = breadth and

h = height of the cuboid.

Ex: Find the length of diagonal of a cuboid 12 m long, 9m broad and 8 m high.

Sol: diagonal = $\sqrt{12^2 + 9^2 + 8^2}$ = $\sqrt{289}$ = 17 m.

5. To find total surface area of a cuboid if the sum of all three sides and diagonal are given. Total surface area = (Sum of all three sides)^2 – (Diagonal)^2

Ex. The sum of length, breadth and height of a cuboid is 25 cm and its diagonal is 15 cm long. Find the total surface area of the cuboid.

Sol: the total surface area

= $(25)^2 - (15)^2$ = 625 – 225 = 400 sq cm.

6. If each edge (or side) of a cube is 'a' units then

(i) Volume of the cube = a^3 cubic units.

(ii) whole surface of the cube = $(6a^2)$ sq units.

(iii) diagonal of the cube = $\left(\sqrt{3}a\right)$ units.

Note : If diagonal of a cube is given, then the volume of the cube is given by $\left(\frac{\text{diagonal}}{\sqrt{3}}\right)^3$

Ex: Find the volume, surface area and the diagonal of a cube, each of whose sides measures 2 cm.

Sol: Volume = $a^3 = (2 \times 2 \times 2) = 8$ cm^3

Surface area = $6a^2 = (6 \times 2 \times 2) = 24$ cm^2

Diagonal = $\sqrt{3} \times a = 2\sqrt{3}$ cm

Cylinder

7. If the radius of the base of cylinder is 'r' units and its hight (or length) is 'h' units, then volume of the cylinder is given by $\left(\pi r^2 h\right)$ cu. units,

Volume of the cylinder = Area of the base of cylinder × Height of the cylinder.

Ex: Find the volume of a cylinder which has a height of 14 metres and a base of radius 3 metres.

Sol: Volume $= \frac{22}{7} \times 3 \times 3 \times 14 = 396$ cu. metres.

8. If the radius of the base of a cylinder is 'r' units and its height (or length) is 'h' units, then curved surface area of the cylinder is $(2\pi rh)$ sq units ie

Surface area = circumference of the base × height

Ex: Find the curved surface area of cylinder which has a hight of 14 metres and a base of radius 3 metres.

Sol: Curved surface area $= 2 \times \frac{22}{7} \times 3 \times 14 = 264$ sq metres.

9. If the radius of the base of a cylinder is 'r' units and its height (or length) is 'h' units, then the total surface area of the cylinder is $(2\pi rh + 2\pi r^2)$ sq units.

Or, Total surface area $= 2\pi r(h + r)$ sq units = Circunference × (height + radius)

Ex: Find the total surface area of a cylinder which has a height of 14 metres and a base of radius 3 metres.

Sol: total surface area

$$= 2 \times \frac{22}{7} \times 14 \times 3 + \frac{22}{7} \times 2 \times 3 \times 3$$

$$= 264 + \frac{396}{7} = 320.57 \text{ sq units.}$$

Sphere

10. If the radius of a sphere is r units, then volume of the sphere is $\left(\frac{4}{3}\pi r^3\right)$ cu units. If diameter is given, then volume of sphere becomes $\left(\frac{1}{6}\pi D^3\right)$ cubic units. [where D = diameter]

Ex: Find the volume of a sphere of diameter 42 cm or radius 21 cm.

Sol: **Case I:** Volume of the sphere

$$= \frac{4}{3} \times \frac{22}{7} \times 21 \times 21 \times 21 = 38808 \text{ cubic cm.}$$

Case II : Volume of the sphere

$$= \frac{1}{6} \times \frac{22}{7} \times 42 \times 42 \times 42 = 38808 \text{ cubic cm.}$$

11. If the radius of a sphere is r units, then the surface area of sphere is $\left(4\pi r^2\right)$ sq units. If in place of radius, diameter of the sphere is given, then

Surface area of a sphere $= 4\pi\left(\frac{D}{2}\right)^2 = \pi D^2$ sq units.

Ex.: Find the surface area of a sphere of diameter 42 cm.

Sol.: surface area of a sphere

$$= \frac{22}{7} \times 42 \times 42 = 5544 \text{ sq cm.}$$

12. If the radius of a sphere is r units, then volume of a hemisphere $= \left(\frac{2}{3}\pi r^3\right)$ cu. units. If diameter is given, then volume of a hemisphere is given by $\left(\frac{\pi}{2}D^3\right)$ cu units.

[Where, D = diameter of the sphere]

Ex.: Find the volume of a hemisphere of radius 21 cm.

Sol. Volume of hemisphere = $\left(\frac{2}{3}\times\frac{22}{7}\times21\times21\times21\right)$

$= 19404 \text{ cm}^3$

13. If the radius of a sphere is r units, then the curved surace area of the hemishere is $(2\pi r^2)$ sq units.

If in place of radius, diameter is given, then the curved suface area of the hemisphere becomes $\left(\frac{\pi}{2}D^2\right)$ sq units.

Ex.: Find the curved surface area of a hemisphere of radius 21 cm.

Sol.: curved surface area = $2\pi r^2$

$= \left(2\times\frac{22}{7}\times21\times21\right) \text{cm}^2$

$= 2772 \text{ cm}^2$

14. If the radius of a sphere is r units, then the whole surface area of the hemisphere is $(3\pi r^2)$ sq units. If in place of radius, diameter is given, then the whole surface area of the hemisphere is given by $\left(\frac{3}{4}\pi D^2\right)$ sq units.

Ex: Find the total surface of a hemisphere of radius 21 cm.

Sol. total surface area

$= 3\pi r^2 = \left(3\times\frac{22}{7}\times21\times21\right) \text{cm}^2 = 4158 \text{ cm}^2$

Note:- When a solid cutted.........

(i) Volume of solid no change

(ii) surface area of solid will be increase

Right Circular Cone

15. To find the slant height of the right circular cone if radius of its base and height of the cone are given.

Slant height (l) = $\left(\sqrt{h^2+r^2}\right)$ units.

Where h = height and r = radius of the base.

Ex: Radius of the base of a right circular cone is 3 cm and height of the cone is 4 cm. Find the slant height of the cone.

Sol. slant height of the right circular cone

$= \sqrt{4^2+3^2} = 5$ cm.

16. To find the volume of the right circular cone, if radius of the base and the height of the cone is given.

Volume of the cone = $\left(\frac{1}{3}\pi r^2 h\right)$ cu. units.

Ex: Radius of the base of a right circular cone is 7 cm and the height of the cone is 3 cm. Find the volume of the cone.

Sol: Volume of the cone = $\frac{1}{3}\times\frac{22}{7}\times7\times7\times3$

$= 154 \text{ cm}^3$

17. To find the curved surface area of the cone,

(i) If its slant height and radius of its base are given.

Curved surface area of the cone = πrl sq units.

(ii) If its height and radius of its base are given,

Curved surface area of the cone

$= \pi r\left(\sqrt{r^2+h^2}\right)$ sq units.

Ex: Radius of the base of a right circular cone is 3 cm and the height of the cone is 4 cm. Find the curved surface area of the cone.

Sol. Curved surface area of the cone

$$= \frac{22}{7} \times 3 \left(\sqrt{4^2+3^2}\right)$$

$$= \frac{22\times3\times5}{7} = 47\frac{1}{7} \text{ sq cm.}$$

18. To find the total surface area of the right circular cone.

(i) If its slant hight and radius of its base are given.

Total surface area of the cone

$= (\pi rl + \pi r^2) = \pi r(l + r)$ sq units.

(ii) If its height and radius of its base are given.

Total surface area of the cone

$$= \pi r\left(\sqrt{h^2+r^2}+r\right) \text{ sq units.}$$

Ex: Radius of the base of a right circular cone is 3 cm and the height of the cone is 4 cm. Find the total surace area of the cone.

Sol: total surace area $= \frac{22}{7} \times 3 \left(\sqrt{4^2+3^2}+3\right)$

$$= \frac{22\times3\times8}{7} = \frac{528}{7} = 75\frac{3}{7} \text{ sq cm.}$$

Frustum of a Right Circular Cone

Frustum: If a cone is cut by a plane parallel to the base so as to divide the cone into two parts.

Lower part is called the frustum of the cone.

let the radius of the base of the frustum = R, the radius of the top of the frustum = r, and slant height of the frustum = l units.

(i) Slant height $(l) = \sqrt{h^2+(R-r)^2}$ units.

(ii) Curved surface area = $\pi(R + r)l$ sq units.

(iii) Total surface area = $\pi[(R + r)l + r^2 + R^2]$ sq units.

(iv) volume of the frustum = $\frac{\pi h}{3}(r^2 + R^2 + rR)$ cu units.

Ex: A frustum of a right circular cone has a diameter of base 10 cm of top 6 cm and a height of 5 cm. Find

(i) slant height

(ii) curved surface area

(iii) Total surface area and

(iv) volume of the frustum.

Sol: Here, $r = \frac{6}{2} = 3$cm; $R = \frac{10}{2} = 5$ cm; $h = 5$ cm

(i) slant height $= \sqrt{h^2+(R-r)^2}$

$= \sqrt{5^2+(5-3)^2} = \sqrt{29}$ cm = 5.385cm

(ii) curved surface area

$= \pi(R+r)l = \frac{22}{7} \times 8 \times 5.385 = 135.4$ sq cm.

(iii) Total surface area of the frustum

$= \pi\left[(R+r)l+r^2+R^2\right]$

$= \frac{22}{7}[8 \times 5.385 + (3)^2 + 5^2]$

$= \frac{22}{7}[43.08 + 9 + 25] = 242.25$ sq cm.

(iv) Volume of the frustum $= \frac{\pi h}{3}(r^2 + R^2 + rR)$

$= \frac{22}{7} \times \frac{5}{3}[5^2 + 3^2 + 5 \times 3] = 256.67$ cu. cm.

20. To find number of bricks when the dimensions of brick and wall are given.

Required no. of bricks $= \frac{\text{Volume of wall}}{\text{Volume of one brick}}$

Ex: A brick measures 20 cm by 10 cm by $7\frac{1}{2}$ cm. How many bricks will be requiured for a wall 25 m long, 2 m high and $\frac{3}{4}$ m thick?

Sol: Volume of wall = $25 \times 2 \times \frac{3}{4}$ cu. m.

volume of one brick

$$= \frac{20}{100} \times \frac{10}{100} \times \frac{15}{200} = \frac{3}{2000} \text{ cu. m.}$$

Required number of bricks

$$= \left(25 \times 2 \times \frac{3}{4}\right) \div \frac{3}{2000} = 25000$$

21. To find capacity. volume of material and weight of material of a closed box, when external dimensions (ie length, breadth and height) and thickness of material of which box is made, area given.

(i) Capacity of box = (External length – 2 × thickness) × (External breadth – 2 × thickness) × (External height – 2 × thickness)

(ii) Volume of material = External Volume – Capacity

(iii) Weight of wood = Volume of wood × Density of wood.

Ex: A closed wooden box measures externally 9 cm long. 7 cm broad. 6 cm high. If the thickness of the wood is 0.5 cm, find (i) the capacity of the box and (ii) the weight supposing that one cubic cm. of wood weighs 0.9 gm.

Sol. Capacity = (external length – 2 × thickness) × (external breadth – 2 × thickness) × (external height – 2 × thickness)

Volume of material = external volume – capacity

∴ In this question,

Capacity = (9 – 2 × 0.5) (7 – 2 × 0.5) (6 – 2 × 0.5) = 8 × 6 × 5 = 240 cm^3

∴ Volume of wood = external volume – capacity

= 9 × 7 × 6 – 240 = 138 cu. cm.

∴ Weight of wood = Volume of wood × density of wood = 138 × 0.9 = 124.2 g.

22. To find the volume of a cube if the surface area of the cube is given.

$$\text{Volume of cube} = \left(\sqrt{\frac{\text{surface area}}{6}}\right)^3$$

Ex: The surface area of a cube is $30\frac{3}{8}$ sq metres. Find its volume.

Sol. $$\text{Volume} = \left(\sqrt{\frac{\frac{243}{8}}{6}}\right)^3 = 11\frac{25}{64} \text{ cu m.}$$

23. To find the volume of rain water at a place if the annual rainfall of that place is given.

Volume of rain water = Height (or level) of water (ie Annual rainfall) × Base area (ie area of the place)

Ex: The annual rainfall at a places is 43. Find the weight in metric tonnes of the annual rainfall there on a hectare of land, taking the weight of water to be 1 metric tone for 1 cubic metre.

Sol. Volume of water = height (level) of water × base area In the given question, level of rainfall is 43 cm.

∴ volume of water = $\frac{43}{100}$ m × 10000 sq m

= 4300 cu m.

(As 1 hectare = 10,000 sq m).

∴ weight of water = 4300 × 1 = 4300 metric tonnes.

24. A rectangular tank is 'l' metres long and 'h' metres deep. If 'x' cubic metres of water be drawn off the tank, the level of the water in the tank goes down by 'd' metres, then the amount of water (in cubic metres) the tank can hold is given by $\left(\frac{x \times h}{d}\right)$ cubic metres and the breadth of the tank is $\left(\frac{x}{ld}\right)$ metres.

Ex: A rectangular tank is 50 metres long and 29 metres deep. If 1000 cubic metres of water be drawn off the tank, the level of the water in the tank goes down by 2 metres. How many cubic metres of water can the tank hold? And also find the breadth of the tank.

Sol: Let the breadth of the tank be x metres. Volume of the tank = $50 \times 29 \times x$ cubic metres.

From the question,

$50 \times 29 \times x = 1000 + 50 \times (29 - 2) \times x$

or, $x \times 50 \times 2 = 1000 \;\therefore x = 10$ metres.

$\therefore$ Breadth of the tank is 10 metres.

Volume of the tank = $50 \times 29 \times 10 = 14500$ cubic metres.

Method:2

Volume of the tank = $\frac{1000 \times 29}{2} = 14500$ cub m.

Note: we may conclude that even if the length of the rectangular tank is not given. Volume of the tank can be calculated. To find breadth of the tank length is needed but not the height of the tank.

$\therefore$ Breadth of the tank = $\frac{1000}{50 \times 2} = 10$ metres.

25. x cubic metres of copper weighing y kiligrams is rolled into a square bar l metres long. An exact cube is cut off from the bar. Weight of the cube is given by $\left[\left(\sqrt{\frac{x}{l}}\right)^3 \times y\right] kg.$

Ex: A cubic metre of copper weighing 9000 kilograms is rolled into a square bar 9 metres long. An exact cube is cut off from the bar. How much does it weigh?

Sol: In question volume of copper is rolled into a square bar (basically a cuboid with square base) of given length. Then an exact cube is cut off from this square bar. Clearly, the exact cube should have the same dimensiosn as that of the square base of the square bar.

Now, given volume = 1 cu m.

= Area of square base × length

$\Rightarrow$ Area of square base × length = 1

$\Rightarrow$ Area of square base = $\frac{1}{\text{length}} = \frac{1}{9}$

$\therefore$ side of square base = $\sqrt{\frac{1}{9}} = \frac{1}{3}$

$\therefore$ Vol. of the cut of cube

= (side of the square base)^3 = $\left(\frac{1}{3}\right)^3 = \frac{1}{27}$

$\therefore$ weight of cube = $\frac{1}{27} \times 9000 = 333.3$ kg.

Method:2

Volume of cube cut off

$$= \left(\sqrt{\frac{\text{Volume of original solid}}{\text{length of the solid}}}\right)^3$$

$\therefore$ Volume = $\left(\sqrt{\frac{1}{9}}\right)^3 = \frac{1}{27}$

$\therefore$ Weight = $\frac{9000}{27} = 333.3$kg.

Note: We can also solve directly.

$\therefore$ Weight of cube

$$= \left(\sqrt{\frac{1}{9}}\right)^3 \times 9000 = \frac{9000}{27} = 333.3 \text{ kg.}$$

26. When many cubes integrate into one cube,the side of the new cube is given by side

$$= \sqrt[3]{\text{Sum of cubes of sides of all the cubes}}$$

Ex: Three cubes of metal whose edges are 3, 4 and 5 cm respectively are melted and formed into a single cube. If there be no loss of metal in the process find the side of the new cube.

Sol: Volume of the first cube = $(3)^3$ = 27 cubic cm.

Volume of the second cube = $(4)^3$ = 64 cu. cm

Volume of third cube = $(5)^3$ = 125 cu. cm.

$\because$ Volume remains unchanged.

$\therefore$ Volume of the new cube = 27 + 64 + 125 = 216 cu cm.

$\therefore$ Side of the new cube = $\sqrt[3]{216}$ = 6 cm.

Method:2 side = $\sqrt[3]{3^3+4^3+5^3} = \sqrt[3]{27+64+125}$

$= \sqrt[3]{216} = 6$ cm.

27. Total volume of a solid does not change when its shape is changed.

$\therefore$ Old volume = New volume

Ex: A cubic metre of gold is extended by hammering so as to cover an area of 6 hectares. Find the thickness of the gold.

Sol:

$\Rightarrow$ 1 cu m = 60000 × thickness

$\Rightarrow$ thickness = $\frac{1}{60000}$ m = 0.0017 cm.

28. To find the number of possible cubes when disintegration of a cube into identical cubes.

Number of cubes = $\left(\frac{\text{Original length of side}}{\text{New length of side}}\right)^3$

Ex. Find t

Ex: A cube of sides 3 cm is melted and smaller cubes of sides 1 cm each are fromed. How many such cubes are possible?

Sol: Number possible =

$\left(\frac{\text{original length of side}}{\text{new length of side}}\right)^3$

$\therefore$ possible number of cubes

$= \left(\frac{3}{1}\right)^3 = 27$

29. A hollow cylindrical tube open at both ends is made of a thick metal. If the internal diameter or radius and length of the are given, then the volume of metal is given by [π × height × (2 × Internal radius + thickness) × thickness] cu. units.

Ex. A holow cylindrical tube open at both ends is made of iron 2 cm thick. If the internal diameter be 50 cm and the length of the tube be 140 cm, find the volume of iron in it.

Sol: Here, internal diameter = 50 cm

$\therefore$ internal radius = $\frac{50}{2}$ = 25 cm

Required volume = $\frac{22}{7} \times 140 \times (25 \times 2 + 2) \times 2$

$= \frac{22}{7} \times 140 \times 52 \times 2$

= 45760 cu cm.

30. A hollow cylindrical tube open at both ends is made of a thich metal. If the internal and external diameter or radius of the tube are given. Then the volume of metal is given by π × height × [(External radius)^2 – (Internal radius)^2] cu. cm.

Ex: A hollow cylindrical tube open at both ends is made of iron. If the external and internal radius of the tube are 25 cm and 23 cm respectively, find the volume of iron in it.

Sol. volume of iron

$= \frac{22}{7} \times 140 \times (25^2 - 23^2) = 42240$ cu. cm.

31. A hollow cylindrical tube open at both ends is made of a thick metal. If the external diameter or radius and length of the tube are given, then volume of metal is given by

[π × height × (2 × outer radius – thickness) × thickness] cu. units.

Ex: A hollow cylindrical tube open at both ends is made of iron 2 cm thick. If the external diameter be 50 cm and the length of the tube be 140 cm, find the volume of iron in it.

Sol: external diameter = 50 cm

$\therefore$ external radius = $\frac{50}{2}$ = 25 cm .

Required volume = $\frac{22}{7}$ × 140 (25 × 2 – 2) × 2

= $\frac{22}{7}$ × 140 × 48 × 2 = 42240 cu cm.

32. If a rectangular sheet is rolled into a cylinder so that the one side becomes the height of the cylinder then the volume of the cylinder so formed is given by

$$\frac{\text{height}\times(\text{other side of the sheet})^2}{4\pi}$$

Ex: A rectangular sheet with dimension 22m × 10 m is rolled into a cylinder so that smaller side becomes the height of the cylinder. What is the volume of the cylinder so formed?

Sol. Volume = $\frac{10\times(22)^2}{4\times\frac{22}{7}}$ = 385 cu m.

33. If a sphere of certain diameter or radius is drawn into a cylinder of certain diameter or radius, then the length or height of the cylinder is given by

$$\frac{4\times(\text{radius of spere})^3}{3\times(\text{radius of cylinder})^2}$$

Ex: A copper sphere of diameter 18 cm is drawn into a wire of diameter 4 mm. Find the lenght of the wire.

Sol: When a sphere is converted into a cylinder (Note that wire is basically a cylinder) the length of the wire is

length of cylinder =

$$\frac{4\times(\text{radius of sphere})^3}{3\times(\text{radius of cylinder})^2}$$

$\therefore$ length = $\frac{4\times(90)^3}{3\times(2)^2}$

= 243000 mm = 24300 cm.

34. A Sphere is converted into a cylinder, If the length and the radius of the cylinder are given, then the radius of the sphere is

$$\sqrt[3]{\frac{3}{4}(\text{length of cylinder})(\text{radius of cylinder})^2}$$

Ex: A cylinder of radius 2 cm and height i5 is melted and the same mass is used to create a sphere . What will be the radius of the sphere?

Sol: the radius of sphere = $\sqrt[3]{\frac{3}{4}\times15\times2\times2}$ = $\sqrt[3]{45}$

35. If a sphere of certain diameter or radius is drawn into a cylinder of certain height or length, then the radius of cylinder is given by =

$$\sqrt{\frac{4\times(\text{radius of sphere})^3}{3\times(\text{length of cylinder})}}$$

Ex: A copper sphere of 36 m diameter is drawn into a cylindrical wire of length 7.29 km. What is the radius of wire.

Sol: radius of wire

$$= \sqrt{\frac{4\times\left(\frac{36}{2}\right)^3}{3\times7.29\times1000}} = \sqrt{\frac{4\times18\times18\times18}{3\times7290}}$$

= 1.03 m.

36. If sphere is melted to form a cylinder whose height is n times its radius, then the ratio of radii of sphere to the cylinder is $\left(\frac{3}{4}\times n\right)^{1/3}$

Ex: A sphere is melted to form a cylinder whose height is $4\frac{1}{2}$ times its radius. What is the ratio fo radii of sphere to the cylinder?

Sol: Let the radius of the sphere and cylinder be 'R' and 'r' respectively.

volume of the cylinder

$$= \pi r^2 h = \pi r^2\left(\frac{9}{2}r\right)\left[\because h=\frac{9}{2}r\right] = \frac{9}{2}\pi r^3$$

Volume of the sphere = $\frac{4}{3}\pi R^3$

Volume of the sphere = Volume of the cylinder

or, $\frac{4}{3}\pi R^3 = \frac{9}{2}\pi r^3$ or, $\left(\frac{R}{r}\right)^3 = \frac{27}{8}$

$\therefore$ R : r = 3 : 2

Method:2

$$\text{the required ratio} = \left(\frac{9}{2}\times\frac{3}{4}\right)^{1/3} = \left(\frac{27}{8}\right)^{1/3}$$

$$= \frac{3}{2} = 3 : 2$$

37. If a cone, whose height is n times of its radius, is melted to form a sphere, assuming that there is no loss of material in this process, ratio fo radius of the sphere to that of the cone is $\left(\frac{n}{4}\right)^{1/3}$

Ex: A cone, whose height is half of its radius, is melted to form a sphere, Find the ratio of radius of the sphere to that of cone.

Sol: Let the raidus of sphere and cylinder be R and r respectively.

$$\text{Volume of the cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2\left(\frac{r}{2}\right)$$

$$= \frac{\pi r^3}{3\times 2} r^2$$

Volume of the sphere = $\frac{4}{3}\pi R^3$

Volume of the sphere = Volume of the cone

or, $\frac{4}{3}\pi R^3 = \frac{\pi}{3}\times\frac{r^3}{3}$

or, $\left(\frac{R}{r}\right)^3 = \frac{1}{8}$

$\therefore$ R : r = 1 : 2

Method -2

$$\text{Required ratio} = \left[\frac{1/2}{4}\right]^{1/3} = \left[\frac{1}{8}\right]^{1/3} = \frac{1}{2} = 1 : 2$$

38. When one cylinder is converted into many small spheres, then the number of small spheres is given by the following formula.

$$\text{Number of small spheres} = \frac{\text{Volume of cylinder}}{\text{Volume of 1 sphere}}$$

Ex: How many bullets can be made out of a lead cylinder 28 cm high and 6 cm radius, each bullet being 1.5 cm in diameter?

Sol: one cylinder is not converted into just one sphere but many spheres area being made.

$$\text{Number of bullets} = \frac{\text{Volume of cylinder}}{\text{Volume of 1 bullet}}$$

$$= \frac{\pi\times 6\times 6\times 28}{\frac{4}{3}\times\pi\times 0.75\times 0.75\times 0.75} = 1792$$

39. If a sphere of radius R units has a spherical cavity of radius r units, then the volume of the spherical shell is

$\left[\frac{4}{3}\pi\left(R^3 - r^3\right)\right]$ cubic units.

Ex: A sphere of radius 5 cm has a spherical cavity of radius 3 cm. Find the volume of the spherical shell.

Sol: Volume of the spherical shell = $\frac{4}{3} \times \frac{22}{7} \times \left[5^3 - 3^3\right]$

$= \frac{4}{3} \times \frac{22}{7} \times 98 = 410\frac{2}{3}\ \text{cm}^3$

40. The radius of a greatest sphere that can be covered out of a cone of radius *r* and height *h* is $\left(\frac{rh}{r+l}\right)$. Where, *l* = slant height of the cone.

$= \sqrt{r^2 + h^2}$

Ex: There is a cone of radius 3 metres and height 4 metres. Find the radius of the greatest sphere that can be covered out of that cone.

Sol: slant height = $\sqrt{3^2 + 4^2} = \sqrt{25} = 5$ and

radius of the greatest sphere

$= \frac{3 \times 4}{3+5} = \frac{12}{8} = 1\frac{1}{2} = 1.5$ metres.

41. When a sphere disintegrates into many indentical spheres, then number of smaller identical are given by $\left(\frac{\text{bigger radius}}{\text{smaller radius}}\right)^3$

Ex: Find the number of lead balls of diameter 1 cm each that can be made from a sphere of diameter 16 cm.

Sol. Number of balls $= \frac{\text{Volume of big sphere}}{\text{Volume of 1 small sphere}}$

$= \frac{\frac{4}{3}\pi \times 8 \times 8 \times 8}{\frac{4}{3}\pi \times 0.5 \times 0.5 \times 0.5} = 4096$

Method :2

the required number = $\left(\frac{8}{0.5}\right)^3 = 16^3 = 4096$

42. If the ratio of surface areas of the two spheres are given, then the ratio of their volumes will be

(Ratio of the surface areas)^3 = (Ratio of volumes)^2

Ex: The curved surface areas of two spheres are in the ratio 1 : 4. Find the ratio of their volumes.

Sol: $(1:4)^3$ = (ratio of volumes)^2

or, (1 : 64) = (ratio of volumes)^2

or, $\sqrt{1:64} = 1 : 8$ = ratio of volumes.

43. If the ratio of the radii of two spheres are given, then the ratio of their surface areas will be obtained from the following result.

(Ratio of radii)^2 = Ratio of surface areas.

Ex: The radii of two sphere are in the ratio of 1 : 2. What is the ratio of their surface areas?

Sol: (ratio of surface areas) = (ratio of radii)^2

$= (1:2)^2 = 1 : 4$

44. If the ratio of the radii of two spheres area given, then the ratio of their volumes will be

(Ratio of radii)^3 = Ratio of volumes

Ex: The radii of two spheres are in the ratio of 1 : 2. What is the ratio of their volumes?

Sol: the ratio of volumes

= (Ratio of radii)^3 = $(1:2)^3 = 1 : 8$

Two Cylinders

A. When volumes are equal

45. If the ratio of the heights of two circular cylinders of equal volume are given, then the ratio of their radii is

Ratio of radii = $\sqrt{\text{inverse ratio of heights}}$

Ex: Tow circular cylinders of equal volume have their heights in the ratio of 1 : 2. Ratio of their radii is?

Sol: the ratio of radii

$= \sqrt{\text{inverse ratio of heights}} = \sqrt{2:1} = \sqrt{2} : 1.$

46. If the ratio of curved surface areas of two circular cylinders of equal volume are given, then the ratio of their heights is

Ratio of curved surface areas = $\sqrt{\text{ratio of heights}}$

Ex: Two circular cylinders of equal volume have their heights in the ratio of 1 : 2 . Find the ratio of their curved surface areas.

Sol:

ratio of curved surface areas = $\sqrt{\text{Ratio of heights}}$

$\therefore$ Ratio of curved surface areas = $\sqrt{1:2}$ = 1 : $\sqrt{2}$

47. If the ratio of radii of two circular cylinders of equal volume are given, then the ratio of their curved surface areas are

Ratio of curved surface areas = inverse ratio of radii.

Ex: Two circular cylinders of equal volume have their radii in the ratio of 1 : 2, Find the ratio of their curved surface areas.

Sol: the ratio of curved surface areas

= inverse ratio of radii = 2 : 1

B. When radii are equal

48. If the ratio of heights of two circular cylinders of equal radii are given then the ratio of their volumes area

Ratio of volumes = Ratio of heights.

Ex: Two circular cylinders of equal radii have height in the ratio of 1 : 2. Find the ratio of their volumes.

Sol: Applying the above theorem, we have

the ratio of volumes = ratio of heights

$\therefore$ required answer = 1 : 2

49. If the ratio of heights of two circular cylinders of equal radii are given then the ratio of their curved surface areas are

Ratio of curved surface areas = Ratio of heights.

Ex: Two circular cylinders of equal radii have their height in the ratio of 1 : 2. Find the ratio of their curved surface areas.

Sol: ratio of curved surface areas = 1 : 2.

50. If the ratio of volumes of two circular cylinders of equal radii are given then the ratio of their curved surface areas are

Ratio of volume = Ratio of curved surface areas.

Ex: Two circular cylinders of equal radii have their volumes in the ratio of 3 : 1. Find the ratio of their curved surface areas.

Sol: ratio of curved surface areas = Ratio of volumes = 3 : 1.

C. When heights are equal

51. If the ratio of radii fo two circular cylinders of equal heights are given, then the ratio of their volumes is

Ratio of volumes = $(\text{Ratio of radii})^2$

Ex: Two circular cylinders of equal heights have their radii in the ratio of 2 : 3 . Ratio fo their volumes is ?

Sol:

ratio of volumes = $(\text{Ratio of radii})^2 = (2 : 3)^2$

= 4 : 9

52. If the ratio of radii of two circular cylinders of equal heights are given, then the ratio of their curved surface areas is

Ratio of curved surface areas = ratio of radii

Ex: Two circular cylinders of equal heights have their radii in the ratio of 2 : 3 . Find the ratio of their curved surface areas.

Sol:

ratio of curved surface areas = ratio of radii

= 2 : 3

53. If the ratio of curved surface areas of two circular cylinders of equal heights are given, then the ratio of their volumes is

Ratio of volumes = $(\text{Ratio of curved surface areas})^2$

Ex: Two circular cylinders of equal heights have their curved surface areas in the ratio of 2 : 3. Find the ratio of their volumes.

Sol: Ratio of volumes = (Ratio of curved surface areas)^2 = $(2 : 3)^2$ = 4 : 9

D. When curved surface areas are equal.

54. If the ratio of radii of two circular cylinders of equal curved surface areas are given, then the ratio of volumes is calculated from the following result.

Ratio of volumes = Ratio of radii

Ex: Two circular cylinders of equal curved surface areas have their radii in the ratio of 3 : 4. Find the ratio of their volumes.

Sol: Ratio of volumes = Ratio of radii = 3 : 4

55. If the ratio of heights of two circular cylinders of equal curved surface areas are given, then the ratio fo their volumes is

Ratio of volumes = inverse ratio of heights.

Ex: Two circular cylinders fo equal curved surface areas have their heights in the ratio of 3 : 4. Find the ratio of their volumes.

Sol: Ratio of volumes

= Inverse ratio of heights = $\frac{1}{3} : \frac{1}{4}$ = 4 : 3

Two Cubes

58. If the ratio of sides of two cubes is given, then the ratio of their volumes–

Ratio of volumes = (Ratio of sides)^3

Ex: Sides of two cubes are in the ratio 2 : 3. Find the ratio of their volumes.

Sol:

Ratio of volumes

= (Ratio of sides)^3 = $(2 : 3)^3$ = 8 : 27

59. If the ratio of sides of two cubes is given, then the ratio of their surface ares is

Ratio of surface areas = (Ratio of sides)^2

Ex: Sides of two cubes are in the ratio of 2 : 3. Find the ratio of their surface areas.

Sol: Ratio of surface areas.

= (Ratio of sides)^2 $(2 : 3)^2$ = 4 : 9

60. If the ratio of volumes of two cubes the given, then the ratio fo their surface areas is

(ratio of surface areas)^3 = (ratio of volumes)^2

Ex: Volumes of two cubes are in the ratio of 1 : 8. Find the ratio of their surface areas.

Sol:

(Ratio of surface areas)^3 = (Ratio of volumes)^2 = $(1 : 8)^2$ = 1 : 64

$\therefore$ Ratio of surface areas = $\sqrt[3]{1:64}$ = 1 : 4

61. If the ratio of heights (not slant height) and the ratio of diameters or radii of two right circular cones are given, then the ratio of their volumes –

Ratio of volumes = (Ratio of radii)^2 × (ratio of heights)

Ex: If the heights of two cones are in the ratio 1 : 4 and their diameters in the ratio 4 : 5, what is the ratio of their volumes?

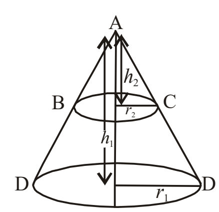
Sol:

Ratio of volumes = $(4 : 5)^2$ (1 : 4) = $\frac{16}{25} \times \frac{1}{4}$

= 4 : 25

[ratio of diameters = ratio of radii]

Concept:



If a plance BC cut the cone and V_1 and V_2 are the Volume of upper cone and orginal cone then,

$$\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3$$ (After using similarity)

62. If the ratio of radii and the ratio of volumes of two right circular cones are given, then the ratio of their heights –

Ratio of heights = (inverse ratio of radii)^2 (ratio of volumes)

Ex: If the radii of two cones are in the ratio 1 : 4 and their volumes in the ratio 4 : 5, what is the ratio of their heights?

Sol:

$$\text{Ratio of heights} = \left(\frac{1}{1}:\frac{1}{4}\right)^2 (4:5) = 16 \times \frac{4}{5}$$

$$= 64 : 5$$

63. If the ratio of volumes and the ratio of heights of two right circular cones are given, then the ratio their radii is

ratio of radii

$$= \sqrt{(\text{ratio of volumes})(\text{inverse ratio of heights})}$$

Ex: If the volumes of the two cones are in the ratio 4 : 1 and their heights in the ratio 4 : 9, what is the ratio of their radii?

$$\text{Ratio of radii} = \sqrt{(4:1)\left(\frac{1}{4}:\frac{1}{9}\right)} = \sqrt{(4:1)(9:4)}$$

$$= 3 : 1$$

64. If the ratio of heights and the ratio fo radii of two circular cylinders are given, then the ratio of their curved surface areas is

Ex: If the heights and the radii of two circular cylinders are in the ratio 2 : 3 and 1 : 2 respectively. Find the ratio of their curved surface areas.

Sol: required ratio = (2 : 3) (1 : 2) = 1 : 3

65. If the ratio of radii and the ratio of curved surface areas of two circular cylinders are given then the ratio of their heights are

(Ratio of curved surface areas) =(Inverse ratio of radii)

Ex: If the radii and the curved surface areas of two circular cylinders are in the ratio 3 : 5 and 6 : 7 respectively. Find the ratio of their heights.

Sol:

the required ratio

$$= (6:7)\left(\frac{1}{3}:\frac{1}{5}\right) = (6:7)(5:3) = \frac{30}{21} = 10:7$$

66. If the ratio of heights and the ratio of curved surface areas of two circular cylinders are given, then the ratio of their radii is (Ratio of curved surface areas) (Inverse ratio of heights)

Ex: If the heights and the curved surface areas of two circular cylinders are in the ratio 1 : 3 and 4 : 5 respectively. Find the ratio of their radii.

Sol:

$$\text{required ratio} = (4:5)\left(1:\frac{1}{3}\right) = (4:5)(3:1)$$

$$= 12 : 5$$

67. x units of rain has fallen on $a\ y$ square units of land. Assuming that R% of the raindrops could have been collected and contained in a pool having $a\ x_1$ units × y_1 units base, the level, by which the water level in the pool would have increased,

is given by $\frac{R}{100}\left(\frac{xy}{x_1y_1}\right)$ units.

Ex: Two cm of rain has fallen on a square km of land. Assuming that 50% of the raindrops could have been collected and contained in a pool having a 100 m × 10 m base, by what level would the water level in the pool have increased?

Sol:

Volume of rain water = Area × height

$= (1\text{km})^2 \times 2\text{cm} = (1000\text{ m})^2 \times 0.02\text{ m}$

$= 20{,}000\text{ m}^3$

Quantity of collected water

$= 50\%\text{ of }20{,}000\text{ m}^3 = \frac{1}{2} \times 20{,}000 = 10{,}000\text{ m}^3$

$\therefore$ increased level in pool

$$= \frac{\text{Volume collected}}{\text{Base area of pool}} = \frac{10000}{10 \times 100} = 10 \text{ m}.$$

Method :2

increased level in pool

$$= \left[\frac{(1000)2 \times 0.02}{100 \times 10}\right] \frac{50}{100} = \frac{20000}{1000} \times \frac{1}{2} = 10 \text{ m}.$$

68. If the radius of a cylinder becomes 'x' times and the height becomes 'y' times, then the ratio between the new volume and the previous volume is (x^2y).

Ex: If the radius of a cylinder is doubled and the height is half, what is the ratio between the new volume and the previous volume?

Sol:

Let the initial radius and height of the cylinder be r cm and h cm respectively.

Then, $V_1 = \pi r^2h$ and $V_2 = \pi (2r)^2 \frac{h}{2} = 2\pi r^2h$

$$\frac{\text{New volume}}{\text{Previous volume}} = \frac{2\pi r^2 h}{\pi r^2 h} = \frac{2}{1} = 2 : 1$$

Method-2

$x = 2$ and $y = \frac{1}{2}$

$\therefore$ required ratio $= (2)^2 \times \frac{1}{2} = 2 : 1$

69. If the radius of a circular cylinder becomes x times and the height becomes y times, then the ratio between the new curved surface area and the previous curved surface area of the cylinder is given by (xy).

Ex: If the radius of a cylinder is doubled and the height is half, what is the ratio between the new curved surface area and the previous curved surface area of the cylinder.

Sol: Let the initial radius and height of the cylinder be r cm and h cm respectively.

Then, curved surface area of the original cylinder $= 2\pi rh$ and

curved surface area of the new cylinder

$$= 2\pi(2r) \times \frac{h}{2} = 2\pi rh$$

$\therefore$ required ratio

$$= \frac{\text{New curved surface area}}{\text{Previous curved surface area}} = \frac{2\pi rh}{2\pi rh} = 1 : 1$$

Method :2

Here, $x = 2$ and $y = \frac{1}{2}$

$\therefore$ Required ratio $= 2 \times \frac{1}{2} = 1 : 1$

70. A well of 'D' m diameter or radius 'r' metre (here, $r = \frac{D}{2}$) is dug 'h' m deep. If the earth taken out has been spread all round it to a width of 'w' m to form a circular embankment, then the height of this embankment is given

by $\left[\frac{r^2 h}{w(w+D)}\right]$ metres.

Ex: A well of 11.2 m diameter is dug 8 m deep. The earth taken out has been spread all round it to a width of 7 m to form a circular embankment. Find the height of this embankment.

Sol: Volume of earth dug out

$$\pi r^2 h = \frac{22}{7} \times \left(\frac{11.2}{2}\right)^2 \times 8$$

$$= \frac{22}{7} \times 5.6 \times 5.6 \times 8 = 788.48 \text{ m}^3$$

Area of embankment = $\pi(5.6+7)^2 - \pi(5.6)^2$

$= \pi[(5.6+7)^2 - (5.6)^2]$

$= \pi[(5.6+7-5.6)(5.6+5.6+7)]$

$= \frac{22}{7} \times 7 \times 18.2 = 400.4 \text{ m}^2$

$\therefore$ height of the embankment $= \frac{788.48}{400.4}$

$= 1.97$ metres

Method:2

the height of embankment

$= \frac{5.6 \times 5.6 \times 8}{7(7+11.2)} = \frac{6.4 \times 5.6}{18.2} = 1.97$ metres.

71: A right-angled triangle having base x metres and height equal to y metres is turned around the height, a right circular cone is formed. Then,

(i) the volume of the cone $= \left[\frac{\pi}{3}x^2y\right]$ cubic metres and

(ii) the surface area of the cone

$= \left[\pi x\left(\sqrt{x^2+y^2}\right)\right]$ sq metres.

Ex: A right-angle triangle having base 3 metres and height equal to 4 metres, is turned around the height. Find the volume of the cone thus formed. Also find the surface area.

Sol: Required volume

$= \frac{\pi}{3}x^2y = \frac{\pi}{3} \times 3 \times 3 \times 4 = 12\pi$ cubi m.

Required surface area $= \pi x\left(\sqrt{x^2+y^2}\right)$

$= \pi \times 3 \times \sqrt{3^2+4^2} = 15\pi$ sq metres.

72. A right-angled triangle having base x metres and height equal to y metres is turned around the base, a right circular cone is formed. Then,

(i) the volume of the cone $= \left[\frac{\pi}{3}xy^2\right]$ cubic metres and

(ii) the surface area of the cone $= \pi y\left(\sqrt{x^2+y^2}\right)$ sq metres.

Ex: A right- angled triangle having base 3 metres and height equal to 4 metres, is turned around the base. Find the volume of the cone thus formed. Also find the surface area.

Sol:

Volume of the cone $= \frac{\pi}{3} \times 3 \times 4 \times 4 = 16\pi$ cu.m.

Surface area of the cone $= \pi \times 4 \times 5 = 20\pi$ sq m.

Radius = Height of the triangle

Slant height = Hypotenuse of the triangle.

73. If length, breadth and height of a cuboid is increased by x%, y% and z% respectively, then its volume is increased by

$$= \left[x+y+z+\frac{xy+xz+yz}{100}+\frac{xyz}{(100)^2}\right]\%$$

Ex: Two cubes each of edge 10 cm are joined to form a single cuboid. What is the surface area of the new cuboid so formed is given by ($10a^2$) sq metres.

Sol: Breadth and height of the new cuboid will remain as the edge of the cube but length of the cuboid will be doubled. Then for the cuboid;

length (l) = 2 × 10 = 20 cm.

breadth (b) = 10 cm

height (h) = 10 cm

$\therefore$ surface area of cuboid $= 2[lb + bh + lh]$

$= 2[20 \times 10 + 10 \times 10 + 20 \times 10]$

$= 2[500] = 1000 \text{ cm}^2$

Ex: A circular wire of radius 42 cm is cut and bent in the form of a rectangle whose sides are in the ratio of 6 : 5. Find the smaller side of the rectangle.

Sol: Length of the wire = circumference of the circle

$$= 2\pi \times 42 = \frac{2\times22\times42}{7} = 264 \text{ cm}$$

Now, perimeter of the rectangle = 264 cm

Since, perimeter includes double the length and breadth, while finding the sides we divide by double the sum of ratio.

Therefore, length = $\frac{264}{2(6+5)} \times 6 = 72$ cm

and breadth = $\frac{264}{2(6+5)} \times 5 = 60$ cm

Ex: A right circular cone is exactly fitted inside a cube in such a way that the edges of the base of the cone are touching the edges of one of the faces of the cube and the vertex is on the opposite face of the cube . If the volume of the cube is 343cc, what approximately is the volume of the cone?

Sol: Edge of the cube = $\sqrt[3]{343}$ = 7cm

$\therefore$ radius of cone = 3.5 cm.

*****

Exercise (Area & Perimeter)

1. The diagonal of a square is $4\sqrt{2}$ cm. The diagonal of another square whose area is double that of the first square is :

(a) $8\sqrt{2}$ cm (b) 16 cm

(c) $\sqrt{32}$ cm (d) 8 cm

2. If the diagonals of two squares are in the ratio of 2 : 5, their areas will be in the ratio of

(a) $\sqrt{2} : \sqrt{5}$ (b) 2 : 5

(c) 4 : 25 (d) 4 : 5

3. If the ratio of areas of two squares is 225 : 256. then the ratio of their perimeter is :

(a) 225 : 256 (b) 256 : 225

(c) 15 : 16 (d) 16 : 15

4. From four corners of a square sheet of side 4 cm. four pieces, each in the shape of arc of a circle with radius 2 cm. are cut out. The area of the remaining portion is:

(a) $(8 - \pi)$ sq.cm.

(b) $(16–4\pi)$ sq.cm.

(c) $(16 - 8\pi)$ sq.cm.

(d) $(4 - 2\pi)$ sq.cm.

5. How many tiles, each 4 decimetre square, will be required to cover the floor of a room 8 m long and 6 m broad ?

(a) 1200 (b) 1260

(c) 1280 (d) 1300

6. A circular wire of diameter 42 cm is folded in the shape of a rectangle whose sides are in the ratio 6 : 5. Find the area enclosed by the rectangle. (Take $\pi = \frac{22}{7}$)

(a) 540 cm^2 (b) 1080 cm^2

(c) 2160 cm^2 (d) 4320 cm^2

7. A took 15 sec. to cross a rectangular field diagonally walking at the rate of 52 m/min. and B took the same time to cross the same field along its sides walking at the rate of 68 m/min. The area of the field is :

(a) 30 m^2 (b) 40 m^2

(c) 50 m^2 (d) 60 m^2

8. A path of uniform width runs round the inside of a rectangular field 38 m long and 32 m wide. If the path occupies 600 m^2, then the width of the path is

(a) 30 m (b) 5 m

(c) 18.75 m (d) 10 m

9. The length and breadth of a rectangle are increased by 20% and 25% respectively. The increase in the area of the resulting rectangle will be:

(a) 60% (b) 50%

(c) 40% (d) 30%

10. A street of width 10 metres surrounds from outside a rectangular garden whose measurement is 200 m × 180 m. The area of the path (in square metres) is

(a) 8000 (b) 7000

(c) 7500 (d) 8200

11. In measuring the sides of a rectangle, there is an excess of 5% on one side and 2% deficit on the other. Then the error percent in the area is

(a) 3.3 (b) 3.0

(c) 2.9 (d) 2.7

12. A lawn is in the form of a rectangle having its length and breadth in the ratio 3 : 4. The area of the lawn is $\frac{1}{12}$ hectare. The length of the lawn is

(a) 25 metres (b) 50 metres

(c) 75 metres (d) 100 metres

13. The length and breadth of a rectangular field are in the ratio 7 : 4. A path 4 m wide running all round outside has an area of 416 m^2 . The breadth (in m) of the field is

(a) 28 (b) 14
(c) 15 (d) 16

14. If the area of a triangle is 1176 cm^2 and base : corresponding altitude is 3 : 4, then the altitude of the triangle is :

(a) 42 cm (b) 52 cm
(c) 54 cm (d) 56 cm

15. The sides of a triangle are in the ratio $\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$. If the perimeter of the triangle is 52 cm, the length of the smallest side is :

(a) 24 cm (b) 18 cm
(c) 12 cm (d) 9 cm

16. In a triangular field having sides 30 m, 72 m and 78 m, the length of the altitude to the side measuring 72 m is:

(a) 25 m (b) 28 m
(c) 30 m (d) 35 m

17. ABC is an equilateral triangle of side 2 cm. With A, B, C as centres and radius 1 cm three arcs are drawn. The area of the region within the triangle bounded by the three arcs is

(a) $\left(3\sqrt{3}-\frac{\pi}{2}\right)$ cm^2

(b) $\left(\sqrt{3}-\frac{3\pi}{2}\right)$ cm^2

(c) $\left(\sqrt{3}-\frac{\pi}{2}\right)$ cm^2

(d) $\left(\frac{\pi}{2}-\sqrt{3}\right)$ cm^2

18. In an equilateral triangle ABC of side 10cm, the side BC is trisected at D & E. Then the length (in cm) of AD is

(a) $3\sqrt{7}$ (b) $7\sqrt{3}$

(c) $\frac{10\sqrt{7}}{3}$ (d) $\frac{7\sqrt{10}}{3}$

19. If the numerical value of the perimeter of an equilateral triangle is $\sqrt{3}$ times the area of it, then the length of each side of the triangle is

(a) 2 unit (b) 3 unit
(c) 4 unit (d) 6 unit

20. The ratio of sides of a triangle is 3 : 4 : 5 and area of the triangle is 72 square unit. Then the area of an equilateral triangle whose perimeter is same as that of the previous triangle is

(a) $32\sqrt{3}$ square unit

(b) $48\sqrt{3}$ square unit

(c) $24\sqrt{3}$ square unit

(d) $64\sqrt{3}$ square unit

21. An isosceles right angled triangle is inscribed in a semi-circle of radius 7 cm. The area enclosed by the semi-circle but exterior to the triangle is

(a) 14 cm^2 (b) 28 cm^2
(c) 44 cm^2 (d) 68 cm^2

22. The altitude drawn to the base of an isosceles triangle is 8 cm and its perimeter is 64 cm. The area (in cm^2) of the triangle is

(a) 240 (b) 180
(c) 360 (d) 120

23. The diagonals of a rhombus are 24 cm and 10 cm. The perimeter of the rhombus (in cm) is :
(a) 68 (b) 65
(c) 54 (d) 52

24. The perimeter of a rhombus is 40 cm. If one of the diagonals be 12 cm long, what is the length of the other diagonal?
(a) 12 cm (b) $\sqrt{136}$ cm
(c) 16 cm (d) $\sqrt{44}$ cm

25. The area of a field in the shape of a trapezium measures 1440 m^2. The perpendicular distance between its parallel sides is 24 m. If the ratio of the parallel sides is 5 : 3, the length of the longer parallel side is:
(a) 75 m (b) 45 m
(c) 120 m (d) 60 m

26. The length of one side of a rhombus is 6.5 cm and its altitude is 10 cm. If the length of one of its diagonals be 26 cm, the length of the other diagonal will be :
(a) 5 cm (b) 10 cm
(c) 6.5 cm (d) 26 cm

27. A parallelogram has sides 15 cm and 7 cm long. The length of one of the diagonals is 20 cm. The area of the parallelogram is
(a) 42 cm^2 (b) 60 cm^2
(c) 84 cm^2 (d) 96 cm^2

28. A parallelogram ABCD has sides AB = 24 cm and AD = 16 cm. The distance between the sides AB and DC is 10 cm. Find the distance between the sides AD and BC.
(a) 16 cm. (b) 18 cm.
(c) 15 cm. (d) 26 cm.

