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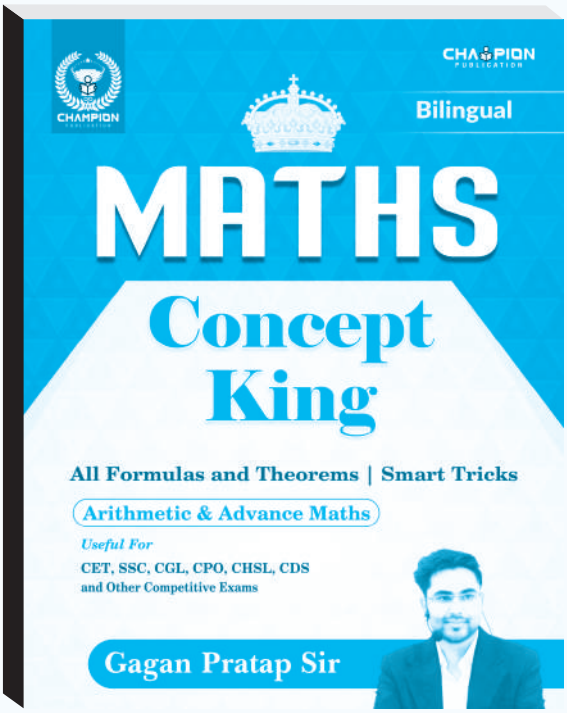

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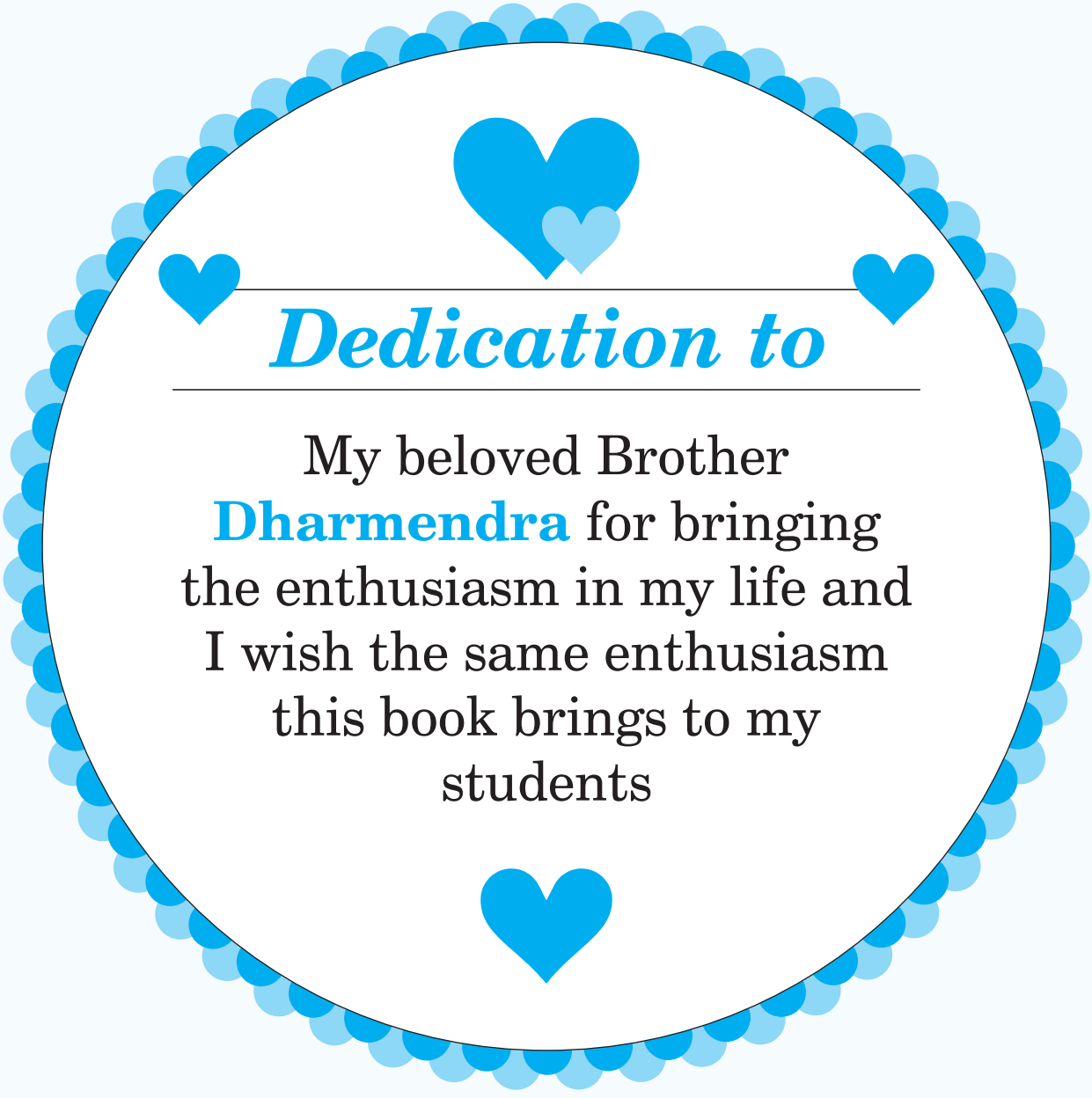

Dedication to

My beloved Brother
Dharmendra for bringing
the enthusiasm in my life and
I wish the same enthusiasm
this book brings to my
students

Preface

The main aim of publishing this book is to spread the knowledge in the easiest way amongst learners. There are manifold purposes of writing this book on the subject. Basically, it caters to the needs of the candidates aspiring for competitive examinations, and for the beginners to understand the intricacies of the subject.

It is observed that the very name of the subject, Mathematics evokes fear in the minds of the students. Through this book an effort has been made to dispel that fear. MCQs at the end of every unit will help the students to make a self-assessment of the knowledge assimilated by going through the Chapters. The answers have been given for MCQs along with explanations.

As a professional it is observed that the books in the market are providing useful information to the students. They provide very useful multiple choice questions with their correct answers. The inquisitive mind of the student is still left high and dry as he is at a loss to know as to why a particular answer and why not otherwise. Through this book, the author has made an effort to provide rationale for the solutions. The book, therefore, meets the expectations of the students as it answers the demand and the quest in their mind.

The book is user-friendly and provides content in a well structured manner. It provides comprehensive and critical study of the various concepts of the subject matter. A word or suggestion from your side may add another feather to the cap of the subject matter of the book. The author looks forward to the comments, suggestions and criticism from the readers. Constructive suggestions and feedback from users would be highly appreciated, gratefully acknowledged and suitably incorporated.

Striving to serve the student community and to impart quality education.

With best wishes

Champion Publication

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Table of Symbols (प्रतीकों की तालिका)

Appendix

Symbol	Reference
=	equal to
≠	not equal to
≡	identity
≃	approximately equal to
≅	congruent to
→	approaches, ray
∝	proportional to
<	less than
≮	not less than
>	greater than
≯	not greater than
≤	less than or equal to
≥	greater than or equal to
<<	much less than
>>	much greater than
∞	infinity
σ or Σ	sigma (Summation)
%	percentage
+	plus, positive
–	minus, negative
±	plus or minus
$\begin{cases} a \times b \\ a \cdot b \end{cases}$	multiplication
$\begin{cases} a \div b \\ a / b \end{cases}$	division
∴	therefore
∵	since
—	line segment
∠	acute angle
⊥	perpendicular
||	parallel
Δ	triangle
▭	rectangle
□	square
$\log_b a$	logarithm (to base b)
$\log_{10} a$	common logarithm
$\log_e a$ or $\ln a$	natural logarithm
∧	conjunction (and)
∨	disjunction (or)
⇒, ⇔	implication
~	negation, equivalence, relation
∃, ∀	quantifier
{ }	set
ϕ	empty set, void set, null set
!	Factorial
i	Imaginary unit
∪	Union
⊂	Subset of
⊃	Superset
∩	Intersection
⊆	Subset or equal to
μ	Mean (average)

Conversion of Units

Conversion of Length

10 millimetres (mms)	= 1 centimetre (cm)
10 centimetres	= 1 decimetre (dm)
10 decimetres	= 1 metre (m)
10 metres	= 1 decametre (dam)
10 decametres	= 1 hectometre (hm)
10 hectometres	= 1 kilometre (km)

Conversion of Area

100 square millimetres	= 1 square centimetre
100 square centimetres	= 1 square decimetre
100 square decimetres	= 1 square metre
100 square metres	= 1 square decametre
100 square decametres	= 1 square hectometre
100 square hectometres	= 1 square kilometre
1 hectare	= 10000 square metres

Conversion of Volume

1000 cubic millimetres	= 1 cubic centimetre
1000 cubic centimetres	= 1 cubic decimetre
1000 cubic decimetres	= 1 cubic metre
1000 cubic metres	= 1 cubic decametre
1000 cubic decametres	= 1 cubic hectometre
1000 cubic hectometres	= 1 cubic kilometre

Conversion of Capacity

10 millilitres	= 1 centilitre
10 centilitres	= 1 decilitre
10 decilitres	= 1 litre
10 litres	= 1 decalitre
10 decalitres	= 1 hectolitre
10 hectolitres	= 1 kilolitre