29. One of the four angles of a rhombus is 60°. If the length of each side of the rhombus is 8 cm, then the length of the longer diagonal is
(a) $8\sqrt{3}$ cm (b) 8 cm
(c) $4\sqrt{3}$ cm (d) $\frac{8}{\sqrt{3}}$ cm

30. The length of each side of a rhombus is equal to the length of the side of a square whose diagonal is $40\sqrt{2}$ cm. If the lengths of the diagonals of the rhombus are in the ratio 8 : 9, then longer diagonal (in cm) is
(a) 30 (b) 60
(c) 20 (d) 40

31. Each side of a regular hexagon is 1 cm. The area of the hexagon is
(a) $\frac{3\sqrt{3}}{2}$ cm^2 (b) $\frac{3\sqrt{3}}{4}$ cm^2
(c) $4\sqrt{3}$ cm^2 (d) $3\sqrt{2}$ cm^2

32. The diameter of a circular wheel is 7 m. How many revolutions will it make in travelling 22 km?
(a) 100 (b) 400
(c) 500 (d) 1000

33. The area of the ring between two concentric circles, whose circumferences are 88 cm and 132 cm. is:
(a) 780 cm^2 (b) 770 cm^2
(c) 715 cm^2 (d) 660 cm^2

34. A can go round a circular path 8 times in 40 minutes. If the diameter of the circle is increased to 10 times the original diameter, the time required by A to go round the new path once travelling at the same speed as before is:
(a) 25 min (b) 20 min
(c) 50 min (d) 100 min

35. Three circles of radius 3.5 cm each are placed in such a way that each touches the other two. The area of the portion enclosed by the circles is

(a) 1.905 cm^2 (b) 1.985 cm^2

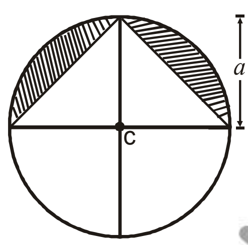
(c) 19.67 cm^2 (d) 21.21 cm^2

36. The area of a circle is increased by 22 cm. Its radius is increased by 1 cm. The original radius of the circle is

(a) 6 cm (b) 3.2 cm

(c) 3 cm (d) 3.5 cm

37. The area of the shaded region in the figure given below is



(a) $\frac{a^2}{2}\left(\frac{\pi}{2}-1\right)$ sq. units

(b) $a^2(\pi - 1)$ sq. units

(c) $a^2\left(\frac{\pi}{2}-1\right)$ sq. units

(d) $\frac{a^2}{2}(\pi-1)$ sq. units

38. The wheel of a motor car makes 1000 revolutions in moving 440 m. The diameter (in metre) of the wheel is

(a) 0.44 (b) 0.14

(c) 0.24 (d) 0.34

39. The circumference of a circle is 11 cm and the angle of a sector of the circle is 60°. The area of the sector is (Use $\pi = \frac{22}{7}$)

(a) $1\frac{29}{48}$ cm^2 (b) $2\frac{29}{48}$ cm^2

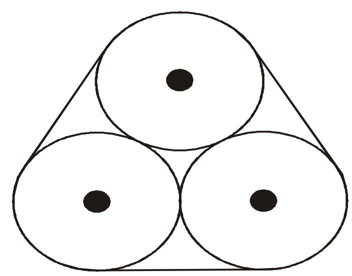
(c) $1\frac{27}{48}$ cm^2 (d) $2\frac{27}{48}$ cm^2

40. A gear 12 cm in diameter is turning a gear 18 cm in diameter. When the smaller gear has 42 revolutions, how many has the larger one made?

(a) 28 (b) 20

(c) 15 (d) 24

41. Three circles of diameter 10 cm each, are bound together by a rubber band, as shown in the figure.



The length of the rubber band, in cm, if it is stretched as shown, is

(a) 30 (b) $30 + 10\pi$

(c) 10π (d) $60 + 20\pi$

42. A 7 m wide road runs outside around a circular park, whose circumference is 176 m. The area of the road is: (use $\pi = \frac{22}{7}$)

(a) 1386 m^2 (b) 1472 m^2

(c) 1512 m^2 (d) 1760 m^2

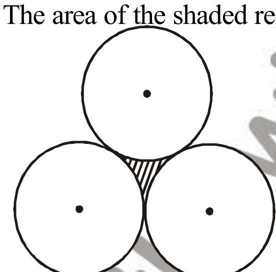
43. A circular road runs around a circular ground. If the difference between the circumferences of the outer circle and the inner circle is 66 metres, the width of the road is: (Take $\pi = \frac{22}{7}$)

(a) 10.5 metres (b) 7 metres
(c) 5.25 metres (d) 21 metres

44. A person observed that he required 30 seconds less time to cross a circular ground along its diameter than to cover it once along the boundary. If his speed was 30 m/minute, then the radius of the circular ground is (Take $\pi = \frac{22}{7}$)

(a) 5.5 m (b) 7.5 m
(c) 10.5 m (d) 3.5 m

45. Three circles of equal radius 'a' cm touch each other. The area of the shaded region is :



(a) $\left(\frac{\sqrt{3}+\pi}{2}\right)a^2$ sq. cm.

(b) $\left(\frac{6\sqrt{3}-\pi}{2}\right)a^2$ sq. cm.

(c) $\left(\sqrt{3}-\pi\right)a^2$ sq. cm.

(d) $\left(\frac{2\sqrt{3}-\pi}{2}\right)a^2$ sq. cm.

46. The area of the greatest circle, which can be inscribed in a square whose perimeter is 120 cm, is:

(a) $\frac{22}{7}\times(15)^2$ cm^2

(b) $\frac{22}{7}\times\left(\frac{7}{2}\right)^2$ cm^2

(c) $\left(\sqrt{3}-\pi\right)a^2$ cm^2

(d) $\left(\frac{2\sqrt{3}-\pi}{2}\right)a^2$ cm^2

47. The circumference of a circle is 100 cm. The side of a square inscribed in the circle is

(a) $\left(\frac{100\sqrt{2}}{\pi}\right)$ cm (b) $\left(\frac{50\sqrt{2}}{\pi}\right)$ cm

(a) $\frac{100}{\pi}$ cm (d) $50\sqrt{2}$ cm

48. The area of the largest circle, that can be drawn inside a rectangle with sides 18 cm by 14 cm, is

(a) 49 cm^2 (b) 154 cm^2
(c) 378 cm^2 (d) 1078 cm^2

49. The sides of a triangle are 6 cm, 8 cm and 10 cm. The area of the greatest square that can be inscribed in it, is

(a) 18 cm^2 (b) 15 cm^2

(c) $\frac{2304}{49}$ cm^2 (d) $\frac{576}{49}$ cm^2

50. The radius of the in-circle of a triangle is 2 cm. If the area of the triangle is 6 cm^2, then its perimeter is
(a) 2 cm (b) 3 cm
(c) 6 cm (d) 9 cm

51. A circle is inscribed in an equilateral triangle and a square is inscribed in that circle. The ratio of the areas of the triangle and the square is
(a) $\sqrt{3} : 4$ (b) $\sqrt{3} : 8$
(c) $3\sqrt{3} : 2$ (d) $3\sqrt{3} : 1$

52. The perimeter of a rectangle is 160 metre and the difference of two sides is 48 metre. Find the side of a square whose area is equal to the area of this rectangle?
(a) 32 m (b) 8 m
(c) 4 m (d) 16 m

53. If the radius of a circle is increased by 50%, its area is increased by:
(a) 125% (b) 100%
(c) 75% (d) 50%

54. The cost of cultivating a square field at the rate of Rs. 160 per hectare is Rs. 1440. The cost of putting a fence around it at the rate fo 75 paise per metre is :
(a) Rs. 900 (b) Rs. 1800
(c) Rs. 360 (d) Rs. 810

55. The length of a rectangle is twice its breadth. If its length is decreased by 5 cm and breadth is increased by 5 cm, the area of the rectangle is increased by 75 cm^2. Then the length of the rectangle is :
(a) 20 cm (b) 30 cm
(c) 40 cm (d) 50 cm

56. The dimensions of the floor of a rectangular hall are 4 m × 3 m. The floor of the hall is to be tiled fully with 8 cm × 6 cm rectangular tiles without breaking tiles to smaller size. The number of tiles required is
(a) 4800 (b) 2600
(c) 2500 (d) 2400

57. A square lawn with side 100 m long has a circular flower bed in the centre. If the area of the lawn, excluding the flower bed is 8614 m^2, the radius of the circular flower bed is :
(a) 31 m (b) 21 m
(c) 41 m (d) None of these

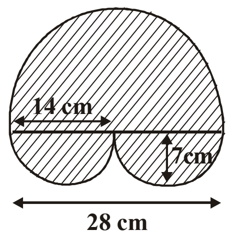
58. A paper is in the form of a square of side 20 m. Semi-circles are drawn inside the square paper on two opposite sides as diameter. The semi-circular portions are cut off. The area of the remaining paper is :
(a) $(40 - 2\pi)m^2$
(b) $(400 - 200\pi)m^2$
(c) $(400 - 100\pi)m^2$
(d) $200\pi\ m^2$

59. If the area of a circle be equal to that of a square, then the ratio of the side of the square and the radius of the circle is :
(a) $\sqrt{\pi} : 1$ (b) $1 : \sqrt{\pi}$
(c) $1 : \pi$ (d) $\pi : 1$

60. A wire is in the form of an equilateral triangle with area 5 m^2. If it is changed into a circle, the radius will be
(a) $\frac{3\sqrt{20}}{2\pi}$ m (b) $3^{3/4}\frac{\sqrt{20}}{2\pi}$ m
(c) $\sqrt{3}\frac{\sqrt{20}}{2\pi}$ m (d) $\frac{3^{3/4}\sqrt{20}}{2\pi}$ m

61. The area from an arc to the centre of circle is 12.4 sq. cm. If this arc subtends an angle of 60° at the centre, what will be the area of the remaining part of the circle?
(a) 70 sq.cm (b) 80 sq.cm
(c) 62 sq.cm (d) 85 sq.cm

62. In the given figure, the area of the shaded region (in cm^2), is

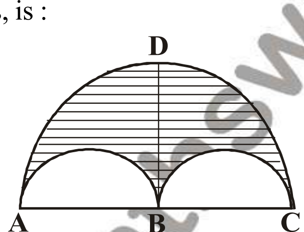


(a) 324 (b) 428
(c) 462 (d) 500

63. A circle of radius a is divided into 6 equal sectors. An equilateral triangle is dranw on the chord of each sector to lie outside the circle. Area of the resulting figure is

(a) $\frac{3\sqrt{3}\pi a^2}{2}$ (b) $\frac{3\sqrt{3}a^2}{2}$
(c) $3\pi a^2$ (d) $3\sqrt{3}\,a^2$

64. Let AB = 4 cms & BD = 4 $\sqrt{3}$ cms. Then the area (shaded) bounded by three semi-circles are shown in the figure, in square cms, is :

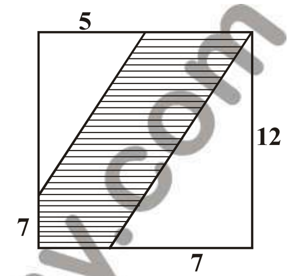


(a) 48π (b) 24π
(c) 16π (d) 12π

65. The length of the diagonal of a rhombus is 80% of the length of the other diagonal. Then, the area of the rhombus is how many times the square of the side of the longer diagonal?

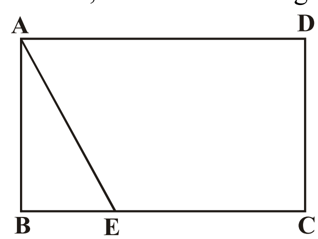
(a) $\frac{4}{5}$ (b) $\frac{2}{5}$
(c) $\frac{3}{4}$ (d) $\frac{1}{4}$ vw

66. The area of the shaded portion in the given figure is



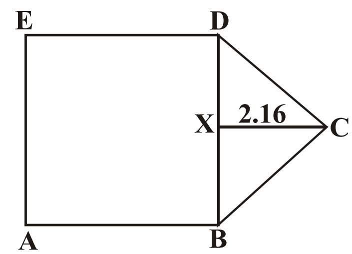
(a) 77 sq. units (b) 89.5 sq. units
(c) 72 sq. units (d) 69 sq. units

67. In the adjoining figure, ABCD is a rectangle and area of Δ ABE = 15 cm^2. If EC = 2 BE, area of the rectangle in cm^2 is



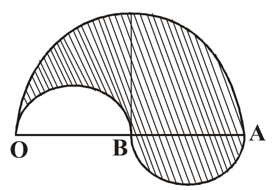
(a) 24 (b) 48
(c) 90 (d) 120

68. In the given figure, AB = BD = DE = EA = 2.5 cm. If BX = XD, then the area of the figure (in sq cm) is



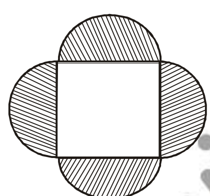
(a) 18.41 cm^2 (b) 15.40 cm^2
(c) 12.70 cm^2 (d) 8.95 cm^2

69. The boundary of the shaded region in the given diagram consists of three semi-circular arcs, the smaller ones being equal. If the diameter of the larger arc is 10 cm, the area of the shaded region is ($\pi = 3.14$):



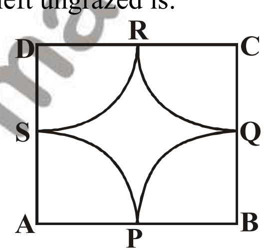
(a) 39.25 cm^2 (b) 46.45 cm^2
(c) 35.60 cm^2 (d) 37.95cm^2

70. A bed of roses is like the adjoining diagram. In the centre is a square and on each side there is a semi-circle. The side of the square is 21 metres. If each rose plant needs 6m^2 of space, the number of plants is



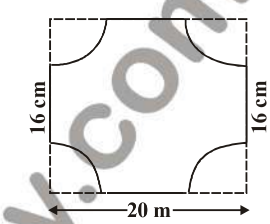
(a) 176 (b) 163
(c) 168 (d) 189

71. Four horses are tethered at four corners of a square plot of side 63 metres, so that they just cannot reach one another. The area left ungrazed is:



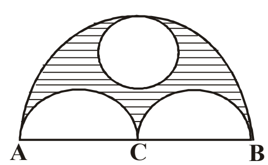
(a) 675.5 m^2 (b) 780.6 m^2
(c) 785.8 m^2 (d) 850.5 m^2

72. A rectangular piece is 20 m long and 16 m wide. From its four corners, quadrants of radii 3.5 metres have been cut. The area of the remaining part is



(a) 281.5 m^2 (b) 276.4 m^2
(c) 265.6 m^2 (d) 264.8 m^2

73. In the given figure, the diameter of the biggest semi-circle is 56 cms. and the radius of the smallest circle is 7 cms. The area of the shaded portion is:



(a) 482 cm^2 (b) 462 cm^2
(c) 654 cm^2 (d) 804 cm^2

74. The perimeters of five squares are 24 cm, 32 cm, 40 cm, 76 cm and 80 cm respectively. The perimeter of another square equal in area to sum of the areas of these squares is:
(a) 31 cm (b) 62 cm
(c) 124 cm (d) 961 cm

75. The perimeter (in metres) of a semicircle in numerically equal to its area (in square metres). The length of its diameter is (Take $\pi = \frac{22}{7}$)

(a) $3\frac{3}{11}$ metres (b) $5\frac{6}{11}$ metres

(c) $6\frac{6}{11}$ metres (d) $6\frac{2}{11}$ metres

76. The circumference of a circle is 11 cm and the angle of a sector of the circle is 60°. The area of the sector is (Take $\pi = \frac{22}{7}$)

(a) $1\frac{29}{48}$ cm^2 (b) $2\frac{29}{48}$ cm^2
(c) $1\frac{27}{48}$ cm^2 (d) $2\frac{27}{48}$ cm^2

77. The area of a circle is proportional to the square of its radius. A small cirlce of radius 3 cm is drawn within a larger circle of radius 5 cm. Find the ratio of the area of the annular zone to the area of the larger circle. (Area of the annular zone is the difference between the area of the larger circle and that of the smaller circle).

(a) 9 : 16 (b) 9 : 25
(c) 16 : 25 (d) 16 : 27

78. If the difference between areas of the circumcircle and the incircle of an equilateral triangle is 44 cm^2, then the area of the triangle is (Take $\pi = \frac{22}{7}$)

(a) 28 cm^2 (b) $7\sqrt{3}$ cm^2
(c) $14\sqrt{3}$ cm^2 (d) 21 cm^2

79. The perimeters of a square and a circular field are the same. If the area of the circular field is 3850 sq metres, what is the area (in m^2) of the square?

(a) 4225 (b) 3025
(c) 2500 (d) 2025

80. The areas of a square and a rectangle are equal. The length of the rectangle is greater than the length of any side of the square by 5 cm and the breadth is less by 3 cm. Find the perimeter of the rectangle.

(a) 17 cm (b) 26 cm
(c) 30 cm (d) 34 cm

81. Four equal sized maximum circular plates are cut off from a square paper sheet of area 784 sq. cm. The circumference of each plate is (Take p = $\frac{22}{7}$)

(a) 22 cm (b) 44 cm
(c) 66 cm (d) 88 cm

82. A cow is tied on the corner of a rectangular field of size 30 m x 20 m by a 14 m long rope. The area of the region, that she can graze, is (Take $\pi = \frac{22}{7}$)

(a) 350 m^2 (b) 196 m^2
(c) 154 m^2 (d) 22 m^2

83. A roller 150 cm long has diameter 70 cm. To level a playground, it takes 750 complete revolutions. The cost of levelling the playground at the rate of Rs. 2 per m^2 is

(a) Rs. 5000 (b) Rs. 2950
(c) Rs. 4500 (d) Rs. 4950

84. A circular swimming pool with a diameter of 28 ft has a deck of uniform width built around it. If the area of the deck is 60 π sq ft, find its width.

(a) 3 ft (b) 2.8 ft
(c) 2 ft (d) 2.5 ft

85. Length of a rectangular blackboard is 8 m more than that of its breadth. If its length is increased by 7m and its breadth is decreased by 4m , its area remains unchanged. The length and breadth of the rectangular blackboard is

(a) 24 m, 16 m (b) 20 m, 24 m
(c) 28 m, 20 m (d) 28 m, 16 m

86 A rectangular farm has to be fenced on one long side, one short side and the diagonal. If the cost of fencing is Rs. 100 per m, the area of the farm is 1200 m^2 and the short side is 30 m long. How much would the job cost?

(a) Rs. 14000 (b) Rs. 12000
(c) Rs. 7000 (d) Rs. 15000

87. A lawn is in the form of an isosceles triangle . The cost of turfing it came to Rs. 1200 at Rs. 4 per m^2 . If the base be 40m long. Find the length of equal side.

(a) 25 m (b) 24 m
(c) 26 m (d) None of these

88. The length and breadth of a playground are 36 m and 21 m respectively, Poles are required to be fixed all along the boundary at a distance 3 m apart. The number of poles required will be

(a) 39 (b) 38
(c) 37 (d) 40

89. Four sheets 50 cm × 5 cm are arranged without overlapping to form a square having side 55 m. What is the area of inner square so formed?

(a) $2500\,cm^2$ (b) $2025\,cm^2$
(c) $1600\,cm^2$ (
d) None of these

90. Four horses are tied on the four corners of a square field of 14 m length so that each horse can just touch the other two horses. They were able to graze in the area accessible to them for 11 days. For how many days is the ungrazed area sufficient for them?

(a) 3 days (b) 4 days
(c) 5 days (d) 2 days

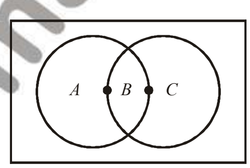
91. The circumference of a circular ground is 88 m. A strip of land, 3 m wide, inside and along circumference of the ground is to be levelled. What is the budget expenditure if the levelling cost is Rs. 7 per sq m?

(a) Rs. 1050 (b) Rs. 1125
(c) Rs. 1325 (d) Rs. 1650

92. A ladder 15 m long reaches a window which is 9 m above the ground on one side of a street. Keeping its foot at the same point, the ladder is turned to the other side of the street to reach a window 12 m high. Find the width of the street.

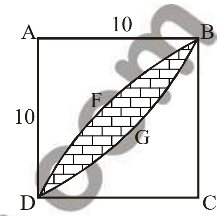
(a) 19 m (b) 21 m
(c) 20 m (d) 22 m

93. In the given diagram, two circles pass through each other's centre. If the radius of each circle is 2, then what is the perimeter of the region marked B?



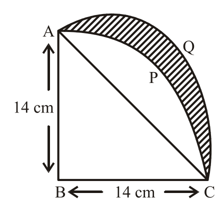
(a) $(8/3)\pi$ (b) $(4/5)\pi$
(c) 4π (d) $(5/3)\pi$

94. In the figure, ABCD is a square with side 10. BFD is an arc of a circle with centre C. BGD is an arc of a circle with centre A. What is the area of the shaded region?



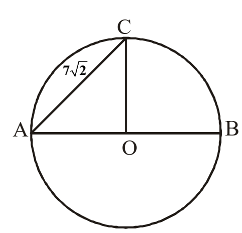
(a) $100-50\pi$ (b) $100-25\pi$
(c) $50\pi-100$ (d) $25\pi-100$

95. ABCP is a quadrant of a circle of radius 14 cm. With AC as diameter, a semi-circle is drawn. Find the area of the shaded portion .



(a) $49\,cm^2$ (b) $196\,cm^2$
(c) $98\,cm^2$ (d) None of these

96. In the accompanying figure, AB is one of the the diameters of the circle and OC is perpendicular to it through the centre O. If AC is $7\sqrt{2}$ cm, then what is the area of the circle in sq cm?



(a) 24.5 (b) 49
(c) 98 (d) 154

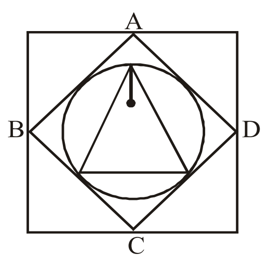
97. The cross section of a canal is in the form of a trapezium. If the top is 10 m wide, the bottom is 6 m wide and the area of the cross section is 72 m^2, then the depth of the canal is

(a) 10 m (b) 7 m
(c) 6 m (d) 9 m

98. Semi-circular lawns are attatched to both the edges of a rectangular field measuring 42× 35 m. The area of the total field is

(a) 3818.5 m^2 (b) 8318 m^2
(c) 5813 m^2 (d) 1358 m^2

99. What is the area of the inner equilateral triangle, if the side of the outermost square is '*a*'? (ABCD is a square)

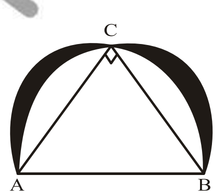


(a) $\frac{\sqrt{3}a}{2\sqrt{2}}$ (b) $\frac{2\sqrt{3}a}{3}$

(c) $\frac{2\sqrt{2}a}{\sqrt{3}}$

(d) None of these

100. In given figure, ABC is a right traingle in which C is the right angle. Three semi circles are drawn on diameter AB, BC and AC. The area of triangle ABC is 37 sq. unit then the area of shaded portion.



(a) 24 sq. unit (b) 37 sq. unit
(c) (18.5π +2 sq. unit (d) 18.5 sq. unit

101. A circle and a rectangle have the same perimeter. The sides of the rectangle are 26 cm and 18 cm. The area of the circle is [taken $\pi = \frac{22}{7}$]

(a) 125 cm^2 (b) 230 cm^2
(c) 550 cm^2 (d) 616 cm^2

102. A playground is in the shape of a rectangle. A sum of Rs. 1,000 was spent to make the ground usable at the rate of 25 paise per sq. m. The breadth of the ground is 50 m. If the length of the ground is increased by 20 m, what will be the expenditure in rupees at the same rate per sq. m.?

(a) 1,250 (b) 1,000
(c) 1,500 (d) 2,250

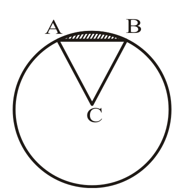
103. It is proposed to do marble flooring of a showroom of dimensions 60 m long and 20 m broad. The marble blocks are available in the size $3m \times 2m$ at the rate of Rs. 100 per piece. Find the number of marble blocks required and the total cost.

(a) Rs. 20,000 & 200 (b) Rs. 30,000 & 300
(c) Rs. 15,000 & 150 (d) Rs. 18,000 & 180

104. Certain number of paving stones each measuring $3m \times 2m$ are required to pave a rectangular courtyard 40 m long and 13.5 m wide. What amount needs to be spent if the tiles of the a fore-said dimension are available at Rs. 3 per piece?

(a) Rs. 270 (b) Rs. 270
(c) Rs. 370 (d) Rs. 240

105. In the given figure ABC is an equilateral triangle and C is the centre of the circle. A &B are points on the circle what is the area of the shaded region if diameter of a circle is 28 cm?



(a) $\left(102\frac{2}{3} - 49\sqrt{3}\right)$ cm^2

(b) $\left(102\frac{2}{3} - 48\sqrt{3}\right)$ cm^2

(c) $\left(109 - 98\sqrt{3}\right)$ cm^2

(d) None of these

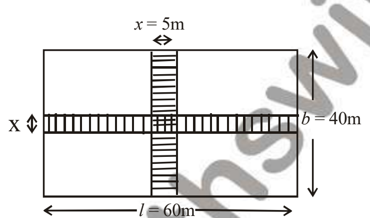
106. A rectangular field is (25m $\times$ 15m). Two mutually perpendicular passages of width 2m have been left in its central and grass has been grown rest of the field. The area under the grass is

(a) 295 m^2 (b) 299 m^2
(c) 200 m^2 (d) 375 m^2

107. The area of a rectangle is 4 times of the area of a square. The length of the rectangle is 90cm and the breadth of the rectangle is $\frac{2}{3}$ rd of the side of the square. What is the side of the square?

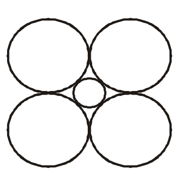
(a) 10 cm (b) 15 cm
(c) 20 cm (d) Couldn't be determined

108. A rectangular lawn 60×40 m^2 has two roads each 5m wide running between the park. One is parallel to width. Cost of gravelling is 60 paise/m^2. Find the total cost of gravelling?



(a) Rs. 285 (b) Rs. 300
(c) Rs. 275 (d) Rs. 270

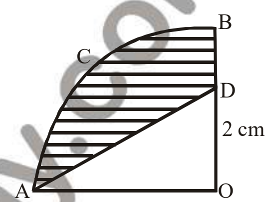
109. In the given figure, when the outer circles all have radii 'R' then the radius of the inner circle will be:



(a) $\frac{2}{(\sqrt{2}+1)R}$ (b) $\frac{1}{\sqrt{2}}R$

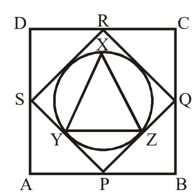
(c) $(\sqrt{2}-1)R$ (d) $\sqrt{2}R$

110. In the adjoining figure, AOBCA represents a quadrant of a circle of radius 4 cm with centre O. Calculate the area of the shaded portion.



(a) 8.56cm^2 (b) 7.35cm^2
(c) 8.45cm^2 (d) 9cm^2

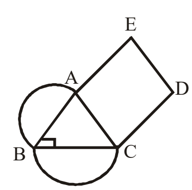
111. In the given figure ABCD is a square and PQRS is also a square made by joining the mid-points of the sides of the larger square ABCD. There is a inscribed the circle. In PQRS and an equilateral Δ XYZ inscribed in the circle
Find the ratio of the square ABCD to the side of the equilateral triangle XYZ.



(a) $\sqrt{2}:\sqrt{3}$ (b) $2\sqrt{2}:3$

(c) $2\sqrt{2}:\sqrt{3}$ (d) None of these

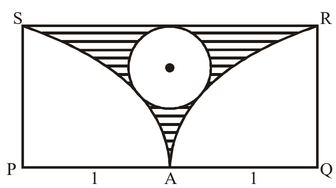
112. The area of the square on AC as a side is 128 cm^2. What is the sum of the area of semicircles drawn on AB and AC as diameters, given ABC is an isoceles right angled triangle and AC is its hypotenuse.



(a) 32π cm^2 (b) 16π cm^2

(c) 16cm^2 (d) 32cm^2

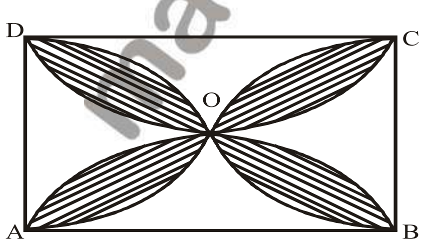
113. In the following figure PQRS is a rectangle with PS and RS equal to 1 and 2 units respectively. Two quarter circles are drawn with centres at Q and P respectively. Now a circle is drawn touching both the quarter circles and done of the sides of the rectangle. Find the area of the shaded region:



(a) $\frac{32}{115}$ square units (b) $\frac{13}{56}$ square units

(c) $\frac{16}{83}$ square units (d) $\frac{7}{20}$ square units

114. In the given figure ABCD is a square. Four equal semicircles are drawn in such a way the they meet each other at 'o'. Sides AB, BC, CA and DA are the reprective diameters of the four simicircles. Each of the side of the square is simicircles. Each of the side of the square is 8cm. Find the area of the shaded region.

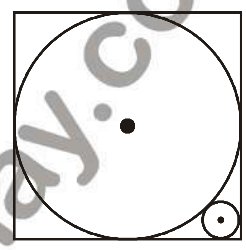


(a) $32(\pi - 2)$ cm^2 (b) $16(\pi - 2)$cm^2

(c) $(2\pi - 8)$ cm^2 (d) $\left(\frac{3}{4}\pi - 4\right)$cm^2

115 In the given figure, find the radius of smaller circle (r),

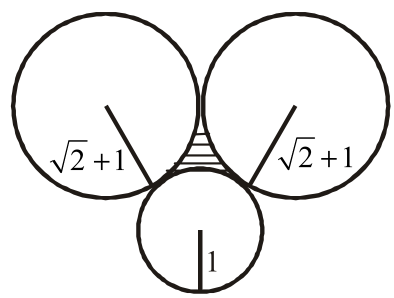
If the radius of larger circle is R.



(a) $\left(\sqrt{3} - 2\sqrt{2}\right)$R (b) $2\left(\sqrt{2} - 1\right)$R

(c) $\left(3 - 2\sqrt{2}\right)$R (d) None of these

116 Three circles of radius $\sqrt{2}+1, \sqrt{2}+1$ and 1 unit, touch each other externally, then find the perimeter of the sorroundinded part by three circles.



(a) $\frac{\pi}{2}\left(2\sqrt{2}+2\right)$ (b) $\frac{\pi}{2}\left(\sqrt{2}+2\right)$

(c) $\pi\left(\sqrt{2}+2\right)$ (d) None of these

Answer

1.	(d)	2.	(c)	3.	(c)	4.	(b)	5.	(a)	6.	(b)	7.	(d)	8.	(b)	9.	(b)
10.	(a)	11.	(c)	12.	(a)	13.	(d)	14.	(d)	15.	(c)	16.	(c)	17.	(c)	18.	(c)
19.	(c)	20.	(b)	21.	(b)	22.	(d)	23.	(d)	24.	(c)	25.	(a)	26.	(a)	27.	(c)
28.	(c)	29.	(a)	30.	(b)	31.	(a)	32.	(c)	33.	(b)	34.	(c)	35.	(b)	36.	(c)
37.	(c)	38.	(b)	39.	(a)	40.	(a)	41.	(b)	42.	(a)	43.	(a)	44.	(d)	45.	(d)
46.	(a)	47.	(b)	48.	(b)	49.	(d)	50.	(c)	51.	(c)	52.	(a)	53.	(a)	54.	(a)
55.	(c)	56.	(c)	57.	(b)	58.	(c)	59.	(a)	60.	(d)	61.	(c)	62.	(c)	63.	(d)
64.	(d)	65.	(b)	66.	(b)	67.	(c)	68.	(d)	69.	(a)	70.	(d)	71.	(d)	72.	(a)
73.	(b)	74.	(c)	75.	(c)	76.	(a)	77.	(c)	78.	(c)	79.	(b)	80.	(d)	81.	(b)
82.	(c)	83.	(d)	84.	(c)	85.	(c)	86.	(b)	87.	(a)	88.	(a)	89.	(b)	90.	(a)
91.	(d)	92.	(b)	93.	(a)	94.	(c)	95.	(c)	96.	(d)	97.	(d)	98.	(a)	99.	(a)
100.	(b)	101.	(d)	102.	(a)	103.	(a)	104.	(b)	105.	(a)	106.	(b)	107.	(b)	108.	(a)
109.	(c)	110.	(a)	111.	(c)	112.	(b)	113.	(b)	114.	(a)	115.	(c)	116.	(b)		

Solution & Hints

Sol 1. diagonal of first square $= 4\sqrt{2} = a\sqrt{2}$

then side $a = 4$

hence area of Ist square $= a^2 = 16$

area of second square $= 2 \times 16$

$A^2 = 32$

$\Rightarrow \quad A = 4\sqrt{2}$

diagonal $= A\sqrt{2} = 8$

Or,

area $\propto$ (diagonal)^2

$$\frac{A_1}{A_2} = \frac{d_1^{\,2}}{d_2^{\,2}} \Rightarrow \frac{A_1}{2A_1} = \frac{d_1^{\,2}}{d_2^{\,2}}$$

$$\Rightarrow \frac{d_1}{d_2} = \frac{1}{\sqrt{2}} \Rightarrow d_2 = d_1\sqrt{2} = 8$$

Sol 2. $\frac{d_1}{d_2} = \frac{2}{5} \Rightarrow \frac{A_1}{A_2} = \left(\frac{d_1}{d_2}\right)^2 = \frac{4}{25}$

Sol 3. area $\propto$ (perimeter)^2

$$\frac{A_1}{A_2} = \left(\frac{P_1}{P_2}\right)^2 \Rightarrow \frac{P_1}{P_2} = \sqrt{\frac{A_1}{A_2}}$$

$$\frac{P_1}{P_2} = \sqrt{\frac{225}{256}} = \frac{15}{16}$$

Sol 4.

area of remaining portion

= area of square – area of four sector

$= (4)^2 - 4\left(\frac{1}{4}\pi(2)^2\right)$

$= 16 - 4\pi$

Sol 5. No. of tiles required $= \frac{\text{area of floor}}{\text{area of 1 tile}}$

$$= \frac{8 \times 6}{4 \times 10^{-2}} = 1200$$

($\because$ $1m = 10$ decimeter
$1m^2 = 100\ dm^2$)

Sol 6. length of wire $= 2\pi r$

$= 2 \times \frac{22}{7} \times 21$

$= 132$ cm

let length $= 6k$, breath $= 5k$

$2(l + b) = 132$

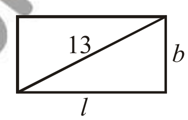
$2(11k) = 132 \Rightarrow k = 6$

$\Rightarrow$ area $= 6k.\ 5k = 36 \times 30 = 1080\ cm^2$

Sol 7. length of diagonal

= distance covered by A in 15 second.

$= 52 \times \frac{15}{60} = 13$ m



sum of length and breadth

= distance covered by B in 15 second

$= 68 \times \frac{15}{60} = 17$ m

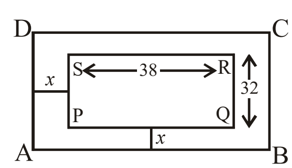
$l + b = 17$

we know that hypotenuse in a right angle triangle is 13 then l and b may be 5 and 12

($\because$ $l + b = 17$)

$\Rightarrow$ area $= 5 \times 12 = 60\ m^2$

Sol 8.



area of path = area of rectangle ABCD – area of rectangle PQRS

$600 = (38 \times 32) - (38 - 2x)(32 - 2x)$

$600 = 1216 - 1216 + 76x + 64x - 4x^2$

$4x^2 - 140x + 600 = 0$

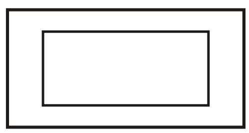
$x^2 - 35x + 150 = 0$
$x^2 - 30x - 5x + 150 = 0$
$x(x-30) - 5(x-30) = 0$
$(x-30)(x-5) = 0$
$x = 5 \quad (\because x \neq 30)$
other wise inner rectangale is not possible

Sol 9. Effect on area $= l + b + \frac{lb}{100}$

$= 20 + 25 + \frac{20 \times 25}{100}$

$= 50\%$

Sol 10.



$xy = AB$
$= 200 + 10 + 10$
$= 220$
area of path = $2 \times$ area of rectangle XABY
$+ 2 \times$ area of rectangle PQYZ
$= 2 \times 220 \times 10 + 2 \times 180 \times 10 = 8000\ m^2$

Sol 11. Effect on area $= l + b + \frac{lb}{100}$

$= 5 - 2 + \frac{5(-2)}{100}$

$= 3 - 0.1 = 2.9\%$

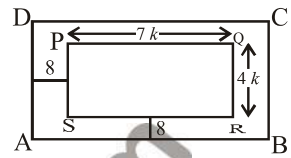
Sol 12. 1 hecto meter = 100 meter
1 hectare = 1 hectometer square
$= 100 \times 100\ m^2$
let length = $3k$, breath = $4k$

area $= \frac{1}{12}$ hectore $= \frac{10000}{12}$

$k^2 = \left(\frac{100}{12}\right)^2 \Rightarrow k = \frac{100}{12} = \frac{25}{3}$

length $= 3k = 25\ m$

Sol 13.



Area of path = area of rectangle ABCD – area of rectangle PQRS
$416 = (7k + 8)(4k + 8) - 7k \cdot 4k$
$416 - 64 = 88k$
$352 = 88k$
$k = 4$
breath $= 4k = 16$ m

Sol 14. let base $= 3k$, altitude $= 4k$

area of triangle $= \frac{1}{2}$ base $\times$ altitude

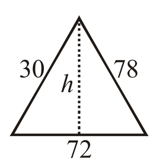
$1176 = \frac{1}{2} \times 3k \times 4k$

$k^2 = 196$
$k = 14$
hence altitude $= 4k = 56$ cm

Sol 15. Ratio of height $= \left(\frac{1}{2} : \frac{1}{3} : \frac{1}{4}\right) \times 12$

$= 6 : 4 : 3$
Permeter = (6 + 4 + 3) unit
52cm = 13 unit
1 unit = 4 cm
Smallest side = 3 unit
$= 3 \times 4 = 12$ cm

Sol 16.



$s = \frac{a+b+c}{2} = 90$

we will equal area by two method

$= \sqrt{s(s-a)(s-b)(s-c)} = \frac{1}{2} \times \text{base} \times \text{hight}$

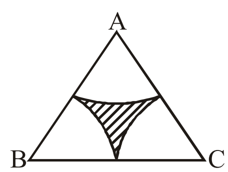
$= \sqrt{90(90-30)(90-72)(90-78)}$

$= \frac{1}{2} \times 72 \times h$

$\Rightarrow 1080 = \frac{1}{2} \times 72 \times h$

$\Rightarrow h = 30\,\text{cm}$

Sol 17.

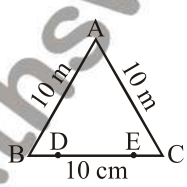


region bounded by three arcs
= area of triangle – area of all three sector

$= \frac{\sqrt{3}}{4}(2)^2 - 3\left(\frac{1}{6}\pi \times (1)^2\right)$

$= \left(\sqrt{3} - \frac{\pi}{2}\right) \text{cm}^2$

Sol 18.



$BD = DE = EC = \frac{10}{3}$

$DO = \frac{10}{3 \times 2} = \frac{5}{3}$

height $AO = \frac{\sqrt{3}}{2} a = \frac{\sqrt{3}}{2} \times 10 = 5\sqrt{3}$

in Δ ADO
$AD^2 = AO^2 + DO^2$

$= \left(5\sqrt{3}\right)^2 + \left(\frac{5}{3}\right)^2 = 75 + \frac{25}{9}$

$AD = \sqrt{25\left(3 + \frac{1}{9}\right)} = \frac{5\sqrt{28}}{3} = \frac{10\sqrt{7}}{3}$

Sol 19. Perimeter $= \sqrt{3}$ area

$3a = \sqrt{3} \times \frac{\sqrt{3}}{4} a^2$

$1 = \frac{a}{4}$

$\Rightarrow a = 4$ unit

Sol 20. Perimeter $= 3k + 4k + 5k = 12k$

$s = \frac{12k}{2} = 6k$

Area of $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$72 = \sqrt{6k \times 3k \times 2k \times k}$

$72 = k^2 \times 6$

$k^2 = 12$

$k = 2\sqrt{3}$

Perimeter $= k(3 + 4 + 5)$

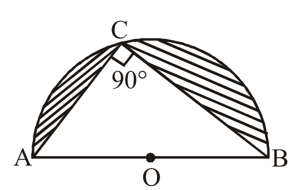
$= 2\sqrt{3} \times 12$

$= 24\sqrt{3} = 3a \Rightarrow a = 8\sqrt{3}$

Area of equilateral triangle $= \frac{\sqrt{3}}{4} a^2$

$= 48\sqrt{3}$ sq. unit

Sol 21.



AC = CB = x cm (isoceles triangle)

AB = 14 cm

$AC^2 + BC^2 = AB^2$

$x^2 + x^2 = 14^2$

$x = \sqrt{14 \times 7} = 7\sqrt{2}$ cm

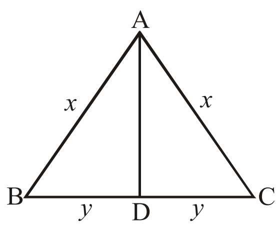
Area of $\Delta ABC = \frac{1}{2} \times 7\sqrt{2} \times 7\sqrt{2} = 49 \text{cm}^2$

Area of semicircle =

$\frac{\pi r^2}{2} = \frac{22}{7 \times 2} \times 7 \times 7 = 77 \text{cm}^2$

Area of shaded region = 77 – 49 = 28 cm^2

Sol 22.



Perimeter = 64

$x + x + 2y = 64$

$x + y = 32$ …(1)

$AB^2 = BD^2 + AD^2$ [By Pythogoras theorem]

$x^2 + x^2 = 14^2$ …(2)

Divide eqn. (2) by eqn. (1)

$\frac{x^2 - y^2}{x - y} = \frac{64}{32} \Rightarrow x + y = 2$ …(3)

From eqn. (1) and (3)

$2x = 34 \Rightarrow x = 17$ cm

$y = 15$ cm

Area of $\Delta ABC = \frac{1}{2} \times (15 + 15) \times 8 = 120$ cm^2

Sol 23. $4a^2 = d_1^2 + d_2^2$

$4a^2 = (24)^2 + (10)^2$

$4a^2 = 576 + 100$

$4a^2 = 676$

$a^2 = 169$

$a = 13$

Perimeter of Rhombus

$= 4a$

$= 4 \times 13$

= 52 cm

Sol 24. $4a^2 = d_1^2 + d_2^2$

$4a = 40$ cm

$2a = 20$ cm

$(2a)^2 = d_1^2 + d_2^2$

$(20)^2 = (12)^2 + d_2^2$

$d_2^2 = (20 + 12)(20 - 12)$

$d_2 = \sqrt{4 \times 8 \times 8}$

= 16 cm

Sol 25. Let parallel sides are 5 K & 3 K

Area = $\frac{1}{2}(5K + 3K) \times 24$

$\Rightarrow 96k = 1440$

K = 15

longer side = 5 K = 75 m

Sol 26. Area of rhombus = $\frac{1}{2} d_1 d_2$

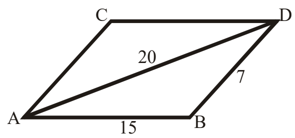
= side × altitude

$$\Rightarrow d_2 = \frac{2 \times \text{side} \times \text{altitude}}{d_1}$$

$$= \frac{2 \times 6.5 \times 10}{26}$$

$$= 5 \text{ cm}$$

Sol 27.



$$s = \frac{20 + 7 + 15}{2} = 21$$

area of parallelogram ABCD

= 2 × area of triangle ABC

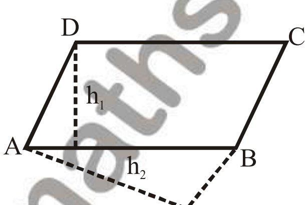
$$= 2\sqrt{s(s-a)(s-b)(s-c)}$$

$$= 2\sqrt{21(21-20)(21-15)(21-7)}$$

$$= 2 \times \sqrt{21 \times 1 \times 16 \times 14}$$

$= 84 \text{ cm}^2$

Sol 28.



AB = 24 cm AD = BC = 16 cm

$h_1 = 10$ cm

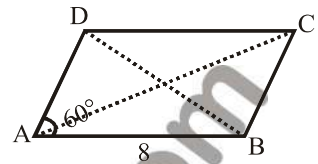
area of parallelogram ABCD

$\Rightarrow \quad h_1.AB = h_2.BC$

$\Rightarrow \quad 10 \times 24 = h_2 \times 16$

$\Rightarrow \quad h_2 = 15$ cm

Sol 29.