Conversion of Weight

10 milligrams	= 1 centigram
10 centigrams	= 1 decigram
10 decigrams	= 1 gram (g)
10 grams	= 1 decagram
10 decagrams	= 1 hectogram
10 hectograms	= 1 kilogram (kg)
100 kilograms	= 1 quintal
10 quintals or 1000 kg	= 1 metric tonne

Conversion of Time

60 seconds	= 1 minute
60 minutes	= 1 hour
24 hours	= 1 day
7 days	= 1 week
15 days	= 1 fortnight
28, 29, 30 or 31 days	= 1 month
12 months	= 1 year
365 days	= 1 year
366 days	= 1 leap year
10 years	= decade
25 years	= silver jubilee
50 years	= golden jubilee
60 years	= diamond jubilee
75 years	= radium jubilee or platinum jubilee
100 years	= century
1000 years	= 10 centuries or 1 millennium

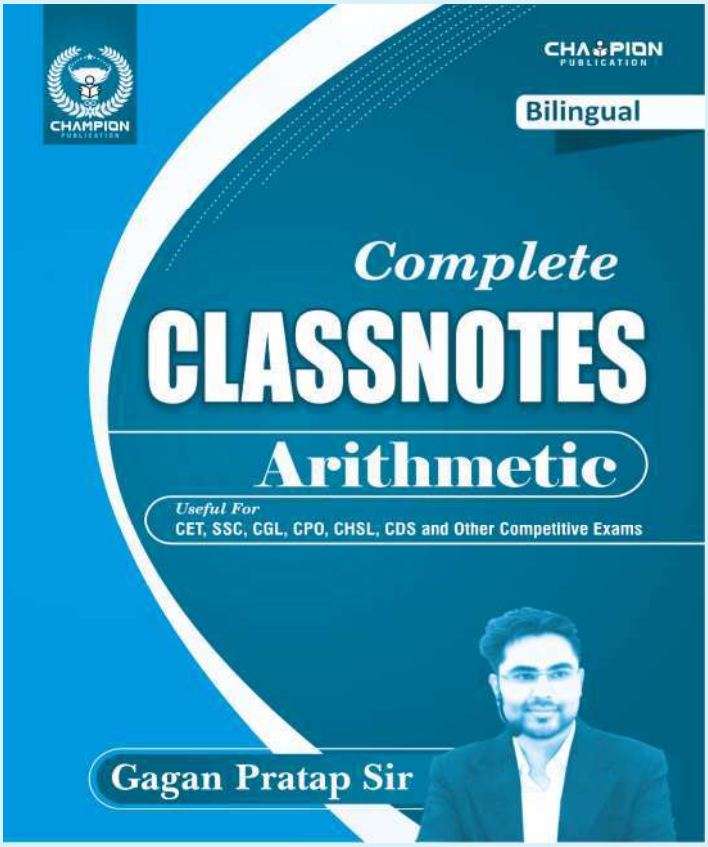
Equivalents of Units

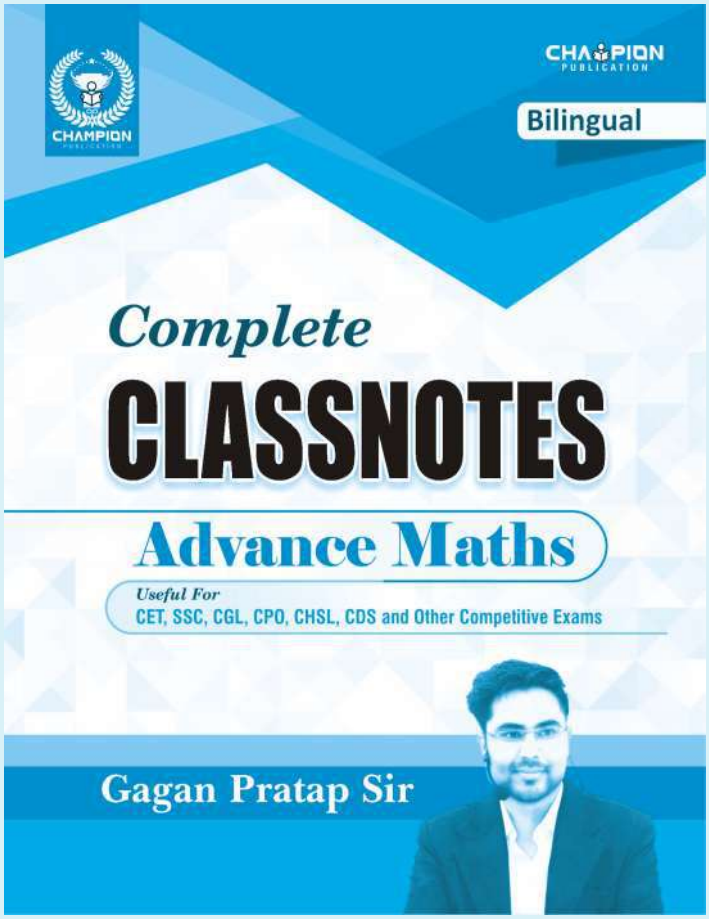
Units of Lengths

12 inches	= 1 feet (ft) = 0.3048 metres
3 feet	= 1 yard (d)
1 yard	= 0.9144 metres
22 yards	= 1 chain
1 kilometre	= 0.621 mile or 10^3 metres
1 mile	= 1.6093 kilometres or 1760 yards
1 inch	= 2.54 centimetres
1 hectare	= 2.471 acres
1 mile	= 5280 feet

Units of Area

1 square feet	= 144 square inches
	= 0.0929 square metres
1 square metre	= 1.196 square yards
1 square yard	= 0.836 square metres
1 square kilometre	= 0.3861 square miles
	= 1000 hectares
1 square mile	= 2.59 square kilometres
	= 640 acres
1 acre	= 4840 square yards
	= 4046.86 square metres
1 hectare	= 10000 square meters





Geometry (Line & Angle) मे ं क्ि ोएिक) बिं

Line and Angle (रेखा और कोण)

❖ **Point**(•): Zero dimension figure or a circle with zero radius.

(प्रतीक शून्य आयामी आकृति या शून्य त्रिज्या वाला एक वृत्त।

➢ **Types of point/बिन्दु के प्रकार**

(i) Collinear point/सरेंख बिन्दु

If 3 or more than 3 points lie on a line close to or far from each other, then they are said to be collinear.

यदि 3 या 3 से अधिक बिंदु एक रेखा पर एक दूसरे के निकट या दूर स्थित हो, तो वे सरेंख कहलाते है।

Ex. Point P, Q, R, S are collinear/P, Q, R, S सरेंख बिंदु है।

P Q R S

(ii) Non-collinear point/असरेंख बिंदु

In 3 or more points are not situated on a straight line, these all point are called non-collinears point.

यदि 3 या अधिक बिन्दु एक सीधी रेखा पर स्थित नहीं हैं, तो ये सभी बिंदु असरेंख बिंदु कहलाते हैं।

Ex. ∴

❖ **Line:** (One dimension figure) line is a set of points having only length with no ends. ⟷

∵ क्ि (एक आयामी आकृति) रेखा बिंदुओं का एक समूह है जिसमें केवल लंबाई होती है जिसका कोई अंत नहीं होता है।

❖ **Line segment:** A line with a fixed length.

∵ क्ि ंलक एक निश्चित लंबाई वाली रेखा।

P——Q

❖ **Ray**: A line with uni-direction length. A⟶B

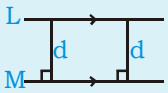
( ) बकि एक दिशा लंबाई वाली रेखा।

❖ **Parallel lines:** two or more line that never intersects L ∥ M

दु ि:ररि केंं ख्क्ि दो या अधिक रेखाएँ जो एक दूसरे को कभी नहीं काटती हैं। L ∥ M

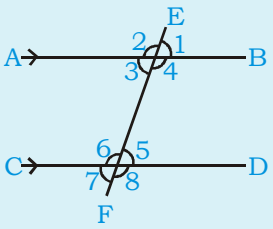
❖ **Transversal Line :** A line which intersects (touches) two or more lines at distinct point is called transversal lines of the given lines.

(रण्से केंं क्ि्कह रेखा जो दो या दो से अधिक रेखाओं को अलग-अलग बिंदु पर काटती (स्पर्श) है, दी गई रेखा की तिर्यक रेखा कहलाती है।



AB ∥ CD and EF is transversal line

AB ∥ CD और EF एक तिर्यक रेखा है



Corresponding angles / संगत कोण ⇒ ∠1 = ∠5, ∠4 = ∠8

∠2 = ∠6, ∠3 = ∠7

Alternate Angles / एकांतर कोण ⇒ ∠3 = ∠5, ∠4 = ∠6

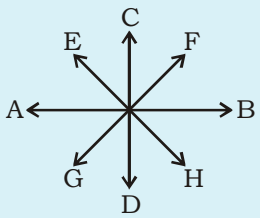
∠4 + ∠5 = 180°

∠3 + ∠6 = 180°

➢ **Concurrent line/समवर्ती रेखा**

Three or more than three lines, which pass from a single point is called concurrent lines.

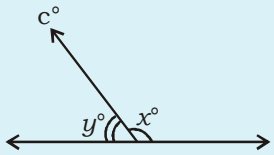
तीन या तीन से अधिक रेखाएं, जो एक बिंदु से होकर गुजरती हैं, समवर्ती रेखाएं कहलाती हैं।



➢ **Linear pair angle/रैखिक युग्म कोण**

A linear pair is a pair of adjacent angle whose non-common sides are opposite rays.

एक रेखीय युग्म आसन्न कोणों का युग्म होता है, जिसकी गैर उभयनिष्ठ भुजाएं विपरीत रेखा होती हैं।



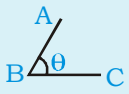
∠x + ∠y = 180°

⇒ Linear pair angle are supplementry.

❖ **Angle**: inclination between two lines is called angle.

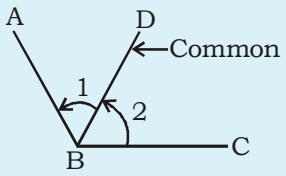
कोण: दो रेखाओं के बीच के झुकाव को कोण कहते हैं।

$\angle ABC = \theta$



➢ **Adjacent angles/आसन्न कोण**

Two angles are said to be adjacent if



$\angle 1$ and $\angle 2 \rightarrow$ adjacent angle.

(i) They have a common vertex (vertex B)
उनका एक उभयनिष्ठ शीर्ष है (शीर्ष B)

(ii) They have a common arm. (BD is common)
उनकी एक ही भुजा है। (BD सामान्य है)

➢ There is one and only one line passing through two distinct point.
Two or more line are said to be coplanar if they lie in the same plane, otherwise they are said to be non-coplanar.
दो विभिन्न बिन्दुओं से होकर जाने वाली एक और केवल एक ही रेखा है। दो या दो से अधिक रेखाएं समतलीय कहलाती हैं यदि वे एक ही तल में स्थित हों, अन्यथा उन्हें गैर-समतलीय कहा जाता है।

❖ **Complementary Angle :** If sum of two angles is 90° then they are Complementary to each other.
यें) क) बिकि यदि दो कोणों का योग 90° हो तो वे एक दूसरे के पूरक होते हैं।

❖ Complementary Angle/ (पूरक कोण) 

α+β = 90°

Supplementary Angle : If sum of two angles is 180° then they are supplementary to each other.
द्र्ये) क) बिकि यदि दो कोणों का योग 180° हो तो वे एक दूसरे के पूरक होते हैं।

Supplementary Angles/(संपूरक कोण) 

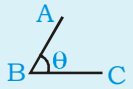
α+β = 180°

❖

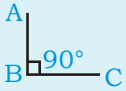
Angle	Complementary	Supplementary
कोण	पूरक	संपूरक
43°	47° $\xrightarrow{+90}$	137°
12°	78° $\xrightarrow{+90}$	168°
θ	90° – θ $\xrightarrow{+90}$	180° – θ

Supplementary angle of an angle is 90° more than complementary angle.
एक कोण का संपूरक कोण पूरक कोण से 90° अधिक होता है।

Types of Angles (कोणों के प्रकार)

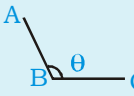
❖ Acute Angle / न्यून कोण → 

0° < θ < 90°

❖ Right Angle / समकोण → 

AB ⊥ BC **θ=90°**

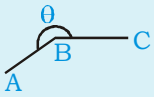
❖ Obtuse Angle / अधिक कोण →



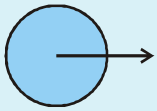
90°<θ<180°

❖ Straight or line angle / सीधा या रेखा कोण →

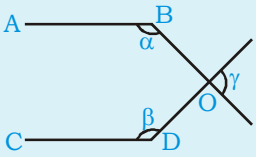
θ = 180°

❖ Reflex Angle / वृहत्तकोण → 

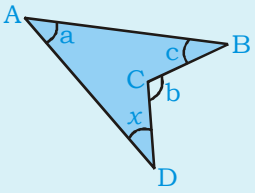
180°<θ<360°

❖ Complete Angle / पूर्ण कोण → 
θ = 360°

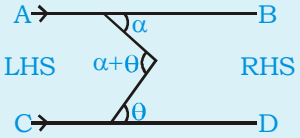
❖ If AB ∥ CD then find the value of α+β+γ?
अगर AB ∥ CD तो α+β+γ का मान ज्ञात करें?



α + β + γ = 360°

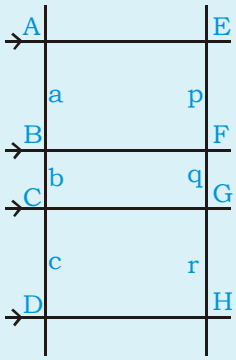
❖ 

b = a + c + x

❖ 

Sum of angle on RHS = LHS
(RHS = LHS पर कोणों का योग)
α+θ = α+θ

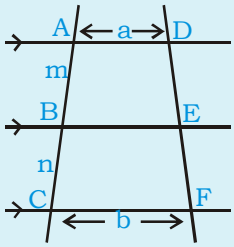
❖



a : b : c = p : q : r

$$\frac{a}{a+b+c} = \frac{p}{p+q+r}$$

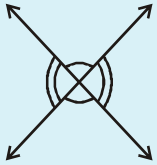
❖

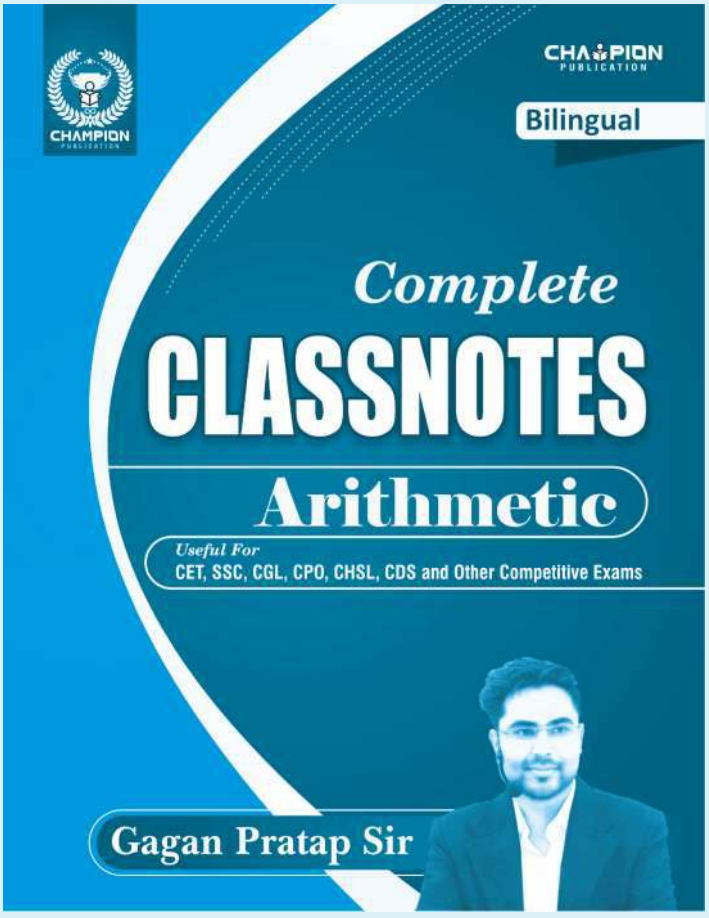


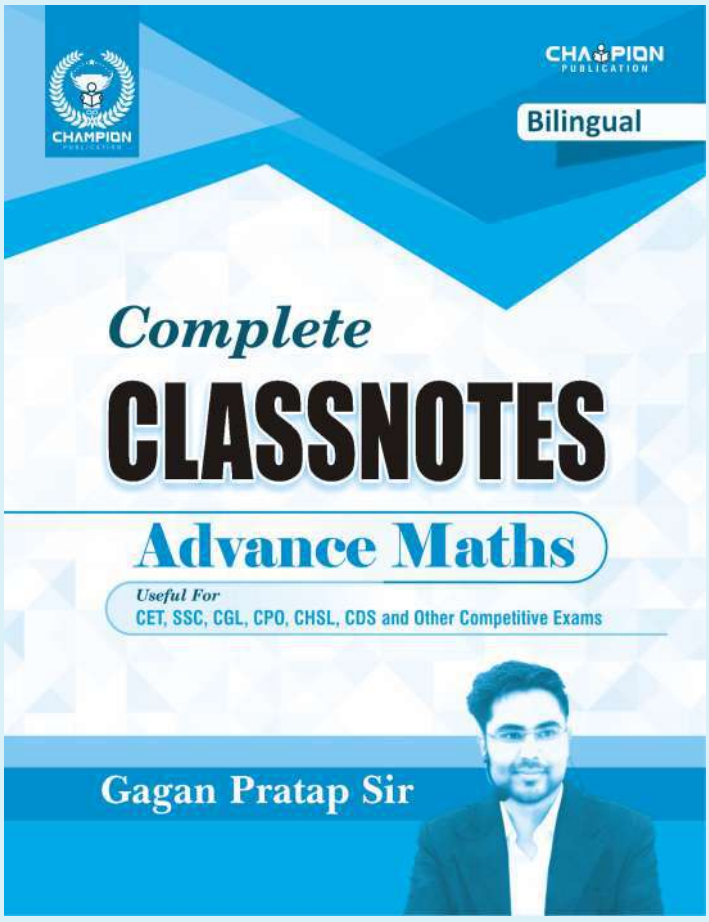
$$\frac{AB}{BC} = \frac{DE}{EF} = \frac{m}{n}$$

$$BE = \frac{an+bm}{m+n}$$

❖ Vertically opposite/(शीर्षभिमुख कोण)







Types of Triangles म(अैं ी क)ं क्म>)ें

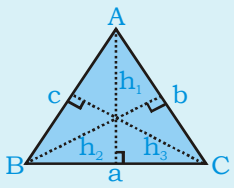
Triangle (त्रिभुज)

❖ A triangle is a 3-sided polygon that consists of three edges and three vertices.