AB = AD

$$\angle ABD = \angle ADB = \frac{180° - 60°}{2} = 60°$$

ΔABD is an equilateral triangle

hence area of rhombus

$= \quad 2 \times \text{area of } \Delta ABD = \frac{1}{2} d_1 d_2$

$$\Rightarrow \quad 2 \times \frac{\sqrt{3}}{4}(8)^2 = \frac{1}{2} \times 8 \times d_2$$

$$\Rightarrow \quad d_2 = 8\sqrt{3}$$

Sol 30. diagonal of square = $40\sqrt{2}$ cm

then side = 40 cm

$\therefore$ area of square = 1600 cm^2

let diagonal of rhombus are 8 K and 9 K

area of rhombus = area of square

$$\frac{1}{2}.8K.9K = 1600$$

$$36K^2 = 1600$$

$$K = \frac{40}{6} = \frac{20}{3}$$

hence longer diagonal = 9K

$$= 9 \times \frac{20}{3}$$

$$= 60 \text{ cm}$$

Sol 31. Area of hexagon = $6\times\frac{\sqrt{3}}{4}a^2$

$$=6\times\frac{\sqrt{3}}{4}(1)^2$$

$$=\frac{3\sqrt{3}}{2}\text{cm}^2$$

Sol 32. Circumference of wheel = $2\pi r$

$$=2\times\frac{22}{7}\times 7$$

$$=44\text{ cm}$$

distance of 44 m will be covered in 1 revolution

distance of 1 m will be covered in = $\frac{1}{44}$ revolution

22 km = 22000 m will be covered = $\frac{1}{44}\times 22000$

= 500 revolution

Sol 33. $2\pi r_1 = 88\text{cm} \Rightarrow r_1 = 14\text{cm}$

$2\pi r_2 = 132\text{cm} \Rightarrow r_2 = 21\text{cm}$

Area of ring = $\pi\left(r_2^2 - r_1^2\right)$

$$=\pi(r_2 + r_1)(r_2 - r_1)$$

$$=\pi(35)(7)$$

$$=770\text{ cm}^2$$

Sol 34. Time is proportional to distance

time $\propto$ distance

time $\propto$ (circumfence)

time $\propto$ diameter

Initial 8 times in 40 min means

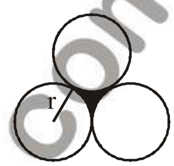
Once in 5 min.

$$\Rightarrow \frac{t_1}{t_2} = \frac{d_1}{d_2}$$

$$\Rightarrow \frac{5}{t_2} = \frac{d_1}{10d_2} = \frac{1}{10}$$

$t_2 = 50$ min

Sol 35. Area enclosed by three circles



$$=\frac{\sqrt{3}}{4}(2r)^2 - \frac{\pi r^2}{2}$$

$$=\left(\sqrt{3}-\frac{\pi}{2}\right)r^2 = (0.162)r^2$$

$$=0.162\times(3.5)^2$$

$$=1.985\text{ cm}^2$$

Sol 36. let radius = r

$$\pi(1+r)^2 - \pi r^2 = 22$$

$$\pi\left\{(1+r)^2 - r^2\right\} = 22$$

$$\frac{22}{7}(2r+1)(1) = 22$$

$$2r+1 = 7$$

$r = 3$ cm

Sol 37. Area of shaded region = area of semicircle – area of triangle

$$=\frac{1}{2}\pi a^2 - \frac{1}{2}.2a.a$$

$$=a^2\left(\frac{\pi}{2}-1\right)$$

Sol 38. Distance cover in 1000 revolution = 440 m

Circumference of wheel = distance in one revolution.

$$2\pi r = \frac{440}{1000}$$

$$2 \times \frac{22}{7} \times r = \frac{440}{1000}$$

$$r = \frac{7}{100} = .07$$

$\text{diameter} = 2r = 0.14$

Sol 39. $\because \quad 2\pi r = 11 \quad \Rightarrow \ r = \frac{7}{4}$ cm

$$\text{Area of sector} = \frac{\pi r^2}{360°} \times 60$$

$$= \frac{1}{6} \times \frac{22}{7} \times \left(\frac{7}{4}\right)^2$$

$$= \frac{77}{48} = 1\frac{29}{48}\text{cm}^2$$

Sol 40. Small diameter – loose no. of revolution

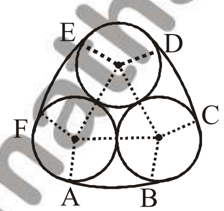
big diameter – loose no. of revolution

d_1 × of revolution = d_2 × no. of revolution

$12 \times 42 = 18 \times n$

$n = 28$ revolution

Sol 41.



$AB = CD = EF = 2\,r$

length of rubber band = AB + CD + EF + 3 × length of arc

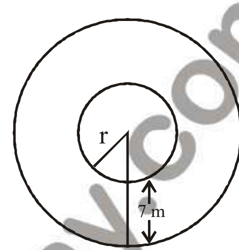
$$= 6r + 3 \times \frac{2\pi r}{360°} \times 120°$$

$$= 6r + 2\pi r$$

$$= 6 \times 5 + 2\pi \times 5$$

$$= 30 + 10\pi$$

Sol 42.



$2\pi r = 176 \quad \Rightarrow \ r = 28\text{ m}$

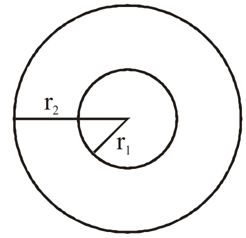
$$\text{area of road} = \pi\left((r+7)^2 - r^2\right)$$

$$= \pi(2r+7)(7)$$

$$= \frac{22}{7} \times 63 \times 7$$

$$= 1386\text{ m}^2$$

Sol 43.



$$2\pi r_2 - 2\pi r_1 = 66$$

$$2 \times \frac{22}{7}(r_2 - r_1) = 66$$

$$\text{width} = r_2 - r_1 = \frac{21}{2} = 10.5\text{m}$$

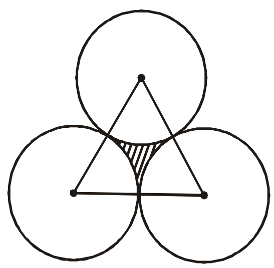
Sol 44. $\text{Time distance} = \frac{2\pi r}{\text{Speed}} - \frac{2r}{\text{Speed}}$

$$\Rightarrow \ 30\sec = \frac{30}{60}\min = \frac{2r}{\text{speed}}(\pi - 1)$$

$$\Rightarrow \frac{1}{2} = \frac{2\times r}{30}\left(\frac{22}{7}-1\right) = \frac{r}{15}\times\frac{15}{7}$$

$$\Rightarrow r = 3.5 \text{ m}$$

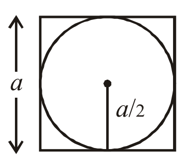
Sol 45. Area of shaded region = area of triangle ABC – area of all three sector



$$= \frac{\sqrt{3}}{4}(2a)^2 - 3\left(\frac{\pi r^2}{360^\circ}\times 60\right)$$

$$= \left(\sqrt{3}-\frac{\pi}{2}\right)a^2 \text{ sq. cm}$$

Sol 46.



$4a = 120\text{cm}$

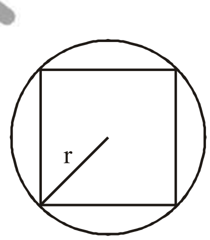
$a = 30 \text{ cm}$

radius $r = \frac{a}{2} = \frac{30}{2} = 15\text{cm}$

Area of circle $= \pi r^2$

$= \frac{22}{7}\times(15)^2 \text{ cm}^2$

Sol 47.

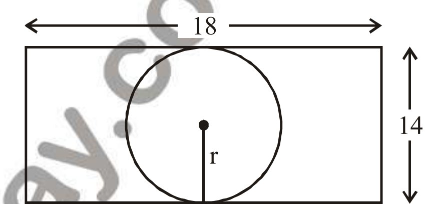


$2\pi r = 100 \text{ cm}$

$2r = \frac{100}{\pi}\text{cm}$ = length of diagonal = $a\sqrt{2}$.

$a = \frac{100}{\pi\sqrt{2}} = \frac{50\sqrt{2}}{\pi}\text{cm}$

Sol 48.

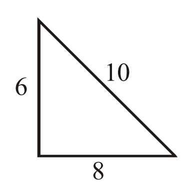


$r = \frac{14}{2} = 7\text{cm}$

area of this circle $= \frac{22}{7}\times(7)^2$

$= 154 \text{ cm}^2$

Sol 49. In a right angle triangle side of square having maximum



$$\text{Area} = \frac{ab}{a+b} = \frac{6\times 8}{6+8} = \frac{24}{7}$$

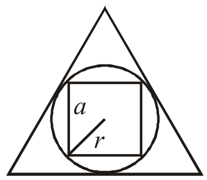
hence Area $= \frac{576}{49}\text{cm}^2$

Sol 50. In radius $r = \frac{\text{area of triangles}}{\text{semiperimeters}}$

$$2 = \frac{6}{s} \Rightarrow s = 3$$

hence perimeter = 2s = 6 cm

Sol 51. Side of square = a



$2r = a\sqrt{2} \qquad \Rightarrow \; r = \frac{a}{\sqrt{2}}$

In radius of triangle = $\frac{A}{2\sqrt{3}} = \frac{a}{\sqrt{2}}$

hence $\frac{\text{area of triangle}}{\text{area of square}} =$

$\frac{\sqrt{3}}{4}\frac{A^2}{a^2} = \frac{\sqrt{3}.6a^2}{4a^2} = \frac{3\sqrt{3}}{2}$

Sol 52. $2(l+b) = 160 \quad \Rightarrow l + b = 80$

$l - b = 48$

on solving l = 64, b = 16

area of square = area of rectangle

$a^2 = 16 \times 64$

$a = \sqrt{16 \times 64} = 32$ sq.

Sol 53. Affect of area of circle $= r + r + \frac{r \times r}{100}$

$= 50 + 50 + \frac{50 \times 50}{100} = 125\%$

Sol 54. Area of square = $\frac{1440}{160} = 9$ hectare

9 hectare = 90000 m^2

hence cost of fence around it = 75 paise × perimeter

$= \frac{3}{4} \times 4 \times 300$

= 900 Rs.

Sol 55. Let length = $2b$, breadth = b

Area = $2b.\,b = 2b^2$

$\Rightarrow \quad (2b-5)(b+5) = 2b^2 + 75$

$\Rightarrow \quad 2b^2 + 10b - 5b - 25 = 2b^2 + 75$

$5b = 100$

$\Rightarrow \quad b = 20$ cm

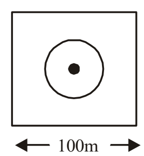
length = 40 cm

Sol 56. No. of tiles = $\frac{\text{area of floor}}{\text{area of 1 tile}}$

$= \frac{400\text{cm} \times 30\text{cm}}{8\text{cm} \times 6\text{cm}}$

= 2500 tile

Sol 57.



Area of the lawn excluding the flower bed = 8614m^2

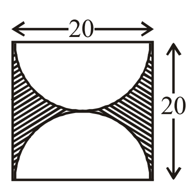
$(100)^2 - \pi r^2 = 8614$

$\pi r^2 = 10000 - 8614 = 1386$

$r^2 = \frac{1386 \times 7}{22}$

$r = 21$ cm

Sol 58



Area of shaded paper = Area of square – area of 2 semi circle = 20 × 20 – 2 × $\frac{\pi \times (10)^2}{2}$ =

$(400 - 100\pi)\text{m}^2$

Sol 59. Area of circle A = πr^2

$$\text{(radius) } r = \sqrt{\frac{A}{\pi}}$$

Area of square A = a^2

$$\text{(side) } a = \sqrt{A}$$

$$\frac{\text{side of square}}{\text{radius of circle}} = \frac{\sqrt{A}}{\sqrt{\frac{A}{\pi}}}$$

$$= \frac{\sqrt{\pi}}{1} = \sqrt{\pi} : 1$$

Sol 60. Area of equilateral triangle = 5m²

$$\frac{\sqrt{3}}{4}a^2 = 5$$

$$a = \sqrt{\frac{20}{\sqrt{3}}}$$

$$a = \frac{\sqrt{20}}{(3)^{\frac{1}{4}}}$$

Perimeter of equlateral triangle = $3a$

$$= \frac{3 \times \sqrt{20}}{(3)^{\frac{1}{4}}}$$

$$= 3^{\frac{3}{4}} \times \sqrt{20}$$

$\therefore$ Circumference of circle = $3^{\frac{3}{4}} \times \sqrt{20}$

$$2\pi r = 3^{\frac{3}{4}} \times \sqrt{20}$$

$$\therefore \ r = \frac{3^{\frac{3}{4}}\sqrt{20}}{2\pi} \text{ m}$$

Sol 61. Area of sector = $\pi r^2 \times \frac{60°}{360°} = 12.4\text{cm}^2$

$$\pi r^2 = 12.4 \times 6$$

Area of remaining circle = $\pi r^2 - \frac{1}{6}\pi r^2 = \frac{5}{6}\pi r^2$

$$= \frac{5}{6} \times 12.4 \times 6 = 62 \text{ cm}^2$$

Sol 62 Area of shaded region

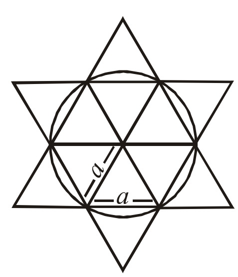
$$= \frac{1}{2}\pi(14)^2 + \frac{1}{2}\pi(7)^2 + \frac{1}{2}(7)^2$$

$$= \frac{1}{2}\pi(196 + 49 + 49)$$

$$= \frac{1}{2} \times \frac{22}{7} \times 294$$

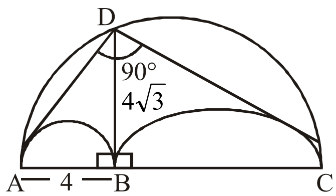
$$= 462 \text{ cm}^2$$

Sol 63 Area of resultant figure = area of 12 equilateral triangle of side a



$= \quad 12\times\frac{\sqrt{3}a^2}{4} = 3\sqrt{3}a^2$

Sol 64.



In right angle triangle ABD

$\tan A = \frac{BD}{AB} = \frac{4\sqrt{3}}{4} = \sqrt{3} = \tan 60°$

$\Rightarrow \angle A = 60°$ hence

$\angle C = 180° - 90° - 60° = 30°$

In right angle triangle CBD

$\tan C = \tan 30° = \frac{1}{\sqrt{3}} = \frac{BD}{BC} = \frac{4\sqrt{3}}{BC}$

$\Rightarrow$ BC = 12 cm $\Rightarrow$ AC = 4 + 12 = 16 cm.

area of shaded region = area of larger semi circle – area of two smaller circle

$= \frac{1}{2}\pi.(8)^2 - \frac{1}{2}\pi.(2)^2 - \frac{1}{2}\pi.(6)^2$

$= 12\pi$

Sol 65. Let longer diagonal = 100

other diagonal = 80

then area of rhombus = $\frac{1}{2}\times 100\times 80 = 4000$

area of square = (longer diagonal)^2 = 10000

ratio of area of rhombus to square = $\frac{40000}{10000} = \frac{2}{5}$

Sol 66. Area of shaded portion = 12 × 12 – $\frac{1}{2}\times 12\times 7 - \frac{1}{2}\times 5\times 5$

$= 144 - 42 - 12.5$

$= 89.5$ sq. units

Sol 67. Let BE = x

then EC = $2x$

Side BC = $3x$

area of triangle ABC = $\frac{1}{2}\times AB\times BE = 15$

$\Rightarrow AB\times x = 30$

area of rectangle = AB. BC = AB.$3x$ = 3×30 = 90cm^2

Sol 68. area of figure = area of square ABDE + area of ΔBCD

$= 2.5\times 2.5 + \frac{1}{2}\times 2.16\times 2.5$

$= 6.25 + 1.08\times 2.5 = 8.95$ cm^2

Sol 69. In the figure we can see shaded region will be equal to only larger semi circle because both smaller semi circle are equal and one unshaded by inside and one shaded is outside

So, area of shaded region $= \frac{1}{2}\pi(5)^2$

$= 1.57\times 25$

$= 39.25$ cm^2

Sol 70. Area of total figure = 4 × area of semi circle + rea of square

$= 4\times\frac{1}{2}\pi\left(\frac{21}{2}\right)^2 + 21\times 21$

$= 1134$ m^2

No. of plants $= \frac{\text{total area}}{\text{area needed of each rose plant}}$

$$= \frac{1134}{6} = 189$$

Sol 71. Ungrazed area = $(4-\pi)\frac{63^2}{4} = \frac{6}{7}\times\frac{63\times 63}{4}$

$$= 850.5\ m^2$$

Sol 72. Area of remaining part =

Area of rectangle – 4 × area of quarter circle

$$= 20\times 16 - 4\times\frac{1}{4}\times\frac{22}{7}\times(3.5)^2 = 281.5\ m^2$$

Sol 73. Area of shaded protion

$$= \frac{1}{2}\times\pi(28)^2 - \frac{1}{2}\times\pi(14)^2\times 2 - \pi(7)^2$$

$$= 462\ cm^2$$

Sol 74. Side of all five squares is 6 cm, 8 cm, 10 cm, 19 cm, 20 cm, respectively.

hence,

area of new square $= (6)^2 + (8)^2 + (10)^2 + (19)^2 + (20)^2$

$$A^2 = 961\ cm^2$$

$$\Rightarrow A = \sqrt{961} = 31$$

hence perimeter = 31 × 4 = 124 cm

Sol 75. Perimeter of semicircle = area of semicircle

$$2r + \pi r = \frac{\pi}{2}r^2$$

$$\pi + 2 = \frac{\pi}{2}r \Rightarrow r = 2\times\frac{(\pi+2)}{\pi} = 2 + \frac{4}{22}\times 7$$

diameter $= 6\frac{6}{11}$ metres

Sol 76. Circumference of circle = $2\pi r$

$$2\pi r = 11$$

$$r = \frac{11\times 7}{2\times 22} = \frac{7}{4}$$

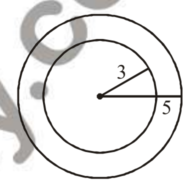
Area of sector = $\pi r^2 \times \frac{\theta}{360}$

$$= \frac{22}{7}\times\frac{7}{4}\times\frac{7}{4}\times\frac{60}{360}$$

$$= \frac{77}{48} = 1\frac{29}{48}\ cm^2$$

Sol 77. Area of annular zone = area of shaded region

$$= \pi\ (5^2 - 3^2) = 16\pi$$



$$\frac{\text{area of annular zone}}{\text{area of larger circle}} = \frac{16\pi}{25\pi} = \frac{16}{25}$$

Sol 78. circumradius = $\frac{a}{\sqrt{3}}$ inradius = $\frac{a}{2\sqrt{3}}$

difference of area of circumcircle and incircle

$$= \pi\left(\frac{a}{\sqrt{3}}\right)^2 - \pi\left(\frac{a}{2\sqrt{3}}\right)^2 = 44\ cm^2$$

$$\frac{22}{7}\left(\frac{a^2}{3} - \frac{a^2}{12}\right) = 44 \Rightarrow \frac{a^2}{4} = 14$$

$$a^2 = 56$$

hence area of triangle = $\frac{\sqrt{3}}{4}a^2 = 14\sqrt{3}\ cm^2$

Sol 79. $\pi r^2 = 3850$

$\Rightarrow r = 35$ m

Perimeter of square = perimeter of circle = $2\pi r$

$$= 2\times\frac{22}{7}\times 35$$

$$4a = 220\ m$$

$$a = 55\ m$$

area of square = $a^2 = (55)^2 = 3025\ m^2$

Sol 80. Let side of square = a

length breadth of rectangle will be $a + 5$ and $a - 3$ respectively

area of rectangle & square is equal.

$\Rightarrow (a + 5)(a - 3) = a^2$

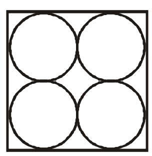
$\Rightarrow a^2 + 2a - 15 = a^2$

$\Rightarrow a = 7.5$ cm

$l = 12.5$, b = 4.5

perimeter = $2(l + b) = 34$cm .

Sol 81.



Area of square = 784

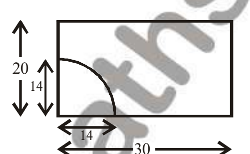
$a^2 = 784$

$a = 28$

Radius of circle = $\frac{28}{4} = 7$cm

Circumference of each plate = $2 \times \frac{22}{7} \times 7 = 44$cm

Sol 82.



Area of grazing part

$= \pi r^2 \times \frac{\theta}{360}$

$= \frac{22}{7} \times 14 \times 14 \times \frac{90°}{360°}$

$= 154 m^2$

Sol 83. Cost $= 2\pi rh \times 750 \times 2 = 2 \times \frac{22}{7} \times \frac{35 \times 150}{100 \times 100} \times 1500$

= Rs. 4950

Sol 84. $\pi[R^2 - (14)^2] = 60\pi$

R = 16 feet

then width 16 – 14 = 2 feet

Sol 85. breadth = x and length = $x + 8$

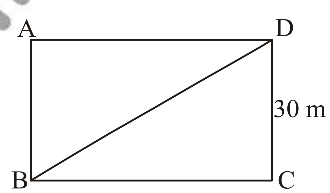
$x(x + 8) = (x + 8 + 7)(x - 4)$

on solving,

$x = 20$m

hence breadth = 20m & length = 28m

Sol 86.



Area of rectangle = 1200 m²

Length × 30 = 1200

Length = 40m

Diagonal = $\sqrt{(30)^2 + (40)^2} = 50$ m

Perimeter of fencing part = 30 + 40 + 50

= 120 m.

Job cost = 120 × 100 = Rs. 12000

Sol 87. Area of triangle = $\frac{1200}{4} = 300m^2$

Area of half triangle = 150 m²

$\frac{1}{2} \times 20 \times x = 150$

height = x = 15

Equal Side = $\sqrt{(20)^2 + (15)^2} = 25$ m

Sol 88. Perimeter of playground

$= 2(l + b)$

$= 2(36 + 21)$

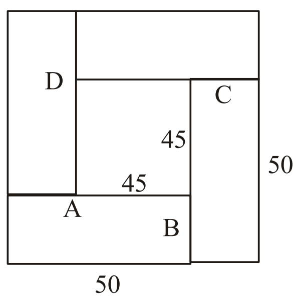
$= 2 \times 57$

No. of poles fixed at a distance 3 m apart

$= \frac{2 \times 57}{3} = 38$

No. of poles $= 38 + 1 = 39$

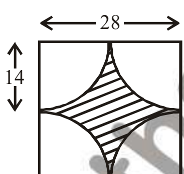
Sol 89.



Area of inner part ABCD $= 45 \times 45$

$= 2025\ cm^2$

Sol 90.



Area of square $= 28 \times 28 = 784\ cm^2$

Area of 4 sectors

$\frac{22}{7} \times 14 \times 14 \times \frac{90}{360} = 616\,cm^2$

Area grazed in 11 days = 616

1 day $= \frac{616}{11}$

$= 56$

Remaining area grazed in $= \frac{168}{56} = 3$ days

Sol 91. Circumference of grounds = 88m

$\therefore\ 2\pi r = 88$

$\Rightarrow r = 14\ m$ (radius of outer circle)

radius of inner circle $R = 14 - 3 = 11\ m$

Now area of strip $= \pi r^2 - \pi R^2$

$= \pi \times (14)^2 - \pi (7)^2$

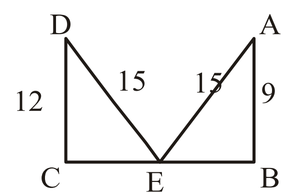
$= \frac{22}{7} \times 75\ m^2$

cost of levelling $= \frac{22}{7} \times 75 \times 7$

($\therefore$ 7 Rs per sq. m)

= Rs. 1650

Sol 92.



In Δ AEB,

$EB = \sqrt{(15)^2 - 9^2} = 12m$

In Δ DEC,

$CE = \sqrt{(15)^2 - (12)^2} = 9m$

$\therefore$ CB $= 9 + 12 = 21$ m

Sol 93. Perimeter of the region marked B

$= \frac{4}{3}\pi r = \frac{4}{3}\pi \times 2$

$= \frac{8}{3}\pi$

Sol 94. Area of shaded region = (area of sector ADB) + (area of sector BCD – (area of square ABCD)

$$= \frac{1}{4}\pi(10)^2 + \frac{1}{4}\pi(10)^2 - (10)^2$$

$$= \frac{\pi}{2} \times (100) - (100)$$

$$= 50\pi - 100$$

Sol 95. $AC = \sqrt{14^2 + 14^2}$

$$= 14\sqrt{2} \text{ cm}$$

$\Rightarrow$ Area of Quadrant ABCP = $\frac{1}{4}\pi r^2$

$$= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14 = 154 \text{ cm}^2$$

$\Rightarrow$ Area of small semicircle APC

$$= 154 - \frac{1}{2} \times 14 \times 14$$

$$= 154 - 98 = 56 \text{ cm}^2$$

$\Rightarrow$ Area of big semicircle ACQ = $\frac{1}{2}\pi \times \left(7\sqrt{2}\right)^2$

$$= 49 \times \frac{22}{7} = 154$$

$\therefore$ Area of shaded region = 154 – 56 = 98 cm²

Or

Area of shaded region

= (area of semicircle AQC) + (area of Δ ABC) – (area of quarter circle APC)

$$= \frac{1}{2}\pi(7\sqrt{2})^2 + \frac{1}{2} \times 14 \times 14 - \frac{1}{4}\pi(14)^2$$

$$= \frac{1}{2} \times 14 \times 14 = 98 \text{ cm}^2$$

Sol 96. OC = OA = r

In Δ AOC

$$r^2 + r^2 = \left(\sqrt[7]{2}\right)^2$$

$$2r^2 = 49 \times 2$$

$$r = 7 \text{ cm}$$

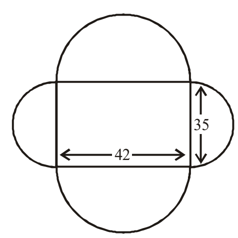
$\therefore$ Area = $\pi r^2 = \frac{22}{7} \times 7 \times 7 = 154 \text{ cm}^2$

Sol 97. Area of trapezium = $\frac{1}{2} \times (10 + 6) \times h$

$$72 = \frac{1}{2} \times 16 \times h$$

$\therefore$ h = 9m

Sol 98. Area of total field



= (area of rectangle) + (area of all semicircles)

$$= (42 \times 35) + \left[\frac{1}{2}\pi\left(\frac{42}{2}\right)^2 + \frac{1}{2}\pi\left(\frac{35}{2}\right)^2 + \frac{1}{2}\pi\left(\frac{42}{2}\right)^2 + \frac{1}{2}\pi\left(\frac{35}{2}\right)^2\right]$$

$$= (42 \times 35) + \left(\pi \times 21 \times 21 + \pi \times \frac{35}{2} \times \frac{35}{2}\right)$$

$$= 1470 \times 1386 + 962.5$$

$$= 3818.5 \text{ m}^2$$

Sol 99. Area of bigger square = a^2

diagonal of smaller square ABCD = a

Side of smaller square = $a / \sqrt{2}$

Now,

side of smaller square = diameter of circle = $a / \sqrt{2}$

$\Rightarrow$ height of equilateral triangle = $\frac{3}{4} \times \frac{a}{\sqrt{2}} = \frac{3a}{4\sqrt{2}}$

$\Rightarrow$ side of equilateral triangle = $\frac{\sqrt{3}\,a}{2\sqrt{2}}$

Sol 100. Area of shaded region

$$= \frac{1}{2}\pi\left(\frac{AC}{2}\right)^2 + \frac{1}{2}\pi\,(BC)^2 -$$

$$\left[\frac{1}{2}\pi\left(\frac{AB}{2}\right)^2 - \frac{1}{2}(AC)\times(BC)\right]$$

$[\therefore AB^2 = AC^2 + BC^2]$

$$= \frac{1}{2}\,(AC)\times(BC)$$

= area of triangle ABC = 37 sq. unit

Sol 101. Perimeter of circle = Perimeter of rectangle

$$2\pi r = 2(l+b)$$

$$\frac{22}{7}\times r = (26+18)$$

$$r = \frac{44}{22}\times 7 = 14\text{cm}$$

Area of circle = πr^2

$$= \frac{22}{7}\times 14\times 14 = 616\,\text{cm}^2$$

Sol 102. Let the initial length of play ground be l

area of play ground = $lb = 50l$

expenditure = $50 \times l \times \frac{25}{100} = 100$

$= l = 80$ m

Now, length is increased by 20m

new length $l = 80 + 20 = 100$m

area of play ground = lb

$= 100 \times 50 = 5000\,\text{m}^2$

expenditure to make ground = $5000 \times \frac{25}{100}$

= 1250 Rs.

Sol 103. Number of marbles blocks = $\frac{60\times 20}{3\times 2} = 200$

Cost of marbles = 200 × 100 = Rs. 20000

Sol 104. No. of tiles = $\frac{40\times 13.5}{3\times 2} = 90$

cost of 1 tile = 3 Rs.

cost of 90 tiles = 90 × 3 = 270 Rs.

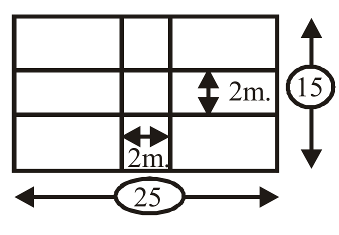
Sol 105. Area of shaded region = Area of sector – Area of equlateral triangle

$$= \frac{\pi r^2\theta}{360} - \frac{\sqrt{3}}{4}r^2$$

$$= \frac{22}{7}\times 14\times 14\times\frac{60'}{360^\circ} - \frac{\sqrt{3}}{4}\times 14\times 14$$

$$= \left(102\frac{2}{3} - 49\sqrt{3}\right)\text{cm}^2$$

Sol 106.



$\therefore$ Area under the grass

= 25 × 15 – (25 × 2 + 15 × 2 – 2 × 2)

= 375 –(50 + 30–4)

= 375 – 76 = 299m²

Sol 107. Area of rectangle = 4 × Area of square

$l \times b = 4a^2$

$90 \times \frac{2}{3} \times a = 4a^2$

$a = 15$ cm

Sol 108. Area of the roads

$= 60 \times 5 + 40 \times 5 - 5 \times 5$

$= 475$ cm^2

cost of gravelling $= 475 \times 0.60 = 285$ Rs.

Sol 109. Let the radius of smaller circle be r.

$r = \frac{1}{2}$ (diagonal of square) – (radius of bigger circle)

$= \frac{1}{2}(2\sqrt{2}\,\text{R}) - \text{R}$

$r = (\sqrt{2} - 1)\text{R}$

Sol 110. Area of shaded region = Area of sector AOBC – area of Δ AOD

$= \frac{\pi(4)^2}{4} - \frac{1}{2} \times 2 \times 4$

$= 4\pi - 4$

$= 8.56$ cm^2

Sol 111. Let AB = 2A

$\text{PQ} = \sqrt{a^2 + a^2} = \sqrt{2}\text{a}$

Radius of circle $= \frac{\sqrt{2}\text{a}}{2} = \frac{\text{a}}{\sqrt{2}}$

Let side of triangle $= x$

$\Rightarrow$ Circum radius $\text{R} = \frac{x}{\sqrt{3}}$

$\Rightarrow \frac{x}{\sqrt{3}} = \frac{a}{\sqrt{2}}$

$\Rightarrow x = \frac{\sqrt{3}a}{\sqrt{2}}$

$\therefore \frac{\text{Side of the square ABCD}}{\text{side of the equilateral triangle XYZ}} =$

$\frac{2\text{a}}{\frac{\sqrt{3}\text{a}}{\sqrt{2}}} = \frac{2\sqrt{2}}{\sqrt{3}}$

Sol 112. Area of square ACDE = 128

side $\Rightarrow$ AC = $8\sqrt{2}$

In right angle triangle ABC

AB = BC = 8 cm.

area of semicircles $= \frac{1}{2}\pi(4)^2 + \frac{1}{2}\pi(4)^2$

$= 16\pi$ cm^2

Sol 113. Let the radius of smaller circle be r

$(1 + r)^2 = (1)^2 + (1 - r)^2$

$r = 1/4$

area of shaded regin = area of rectangle PQRS – [(area of both quarter circles) + (area of smaller circle)]

$= 2 \times 1 - \left[\left(\frac{\pi}{4}(1)^2 + \frac{\pi}{4}(1)^2\right) + \pi\left(\frac{1}{4}\right)^2\right]$

$= 2 \times 1 - \left(\frac{\pi}{2} + \frac{\pi}{16}\right)$

$= \frac{13}{56}$ sq. unit

Sol 114. Area of shaded region

= [(area of semicircel AOB) + (area of semicircle BOC) +(area of semicircle COD) + (area of semicircle AOD) – (area of square ABCD)

$= \left[\frac{\pi(4)^2}{4} + \frac{\pi(4)^2}{4} + \frac{\pi(4)^2}{4} + \frac{\pi(4)^2}{4}\right] - (8)^2$

$= \quad 32\pi - 64$

$= \quad 32(\pi - 2)\ \text{cm}^2$

Sol 115. Let the radius of smaller circle be r

$$OA = \frac{1}{2}(\text{diagonal of square}) = \frac{2R\sqrt{2}}{2} = R\sqrt{2}$$

Now,

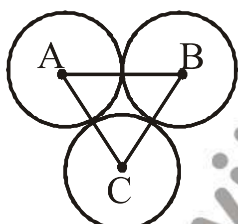
$$OA = OC + r + r\sqrt{2}$$

$$\Rightarrow \quad R\sqrt{2} = R + r(\sqrt{2} + 1)$$

$$\Rightarrow \quad r = \frac{(\sqrt{2}-1)R}{\sqrt{2}+1}$$

$$r = (3 - 2\sqrt{2})R$$

Sol 116. Let the centres of three circles be A, B and C



In Δ ABC

$AB = \sqrt{2} + 2, BC = \sqrt{2} + 2$

$AC = 2\sqrt{2} + 2 = \sqrt{2}(2 + \sqrt{2})$

$(AC)^2 = (AB)^2 + (BC)^2$

Hence ABC is an isoceles right angle triangle where angles are 90º, 45º and 45º

Perimeter of shaded region

$$= \frac{45^\circ}{360^\circ} \times 2\pi(\sqrt{2} + 1) + \frac{45^\circ}{360^\circ} \times 2\pi(\sqrt{2} + 1) + \frac{90^\circ}{360^\circ}(1)2\pi$$

$$= \frac{\pi}{2}(\sqrt{2} + 2)$$

☆☆☆☆☆

Exercise - (Cube and Cuboid)

1. The edges of a cuboid are in the ratio 1 : 2 : 3 and its surface area is 88 cm^2. The volume of the cuboid is:

(a) 120 cm^3 (b) 64 cm^3
(c) 48 cm^3 (d) 24 cm^3

2. Find the length of the largest rod that can be placed in a room of 16 m long, 12 m broad and $10\frac{2}{3}$ m high.

(a) 23 m (b) 68 m
(c) $22\frac{2}{3}$ m (d) $22\frac{1}{3}$ m

3. How many cubes, each of edge 3 cm, can be cut from a cube of edge 15 cm?

(a) 25 (b) 27
(c) 125 (d) 144

4. What is the volume of a cube (in cubic cm) whose diagonal measures $4\sqrt{3}$ cm?

(a) 16 (b) 27
(c) 64 (d) 8

5. A cuboidal block of 6 cm × 9 cm × 12 cm is cut up into exact number of equal cubes. The least possible number of cubes will be

(a) 6 (b) 9
(c) 24 (d) 30

6. A soap cake is, of size 8 cm × 5 cm × 4 cm. The number of such soap cakes that can be packed in a box measuring 56 cm × 35 cm × 28 cm is :

(a) 49 (b) 196
(c) 243 (d) 343

7. The perimeter of the floor of a room is 18 m. What is the area of the walls of the room, if the height of the room is 3 m ?

(a) 21 m^2 (b) 42 m^2
(c) 54 m^2 (d) 108 m^2

8. Water flows into a tank which is 200 m long and 150 m wide, through a pipe of cross-section 0.3 m × 0.2 m at 20 km/hour. Then the time (in hours) for the water level in the tank to reach level of 8 m is

(a) 50 (b) 120
(c) 150 (d) 200

9. A rectangular sheet of metal is 40 cm by 15 cm. Equal squares of side 4 cm are cut off at the corners and the remainder is folded up to form an open rectangular box. The volume of the box is

(a) 896 cm^3 (b) 986 cm^3
(c) 600 cm^3 (d) 916 cm^3

10. The areas of three consecutive faces of a cuboid are 12 cm^2, 20 cm^2 and 15 cm^2, then the volume (in cm^3) of the cuboid is:

(a) 3600 (b) 100
(c) 80 (d) 60

11. A hall of 25 metres long and 15 metres broad is surrounded by a verandah of uniform width of 3.5 metres. The cost of flooring the verandah, at Rs. 27.50 per square metre is

(a) Rs. 9149.50 (b) Rs. 8146.50
(c) Rs. 9047.50 (d) Rs. 4186.50

12. Three solid iron cubes of edges 4 cm, 5 cm and 6 cm are melted together to make a new cube. 62 cm^3 of the melted material is lost due to improper handling. The area (in cm^2) of the whole surface of the newly formed cube is

(a) 294 (b) 343
(c) 125 (d) 216

13. The length, breadth and height of a cuboid are in the ratio 1 : 2 : 3. If they are increased by 100%, 200% and 200% respectively then, compared to the original volume the increase in the volume of the cuboid will be

(a) 5 times (b) 18 times
(c) 12 times (d) 17 times

14. A cuboid is 20 m × 10 m × 8 m. Find its length of diagonal, surface area and volume.

(a) 27.23 m, 880 m^2, 1600 m^3
(b) 23.75 m, 880 m^2, 1600 m^3
(c) 27 m, 1600 m^2, 880 m^3
(d) 23 m, 1200 m^2, 1600 m^3

15. The internal measurements of a box with dimensions are 20 cm × 12 cm × 10 cm and the wood of which it is made is 1cm thick. Find the volume of wood.

(a) 960 cm^3 (b) 1296 cm^3
(c) 2400 cm^3 (d) 1120 cm^3

16. A rectangular tank, measuring internally $37\frac{1}{3}$ metres in legth, 12 metres in breadth and 8 metres in depth, is full of water. Find the weight of water in metric tons, given that one cubic metre of water weights 1000 kilograms.

(a) 3584 metric tons
(b) 3685 metric tons
(c) 3758 metric tons
(d) 3868 metric tons

17. A cistern of capacity 8000 litres measures externally 3.3. m by 2.6 m by 1.1 m and its walls are 5 cm thick. The thickness of the bottom is :

(a) 90 cm (b) 1 dm
(c) 1 m (d) 1.1 m

18. If the areas of the three adjacent faces of a cuboidal box are P cm^2, Q cm^2 and R cm^2 respectively, then find the volume of the box.

(a) $\sqrt{PQ^2 + QR^2 + RP^2}$
(b) $\sqrt{PQ} + \sqrt{QR} + \sqrt{RP}$
(c) $\sqrt{(P^2 + Q^2 + R^2)(P + Q + R)}$
(d) $\sqrt{PQR}$

19. A field in the form of a rectangle having length 20 m and breadth 25 m. There is a square pit having dimension 15 × 15 m. This pit is to be filled uniformly upto a height of 4 m with the soil taken out be digging the rectangular field. Find out the depth upto which the rectangular field must be dug if the soil is to fill the pit?

(a) $\frac{9}{5}$ cm (b) $\frac{9}{2}$ cm
(c) $\frac{9}{7}$ cm (d) $\frac{9}{4}$ cm

20. A cuboid of dimension *a, b, c* and its volume V and total surface area S then $\frac{4}{5}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ equal to :

(a) $\frac{8S}{5V}$ (b) $\frac{2S}{5V}$
(c) $\frac{4S}{5V}$ (d) $\frac{S}{5V}$

21. The side of a hollow cube is 4 m. What is the length of the largest pole that can be fit into it?

(a) $4\sqrt{3}$m (b) $3\sqrt{4}$m
(c) $5\sqrt{3}$m (d) $3\sqrt{5}$m

22. A cubic metre of gold is extended by hammering so as to cover an area of 6 hectares. Find the thickness of the gold.

(a) 0.0015 cm (b) 0.0017 cm
(c) 0.0019 cm (d) 0.0021 cm

23. The Cost of painting the whole surface area of a cube at the rate of paise 13 paise persq. cm is Rs. 343.98 Then the volume of the cube is :

(a) 8500 cm^3 (b) 9000 cm^3
(c) 9250 cm^2 (d) 9261 cm^3

24. How many bricks each measuring 25 cm × 11.5 cm × 6 cm will be needed to construct a wall 8 m long, 6 m high and 22.5 cm thick?

(a) 8100 (b) 6400
(c) 7400 (d) 9400

25. Three cubes of sides 8 cm, 6 cm and 1 cm are melted to form a new cube. Find the surface area of the cube so formed.

(a) $648 cm^2$ (b) $864 cm^2$
(c) $486 cm^2$ (d) $946 cm^2$

26. If the areas of the adjacent faces of a rectangular block are in the ratio 2 : 3 : 4 and its volume is 9000 cm^3, then the length of the shortest side is :

(a) 30 cm (b) 20 cm
(c) 15 cm (d) 10 cm

27. The number of bricks, each measuring 25 cm × 12.5 cm × 7.5 cm required to construct a wall 6 m long, 5 m high and 0.5 m thick, while the mortar occupies 5% of the volume of the wall, is

(a) 6080 (b) 5740
(c) 3040 (d) 8120

28. The dimensions of an open box are 52 cm, 40 cm and 29 cm. Its thickness is 2 cm. If 1 cm^3 of metal used in the box weights 0.5 gms, the weight of the box is :

(a) 8.56 kg (b) 7.76 kg
(c) 7.576 kg (d) 6.832 kg

29. A rectangular water tank is open at the top. Its capacity is 24 m^3. Its length and breadth are 4 m and 3 m respectively. Ignoring the thickness of the material used for building the tank, the total cost of painting the inner and outer surfaces of the tank at the rate of Rs. 10 per m^2 is:

(a) Rs. 400 (b) Rs. 500
(c) Rs. 600 (d) Rs. 800

30. If V be the volume and S the surface area of a cuboid of dimensions *a, b, c* then $\frac{1}{V}$ is equal to:

(a) $\frac{S}{2}(a+b+c)$ (b) $\frac{2}{S}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

(c) $\frac{2S}{a+b+c}$ (d) $2S(a+b+c)$

31. The length of a hall is 15 m and width 12 m. The sum of the areas of the floor and the flat roof is equal to the sum of the areas of the four walls. The volume of the hall is:

(a) $1800 m^3$ (b) $1200 m^3$
(c) $900 m^3$ (d) $720 m^3$

32. An open box is made of wood 3 cm thick. Its external length is 1.46 m, breadth 1.16 m and height 8.3 dm. The cost of painting the inner surface of the box at 50 paise per 100 cm^2 is :

(a) Rs. 138.50 (b) Rs. 277
(c) Rs. 415.50 (d) Rs. 554

33. A cuboidal water tank has 216 litres of water. Its depth is 1/3 of its length and breath is 1/2 of 1/3 of the difference of length and depth. The length of the tank is:

(a) 72 dm (b) 18 dm
(c) 6 dm (d) 2 dm

34. The areas of three adjacent faces of a cuboid are *x, y* & *z* square units respectively. If the volume of the cuboid be *v* cubic units, then the correct relation between *v, x, y, z* is:

(a) $v^2 = xyz$ (b) $v^3 = xyz$

(c) $v^2 = x^3y^3z^3$ (d) $v^3 = x^2y^2z^2$

35. A square of side 3 cm is cut off from each corner of a rectangular sheet of length 24 cm and breadth 18 cm and the remaining sheet is folded to form an open rectangular box. The surface area of the box is:

(a) $468 cm^2$ (b) $396 cm^2$
(c) $612 cm^2$ (d) $423 cm^2$

36. 1 m^3 piece of copper is melted and recast into a square cross section bar 36 m long. An exact cube is cut off from this bar. If one m^3 of copper cost Rs. 108, then the cost of the cube is.

(a) 50 paise (b) 25 paise
(c) 75paise (d) 1 paise

37. The volume of a rectangular block of stone is 10368 dm^2 , its dimensions are in the ratio of 3:2:1, If its entire surface is polished at 2 paise per dm^2, then what is the total cost?
(a) Rs. 31.68 (b) Rs 31.50
(c) Rs. 63 (d) Rs. 63.36

38. 2 cm of rain has fallen on a square km of land. Assuming that 50% of raindrops could have been collected and contained in a pool having a 100 m × 10 m base, by what level would the water level in the pool have increased?
(a) 15 m (b) 20 m
(c) 10 m (d) 25 m

39. A solid cube with an edge of 10 cm is melted to form two equal cubes. The ratio of the edge of the smaller cube to the bigger cube is.
(a) $\left(\frac{1}{3}\right)^{1/3}$ (b) $\frac{1}{2}$
(c) $\left(\frac{1}{2}\right)^{1/3}$ (d) $\left(\frac{1}{4}\right)^{1/3}$

40. How many small cubes, each of 96 cm^2 surface area can be formed from the material obtained by melting a larget cube if 384 cm^2 surface area?
(a) 8 (b) 5
(c) 800 (d) 8000

41. A rectangular tank measuing 5 m × 4.5 m × 2.1 m is dug in the centre of the field measuring 13.5 × 2.5 m. The earth dug out is spread evenly over the remaining portion of the field. How much is the level of the field raised?
(a) 4.0 m (b) 4.1 m
(c) 4.2 m (d) 4.3 m

42. The dimensions of a rectangular box are in the ratio 1 : 2 : 4 and the difference between the costs of covering in with the cloth and sheet at the rate of Rs. 20 and Rs. 20.5 per sq m respectively is Rs. 126. Find the dimensions of the box.
(a) 3 m, 6 m, 12 m (b) 6 m, 12 m, 24 m
(c) 1 m, 2 m, 4 m (d) None of these

43. The volume of a cube is numerically equal to the sum of its edges. What is its total surface area in square units?
(a) 66 (b) 183
(c) 36 (d) 72

44. A cistern of dimensions 2.4 m × 2 m × 1.5m takes 2 h 30 min to get filled with water. The rate at which water flower into the cistern is
(a) 0.48000 cu.m/h (b) 2.88 cu.m/h
(c) 800 cu.m/s (d) 80 cu.m/min

45. A water tank in the room of a cuboid has its base 20 m long, 7 m wide and 10 m deep. Initially, the tank is full but later when water is taken out of it, the level of water in the tank reduces by 2 m. The volume of the water left in the tank is
(a) 1120 m^3 (b) 400 m^3
(c) 280 m^3 (d) 140 m^3

46. A tank 30m long, 20 m wide and 12 m deep is dug in a field 500m long and 30 m wide. By how much will the level of the field rise if the earth dug out of the tank is evenly spread over the field?
(a) 0.33 m (b) 0.5 m
(c) 0.25 m (d) 0.4 m

47. A room of 8 meter long, 6 meter wide and 3 meter high has two windows of the size $1\frac{1}{2}\text{m}\times 1\text{m}$ and a door of the size $2\text{m}\times 1\frac{1}{2}\text{m}$. The cost of papering the walls with paper of width 50 cm at 25 paisa per meter is
(a) Rs. 50 (b) Rs. 45
(c) Rs. 60 (d) Rs. 39

48. Water has been poured into an empty rectangular tank at the rate of 8 cu. ft/min for 2.5 min. The length of the tank is 3 ft and the width is one half of the length. How deep is the water in the tank?
(a) 4 ft (b) 3.86 ft
(c) 3.23 ft (d) 4.44 ft

49. In swimming pool measuring 90 m by 40 m, 150 men take a dip. If the average displacement of water by a man is 8 cu m, what will be the rise in water level?
(a) 33.33 cm (b) 30 cm
(c) 20 cm (d) 25 cm

50. A rectangular water tank measure 15m × 6m at top and is 10m deep. It is full of water. If water is drawn out lowering the level by 1 meter how much of water has been drawn out?
(a) 90,000 litres (b) 45,000 litres
(c) 80,000 litres (d) 40,000 litre

51. A rectangular field is 40m long and 14m broad. In one corner of it, a pit 12m long, 6m wide and 5m deep has been dug out and the earth taken out of it has been evenly spread over the remaining part of the field. Find the rise in level of the field.

(a) 73.77 cm (b) 72.12 cm
(c) 70 cm (d) 75 cm

52. 125 identical cubes are cut from a big cube and all the smaller cubes are arranged in a row to form a long cuboid. What is the percentage increase in the total surface area of the cuboid over the total surface area of the cube?

(a) $234\frac{1}{3}\%$ (b) $234\frac{2}{3}\%$
(c) 117% (d) None of these

53. A rectangular tank is 45 m long and 26 m broad. Water flows into it through a pipe whose cross section is 13 cm^2, at the rate of 9 km/hour. How much will the level of the water rise in the tank in 15 min?

(a) 0.0016 m (b) 0.0020 m
(c) 0.0025 m (d) 0.0018

54. A rectangular water reservoir is 15 m × 12 m at the base. Water flows into it through a pipe whose cross section is 5 cm by 3 cm at the rate of 16m/s. Find the height to which water will rise in the reservoir in 25 minutes.

(a) 0.2 m (b) 2 cm
(c) 0.5 m (d) None of these

55. What is the total surface area of the identical cubes of largest possible volume that are cut from a cuboid of size 85 cm × 17 cm × 51 cm?

(a) 26010 cm^2 (b) 21600 cm^2
(c) 26100 cm^2 (d) None of these

56. 64 small cubes of 1 cm^3 are to be arranged in a cuboidal shape in such a way that the surface area will be minimum. What is the length of diagonal of the larger cuboid?

(a) $8\sqrt{2}$ cm (b) $\sqrt{273}$ cm
(c) $4\sqrt{3}$ cm (d) 4 cm

57. The diagonals of the three faces of a cuboid are x, y and z respectively. Find the volume of cuboid?

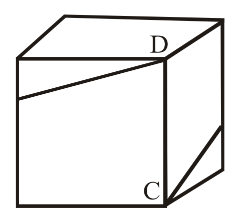
(a) $\frac{xyz}{2\sqrt{2}}$

(b) $\frac{\sqrt{(y^2+z^2-x^2)(z^2+x^2-y^2)(x^2+y^2-z^2)}}{2\sqrt{2}}$

(c) $\sqrt{\frac{(y^2+z^2(z^2+x^2)(x^2+y^2)}{2\sqrt{2}}}$

(d) None of these

58. The same string, when wound on the exterior four walls of a cube of side n cm, starting at point C and ending at point D, can give exactly one turn (see figrue, not drawn to scale). The length of the string, in cm, is



(a) $\sqrt{2}\,n$ (b) $\sqrt{17}\,n$
(c) n (b) $\sqrt{13}\,n$

59. The water in a rectangular reservoir having a base 80 metres by 60 metres is 6.5 metres deep. In what time can the water be emptied by a pipe of which the cross section is a square of side 20 cm, if the water runs through the pipe at the rate of 15 km/h?

(a) 52 h (b) 26 h
(c) 65 h (b) 42 h

60. A swimming bath is 24 m long and 15 broad. When a number of men drive into the bath, the height of the water rises by 1 cm. If the average amount of water displaced by one of the men be 0.1 cubic metre, how many men are there in the bath?

(a) 42 (b) 46
(c) 32 (b) 36

Answer

1.	(c)	**2.**	(c)	**3.**	(c)	**4.**	(c)	**5.**	(c)	**6.**	(d)	**7.**	(c)	**8.**	(d)	**9.**	(a)
10.	(d)	**11.**	(c)	**12.**	(a)	**13.**	(d)	**14.**	(b)	**15.**	(b)	**16.**	(a)	**17.**	(b)	**18.**	(d)
19.	(a)	**20.**	(b)	**21.**	(a)	**22.**	(b)	**23.**	(d)	**24.**	(b)	**25.**	(c)	**26.**	(c)	**27.**	(a)
28.	(a)	**29.**	(a)	**30.**	(b)	**31.**	(b)	**32.**	(b)	**33.**	(b)	**34.**	(a)	**35.**	(c)	**36.**	(a)
37.	(d)	**38.**	(c)	**39.**	(c)	**40.**	(a)	**41.**	(c)	**42.**	(a)	**43.**	(d)	**44.**	(b)	**45.**	(a)
46.	(b)	**47.**	(d)	**48.**	(d)	**49.**	(a)	**50.**	(a)	**51.**	(a)	**52.**	(b)	**53.**	(c)	**54.**	(a)
55.	(a)	**56.**	(c)	**57.**	(c)	**58.**	(b)	**59.**	(s)	**60.**	(d)						

Solution & Hints

Sol 1. Let the edges be $x, 2x, 3x \Rightarrow l = x, b = 2x, h = 3x$

$\Rightarrow 2(2x^2 + 6x^2 + 3x^2) = 88$

$[\therefore \text{TSA} = 2(lb + bh + hl)]$

$\Rightarrow x = 2$

Volume $= x \times 2x \times 3x$ $(\therefore \text{Volume} = lbh)$

$= 6x^3 = 48 \text{ cm}^3$

Sol 2. Length of largest rod can be placed in a room

$= \text{diagonal of room} = \sqrt{l^2 + b^2 + h^2}$

here $l = 16, b = 12, h = \frac{32}{3}$

$\text{diagonal of room} = \sqrt{(16)^2 + (12)^2 + \left(\frac{32}{3}\right)^2}$

$= \frac{68}{3} = 22\frac{2}{3}$ m

Sol 3. Let number of cubes can be cut be n.

Volume of cube of edge 15 cm $= n \times$ (Volume of cube of edge 3 cm)

$\Rightarrow 15 \times 15 \times 15 = n \times 3 \times 3 \times 3$

$n = 125$

Sol 4. diagonal of cube $= \sqrt{3}\, a$ ($\therefore$ a is side of cube)

$\Rightarrow 4\sqrt{3} = \sqrt{3}\, a$

$\Rightarrow a = 4$

Volume $= a^3 = 64$

Sol 5. HCF of 6, 9, 12 is 3

Side of cube = 3 cm

Volume of cuboid $= n \times$ (Volume of cube)

($\therefore$ n is no. of cubes)

$6 \times 9 \times 12 = n \times (3)^3$

$\Rightarrow n = 24$

Sol 6. Let the number of soap cake be n

$\therefore$ volume of box $= n \times$ (Volume of soap cake)

$\Rightarrow 56 \times 35 \times 28 = n \times (8 \times 5 \times 4)$

$\Rightarrow n = 343$

Sol 7. Let the length of and breadth of a room be l and b

$\therefore$ Perimeter of floor $= 2(l + b)$

$\Rightarrow 2(l + b) = 18$

$\Rightarrow (l + b) = 9$ m

$\because$ Area of walls $= 2(l + b)h$

$= 2 \times (9) \times 3 = 54 \text{ m}^2$

Sol 8. height of the pipe $= \frac{\text{Volume of tank}}{\text{cros-section area of pipe}}$

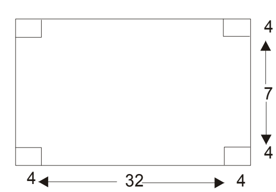
$= \frac{200 \times 150 \times 8}{0.3 \times 0.2} = 40{,}00{,}000 \text{ m}$

time taken by pipe =

$\frac{\text{height of pipe}}{\text{speed of water flowing in pipe}}$

$= \frac{40{,}00000 (\text{m})}{20 \times 1000 (\text{m/hr})} = 200$ hour.

Sol 9. When four square of 4 cm are removed from four corners of rectangular sheet



length and breadth of remaining rectangular sheet will be 32 cm and 7 cm and height of sheet will be 4 cm.

Volume of open rectangular box

= lenght × breadth × height

$= 32 \times 7 \times 4 = 896 \text{ cm}^3$

Sol 10. Let the length, breadth and height of the cuboid be l, b and h respectively.

$\therefore$ $lb = 12\ cm^2$ …(i)

$bh = 20\ cm^2$ …(ii)

$hl = 15\ cm^2$ …(iii)

Volume of cuboid = lbh

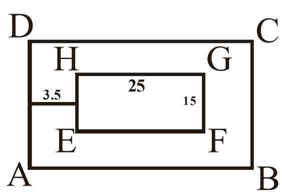
on multiplying eq. (i), (ii) and (iii)

$\therefore$ $(lbh)^2 = 12 \times 20 \times 15 \Rightarrow lbh = \sqrt{12 \times 20 \times 15}$

$\Rightarrow$ $lbh = 60\ cm^3$

Volume of cuboid = $60\ cm^3$

Sol 11. Area of EFGH = 25 × 15 = 375 cm^2



Area of ABCD = (25 + 7) × (15 + 7)

= 32 × 22

= 704 m^2

Area of Verandah = 704 – 375 = 329 m^2

cost of flooring the verandah = 329 × 27.50

= Rs. 9047.50

Sol 12. Total volume of 3 cubes which are melted

= volume of cube with side 4 cm + volume of cube with 5 cm side + volume of side cube with side 6 cm.

($\therefore$ Volume of cube = a^3 where a is side of cube)

$\Rightarrow$ Total volume = $(4)^3 + (5)^3 + (6)^3$

= 405 cm^3

As 62 cm^3 is lost

remaining volume = 405 – 62 = 343

$\therefore$ Volume of remaining material = volume of new cube

$\therefore$ $343 = b^3$ (let side of new cube be b)

$\Rightarrow$ $b = 7$ cm

whole surface area of new cube = $6a^2$

= 6 × 7 × 7 = 294 cm^2

Sol 13. Let length, breadth and height of cuboid be 100, 200 and 300 respectively.

$\Rightarrow$ Volume of cuboid (V_1) = 100 × 200 × 300

= 6000000

$\because$ Length increased by 100%, width by 200%, and height by 200%

Now, new length, width and height will be 200, 600, 900 respectively

$\Rightarrow$ new volume of cuboid (V_2) = 108000000

increase in volume = $\dfrac{V_2 - V_1}{V_1}$

$= \dfrac{102\,000000}{6000000} = 17$

$\Rightarrow$ increase in volume = 17 times

Sol 14. $l = 20\ m, b = 10\ m, h = 8\ m$

$\therefore$ diagonal = $\sqrt{l^2 + b^2 + h^2}$

$= \sqrt{(20)^2 + (10)^2 + (8)^2}$

$= \sqrt{564} = 23.75$m

surface area = 2 ($lb + bh + hl$)

= 2 (20 × 10 + 10 × 8 + 20 × 8)

= 880 m^2

$\because$ Volume = lbh

= 20 × 10 × 8 = 1600 m^2

Sol 15. Let lenght of inner surface (l_1) = 20 cm

$\Rightarrow$ Length of outer surface(l_2) = 22 cm

Let breadth of inner surface (b_1) = 12 cm

$\Rightarrow$ Breadth of outer surface (b_2) = 14 cm

Let height of inner surface (h_1) = 10 cm

$\Rightarrow$ height of outer surface (h_2) = 12 cm

Volume of wood =

volume of outer surface – Volume of inner surface

$= l_2 b_2 h_2 - l_1 b_1 h_1$

= 22 × 14 × 12 – 20 × 12 × 10

= 3696 – 2400

= 1296 cm^3

Sol 16. Volume of water = volume of tank = lbh

$$= \frac{112}{3} \times 12 \times 8 = 3584\ m^3$$

$\therefore$ 1 m^3 = 1000 kg

3584 m^3 = 3584 × 1000 kg

= 3584 metric ton (1 metric ton = 1000 kg)

Sol 17. Volume of cistern = 8000 × 1000 cm^3

internal length = 330 – 10 = 320 cm

internal width = 260 – 10 = 250 cm

internal height = h

$320 \times 250 \times h = 8000 \times 1000$

$h = 100$ cm

thickness of bottom = 110 – 100 = 10 cm = 1 dm

Sol 18. Let the length, breadth and height of cuboid be l, b and h respectively.

$\Rightarrow$ P = lb

$\Rightarrow$ Q = bh

$\Rightarrow$ R = lh

Volume = $lbh = \sqrt{PQR}$ $[\because PQR = (lbh)^2]$

Sol 19. Volume of soil digged out from field = volume of soil filled into square pit.

$\therefore$ $20 \times 25 \times h = 15 \times 15 \times 4$

($\therefore$ h = depth of rectangular field upto which field must be dug)

$$\Rightarrow h = \frac{15 \times 15 \times 4}{20 \times 15} = \frac{9}{5} \text{ cm.}$$

Sol 20. Volume of cuboid (V) = abc

total surface area (S) = 2 ($ab + bc + ca$)

$$\Rightarrow S = 2abc\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

$$\Rightarrow S = 2V\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

$$\Rightarrow \frac{S}{2V} = \left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

$$\Rightarrow \frac{4}{5}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) = \frac{4}{5} \times \frac{S}{2V} = \frac{2S}{5V}$$

Sol 21. Length of largest pole = length of diagonal = $a\sqrt{3}$ ($\therefore$ a is side of cube) = $4\sqrt{3}$ m

Sol 22. Initial volume of gold = final volume of gold

$1m^3 = 6 \times 10{,}000\ m^2 \times h$

($\therefore$ h = thickness) (1 hectare = 10, 000 m)

$$\Rightarrow h = \frac{0.17}{10{,}000} \text{ m}$$

$\Rightarrow$ $h = 0.0017$ cm

Sol 23. Cost of painting the whole surface area

= Rs. 343.98 = 34398 paise

$$\text{Total suface area} = \frac{34398}{13} = 2646\ cm^2$$

$6a^2 = 2646$

$a^2 = 441$

$a = 21$ cm

So, volume = 21 × 21 × 21 = 9261 cm^3

Sol 24. Volume of wall = n × Volume of one brick

($\therefore$ n = no. of bricks)

$\Rightarrow$ $800 \times 600 \times 22.5 = 25 \times 11.5 \times 6 \times n$

$$\Rightarrow n = \frac{800 \times 600 \times 22.5}{25 \times 11.5 \times 6} = 6400$$

Sol 25. Volume of new cube = summation of volume of three cubes

$a^3 = (8)^3 + (6)^3 + (1)^3 = 729$

(a = side of new cube)

$\Rightarrow$ $a = 9$

Surface area of new cube = $6a^2$

= $6 \times (9)^2$

= 486 cm^2

Sol 26. Let the three sides of cube be l, b and h and area of three adjacent faces be $2x$, $3x$ and $4x$

Volume of cuboid = $lbh = \sqrt{(2x)(3x)(4x)}$

($\therefore$ from que 18)

$\therefore$ $9000 = \sqrt{24x^3}$

$\Rightarrow$ $x = 150$

$\therefore$ area of surfaces $\Rightarrow$ $2x = 300\ \text{cm}^2 = lb$ ....(i)

$3x = 450\ \text{cm}^2 = bh$ ....(ii)

$4x = 600\ \text{cm}^2 = \text{lh}$ ....(iii)

$\because$ Volume of cuboid = $lbh = 9000\ \text{cm}^3$ ....(iv)

divide (iv) by (i),

$h = 30$ cm

divide (iv) by (ii)

$b = 20$ cm

divide (iv) by (iii)

$l = 15$ cm`

Length of shortest side = 15 cm

Sol 27. $\therefore$ (Volume of brick) × (no of bricks) = (Volume of wall) – (Volume occupied by mortar)

$\Rightarrow$ Let no of bricks be x.

$\therefore$ Volume of brick = lbh

$= 25 \times 12.5 \times 7.5$

$= 2343.75\ \text{cm}^3$

$\therefore$ Volume of wall = $600 \times 500 \times 50$

$= 15000000\ \text{cm}^3$

Volume occupied by mortar = 5% of volume of wall

$= \frac{5}{100} \times 15000000 = 750000\ \text{cm}^3$

$\Rightarrow$ $(2343.75) \times n = 15000000 - 750000$

$= 14250000$

$n = 6080$ Bricks

Sol 28. Volume of metal

= Volume of outer surface – volume of inner surface

$= 52 \times 40 \times 29 - 48 \times 36 \times 25$

$= 60320 - 43200$

$= 17120\ \text{cm}^3$

$\therefore$ $1\ \text{cm}^3 = 0.5$gms (given)

$\Rightarrow$ $17120\ \text{cm}^3 = 17120 \times 0.5\ \text{gms} = 8560\ \text{gms}$

$= 8.56$ kg

Sol 29. Length of tank (l) = 4 m

Breadth of tank (b) = 3 m

Depth of tank (d) = h

$\therefore$ Volume = lbh

$24 = 4 \times 3 \times h$

$\Rightarrow$ $h = 2$ m

(As tank is open from upper sides remaining five sides of the tank will be painted)

Area of 5 sides = $2(hb + lh) + lb$

$= 2(2 \times 3 + 2 \times 4) + 4 \times 3$

$= 28 + 12 = 40\ \text{m}^2$

$\therefore$ Total area to be painted inner and outer side $= 40\text{m}^2$ cost of painting $= 40 \times 10 = 400$ Rs.

Sol 30. $\therefore$ $V = abc$

$\therefore$ $S = 2(ab + bc + ca)$

$\Rightarrow$ $S = 2abc\left(\frac{1}{c}+\frac{1}{a}+\frac{1}{b}\right)$

$\Rightarrow$ $S = 2V\left(\frac{1}{c}+\frac{1}{a}+\frac{1}{b}\right)$

$\Rightarrow$ $\frac{1}{V} = \frac{2}{S}\left(\frac{1}{c}+\frac{1}{a}+\frac{1}{b}\right)$

Sol 31. $\therefore$ Sum of area of floor and roof = 2 (length × breadth)

$\therefore$ Sum of area of four walls = 2 (length × height + breadth × height)

$\Rightarrow$ $2(l \times b) = 2(l \times h + b \times h)$

$\Rightarrow$ $lb = (l + b)h$

$\Rightarrow$ $15 \times 12 = (15 + 12)h$

$\Rightarrow h = \dfrac{15 \times 12}{27} = \dfrac{20}{3}$ m

$\therefore$ Volume of room = lbh

$= 15 \times 12 \times \dfrac{20}{3}$

$= 1200\ m^3$

Sol 32. $\therefore$ External length (L) = 1.46 m = 146 cm

$\therefore$ External breadth (B) = 1.16 m = 116 cm

$\therefore$ External Height (H) = 8.3 dm = 83 cm

$\Rightarrow$ Internal length (l) = 146 – 6 = 140 cm

$\Rightarrow$ Internal breadth (b) = 116 – 6 = 110 cm

$\Rightarrow$ Internal Heigth (h) = 83 – 3 = 80 cm

Total surface area of inner walls

$= 2h(l + b) + lb$

$= 2 \times 80\,(140 + 110) + 140 \times 110$

$= 40{,}000 + 1540$

$= 55400\ cm^2$

Cost of painting 100 cm^2 = 50 paise = 0.5 Rs.

cost of painting 55400 cm^2 = 554 × 0.5 Rs

= Rs. 277 Rs.

Sol 33. Volume = 216 litre

$\therefore$ depth $h = \dfrac{l}{3}$,

$\therefore\ b = \dfrac{1}{2} \times \dfrac{1}{3}(l - h) = \dfrac{1}{2}\left(l - \dfrac{l}{3}\right) = \dfrac{l}{9}$

$\therefore$ Volume = lbh

$\Rightarrow l \times \dfrac{l}{9} \times \dfrac{l}{3} = 216 \times 1000\ cm^3$

$l = \sqrt[3]{27 \times 216 \times 1000}$

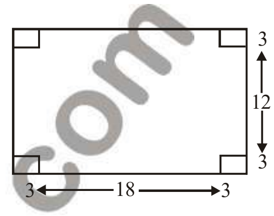
= 180 cm = 18 dm.

Sol 34. Hint: Same as Question 18

Sol 35. Length remained after square is removed (l) = 18 cm

Breadth remained after square is removed (b) = 12 cm

Height of box such formed (h) = 3 cm



$\therefore$ Surface area of box = $2(lb + bh + hl)$

$= 2(18 \times 12 + 12 \times 3 + 18 \times 3)$

$= 612\ cm^2$

Sol 36. Volume of bar = piece of copper

area of cross section of square × length = 1m^3

$a^2 \times 36 = 1$

$\Rightarrow a = \dfrac{1}{6}$ m

then volume of cube = $\dfrac{1}{6} \times \dfrac{1}{6} \times \dfrac{1}{6} = \dfrac{1}{216}\ m^3$

cost of cube = Volume × cost of 1m^3

$= \dfrac{1}{216} \times 108 = \dfrac{1}{2}$ Rs. = 50 paise

Sol 37. Let the length , breadth and height of block be $3x$, $2x$ and x

$\therefore$ Volume of block = lbh

$3x \times 2x \times x = 10368$

$x = 12$ dm

Now, length, breadth and height of block will be 36 dm, 24 dm and 12 dm respectively.

$\therefore$ Total surface area of block = $2(lb + bh + hl)$

$= 2(36 \times 24 + 24 \times 12 + 36 \times 12)$

$= 3168\ dm^2$

$\Rightarrow$ cost of polishing entire surface = 3168 × 0.02

= Rs. 63.36

Sol 38. Volume of water in the land = 1 sq km × 2 cm

$= 1000000 \times 0.02 = 20000\ m^3$

$\therefore$ 50% raindrops are collected

$\Rightarrow$ Volume of raindrops = 50% of 20000 m^3 = 10000 m^3

$\therefore$ Volume of water in the pool = Volume of raindrops collected

$\Rightarrow 100 \times 10 \times h = 10000$

$\Rightarrow h = 10$ m

level increased is water pool = 10 m

Sol 39. Volume of larger cube = summation of volume of smaller cubes

$\therefore$ Let the edge of smaller cubes be a

$\Rightarrow (10)^3 = (a)^3 + (a)^3$

$\Rightarrow (10)^3 = 2\,(a^3)$

$$\Rightarrow a = \frac{10}{(2)^{1/3}}$$

ratio of edge of smaller cube to larger cube

$$\Rightarrow \frac{a}{10} = \left(\frac{1}{2}\right)^{1/3}$$

Sol 40. Surface area of smaller cubes = $6a^2 = 96$

$\Rightarrow a = 4$ cm

$\therefore$ Volume of one smaller cube = $a^3 = 64\ cm^3$

Surface area of larger cube = $6b^2 = 384$

$= b = 8$

Volume of larger cube = $b^3 = 512 cm^3$

Volume of largest cube = n × (Volume of one smaller cube)

($\therefore$ n = no. of smaller cubes) $512 = n \times 64$

$\Rightarrow n = 8$

Sol 41. Area of shaded region where soil has to be spread = (Area of field) – (Area of tank)

$= 13.5 \times 2.5 - 5 \times 4.5 = 11.25$

Volume of soil dugged from tank = volume of soil spread out in remaining field

$\Rightarrow 5 \times 4.5 \times 2.1 = 11.25 \times h$

($\therefore$ h = level of field raised)

$\Rightarrow h = 4.2$ m

Sol 42. Let the length, breath and height of box be x, $2x$ and $4x$

$\therefore$ outer surface area of box = $2\,(lb + bh + hl)$

$= 2\,(x \times 2x + 2x \times 4x + x \times 4x) = 28\,x^2$

$\therefore$ cost of cloth covering box = $20 \times 28\,x^2$ Rs.

$\therefore$ cost of sheet covering box = $20.5 \times 28x^2$ Rs.

$\therefore$ difference between their cost = 126 Rs.

$\Rightarrow 20.5 \times 28x^2 - 20 \times 28x^2 = 126$

$\Rightarrow 0.5 \times 28 \times x^2 = 126$

$$\Rightarrow x^2 = \frac{126}{14} = 9$$

$x = 3$

dimensions of box $\Rightarrow x = 3$m, $2x = 6$m, $4x = 12$m

Sol 43. Let the edge of cube = a

sum of edges = $12a$

$\therefore$ Volume of cube = a^3

According to question :-

$a^3 = 12a$

$\Rightarrow a^2 = 12$

Surface area = $6a^2 = 6 \times 12 = 72$

Sol 44. Time = 2 hour 30 min

= 2.5 hour

$$\text{Rate} = \frac{\text{Volume of water}}{\text{time}} = \frac{2.4 \times 2 \times 1.5}{2.5}$$

$$= \frac{24 \times 3}{25} = 2.88 \text{ cu.m/h}$$

Sol 45. Volume of water left in the tank = Initial volume of water in tank – Volume of water taken out

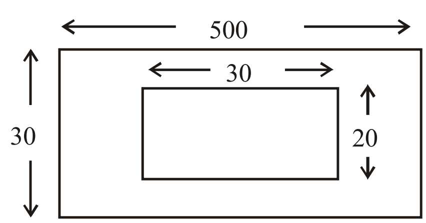
($\therefore$ Volume of cuboid = lbh)

$\Rightarrow 20 \times 7 \times 10 - 20 \times 7 \times 2$

$\Rightarrow 1120\ m^3$

Sol 46. Area in which soil has to be spread

= Area of field – Area of tank



$= 500 \times 30 - 30 \times 20$

$= 14400\ m^2$

Volume of soil dugged out = volume of spread in remaining field

$30 \times 20 \times 12 = 14400 \times h$

($\therefore h$ = rise in level)

$h = 0.5$ m.

Sol 47. Area of four walls

$= 2h(l + b) = 2 \times 3(8 + 6) = 84m^2$

Area of two windows and one door

$$= 2\left(\frac{3}{2}\times 1\right)+1\left(2\times\frac{3}{2}\right)=6\ m^2$$

$\Rightarrow$ Area of walls to be papering = 84 – 6

$= 78\ m^2$

cost of 50cm × 1m = 25paise

cost of 1m × 1m = 50paise = $\frac{1}{2}$

$\Rightarrow$ cost of papering = $78 \times \frac{1}{2}$

= Rs.39

Sol 48. Volume of water coming out from Pipe

= 8 cu.ft/min × 2.5 min

= 20 cu. fit

$\therefore$ Let the depth of water in tank be h.

width of tank = $\frac{3}{2}$ = 1.5 ft.

$\Rightarrow$ Volume of water in tank = Volume of water coming out from pipe

$\Rightarrow 3 \times 1.5 \times h = 20$

$\Rightarrow h = 4.44$ ft.

Sol 49. Volume of water displaced by 150 men = volume of water is came out

(Let the height raised in water = h)

$\Rightarrow 8 \times 150 = 90 \times 40 \times h$

$\Rightarrow h = \frac{1}{3}$ m = 33.33 cm

Sol 50. Volume of water drawn out = (Volume of water initially in tank) – (Volume of water remained in tank)

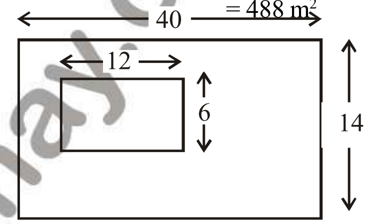
$= 15 \times 6 \times 10 - 15 \times 6 \times 9 = 90\ m^3$ = 90,000 litre

($1\ m^3$ = 1000litre)

Sol 51. Area of shaded region where soil has to be placed = Area of field – area of pit.

$= 40 \times 14 - 12 \times 6$

$= 488\ m^2$



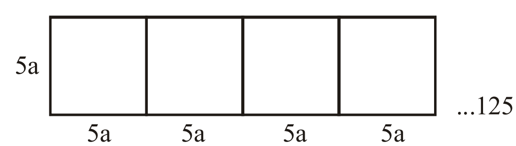
$\therefore$ Let the level of field = h

$\therefore$ Volume of soil spread in field = Volume of soil taken out from fit.

$\Rightarrow 12 \times 6 \times 5 = 488 \times h$

$h = 0.7377$ m = 73.77 cm.

Sol 52. Let side of small cubes be a, and side of larger cube be b.



$\therefore$ Volume of larger cube = 125 × volume of smaller cube

$\Rightarrow b^3 = 125a^3$

$\Rightarrow b = 5a$ ....(i)

Total surface, area of larger cube = $6b^2$

$= 6(5a)^2$

$= 150\ a^2$

When these 125 cubes are arranged to form a cuboid

$\Rightarrow$ Breadth and height of cuboid = a

Length of cuboid = $125a$

Now, total surface of cuboid = $2(lb + bh \times lh)$

$= 2(125a^2 + a^2 + 125a^2)$

$= 502\ a^2$

Increase in total surface area

$= \frac{502a^2 - 150a^2}{150a^2} \times 100 = \frac{352}{150} \times 100$

$= \frac{704}{3} = 234\frac{2}{3}\%$

Sol 53. Volume of water will come out of pipe in 15 min

= Cross section area × Length per 15 min

$= \frac{13}{10000} \times \frac{9000}{60} \times 15 = 2.925 \text{ m}^3$

$\therefore$ Volume of water filled in tank in 15 min

= Volume of water came out of pipe in 15 min

$\Rightarrow 45 \times 26 \times h = 2.925$

($\therefore h$ = level of water rise in tank)

$\Rightarrow h = 0.0025$ m

Sol 54. Volume of water comes out from pipe in 1 sec

$= \frac{5}{100} \times \frac{3}{100} \times 16 \text{ m}^3 = 0.0240 \text{ m}^3$

Volume of water comes out from pipe in 25 min

$= 0.0240 \times 25 \times 60 = 36 \text{ m}^3$

$\Rightarrow$ Volume of water pour into tank = volume of water comes out from pipe

$\therefore 15 \times 12 \times h = 36$

($\therefore h$ = rise in level of water)

$\Rightarrow h = 0.2$ m

Sol 55. Cube of largest length can be cut of 17 cm. (HCF of 85, 17, 51 is 17)

$\therefore$ total no. of cubes $= \frac{\text{Volume of cuboid}}{\text{Volume of one cube}}$

$= \frac{85 \times 17 \times 51}{17 \times 17 \times 17} = 15$ cubes

$\therefore$ surface of one cube $= 6a^2$

$= 6 \times 17 \times 17 = 1734 \text{ cm}^2$

$\Rightarrow$ Surface area of 15 cube = 15 × 1734

$= 26010 \text{ cm}^2$

Sol 56. For surface area to be minimum 64 cubes will be arranged such that each edge of cube will contain 4 small cubes

$\therefore$ side of cube = a = 4 cm

Length of diagonal $= a\sqrt{3}$

$= 4\sqrt{3}$ cm

Sol 57. Let the length, breadth and height of cuboid be l, b and h

$\Rightarrow x^2 = l^2 + b^2$ ....(i)

$y^2 = b^2 + h^2$ ....(ii)

$z^2 = l^2 + h^2$ ....(iii)

adding (i), (ii), (iii) we get,

$x^2 + y^2 + z^2 = 2(l^2 + b^2 + h^2)$

$\Rightarrow l^2 + b^2 + h^2 = \frac{x^2 + y^2 + z^2}{2}$ . ... (iv)

subtracting (i) from (iv) we get,

$h^2 = \frac{y^2 + z^2 - x^2}{2}$

$\Rightarrow h = \sqrt{\frac{y^2 + z^2 - x^2}{2}}$

Subtracting (ii) from (iv) we get,

$l^2 = \frac{x^2 + z^2 - y^2}{2} \Rightarrow l = \sqrt{\frac{x^2 + z^2 - y^2}{2}}$

subtracting (iii) from (iv) we get,

$\therefore b^2 = \frac{x^2 + y^2 - z^2}{2}$

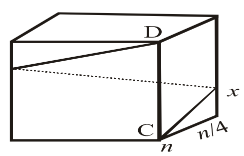
$\Rightarrow b = \sqrt{\frac{x^2 + y^2 - z^2}{2}}$

$\therefore$ Volume = lbh

$= \sqrt{\frac{(x^2 + y^2 - z^2)(y^2 + z^2 - x^2)(x^2 + z^2 - y^2)}{8}}$

$= \frac{\sqrt{(x^2 + y^2 - z^2)(y^2 + z^2 - x^2)(x^2 + z^2 - y^2)}}{2\sqrt{2}}$

Sol 58. The string of minimum length, if starting from C, touches next corner at height $n/4$ on the completion of one turn, starting from height $n/4$ touches next corner at height $n/2$ in the second turn, and so on.



$$CX = \sqrt{n^2 + \left(\frac{n}{4}\right)^2} = \frac{\sqrt{17} \times n}{4}$$

$\therefore$ Length of string $= 4 \times \frac{\sqrt{17} \times n}{4} = \sqrt{17}n$.

Aliter (Quircker method) : Opening up the four vertical sides of the cubs of side n,

Length of string $= \sqrt{(4n)^2 + (n)^2}$

$= \sqrt{17} \times n$

Sol 59. Cross section area of pipe $= 20 \times 20 = 400\ cm^2$

$= 0.04\ m^2$

Volume of water emptied by pipe in 1 hour

$= 0.04 \times 15000 = 600\ m^3$

[$\therefore$ water coming at the rate of 15000m/hr]

Volume of water in rectangular reservoir

$= 80 \times 60 \times 6.5$

$= 31200\ m^3$

Time taken to emptied the take $= \frac{31200}{600}$

$= 52$ hour.

Sol 60. Volume of water rained = volume of water displaced by n people

$\Rightarrow$ volume of water raised $= 24\ m \times 15\ m \times 0.01\ m$

$= 3.6\ m^3$

$\Rightarrow 3.6 = 0.1 m^3 \times n$

$n = 36$

☆☆☆☆☆

Exercise (Cylinder)

1. Two right circular cylinders of equal volume have their heights in the ratio 1 : 2. The ratio of their radii is:

(a) $\sqrt{2}$: 1 (b) 2 : 1
(c) 1 : 2 (d) 1 : 4

2. Water is being pumped out through a circular pipe whose internal diameter is 7 cm. If the flow of water is 12 cm per second, how many litres of water is being pumped out in one hour?

(a) 1663.2 (b) 1500
(c) 1747.6 (d) 2000

3. The base radii of two cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 : 3. The ratio of their volumes is:

(a) 27 : 20 (b) 20 : 27
(c) 9 : 4 (d) 4 : 9

4. A hollow cylindrical tube 20 cm long, is made of iron and its external and internal diameters are 8 cm and 6 cm respectively. The volume of iron used in making the tube is $\left(\pi = \frac{22}{7}\right)$

(a) 1760 cu.cm. (b) 880 cu.cm.
(c) 440 cu.cm. (d) 220 cu.cm

5. A hollow iron pipe is 21 cm long and its exterior diameter is 8 cm. If the thickness of the pipe is 1 cm and iron weighs 8 g/cm^3, then the weight of the pipe is (Take $\pi = \frac{22}{7}$)

(a) 3.696 kg (b) 3.6 kg
(c) 36 kg (d) 36.9 kg

6. The curved surface of a cylindrical pillar is 264 m^2 and its volume is 924 m^3. The ratio of its diameter to its height is $\left[\text{use } \pi = \frac{22}{7}\right]$

(a) 7 : 6 (b) 6 : 7
(c) 3 : 7 (d) 7 : 3

7. If the height of a cylinder is increased by 15 percent and the radius of its base is decreased by 10 percent then by what percent will its curved surface area change?

(a) 3.5 percent decrease
(b) 3.5 percent increase
(c) 5 percent increase
(d) 5 percent decrease

8. A right circular cylinder is formed by rolling a rectangular paper 12 cm long and 3 cm wide along its length. The radius of the base of the cylinder will be

(a) $\frac{3}{2\pi}$ cm (b) $\frac{6}{\pi}$ cm
(c) $\frac{9}{2\pi}$ cm (d) 2π cm

9. Water flows through a cylindrical pipe, whose radius is 7 cm at 5 metres per second. The time, it takes to fill an empty water tank, with height 1.54 metres and area of the base (3 × 5) square metres, is $\left[\text{take } \pi = \frac{22}{7}\right]$

(a) 6 minutes (b) 5 minutes
(c) 10 minutes (d) 9 minutes

10. Two solid cylinders of radii 4 cm and 5 cm and lengths 6 cm and 4 cm respectively are recast into cylindrical disc of thickness 1 cm. The radius of the disc is

(a) 7 cm (b) 14 cm
(c) 21 cm (d) 28 cm

11. The lateral surface area of a cylinder is 1056 cm^2 and its height is 16 cm. Find its volume.

(a) 4545 cm^3 (b) 4455 cm^3
(c) 5445 cm^3 (d) 5544 cm^3

12. A solid cylinder has total surface area of 462 sq. cm. Its curved surface area is $\frac{1}{3}$rd of the total surface area. Then the radius of the cylinder is

(a) 7 cm (b) 3.5 cm
(c) 9 cm (d) 11 cm

13. The radius and height of a cylinder are in the ratio 5 : 7 and its volume is 550 cm^3. Calculate its curved surface area in sq. cm.

(a) 110 (b) 444
(c) 220 (d) 616

14. If the radius of a cylinder is de-creased by 50% and the height is increased by 50% to form a new cylinder, the volume will be decreased by

(a) 0% (b) 25%
(c) 62.5% (d) 75%

15. Find the volume, curved surface area and the total surface area of a cylinder with diameter of base 7 cm. and height 40 cm.

(a) 1540 cm^3. 880 cm^2, 957 cm^2
(b) 1740 cm^3. 1080 cm^2, 880 cm^2
(c) 2040 cm^3, 1740 cm^2, 1540 cm^2
(d) 2420 cm^3. 1470 cm^2, 1880 cm^2

16. There is a cylindrical pipe. Its height is 20 m, outer radius is 2 m and inner radius is 1.5 m. Find the volume of the metal used in the pipe.

(a) 110 m^3 (b) 210 m^3
(c) 180 m^3 (d) 220 m^3

17. A reservoir is supplied water by a pipe 6 cm in diameter. How many pipes of 1.5 cm diameter would discharge the same quantity, supposing the velocity of water is same?

(a) 8 (b) 12
(c) 16 (d) 20

18. A right circular cylindrical tank has the storage capacity of 38808 ml. If the radius of the base of the cylinder is threefourth of the height, what is the diameter of the base?

(a) 28 cm (b) 56 cm
(c) 21 cm (d) 42 cm

19. A cylinderical tank of diameter 35 cm is full of water. If 11 litres of water is drawn off, the water level in the tank will drop by :

(a) $10\frac{1}{2}$ cm. (b) $11\frac{3}{7}$ cm.
(c) $12\frac{6}{7}$ cm. (d) 14 cm.

20. Water flows through a cylinderical pipe of internal diameter 7 cm at 2 m per second. If the pipe is always half full, then what is the volume of water (in litres) discharged per hour.

(a) 2310 (b) 3850
(c) 4620 (d) 9240

21. A cylindrical tube open at both ends in made of metal. The internal diameter of the tube is 11.2 cm and its length is 21 cm. The metal every where is 0.4 cm thick. The volume of the metal is:

(a) 280.52 cm^3 (b) 306.24 cm^3
(c) 310 cm^3 (d) 316 cm^3

22. The volume of a right circular cylinder is V, total surface area S, height h and radius of base then the ratio of $V\left(\frac{1}{h}+\frac{1}{r}\right)$: S is :

(a) 1 : 4 (b) 2 : 1
(c) 1 : 2 (d) 1 : 1

23. If the diameter of the base of a cylindrical pillar is 4 m and its height is 21 m, then the cost of construction of the pillar at Rs. 1.50 per cubic metre is:

(a) Rs. 396 (b) Rs. 400
(c) Rs. 410 (d) Rs. 420

24. Given that 1 cm^3 of a metal weights 5 gms, the weight of a cylindrical metal container with base radius 10.5 cm and height 600 cm, is :

(a) 97.65 kg (b) 48.75 kg
(c) 1039.5 kg (d) 1024.5 kg

25. A right circular cylindrical tunnel of diameter 2 m and length 40 m is to be constructed from a sheet of iron. The area of the iron sheet required in m^2, is:

(a) 40π (b) 80π
(c) 160π (d) 200π

26. The ratio between the radius of the base and the height of a cylinder is 2 : 3. If its volume is 1617 cm^3, the total surface area of the cylinder is :

(a) 308 cm^2 (b) 462 cm^2
(c) 540 cm^2 (d) 770 cm^2

27. The sum of the radius of the base and the height of a solid cylinder is 37 m. If the total surface area of the cylinder be 1628 m^2, its volume is :

(a) 3180 m^3 (b) 4620 m^3
(c) 5240 m^3 (d) None of these

28. The ratio between the curved surface area and the total surface area of a right circular cylinder is 1 : 2. If the total surface area is 616 cm^2. the volume of the cylinder is :

(a) 1232 cm^3 (b) 1078 cm^3
(c) 1848 cm^3 (d) None of these

29. The altitude of a circular cylinder is increased six times and the base area is decreased to one-ninth of its value. The factor by which the lateral surface of the cylinder increases, is :

(a) $\frac{2}{3}$ (b) $\frac{1}{2}$
(c) $\frac{3}{2}$ (d) 2

30. Two cylindrical vessels with radii 15 cm & 10 cm and heights 35 cm & 15 cm respectively are filled with water. If this water is poured into a cylindrical vessel 15 cm in height, then the radius of the vessel is :

(a) 25 cm (b) 20 cm
(c) 17.5 cm (d) 18 cm

31. A circular pipe is to be designed in such a way that water flowing through it at a velocity of 7 m per min. is collected at its open end at the rate of 11 cubic metre per min. what should be the inner radius of the pipe

(a) $\sqrt{2}$ m (b) 2 m
(c) $\frac{1}{\sqrt{2}}$ m (d) $\frac{1}{2}$ m

32. Given a solid cylinder of radius 10 cm and length 1000 cm, a cylindrical hole is made into it to obtain a cylindrical shell of uniform thickness and having volume equal to one-fourth of the original cylinder. The thicknes sof the cylindrical shell is:

(a) $5\left(\sqrt{5}-2\right)$ cm (b) $10\left(2-\sqrt{3}\right)$cm
(c) 5 cm (d) $5\sqrt{2}$ cm

33. A well 20 m in diameter is dug 14 m deep and the earth taken out is spread all around it to a width of 5 m to form an ambankment. The height of the embankment is:

(a) 10 m (b) 11 m
(c) 11.2 m (d) 11.5 m

34. A well of radius *'r'* is dug 20 m deep and the earth taken out is spread all around it to a width of 1 m to form an embankment. The height of the embankment is 5 m then find the value of 'r' :

(a) $\frac{1+\sqrt{5}}{2}$ (b) $\frac{1+\sqrt{5}}{4}$
(c) $\frac{\sqrt{5}-1}{2}$ (d) $\frac{\sqrt{5}-1}{4}$

35. A right circular cylinder of height 16 cm is covered by a rectangular tin foil of size 16 cm × 22 cm. The volume of the cylinder is:

(a) 352 cm^3 (b) 308 cm^3
(c) 616 cm^3 (d) 176 cm^3

36. The volume of the metal of a cylindrical pipe is 748 cm^3. The length of the pipe is 14 cm and its external radius is 9 cm. Its thickness is (Take $\pi = \frac{22}{7}$)

(a) 1 cm (b) 5.2 cm

(c) 2.3 cm (d) 3.7 cm

37. The radii of the bases of two cylinders A and B are in the ratio 3 : 2 and their heights in the ratio $n:1$. If the volume of cylinder A is 3 times that of cylinder B, the value of n is:

(a) $\frac{4}{3}$ (b) $\frac{2}{3}$

(c) $\frac{3}{4}$ (d) $\frac{3}{2}$

38. A cylinder has '*r*' as the radius of the base and '*h*' as the height. The radius of base of another cylinder, having double the volume but the same height as that of the first cylinder must be equal to:

(a) $\frac{r}{\sqrt{2}}$ (b) $2r$

(c) $r\sqrt{2}$ (d) $\sqrt{2r}$

39. The height of a solid right circular cylinder is 6 metres and three times the sum of the areas of its two end faces is twice the area of its curved surface. The radius of its base, in metre, is:

(a) 4 (b) 2

(c) 8 (d) 10

40. The radius of a cylinder is 10 cm and height is 4 cm. The number of centimeters that may be added either to the radius or to the height to get the same increase in the volume of the cylinder is:

(a) 5 (b) 4

(c) 25 (d) 16

41. The area of the curved surface and the area of the base of a right circular cylinder are *a* cm^2 and *b* cm^2 respectively. The height of the cylinder is:

(a) $\frac{2a}{\sqrt{\pi b}}$ cm (b) $\frac{a\sqrt{b}}{2\sqrt{\pi}}$ cm

(c) $\frac{a}{2\sqrt{\pi b}}$ cm (d) $\frac{a\sqrt{\pi}}{2\sqrt{b}}$ cm

42. It is required to design a circular pipe such that water flowing through it at a speed of 7m/min fills a tank of capacity 440 cu m in 10 min. The inner radius of the pipe should be.

(a) 2m (b) $\sqrt{2}$ m

(c) 1/2 m (d) $\frac{1}{\sqrt{2}}$

43. A cylinder is filled to 4/5th of volume . It is then tilted so that the level of water coincides with one half edge of its bottom and top edge of the opposite side. In the process, 30 litre of the water is spilled. What is the volume of the cylinder?

(a) 75 litre (b) 96 litre

(c) Data insufficient (d) 100 litre

44. A monument has 50 cylindrical pillars each of diameter 50 cm and height 4m . what will be the labour charges for getting these pillars cleared at the rate of 50 paise per m^2 (Use $\pi = 3.14$).

(a) Rs. 237 (b) Rs. 157

(c) Rs. 257 (d) Rs. 353

45. The inner diameter of a circular building is 54 cm and the base of the wall occupies space of 352 cm^2. The thickness of the wall is

(a) 29 cm (b) 2 cm

(c) 4 cm (d) 58 cm

46. A right circular cyindrical tank has the storage capacity of 38808 ml. If the radius of the base of the cylinder is three fourth of the height what is the radius of base?

(a) 28 cm (b) 56 cm

(b) 21cm (d) 42 cm

47. A rectangular piece of iron sheet measuring 50 cm and 100 cm is rolled into cylinder of height 50 cm. If the cost of painting the cylinder is Rs. 50 per square meter, then what will be the cost of painting the surface of the cylinder?

(a) Rs. 2500 (b) Rs. 37.50

(c) Rs. 75.00 (d) Rs. 87.50

48. The internal radius and thickness of a hollow metallic pipe are 24 cm and 1 cm respectively. It is melted and recast into a solid cylinder of equal length. The diameter of the solid cylinder will be:

(a) 7 cm (b) 14 cm

(c) 960 cm^3 (d) 980 cm

49. A solid cylinder has total surface area of 462 sq cm. Curved curface area is $\frac{1}{3}$rd of it's total surface area. Volume of the cylinder is–

(a) 530 cm^3 (b) 536 cm^3

(c) 539 cm^3 (d) 545 cm^3

50. The thickness of a hollow wooden cylinder is 2 cm. It is 35 cm long and its inner radius is 12 cm. Find the volume of the wood required to make the cylinder, assuming it is open at either end.

(a) 5720 cm^3 (b) 5770 cm^3

(c) 7520 cm^3 (d) 5570 cm^3

51. Sixteen cylindrical cans, each with a radius of 1 unit, are placed inside a cardboard box four in a row. If the cans touch the adjacent cans and or the walls of the box, then which of the following could be the interior area of the bottom of the box in square units?

(a) 16 (b) 32

(c) 64 (b) 128

Answer

1. (a)	**2.** (a)	**3.** (b)	**4.** (c)	**5.** (a)	**6.** (d)	**7.** (b)	**8.** (b)	**9.** (a)			
10. (b)	**11.** (d)	**12.** (a)	**13.** (c)	**14.** (c)	**15.** (a)	**16.** (a)	**17.** (c)	**18.** (d)			
19. (b)	**20.** (c)	**21.** (b)	**22.** (c)	**23.** (a)	**24.** (c)	**25.** (b)	**26.** (d)	**27.** (b)			
28. (b)	**29.** (d)	**30.** (a)	**31.** (c)	**32.** (c)	**33.** (c)	**34.** (b)	**35.** (c)	**36.** (a)			
37. (a)	**38.** (c)	**39.** (a)	**40.** (a)	**41.** (c)	**42.** (b)	**43.** (d)	**44.** (b)	**45.** (b)			
46. (c)	**47.** (a)	**48.** (b)	**49.** (c)	**50.** (a)	**51.** (c)						

Solution & Hints

Sol 1. $V_1 = V_2$ ....(given)

$\Rightarrow \pi r_1^2 h_1 = \pi r_2^2 h_2$

$$\Rightarrow \left(\frac{r_1}{r_2}\right)^2 = \frac{h_2}{h_1} = \frac{2}{1}$$

$$\Rightarrow \frac{r_1}{r_2} = \frac{\sqrt{2}}{1}$$

Sol 2. Volume of water poured out in one second = $\pi r^2 h$

$$= \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 12 = 462 \text{ cm}^3$$

Hence, water poured out in one hour = 462 × 3600

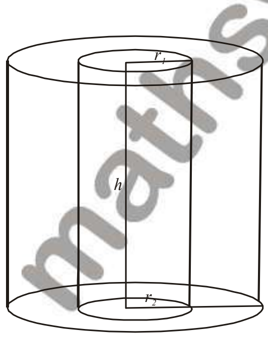
= 1663200 cm³ (1h = 3600 sec)

= 1663.2 litre (1 l = 1000 cm³)

Sol 3. $\frac{r_1}{r_2} = \frac{2}{3}, \frac{h_1}{h_2} = \frac{5}{3}$ ....**(given)**

$$\frac{V_1}{V_2} = \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2} = \left(\frac{2}{3}\right)^2 \times \frac{5}{3} = \frac{20}{27}$$

Sol 4.



Volume of Iron = $\pi (r_1^2 - r_2^2) h$

$$= \frac{22}{7} [(4)^2 - (3)^2] \times 20 = 440 \text{ c.u. cm.}$$

Sol 5. $\therefore$ Volume of metal = $\pi (r_1^2 - r_2^2) h$

$$= \frac{22}{7}(4^2 - 3^2) \times 21 = 462 \text{ cm}^3$$

weight of pipe = 462 × 8 = 36960 gram

= 3.696 kg.

Sol 6. $\therefore$ Curved surface area = $2\pi rh = 264\text{m}^2$ …(1)

$\therefore$ Volume = $\pi r^2 h = 924 \text{ m}^3$ …(2)

Divide (2) by (1) we get,

$$\Rightarrow \frac{\pi r^2 h}{2\pi rh} = \frac{924}{264}$$

$$\Rightarrow \frac{r}{2} = 3.5$$

$\Rightarrow r = 7$ m

Putting value of r in equation (1) we get,

$$\Rightarrow 2 \times \frac{22}{7} \times 7 \times h = 264$$

$\Rightarrow h = 6$ m

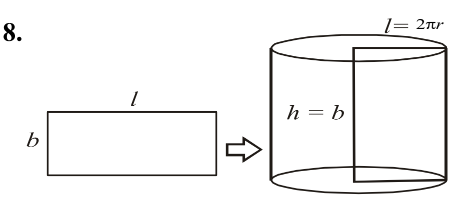
Hence, $\frac{D}{h} = \frac{2r}{h} = \frac{14}{6} = \frac{7}{3}$

Sol 7. $\therefore$ Curved Surface area $\propto r \times h$

effect on Curved Surface area = $a + b + \frac{ab}{100}$

$$= 15 - 10 + \frac{15 \times (-10)}{100} = 3.5\% \text{ increase}$$

Sol 8.



$\therefore$ Rectangle is rolled along its length then,
height = breadth
curcumference of base = length

$h = 3$ cm,

$2\pi r = 12\text{cm}$

$\Rightarrow \ r = \frac{6}{\pi}\text{cm}$

Sol 9. Time taken to empty the tank

$= \frac{\text{Volume of tank}}{\text{Volume of water flows through pipe in one hour}}$

$= \frac{3\times 5\times 1.54}{\pi(7)^2\times 5} = 300 \text{ second} = 6 \text{ min.}$

Sol 10. Volume of disc = Summation of Volume of both cylinder

$\pi r^2\times 1 = \pi(4)^2\times 6 + \pi(5)^2\times 4$

$\Rightarrow \ r^2 = 96 + 100$

$\Rightarrow \ r = 14 \text{ cm}$

Sol 11. Lateral surface area = 1056, h=16 cm..**(given)**

$\therefore \ 2\pi rh = 1056$

$\Rightarrow \ 2\times\frac{22}{7}\times r\times 16 = 1056$

$\Rightarrow \ r = 10.5$

$\therefore$ Volume = $\pi r^2 h$

$= \frac{22}{7}\times 10.5\times 10.5\times 16 = 5544 \text{ cm}^2$

Sol 12. $\therefore$ Total surface area $= 2\pi r(r+h) = 462$ sq. cm.