एक त्रिभुज एक 3-भुजाओं वाला बहुभुज है जिसमें तीन किनारे और तीन शीर्ष होते हैं।

❖ 3 sides, 3 vertices, 3 altitudes, 3 angles

3 भुजाएँ, 3 शीर्ष, 3 ऊँचाई, 3 कोण



$\angle A+\angle B+\angle C=180°$

Area $\Rightarrow \frac{1}{2}\times a\times h_1 = \frac{1}{2} bh_2 = \frac{1}{2} ch_3 = \frac{1}{2} \times$ Base × Corresponding height.

$\Rightarrow ah_1 = bh_2 = ch_3 =$ constant

$$\mathbf{h_1 : h_2 : h_3 = \frac{1}{a} : \frac{1}{b} : \frac{1}{c}}$$

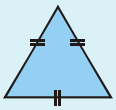
Type of Triangle (त्रिभुज के प्रकार)

By side — By Angle

' ी कि)ं क्म भिेंि क्मे —) बिकि)ं क्म भिेंि क्मे

By side (भुजाओं के आधार पर)

❖ **Equilateral Triangle** जक्द्दु पधिक्रअैं ी



Equilateral triangle has 3 equal sides, each angle equals 60°.

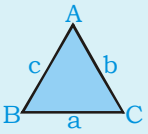
समबाहु त्रिभुज में प्रत्येक कोण 60° तथा तीनों भुजाएँ समान होती है।

❖ **Isosceles Triangle** जक्द्दु (/पधिक्रअैं ी

two equal sides / दो समान भुजाएँ

two angle same / दो कोण समान

❖ **Scalene Triangle** जक्हद्दु पधिक्रअैं ी



three unequal sides / तीन असमान भुजाएँ

three unequal angles / तीन असमान कोण

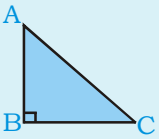
$\angle A\neq\angle B\neq\angle C$ & $a\neq b\neq c$

By Angle (कोणों के आधार पर)

❖ **Acute Angle Triangle** जक्द्दु : क) बिकि्रअैं ी

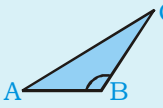
All three angles < 90° / तीनों कोण < 90°

❖ **Right Angle Triangle** जक्द्दु) बिकि्रअैं ी



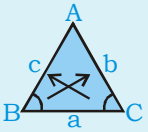
One angle is 90°. $\angle B = 90°$ and $\angle A + \angle C = 90°$

❖ **Obtuse Angle Triangle** जक्त (भ) क) बिकि्रअैं ी



One angle > 90° / एक कोण > 90°

❖ Let $\angle B$ = largest angle / माना $\angle B$ = सबसे बड़ा कोण



∴ side AC = b = largest side

भुजा AC = b = सबसे बड़ी भुजा

Let $\angle C$ = smallest angle (माना $\angle C$ = सबसे छोटा कोण)

∴ side AB = c = smallest side

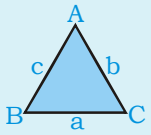
भुजा AB = c = सबसे छोटी भुजा

❖ **Inequality of triangle** म(अैं ी क) प्क्त द( ) ख्रिं

The triangle inequality states that for any triangle the sum of the lengths of any two sides must be greater than the length of the remaining side.

त्रिभुज असमानता बताती है कि किसी भी त्रिभुज के लिए किसी भी दो भुजाओं की लंबाई का योग शेष भुजा की लंबाई से अधिक होना चाहिए।

❖ **Conditions for formation of triangle:→**



$|b-c| < a < b+c$

$|a-c| < b < a+c$

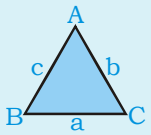
$|a-b| < c < a+b$

❖ Eg→ 4, 9, 15 Δ not possible / त्रिभुज संभव नहीं ∵ $4+9<15$

5, 10, 15 Δ not possible / त्रिभुज संभव नहीं ∵ $5+10=15$

7, 12, 15 Δ is possible / त्रिभुज संभव है ∵ $7+12>15$

OR $7+15>12$ OR $12+15>7$



1. Sum of any two sides is always greater than 3rd side.

   किन्हीं भी दो भुजाओं का योग हमेशा तीसरी भुजा से बड़ा होता है।

   $a+b > c$

   $b+c > a$

   $c+a > b$

2. Difference of any two sides is always less than 3rd side.

   किन्हीं दो भुजाओं का अंतर सदैव तीसरी भुजा से कम होता है।

   $\Rightarrow b > c-a$

   $\Rightarrow b > a-c$

   $\Rightarrow \therefore |c-a| < b < c+a$

❖ If 10, 17, x are sides of a Δ, $x \rightarrow$ integer

Then $7 < x < 27$

$\therefore x \rightarrow \{8, 9, 10, \ldots. 26\}$

$x_{min} = 8$, $x_{max} = 26$

x_{total} = 19 values possible / 19 मान संभव

∴ 19 Δ's possible / 19 त्रिभुज संभव है।

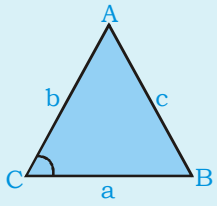
Possible values of $x = 2\times$small side -1

$\Rightarrow 2\times10-1 = 19$

x के संभावित मान = 2×छोटी भुजा −1 ⇒ 2×10−1 = 19

Relation between 3 sides of Triangle
(त्रिभुज की 3 भुजाओं के बीच संबंध)

I. Acute Angle Triangle जक्कर्ण : क) बिक्कैंजे ॅी

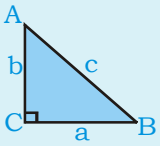


∠C = largest / ∠C = सबसे बड़ा कोण

side c = largest / भुजा c = सबसे बड़ा

$c^2 < a^2+b^2$

II. Right Angle Triangle जक्कद्दु) बिक्कैंजे ॅी

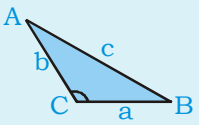


∠C = largest / ∠C = सबसे बड़ा कोण

side c = largest / भुजा c = सबसे बड़ा

$c^2 = a^2 + b^2$

III. Obtuse Angle Triangle जक्का (ध) क) बिक्कैंजे ॅी



∠C = largest / ∠C = सबसे बड़ा कोण

side c = largest / भुजा c = सबसे बड़ा

$c^2 > a^2 + b^2$

❖ sides of triangle : 11.7, 16.9, 23.4. which type of Δ it is?

त्रिभुज की भुजाएँ : 11.7, 16.9, 23.4. यह किस प्रकार का Δ है?

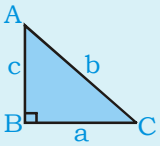
Take ratio of sides 11.7 : 16.9 : 23.4

9 : 13 : 18

$18^2 > 9^2 + 13^2$ ∴Δ is obtuse angle triangle.

Pythagoras Triplets (पाइथागोरस त्रिगुण/त्रिक)

❖



$b^2 = c^2 + a^2$

(3,4,5), (5,12,13), (7,24,25),
(8,15,17), (9,40,41), (11,60,61),
(12,35,37), (16,63,65), (13,84,85),
(20,21,29), (28,45,53), (33,56,65),
(39,80,89), (36,77,85), (65, 72, 97),
(20, 99, 101)

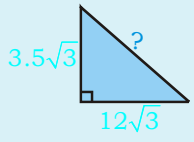
multiplication and division on these triplets will also result in triplets.

इन त्रिक पर गुणा और भाग का परिणाम भी त्रिक होगा।

$(5,12,13) \xrightarrow{\times 2} (10, 24, 26)$

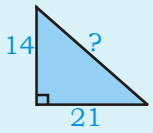
$(3,4,5) \rightarrow (6,8,10), (9,12,15), (12,16,20), (15,20,25)$

Ex: 1→



7 24 25
↓÷2 ↓÷2 ↓÷2
3.5 12 $12.5 \Rightarrow 3.5\sqrt{3}, 12\sqrt{3}, 12.5\sqrt{3}$

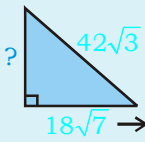
Ex: 2→



$\sqrt{3^2+2^2} = \sqrt{13}$

⑦×2 ⑦×3 $⑦\times\sqrt{13} = 7\sqrt{13}$

Ex: 3→

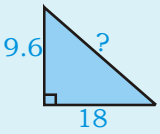


$42\sqrt{3} \rightarrow 6\times\sqrt{7}\times\sqrt{7}\times\sqrt{3} = 6\sqrt{7}\times\sqrt{21}$

$18\sqrt{7} \rightarrow 6\sqrt{7}\times 3$

3rd side = $6\sqrt{7}\times\sqrt{21-9} = 6\sqrt{7}\times\sqrt{12}$ = **$12\sqrt{21}$**

Ex: 4→



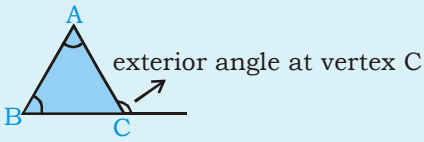
×1.2 (9.6 : 18 / 8 : 15) : 17 $\xrightarrow{\times 1.2}$ 20.4

❖ Exterior angle is equal to sum of opposite interior angles.