$\therefore$ Curved surface

$2\pi rh = \frac{1}{3}\times 462 = 154 \text{ sq.cm}$

$\Rightarrow \ 2\pi r^2 + 2\pi rh = 462$

$\Rightarrow \ 2\pi r^2 + 154 = 462$

$\Rightarrow \ 2\pi r^2 = 308$

$\Rightarrow \ r = 7 \text{ cm}$

Sol 13. Let the radius and height be $5x$ and $7x$.

$\therefore$ Volume = $\pi r^2 h$

$\Rightarrow \ \frac{22}{7}\times(5x)^2\times(7x) = 550$

$\Rightarrow \ x = 1$

Radius = $5x = 5$ cm

Height = $7x = 7$ cm

$\therefore$ Curved surface area $= 2\pi rh$

$= 2\times\frac{22}{7}\times 5\times 7 = 220\text{cm}^2$

Sol 14. Let the radius and height of cylinder be r and h.

Volume = $\pi r^2 h$

When radius is decreased by 50%

$\Rightarrow$ new radius = 0.5 r

and height increased by 50%

$\Rightarrow$ new height = 1.5 h

new volume of cylinder

$= \pi(0.5r)^2\times(1.5)h$

$= 0.375\,\pi r^2 h$

% Decrease in volume $= \frac{\pi r^2 h - 0.375\pi r^2 h}{\pi r^2 h}\times 100$

$= 62.5\%$

Sol 15. $r = \frac{7}{2}$cm, $h = 40$ cm **.......(given)**

$\therefore$ Volume=

$\pi r^2 h = \frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times 40 = 1540\text{cm}^3$

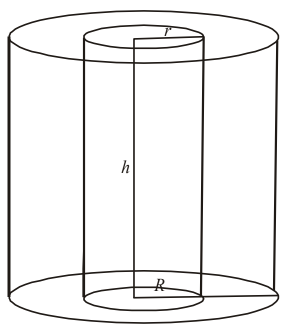
$\therefore$ Curved surface area = $2\pi\text{rh}$

$= 2\times\frac{22}{7}\times\frac{7}{2}\times 40 = 880 \text{ cm}^2$

$\therefore$ Total surface area = $2\pi r(r+h)$

$$2\times\frac{22}{7}\times\frac{7}{2}\left(\frac{7}{2}+40\right) = 957\text{ cm}^2$$

Sol 16.



$\therefore$ Outer radius = R = 2 m,

Inner radius = r = 1.5 m, and height = h = 20 m

$\therefore$ Volume of metal = $\pi\left(R^2 - r^2\right)h$

$$= \frac{22}{7}\left(2^2-(1.5)^2\right)\times 20$$

$= 110\text{ m}^3$

Sol 17. Volume of supplied water by diameter of 6 cm

= n × volume of water supplied by diameter 1.5 cm

$$\Rightarrow \quad \pi\times\left(\frac{6}{2}\right)^2\times h = n\times\pi\times\left(\frac{1.5}{2}\right)^2\times h \quad (n = \text{no. of pipes})$$

$\Rightarrow$ $n = 16$

Sol 18. Let the height be h.

radius (r) = $\frac{3}{4}h$ **....(given)**

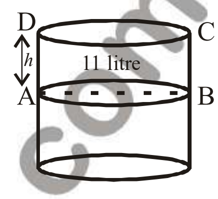
$\therefore$ Volume = $\pi r^2 h$

$$\Rightarrow \frac{22}{7}\times r^2\times\frac{4}{3}r = 38880\text{ml} = 38808\text{cm}^3$$

$\Rightarrow$ $r = 21$ cm

Hence, diameter = $2r = 42$ cm

Sol 19. Level of water dropped in tank will depend upon volume of water which is drawn off



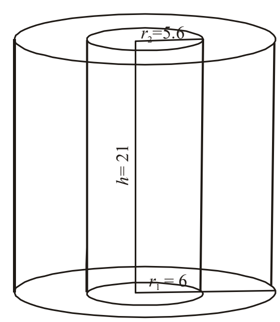
Volume of cylinder ABCD = 11 lt

$$\Rightarrow \pi\left(\frac{35}{2}\right)^2\times h = 11\times 1000 \quad (1 \text{ litre} = 1000\text{cm}^3)$$

$$h = \frac{11000\times 4\times 7}{22\times 35\times 35} = 11\frac{3}{7}\text{cm}$$

Sol 20. Hint : Volume of water will be equal to the volume of cylinder.

Sol 21.



Internal Radius = 11.2 = 5.6 cm

External radus = 5.6 + 0.4 = 6 cm

$\therefore$ Volume of metal = $\pi\left(r_1^2 - r_2^2\right)^2 h$

$$= \frac{22}{7}\left((6)^2-(5.6)^2\right)\times 21$$

$= 306.24\text{ cm}^3$

Sol 22. $\therefore$ Volume (V) = $\pi r^2 h$,

$\therefore$ Total surface area (S) = $2\pi r(r+h)$

$$V \times \left(\frac{1}{h}+\frac{1}{r}\right) = \pi r^2 h\left(\frac{r+h}{hr}\right)$$

$$= \pi r(r+h) = \frac{S}{2}$$

$$\Rightarrow \frac{V\left(\frac{1}{h}+\frac{1}{r}\right)}{S} = 1:2$$

Sol 23. $r = 2\text{m},\ h = 21\text{ m}$

$\therefore$ Volume of pillar = $\pi r^2 h$

$$= \frac{22}{7}\times 2\times 2\times 21 = 264\text{ m}^3$$

Cost of construction $= 264 \times 1.5 = \text{Rs. } 396$

Sol 24. $r = 10.5\text{cm}$ and $h = 600\text{ cm}$

$\therefore$ Volume of container = $\pi r^2 h$

$$= \frac{22}{7}\times 10.5\times 10.5\times 600 = 207900\text{ cm}^3$$

Weight of metal = $207900 \times \frac{5}{1000}$ kg

$= 1039.5$ kg

Sol 25. $r = 1$ m, and $h = 40$ m

$\therefore$ Area of sheet = $2\pi rh$

$= 2\times\pi\times 1\times 40$

$= 80\pi\text{ m}^2$

Sol 26. Let radius and height be $2k$ and $3k$

$\therefore$ Volume = $\pi r^2 h$

$$\Rightarrow \frac{22}{7}\times(2k)^2\times(3k) = 1617$$

$\Rightarrow k = 3.5$

$\Rightarrow$ Radius $= r = 2k = 7$ cm

$\Rightarrow$ height $= h = 3k = 10.5$ cm

$\therefore$ total surface area = $2\pi r(r+h)$

$$= 2\times\frac{22}{7}\times 7(7+10.5) = 770\text{cm}^2$$

Sol 27. $r + h = 37$ cm

$\therefore$ Total surface area = $2\pi r(r+h) = 1628$

$\Rightarrow 2\pi r\times 37 = 1628$

$\Rightarrow r = 7$ cm

$h = 37 - 7 = 30$ cm

$$\therefore \text{Volume} = \pi r^2 h = \frac{22}{7}\times 7\times 7\times 30 = 4620\text{ m}^3$$

Sol 28. $\frac{\text{Curved surface area}}{\text{Total surface area}} = \frac{1}{2}$

$$\text{CSA} = \frac{1}{2}\times\text{TSA} = \frac{1}{2}\times 616 = 308\text{cm}^2$$

$\Rightarrow 2\pi rh = 308$ ....(i)

$\Rightarrow$ T.S.A. = $2\pi r(r+h) = 616$

$\Rightarrow 2\pi r^2 = 616 - 308 = 308$

$\Rightarrow r = 7$ cm

Putting value of r in eq. (i) we get, $h = 7$ cm

$\therefore$ Volume = $\pi r^2 h$

$$= \frac{22}{7}\times 7\times 7\times 7 = 1078\text{cm}^3$$

Sol 29. Original altitude = h

New altitude = $6h$

Original base area = πr^2

New base area = $\frac{1}{9}\pi r^2$

New radius of bar is $\frac{1}{3}$ rd of original radius

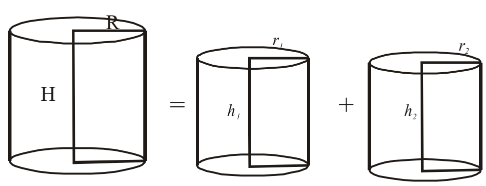
Original curved surface area = $2\pi rh$

$\therefore$ New curved surface area = $2\pi \times \frac{1}{3} r \times 6h$

$= 2\pi rh \times 2$

$\Rightarrow$ Surface area is increased by factor 2

Sol 30.



Volume of third vessel = sum of volume of first two vessel

$\Rightarrow \pi R^2 H = \pi r_1^2 h_1 + \pi r_2^2 h_2$

$\Rightarrow R^2 \times (15) = (15)^2 \times 35 + (10)^2 \times 15$

$\Rightarrow R^2 = 625$

$\Rightarrow R = 25$ cm

Sol 31. Volume of water discharged by pipe per min

= Volume of water collected

$\pi r^2 \times 7 = 11$

$\Rightarrow \frac{22}{7} \times r^2 \times 7 = 11$

$\Rightarrow r = \frac{1}{\sqrt{2}}$ m

Sol 32. Volume of cylindrical hole = $\frac{1}{4} \times$ volume of solid cylinder

$\therefore \frac{1}{3}\pi r^2 \times 1000 = \frac{1}{4} \times \frac{1}{3}\pi \times 10 \times 10 \times 1000$

$\Rightarrow r = 5$ cm

thickness of cylinder = radius of solid cylinder – radius of cylindrical hole = 10 – 5 = 5 cm

Sol 33. Volume of earth taken out =

$\pi \times (10)^2 \times 14 = 1400\pi$ m^3

Area of embankment = $\pi\left(R^2 - r^2\right)$

Volume of earth taken out = volume of embankment

Volume of earth taken out = Area of embankmenet $\times$ height of embankment

$\Rightarrow \pi(15^2 - 10^2) = 125\pi \times$ height of embankment

$\Rightarrow$ Height of embankment = $\frac{1400\pi}{125\pi} = 11.2$ m

Sol 34. Let the radius of well = r

volume of embankment (hollow cylinder)

= volume of earth taken out

$\Rightarrow \pi\left((r+1)^2 - r^2\right) \times 5 = \pi r^2 \times 20$

$\Rightarrow \left[(r+1+r)(r+1-r)\right] = r^2 \times 4$

$\Rightarrow (2r+1) \times 1 = 4r^2$

$\Rightarrow 4r^2 - 2r - 1 = 0$

$r = \frac{2 \pm \sqrt{4+16}}{8} = \frac{\sqrt{5}+1}{4}$ m

Sol 35. $h = 16$ cm

$\therefore$ Curved surface area of cylinder = area of rectangular foil.

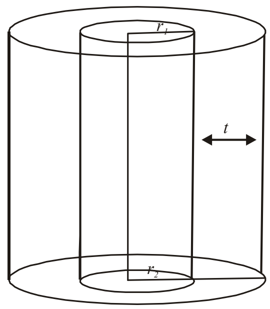
$\Rightarrow 2\pi rh = 16 \times 22$ cm^2

$\Rightarrow r = \frac{16 \times 22}{2 \times \frac{22}{7} \times 16} = \frac{7}{2}$

$\therefore$ Volume of cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 16 = 616 \text{ cm}^3$$

Sol 36.



Let t will be thickness of pipe

$\therefore$ Volume of metal of pipe $= \pi\left(r_1^2 - r_2^2\right) \times h$

$$\Rightarrow 748 = \frac{22}{7} \times \left[(9)^2 - (9-t)^2\right] \times 14$$

$$\Rightarrow 748 = \frac{22}{7} \times \left[(9+9-t) - (9-9+t)\right] \times 14$$

$$\Rightarrow 748 = \left[(18-t)t\right] \times 44$$

$$\Rightarrow t^2 - 18t + 17 = 0$$

$$\Rightarrow (t-17)(t-1) = 0$$

$t = 1$, or $t = 17$ (not possible) Hence, thickness = 1 cm

Sol 37. $\frac{r_A}{r_B} = \frac{3}{2}, \frac{h_A}{h_B} = \frac{n}{1}$ **......(given)**

$V_A = 3V_B \Rightarrow \frac{V_A}{V_B} = \frac{3}{1}$

$$\Rightarrow \frac{\pi r_A^2 h_A}{\pi r_B^2 h_B} = \frac{3}{1}$$

$$\Rightarrow \left(\frac{r_A}{r_B}\right)^2 \times \left(\frac{h_A}{h_B}\right) = \frac{3}{1}$$

$$\Rightarrow \frac{9}{4} \times \frac{n}{1} = \frac{3}{1}$$

$$\Rightarrow n = \frac{4}{3}$$

Sol 38. Let volume, height, radius of first cylinder be V_1, r_1 and h_1.

and volume, height, raidus of second cylinder be V_2, r_2 and h_2.

$h_1 = h_2$, $V_2 = 2V_1$

$r_1 = r$, $r_2 = ?$

$$\therefore \quad \pi r_2^2 h_2 = 2\pi r_1^2 h_1$$

$$\Rightarrow r_2^2 = 2r_1 = 2r$$

$$\Rightarrow r_2 = r\sqrt{2}$$

Sol 39. $h = 6$ m **....(given)**

$\therefore$ area of base $= 2\pi r$

$\therefore$ curved surface area $= 2\pi r^2 h$

According to question,

$$3 \times \left(2\pi r^2\right) = 2 \times 2\pi r^2 h$$

$3r = 2h = 2 \times 6$

$r = 4$ m

Sol 40. Let t cm is increased in radius

then volume $= \pi(10+t)^2 \times 4$

If t cm is increased in height

then volume $= \pi(10)^2 \times (4+t)$

volume should be equal in both cases

$$\pi(10+t)^2 \times 4 = \pi(10)^2 \times (4+t)$$

$\Rightarrow$ $(t+10)^2 = 25(4+t)$

$\Rightarrow$ $t^2 + 100 + 20t = 100 + 25t$

$\Rightarrow$ $t^2 = 5t$

$\Rightarrow$ $t = 5$ cm

Sol 41. $\therefore$ Area of base = $\pi r^2 = b \Rightarrow r = \dfrac{\sqrt{b}}{\sqrt{\pi}}$

$\therefore$ curved surface area = $2\pi rh = a$

$$\Rightarrow h = \frac{a}{2\pi r} = \frac{a\sqrt{\pi}}{2.\pi.\sqrt{b}} = \frac{a}{2\sqrt{\pi b}} \text{ cm}$$

Sol 42. Volume of water flowing in 10 min = 440 m^3

Volume of water flowing in 1 min = 44 m^3

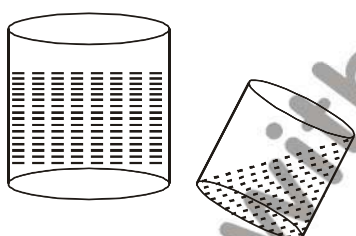
$\therefore$ $\pi r^2 h = 44$

$\Rightarrow$ $\dfrac{22}{7} \times r^2 \times 7 = 44$

$\Rightarrow$ $r^2 = 2$

$\Rightarrow$ $r = \sqrt{2}$ m

Sol 43.



Let total volume of cylinder = $5x$ litre

Volume of water = $\dfrac{4}{5} \times 5x = 4x$ litre

After tilting, there will be water half of total volume = $\dfrac{5x}{2}$

Spilled water = $4x - \dfrac{5x}{2}$

$\Rightarrow$ $30 = \dfrac{3x}{2}$

$\Rightarrow$ $x = 20$ litre.

Hence, volume of cylinder = $5x$ = 100 litre.

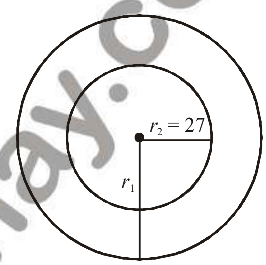
Sol 44. $\therefore$ Curved surface area = $2\pi rh$

Curved surface area of 50 pillars = 50 × $2\pi rh$

$$= 50 \times 2 \times \frac{22}{7} \times \frac{50}{2 \times 100} \times 4 = 314 \text{ m}^2$$

Labour charge for getting these pillars cleared = 314×0.5 = Rs. 157

Sol 45.



thickness of wall = $r_1 - r_2$

$\pi\left(r_1^2 - r_2^2\right) = 352$

$\Rightarrow$ $r_1^2 - r_2^2 = \dfrac{352}{22} \times 7 = 112$

$\Rightarrow$ $r_1^2 - (27)^2 = 112$

$\Rightarrow$ $r_1 = \sqrt{841} = 29$ cm

thickness of wall = 29 – 27 = 2 cm

Sol 46. $r = \dfrac{3}{4}h$ ...(given)

$\Rightarrow$ $h = \dfrac{4r}{3}$

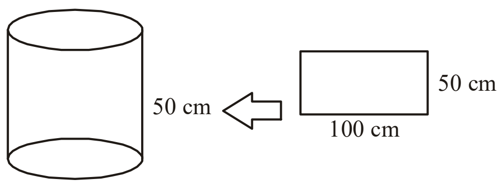
$\therefore$ Volume = $\pi r^2 h$ = 38808 ml = 38.808 l

= 38.808 × 1000 cm^3

$\Rightarrow$ $\dfrac{22}{7} \times r^2 \times \left(\dfrac{4r}{3}\right) = 38808$

$\Rightarrow$ $r^3 = 9261 \Rightarrow r = 21$ cm

Sol 47.



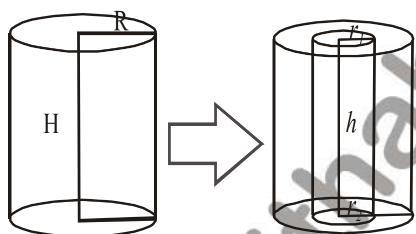
$2\pi r = 100\text{cm}$

Then, total surface area= $2\pi rh$

$100 \times 50 = 5000 \text{ cm}^2$

Cost of painting $= 5000 \times \dfrac{50}{100} =$ Rs. 2500

Sol 48.



Volume of solid cylinder = volume of hollow
metallic pipe

$\Rightarrow \quad \pi R^2 H = \pi\left(r_1^2 - r_2^2\right)h \qquad \{H=h\}$

$\Rightarrow \quad R^2 = r_1^2 - r_2^2$

$\Rightarrow \quad R^2 = (25)^2 - (24)^2$

$\Rightarrow \quad R^2 = 49$

$\Rightarrow \quad R = 7$

Hence, diameter = 14 cm

Sol 49. $\therefore$ Total surface area $= 2\pi r(r+h) = 462$ sq. cm.

$\therefore$ Curved surface area =

$2\pi rh = \dfrac{1}{3} \times 462 = 154$ sq.cm

$\Rightarrow \quad \pi rh = 77 \text{ cm}^2 \qquad \ldots\text{(i)}$

$2\pi r(r+h) = 462$ sq. cm.

$\Rightarrow \quad 2\pi r^2 + 2\pi rh = 462$

$\Rightarrow \quad 2\pi r^2 + 154 = 462$

$\Rightarrow \quad 2\pi r^2 = 308$

$\Rightarrow \quad r = 7$ cm

Multiply by r to eqn (i)

Hence, volume $= \pi r^2 h = \pi rh \times r$

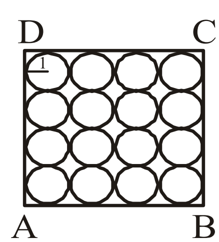
$= 77 \times 7 = 539 \text{ cm}^3$

Sol 50. $\therefore$ Volume $= \pi\left(r_1^2 - r_2^2\right) \times h$

$= \dfrac{22}{7}\left((14)^2 - (12)^2\right) \times 35$

$= 110 \times 26 \times 2 = 5720 \text{ cm}^3$

Sol 51.



Diameter of each circle = 2cm

$\therefore$ AB = 2 × 4 = 8

Similarly, CD = 8

Area of Bottom = AB × CD = 64 cm²

☆☆☆☆☆

Exercise - Cone

1. The slant height of a conical mountain is 2.5 km and the area of its base is 1.54 km^2. the height of the mountain is $\left(\pi = \frac{22}{7}\right)$

(a) 2.2 km (b) 2.4 km
(c) 3 km (d) 3.11 km

2. The ratio of the volumes of two cones is 2 : 3 and the ratio of radii of their bases is 1 : 2. The ratio of their heights is

(a) 3 : 8 (b) 8 : 3
(c) 4 : 3 (d) 3 : 4

3. If a right circular cone of height 24 cm has a volume of 1232 cm^3, then the area of its curved surface is $\left(\pi = \frac{22}{7}\right)$

(a) 1254 cm^2 (b) 704 cm^2
(c) 550 cm^2 (d) 154 cm^2

4. If the height of a given cone be doubled and radius of the base remains the same, the ratio of the volume of the given cone to that of the second cone will be

(a) 2 : 1 (b) 1 : 8
(c) 1 : 2 (d) 8 : 1

5. The radius of base and slant height of a cone are in the ratio 4 : 7. If its curved surface area is 792 cm^2, then the radius (in cm) of its base is $\left(\pi = \frac{22}{7}\right)$

(a) 8 (b) 12
(c) 14 (d) 16

6. The height of the cone is 30 cm. A small cone is cut off at the top by a plane parallel to its base. If its volume is $\frac{1}{27}$ of the volume of the cone, at what height, above the base, is the section made?

(a) 6 cm (b) 8 cm
(c) 10 cm (d) 20 cm

7. The diameter of the base of a right circular cone is 4 cm and its height is $2\sqrt{3}$ cm. The slant height of the cone is

(a) 5 cm (b) 4 cm
(c) $2\sqrt{3}$ cm (d) 3 cm

8. A semi-circular sheet of metal of diameter 28 cm is bent into an open conical cup. The depth of the cup is approximately

(a) 11 cm (b) 12 cm
(c) 13 cm (d) 14 cm

9. The ratio of height and the diameter of a right circular cone is 3 : 2 and its volume is 1078 cc, then (taking $\pi = \frac{22}{7}$) its height is :

(a) 7 cm (b) 14 cm
(c) 21 cm (d) 28 cm

10. The radius of the base of a conical tent is 16 metre. If $427\frac{3}{7}$ sq. metre canvas is required to construct the tent, then the slant height of the tent is (Take $\pi = \frac{22}{7}$)

(a) 17 metre (b) 15 metre
(c) 19 metre (d) 8.5 metre

11. The radius of the base of a right circular cone is doubled keeping its height fixed. The volume of the cone will be:

(a) three times of the previous volume
(b) four times of the previous volume
(c) $\sqrt{2}$ times of the previous volume
(d) double of the previous volume

12. The perimeter of the base of a right circular cone is 8 cm. If the height of the cone is 21 cm, then its volume is:

(a) $108\,\pi$ cm^3 (b) $\frac{112}{\pi}$ cm^3
(c) $112\,\pi$ cm^3 (d) $\frac{108}{\pi}$ cm^3

13. A right circular cone is 3.6 cm high and radius of its base is 1.6 cm. It is melted and recast into a right circular cone with radius of its base as 1.2 cm. Then the height of the cone (in cm) is

(a) 3.6 (b) 4.8
(c) 6.4 (d) 7.2

14. If the radii of the circular ends of a truncated conical bucket which is 45 cm high be 28 cm and 7 cm, then the capacity of the bucket in cubic centimetre is $\left(\pi = \frac{22}{7}\right)$

(a) 48510 (b) 45810
(c) 48150 (d) 48051

15. A cone is cut at mid-point of its height by a frustum parallel to its base. The ratio between the two parts of cone would be

(a) 1 : 1 (b) 1 : 8
(c) 1 : 4 (d) 1 : 7

16. Each of the height and base-radius of a cone is increased by 100%. The percentage increase in the volume of the cone is

(a) 700% (b) 400%
(c) 300% (d) 100%

17. Find the slant height, volume, curved surface area and the whole surface area of a cone of radius 7 cm. and height 24 cm.

(a) 25 cm,. 1232 cm^3, 550 $cm^{3,}$ 704 cm^2
(b) 32 cm,12936 cm^3, 2310 cm^2, 3696 cm^2
(c) 45 cm, 12636 cm^3, 2310 cm^2, 3696 cm^2
(d) 15 cm, 12936 cm^3, 2022 cm^2, 9636 cm^2

18. The slant height and diameter of a conical tomb is 13 m and 10 m respectively. Find the cost of constructing tomb at the rate of Rs. 7 per m^2.

(a) Rs. 2200 (b) Rs. 1800
(c) Rs. 1430 (d) Rs. 1200

19. In a right circular cone, the radius of its base is 7 cm and its height 24. A cross-section is made through the midpoint of the height parallel to the base. The volume of the upper portion is -

(a) 168 cm^3 (b) 154 cm^3
(c) 1078 cm^3 (d) 800 cm^3

20. Find the volume of a right circular cone formed by joining the edges of a sector of a circle of radius 4 cm where the angle of the sector is 90°.

(a) $\frac{2\sqrt{3}}{\pi}$ cm^3 (b) $\frac{2\sqrt{2}\pi}{3}$ cm^3
(c) $\frac{\pi\sqrt{5}}{\sqrt{3}}$ cm^3 (d) $\frac{\sqrt{3}}{\pi}$ cm^3

21. A sector of circle of radius 3 cm has an angle of 120°. If it is modulated into a cone, find the volume of the cone.

(a) $\frac{\pi}{\sqrt{3}}$ cm^3 (b) $\frac{2\sqrt{2}\pi}{3}$ cm^3
(c) $\frac{2\sqrt{3}}{\pi}$ cm^3 (d) $\frac{\sqrt{3}}{\pi}$ cm^3

22. If the radius of the base of a right circular cone is $3r$ and its height is r, then its volume is :

(a) $\frac{1}{3}\pi r^3$ (b) $\frac{2}{3}\pi r^3$
(c) $3\pi r^3$ (d) $9\pi r^3$

23. The radius and height of a right circular cone are in the ratio of 5 : 12 and its volume is 2512 cm^3. The slant height of the cone is: (Take $\pi = 3.14$)

(a) 14 cm (b) 16 cm
(c) 24 cm (d) 26 cm

24. How many metres of cloth 2.5 m wide with the required to make a conical tent whose base radius is 7 m and height is 24 m?

(a) 120 m (b) 180 m
(c) 220 m (d) 550 m

25. The length of canvas 1.1 m wide required to build a conical tent of height 14 m and the floor area 346.5 m^2, is

(a) 665 m (b) 525 m
(c) 490 m (d) 860 m

26. A conical tent is to accomodate 11 persons such that each person occupies 4 m^2 space on the ground and has 220 m^3 of air to breathe. The height of the cone is :

(a) 145 m (b) 155 m
(c) 165 m (d) 205 m

27. If the radius of the base of a cone is halved, keeping the height same, what is the ratio. the volume of the reduced cone to that of the original cone ?

(a) 1 : 2 (b) 1 : 3
(c) 1 : 4 (d) 2 : 3

28. If the slant height and the radius of the base of a right circular cone are H and r respectively, then the ratio of the areas of the lateral surface and the base is :

(a) 2H : r (b) H : r
(c) H : 2r (d) $H^2 : r^2$

29. The diameter of two cones are equal and their slant heights are in the ratio 5 : 4. If the curved surface of the smaller cone is 200 cm^2, then the curved surface of the bigger cone (in cm^2) is :

(a) 200 (b) 250
(c) 400 (d) 500

30. From a circular sheet of paper of radius 10 cm, a sector of area 40% is removed. If the remaining part is used to make a conical surface, then the ratio fo the radius & the height of the cone will be

(a) 1 : 2 (b) 1 : 1
(c) 3 : 4 (d) 4 : 3

31. The height of a conical tank is 60 m and the diameter of its base is 64 m. The cost of painting it from outside at the rate of Rs. 35 per sq. m. is:

(a) Rs. 52.00 approx. (b) Rs. 39.20 approx.
(c) Rs. 35.20 approx. (d) Rs. 23.94 approx.

32. If S denotes the area of the curved surface of a right circular cone of height h and semi-vertical angle α then S equals.

(a) $\pi h^2 \tan^2 \alpha$ (b) $\frac{1}{3}\pi h^2 \tan^2 \alpha$
(c) $\pi h^2 \sec\alpha \tan\alpha$ (d) $\frac{1}{3}\pi h^2 \sec\alpha \tan\alpha$

33. The height and the radius of the base of a right circular cone are 12 cm and 6 cm respectively. The radius of the circular cross-section of the cone cut by a plane parallel to its base at a distance of 3 cm from the base is:

(a) 4 cm (b) 5.5 cm
(c) 4.5 cm (d) 3.5 cm

34. The radius of base and slant height of a cone are in the ratio 4 : 7. If its curved surface area is 792 cm^2, then the radius (in cm) of its base is $\left(\pi = \frac{22}{7}\right)$

(a) 8 (b) 12
(c) 14 (d) 16

35. The radius of the base and height of a right circular cone are in the ratio $5:12$. If the volume of the cone is $314\frac{2}{7}$ cm^3, the slant height (in cm) of the cone will be

(a) 12 (b) 13
(c) 15 (d) 17

36. The ratio of radii of two cones is 3 : 4 and the ratio of their heights is 4 : 3. Then the ratio of their volumes will be:

(a) 3 : 4 (b) 4 : 3
(c) 9 : 16 (d) 16 : 9

37. If a right circular cone is separated into solids of volumes V_1, V_2, V_3 by two planes parallel to the base, which also trisec the altitude, then V_1:V_2:V_3 is:

(a) 1 : 2 : 3 (b) 1 : 4 : 6
(c) 1 : 6 : 9 (d) 1 : 7 : 19

38. If the radii of the circular ends of a truncated conical bucket which is 90 cm high be 14 cm and 7 cm, then the capacity of the bucket in cubic centimetre is

(use $\pi = \frac{22}{7}$)

(a) 9485 (b) 4581
(c) 4815 (d) 4805

39. A right angled sector of radius r cm is rolled up into a cone in such a way that the two binding radii are joined together. Then the curved surface area of the cone is:

(a) πr^2 cm^2 (b) $4\pi r^2$ cm^2

(c) $\frac{\pi r^2}{4}$ cm^2 (d) $2\pi r^2$ cm^2

40. If h, c, v are respectively the height, curved surface area and volume of a right circular cone, then the value of $3\pi v h^3 - c^2 h^2 + 9v^2$ is:

(a) 2 (b) –1

(c) 1 (d) 0

41. A reservoir is in the shape of a frustum of a right circular cone. It is 8 m across at the top and 4 m across the bottom. It is 6 m deep. Find the area of its curved surface.

(a) 118.4 m^2 (b) 162.3 m^2

(c) 452 m^2 (d) 119.26 m^2

42. A hollow cone is cut by a plane parallel to the base and the upper portion is removed. If the curved surface area of the remainder is 8/9th of the curved surface of the whole cone, the ratio of the line segments into which the cone's altitude is divided by the plane is given by

(a) 2 : 3 (b) 1 : 3

(c) 1 : 2 (d) 1 : 4

43. The radius of the base of a conical tent is 5 m. If the tent is 12 m high, then the area of the canvas required in making the tent is

(a) 300 π m^2 (b) 60 π m^2

(c) 90 π m^2 (d) 65 π

44. How many metres of cloth 5 m wide will be required to make a conical tent, the radius of whose base is 7 m and whose height is 24 m $\left(\pi = \frac{22}{7}\right)$

(a) 108 m (b) 110 m

(c) 112 m (d) 115 m

45. The weight of a solid cone having diameter 14 cm and verticle height 51 cm is .... if the material of solid cone weights 10 g/cm^3.

(a) 16.18 kg (b) 17.25 kg

(c) 26.18 kg (d) 71.40 kg

46. A right circular cone has base radius 7 cm and its height is 24 cm. A section is made by a plane parallel to its base through a height of half the height of the cone. Find the volume of the upper part.

(a) 168 cm^3 (b) 154 cm^3

(c) 1078 cm^3 (d) 800 cm^3

47. A sector of a circle of radius 15 cm has the angle 120°. It is rolled up so that two bounding radii are joined together to form a cone. The volume of the cone is

(a) $\left(250\sqrt{2}\right)\pi$cm^3

(b) $\left(100\sqrt{2}\right)\pi$cm^3

(c) $\left[\left(250\sqrt{2}\right)\pi/3\right]$cm^3

(d) $\left[\left(100\sqrt{2}\right)\pi/3\right]$cm^3

48. The height of a cone is 40cm. The cone is cut parallel to its base such that the volume of the small cone is $\frac{1}{64}$ of the cone. Find at which height the cone is cut?

(a) 20 cm (b) 30 cm

(c) 25 cm (d) 22.5cm

49. The base radius and height of a cone is 5cm and 25 cm respectively. If the cone is cut parallel to its base at a height of h from the base. If the volume of this frustum is 110 cm^3. Find the radius of smaller cone?

(a) $(104)^{1/3}$cm (b) $(104)^{1/2}$cm

(c) 5 cm (d) None of these

50. The height of a right circular cone and the radius of its circular base are respectively 9cm and 3cm. The cone is cut by a plane parallel to its base so as to divide it into two parts. The volume of the frustum (i.e., the lower part) of the cone is 44 cu cm. The radius of the upper circular surface of the frustum $\left(\pi = \frac{22}{7}\right)$ is–

(a) $\sqrt[3]{12}$ cm (b) $\sqrt[3]{13}$ cm

(c) $\sqrt[3]{6}$ cm (d) $\sqrt[3]{20}$ cm

51. A plane divides a cone into two parts of equal volume. If the plane is parallel to the base, then the ratio in which the height of the cone is divided, is-

(a) $1 : \sqrt{2}$ (b) $1 : \sqrt[3]{2} - 1$

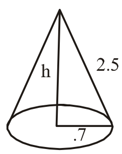
(c) $1 : \sqrt[3]{2}$ (d) $1 : \sqrt[3]{2} + 1$

Answer

1.	(b)	**2.**	(b)	**3.**	(c)	**4.**	(a)	**5.**	(b)	**6.**	(d)	**7.**	(b)	**8.**	(b)	**9.**	(c)
10.	(d)	**11.**	(b)	**12.**	(b)	**13.**	(c)	**14.**	(a)	**15.**	(d)	**16.**	(a)	**17.**	(a)	**18.**	(c)
19.	(b)	**20.**	(c)	**21.**	(b)	**22.**	(c)	**23.**	(d)	**24.**	(c)	**25.**	(b)	**26.**	(c)	**27.**	(c)
28.	(b)	**29.**	(b)	**30.**	(c)	**31.**	(d)	**32.**	(c)	**33.**	(c)	**34.**	(b)	**35.**	(b)	**36.**	(a)
37.	(d)	**38.**	(a)	**39.**	(c)	**40.**	(d)	**41.**	(d)	**42.**	(c)	**43.**	(d)	**44.**	(b)	**45.**	(c)
46.	(b)	**47.**	(c)	**48.**	(b)	**49.**	(a)	**50.**	(b)	**51.**	(b)						

Solution & Hints

Sol 1. slant height $(l) = 2.5$ km, **...(given)**



area of base $= \pi r^2 = 1.54\,\text{km}^2$ **...(given)**

$\Rightarrow \quad r = .7$ km

height $(h) = \sqrt{(2.5)^2 - (.7)^2}$

$= \sqrt{(2.5+.7)(2.5-.7)} = 2.4$ km

Sol 2. Let volumes of two cones be V_1 and V_2

$\frac{V_1}{V_2} = \frac{2}{3}$ & $\frac{r_1}{r_2} = \frac{1}{2}$ **...(given)**

$$\Rightarrow \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{2}{3}$$

$$\Rightarrow \left(\frac{r_1}{r_2}\right)^2 \times \frac{h_1}{h_2} = \frac{2}{3}$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{2}{3}\left(\frac{2}{1}\right)^2 = \frac{8}{3}$$

Sol 3. Height $(h) = 24$ cm and

Volume $= \frac{1}{3}\pi r^2 h = 1232$ cm³ **...(given)**

$$\Rightarrow r^2 = \frac{1232 \times 3 \times 7}{24 \times 22} = 49$$

$\Rightarrow \quad r = 7$ cm

slant height $(l) = \sqrt{r^2 + h^2} = \sqrt{(7)^2 + (24)^2}$

$= 25$ cm

curved surface area =

$$\pi rl = \frac{22}{7} \times 7 \times 25 = 550\,\text{cm}^2$$

Sol 4. Let radius and height of original cone be r_1 and h_1

when height of cone is doubled $h_2 = 2h_1$

and radius remained same $r_2 = r_1$

$$\Rightarrow \frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{h_1}{h_2} = \frac{2}{1}$$

Sol 5. $\frac{r}{l} = \frac{4}{7}$ **...(given)**

$$\Rightarrow l = \frac{7r}{4}$$

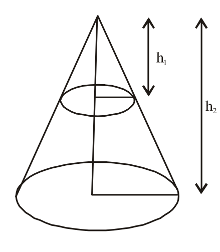
$\therefore$ Curved surfaced area $= \pi rl = 792$

$$\Rightarrow \frac{22}{7} \times r \times \frac{7r}{4} = 792$$

$\Rightarrow \quad r^2 = 144$

$\Rightarrow \quad r = 12$ cm

Sol 6. Ratio of volume of smaller cone to larger cone



$$\frac{V_1}{V_2} = \frac{1}{27} = \left(\frac{h_1}{h_2}\right)^3$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{1}{3}$$

$$\Rightarrow h_1 = \frac{h_2}{3} = \frac{30}{3} = 10\text{cm}$$

Then, height above the base is section made

$h_1 - h_2 = 30 - 10 = 20$ cm

Sol 7. Radius $(r) = \frac{4}{2} = 2\text{cm}$, and

Height $(h) = 2\sqrt{3}$ cm ....**(given)**

$l = \sqrt{r^2 + h^2}$

$l = \sqrt{(2)^2 + \left(2\sqrt{3}\right)^2} = \sqrt{16} = 4\text{cm}$

Sol 8. When semi-circular sheet of radius R is bent into a cone then

radius base of cone $= \frac{R}{2}$ $(\because\ 2\pi r = \pi R)$

$\Rightarrow r = \frac{14}{2} = 7\text{cm}$, and slant height *(l)* = R

$\Rightarrow l = 14$ cm

$h = \sqrt{l^2 - r^2} = \sqrt{(14)^2 - (7)^2} = 12$ cm (App.)

Sol 9. Height and diameter of cone are *h* and *d* respectively $\frac{h}{d} = \frac{3}{2}$ ....**(given)**

$\Rightarrow \frac{h}{2r} = \frac{3}{2}$

$\Rightarrow r = \frac{h}{3}$

$\therefore$ Volume $= \frac{1}{3}\pi r^2 h = 1078\ \text{cm}^3$

$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times \frac{h^2}{9} \times h = 1078$

$\Rightarrow h^3 = 9261$

$\Rightarrow h = 21$ cm

Sol 10. Radius $(r) = 16$m ....**(given)**

$\therefore$ Curved surface area $= \pi r l = 427\frac{3}{7}$

$\Rightarrow \frac{22}{7} \times 16 \times l = 427\frac{3}{7}$

$l = 8.5$ m

Sol 11. Volume $\propto$ (radius)^2

If radius is doubled then volume will be four times.

Sol 12. Perimeter of base $= 2\pi r = 8$ ....**(given)**

$\Rightarrow r = \frac{4}{\pi}$cm and $h = 21$ cm

$\therefore$ Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times \frac{16}{\pi^2} \times 21 = \frac{112}{\pi}\text{cm}^3$

Sol 13. Let height and radius of melted cone be h_1 and r_1 and height and radius of formed cone be h_2 and r_2

$h_1 = 3.6$ cm, $r_1 = 1.6$ cm, $r_2 = 1.2$ cm, $h_2 = ?$

If any solid is melted and new solid is formed then volume of both the solids will be equal.

$\Rightarrow \frac{1}{3}\pi r_1^2 h_1 = \frac{1}{3}\pi r_2^2 h_2$

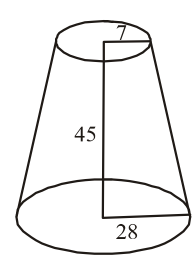
$\Rightarrow \left(\frac{r_1}{r_2}\right)^2 = \frac{h_2}{h_1}$

$\Rightarrow \left(\frac{1.6}{1.2}\right)^2 = \frac{h_2}{3.6}$

$\Rightarrow h_2 = 6.4$ cm

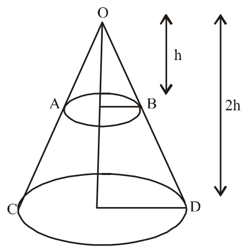
Sol 14. Capacity of bucket = volume of frustum

$= \frac{1}{3}\pi\left(r^2 + R^2 + rR\right)h$



$= \frac{1}{3} \times \frac{22}{7}\left\{7^2 + (28)^2 + (7)(28)\right\} \times 45 = 48510\ \text{cm}^3$

Sol 15.



$$\frac{\text{Volume of cone OAB}}{\text{Volume of cone OCD}}=\left(\frac{h}{2h}\right)^2=\frac{1}{8}$$

Volume of lower part ABCD = Volume of Con – Volume of Con = 7

then ratio of volume of upper part to lower part = 1 : 7

Sol 16. Let initial height and radius of cone is h and r

Initial volume $(V_1)=\frac{1}{3}\pi r^2 h$

When, both radius and height increased by 100%

then, Final radius = $2r$ and final height = $2h$

Final volume $(V_2)=\frac{1}{3}\pi(2r)^2\times 2h=8\times V_1$

% increased in volume $=\frac{V_2-V_1}{V_1}\times 100$

$=\frac{8V_1-V_1}{V_1}\times 100=700\%$

Sol 17. Radius (r) = 7 cm,

Height (h) = 24 cm **.........(given)**

slant height $(l)=\sqrt{r^2+h^2}$

$=\sqrt{(7)^2+(24)^2}=25$ cm

Volume $=\frac{1}{3}\pi r^2 h=\frac{1}{3}\times\frac{22}{7}\times(7)^2\times 24=1232\text{ cm}^3$

Curved surface area = πrl

$=\frac{22}{7}\times 7\times 25=550\text{ cm}^2$

Whole surface area = $\pi r(l+r)$

$=\frac{22}{7}\times 7\times(25+7)=704\text{ cm}^2$

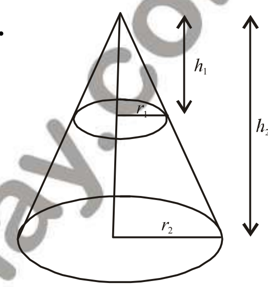
Sol 18. Slant height (l) = 13 m, radius (r) = 5 m **...(given)**

$\therefore$ Lateral surface area = $\pi rl=\frac{22}{7}\times 13\times 5$

$\Rightarrow$ Cost of constructing tomb

$=\frac{22}{7}\times 13\times 5\times 7=\text{Rs. }1430$

Sol 19.



$\therefore\quad\frac{r_1}{r_2}=\frac{h_1}{h_2}=\frac{1}{2}$

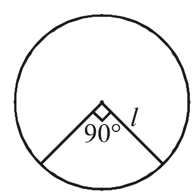
$\Rightarrow\quad h_1=\frac{h_2}{2}=12$ cm

$\Rightarrow\quad r_1=\frac{r_2}{2}=\frac{7}{2}$ cm

Volume of upper portion

$=\frac{1}{3}\pi r_1^2 h_1=\frac{1}{3}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times 12=154\text{ cm}^3$

Sol 20.



Slant height (l) = radius of circle = 4 cm

r = radius of cone

$\therefore$ Perimeter of base of cone = $2\pi r$

$\Rightarrow 2\pi r=\frac{90°}{360°}\times$ (perimeter of circle)

$\Rightarrow 2\pi r=\frac{1}{4}\times 2\pi\times 4$

$\Rightarrow \quad r = 1$ cm.

height of cone (h) = $\sqrt{l^2 - r^2} = \sqrt{16-1} = \sqrt{15}$

$\therefore$ Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times (1)^2 \times \sqrt{15}$

$= \frac{\pi\sqrt{5}}{\sqrt{3}}$ cm³

Sol 21. Same as Q. No. 20

Sol 22. Radius = $3r$, **........(given)**

Height = r

$\therefore$ Volume $= \frac{1}{3}\pi R^2 H$

(Where R and H are radius and height of cone respectively)

$= \frac{1}{3} \times \pi \times 9r^2 \times r = 3\pi r^3$

Sol 23. Let the radius and height of cone b $5x$ and $12x$.

$\therefore$ Volume $= \frac{1}{3}\pi r^2 h$

$\Rightarrow 2512 = \frac{1}{3} \times 3.14 \times (5x)^2 \times (12x)$

$\Rightarrow x = 2$

$\Rightarrow$ radius = 5 × 2 = 10 cm and height = 12 × 2 = 24cm

$l = \sqrt{r^2 + h^2} = \sqrt{(10)^2 + (24)^2} = 26$cm

Sol 24. Radius (r) = 7 m, height (h) = 2**......(given)**

$l = \sqrt{(7)^2 + (24)^2} = 25$

Curved surface area =

$\pi rl = \frac{22}{7} \times 7 \times 25 = 550\text{cm}^2$

cloth required $= \frac{\text{curved surface area of cone}}{\text{width of cloth}}$

$= \frac{550}{2.5} = 220$m

Sol 25. Floor area = $\pi r^2 = 346.5$ **.........(given)**

$\Rightarrow \quad r = 10.5$

$\therefore \quad l = \sqrt{r^2 + h^2}$

$\Rightarrow \quad l = \sqrt{(10.5)^2 + (14)^2} = 17.5$

Curved surface area = $\pi rl = \frac{22}{7} \times 10.5 \times 17.5$

= 577.5m²

Length of canvas $= \frac{\text{curved surface area}}{\text{width of canvas}}$

$= \frac{577.5}{11} = 525$ m

Sol 26. Area of floor = $\pi r^2 = 4\text{m}^2$

$\therefore$ Volume of tent $= \frac{1}{3}\pi r^2 h = 220$

$\frac{1}{3} \times 4 \times h = 220$

$\Rightarrow$ height of tent (h) = 165 m

Here, No. of People will not be matter.