बाहरी कोण विपरीत आंतरिक कोणों के योग के बराबर है।

A+B+C = 180°

A+B = 180° – C

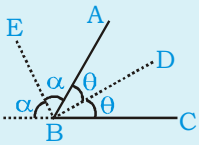


sum of all exterior angles = 360°

सभी बाहरी कोणों का योग = 360°

Angle Bisector (कोण द्विभाजक)

❖



BE → exterior angle bisector of ∠ABC

BE → ∠ABC का बाह्य कोण समद्विभाजक

2α + 2θ = 180°

α+θ = 90° = ∠EBD

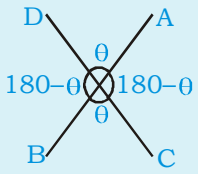
∴ Angle between internal angle bisector and external angle bisector of an angle is 90°.

किसी कोण के आंतरिक कोण समद्विभाजक और बाह्य कोण समद्विभाजक के बीच का कोण 90° होता है।

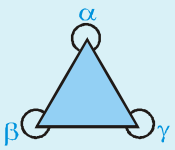
BD is interior angle bisector of ∠ABC

BD, ∠ABC का आंतरिक कोण समद्विभाजक है

❖ **Vertically Opposite Angle** जक्मन-ष्ठिीसिु ेकि) बिों

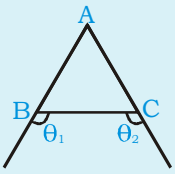


❖ **Some other properties** जक) ेिक्ता वाकलेबि



α+β+γ = 3×360° – 180° = 900°

❖



$\theta_1 + \theta_2 = 180° + \angle A$

❖ If angles of a Δ are in A.P., middle angle is always 60°/ (यदि Δ के कोण समांतर श्रेणी में हैं, तो मध्य कोण हमेशा 60° होता है)

(a–d), a, (a+d)

∴ a–d+a+a+d = 180°

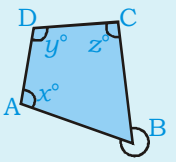
3a = 180°

a = 60°

∴ ∠A + ∠C = 120° & ∠B = 60°

∠A ∠B ∠C
↓ ↓ ↓
60°– 60° 60°+

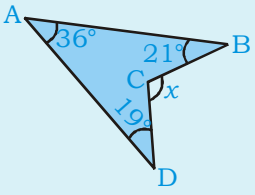
❖



∠B (internal / आंतरिक) = 360° – (x+y+z)

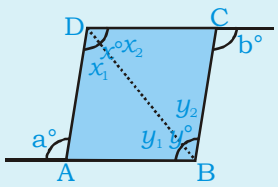
∠B (external / बाहरी) = x°+y°+z°

❖



x° = 36° + 21° + 19° = 76°

❖



a+b = x+y

$x_1 + y_1 = a°$

$x_2 + y_2 = b°$

$x_1 + x_2 + y_1 + y_2 = a+b$

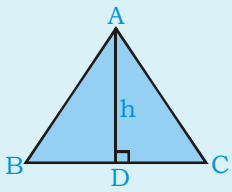
$\boldsymbol{x+y = a+b}$

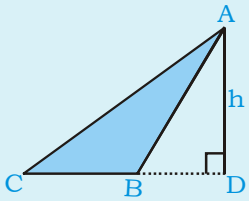
❖ **Altitude / Height / Perpendicular**

ऊँचाई / लम्ब / पाद / पक्ष का चढ़ाव / शीर्षक पहर

The perpendicular drawn from the vertex of the triangle to the opposite side.

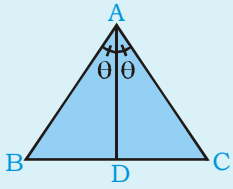
त्रिभुज के शीर्ष से विपरीत दिशा में खींचा गया लंब।





❖ A line that splits an angle into two equal angles.

एक रेखा जो एक कोण को दो बराबर कोणों में विभाजित करती है।



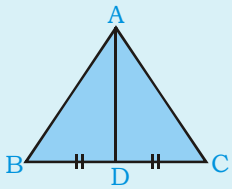
AD is the angle bisector of ∠BAC, BD and DC need not be equal

AD, ∠BAC का कोण समद्विभाजक है और BD, DC का बराबर होना आवश्यक नहीं है

Median (माध्यिका)

❖ Line drawn from a vertex to opposite side which divides the opposite side into equal parts.

किसी शीर्ष से विपरीत दिशा में खींची गई रेखा जो विपरीत भुजा को समान भागों में विभाजित करती है।



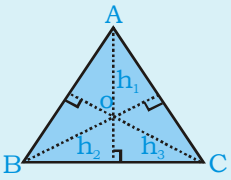
AD is the median of side BC. / AD भुजा BC की माध्यिका है

BD = DC

Perpendicular bisector (लंब द्विभाजक)

❖ A Perpendicular bisector is a line that bisects a line segment in two equal parts and makes an angle of 90° at the point of intersection.

लंब समद्विभाजक एक रेखा है जो एक रेखा खंड को दो बराबर भागों में विभाजित करती है और प्रतिच्छेदन बिंदु पर 90° का कोण बनाती है।

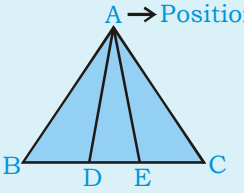


O = circumcentre / परिकेन्द्र

Cevian (केवियन)

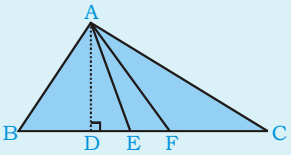
❖ Cevian → Any line which joins vertex to opposite side.

केवियन → कोई भी रेखा जो शीर्ष को विपरीत भुजा से जोड़ती है



AD, AE are cevians / AD, AE केवियन हैं

❖ Δ ABC is scalene Δ / Δ ABC विषमबाहु Δ है



AC > AB

∴ ∠B > ∠C

⊥ AD will be near to largest among ∠B and ∠C i.e angle ∠B and far from small angle ∠C.

⊥ AD, ∠B और ∠C कोण में से सबसे बड़े ∠B के निकट होगा और छोटे कोण ∠C से दूर होगा।

AE → Angle bisector of ∠A

AE → ∠A का कोण द्विभाजक

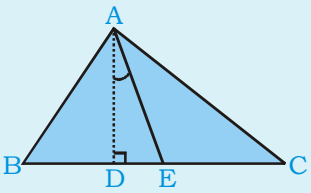
AF → median i.e. BF = FC

AF → माध्यिका यानी BF = FC

❖ AD → Altitude / AD → ऊंचाई

AE → Angle bisector of ∠A

AE → ∠A का कोण द्विभाजक



$$\boldsymbol{\angle DAE = \frac{\angle B - \angle C}{2}}$$

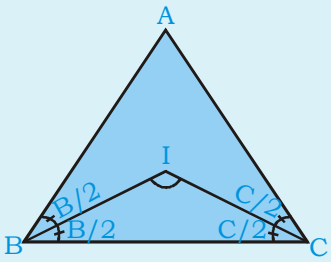
$$\angle DAE = \frac{\angle A}{2} - 90° + \angle B$$

$$= \frac{\angle A}{2} - \frac{\angle A}{2} - \frac{\angle B}{2} - \frac{\angle C}{2} + \angle B$$

$$\because \frac{\angle A}{2} + \frac{\angle B}{2} + \frac{\angle C}{2} = 90°$$

$$= \frac{\angle B}{2} - \frac{\angle C}{2} = \frac{\angle B - \angle C}{2}$$

❖ A+B+C = 180°



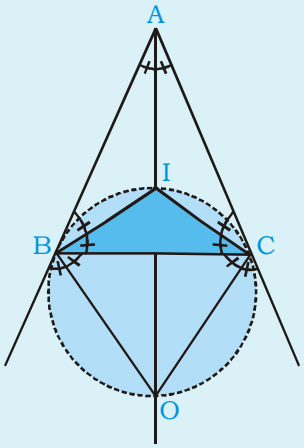
$$\angle A = 180° - (\angle B + \angle C) \Rightarrow \frac{\angle B + \angle C}{2} = \frac{180° - \angle A}{2}$$

$$\angle BIC = 180° - \left(\frac{\angle B + \angle C}{2}\right)$$

$$= 180° - \left(\frac{180° - \angle A}{2}\right) = 90° + \frac{\angle A}{2}$$

$$\therefore \mathbf{\angle BIC = 90° + \frac{\angle A}{2}}$$

❖



$$\mathbf{\angle BIC = 90° + \frac{\angle A}{2}}$$

$$\mathbf{\angle BOC = 90° - \frac{\angle A}{2}}$$

AIO will be a straight line and bisect angle A.

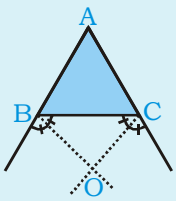
AIO एक सीधी रेखा और कोण A द्विभाजित होगा।

∠BIC + ∠BOC = 180°

BICO will be a cyclic quadrilateral.

BICO एक चक्रीय चतुर्भुज होगा

❖

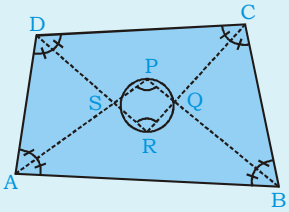


$$\mathbf{\angle BOC = 90° - \frac{\angle A}{2}}$$

$\angle A = 2(90° - \angle BOC)$

❖ In any quadrilateral bisector of ∠A & ∠B meet at P.

किसी चतुर्भुज में ∠A व ∠B के समद्विभाजक P पर मिलते है।



$$\mathbf{\angle APB = \frac{\angle C + \angle D}{2}}$$

$$\angle APB = 180° - \left(\frac{A}{2} + \frac{B}{2}\right)$$

$$\angle APB = \frac{C}{2} + \frac{D}{2}$$

$\angle A + \angle B + \angle C + \angle D = 360°$

$$\frac{\angle A}{2} + \frac{\angle B}{2} + \frac{\angle C}{2} + \frac{\angle D}{2} = \frac{360°}{2} = 180°$$

$$180° - \left(\frac{\angle A}{2} + \frac{\angle B}{2}\right) = \frac{C}{2} + \frac{D}{2}$$

Bisector of ∠C and ∠D meet at R

$$\therefore \mathbf{\angle DRC = \frac{\angle A + \angle B}{2}}$$

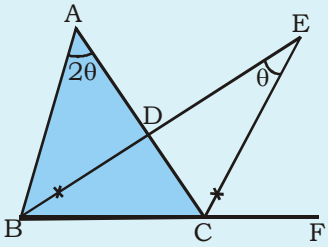
$\angle P + \angle R = 180°$

$\angle S + \angle Q = 180°$

∴ □ PQRS will be a cyclic quadrilateral.

□ PQRS एक चक्रीय चतुर्भुज होगा।

❖



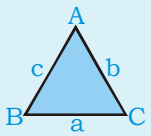
$$\angle BEC = \frac{\angle A}{2}$$

Area side properties म(अैा ूा क) विक्षअिक-ँ

Area of triangle (त्रिभुज का क्षेत्रफल)

❖ Area of $\Delta = \frac{1}{2} \times$ base $\times$ height

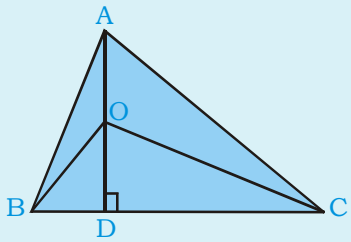
Δ का क्षेत्रफल $= \frac{1}{2} \times$ आधार $\times$ ऊँचाई



$s = \frac{a+b+c}{2}$

Area of $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

❖ In any ΔABC, $AD \perp BC$

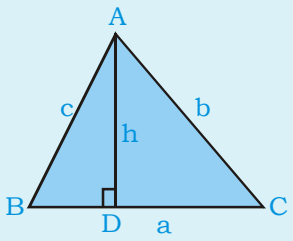


$AB^2 + OC^2 = OB^2 + AC^2$

O is any point on altitude / O ऊंचाई पर कोई बिंदु है

Sine Rule (ज्या नियम)

❖



$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = K$ (constant)

a:b:c = KsinA : KsinB : KsinC

a:b:c = sinA : sinB : sinC

Area of $\Delta ABC = \frac{1}{2} \times$ base $\times$ height

Area of $\Delta = \frac{1}{2} \times a \times h$

$= \frac{1}{2} \times a \times c \sin B$

$\sin B = \frac{h}{c} \Rightarrow h = c \sin B$

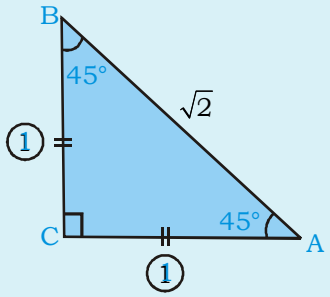
$\sin C = \frac{h}{b} \Rightarrow h = b \sin C$

$C \sin B = b \sin C \Rightarrow \frac{c}{\sin C} = \frac{b}{\sin B}$

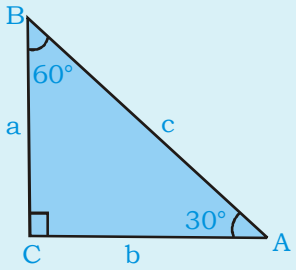
$\therefore \Delta = \frac{1}{2} ac \sin B = \frac{1}{2} ab \sin C = \frac{1}{2} bc \sin A$

Side-Angle ratio of some triangles (कुछ त्रिभुजों का भुजा-कोण अनुपात)

❖



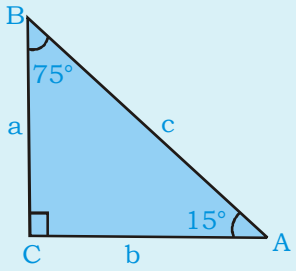
45°	45°	90°
↓	↓	↓
sides ⇒ **1 :**	**1 :**	**$\sqrt{2}$**



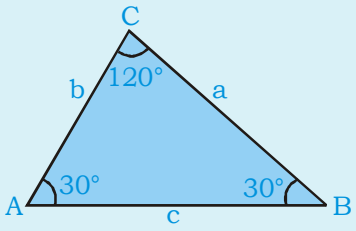
a:b:c = sin30° : sin60° : sin90°

$\frac{1}{2} : \frac{\sqrt{3}}{2} : 1$

a : b : c = 1 : $\sqrt{3}$: 2



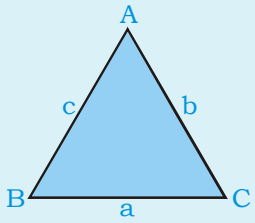
⇒ **a : b : c = $\sqrt{3}-1$: $\sqrt{3}+1$: $2\sqrt{2}$**



$\Rightarrow$ **a : b : c = 1 : 1 : $\sqrt{3}$**

Cosine Rule (कोज्या नियम)

❖



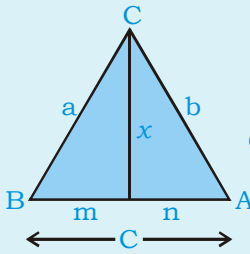
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \Rightarrow a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\cos B = \frac{c^2 + a^2 - b^2}{2ca} \Rightarrow b^2 = a^2 + c^2 - 2ac\cos B$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} \Rightarrow c^2 = a^2 + b^2 - 2ab\cos C$$

Stewarts Theorem (स्टीवर्ट प्रमेय)

❖



C = m + n

$a^2n + b^2m = x^2c + mnc$

In isosceles triangle a = b

$a^2n + a^2m = x^2c + mnc$

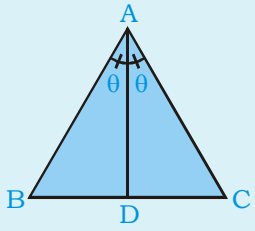
$a^2(m+n) = c(x^2 + mn)$

$a^2 = x^2 + mn$

$x^2 = a^2 - mn$

Interior Angle bisector theorem (आंतरिक कोण समद्विभाजक प्रमेय)

❖

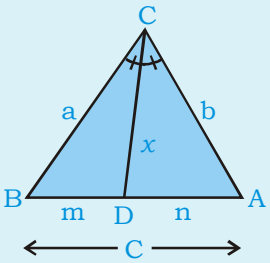


$$\frac{AB}{AC} = \frac{BD}{DC}$$

Length of Angle bisector (कोण द्विभाजक की लंबाई)

❖ CD is angle bisector of ∠BCA

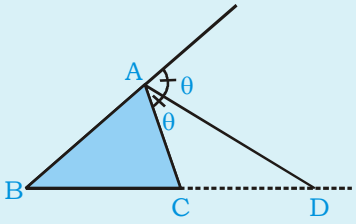
CD, ∠BCA का कोण समद्विभाजक है



$x^2 = ab - mn$

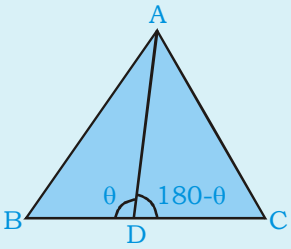
Exterior Angle bisector theorem (बाह्य कोण समद्विभाजक प्रमेय)

❖



$$\frac{AB}{AC} = \frac{DB}{DC}$$

❖



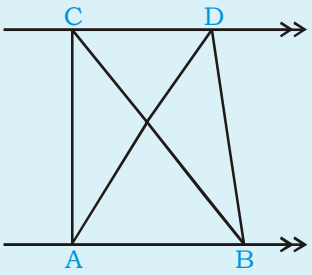
$$\frac{Ar\Delta ADB}{Ar\Delta ADC} = \frac{\frac{1}{2} \times AD \times BD \times \sin\theta}{\frac{1}{2} \times AD \times DC \times \sin(180 - \theta)} = \frac{BD}{DC}$$

∴ जिस ratio में cevian base को divide करेगी, Area भी उसी ratio में divide होगा।

If AD is median BD = DC

∴ Ar ΔADB = Ar ΔADC

❖



Ar ΔABC = Ar ΔABD AB || CD

If AB || CD, same parallel line के बीच same base पर बने Δ का Area बराबर होता है।

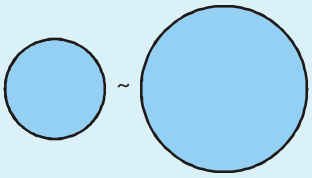
Similarity of triangles म(अैंॅी क) ष्क्रु रूयरों व(~)

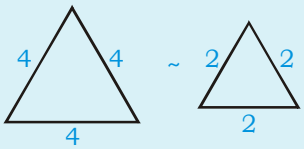
❖ **Similarity of triangles :** Two triangles are similar if they have the same ratio of corresponding sides and equal pair of corresponding angles.