Sol 27. Initial volume of cone $V_1 = \frac{1}{3} \times \pi (r_1)^2 h$

When radius is halved

$\Rightarrow \quad r_2 = \frac{r_1}{2}$

Final volume of cone $V_2 = \frac{1}{3}\pi (r_2)^2 h$

$= \frac{1}{3}\pi \left(\frac{r_1}{2}\right)^2 h$

$\therefore$ Ratio of volumes $= \frac{V_1}{V_2} = \frac{\frac{1}{3}\pi \frac{r_1^2}{4} h}{\frac{1}{3}\pi r_1^2 h} = 1:4$

Sol 28. Slant height = H, and radius = r

$\Rightarrow \quad \frac{\text{curved surface area}}{\text{area of base}} = \frac{\pi r H}{\pi r^2} = H : r$

Sol 29. Let the slant height of two cones be $5l$ and $4l$.
as diameter is equal their radius will also be equal

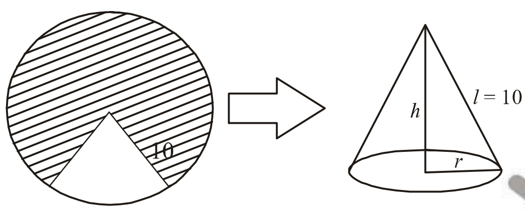
$$\therefore \quad \frac{\text{curved surface area of smaller cone}}{\text{curved surface area of larger cone}} = \frac{\pi r(4l)}{\pi r(5l)}$$

curved surface area of larger cone

$$= \frac{5}{4} \times (\text{curved surface area of smaller cone})$$

$$= \frac{5}{4} \times 200 = 250\,\text{cm}^2$$

Sol 30.



$$\therefore \quad \pi rl = 60\% \text{ of circular sheet}$$

$$\Rightarrow \pi r \times 10 = \frac{60}{100} \times \pi \times 10 \times 10$$

$$\Rightarrow r = 6\text{ cm}$$

Now, $l = \sqrt{r^2 + h^2}$

$$\Rightarrow 10 = \sqrt{l^2 + h^2}$$

$$\Rightarrow h = 8\text{ cm}$$

Since ratio of radius and height $= \frac{6}{8} = \frac{3}{4}$

Sol 31. height $(h) = 60$ m, radius $(r) = 32$ m..... **(given)**

$$\Rightarrow \quad l = \sqrt{h^2 + r^2}$$

$$= \sqrt{(60)^2 + (32)^2} = 68\text{ m}$$

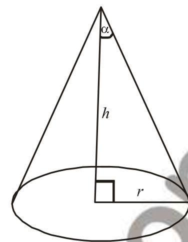
$\therefore$ Curved surface area $= \pi rl$

$$= \frac{22}{7} \times 32 \times 68$$

$$= 6838.85 = 0.6838\text{ m}^2$$

Cost of painting $= 35 \times 0.6838 =$ Rs. 23.94

Sol 32.



$$\tan \alpha = \frac{r}{h} \Rightarrow r = h \tan \alpha$$

$$l = \sqrt{r^2 + h^2} = \sqrt{h^2 \tan^2\alpha + h^2}$$

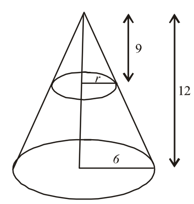
$$= \sqrt{h^2(\tan^2\alpha + 1)} = h \sec \alpha$$

$\therefore$ Curved surface area (S) $= \pi rl$

$$= \pi \times h \tan \alpha \times h \sec \alpha$$

$$= \pi\, h^2 \sec \alpha \tan \alpha$$

Sol 33.



height of smaller cone from base = 3 cm
height of smaller cone from top = 12 – 3 = 9 cm

$$\therefore \quad \frac{r}{6} = \frac{9}{12} \quad \Rightarrow r = 4.5\text{ cm}$$

Sol 34. Let the radius and slant height be $4x$ and $7x$
Curved surface area $= \pi rl$

$$= \frac{22}{7} \times 4x \times 7x = 792$$

$$\Rightarrow x^2 = 9$$

$$\Rightarrow x = 3$$

Hence, radius $= 4x = 4 \times 3 = 12$ cm

Sol 35. Let radius and height be $5x$ and $12x$
$r = 5x$, $h = 12x$

slant height $= l = \sqrt{r^2 + h^2}$

$$= \sqrt{(5x)^2 + (12x)^2} = 13x$$

$\therefore$ Volume $= \frac{1}{3}\pi r^2 h$

$\times \frac{22}{7} \times (15x)^2 \times (12x) = \frac{2200}{7}$

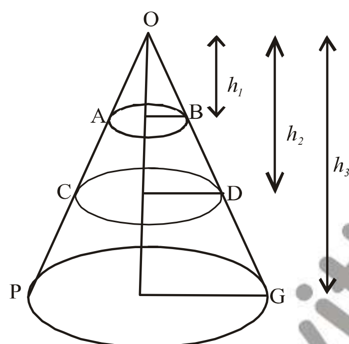
$=1$

$l = 13x = 13\text{cm}$

Sol 36. Let the radius be $3x$ and $4x$ and height be $4h$ and $3h$

$$\frac{V_1}{V_2} = \frac{\frac{1}{3}\pi(3x)^2 \times (4h)}{\frac{1}{3}\pi(4x)^2 \times (3h)} = 3:4$$

Sol 37. As the altitude is trisected, it will be divided into three equal parts



$\therefore$ Volume of cone OAB $= v_1$ with height = 1 (let)

$\therefore$ Volume of cone OCD $= v_2$ with height = 2

$\therefore$ Volume of cone OPG $= v_3$ with height = 3

$$\frac{v_1}{v_2} = \left(\frac{h_1}{h_2}\right)^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

Volume fo ABCD (V_2)

= Volume of cone OCD – Volume of cone OAB

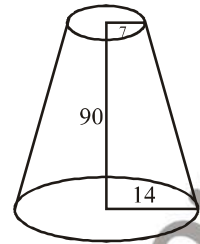
$= 8 - 1 = 7$

Similarly, $\left(\frac{v_2}{v_3}\right)^2 = \left(\frac{h_2}{h_3}\right)^3 = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$

Volume of CFGD (V_3) $= v_3 - v_2 = 27 - 8 = 19$

$\Rightarrow V_1 : V_2 : V_3 = 1 : 7 : 19$

Sol 38.



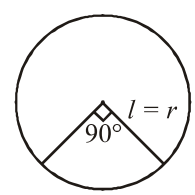
$r = 7, R = 14, h = 90$ cm

$\therefore$ Volume $= \frac{1}{3}\pi(r^2 + R^2 + rR)h$

$\frac{1}{3} \times \frac{22}{7}[(7)^2 + (14)^2 + 7 \times 14)90 = 9485\text{ cm}^3$

Sol 39. Radius of sector will be slant height of cone.

$\therefore$ perimeter of base of cone and sector will be same



$2\pi R = \frac{2\pi r}{4}$

$\Rightarrow$ $R = \frac{r}{4}$ (Where, R is radius of cone) $l = r$

$\therefore$ Curved surface area = π × (radius) × (slant height)

$= \pi \times \frac{r}{4} \times r = \frac{\pi r^2}{4}$

Sol 40. r = radius of cone. $\therefore$ $V = \frac{1}{3}\pi r^2 h$

(Squaring both the sides)

$\Rightarrow 9V^2 = \pi^2 r^4 h^2$.............(i)

$\Rightarrow$ curved surface area (C) = πrl

$= \pi r\sqrt{r^2 + h^2}$

(Squaring both the sides)

$C^2 = \pi^2 r^2 (r^2 + h^2)$

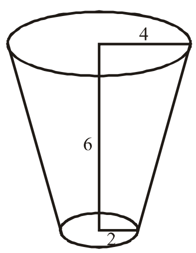
$C^2 = \pi^2 r^4 + \pi^2 r^2 h^2$

$\Rightarrow C^2 = \dfrac{9V^2}{h^2} + \dfrac{3V\pi}{h} \times h$ [from (i)]

$\therefore C^2 = \dfrac{9V^2}{h^2} + 3\pi h V$

$\Rightarrow 3Vh^3 - C^2h^2 + 9V^2 = 0$

Sol 41.



$r = 2\text{m}, R = 4\text{m}, h = 6\text{m}$

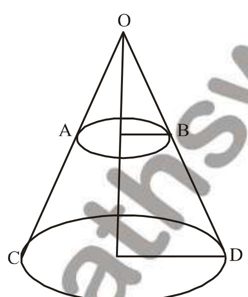
$\therefore l = \sqrt{h^2 + (R-r)^2}$

$= \sqrt{6^2 + (4-2)^2} = \sqrt{40}$

$\therefore$ Curved surface area $= \pi (r + R) l$

$= \dfrac{22}{7}(2+4) \times \sqrt{40} = 119.26 \text{ m}^2$

Sol 42.



Curved surface area of ABCD $= \dfrac{8}{9}$ (C.S.A of OCD)

$\Rightarrow$ Curved surface area of OAB $= \dfrac{1}{9}$ (C.S.A. of OCD)

$\dfrac{\text{C.S.A. of OAB}}{\text{C.S.A of OCD}} = \left(\dfrac{OA}{OC}\right)^2 = \dfrac{1}{9}$

$\Rightarrow \dfrac{OA}{OC} = \dfrac{1}{3}$

$\Rightarrow \dfrac{OA}{OA + AC} = \dfrac{1}{3}$

$\Rightarrow \dfrac{OA}{AC} = 1:2$

$\Rightarrow OA : AC = 1:2$

Sol 43. $r = 5$ m, $h = 12$m **......(given)**

$l = \sqrt{r^2 + h^2} = \sqrt{5^2 + 12^2} = 13$ m

area of canvas required =

curved surface area $= \pi r l = \pi \times 5 \times 13 = 65\pi$

Sol 44. $r = 7$m, $h = 24$m **......(given)**

$\therefore l = \sqrt{r^2 + h^2}$

$= \sqrt{7^2 + (24)^2} = 25$ m

$\therefore$ Curved surface area $= \pi r l$

$= \dfrac{22}{7} \times 7 \times 25 = 550 \text{ m}^2$

Length of cloth required $= \dfrac{550}{5} = 110$m

Sol 45. $\therefore$ $r = 7$cm, $h = 51$ cm **.........(given)**

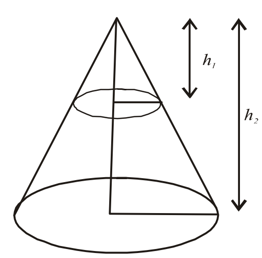
$\therefore V = \dfrac{1}{3}\pi r^2 h = \dfrac{1}{3} \times \dfrac{22}{7} \times 7 \times 7 \times 51$

$= 2618 \text{ cm}^3$

weight of cone $= 2618\text{cm}^3 \times 10 \text{ g/cm}^3$

$= 26180 \text{ gm} = 26.18$ kg.

Sol 46.



$r_1 = 7$ cm, $h_1 = 24$ cm

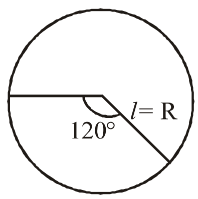
$V_1 = \dfrac{1}{3}\pi r^2 h = \dfrac{1}{3} \times \dfrac{22}{7} \times 7 \times 7 \times 24 = 1232 \text{ cm}^3$

$h_2 = \dfrac{h_1}{2} = 12$ cm

$$\therefore \frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3 = \left(\frac{24}{12}\right)^3 = 8$$

$$V_2 = \frac{V_1}{8} = \frac{1232}{8} = 154 \text{ cm}^3$$

Sol 47. Radius of cone = Slant height of Cone = 15 cm.



$\therefore$ Perimeter of base of sector = perimeter of base of cone

$$\Rightarrow \frac{120^\circ}{360^\circ} \times 2\pi(15) = 2\pi r \quad \text{(where, } r \text{ is radius of cone)}$$

$\Rightarrow \quad r = 5\text{cm}$

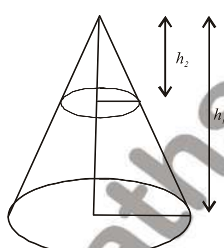
$$h = \sqrt{l^2 - r^2} = \sqrt{(15)^2 - (5)^2} = \sqrt{200} = 10\sqrt{2}$$

$$\therefore \text{ Volume } = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \pi \times 5 \times 5 \times 10\sqrt{2}$$

$$= [(250\sqrt{2})\pi/3] \text{ cm}^3$$

Sol 48.



$$\therefore \frac{V_2}{V_1} = \left(\frac{h_2}{h_1}\right)^3$$

$$\Rightarrow \frac{1}{64} = \left(\frac{h_2}{40}\right)^3$$

$$\Rightarrow \frac{1}{4} = \frac{h_2}{40}$$

$\Rightarrow h_1 = 10$ cm

$\Rightarrow$ Height from base $= h_1 - h_2 = 40 - 10 = 30$ cm.

Sol 49. Volume of cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 25 = 654.76$$

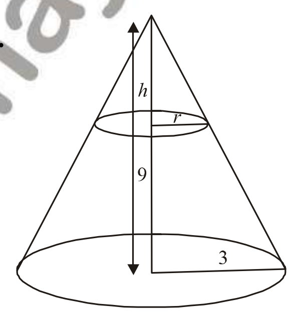
Volume of smaller cone = 654.76 – 110 = 544.761

$$\therefore \frac{\text{Volume of smaller cone}}{\text{Volume of larger cone}} = \left(\frac{\text{radius of smaller cone}}{\text{radius of large cone}}\right)^3$$

$$\Rightarrow \frac{544.761}{654.761} = \left(\frac{r}{5}\right)^3$$

$\Rightarrow r = (104)^{1/3}$ cm

Sol 50.



$$\text{Volume of cone V} = \frac{1}{3}\pi R^2 H$$

$$= \frac{1}{3} \times \pi \times 3 \times 3 \times 9 = 27\pi$$

Volume of lower part $v = 44 = 14\pi$

Volume of Upper part = V – v

$= 27\pi - 14\pi$

$$\text{Volume of upper part} = \frac{1}{3}\pi r^2 h = 13\pi$$

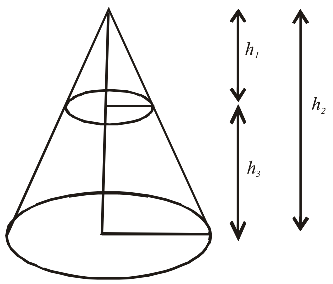
$$\left(\frac{h}{9} = \frac{r}{3} \Rightarrow h = 3r\right)$$

$$\Rightarrow 13\pi = \frac{1}{3}\pi r^2 (3r)$$

$\Rightarrow r^3 = 13$

$\Rightarrow r = \sqrt[3]{13}$

Sol 51.



$$V_1 = \frac{V_2}{2}$$

$$\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3$$

$$\left(\frac{1}{2}\right)^{1/3} = \frac{h_1}{h_2}$$

$\Rightarrow h_2 = 2^{1/3} h_1$

ratio is which height is divided

$= h_1 : h_3$ (Where, $h_3 = h_2 - h_1$)

$$= \frac{h_1}{h_2 - h_1} = \frac{h_1}{\left(\sqrt[3]{2} - 1\right) h_1}$$

$$\frac{h_1}{h_3} = \frac{1}{\left(\sqrt[3]{2} - 1\right) h_1}$$

★★★★★

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Exercise -(Sphere)

1. Three solid metallic spheres of diameters 6 cm, 8 cm and 10 cm are melted and recast into a new solid sphere. The diameter of the new sphere is:

(a) 4 cm (b) 6 cm
(c) 8 cm (d) 12 cm

2. A sphere of radius 2 cm is put into water contained in a cylinder of base-radius 4 cm. If the sphere is completely immersed in the water, the water level in the cylinder rises by

(a) $\frac{1}{3}$ cm (b) $\frac{1}{2}$ cm
(c) $\frac{2}{3}$ cm (d) 2 cm

3. A hollow spherical metallic ball has an external diameter 6 cm and is $\frac{1}{2}$ cm thick. The volume of the ball (in cm^3) is (Take $\pi=\frac{22}{7}$)

(a) $41\frac{2}{3}$ (b) $37\frac{2}{3}$
(c) $47\frac{2}{3}$ (d) $40\frac{2}{3}$

4. A copper sphere of radius 3 cm is beaten and drawn into a wire of diameter 0.2 cm. The length of the wire is:

(a) 9 m (b) 12 m
(c) 18 m (d) 36 m

5. 12 spheres of the same size are made by melting a solid cylinder of 16 cm diameter and 2 cm height. The diameter of each sphere is :

(a) 2 cm (b) 4 cm
(c) 3 cm (d) $\sqrt{3}$ cm

6. By melting a solid lead sphere of diameter 12 cm, three small spheres are made whose diameters are in the ratio 3 : 4 : 5. The radius (in cm) of the smallest sphere is

(a) 3 (b) 6
(c) 1.5 (d) 4

7. The total surface area of a metallic hemisphere is 1848 cm^2. The hemisphere is melted to form a solid right circular cone. If the radius of the base of the cone is the same as the radius of the hemisphere, its height is

(a) 42 cm (b) 26 cm
(c) 28 cm (d) 30 cm

8. The total surface area of a solid hemisphere is 108π cm^2. The volume of the hemisphere is

(a) 72π cm^3 (b) 144π cm^3
(c) 108π cm^3 (d) 54π cm^3

9. A solid metallic sphere of radius 8 cm is melted to form 64 equal small solid spheres. The ratio of the surface area of this sphere to that of a small sphere is

(a) 4 : 1 (b) 1 : 16
(c) 16 : 1 (d) 1 : 4

10. A solid metallic sphere of radius 3 decimetres is melted to form a circular sheet of 1 milimetre thickness. The diameter of the sheet so formed is

(a) 26 metres (b) 24 metres
(c) 12 metres (d) 6 metres

11. A copper wire of length 36 m and diameter 2 mm is melted to form a sphere. The radius of the sphere (in cm) is

(a) 2.5 (b) 3
(c) 3.5 (d) 4

12. A child reshapes a cone made up of clay of height 24 cm and radius 6 cm into a sphere. The radius (in cm) of the sphere is

(a) 6 (b) 12
(c) 24 (d) 48

13. A sphere and a hemisphere have the same volume. The ratio of their curved surface areas is:

(a) $2^{\frac{3}{2}}:1$ (b) $2^{\frac{2}{3}}:1$

(c) $4^{-\frac{2}{3}}:1$ (d) $2^{\frac{1}{3}}:1$

14. A solid sphere of 6 cm diameter is melted and recast into 8 solid spheres of equal volume. The radius (in cm) of each small sphere is

(a) 1.5 (b) 3

(c) 2 (d) 2.5

15. If the total surface area of a hemisphere is 27 π square cm, then the radius of the base of the hemisphere is

(a) $9\sqrt{3}$ cm (b) 3 cm

(c) $3\sqrt{3}$ cm (d) 9 cm

16. Assume that a drop of water is spherical and its diameter is one- tenth of a cm. A conical glass has a height equal to the diameter of its rim. If 32,000 drops of water fill the glass completely, then the height of the glass, in cm, is

(a) 1 (b) 2

(c) 3 (d) 4

17. A solid metallic spherical ball of diameter 6 cm is melted and recasted into a cone with diameter of the base as 12 cm. The height of the cone is

(a) 6 cm (b) 2 cm

(c) 4 cm (d) 3 cm

18. A solid spherical copper ball, whose diameter is 14 cm is melted and converted into a wire having diameter equal to 14 cm. The length of the wire is

(a) 27 cm (b) $\frac{16}{3}$ cm

(c) 15 cm (d) $\frac{28}{3}$ cm

19. Find the volume, and total surface area of a sphere of radius 21 cm.

(a) 38808 cm^3. 5544 cm^2.

(b) 11646 cm^3. 4838 cm^2

(c) 32256 cm^3. 6758 cm^2

(d) 41456 cm^3. 5248 cm^2

20. There are three spherical balls having a radius of 3, 4, 5 m. By melting both balls a bigger spherical ball is made. Find the radius of the new ball.

(a) 5 m (b) 6 m

(c) 8 m (d) 10 m

21. A metallic hemisphere is melted and recast in the shape of cone with the same base radius (R) as that of the hemisphere. If H is the height of the cone, then :

(a) H = 2 R (b) H = 3 R

(c) H = $\sqrt{3}$ R (d) H = $\frac{3}{2}$ R

22. A sphere of maximum volume is cut out from a solid hemisphere of radius *r*. The ratio of the volume of the hemisphere to that of the cut out sphere is :

(a) 3 : 2 (b) 4 : 1

(c) 4 : 3 (d) 7 : 4

23. The volume of a spherical shell whose internal and external diameters are 8 cm and 10 cm respectively (in cubic cm) is:

(a) $\frac{122\pi}{3}$ (b) $\frac{244\pi}{3}$

(c) 212 (d) 257

24. Three solid spheres of a metal whose radii are 1 cm, 6 cm and 8 cm are melted to form an other solid sphere. The radius of this new shpere is:

(a) 10.5 cm (b) 9.5 cm

(c) 10 cm (d) 9 cm

25. Spheres A and B have their radii 40 cm and 10 cm respectively. Ratio of surface area of A to the surface area of B is:

(a) 1 : 16 (b) 4 : 1

(c) 1 : 4 (d) 16 : 1

26. The volume of a sphere is $\frac{88}{21}\times(14)^3$ cm^3. The curved surface of the sphere is (Take $\pi = \frac{22}{7}$)

(a) 2424 cm^2 (b) 2446 cm^2

(c) 2484 cm^2 (d) 2464 cm^2

27. If S_1 and S_2 be the surface area of a sphere and the curved surface area of the circumscribed cylinder respectively, then S_1 is equal to:

(a) $\frac{3}{4}S_2$ (b) $\frac{1}{2}S_2$

(c) $\frac{2}{3}S_2$ (d) S_2

28. A spherical lead ball of radius 10 cm is melted and small lead balls of radius 5 mm are made. The total number of possible small lead balls is: (Take $\pi = \frac{22}{7}$)

(a) 8000 (b) 400

(c) 800 (d) 125

29. If surface area and volume of a sphere are S and V respectively, then value of $\frac{S^3}{V^2}$ is:

(a) 32 π unit (b) 9 π unit

(c) 18 π unit (d) 36 π unit

30. If the total surface area of a hemisphere is 27π square cm, then the radius of the base of the hemisphere is:

(a) $9\sqrt{3}$ cm (b) 3 cm

(c) $3\sqrt{3}$ cm (d) 9 cm

31. The total surface area of a sphere is 8π square unit. The volume of the sphere is:

(a) $\frac{8\sqrt{2}}{3}\pi$ cubic unit (b) $\frac{8}{3}\pi$ cubic unit

(c) $8\sqrt{3}\pi$ cubic unit (d) $\frac{8\sqrt{3}}{5}\pi$ cubic unit

32. The total number of spherical bulets, each of diameter 5 decimeter, that can be made by utilizing the maximum of a rectangular block of lead with 11 meter length, 10 metre breadth and 5 metre width is (assume that $\pi > 3$)

(a) equal to 8800 (b) less than 8800

(c) equal to 8400 (d) greater than 9000

33. A cylindrical rod of iron whose height is eight times its radius is melted and cast into spherical balls each of half the radius of the cylinder. The number of such spherical is:

(a) 12 (b) 16

(c) 24 (d) 48

34. The ratio of the volume of a cube and of a solid sphere is 363: 49. The ratio of an edge of the cube and the radius of the sphere is (Take $\pi = \frac{22}{7}$)

(a) $7:11$ (b) $22:7$

(c) $11:7$ (d) $7:22$

35. A sphere of diameter 6 cm is dropped in a right circular cylindrical vessel party filled with water. The diameter of the cylindrical vessel is 12 cm. If the sphere is just completely submerged in water, then the rise of water level in the cylindrical vessel is

(a) 2 cm (b) 1 cm

(c) 3 cm (d) 4 cm

36. A copper sphere of diameter 18cm is drawn into a wire of diameter 4 mm. The length of the wire, in metre, is:

(a) 2.43 (b) 243

(c) 2430 (d) 24.3

37. A spherical ball of lead, 3 cm in diameter is melted and recast into three spherical balls. The diameter of two of these are 1.5 cm and 2 cm respectively. The diameter of the third ball is

(a) 3 cm (b) 2.66 cm

(c) 2.5 cm (d) 3.5 cm

38. A hemispherical bowl is 176 cm round the brim. Supposing it to be half full, how many persons may be served from it in hemispherical glasses 4 cm in diameter at the top?

(a) 1372 (b) 1272

(c) 1172 (d) 1472

39. A hemispherical bowl is made of steel 0.5 cm thick. The inner radius of the bowl is 4 cm.The volume of steel used in making the bowl is

(a) 56.83 cm^3 (b) 55.83 cm^3

(c) 57.83 cm^3 (d) 58.83 cm^3

40. A sphere of radius 3 cm is dropped into a cylindrical vessel partly filled with water. The radius of the vessel is 6 cm. If the sphere is submerged completely, then the surface of the water is raised by

(a) $\frac{1}{4}$ cm (b) $\frac{1}{2}$ cm

(c) 1 cm (d) 2 cm

41. A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. If the right circular cylinder circumscribes the toy, find the difference of the volumes of the cylinder and the toy. (Take $\pi = 3.14$)

(a) 25.12 cm^3 (b) 2.512 cm^3

(c) 251.2 cm^3 (d) 0.2512 cm^3

42. A metallic sphere of radius 10.5 cm is melted and recast into small right circular cones, each of base radius 3.5 cm and height 3 cm. The number of cones so formed is

(a) 105 (b) 135

(c) 126 (d) 113

43. A hemispherical bowl of internal radius 9cm contains a liquid. This liquid is to be f illed into cylindrical shaped small bottles of diameter 3cm and height 4cm . How many bottles will be needed of empty the bowl?

(1) 54 (2) 63

(3) 27 (4) 72

44. A hemispherical bowl is filled to the brim with a beverage. The contents of the bowl area transfersed into a cylindrical vessel whose radius is 50% more than its height. If the diameter is same for both of the beverage in the cylinderical vessel will be

(1) $66\frac{2}{3}$ % (2) 78.5 %

(3) 78 % (4) 100 %

45. A sphere of 27 cm. radius is dropped into a cylinderical vessel of 60 cm diameter, which is partly filled with water, then its level rises by x cm. Find x :

(a) $11\frac{21}{25}$ cm. (b) $22\frac{4}{25}$ cm.

(c) $29\frac{4}{25}$ cm. (d) $11\frac{23}{25}$ cm.

46. A spherical ball of lead 6 cm in radius is melted and recast. into three spherical balls. The radii of two of these balls are 3 cm and 4 cm. What is the radius of the third sphere?

(a) 6 cm (b) 6.5cm

(c) 5.5cm (d) 5cm

47. A metal sphere, 14 cm in diameter, is dropped into a rectangular cistern whose base measures $49\text{cm} \times \frac{44}{3}$ cm. If the sphere is totally submerged, by how much will surface of the water be raised?

(a) 2 cm (b) 1 cm
(c) 4 cm (d) 3 cm

48. A ball of lead 4 cm in diameter is covered with gold. If the volume of the gold and lead are equal, then the thickness of gold is approximately. [given $\sqrt[3]{2} = 1.259$]

(a) 5.038 cm (b) 5.190 cm
(c) 1.038 cm (d) 0.518 cm

49. A large solid sphere is melted and moulded to form identical right circular cones with base radius and height same as the radius of the sphere. One of these cones is melted and moulded to form a smaller solid sphere. Then the ratio of the surface area of the smaller to the surface area of the larger sphere is–

(a) $1 : 3^{\frac{4}{3}}$ (b) $1 : 2^{\frac{3}{2}}$
(c) $1 : 3^{\frac{2}{3}}$ (d) $1 : 2^{\frac{4}{3}}$

50. Let A and B be two solid spheres such that the surface area of B is 300% higher than the surface area of A. The volume of A is found to be k% lower than the volume of B. The value of k must be

(a) 85.5 (b) 92.5
(c) 90.5 (b) 87.5

Answer

1. (d)	**2.** (c)	**3.** (c)	**4.** (d)	**5.** (b)	**6.** (a)	**7.** (c)	**8.** (b)	**9.** (c)	
10. (d)	**11.** (b)	**12.** (a)	**13.** (d)	**14.** (a)	**15.** (b)	**16.** (d)	**17.** (d)	**18.** (d)	
19. (a)	**20.** (b)	**21.** (a)	**22.** (b)	**23.** (b)	**24.** (d)	**25.** (d)	**26.** (d)	**27.** (d)	
28. (a)	**29.** (d)	**30.** (b)	**31.** (a)	**32.** (b)	**33.** (d)	**34.** (b)	**35.** (b)	**36.** (b)	
37. (c)	**38.** (a)	**39.** (a)	**40.** (c)	**41.** (a)	**42.** (c)	**43.** (a)	**44.** (d)	**45.** (c)	
46. (d)	**47.** (a)	**48.** (d)	**49.** (d)	**50.** (d)					

Solution & Hints

Sol 1. Volume of new sphere

= summation of volume of all three spheres.

$$\frac{4}{3}\pi (R)^3 = \frac{4}{3}\pi\left(\frac{6}{2}\right)^3 + \frac{4}{3}\pi\left(\frac{8}{2}\right)^3 + \frac{4}{3}\left(\frac{10}{2}\right)^3$$

$\therefore$ $R^3 = 3^3 + 4^3 + 5^3$

R = 6 cm

Diameter = 2R = 12 cm.

Sol 2. Volume of Cylinder = Volume of Sphere

$$\pi r^2 h = \frac{4}{3}\pi R^3$$

$$\pi\times(4)^2\times h = \frac{4}{3}\times\pi\times(2)^3$$

$\therefore$ Water Level rise (h) $= \frac{2}{3}$ cm

Sol 3. External radius (R) = 3 cm.

internal radius (r) = 3 – 0.5 = 2.5 cm

Volume of ball = $\frac{4}{3}\pi\ (R^3 - r^3)$

$= \frac{4}{3}\times\frac{22}{7}\ [(3)^3 - (2.5)^3]$

$= 47\frac{2}{3}$ cm^3

Sol 4. Volume of wire = Volume of sphere

$$\pi r^2 h = \frac{4}{3}\pi R^3$$

$$\pi\times\left(\frac{0.2}{2}\right)^2\times h = \frac{4}{3}\times\pi\times(3)^3$$

$\therefore$ Length of Wire (h) = 3600 cm = 36 m

Sol 5. Volume of 12 sphere = Volume of solid Cylinder

$$\Rightarrow\quad 12\times\frac{4}{3}\times\pi\times(R)^3 = \pi\times\left(\frac{16}{2}\right)^2\times 2$$

$R^3 = 8$

R = 2 cm

$\therefore$ Diameter of each sphere = 2R

= 2 × 2 = 4 cm

Sol 6. Let the diameters of small spheres be 3k, 4k and 5k

$\therefore$ radius of small sphere = $\frac{3k}{2}, \frac{4k}{2}, \frac{5k}{2}$

Volume of large sphere = volume of three sphere

$$\therefore \frac{4}{3}\pi R^3 = \frac{4}{3}\pi\left(\frac{3k}{2}\right)^3 + \frac{4}{3}\pi\left(\frac{4k}{2}\right)^3 + \frac{4}{3}\pi\left(\frac{5k}{2}\right)^3$$

$$\Rightarrow\quad (6)^3 = \left(\frac{3k}{2}\right)^3 + \left(\frac{4k}{2}\right)^3 + \left(\frac{5k}{2}\right)^3$$

$$\Rightarrow\quad 216 = \frac{27k^3}{8} + \frac{64k^3}{8} + \frac{125k^3}{8}$$

$\Rightarrow$ $K = 2$

Radius of spheres = $\frac{3k}{2}$ = 3 cm

$\frac{4k}{2}$ = 4 cm

$\frac{5k}{2}$ = 5 cm

$\therefore$ Smallest radius = 3 cm

Sol 7. Total surface area of hemispehre = 1848 cm³

$\Rightarrow$ $3\pi r^2 = 1848$

$\Rightarrow$ $r^2 = 28 \times 7$

$\therefore$ Radius of hemisphere (r) = 14 cm

Volume of cone = Volume of hemisphere

$$\frac{1}{3}\times\pi\times(14)^2\times h = \frac{2}{3}\pi\times(14)^3$$

$\therefore$ Height of cone (h) = 28 cm

Sol 8. Total surface area = $3\pi r^2 = 108\pi$

$r = 6$ cm

$$\text{Volume} = \frac{2}{3}\pi r^3$$

$$= \frac{2}{3} \times \pi \times (6)^3 = 144\pi \text{ cm}^3$$

Sol 9. Volume of solid sphere = 64 × Volume of small solid sphere

$$\therefore \quad \frac{4}{3}\pi(8)^3 = 64 \times \frac{4}{3}\pi(r)^3$$

$$\Rightarrow \quad r = 2 \text{ cm}$$

$$\frac{\text{Surface area of sphere}}{\text{surface area of small sphare}} = \frac{4\pi(8)^2}{4\pi(2)^2}$$

$$= \quad 16 : 1$$

Sol 10. Radius of solid metallic sphere = 3dm

$$= \frac{3}{10}\text{ m}$$

Thickness of circular sheet = 1mm = $\frac{1}{1000}$ m

Volume of circular sheet = volume of metallic sphere

$$\Rightarrow \quad \pi r^2 \times \frac{1}{1000} = \frac{4}{3}\pi \times \left(\frac{3}{10}\right)^3$$

$$\Rightarrow \quad r^2 = 4 \times 9$$

$$\therefore \quad r = 6 \text{ m}$$

Sol 11. Radius of copper wire = $\frac{2}{2} \times \frac{1}{10}$ cm = $\frac{1}{10}$ cm

Volume of copper wire = Volume of sphere

$$\pi \times \left(\frac{1}{10}\right)^2 \times 3600 = \frac{4}{3}\pi(R)^3$$

$$\Rightarrow \quad R^3 = 27$$

$$\therefore \quad R = 3 \text{ cm}$$

Sol 12. Volume of Cone = Volume of sphere

$$\frac{1}{3} \times \pi \times (6)^2 \times 24 = \frac{4}{3} \times \pi R^3$$

$$\Rightarrow \quad R^3 = 36 \times 6$$

$$\therefore \quad R = 6 \text{ cm}$$

Sol 13. Volume of sphere = volume of hemisphere

$$\frac{4}{3}\pi(R)^3 = \frac{2}{3}\pi r^3$$

($\therefore$ R = radius of sphere, r = radius of hemisphere)

$$\Rightarrow \quad 2R^3 = r^3$$

$$\Rightarrow \quad 3\sqrt{2}\ R = r$$

$$\frac{\text{Curved surface area of sphere}}{\text{Cursed surface area of hemisphere}} = \frac{4\pi R^2}{2\pi r^2}$$

$$= \frac{2R^2}{r^2}$$

$$= \frac{2R^2}{(2)^{2/3}R^2} = 2 : 2^{2/3}$$

$$= 2^{1/3} : 1$$

Sol 14. Volume of solid sphere = Volume of 8 small sphere

$$\frac{4}{3} \times \pi \times \left(\frac{6}{2}\right)^3 = 8 \times \frac{4}{3} \times \pi \times r^3$$

$$\Rightarrow \quad r^3 = \frac{216}{64}$$

$$\Rightarrow \quad r = \frac{6}{4}$$

$\therefore$ Radius of small sphere (r) = 1.5 cm

Sol 15. Total surface area = $3\pi r^2 = 27\pi$

$\therefore$ $r = 3$ cm

Sol 16. Radius of spherical drop (r) = $\frac{1}{2\times10} = \frac{1}{20}$ cm

Volume of conical glass = Volume of 32000 spherical drops

$\Rightarrow \quad \frac{1}{3}\pi R^2 H = 32000 \times \frac{4}{3}\pi r^3$

$\Rightarrow \quad \frac{1}{3}\times\pi\times R^2\times 2R = 32000$

$\times\frac{4}{3}\times\pi\times\left(\frac{1}{20}\right)^3 \qquad [\therefore\ H = 2R]$

$\Rightarrow \quad R = 2$ cm

$\therefore \quad$ Height = 2R = 4 cm

Sol 17. Volume of cone = Volume of spherical ball

$\Rightarrow \quad \frac{1}{3}\times\pi\times\left(\frac{12}{2}\right)^2\times H = \frac{4}{3}\times\pi\times\left(\frac{6}{2}\right)^3$

$\therefore \quad$ Height of cone (H) = 3 cm

Sol 18. Volume of Wire = Volume of Spherical ball

$\Rightarrow \quad \pi\times\left(\frac{14}{2}\right)^2\times H = \frac{4}{3}\times\pi\times\left(\frac{14}{2}\right)^3$

$\therefore$ Length of wire (H) $= \frac{28}{3}$ cm

Sol 19. $R = 21$ c m

Volume = $\frac{4}{3}\pi R^3 = \frac{4}{3}\times\frac{22}{7}\times 21\times 21\times 21 =$ 38808 cm^3

$\therefore \quad$ Total surface area = $4\pi R^2 = 4\times\frac{22}{7}\times$ $21\times 21 = 5544$ cm^2

Sol 20. Volume of new sphere = summation of volumes of small sphere

$\therefore \quad \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3)^3 + \frac{4}{3}\pi(4)^3 + \frac{4}{3}\pi(5)^3$

$\Rightarrow \quad r^3 = 216$

$\Rightarrow \quad r = 6$ m

Sol 21. Volume of cone = Volume of hemisphere

$\Rightarrow \quad \frac{1}{3}\pi R^2 H = \frac{2}{3}\pi R^3$

$\therefore \quad$ H = 2R

Sol 22. Radius of hemisphere = R

radus of sphere = $R/2$

$$\frac{\text{Volume of hemisphere}}{\text{Volume of sphere}} = \frac{\frac{2}{3}\pi R^3}{\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}$$

$= 4 : 1$

Sol 23. Internal radius (r) = 4 cm

External radius (R) = 5 cm

Volume = $\frac{4}{3}\pi(R^3 - r^3)$

$= \frac{4}{3}\pi(5^3 - 4^3) = \frac{244\pi}{3}$ cm^3

Sol 24. Volume of Big sphere = Summation of Volume of Small sphere

$\Rightarrow \quad \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(1)^3 + \frac{4}{3}\pi(6)^3 + \frac{4}{3}\pi(8)^3$

$\Rightarrow \quad R^3 = 729$

$\therefore \quad r = 9$ cm

Sol 25. $\frac{\text{Surface area of A}}{\text{Surface area of B}} = \frac{4\pi(40)^2}{4\pi(10)^2} = 16 : 1$

Sol 26. Volume = $\frac{4}{3}\pi r^3 = \frac{88}{21}(14)^3$

$\Rightarrow \quad \frac{4}{3}\times\frac{22}{7}r^3 = \frac{88}{21}(14)^3$

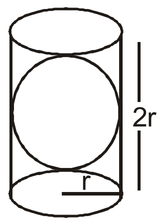
$\Rightarrow \quad r = 14$ cm

Curved surface area = $4\pi r^2$

$= 4\times\frac{22}{7}\times 14\times 14$

$= 2464$ cm^2

Sol 27.



Radius = r, Height = 2r

Surface area of sphere = $S_1 = 4\pi r^2$

Surface area of cylinder

$S_2 = 2\pi rh = 2\pi \times r \times 2r = 4\pi r^2 \quad \therefore S_1 = S_2$

Sol 28. Let the no. of balls be n

Volume of larger sphere = x × Volume of smaller sphere

$$\therefore \frac{4}{3}\pi\,(10 \times 10)^3 = n \times \frac{4}{3}\pi\,(5)^3$$

[$\therefore$ 1 cm = 10 mm]

$$\Rightarrow \frac{100 \times 100 \times 100}{5 \times 5 \times 5} = n$$

$$\Rightarrow n = 8000$$

Sol 29. Volume (V) = $\frac{4}{3}\pi r^3$

Surface area (S) = $4\pi r^2$

$$\Rightarrow \frac{S^3}{V^2} = \frac{(4\pi r^2)^3}{\left(\frac{4}{3}\pi r^3\right)^2} = \frac{(4\pi)^3 r^6 \times 3^2}{(4\pi)^2 r^6} =$$

$4\pi \times 9 = 36\pi$ unit

Sol 30. Total surface area of hemisphere = 27π cm²

$\Rightarrow 3\pi R^2 = 27\pi$

$\Rightarrow$ Radius of hemisphere (R) = 3 cm

Sol 31. $\therefore 4\pi r^2 = 8\pi$

$\Rightarrow r = \sqrt{2}$

$$\text{Volume} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (R)^3$$

$$= \frac{8\sqrt{2}}{3}\pi \text{ cubic unit}$$

Sol 32. Radius of sphere bullet = $\frac{5}{20}$ m. = $\frac{1}{4}$ m.

Total numbers of spherical bullets = $\dfrac{lbh}{\frac{4}{3}\pi r^3}$

$$= \frac{11 \times 10 \times 5}{\frac{4}{3} \times \pi \times \frac{1}{64}} \Rightarrow n\pi = 3 \times 8800 \ (\pi < 3)$$

$n < 8800$

Sol 33. Radius of cylinder = R

Height of cylinder = 8R

Radius of sphere = $\frac{R}{2}$

Number of spherical ball =

$$\frac{\text{Volume of cylinder}}{\text{Volume of sphere}}$$

$$= \frac{\pi R^2 \times 8R}{\frac{4}{3} \times \pi \times \left(\frac{R}{2}\right)^3} = 48 \text{ balls}$$

Sol 34. $\dfrac{\text{Volume of cube}}{\text{Volume of solid sphere}} = \dfrac{363}{49}$

$$\Rightarrow \frac{a^3}{\frac{4}{3}\pi r^3} = \frac{363}{49}$$

$$\Rightarrow \frac{\text{edge of cube}}{\text{radius of sphere}} = \frac{a}{r} = \frac{22}{7}$$

$\therefore$ a : r = 22 : 7

Sol 35. Volume of Cylindrical vessel = Volume of Spherical drop

$$\Rightarrow \pi \times \left(\frac{12}{2}\right)^2 \times h = \frac{4}{3} \times \pi \times \left(\frac{6}{2}\right)^3$$

$\therefore$ level of water rise (h) = 1 cm

Sol 36. Radius of wire = 2mm = $\frac{2}{1000}$ m

Volume of wire = Volume of copper sphere

$$\Rightarrow \quad \pi\times\left(\frac{2}{1000}\right)^2\times h = \frac{4}{3}\pi\times\left(\frac{9}{100}\right)^3$$

$\therefore$ Length of wire = 243 m

Sol 37. Volume of larger sphere = summation of volume of smaller spheres

$$\Rightarrow \frac{4}{3}\pi\left(\frac{3}{2}\right)^3 = \frac{4}{3}\pi\left(\frac{1.5}{2}\right)^3 + \frac{4}{3}\pi\left(\frac{2}{2}\right)^3 + \frac{4}{3}\pi r^3$$

$$\Rightarrow \left(\frac{3}{2}\right)^3 = \left(\frac{1.5}{2}\right)^3 + (1)^3 + r^3$$

$\Rightarrow r = 1.25$cm

Diameter of third ball = $2r$ = 2.5 cm

Sol 38. Brim (Circumference) of hesmispherical bowl = 176

$\Rightarrow 2\pi r = 176$

$\Rightarrow$ r = 28cm

When bowl is half full $= \frac{2}{3}\pi r^3 \times \frac{1}{2}$

$= \frac{1}{3}\pi(28)^3$

Volume of hemispherical glass = $= \frac{2}{3}\pi(2)^3$

No. of persons may be served = $\dfrac{\frac{1}{3}\pi(28)^3}{\frac{2}{3}\pi(2)^3} = 1372$

Sol 39. Inner radius = 4 cm = r

outer radius = 4.5 cm = R

Volume of steel = $\frac{2}{3}\pi(R^3 - r^3)$

$= \frac{2}{3}\times\frac{22}{7}[(4.5)^3 - (4)^3]$

$= 56.83$ cm^3

Sol 40. Volume of cylindrical vessel = Volume of sphere

$$\pi\times(6)^2\times h = \frac{4}{3}\times\pi\times(3)^3$$

$\therefore$ Surface of water rise (h) = 1 cm

Sol 41. Volume of cylinder = $\pi\times(2)^2\times 4 = 16\pi$ cm^3

Volume of toy = $\frac{1}{3}\pi\times(2)^2\times 2 + \frac{2}{3}\pi\times 8$ = 8π cm^2

Difference between volume of cylinder and toy

$= 16\pi - 8\pi = 8\times 3.14$

$= 25.12$ cm^3

Sol 42. Number of cone = $\dfrac{\text{Volume of sphere}}{\text{Volume of cone}}$

$$= \frac{\frac{4}{3}\pi\times(10.5)^3}{\frac{1}{3}\pi\times(3.5)^2\times 3} = 126 \text{ cones}$$

Sol 43. Number of bottles = $\dfrac{\text{Volume of hemisphere}}{\text{Volume of cylindrical bottle}}$

$$= \frac{\frac{2}{3}\pi(9)^3}{\pi\times\left(\frac{3}{2}\right)^2\times 4} = 54$$

Sol 44. Let the height of the vessel be x

Then, Radius of the bowl = radius of the vessel = $\frac{x}{2}$

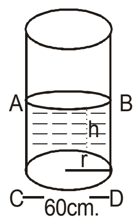
Volume of the bowl, $V_1 = \frac{2}{3}\pi\left(\frac{x}{3}\right)^3 = \frac{1}{12}\pi x^3$

Volume of the vessel, $V_2 = \pi\left(\frac{x}{2}\right)^2 x = \frac{1}{4}\pi x^3$

Since $V_2 > V_1$,

So the vessel can contain 100% of the beverage filled in the bowl.

Sol 45.



r = 30 cm.
R = 27 cm.
Volume of cylinder ABCD = Volume of sphere

$\Rightarrow \pi r^2 h = \frac{4}{3}\pi R^3$

$\Rightarrow 30\times30\times h = \frac{4}{3}\times27\times27\times27$

$\therefore h = \frac{729}{25} = 29\frac{4}{25}$ cm.

Sol 46. Volume of spherical ball = Summation of volume of small spherical balls

$\Rightarrow \frac{4}{3}\pi(6)^3 = \frac{4}{3}\pi(3)^3 + \frac{4}{3}\pi(4)^3 + \frac{4}{3}\pi(r)^3$

$\Rightarrow 216 = 27 + 64 + (r)^3$

$\Rightarrow (r)^3 = 125$

$\therefore r = 5$cm

Sol 47. Let water level is increased by h

l = 49cm. $b = \frac{44}{3}$ cm. h = ?

Volume of cuboid = volume of sphere

$\Rightarrow lbh = \frac{4}{3}\pi r^3$

$\Rightarrow 49\times\frac{44}{3}\times h = \frac{4}{3}\times\frac{22}{7}\times7\times7\times7$

$\therefore h = \frac{4}{2} = 2cm$

Sol 48. Shaded region is the area where gold is coated

$\therefore$ Volume of lead = volume of gold

$\Rightarrow \frac{4}{3}\pi(2)^3 = \frac{4}{3}\pi(R^3 - 2^3)$

$\Rightarrow 8 = R^3 - 8$

$\Rightarrow R = \sqrt[3]{16} = 2\sqrt[3]{2} = 2\times1.259 = 2.518$

$\therefore$ Thickness of gold = R – r
= 2.518 – 2 = 0.518cm

Sol 49. Let total number of cones be n

Volume of solid sphere = n×volume of cones

$\frac{4}{3}\pi r^3 = n\times\frac{1}{3}\pi R^2\times R$

[$\therefore$ Height of cone = R]

n = 4

Volume of one cone = Volume of smaller sphere

$\Rightarrow \frac{1}{3}\pi r^2\times r = \frac{4}{3}\pi R^3$]

$\Rightarrow 4R^3 = r^3$

$\Rightarrow 2^{\frac{2}{3}} R = r$

$\frac{\text{Surface of smaller sphere}}{\text{Surface of larger sphere}} = \frac{4\pi R^2}{4\pi r^2}$

$= \frac{R^2}{2^{\frac{4}{3}}R^2} = \frac{1}{2^{\frac{4}{3}}}$

$\therefore 1 : 2^{\frac{4}{3}}$

Sol 50. Surface area of B = surface area of A + 300% of surface area of A = 4 (Surface of area of A)

Let the radius of A be a and radius of B be b.

$\therefore 4\pi b^2 = 4\times4\pi a^2$

$\Rightarrow b = 2a$

Volume of A = $\frac{4}{3}\pi a^3$

Volume of B = $\frac{4}{3}\pi b^3 = \frac{4}{3}\pi(2a)^3 = \frac{4}{3}\pi 8a^3$

% of volume of A lower than B = k %

$= \frac{\frac{4}{3}\pi 8a^3 - \frac{4}{3}\pi a^3}{\frac{4}{3}\pi 8a^3}\times100$

$= \frac{7}{8}\times100 = 87.5\%$

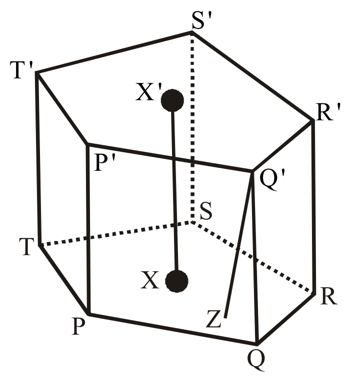
$\therefore k = 87.5$

SURFACE AREA AND VOLUME OF A PRISM

Prism-

A prism is a solid, whose side faces are parallelograms and whose ends (or bases) are congruent parallel rectilinear figures.

In this figure given below, there is a prism whose ends are rectilinear figures PQRST and P'Q'R'S'T'.



Base of Prism-

The end on which a prism may be supposed to stand is called the base of the prism.

In the above figure, PQRST and P'Q'R'S'T' are the bases of the prism. Every prism has two bases.

Height of a prism-

The perpendicular distance between the ends of a prism is called the height of the prism.

In the above figure, Q'Z is the perpendicular distance between the ends PQRST and P'Q'R'S'T'. So, it is the height of the prism shown in the above figure.

Axis of a Prism-

The straight line joining the centres of the ends of a prism is called the axis of the prism.

In the above figure, a straight line passing through X and X' is the axis of the prism.

Lateral Faces-

All faces other than the bases of a prism are known as its lateral faces.

In the above figure, PQQ'P', QRR'Q', RSS'R' etc are lateral faces.

Lateral Edges-

The lines of intersection of the lateral faces of a prism are called the lateral edges of the prism.

In the above figure PP', QQ', RR', SS' and TT' are the lateral edges of the prism.

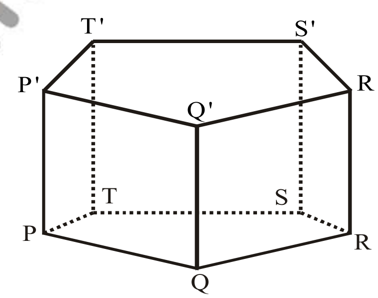
Regular Prism-

If ends are regular figures then prism is called a regular prism.

Right Prism-

A prism is called a right prism if its lateral edges are perpendicular to its ends (bases). Otherwise it is said to be an oblique prism.

The prism shown in figure (I) is an oblique prism whereas the prism shown in figure (II) is a right prism.