(अैंॅी ्रकि) ष्क्रु रूयर किक्को त्रिभुज समरूप होते हैं यदि उनकी संगत भुजाओं का अनुपात समान हो और संगत कोणों का युग्म समान हो।

❖ **Similarity of triangles : size may be different but shape should be same.**

(अैंॅी ्रकि) ष्क्रु रूयर किक्म ि)ैं कैं वि काकिद)रंकाक्(ं( ) : का कि्(र दु ि: काम: ष्क्रा(धख

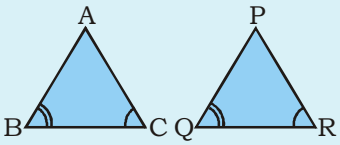




❖ Conditions of Similarity (समरूपता की शर्ते):-

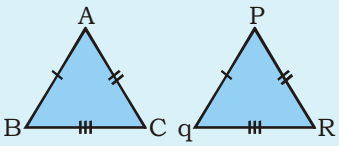
1. A-A (Angle-Angle) (कोण-कोण)

$\Delta ABC \sim \Delta PQR$



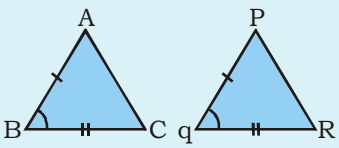
2. S-S-S(Side-Side-Side) (भुजा-भुजा-भुजा)

AB = PQ, BC = QR, AC = PR



3. S-A-S(Side-Angle-Side) (भुजा-कोण-भुजा)

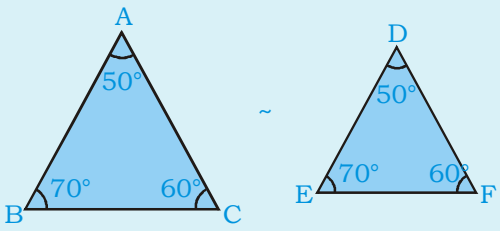
$\angle B = \angle Q$ AB = PQ, QR = BC



❖ If two angle is same in a triangle then third angle will be similar.

यदि एक त्रिभुज में दो कोण समान हैं तो तीसरा कोण भी समान होगा।

$\left.\begin{matrix}\angle A = \angle D \\ \angle B = \angle E \\ \angle C = \angle F\end{matrix}\right]$ corresponding Angles



sides opposite to corresponding angles is called corresponding sides.

संगत कोणों की सम्मुख भुजाएँ संगत भुजाएँ कहलाती हैं।

$\Delta ABC \sim \Delta DEF$

$$\therefore \frac{BC}{EF} = \frac{AC}{DF} = \frac{AB}{DE}$$ (Property)

❖ In similar triangle ratio of each corresponding length is equal.

समरूप त्रिभुज में प्रत्येक संगत लंबाई का अनुपात बराबर होता है।

$$\frac{BC}{EF} = \frac{AC}{DF} = \frac{AB}{DE} = \frac{h_1}{h_2} = \frac{\text{Angle bisector1}}{\text{Angle bisector2}} = \frac{\text{median1}}{\text{median2}}$$

$$= \frac{r_1}{r_2} = \frac{R_1}{R_2} = \frac{\text{perimeter of } \Delta ABC}{\text{perimeter of } \Delta DEF}$$

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{\frac{1}{2}\times BC \times h_1}{\frac{1}{2}\times EF \times h_2} = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DF}\right)^2 = \left(\frac{AB}{DE}\right)^2$$

= Ratio of square of corresponding length.

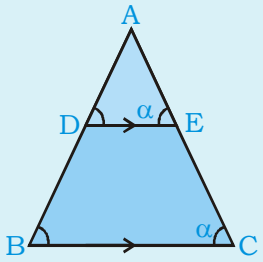
संगत लंबाई के वर्ग का अनुपात।

Thales Theorem (थेल्स प्रमेय)

❖ If a line (DE) is drawn parallel to one side (BC) of triangle then it will divide other two sides in the same ratio. Hence AD : DB = AE : EC

यदि त्रिभुज की एक भुजा (BC) के समांतर एक रेखा (DE) खींची जाए तो वह अन्य दो भुजाओं को समान अनुपात में विभाजित करेगी। अत: AD : DB = AE : EC

DE || BC



ΔADE ~ ΔABC

$$\frac{AD}{AB}=\frac{AE}{AC}=\frac{DE}{BC}$$

If AD : DB = 8:5

$$\text{then } \frac{\text{area }\Delta ADE}{\text{area }\Delta ABC}=\frac{8^2}{13^2}=\frac{64}{169}$$

$$\frac{Ar\Delta ADE}{Ar\square DECB}=\frac{64}{169-64}=\frac{64}{105}$$

Convergence of thales theorem

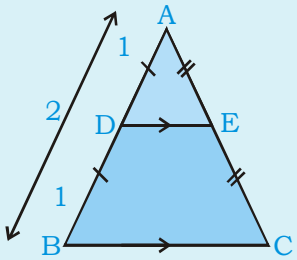
थेल्दिक्प्रु ंक) क्ति (दि बि

If D & E two points on AB and AC such that $\frac{AD}{DB}=\frac{AE}{EC}$ then DE || BC

Mid point theorem (मध्य बिंदु प्रमेय)

❖ The line segment in a triangle joining the mid points of two sides of triangle will be parallel to its third side and is also half of the length of third side.

त्रिभुज की दो भुजाओं के मध्य बिंदुओं को मिलाने वाला रेखाखंड उसकी तीसरी भुजा के समांतर होगा और तीसरी भुजा की लंबाई का आधा भी होगा।



D, E → mid points, AD = DB & AE = EC

DE || BC

$$\Delta ADE \sim \Delta ABC, \qquad DE=\frac{BC}{2}$$

Ar ΔADE : Ar ΔABC = 1 : 4

Ar ΔADE : Ar □ DECB = 1 : 3

❖ **Convergence of mid point theorem**

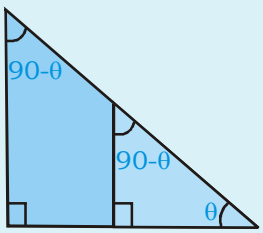
ध्यक्प्रतीक्ष्रु ंक) क्ति (दि बि

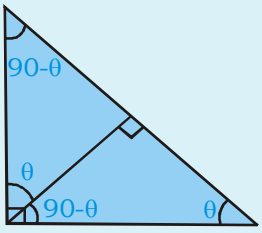
If D is mid point of AB and DE || BC then E will be mid point of AC.

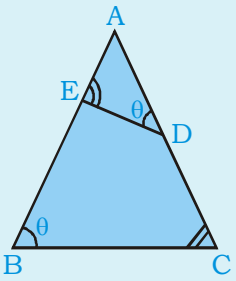
यदि D, AB का मध्य बिंदु है और DE || BC है तो E, AC का मध्य बिंदु होगा।

Similar figures (समरूप आकृतियाँ)

❖







A = common

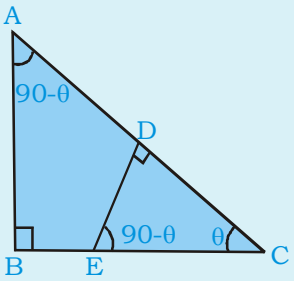
ABC = ADE = θ (given)

∴ 3rd angle will also be equal / तीसरा कोण भी बराबर होगा

∴ **ΔABC ~ ΔADE**

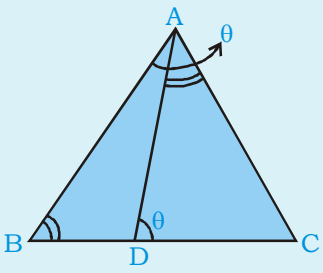
❖ If we make a right angle triangle in an right angle triangle then big and small right triangle are always similar.