In a right prism, length of the prism is same as its height. Also, all lateral edges are of the same length equal to the height of the prism. It is also evident from the definition of a right prism that its all lateral faces are rectangles. The number of lateral edges and lateral faces of a prism is same as the number of sides in the base of the prism.

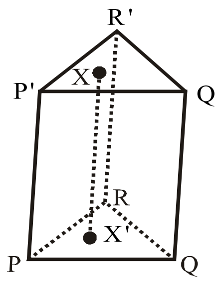
Triangular Prism-

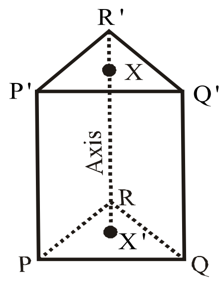
A prism is called a triangular prism if its ends are triangles.

Right Triangular Prism-

A triangular prism is called a right triangular prism if its lateral edges are prependicular to its ends.

The prism shown in figure (III) is a triangular prism whereas the prism shown in figure (IV) is a right triangular prism.



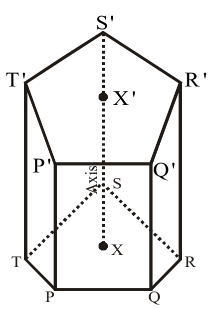


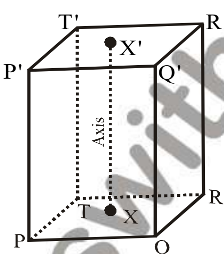
A prism is said to be quadrilateral prism or a pentagonal prism or a hexagonal prism etc according as the number of sides in the rectilinear figure forming the ends (base) is four or five or six etc.

If the ends of a quadrilateral prism are parallelograms, then it is also known as a parallelopiped.

A quadrilateral prism with its ends as squares is called a rectangular solid or a cubiod.

Figure (V) shows a right pentagonal prism and figure (VI) shows a rectangular solid.





Volume and Surface Area of a Right Prism-

(i) Volume of a right prism-
V= Area of the Base × Height

(ii) Lateral Surface area of a right prism-
L.S.A. = Perimeter of the Base × Height

(iii) Total Surface area of a right prism-

T.S.A. = Lateral Surface area + Area of Ends
= Lateral Surface area + 2 (Area of the Base)

For Example : If the base of a right prism is an equilateral triangle of side a and height h, then,

$\Rightarrow$ Lateral surface area = $3\,a \times h$

$$\Rightarrow \text{Total surface area} = 3a \times h + \frac{\sqrt{3}}{2}a^2$$

$$\Rightarrow \text{Volume} = \frac{\sqrt{3}}{4}a^2 \times h$$

Some Examples

Q.1 Find the area of the base of a right triangular prism having volume of 1476 cm^2 and height 18 cm.

Sol. Volume = Area of the base × Height

$$\Rightarrow \text{Area of the base} = \frac{\text{Volume}}{\text{Height}}$$

$$\Rightarrow \text{Area of the base} = \frac{1476}{18}\text{ cm}^2 = 82\text{ cm}^2$$

Q.2 The base of a right prism is an equilateral triangle with a side 10 cm and its height is 25 cm. Find its volume, lateral surface area and total surface area.

Sol. Volume = Area of the base × Height.

Base is an equilateral triangle.

$$\text{Area of the base} = \frac{\sqrt{3}}{4} \times (\text{side})^2$$

$$= \left(\frac{\sqrt{3}}{4} \times 10^2\right)\text{ cm}^2 = 25\sqrt{3}\text{ cm}^2$$

$$\therefore V = \left(25\sqrt{3} \times 25\right)\text{ cm}^3$$

$$= 625\sqrt{3}\text{ cm}^3$$

Lateral surface area
= Perimeter of the base × Height
= $(10 + 10 + 10) \times 25\text{cm}^2 = 750\text{ cm}^2$

Total Surface area
= Lateral Surface area + Area of ends

$$= \left[750 + 2\left(\frac{\sqrt{3}}{4} \times 10^2\right)\right]\text{cm}^2$$

$$= \left[750 + 50\sqrt{3}\right]\text{cm}^2$$

Q.4 A right prism of height 15 cm stands on a triangular base whose sides are 13 cm, 14 cm and 15 cm, find its lateral surface area, total surface area and volume.

Sol. If a, b, c are the length of the sides of a triangle and s is the semi-perimeter, then its area

$$A = \sqrt{s(s-a)(s-b)(s-c)} \quad \text{where } s = \frac{1}{2}(a+b+c)$$

Here, $a = 13$ cm, $b = 14$ cm and $c = 15$ cm.

$$\therefore s = \frac{1}{2}(13+14+15) = 21 \text{ cm.}$$

Perimeter of the base $= 2s = 42$ cm.
Area of the base

$$= \sqrt{21(21-13)\times(21-14)\times(21-15)}$$

$$= \sqrt{21\times8\times7\times6} = \sqrt{7\times3\times8\times7\times3\times2}$$

$$= \sqrt{7^2\times3^2\times4^2} = 7\times3\times4 \text{ cm}^2 = 84 \text{ cm}^2$$

$\therefore$ Lateral surface
= Perimeter of the base × Height
$= 42 \times 15 \text{ cm}^2 = 630 \text{ cm}^2$
Total surface area
= Lateral surface area + 2 (Area of the base)
$= (630 + 2 \times 84) \text{ cm}^2 = 798 \text{ cm}^2$
Volume = Area of the base × Height
$= 84 \times 15 \text{ cm}^3 = 1260 \text{ cm}^3$

Q.5 A right prism stands on a triangular base. The volume of the prism is 606 cm^3 and the sides of the base are 5 cm, 5 cm and 8 cm. Find the height of the prism.

Sol. Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

Here, $a = 5$ cm, $b = 5$ cm and $c = 8$ cm.

$$\therefore s = \frac{1}{2}(5+5+8) = 9 \text{ cm}$$

$$\Rightarrow A = \sqrt{9(9-5)\times(9-5)\times(9-8)}$$

$$= \sqrt{9\times4\times4\times1} = 12 \text{ cm}^2$$

Volume = Area of the base × Height

$$\text{Height} = \frac{\text{Volume}}{\text{Area of the base}} = \frac{606}{12} = 50.5 \text{cm}$$

Q.6 The base of a right prism is an equilateral triangle of side 8 cm. If the lateral surface area of the prism is 960 cm^2. Find its volume.

Sol. Lateral surface area
= Perimeter of the base × Height $\Rightarrow 960 = (8+8+8) \times$ Height

$$\text{Height} = \frac{960}{24} \text{cm} = 40 \text{cm}$$

$$\text{Area of the base} = \frac{\sqrt{3}}{4}\times(\text{side})^2$$

$$= \frac{\sqrt{3}}{4}\times(8)^2 = 16\sqrt{3} \text{ cm}^2$$

$\therefore$ Volume of the prism
= Area of the base × Height

$$= \left(16\sqrt{3}\times40\right) = 640\sqrt{3} \text{ cm}^3$$

Q.7 A right triangular prism of height 18 cm and of base sides 5 cm, 12 cm and 13 cm is transformed into another right triangular prism on a base of sides 9 cm, 12 cm and 15 cm. Find the height of new prism and the change in the whole surfae area.

Sol. Volume of both prism will be same.Both triangle are right angle side according to the length of side.

Here, for first triangle
$a = 5$ cm, $b = 12$ cm & $c = 13$ cm
and height of the prism = 18cm
Here, for first triangle
$a = 9$ cm, $b = 12$ cm and $c = 15$ cm
let the height of second prism is h_2

$$V_1 = V_2$$

$$\left(\frac{1}{2}\times5\times12\right)\times18 = \left(\frac{1}{2}\times9\times12\right)\times h_2$$

$$\Rightarrow h_2 = 10 \text{cm}$$

Let S be the total surface area of the prism.
S = Lateral surface area + 2 (Area of the base)
= Perimeter of the base × height
+ 2 (Area of the base)
$= (30 \times 18 + 2 \times 30) = 600 \text{ cm}^2$
Let h be the height of the new prism.
Let S_1 be the total surface area of the new prism.
S_1 = Perimeter of the base × Height
+ 2 (Area of the base)
$= (36 \times 10 + 2 \times 54) = 468 \text{ cm}^2$
$\therefore$ Change in the whole surface area $= S - S_1$
$= (600 - 468) = 132 \text{ cm}^2$

Q.8 The base of a right triangular prism is an equilateral triangle. If its height is halved and each side of the base is doubled, find the ratio of the volume of the two prisms.

Sol. Let a be the length of each side of the base of the given prism and h be its height. then volume

$$V_1 = \left(\frac{\sqrt{3}}{4}a^2 \times h\right)$$

Let a_1 be the length of each side of the base of the new prism and h_1 be its height. Then,

$a_1 = 2a$ and $h_1 = \dfrac{h}{2}$

If V_2 is the volume of the new prism

$$V_2 = \frac{\sqrt{3}}{4}a_1^2 \times h_1 = \frac{\sqrt{3}}{4}(2a)^2 \times \frac{h}{2} = \left(\frac{\sqrt{3}}{4}a^2 h\right)$$

$$\therefore \frac{V_1}{V_2} = \frac{\frac{\sqrt{3}}{4}a^2h}{\frac{\sqrt{3}}{2}a^2h} = \frac{1}{2}$$

Q.9 The perimeter of the base of right triangular prism is 60 cm and sides of the base are in the ratio 5 : 12 : 13. Find its volume and total surface area, if its height is 40 cm.

Sol. Let a, b, c be the lengths of the sides of the base of the prism.

$a : b : c = 5 : 12 : 13$

a = 5k, b = 12k and c = 13k

a + b + c = 30k $\Rightarrow$ 60 = 30k

[$\because$ a + b + c = 60cm (Given]

$k = 2$

$\therefore a = 10, b = 24$, and $c = 26$

semi-perimeter s = 60/2 = 30.

$\therefore$ Area of the base = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{30(30-10)(30-24)(30-26)}$

$= \sqrt{30 \times 20 \times 6 \times 4} = \sqrt{5 \times 6 \times 5 \times 4 \times 6 \times 4}$

$= 5 \times 6 \times 4 = 120 \text{ cm}^2$

Volume of the prism = Area of the base × Height

$= 120 \times 40 = 4800 \text{ cm}^3$

Total surface ara = Perimeter of the base × Height + 2 (Area of the base)

$= (60 \times 40 + 2 \times 120) = 2640 \text{ cm}^2$

Q.10 The total surface area of a right triangular prism of the height 4 cm is $72\sqrt{3}$ cm². If the base of the prism is an equilateral triangle, find its volume.

Sol. Let each side of the base of the prism be a cm.

total surface area = $72\sqrt{3}$ cm²

Perimeter of the base × height

+ 2 (Area of the base) = $72\sqrt{3}$

$$3a \times 4 + 2\left(\frac{\sqrt{3}}{4}a^2\right) = 72\sqrt{3}$$

$$\sqrt{3}a^2 + 24a - 144\sqrt{3} = 0$$

$$a^2 + 8\sqrt{3} - 144 = 0$$

$$a^2 + 12\sqrt{3}a - 4\sqrt{3}a - 144 = 0$$

$$a\left(a + 12\sqrt{3}\right) - 4\sqrt{3}\left(a + 12\sqrt{3}\right) = 0$$

$$\left(a - 4\sqrt{3}\right)\left(a + 12\sqrt{3}\right) = 0$$

$a - 4\sqrt{3} = 0$ [$\because a + 12\sqrt{3} \neq 0$ as $a > 0$]

$a = 4\sqrt{3}$

$\therefore$ Volume of the prism

= Area of the base × Height

$$= \frac{\sqrt{3}}{4} \times \left(4\sqrt{3}\right)^2 \times 4 = 48\sqrt{3}\text{cm}^3$$

Exercise Prism

1. The base of a prism is a right-angles triangle and the two sides containing the right angle are 8 cm and 15 cm. If its height is 20 cm, then the volume of the prism is

(a) 160 cm^3 (b) 300 cm^3

(c) 1200 cm^3 (d) 600 cm^3

2. The base of a prism is a regular hexagon. If every edge of the prism measures 1 metre and height is 1 metre, then the volume of the prism is

(a) $\frac{3\sqrt{2}}{2}$ cu m (b) $\frac{3\sqrt{3}}{2}$ cu m

(c) $\frac{6\sqrt{2}}{5}$ cu m (d) $\frac{5\sqrt{3}}{2}$ cu m

3. The base of a right prism is a pentagon whose sides are in the raio $1:\sqrt{2}:\sqrt{2}:1:2$ and its height is 10 cm. If the longest side of the base be 6 cm, the volume of the prism is

(a) 270 cm^3 (b) 360 cm^3

(c) 540 cm^3 (d) None of these

4. What is the total surface area of a triangular prism whose height is 30 m and the sides of whose base are 21 m, 20 m and 13 m respectively?

(a) 1872 sq m (b) 1725 sq m

(c) 1652 sq m (d) 1542 sq m

5. The base of a prism is a triangle whose sides are 17 cm, 25 cm & 28 cm and the volume of the prism is 4200 cubic cm. What is the height? Find its lateral area also.

(a) 20 cm, 1400 sq cm

(b) 25 cm, 700 sq cm

(c) 20 cm, 700 sq cm

(d) 10 cm, 1400 sq cm

6. There are two prism, one has equilateral triangle as a base and the other regular hexagon. If both of the prisms have equal heights and volumes, then find the ratio between the length of each side at their bases.

(a) $1:\sqrt{6}$ (b) $\sqrt{6}:1$

(c) $\sqrt{3}:2$ (d) $2:\sqrt{3}$

7. The base of a solid right prism is a triangle whose sides are 9 cm, 12 cm and 15 cm. The height of the prism is 5 cm. Then, the total surface area of the prism is

(a) 180 cm^2 (b) 234 cm^2

(c) 288 cm^2 (d) 270 cm^2

8. The base of a right prism is an equilateral triangle of area 173 cm^2 and the volume of the prism is 10380 cm^3. The area of the lateral surface of the prism is (use $\sqrt{3}$ = 1.73)

(a) 1200 cm^2 (b) 2400 cm^2

(c) 3600 cm^2 (d) 4380 cm^2

9. The base of a right prism is a trapezium. The lengths of the parallel sides are 8 cm and 14 cm and the distance between the parallel sides is 8 cm. If the volume of the prism is 1056 cm^3, then the height of the prism is

(a) 44 cm (b) 16.5 cm

(c) 12 cm (d) 10.56 cm

10. If the altitude of a right prism is 10 cm and its base is an equilateral triangle of side 12 cm, then its total surface area (in cm^2) is

(a) $(5+3\sqrt{3})$ (b) $36\sqrt{3}$

(c) 360 (d) $72(5+\sqrt{3})$

11. The base of a right prism is a right- angled triangle whose sides are 5 cm, 12 cm and 13 cm. If the area of the total surface of the prism is 360 cm^2, then its height (in cm) is

(a) 10 (b) 12

(c) 9 (d) 11

12. If the base is a right rectangular prism is left unchanged and the measure of the lateral edges are doubled, then its volume will be

(a) unchanged (b) tripled
(c) doubled (d) quadrupled

13. The height of a right prism with a square base is 15 cm. If the area of the total surface of the prism is 608 sq. cm, its volume is:

(a) 910 cm^3 (b) 920 cm^3
(c) 960 cm^3 (d) 980 cm^3

14. The base of a right prism is an equilateral triangle of side 8 cm and height of the prism is 10 cm. Then the volume of the prism is:

(a) $320\sqrt{3}$ cm^3 (b) $160\sqrt{3}$ cm^3
(c) $150\sqrt{3}$ cm^3 (d) $300\sqrt{3}$ cm^3

15. A right prism stands on a base 6 cm equilateral triangle and its volume is $81\sqrt{3}$ cm^3. The height (in cms) of the prims is:

(a) 9 (b) 10
(c) 12 (d) 15

16. A prism has as the base a right angled triangle whose sides adjacent to the right angles are 10cm and 12cm long. The height of the prism is 20cm. The density of the material of the prism is 6gm.cubic cm. The weight of the prism is

(a) 6.4 kg (b) 72 kg
(c) 3.4 kg (d) 4.8 kg

17. The perimeter of the triangular base of a right prism is 15 cm and radius of the incircle of the triangular base is 3 cm. If the volume of the prism be 270 cm^3, then the height of the prism is–

(a) 6 cm (b) 7.5 cm
(c) 10 cm (d) 12 cm

18. The base of a prism is a right angled triangle with two sides 5 cm and 12 cm. The height of the prism is 10 cm. The total surface area of the prism is–

(a) 360 sq. cm (b) 300 sq. cm
(c) 330 sq. cm (d) 325 sq. cm

19. The base of a right prism is an equilateral triangle. If the lateral surface area and volume is 120 cm^2, $40\sqrt{3}$ cm^3 respectively then the side of base the prism is.

(a) 4 cm (b) 5 cm
(c) 7 cm (d) 40 cm

20. The base of a right prism is an equilateral triangle. If its height is one-fourth and each side of the base is tripled, then the ratio of the volumes of the old to the new prism is -

(a) 4 : 3 (b) 1 : 4
(c) 1 : 2 (d) 4 : 9

Answer

1.	(c)	2.	(b)	3.	(a)	4.	(a)	5.	(a)	6.	(b)	7.	(c)	8.	(c)	9.	(c)
10.	(d)	11.	(a)	12.	(c)	13.	(c)	14.	(b)	15.	(a)	16.	(b)	17.	(d)	18.	(a)
19.	(a)	20.	(d)														

Solution & Hints

Explanations

Sol 1. Volume of the prism = Area of the base × Height

$$\frac{1}{2}\times 15\times 8\times 20=1200\,\text{cm}^3$$

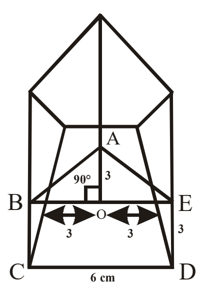
Sol 2. Given prism is a solid with regular hexagonal base

$\therefore$ Its volume = Area of the base × Height

$$=\frac{3\sqrt{3}}{2}\times 1=\frac{3\sqrt{3}}{2}\text{cu m}$$

$$\left[\text{Since, the area of regular hexagon with side } 1\,\text{m}=\frac{3\sqrt{3}}{2}\text{m}^2\right]$$

Sol 3.



$AB = 3\sqrt{2}$
$AE = 3\sqrt{2}$
$\angle ABE = 45°$
$\angle AEB = 45°$

Area of $\square$BCDE = 18 cm^2

Area of $\Delta OAE=\frac{1}{2}\times 3\times 3=4.5\text{cm}^2$

$\therefore$ Area of ΔABE = 9 cm^2

$\therefore$ Area of ABCDE (base) = (18 + 9) = 27 cm^2

Volume of prism = Area of the base × Height

= (27 × 10) = 270 cm^3

Sol 4. Total surface area of prism

= Lateral surface area + 2 ×(Area of base)

Here, s = $\frac{a+b+c}{2}=\frac{20+20+13}{2}=27$

$\therefore$ Required area = (21 + 20 + 13) × 30 + 2 × $\sqrt{27(27-21)(27-20)(27-13)}$

= 54 × 30 + $2\sqrt{27\times 6\times 7\times 14}$

= 1620 + 2 × 126 = 1872 sq m

Sol 5. Let the side a = 17 cm, b = 25 cm, c = 28 cm

Then, $s=\left(\frac{a+b+c}{2}\right)=\left(\frac{17+25+28}{2}\right)=35\text{cm}$

$(s-a)=(35-17)=18$ cm
$(s-b)=(35-25)=10$ cm
$(s-c)=(35-28)=7$ cm

Area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Hence, area of the base = $\sqrt{35\times 18\times 10\times 7}$ sq. cm

Volume of the prism = Area of the base × Height

$\therefore$ Height of the prism = $\left(\frac{4200}{210}\right)=20\text{cm}$

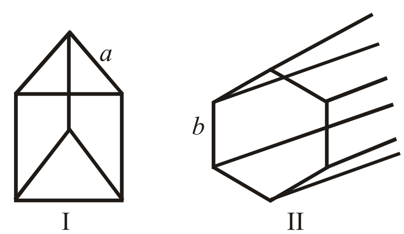
Lateral Area = Perimeter of the base × Height

= (17 + 25 + 28) × 20

= 1400 sq cm

Sol 6. Let the height of each prism be h units and the length of each side of equilateral triangle at thebase of first prism be a units and that the second prism having regular hexagon as base be b units.

(See the figures given below)



According to the question,

Volume of first prism = Volume of second prism

$$\frac{\sqrt{3}}{4}a^2\times h=\frac{\sqrt{3}}{2}b^2\times h$$

$$\Rightarrow \frac{1}{4}a^2=\frac{3}{2}b^2 \Rightarrow a^2=6b^2$$

$a = \sqrt{6}b \Rightarrow \frac{a}{b} = \frac{\sqrt{6}}{1}$

$\therefore a : b = \sqrt{6} : 1$

Sol 7. Base of prism is a right angle triangle

$\therefore$ area of base $= \frac{1}{2} \times 9 \times 12 = 54$ cm^2

Total surface area = Perimeter of box x height + 2 × area of base

= (9 + 12 + 5) × 5 + 2 × 54

= 288 cm^2

Sol 8. Volume = area of base × height

10380 = 173 × h

h = 60 cm

$\therefore$ area of equilateral triangle $= \frac{\sqrt{3}}{4} a^2 = 173$

a = 20 cm

Perimeter of triangle = 3a = 3 × 20 = 60 cm

Lateral surface area = Perimeter of triangle × height

= 60 × 60 = 3600 cm^2

Sol 9. Area of trapezium $= \frac{1}{2}$ × height × (sum of parallel sides)

$= \frac{1}{2} \times 8 \times (8 + 14) = 88$ cm^2

$\therefore$ Volume = area of traperium × height

1056 = 88 × h

h = 12 cm

Sol 10. Total surface area = Permeter base × height + 2 × area of base

$= 36 \times 10 + 2 \times \frac{\sqrt{3}}{4} \times 12 \times 12$

$= 360 + 72\sqrt{3} = 72\left(5+\sqrt{3}\right)$ cm^2

Sol 11. Total surface area = 360

(perimeter base) × height + 2 × area of base = 360

$(5 + 12 + 13) \times h + 2 \times \left(\frac{1}{2}\times5\times12\right) = 360$

$30h = 300 \Rightarrow h = 10$ cm

Sol 12. The base of the prism is rectangular and we are not changing the base so length & breadth will be remain same.If we double the lateral edges it means we are doing double its height so Volume of the prism will be doubled.

Sol 13. Perimeter of square base of side a = 4a

Lateral surface area = 4a × h

Total surface area = Lateral surface area + 2 × area of base

$608 = 4ah + 2a^2$

$608 = 60a + 2a^2$

$a^2 + 30a - 304 = 0$

$a^2 + 38a - 8a - 304 = 0$

$a(a + 38) - 8(a + 38) = 0$

$(a + 38)(a - 8) = 0$

a = 8 cm.

Volume of prism = area of base × height

= 64 × 15

= 960 cm^3

Sol 14. Volume = area of base × height

$= \frac{\sqrt{3}}{4} (8)^2 \times 10$

$= 160\sqrt{3}$ cm.

Sol 15. Area of base of prism $= \frac{\sqrt{3}}{4} \times 6 \times 6$

$= 9\sqrt{3}$ cm^2

$\therefore$ Volume = area of base × height

$81\sqrt{3} = 9\sqrt{3}$ × height

h = 9 cm

Sol 16. Area of base $= \frac{1}{2}$ × base × height

$= \frac{1}{2} \times 10 \times 12 = 60$ cm^2

$\therefore$ Volume of prism = Area of base × height

$= 60 \times 20 = 1200\ \text{cm}^3$

Material used for 1 cubic cm. = 6 gm

Material used for 12 00 $\text{cm}^3 = 1200 \times 6 = 7200$ gm

$= 7.2$ kg

Sol 17. Perimeter of triangle = 15cm

$$\text{semiperimeters} = \frac{15}{2}\ \text{cm}$$

inradius $r = 3$ cm

$$\Delta = r.s = \frac{15}{2} \times 3 = \frac{45}{2}\ \text{cm}$$

Volume of prism = area of base × height

$$270 = \frac{45}{2} \times h$$

$h = 12$ cm

Sol 18. Third side of right angle triangle will be 13 cm.

Tolal surface area.

= (Perimeter of base) × h + 2 × area of base

$$= (5 + 12 + 13) \times 10 + 2 \times \frac{1}{2} \times 5 \times 12$$

$= 300 + 60 = 360$ sq. cm

Sol 19. Volume of prism V = area of base × height

$$= \frac{\sqrt{3}}{4} a^2 \times h$$

Lateral surface area of prism (L.S.A)

= Perimeter of base × height = $3a \times h$

$$\frac{V}{l.S.A} = \frac{\frac{\sqrt{3}}{4} a^2 \times h}{3a \times h} =$$

$$\frac{40\sqrt{3}}{120} = \frac{\sqrt{3}\,a}{12}$$

$a = 4$ cm

Sol 20. $$\frac{V_1}{V_2} = \frac{\frac{\sqrt{3}}{4} {a_1}^2 . h_1}{\frac{\sqrt{3}}{4} {a_1}^2 . h^2} = \left(\frac{a_1}{a_2}\right)^2 . \frac{h_1}{h_2}$$

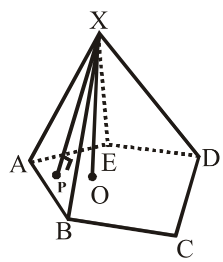
$$= \left(\frac{a_1}{3a_1}\right)^2 . \left(\frac{h_1}{h_2 / 4}\right) = \frac{4}{9} \left(\because a_2 = 3a_1 \ \&\ h_2 = \frac{h_1}{2}\right)$$

☆☆☆☆☆

Pyramid

A pyramid is a solid whose base is a plane rectilinear figure and whose side-faces are triangles having a common vertex outside the plane of the base.

Figure of this below shows a pyramid XABCDE. The base of this pyramid is the pentagon ABCDE and triangles XAB, XBC, XCD, XDE and XEA are five faces. If the base of a pyramid is a triangle, a quadrilateral and a square, then it is called triangular pyramid, quadrilateral pyramid and square pyramid respectively. Similarly, a pyramid is called a pentagonal, hexagonal, septagonal and octagonal according as the number of sides of the base is 5, 6, 7 or 8.



Vertex-

The common vertex of the triangular faces of a pyramid is called the vertex of the pyramid. In the above figure, 'X' is the vertex of the pyramid XABCDE.

Height-

The height of a pyramid is the length of the perpendicular from the vertex to the base.

In the above figure, XP is the height of the pyramid XABCDE.

Axis-

The axis of a pyramid is the straight line joining the vertex to the central point of the base.

In the above figure, XO is the axis of the pyramid XABCDE.

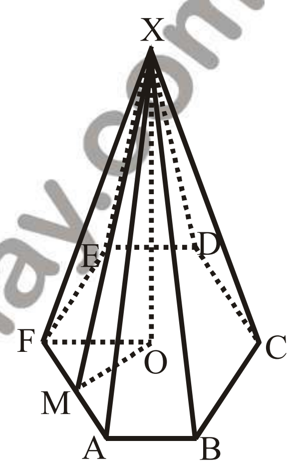
Lateral Edges-

The edges through the vertex of a pyramid are known as its lateral edges.

Slant Height-

The slant height of a regular right-pyramid is the line-segment joining the vertex to the mid-point of anyone of the sides of the base.

The figure given below shows a right regular pyramid, in which O is the centre of the base and XM is the slant height.



Also, in right-angled triangle XOM, we have $XM^2 = OM^2 + XO^2$ (By pythagoras theorem)

$\therefore\ XM = \sqrt{XO^2 + OM^2}$

Right Pyramid-

A pyramid is said to be right pyramid of the perpendicular dropped from the vertex on the base meets the base at its central point, ie the centre of the inscribed or cirumscribed circle. In other words, the vertex of the pyramid lies on the perpendicular to the base drawn through its centre. Otherwise, the pyramid is called an oblique pyramid.

The pyramid shown in the above figue I is an oblique pyramid whereas figure II given above shows a right pyramid.

Regular Pyramid-

A pyramid is said to be a regular if its base is a regular figure. ie all sides of its base are equal.

In case of a right regular pyramid the lateral edges are equal and the lateral faces are congruent triangles.

Volume and Surface Area of a Pyramid-

(i) Volume of a pyramid $= \frac{1}{3} \times$ Area of the Base $\times$ Height

(ii) Lateral surface area of a pyramid

= Sum of areas of all the lateral triangular faces.

L.S.A. $= \frac{1}{2} \times$ perimeter of the base $\times$ slant height.

(iii) Total surface area of a pyramid

= Sum of areas of all lateral faces + Area of the base.

T.S.A. $= \frac{1}{2} \times$ perimeter of the base $\times$ slant height + Area of the base.

For a right pyramid with an equilateral triangle of side 'a' as base and height 'h'.

(i) Lateral edge or Lateral height $= \sqrt{h^2 + \frac{a^2}{3}}$

(ii) Slant height $= \sqrt{h^2 + \frac{a^2}{12}}$

(iii) Lateral surface area

$= \frac{1}{2}$ (Perimeter of the base $\times$ slant height)

(iv) Total surface area $= \frac{1}{2}$ (Perimeter of the base $\times$ slant height) $+ \frac{\sqrt{3}}{4} a^2$

(v) Volume $= \frac{1}{3} \times \frac{\sqrt{3}}{4} \times a^2 \times h = \frac{\sqrt{3}}{12} a^2 h.$

(vi) Area of lateral face $= \frac{1}{2}$ (Length of an edge of the base $\times$ Slant height)

6. Tetrahedron and Regular Tetrahedron

A tetrahedron is a pyramid whose base is a triangle. It has six edges and four triangular faces.

A tetrahedron whose all the edges are of equal length is called a regular tetrahedron. In a regular tetrahedron all the four faces are congruent equilateral triangles.

When the length of each edge of a regular tetrahedron is given 'a' and 'h' is height.

(i) Height of the regural tetrahedron $(h) = \sqrt{\frac{2}{3}} \times a$

(ii) Slant height of the regular tetrahedron $= \frac{\sqrt{3}}{2} \times a.$

(iii) Volume of the regular tetrahedron

$= \frac{\sqrt{2}}{12} \times a^3 = \frac{\sqrt{3}}{8} h^3 \quad \left(\because a = \frac{\sqrt{3}}{2} h\right)$

(iv) Lateral surface area of the regular tetrahedron

$= \frac{3\sqrt{3}}{4} a^2 = \frac{3\sqrt{3}}{2} h^2$

(v) Total surface area of the regular tetrahedron

$= 4 \times \frac{\sqrt{3}}{4} a^2 = \sqrt{3} \times a^2.$

Some Examples

Q. 1 Find the volume of the right pyramid the area of whose base is 60 cm^2 and height 15 cm.

Sol. Volume of a right pyramid.

$= \frac{1}{3}$ (Area of the base) $\times$ (Height)

$\therefore$ Volume of the given pyramid

$= \left(\frac{1}{3} \times 60 \times 15\right) = 300$ cm^3

Q.2 Find the height of the right pyramid whose volume is 750 cm^3 and area of whose base is 250 cm^2.

Sol. Volume of a right pyramid

$= \frac{1}{3}$ (Area of the base × Height)

Let h be the height of the given right pyramid. Then,

$750 = \frac{1}{3} \times 250 \times h$

$h = \frac{750 \times 3}{250} = 9$ cm

Q. 3. A right pyramid has its base as an equilateral triangle of side 40 cm and its height is $24\sqrt{3}$ cm. Find the volume of the pyramid.

Sol. Volume of the pyramid $= \left(\frac{\sqrt{3}}{12} a^2 \times h\right)$

Here, $a = 40$ $h = 24\sqrt{3}$

$\therefore$ Volume of the pyramid $= \left(\frac{\sqrt{3}}{12} \times 40^2 \times 24\sqrt{3}\right)$

$= 9600 \text{ cm}^3$

Q. 4 Find the volume of a regular tetrahedron whose each edge is of $12\sqrt{2}$ cm.

Sol. Volume of a regular tetrahedron $= \frac{\sqrt{2}}{12}$ (edge)^3

$\therefore$ volume of the given tetrahedron $= \frac{\sqrt{2}}{12} \times \left(12\sqrt{2}\right)^3$

$= \frac{\sqrt{2}}{12} \times 1728 \times 2\sqrt{2} = 576 \text{ cm}^3$

Q. 5 Find the lateral surface area and total surface area of a right pyramid in which the base is an equilateral triangle of area $16\sqrt{3}$ cm^3 and length of each lateral edge is 5 cm.

Sol. Let the length of each side of the base is a and h is the height of the pyramid.

Area of the base $= 16\sqrt{3}$ cm^2

$\frac{\sqrt{3}}{4} a^2 = 16\sqrt{3}$

$a^2 = 64$

$\therefore$ $a = 8$ cm

Lateral edge = 5 cm

$\sqrt{h^2 + \frac{a^2}{3}} = 5$ or, $\sqrt{h^2 + \frac{64}{3}} = 5$

$h^2 + \frac{64}{3} = 25 \Rightarrow h^2 = \frac{75-64}{3}$

$h = \sqrt{\frac{11}{3}}$ cm

$\therefore$ Slant height $= \sqrt{h^2 + \frac{a^2}{12}} = \sqrt{\frac{11}{3} + \frac{64}{12}} = \sqrt{9} = 3\text{cm}$

Lateral Surface Area $= \frac{1}{2}$ (Perimeter of the base × Slant height)

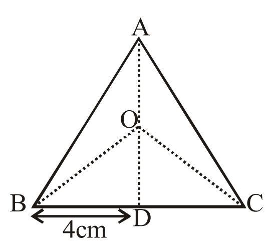
$= \frac{1}{2}(8 + 8 + 8) \times 3 = 36 \text{ cm}^2$

Total surface area

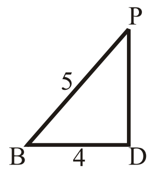
= Lateral surface area + Area of the base

$= \left(36 + 16\sqrt{3}\right)$ cm^2

Method: 2



let base of prism is a triangle of vertics A,B,C. O is a point on the base at which height is standing and P is top of height.



If we can feel the above triangle. Then

PD=$\sqrt{5^2-4^2}$ = 3cm= slant height

Q. 6 If *'a'* be the length of the perpendicular drawn from a vertex of a regular tetrahedron to its opposite face and each edge of length $2b$, show that $3a^2 = 8b^2$.

Sol. a = height of the tetrahedron

$= \sqrt{\frac{2}{3}}$ × (Length of an edge)

$a = \sqrt{\frac{2}{3}} \times 2b \Rightarrow a^2 = \frac{8b^2}{3}$

$3a^2 = 8b^2$

Q. 7 Find the volume of a tetrahedron the sides of whose base are 9 cm, 12 cm and 15 cm and height 15 cm.

Sol. $a = 9$ cm, $b = 12$ cm, $c = 15$ cm.

$s = \frac{a+b+c}{2} = \frac{9+12+15}{2} = 18$

∴ Area of the base = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{18(18-9)(18-12)(18-15)}$

$= \sqrt{18 \times 9 \times 6 \times 3} = 54 \text{ cm}^2$

Volume of the tetrahedron= $\frac{1}{3}$ (Area of the base × Height)

$= \frac{1}{3} \times 54 \times 15 = 270 \text{ cm}^3$

Q. 8 A right pyramid stands on an equilateral triangular base of area $16\sqrt{3}$ cm^2. If the area of one of its lateral faces is 40 cm^2, find the volume of the pyramid.

Sol. Let the length of each side of the base be a cm.

Area of the base = $16\sqrt{3}$ cm^2

$\frac{\sqrt{3}}{4}(a^2) = 16\sqrt{3} \Rightarrow a^2 = 64$

$\therefore a = 8$ cm

Let h be the height of the pyramid and l be its slant height.

$l = \sqrt{h^2 + \frac{a^2}{12}}$

$l^2 = h^2 + \frac{a^2}{12} \Rightarrow l^2 = h^2 + \frac{64}{12}$

$l^2 = h^2 + \frac{64}{12}$ …(i)

Area of one lateral surface = 40 cm^2

$\frac{1}{2}(a \times l) = 40$

$a \times l = 80 \Rightarrow 8 \times l = 80$

$\therefore l = 10$

Put $l = 10$ in eq^n (i)

$100 = h^2 + \frac{16}{3} \Rightarrow h^2 = 100 - \frac{16}{3}$

$\therefore h = \sqrt{\frac{284}{3}}$

Volume of the pyramid= $\frac{1}{3}$ (Area of the base × Height)

$= \frac{1}{3} \times 16\sqrt{3} \times \sqrt{\frac{284}{3}} = \frac{16}{3}\sqrt{284}$ cm^3

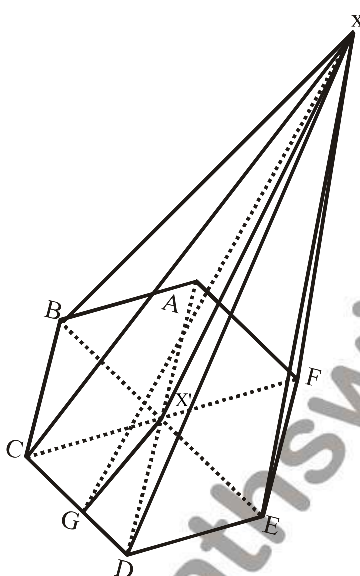
Q.9 Area of a regular hexagon is $216\sqrt{3}$ cm². A pyramid of the height 6 cm is formed upon the regular hexagon. Find the slant surface area of the pyramid.

Sol. Let each side of the regular hexagon be *a* units.

$\therefore$ Area of the regular hexagon = $\dfrac{3\sqrt{3}}{2}a^2$

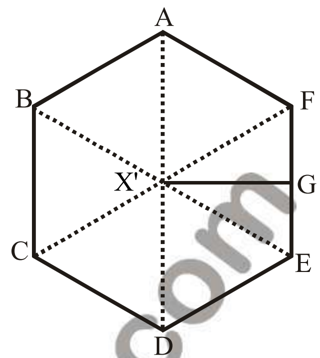
$\dfrac{3\sqrt{3}}{2}a^2 = 216\sqrt{3} \Rightarrow a^2 = 144$

$\therefore a = 12$ cm



Slant surface area = $\dfrac{1}{2}$ (perimeter of base) × slant height

XG is the line joining the mid-point of any side of regular hexagon to point X of the pyramid, i.e. slant height of pyramid.



Let X' be the centre of regular hexagon.

$\angle XX'G = 90°$ and XX'G is the right-angled triangle.

$\therefore XG = \sqrt{(XX')^2 + (X'G)^2}$

XX' is given, now we have to find X'G. Six equal triangles can be drawn in a regular hexagon and area of each triangle is

$= \left(\dfrac{1}{6} \times 216\sqrt{3}\right) = 36\sqrt{3}$ cm²

$\because$ each side of regular hexagon = 12 cm

In Δ X'EF,

$\dfrac{1}{2} \times EF \times X'G = 36\sqrt{3}$

$\dfrac{1}{2} \times 12 \times X'G = 36\sqrt{3}$

$X'G = 6\sqrt{3}$ cm

Slant height of the pyramid = $\sqrt{(XX')^2 + (X'G)^2}$

$= \sqrt{(6)^2 + (6\sqrt{3})^2} = \sqrt{144} = 12$ cm

Slant surface area = $\dfrac{1}{2}$ × perimeter of base × slant height

$= \dfrac{1}{2} \times (12 \times 6) \times 12 = 432$ cm²

Q. 10 Find the lateral surface area, total surface area and volume of a right pyramid with equilateral triangle as a base in which the length of each side of the base is 4 cm and slant height is 5 cm.]

Sol. length of each side of the base $a = 4$ cm and slant height = 5 cm.

Let h be the height of the pyramid,

$$\text{Slant height} = \sqrt{h^2 + \frac{a^2}{12}}$$

$$5 = \sqrt{h^2 + \frac{16}{12}} \Rightarrow 25 = h^2 + \frac{4}{3}$$

$$h^2 = 25 - \frac{4}{3} = \frac{71}{3} \Rightarrow h = \sqrt{\frac{71}{3}} \text{ cm}$$

$$\text{lateral surface area} = \frac{1}{2}\,(\text{Perimeter of the base} \times \text{Slant height})$$

$$= \frac{1}{2}\,(4+4+4) \times 5 = 30 \text{ cm}^2$$

Total surface area

= Lateral surface area + area of the base

$$= \left(30 + \frac{\sqrt{3}}{4} \times 4^2\right)$$

$$= (30 + 4\sqrt{3}) \text{ cm}^2$$

$$\text{Volume of the pyramid} = \frac{1}{3}\,(\text{Area of the base} \times \text{height})$$

$$= \frac{1}{3} \times \left(\frac{\sqrt{3}}{4} \times 4^2\right) \times \sqrt{\frac{71}{3}}$$

$$= \frac{4}{3}\sqrt{71} \text{ cm}^3$$

Q. 11 Find the volume, lateral surface area and total surface area of a right triangular pyramid the lenght of whose edge is 12 cm.

Sol. The pyramid is a tetrahedron whose edge is of length 12 cm.

$$\text{Volume of the pyramid} = \frac{\sqrt{2}}{12} \times (\text{edge})^2$$

$$= \frac{\sqrt{2}}{12} \times (12)^3 = 144\sqrt{2} \text{ cm}^3$$

$$\text{Lateral surface area} = \frac{3\sqrt{3}}{4} \times (\text{edge})^2$$

$$= \frac{3\sqrt{3}}{4} \times (12)^2 = 108\sqrt{3} \text{ cm}^2$$

$$\text{Total surface area} = \sqrt{3} \times (\text{edge})^2$$

$$= \sqrt{3} \times (12)^2 = 144\sqrt{3} \text{ cm}^2$$

Q.12 The base of a right pyramid is an equilateral triangle of side 10 cm and its vertical height is 5 cm find its slant height and area of one side face.

Sol. lenght of each side of the base $a = 10$ cm.

height of the pyramid $h = 5$ cm

$$\therefore \text{Slant height} = \sqrt{h^2 + \frac{a^2}{12}} = \sqrt{25 + \frac{100}{12}}$$

$$= \sqrt{25 + \frac{25}{3}} = \frac{10}{\sqrt{3}} \text{ cm}$$

$$\text{Lateral surface area} = \frac{1}{2}\,(\text{Perimeter of the base} \times \text{Slant height})$$

$$= \frac{1}{2}\,(10+10+10) \times \frac{10}{\sqrt{3}} = \frac{150}{\sqrt{3}} \text{ cm}^2$$

$$\therefore \text{Area of one side face} = \frac{1}{3} \times (\text{Lateral surface area})$$

$$= \frac{1}{3} \times \frac{150}{\sqrt{3}} = \frac{50}{\sqrt{3}} \text{ cm}^2$$

Q.13 The base of a right pyramid is an equilateral triangle each side of which is 2 cm long. Every slant edge is 3 cm long. Find the lateral surface area and the volume of the pyramid.

Sol. $a = 2$ cm and salnt edge = 3 cm.

Let h be the height of the pyramid

$$\text{Slant edge} = \sqrt{h^2 + \frac{a^2}{3}}$$

$$3 = \sqrt{h^2 + \frac{4}{3}} \Rightarrow 9 = h^2 + \frac{4}{3}$$

$$h^2 = 9 - \frac{4}{3} = \frac{23}{3} \Rightarrow h = \sqrt{\frac{23}{3}} \text{ cm}$$

$$\therefore \text{Slant height} = \sqrt{h^2 + \frac{a^2}{12}}$$

$$= \sqrt{\frac{23}{3} + \frac{4}{12}} = \sqrt{8} = 2\sqrt{2} \text{ cm}$$

Lateral surface area = $\frac{1}{2}$ (Perimeter of the base × Slant height)

$$= \frac{1}{2}(2+2+2) \times 2\sqrt{2} = 6\sqrt{2} \text{ cm}^2$$

Volume of the pyramid = $\frac{1}{3}$ (Area of the base × height)

$$= \frac{1}{3} \times \frac{\sqrt{3}}{4} \times (2)^2 \times \frac{\sqrt{23}}{3} = \frac{\sqrt{23}}{3} \text{ cm}^3$$

Q.14 The base of a right pyramid is an equilateral triangle of side 4 cm. The height of the pyramid is half of its slant height. Find the volume and the length of a slant edge of the pyramid.

Sol. length of each side of the base $a = 4$ m.

Let h be the height of the pyramid and l be slant height.

$$h = \frac{l}{2}$$

$$h = \frac{1}{2}\sqrt{h^2 + \frac{a^2}{12}} \Rightarrow 4h^2 = h^2 + \frac{a^2}{12}$$

$$3h^2 = \frac{a^2}{12} \Rightarrow 3h^2 = \frac{16}{12} \; [\because a = 4 \text{ m}]$$

$$h^2 = \frac{4}{9} \Rightarrow h = \frac{2}{3} \text{ m}$$

$$\therefore l = \frac{4}{3} \; [\because l = 2h]$$

$$\text{Lateral edge} = \sqrt{h^2 + \frac{a^2}{3}} = \sqrt{\frac{4}{9} + \frac{16}{3}}$$

$$= \sqrt{\frac{52}{9}} = \frac{2\sqrt{13}}{3} \text{ m}$$

Volume of the pyramid= $\frac{1}{3}$ (Area of the base × Height)

$$= \frac{1}{3} \times \frac{\sqrt{3}}{4} \times (4)^2 \times \frac{2}{3} = \frac{8\sqrt{3}}{9} \text{ m}^3$$

☆☆☆☆☆

Exercise-Pyramid

1. The volume of the pyramid on a square base of side 15 cm and height 10 cm is
(a) 750 cm^3 (b) 700 cm^3
(c) 2250 cm^3 (d) 1125 cm^3

2. The volume of a pyramid whose base is an equilateral trianlge is 12 cm^3. If the height of the pyramid is $3\sqrt{3}$ cm metres, then each side of the base is
(a) 2 cm (b) 3 cm
(c) 4 cm (d) 6 cm

3. If the base of a right pyranmid is a triangle with sides 5 cm. 12 cm and 13 cm respectively and the volume of the pyramid is 240 cubic cm, then the height of the pyramid is
(a) 36 cm (b) 18 cm
(c) 24 cm (d) 72 cm

4. If a regular square pyramid has a base of side 8 cm and height 45 cm, then its volume is
(a) 480 cm^3 (b) 900 cm^3
(c) 640 cm^3 (d) 960 cm^3

5. The base of a pyramid is an equilateral triangle of side 1 cm. If the height of the pyramid is 4 cm, then the volume is
(a) 0.550 cm^3 (b) 0.577 cm^3
(c) 0.678 cm^3 (d) 0.750 cm^3

6. A right pyramid is on a regular hexagonal base. Each side of the base is 10 m and the height is 30 m . The volume of the pyramid is
(a) 2500 m^3 (b) 2550 m^3
(c) 2598 m^3 (d) 5196 m^3

7. There is a pyramid on a base which is a regular hexagon of side $2a$. If every slant edge of this pyramid is of length $\frac{5a}{2}$, then the volume of this pyramid is
(a) $3a^3$ (b) $3a^3\sqrt{2}$
(c) $3a^3\sqrt{3}$ (d) $6a^3$

8. The area of the square base of a right pyramid is 36 cm^2. If the area of each triangle forming the slant surface is 15 cm^3, then the volume of the pyramid is
(a) 64 cm^3 (b) 48 cm^3
(c) 24 cm^3 (d) 36 cm^3

9. If the area of the base of a regular hexagonal pyramid is $96\sqrt{3}$ m^2 and the area of one of its side faces is $32\sqrt{3}$ m^2, then the volume of the pyramid is :
(a) $380\sqrt{3}$ m^3 (b) $382\sqrt{3}$ m^3
(c) $384\sqrt{3}$ m^3 (d) $386\sqrt{3}$ m^3

10. A right pyramid has an equilateral triangular base of side 4 cm. If the numerical value of its total surface area is three times the numerical value of of its volume, then height is
(a) 8 cm (b) 6 cm
(c) 10 cm (d) 12 cm

11. The base of a right pyramid is a square of side 40 cm long. If the volume of the pyramid is 8000 cm^3, then its height is:
(a) 5 cm (b) 10 cm
(c) 15 cm (d) 20 cm

12. The base of a right pyramid is a square of side 16 cm long. If its height be 15 cm, then the area of the lateral surface in square centimetre is:
(a) 136 (b) 544
(c) 800 (d) 1280

13. The base of a right pyramid is an equilateral triangle of side $10\sqrt{3}$ cm. If the total surface area of the pyramid is $270\sqrt{3}$ sq. cm., its height is
(a) $12\sqrt{3}$ cm (b) 10 cm
(c) $10\sqrt{3}$ cm (d) 12 cm

14. If the base of a right pyramid is triangle of sides 5 cm, 12 cm, 13 cm and its volume is 330 cm^3, then its height (in cm) will be

(a) 33 (b) 32
(c) 11 (d) 22

15. A right pyramid stands on a square base of diagonal $10\sqrt{2}$ cm. If the height of the pyramid is 12 cm, the area (in cm^2) of its slant surface is

(a) 520 (b) 420
(c) 360 (d) 260

16. The length of each edge of a regular tetrahedron is 12 cm. The area (in sq. cm) of the total surface of the tetrahedron is

(a) $288\sqrt{3}$ (b) $144\sqrt{2}$
(c) $108\sqrt{3}$ (d) $144\sqrt{3}$

17. If the base of a right pyramid is triangle of sides 5 cm, 12 cm, 13 cm and its volume is 330 cm^3, then its height (in cm) will be

(a) 33 (b) 32
(c) 11 (d) 22

18. The base of right pyramid is a equilateral triangle of side 4 cm. The height of the pyramid is half of its slant height. Its volume is

(a) $\frac{8}{9}\sqrt{2}cm^2$ (b) $\frac{7}{9}\sqrt{3}cm^2$
(c) $\frac{8}{9}\sqrt{3}cm^2$ (d) $\frac{7}{9}\sqrt{2}cm^2$

19. Each edge of the regular tetrahedron is 3 cm.then its volume is

(a) $\frac{9\sqrt{2}}{4}$ c.c (b) $\frac{4\sqrt{2}}{9}$ c.c
(c) $9\sqrt{3}$ c.c (d) $27\sqrt{3}$ c.c

20. A pyramid on a square base has four equilateral triangles on its four other faces, each edges being 10m. Find its volume.

(a) 235.7 m^3 (b) 288.7 m^3
(c) 532.7 m^3 (d) 352.7 m^3

21. A right pyramid stands on a rectangular base 32 cm long and 10 cm in width. If the height of the pyramid is 12 cm. Find its whole surface area.

(a) 933 cm^3 (b) 936 cm^3
(c) 934 cm^3 (d) 935 cm^2

Answer

1.	(a)	2.	(c)	3.	(c)	4.	(d)	5.	(b)	6.	(c)	7.	(c)	8.	(b)	9.	(c)
10.	(a)	11.	(c)	12.	(b)	13.	(d)	14.	(a)	15.	(d)	16.	(d)	17.	(a)	18.	(c)
19.	(a)	20.	(b)	21.	(b)												

Solution & Hints

Sol 1. Area of base = 15 × 15 = 225 cm²

Height = 10 cm

Volume of a pyramid = $\frac{1}{3}$ × Area of the base × Height

$= \frac{1}{3} \times 15 \times 15 \times 10$

$= 750\ \text{cm}^3$

Sol 2. Let a = side of the equilateral triangle

∴ Area of the equilateral triangle = $\frac{\sqrt{3}a^2}{4}$

Now, according to the question,

$\frac{1}{3}\left(\frac{\sqrt{3}}{4}a^2\right) 3\sqrt{3} = 12$

$a^2 = 16$

$\Rightarrow a = 4\ \text{cm}$

Sol 3. Volume of a pyramid

$= \frac{1}{3}$ × Area of the base × Height

Since, $5^2 + 12^2 = 13^2$, the base of the pyramid is right triangle , Now, let the height be h,
Now, according to the question,

$\Rightarrow 240 = \frac{1}{3} \times \left(\frac{1}{2} \times 5 \times 12\right) \times h$

$\Rightarrow h = 24$ cm.

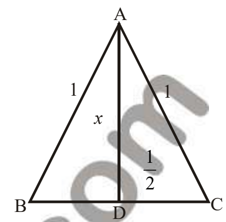
Sol 4. Volume of a pyramid = $\frac{1}{3}$ × Area of the base × Height

$= \frac{1}{3} \times 8 \times 8 \times 45$

$= 960\ \text{cm}^3$

Sol 5. Volume of the pyramid

$= \frac{1}{3}$ × Area of the base × Height



$x^2 = (1)^2 - \left(\frac{1}{2}\right)^2 = \frac{3}{4}$

$\Rightarrow x = \frac{\sqrt{3}}{2}$

∴ Area of Δ ABC = $\frac{\sqrt{3}}{4}$

Volume of the pyramid

$= \frac{1}{3} \times \frac{\sqrt{3}}{4} \times 4 = \frac{\sqrt{3}}{3}$

$= \frac{1.732}{3}$

$= 0.577\ \text{cm}^3$

Sol 6. Volume of a pyramid

$= \frac{1}{3}$ × Area of the base × Height

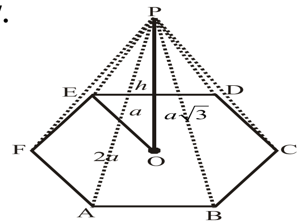
$= \frac{1}{3} \times \frac{3\sqrt{3}}{2} \times (10)^2 \times 30$

$= 2598\ \text{m}^3$

[∵ Area of the regular hexagon of side

$a = \frac{3\sqrt{3}}{2} a^2$ sq units

Sol 7.



$AB = BC = CD = EF = FA = 2a$

$PE = \dfrac{5a}{2}$ and $OE = 2a$

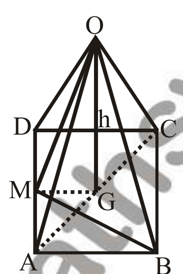
$\therefore\ h = OP = \sqrt{\left(\dfrac{5a}{2}\right)^2 - 4a^2} = \dfrac{3a}{2}$

Volume of the pyramid $= \dfrac{1}{3} \times$ Area of the base $\times$ Height

$= \dfrac{1}{3} \times \dfrac{3a}{2} \times 6 \times \dfrac{1}{2} \times 2a \times a\sqrt{3} = 3a^3\sqrt{3}$

Sol 8. Side of the square base $= \sqrt{36} = 6$ cm

Let G be the centroid of ABCD and M be the mid point of AD.



Slant height $OM = \sqrt{OG^2 + MG^2} = \sqrt{h^2 + 3^2}$

$\therefore$ Area of triangle forming the slant surface

$= \dfrac{1}{2} \times 6 \times \sqrt{h^2 + 9}$

$= 3\sqrt{h^2 + 9}$

Given : $3\sqrt{h^2 + 9} = 15$

$\Rightarrow h = 4$ cm

$\therefore$ Volume of the pyramid $= \dfrac{1}{3} \times$ Area of the base $\times$ Height

$\dfrac{1}{3} \times 36 \times 4 = 48$ cm^3

Sol 9. Area of regular hexagon of side $a = \dfrac{3\sqrt{3}}{2} a^2$

$\Rightarrow \dfrac{3\sqrt{3}}{2} a^2 = 96\sqrt{3} \Rightarrow a = 8$ m

Let h be the height of the pyramid. Then area of one side face of the pyramid $= \dfrac{1}{2} a \times l$, where l is the slant height of the face.

$\Rightarrow \dfrac{1}{2} a \times l = 32\sqrt{3} \Rightarrow l = 8\sqrt{3}$

$\Rightarrow \dfrac{3a^2}{4} + h^2 = l^2 \Rightarrow \dfrac{3 \times 64}{4} + h^2 = 64 \times 3$

$\Rightarrow h^2 = 64 \times 3 \left[1 - \dfrac{1}{4}\right] = 196 \Rightarrow h = 12$ m

$\therefore$ Volume of the pyramid $= \dfrac{1}{3} \times$ Area of the base $\times h$

$= \dfrac{1}{3} \times 96\sqrt{3} \times 12 = 384\sqrt{3}$ m^3

Sol 10. Let a be the length of each side of the base, h be the height and l be the slant height of the pyramid.

$\therefore$ slant height $= \sqrt{h^2 + \dfrac{a^2}{12}}$

$\Rightarrow l = \sqrt{h^2 + \dfrac{16}{12}} \Rightarrow l = \sqrt{h^2 + \dfrac{4}{3}}$

It is given that the numerical value of its total surface area is three times the numerical value of of its volume.

$\therefore$ Lateral surface area + Area of the base = 3 × (Volume)

$$\Rightarrow \frac{1}{2}(4+4+4) \times \sqrt{h^2+\frac{4}{3}} + \sqrt{\frac{3}{4}} \times 4^2$$

$$= 3 \times \frac{1}{3}\left(\frac{\sqrt{3}}{4} \times 4^2 \times h\right)$$

$$\Rightarrow 6\sqrt{h^2+\frac{4}{3}} + 4\sqrt{3} = 4\sqrt{3}\,h$$

$$\Rightarrow 6\sqrt{h^2+\frac{4}{3}} = 4\sqrt{3}\ (h-1)$$

$$\Rightarrow 36\left(h^2+\frac{4}{3}\right) = 48\,(h-1)^2$$

$$\Rightarrow 3\left(h^2+\frac{4}{3}\right) = 4\,(h-1)^2$$

$$\Rightarrow 3h^2 + 4 = 4\,(h^2 - 2h + 1)$$

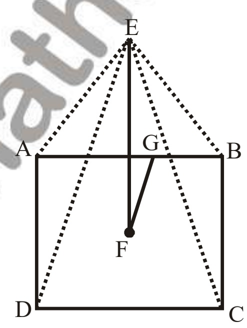
$$\Rightarrow 8h = h^2 \Rightarrow h = 8 \text{ cm}$$

Sol 11. Volume $= \frac{1}{3}$ × area of base × height

$$\Rightarrow 8000 = \frac{1}{3} \times 40 \times 40 \times h$$

$$\Rightarrow h = 15 \text{ cm}$$

Sol 12.



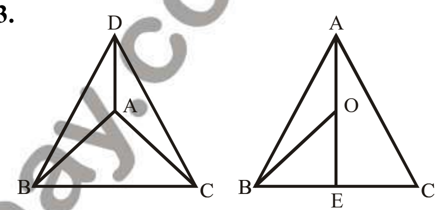
Perimeter of square = $4a = 4 \times 16 = 64$ cm.

$$\text{Slant height} = \sqrt{h^2+\frac{a^2}{4}} = \sqrt{15^2+8^2} = 17 \text{ cm}$$

Lateral surface area $= \frac{1}{2}$ × Perimeter of base × slant height

$$= \frac{1}{2} \times 64 \times 17 = 544 \text{ cm}^2$$

Sol 13.



$AB = 10\sqrt{3}$ cm

$BE = 5\sqrt{3}$

$$AE = \sqrt{\left(10\sqrt{3}\right)^2 - \left(5\sqrt{3}\right)^2} = 15 \text{ cm}$$

$$OE = \frac{1}{3} \times 15 = 5 \text{ cm}$$

If height of pyramid = h cm

Slant height $= \sqrt{h^2+25}$

Total surface area $= \frac{1}{2}$ × area of base × slant height + area of base

$$270\sqrt{3} = \frac{1}{2} \times 30\sqrt{3} \times \sqrt{h^2+25} + \frac{\sqrt{3}}{4} \times \left(10\sqrt{3}\right)^2$$

$h^2 + 25 = 169$

$h = \sqrt{144} = 12$ cm

Sol 14. Area of base $= \frac{1}{2} \times 5 \times 12 = 30 \text{ cm}^2$

$\therefore$ Volume $= \frac{1}{3}$ × area of base × height

$\Rightarrow 330 = 30 \times h \times \frac{1}{3}$

$h = 33$ cm

Sol 15. Side of square base = $\frac{1}{\sqrt{2}} \times 10\sqrt{2} = 10$ cm

slant height = $\sqrt{5^2 + 12^2} = 13$ cm

area of slant surface = $\frac{1}{2}$ × perimeter of base × slant height

$= \frac{1}{2} \times 40 \times 13 = 260$ cm^2

Sol 16. Total surface area of terahedron = $4 \times \frac{\sqrt{3}}{4} a^2$

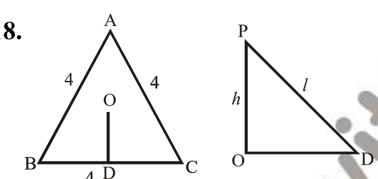
$= \sqrt{3}\,(12)^2 = 144\sqrt{3}$ cm^2

Sol 17. Volume of pyramid = $\frac{1}{3}$ × area of base × height

$330 = \frac{1}{3} \times \frac{1}{2} \times 5 \times 12 \times h$

$h = 33$ cm

Sol 18.



$h = \frac{l}{2}$

$l = 2h$

$OD = \frac{4}{2\sqrt{3}} = \frac{2}{\sqrt{3}}$ (in radius)

In Δ POD

$l^2 = h^2 + OD^2$

$4h^2 = h^2 + \frac{4}{3}$

$3h^2 = \frac{4}{3} \Rightarrow h = \frac{2}{3}$

Volume = $\frac{1}{3}$ × area of base × height

$= \frac{1}{3} \times \frac{\sqrt{3}}{4} \times (4)^2 \times \frac{2}{3} = \frac{8}{9}\sqrt{3}$ cm^2

Sol 19. Volume of tetrahedron = $\frac{\sqrt{2}}{12} a^3$

($\therefore a$ = side of tetrahedron)

$= \frac{\sqrt{2}}{12} \times 3 \times 3 \times 3$

$= \frac{9\sqrt{2}}{4}$ c.c

Sol 20. Slant edge = $a = 10$ m

slant height = $\frac{a\sqrt{3}}{2} = 5\sqrt{3}$

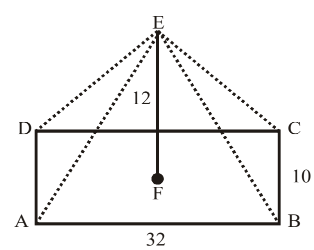
Volume = $\frac{1}{3}$ × area of base × slant height

$= \frac{1}{3} \times (10)^2 \times 5\sqrt{3}$

$= 288.67$m^2

Sol 21. Total surface area of pyramid = lateral surface area + area of base.

slant height for base side AB and CD.



$l_1 = \sqrt{(5)^2 + (12)^2} = 13$ cm

similarly slant height for base side AD and CD

$l_2 = \sqrt{(16)^2 + (12)^2} = 20$ cm.

Area of triangles on side AB and CD

$= 2 \times \frac{1}{2} \times 32 \times 13$

$= 416$ cm^2

Area of triangles on side AD and BC

$= 2 \times \left(\frac{1}{2} \times 20 \times 10\right) = 200$ cm^2

lateral surface area = 416 + 200 = 616 cm^2

area of base = 32 × 10 = 320 cm^2

Total surface area = 616 + 320 = 936 cm^2

Exercise -Miscellaneous

1. From a solid cylinder of height 10 cm and radius of the base 6 cm, a cone of same height and same base is removed. The volume of the remaining solid is :
   (a) 240π cu.cm (b) 5280 cu.cm
   (c) 620π cu.cm (d) 360π cu.cm

2. What part of a ditch, 48 metres long 16.5 metres broad and 4 metres deep can be filled by the sand got by digging a cylindrical tunnel of diameter 4 metres and length 56 metres ? $\left[\text{use } \pi=\frac{22}{7}\right]$
   (a) $\frac{1}{9}$ (b) $\frac{2}{9}$
   (c) $\frac{7}{9}$ (d) $\frac{8}{9}$

3. The size of a rectangular piece of paper is 100 cm × 44 cm. A cylinder is formed by rolling the paper along its length. The volume of the cylinder is (Use $\pi=\frac{22}{7}$)
   (a) 4400 cm^3 (b) 15400 cm^3
   (c) 35000 cm^3 (d) 144 cm^3

4. A cylindrical rod of iron whose height is eight times its radius is melted and cast into spherical balls each of half the radius of the cylinder. The number of such spherical balls is
   (a) 12 (b) 16
   (c) 24 (d) 48

5. A circus tent is cylindrical up to a height of 3 m and conical above it. If its diameter is 105 m and the slant height of the conical part is 63 m, then the total area of the canvas required to make the tent is (take $\pi = \frac{22}{7}$)
   (a) 11385 m^2 (b) 10395 m^2
   (c) 9900 m^2 (d) 990 m^2

6. Water flows at the rate of 10 metres per minute from a cylindrical pipe 5 mm in diameter. How long it lake to fill up a conical vessel whose diameter at the base is 30 cm and depth 24 cm?
   (a) 28 minutes 48 seconds
   (b) 51 minutes 12 seconds
   (c) 51 minutes 24 seconds
   (d) 28 minutes 36 seconds

7. A rectangular sheet of iron foil is 22 cm long and 8 cm wide. A cylinder is made out of it by rolling the foil once along the width. Find the volume of the cylinder.
   (a) 208 cm^3 (b) 308 cm^3
   (c) 408 cm^3 (d) 288 cm^3

8. A rectangular sheet with dimensions 44 m × 10 m is rolled into a cylinder so that the smaller side becomes the height of the cylinder. What is the volume of the cylinder so formed?
   (a) 3500 m^3 (b) 1540 m^3
   (c) 3800 m^3. (d) 4400 m^3

9. A spherical copper ball, whose diameter is 9 cm. is melted and converted into a wire having diameter equal to 2 mm. Find the length of the wire.
   (a) 1512 m (b) 1152 m
   (c) 2512 m (d) 121.5 m

10. Some portion of a cylinderical pot is filled with water. Radius of base of the pot is 6 cm. A sphere of radius 3 cm. is dropped. If the sphere completely submerges in the water, how much water level is increased?
    (a) 5 cm. (b) 8 cm.
    (c) 1 cm. (d) 12 cm.

11. Two rectangular sheets of paper each 30 cm × 18 cm are made into two right circular cylinders, one by rolling the paper along its length and the other along the breadth. The ratio of the volumes of the two cylinders thus formed, is: Δ
    (a) 2 : 1 (b) 3 : 2
    (c) 4 : 3 (d) 5 : 3

12. If a cubic cm of cast iron weights 21 gms, then the weight of a cast iron pipe of length 1 m with a bore of 3 cm and in which the thickness of the metal is 1 cm, is:

(a) 46.2 kg (b) 24.2 kg
(c) 26.4 kg (d) 18.6 kg

13. A circus tent is cylindrical up to a height of 3 m and conical above it. If its diameter is 105 m and the slant height of the conical part is 63 m, then the total area of the canvas required to make the tent is (Take $\pi = \frac{22}{7}$)

(a) 11385 m^2 (b) 10395 m^2
(c) 9900 m^2 (d) 990 m^2

14. From a right circular cylinder of radius 10 cm and height 21 cm, a right circular cone of same base radius is removed. If the volume of the remaining portion is 4400 cm^3, then the height of the removed cone (Take $\pi = \frac{22}{7}$) is:

(a) 15 cm (b) 18 cm
(c) 21 cm (d) 24 cm

15. A right circular cylinder and a cone have equal base radius and equal heights. If their curved surfaces are in the ratio $8:5$, then the radius of the base to the height are in the ratio:

(a) $2:3$ (b) $4:3$
(c) $3:4$ (d) $3:2$

16. A cylindrical can whose base is horizontal and is of internal radius 3.5 cm contains sufficient water so that when a solid sphere is placed inside, water just covers the sphere, the sphere fits in the can exactly. The depth of water in the can before the sphere was put is:

(a) $\frac{35}{3}$ cm (b) $\frac{17}{3}$ cm
(c) $\frac{7}{3}$ cm (d) $\frac{14}{3}$ cm

17. A slab of ice 8 inches in length, 11 inches in breadth, and 2 inches thick was melted and resolidified in the form of a rod of 8 inches diameter. The length of such a rod, in inches, is nearest to.

(a) 3 (b) 3.5
(c) 4 (b) 4.5

18. A cylindrical tub of radius 12 cm contains water up to a depth of 20 cm. A spherical iron ball is dropped into the tub and thus the level of water is raised by 6.75 cm. The radius of the the ball is

(a) 7.25 cm (b) 6 cm
(c) 4.5 cm (d) 9 cm

19. A storage tank consists of a circular cylinder with a hemisphere adjoined on either side. If the external diameter of the cylinder be 14 m and its length be 50 m, then what will be the cost of painting it at the rate of Rs. 10 per sq m?

(a) Rs. 38160 (b) Rs. 28160
(c) Rs. 39160
(d) None of these

20. A cylindrical bucket of height 36 cm and radius 21 cm is filled with sand. The bucket is emptied on the ground and conical heap of sand is formed the height of the conical is 12 cm. The radius of the heap at the base is

(a) 63 cm (b) 53 cm
(c) 56 cm (d) 66 cm

21. A solid cylinder and a solid cone have equal base and equal height. If the radius and the height be in the ratio of 4 : 3, the ratio of the total surface area of the cylinder to that of the cone is in the ratio of

(a) $10:9$ (b) $11:9$
(c) $12:9$ (d) $14:9$

22. A cylindrical vessel of radius 4 cm contains water. A solid sphere of radius 3 cm is lowered into the water until it is completely immersed. The water level in the vessel will rise by

(a) $\frac{9}{2}$ cm (b) $\frac{9}{4}$ cm
(c) $\frac{4}{9}$ cm (d) $\frac{2}{9}$ cm

23. A cylinder is circumscribed about a hemisphere and a cone is inscribed in the cylinder so as to have its vertex at the centre of the one end and other end as its base. The volume of the cylinder, hemisphere and the cone are respectively in the ratio

(a) $2:3:2$ (b) $3:2:1$
(c) $3:1:2$ (d) $1:2:3$

24. The diameter of the iron ball used for the shot-put game is 14 cm. It is melted and then a solid cylinder of height $2\frac{1}{3}$ cm is made. What will be the diameter of the base of the cylinder?

(a) 14 cm (b) 28 cm

(c) $\frac{14}{3}$ cm (d) $\frac{28}{3}$ cm

25. If the area of the circular shell having inner and outer radii of 8 cm and 12 cm respectively is equal to the total surface area of cylinder of radius R_1 and height h, then h, in terms of R_1 will be

(a) $\frac{3R_1^2 - 30}{7R_1}$ (b) $\frac{R_1^2 - 40}{R_1^2}$

(c) $\frac{30 - R_1}{R_1^2}$ (d) $\frac{40 - R_1^2}{R_1}$

26. Water is filled at the rate of 5 cm/hr through a pipe of diameter 14 cm into a rectangular tank which is 50 m long and 44 m wide. Determine the time in which the level of the water in the tank will rise by 7 cm.

(a) 2.1 hr. (b) 2.0 hr

(c) 2.5 hr (d) 2.2 hr

27. A conical cavity is drilled in a circular cylinder of 15 cm height and 16 cm the base diameter. The height and the base diameter of the cone are same as those of the cylinder. Determine the total surface area of the remaining solid.

(a) $440\pi cm^2$ (b) $215\pi cm^2$

(c) $542\pi cm^2$ (d) $376\pi cm^2$

28. A right cylindrical vessel is full with water. How many right cones having the same diameter and height as that of as that of the right cylinder will be needed to store that water? $\left(\pi = \frac{22}{7}\right)$

(a) 4 (b) 2

(c) 3 (d) 5

29. Some solid metallic right circular cones, each with radius of the base 3 cm and height 4 cm, are melted to form a solid sphere of radius 6 cm. The number of right circular cones is

(a) 12 (b) 24

(c) 48 (d) 6

30. A solid metallic cone of height 10 cm, radius of base 20 cm is melted to make spherical balls each of 4 cm diameter. How many such balls can be made?

(a) 25 (b) 75

(c) 50 (d) 125

31. A solid metallic cone is melted and recast into a solid cylinder of the same base as that of the cone. If the height of the cylinder is 7 cm, the height of the cone was

(a) 20 cm (b) 21 cm

(c) 28 cm (d) 24 cm

32. A conical vessel, whose internal radius is 10 cm and height 48 cm, is full of water. If this water is poured into a cylinderical vessel with internal radius 20 cm, find the height to which the water rises in it. $\left[Use \pi = \frac{22}{7}\right]$

(a) 2 cms (b) 4 cms

(c) 6 cms (d) 8 cms

33. The radius of the base of a cone is 2.1 cm and its height is 8.4 cm. It is melted and recast into a sphere. Find the radius of the sphere.

(a) 2.1 cm (b) 4.2 cm

(c) 3.2 cm (d) 1.1 cm

34. A cylindrical vessel 32 cm high and 18 cm as the radius of the base, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, the radius of its base is :

(a) 12 cm (b) 24 cm
(c) 36 cm (d) 48 cm

35. Two solid right cones of equal heights are of radii r_1 and r_2 are melted and made to form a solid sphere of radius R. Then the height of the cone is:

(a) $\frac{4R^2}{r_1^2 + r_2^2}$ (b) $\frac{4R}{r_1 + r_2}$

(c) $\frac{4R^3}{r_1^2 + r_2^2}$ (d) $\frac{R^2}{r_1^2 + r_2^2}$

36. A solid wooden toy is in the shape of a right circular cone mounted on a hemisphere. If the radius of the hemisphere is 4.2 cm and the total height of the toy is 10.2 cm, find the volume of the wooden by (nearly).

(a) 104 cm^3 (b) 162 cm^3
(c) 427 cm^3 (d) 266 cm^3

37. If a solid cone of volume 27π cm^3 is kept inside a hollow cylinder whose radius and height are that of the cone, then the volume of water needed to fill the empty space is:

(a) 3π cm^3 (b) 18π cm^3
(c) 54π cm^3 (d) 81π cm^3

38. The volumes of a cylinder and a cone are in the ratio 3 : 1. Find their diameters and then compare them when their heights are equal

(a) Diameter of cylinder = 2 times of diameter of cone
(b) Diameter of cylinder = Diameter of cone
(c) Diameter of cylinder > Diameter of cone
(d) Diameter of cylinder < Diameter of cone

39. A metallic sphere of radius 10.5 cm is melted and then recast into small cones each of radius 3.5 cm and height 3 cm. The number of cones thus formed is:

(a) 140 (b) 132
(c) 112 (d) 126

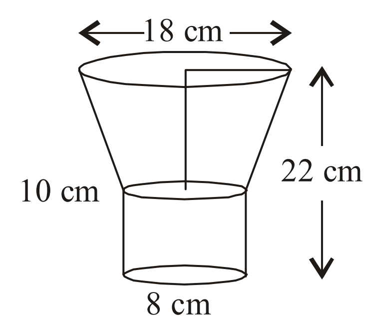
40. A conical vessel of base radius 2 cm and height 3 cm is filled with kerosene. This liquid leaks through a hole in the bottom and collects in a cylindrical jar of radius 2 cm. The kerosene level in the jar is

(a) π cm (b) 1.5 cm
(c) 1 cm (d) 3 cm

41. A conical cavity is drilled in a circular cylinder of 15 m height and 16 cm base diameter. The height and the base diameter of the cone are same as those of the cylinder. Determine the total surface area of the remaining solid.

(a) $440\pi cm^2$ (b) $215\pi cm^2$
(c) $542\pi cm^2$ (d) $376\pi cm^2$

42. An oil funnel made of tin sheet consists of a 10 cm long cylindrical portion attatched to 22 cm, diameter of the cylindrical portion is 8 cm and the diameter of the top of the funnel is 18 cm, find the area of the tin sheet required to make a funnel.



(a) 728.57 cm^3 (b) 782.57 cm^3
(c) 872.57 cm^3 (d) 827.57 cm^3

43. A right circular cone is exactly fitted inside a cube in such a way that the edges of the base of the cone are touching the edges of one of the faces of the cube and the vertex touches the opposite face of the cube. If the volume of the cube is 343cc. What approx is the volume of the cone?

(a) 125 cm^3 (b) 81 cm^3
(c) 90 cm^3 (d) 112.5 cm^3

44 The radius of the base and height of a metallic solid cylinder are r cm and 6cm respectively. It is melted and recast a solid cone of the same radius of box, The height of the cone:

(a) 54cm (b) 27cm
(c) 18cm (d) 9cm

45. A conical flask has base radius a cm and height h cm. It was completely filled with milk. The milk is poured into a cylindrical thermos flask whose base radius is p cm. What will be the height of the solution level in the flask?

(a) $\frac{a^2h}{3p^2}$ cm (b) $\frac{3hp^2}{a^2}$ cm
(c) $\frac{p^2}{3h^2}$ cm (b) $\frac{3a^2}{hP^2}$ cm

46. The ratio of the volume of a cube to that of a sphere which will fit inside the cube is:

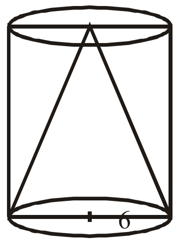
(a) $4 : \pi$ (b) $\pi : 4$
(c) $6 : \pi$ (d) $\pi : 6$

Answer

1.	(a)	**2.**	(b)	**3.**	(b)	**4.**	(d)	**5.**	(a)	**6.**	(a)	**7.**	(b)	**8.**	(b)	**9.**	(d)
10.	(c)	**11.**	(d)	**12.**	(a)	**13.**	(a)	**14.**	(c)	**15.**	(c)	**16.**	(d)	**17.**	(b)	**18.**	(d)
19.	(d)	**20.**	(a)	**21.**	(d)	**22.**	(b)	**23.**	(b)	**24.**	(b)	**25.**	(d)	**26.**	(b)	**27.**	(a)
28.	(c)	**29.**	(b)	**30.**	(d)	**31.**	(b)	**32.**	(b)	**33.**	(a)	**34.**	(c)	**35.**	(c)	**36.**	(d)
37.	(c)	**38.**	(b)	**39.**	(d)	**40.**	(c)	**41.**	(a)	**42.**	(b)	**43.**	(c)	**44.**	(c)	**45.**	(a)
46.	(c)																

Solution & Hints

Sol 1.



Volume of remaining solid = $\pi r^2h - \frac{1}{3}\pi r^2h$

$= \pi r^2h\left(1-\frac{1}{3}\right)$

$= \pi r^2h\ \frac{2}{3}$

$= \pi \times 6 \times 6 \times 10 \times \frac{2}{3}$

$= 240\pi$ cu.m

Sol 2. Volume of tunnel $= \pi r^2h$

$= \frac{22}{7} \times 2 \times 2 \times 56 = 704\ m^3$

Volume of the ditch $= 48 \times 16.5 \times 4 = 3168\ m^3$

Part of the ditch filled $= \frac{704}{3168} = \frac{2}{9}$

Sol 3. Circumference of cylinder = 44

$2\pi r = 44$

$r = 7$ cm.

$h = 100$ cm.

Volume of cylinder $= \pi r^2h = \frac{22}{7} \times 7 \times 7 \times 100$

$= 15400\ cm^3$

Sol 4. Height of cylindrical rod (h) = $8r$

Radius of spherical ball = $\frac{r}{2}$

Number of spherical balls = $\frac{\pi r^2 h}{\frac{4}{3}\pi\left(\frac{r}{2}\right)^3}$

$= \frac{\pi \times r^2 \times 8r}{\frac{4}{3}\times\pi\times\left(\frac{r}{2}\right)^3}$

$= 16 \times 3 = 48$

Sol 5. Height of cylindar = 3 m.

Diameter of cylinder = 105

Radius (r) = $\frac{105}{2}$ m

Slant height of cone (l) = 63

Total surface area of canvas required to make the tent

$= 2\pi rh + \pi rl$

$= \pi r(2h + l)$

$= \pi r(2h + l)$

$= \frac{22}{7} \times \frac{105}{2}(6+63)$

$= \frac{22}{7} \times \frac{5}{2}$mm $\times 69$

$= 11385\ m^2$

Sol 6. Radius of pipe (r) = $\frac{1}{3}$ = $\frac{1}{4}$cm

Radius of cone (R) = 15 cm

vol. passed by pipe = vol of cone

$\pi r^2 \times 1000 \times T = \frac{1}{3}\pi \times 15 \times 15 \times 24$

[$\because$ speed of water = 1000 cm /minute]

$T = \frac{28800}{1000}$ minute = 28 minute 48 sec

Sol 7. Cirumference of cylinder = 22cm

$2\pi r = 22$

$r = 3.5$ cm

$h = 8$ cm

Volume of cylinder $= \pi r^2h$

$= \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10} \times 8$

$= 308\ cm^3$

Sol 8. Cirumference of cylinder = 44

$2\pi r = 44$

$r = 7$ m.

$h = 10$ m.

Volume of cylinder $= \pi r^2h = \frac{22}{7} \times 7 \times 7 \times 10$

$= 1540\ m^3$

Sol 9. Diameter of spherical balls = 9 cm

Radius = 9/2 = 9/200 m

Diameter of wire = 2mm.

Radius = 1mm = 1/1000 m.

$\therefore$ Volume of ball = Volume of Cylinder

$$\frac{4}{3}\pi \times \frac{9}{200} \times \frac{9}{200} \times \frac{9}{200} = \pi \times \frac{1}{1000} \times \frac{1}{1000} \times h$$

$$\frac{243}{2} = h$$

$$\therefore\ h = 121.5 \text{ m}$$

Sol 10. Cylinder radius = 6 cm.

Sphere Radius = 3 cm.

Volume of cylinder = Volume of Sphere

$$\pi\ r^2 h = \frac{4}{3}\pi r^3$$

$$\pi \times 6 \times 6 \times h = \frac{4}{3} \times \pi \times 3 \times 3 \times 3$$

Hence, Level of water increased $h = 1$ cm

Sol 11. **I.** $2\pi r_1 = 30$

Radius of Ist Cylinder $r_1 = \frac{30}{2\pi}$,

Height of Ist Cylinder $h_1 = 18$

II. $2\pi r_2 = 18$

Radius of Ist Cylinder $r_2 = \frac{18}{2\pi}$

Height of Ist Cylinder $h_2 = 30$

Volume of Ist Cylinder = Volume of IInd Cylinder

$$\pi\left(\frac{30}{2\pi}\right)^2 \times 18 = \pi\left(\frac{18}{2\pi}\right)^2 \times 30$$

$\Rightarrow 30 : 18$

$\Rightarrow 5 : 3$

Sol 12. Bore = inner radius = 3 cm

Volume of pipe (hollow cylinder)

$= \pi(4^2 - 3^2) \times 100 = 2200\text{cm}^3$

weight = 2200 × 21 = 46200 gm = 46.2 kg

Sol 13. Same as question no. 5

Sol 14. Volume of remaning portion =

$$\pi r^2 h - \frac{1}{3}\pi r^2 h$$

$$\Rightarrow \frac{2}{3}\pi r^2 h = 4400$$

$$\Rightarrow \frac{2}{3} \times \frac{22}{7} \times 10 \times 10 \times h = 4400$$

Hence, Height of cone $h = 21$ cm

Sol 15. $\frac{\text{C.S.A of cylinder}}{\text{C.S.A of Cone}} = \frac{2\pi rh}{\pi rl} = \frac{8}{5}$

$$\Rightarrow \frac{2\pi rh}{\pi r \times \sqrt{h^2 + r^2}} = \frac{8}{5}$$

$$\Rightarrow \frac{h}{\sqrt{h^2 + r^2}} = \frac{4}{5}$$

Squaring on both side

$$\Rightarrow \frac{h^2}{h^2 + r^2} = \frac{16}{25}$$

$$\Rightarrow \frac{h^2 + r^2}{h^2} = \frac{25}{16}$$

$$\Rightarrow \frac{r^2}{h^2} + 1 = \frac{25}{16}$$

$$\Rightarrow \frac{r^2}{h^2} = \frac{9}{16}$$

$$\Rightarrow \frac{r}{h} = \frac{3}{4}$$

Sol 16. Volume of Cylindrical can = Volume of solid sphere.

$$\pi r^2 h = \frac{4}{3}\pi r^3$$

$$\pi \times 3.5 \times 3.5 \times h = \frac{3}{4} \times \pi \times 3.5 \times 3.5 \times 3.5$$

$$h = \frac{4}{3} \times 3.5$$

Hence, depth of water in the can $h = \frac{14}{3}$ cm.

Sol 17. A.T.Q.

$l \times b \times h = \pi r^2 h_1$

$8 \times 11 \times 2 = \frac{22}{7} \times 4 \times 4 \times h_1$

$h_1 = \frac{7}{2}$

$h_1 = 3.5$

Sol 18. Volume of spherical Ball = Volume of cylinder when depth is 26. 75 cm – Volume of cylinder when depth is 20 cm.

$= \pi r^2 \times 26.75 - \pi r^2 \times 20$

$= \pi r^2 (26.75 - 20)$

$= \pi r^2 (6.75)$

$= \pi \times 12 \times 12 \times 6.75$

But, Volume of spherical ball = $\frac{4}{3} \pi r^3$

$\therefore \frac{4}{3} \pi r^3 = \pi \times 12 \times 12 \times 6.75$

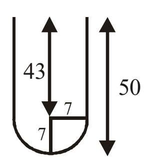
$r^3 = \frac{12 \times 12 \times 6.75 \times 3}{4}$

$r^3 = 729$

$r = 9$ cm

$\therefore$ Radius of spherical ball = 9 cm.

Sol 19. C.S.A. of storage tank = C.S.A. of cylinder + C.S.A. of hemisphere



$= 2\pi rh + 2\pi r^2$

$= 2\pi r(h + r)$

$= 2\pi r(43 + 7)$

$= 2 \times \frac{22}{7} \times 7 \times 50$

$= 2200 \text{ m}^2$

Cost of painting on 1m^2 = Rs.10

Cost of painting of 2200 m^2 = Rs. 22000

Sol 20. Height of bucket = 36 cm

Radius = 21 cm

Height of the heap of sand = 12 cm

Volume of bucket sand = volume of heap of sand

$\Rightarrow \pi r^2 h = \frac{1}{3} \pi R^2 h_2$

$\Rightarrow \pi \times 21 \times 21 \times 36 = \frac{1}{3} \times \pi \times 12 \times R^2$

$\Rightarrow R^2 = 21 \times 21 \times 9$

$\therefore$ Radius of heap $R = 21 \times 3 = 63$ cm

Sol 21. $\frac{r}{h} = \frac{4x}{3x}$

Slant hieght $l = \sqrt{(4x)^2 + (3x)^2}$

$l = 5x$

$\Rightarrow \frac{\text{T.S.A. of cylinder}}{\text{T.S.A. of cone}} = \frac{2\pi rh + 2\pi r^2}{\pi rl + \pi r^2}$

$= \frac{2\pi 3x . 4x + 2\pi (4x)^2}{\pi 3x . 5x + \pi (4x)^2}$

$= \frac{56x^2}{36x^2} = \frac{14}{9}$

$= 14 : 9$

Sol 22. Radius of cylinder $r_1 = 4$ cm

Radius of sphere $r_2 = 3$ cm

A.T.Q.

Volume of cylinder = Volume of sphere

$\pi r_1^2 h = \frac{4}{3} \pi r_2^3$

$4 \times 4 \times h = \frac{4}{3} \times 3 \times 3 \times 3$

Hence, level of water rises by $h = \frac{9}{4}$ cm.

Sol 23. In this case,

Radius of base of all three solid = r

and Height of cone and cylinder will be r.

$$V_1 = \pi r^3,\ V_2 = \frac{2}{3}\pi r^3\ \&\ V_3 = \frac{1}{3}\pi r^3,$$

$V_1 : V_2 : V_3 = 3 : 2 : 1$

Sol 24. Diameter of ball = 14 cm

Radius = 7 cm

Height of solid cylinder = $\frac{7}{3}$ cm

A.T.Q.

Volume of ball = Volume of cylinder

$$\Rightarrow \frac{4}{3}\pi r^3 = \pi r^2 h$$

$$\Rightarrow \frac{4}{3} \times \pi \times 7 \times 7 \times 7 = \pi \times r^2 \times \frac{7}{3}$$

$\Rightarrow r^2 = 49 \times 4$

$\Rightarrow r = 7 \times 2 = 14$ cm

Diameter of the base of the cylinder

$\Rightarrow D = 2r = 2 \times 14 = 28$ cm

Sol 25. Inner radius r = 8 cm

Outer Radius R = 12 cm

Area of Circular shell = $\pi(R^2 - r^2)$

$$= \pi(144 - 64) = 80\pi$$

A.T.Q.

Total surface area of cylinder = area of shell

$\Rightarrow 2\pi R_1(h + R_1) = 80\pi$

$$\Rightarrow h + R_1 = \frac{40}{R_1}$$

$$\Rightarrow h = \frac{40}{R_1} - R_1$$

$$\Rightarrow h = \frac{40 - R_1^2}{R_1}$$

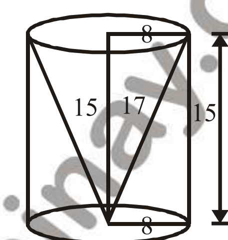
Sol 26. $l \times b \times h$ = Area of cross section × speed × time

$$50 \times 44 \times \frac{7}{100} = \frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} \times 5 \times 1000 \times t$$

t = 2 hours

Sol 27. Slant height of cone (l) = $\sqrt{r^2 + h^2}$

$$\Rightarrow l = \sqrt{15^2 + 8^2}$$



$\Rightarrow l = \sqrt{289}$

$l = 17$

T.S.A. of solid =

Area of base of solid + CSA of cylinder + CSA of cone

$= \pi r^2 + 2\pi rh + \pi rl$

$= \pi r(r + 2h + l)$

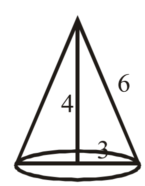
$= \pi r(8 + 30 + 17)$

$= \pi \times 8 \times 55$

$= 440\pi$ cm^2

Sol 28. Number of cones $= \dfrac{\pi r^2 h}{\frac{1}{3}\pi r^2 h} = 3$

Sol 29.

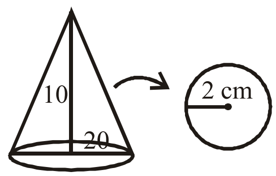


A.T.Q.

$$\Rightarrow \frac{\frac{4}{3}\pi r^3}{\frac{1}{3}\pi r^2 h} = \frac{\frac{4}{3}\pi \times 6 \times 6 \times 6}{\frac{1}{3}\pi \times 3 \times 5 \times 4} = 24$$

Total Number of cones = 24

Sol 30.

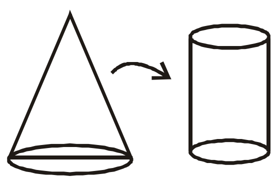


A.T.Q.

$$\Rightarrow \frac{\frac{1}{3}\pi r^2 h}{\frac{4}{3}\pi r^3} = \frac{\frac{1}{3}\pi \times 20 \times 20 \times 10}{\frac{4}{3}\pi \times 2 \times 2 \times 2}$$

Hence, Number of balls = 125

Sol 31.



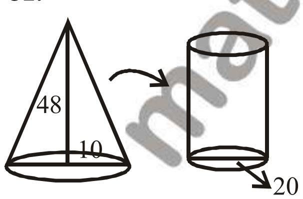
A.T.Q.

$$\frac{1}{3}\pi r^2 h = \pi r^2 h_1$$

$$\frac{1}{3}\pi r^2 h = \pi r^2 \times 7$$

$\therefore$ Height of cone = 21 cm

Sol 32.



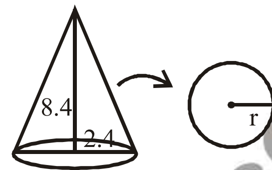
A.T.Q.

$$\Rightarrow \frac{1}{3}\pi r^2 h = \pi r^2 h$$

$$\Rightarrow \frac{1}{3}\pi \times 10 \times 10 \times 48 = \pi \times 20 \times 20 \times h$$

$\Rightarrow$ Hence, level of water rises = 4 cm

Sol 33.

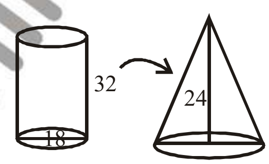


$$\frac{1}{3}\pi r^2 h = \frac{4}{3}\pi r^3$$

$$\frac{1}{3}\pi \times 2.1 \times 2.1 \times 8.4 = \frac{4}{3}\pi r^3$$

$r = 2.1$ cm

Sol 34.



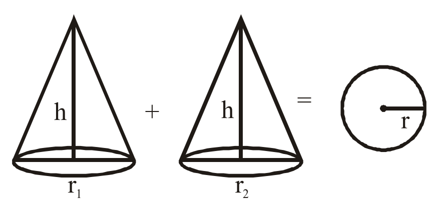
Volume of cylindrical vessel = volume of cone

$$\pi r^2 h = \frac{1}{3}\pi r_1^2 h_1$$

$$\pi \times 18 \times 18 \times 32 = \frac{1}{3}\pi \times r^2 \times 24$$

$r = 36$ cm

Sol 35.

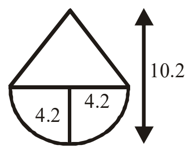


$$\frac{1}{3}\pi r_1^2 h + \frac{1}{3}\pi r_2^2 h = \frac{4}{3}\pi R^3$$

$$\frac{1}{3}\pi h (r_1^2 + r_2^2) = \frac{4}{3}\pi R^3$$

$$h = \frac{4R^3}{r_1^2 + r_2^2}$$

Sol 36.



height of cone

$10.2 - 4.2 = 6$

Volume of woods $= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{\pi r^2}{3}(h + 2r)$

$= \frac{\pi r^2}{3}(6 + 8.4)$

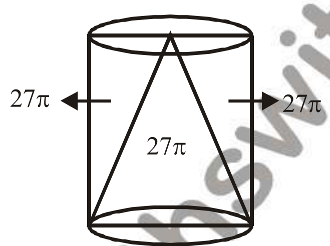
$\Rightarrow \frac{22}{7} \times \frac{4.2 \times 4.2}{3} \times 14.4$

$\Rightarrow 22 \times 0.6 \times 1.4 \times 14.4$

$\Rightarrow 266.112$

$\Rightarrow 266$ (nearly)

Sol 37.



$= 54\pi$

Sol 38. $h_1 = h_2$

$\frac{\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{3}{1}$

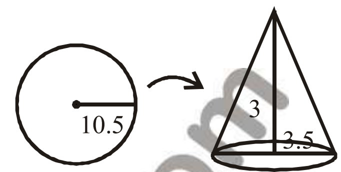
$r_1^2 = r_2^2$

$r_1 = r_2$

$D_1 = D_2$

(b) Diameter of cylendar = diameter of cone

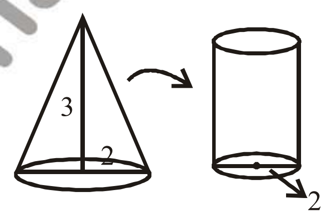
Sol 39.



Number of cone $= \frac{\frac{4}{3}\pi \times 10.5 \times 10.5 \times 10.5}{\frac{1}{3}\pi \times 3.5 \times 3.5 \times 3}$

$= 126$

Sol 40.



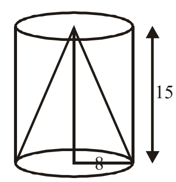
A.T.Q.

$\frac{1}{3}\pi r^2 h = \pi r^2 h$

$\frac{1}{3}\pi \times 2 \times 2 \times 3 = \pi \times 2 \times 2 \times h$

$h = 1$ cm

Sol 41.



$l = \sqrt{15^2 + 8^2}$

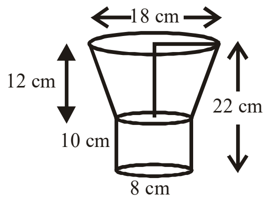
$= 17$ cm

T.S.A. $= 2\pi rh + \pi r^2 + \pi rl$

$= \pi(2 \times 8 \times 15 + (8)^2 + 8 \times 17)$

$= 440\pi$

Sol 42.



$R = 9$

$r = 4$

$l = \sqrt{(12)^2 + (5)^2}$

$l = 13$

Area of funnel

$= 2\pi rh \times \pi(R + r)l$

$= \pi[2 \times 4 \times 10 + (9 + 4) \times 13]$

$= 249\pi$

$= 249 \times \dfrac{22}{7} = 782.57 \text{ cm}^3$

Sol 43. Volume of cube = 343

$a^3 = 343$

$a = 7$

then, $2r = a = 7$

$r = \dfrac{a}{2} = \dfrac{7}{2}$

Volume of cone $= \dfrac{1}{3}\pi r^2 h$

$= \dfrac{1}{3} \times \dfrac{22}{7} \times \dfrac{7}{2} \times \dfrac{7}{2} \times 7$

$= 90 \text{ cm}^3$

Sol 44. $r = r$ cm.

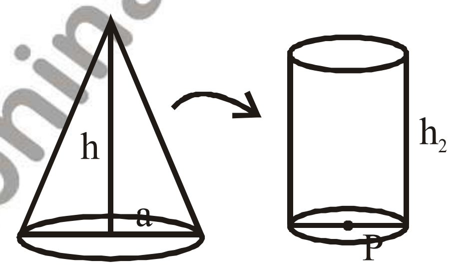
$h = 6$ cm

$\pi r^2 h = \dfrac{1}{3}\pi r^2 h$

$\pi r^2 6 = \dfrac{1}{3}\pi r^2 h$

$h = 18$ cm

Sol 45.



$\pi r^2 h_2 = \dfrac{1}{3}\pi r^2 h$

$\pi P^2 h_1 = \dfrac{1}{3}\pi a^2 h$

$\boxed{h_1 = \dfrac{a^2 h}{3P^2}}$

Sol 46. a = side of cube

radius of sphere

$= r = \dfrac{a}{2}$

Hence, $a^3 : \dfrac{4}{3}\pi\left(\dfrac{a}{2}\right)^3$

$= 6 : \pi$