यदि हम एक समकोण त्रिभुज में एक समकोण त्रिभुज बनाते हैं तो बड़ा और छोटा समकोण त्रिभुज हमेशा समरूप होते हैं।



ΔABC ~ ΔEDC

❖



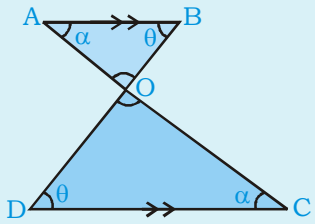
∠C = common

∠A = ∠ADC = θ (given)

∴ 3rd angle will be equal ∠ABC = ∠DAC

∴ **ΔABC ~ ΔDAC**

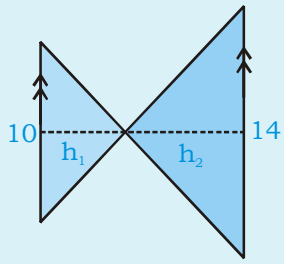
❖



AB || CD

ΔAOB ~ ΔCOD

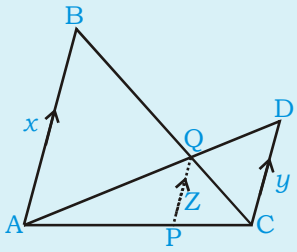
❖



$\frac{h_1}{h_2} = \frac{10}{14}$

$h_1 : h_2 \Rightarrow 5 : 7$

❖



AB || PQ || CD

$Z = \frac{xy}{x+y}$

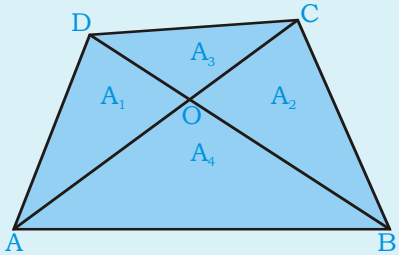
$\frac{z}{x} = \frac{CP}{CA}$ & $\frac{z}{y} = \frac{AP}{AC}$

$\frac{z}{x} + \frac{z}{y} = \frac{CP + AP}{AC} \Rightarrow \frac{z}{x} + \frac{z}{y} = \frac{AC}{AC} = 1$

$\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$ (Result)

In any quadrilateral (किसी चतुर्भुज में)

❖



$\frac{AO}{OC} = \frac{A_1}{A_3} = \frac{A_4}{A_2}$

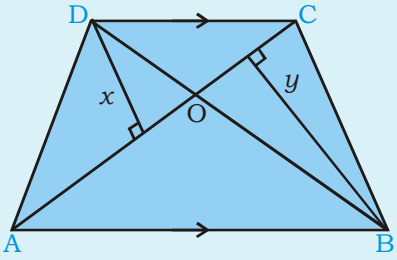
$\mathbf{A_1 \times A_2 = A_3 \times A_4}$

$\frac{A1}{A3} = \frac{A4}{A2}$

A_1, A_2, A_3 and A_4 are the areas of respective triangles.

A_1, A_2, A_3 and A_4 क्रमश: त्रिभुजों के क्षेत्रफल है।

Proof:-



x and y is the heights of triangles.

Area of $\Delta = \frac{1}{2} \times$ Base $\times$ height

Area of triangle $A_1 = \frac{1}{2} \times AO \times x$ ...(1)

Area of triangle $A_2 = \frac{1}{2} \times OC \times y$ ...(2)

Area of triangle $A_3 = \frac{1}{2} \times CO \times x$ ...(3)

Area of triangle $A_4 = \frac{1}{2} \times AO \times y$ ...(4)

Multiply eq (1) and eq (2)

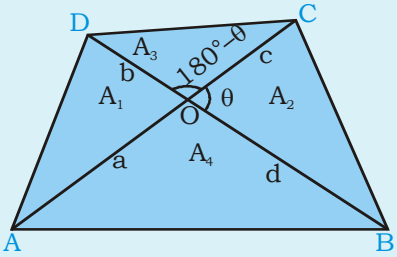
$A_1 \times A_2 = \left(\frac{1}{2} \times AO \times x\right)\left(\frac{1}{2} \times CO \times y\right)$

By commutative property

$A_1 \times A_2 = \left(\frac{1}{2} \times CO \times x\right)\left(\frac{1}{2} \times AO \times y\right)$

$\mathbf{A_1 \times A_2 = A_3 \times A_4}$ Hence proved

Alternatively:-



[∵ sin (180° – θ) = sinθ]

$A_1 = \frac{1}{2} \times a \times b \sin\theta$ ...(1)

$A_2 = \frac{1}{2} \times c \times d \sin\theta$ ...(2)

$A_3 = \frac{1}{2} \times b \times c \sin\theta$ ...(3)

$A_4 = \frac{1}{2} \times a \times d \sin\theta$ ...(4)

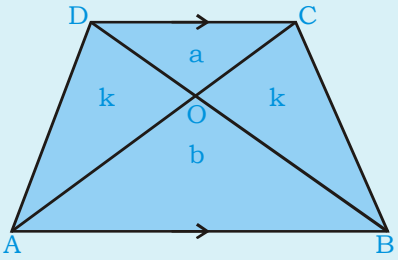
[Multiply eq. (1) and (2) or (3) and (4)]

$\frac{1}{2} \times abcd \sin^2\theta = \frac{1}{2} \times abcd \sin^2\theta$

$A_1 \times A_2 = A_3 \times A_4$ Hence proved

In a trapezium (एक समलम्ब चतुर्भुज में)

❖



Ar Δ ADB = Ar Δ ACB

Common Area = Δ AOB

ΔAOB ~ ΔCOD

∴ Ar Δ AOD = Ar Δ BOC

(In a trapezium the triangle formed on non-parallel sides have equal area)

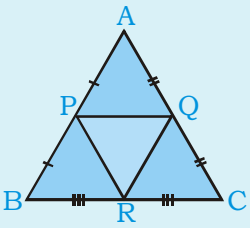
(एक समलंब चतुर्भुज में, असमांतर भुजाओं पर बने Δ का क्षेत्रफल बराबर होता है।)

$K \times K = a \times b$

$K = \sqrt{ab}$

Medial Triangle (मध्य का त्रिभुज)

❖



P, Q, R are mid points / P, Q, R मध्य बिंदु हैं

Perimeter of Δ PQR = $\frac{1}{2}$ × perimeter of ΔABC

Area of Δ PQR = $\frac{1}{4}$ × Area of ΔABC

Congruency of triangle म(अैिू क)ष्वद्महछिद्रु रॉ

Congruency of triangle (त्रिभुज की सर्वांगसमता)

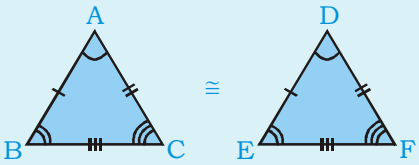
❖ Two triangles are called congruent if all three corresponding sides are equal and all the three corresponding angles are equal.

दो त्रिभुज सर्वांगसम कहलाते हैं यदि तीनों संगत भुजाएँ बराबर हों और तीनों संगत कोण बराबर हों।

❖ Congruency of triangle → size and shape is same
$\Delta ABC \cong \Delta DEF$

त्रिभुज की सर्वांगसमता → आकार व आकृति समान होती है।

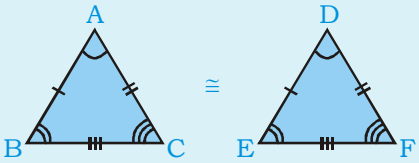
$\Delta ABC \cong \Delta DEF$



Condition of congruency (सर्वांगसमता की शर्त)

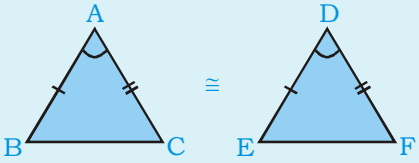
(i) SSS (side-side-side) → Corresponding sides are equal.

SSS क्मै ू गि ू गि ू ि क→ संगत भुजाएं बराबर हो।



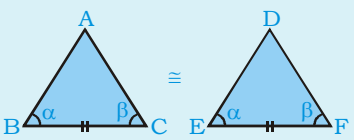
(ii) SAS (side-angle-side) → Two side and angle between them is equal.

SASक्मै ू ि-) बिं- ू ि क→ दो भुजाएं और उनके बीच का कोण बराबर होता है।



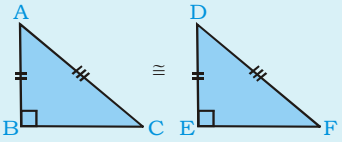
(iii) ASA (Angle-side-angle) → Two angle and side between them is equal.

ASA म) बिगि ू गि) बिं क→ दो कोण और उनके बीच की भुजा बराबर होती है।



(iv) RHS (Right angle-hypotenuse-side) →

RHSक्मदु) बिगि) बिंक्ि ू ि क→



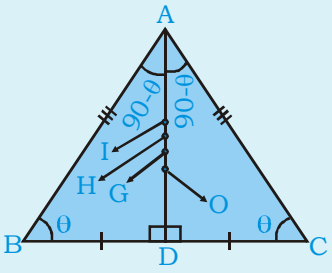
$\Delta ADB \cong \Delta ADC$

AD → ⊥ angle bisector of ∠A / AD → कोण का ⊥ द्विभाजक

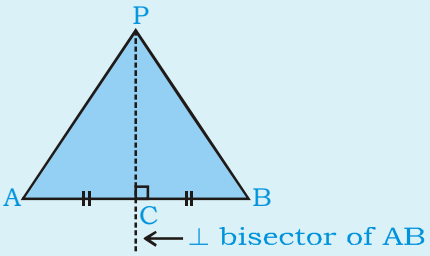
AD → Median (माध्यिका)

AD → ⊥ bisector of BC

AD → All 4 centres lie on AD.



❖



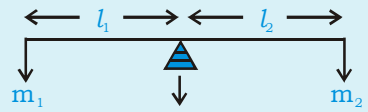
$\Delta PCA \cong \Delta PCB$

PA = PB

P is any point

Mass point geometry (द्रव्यमान बिंदु ज्यामिति)

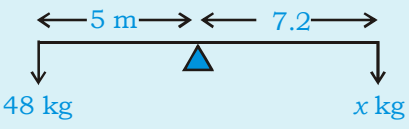
❖



Center of mass(द्रव्यमान केन्द्र)

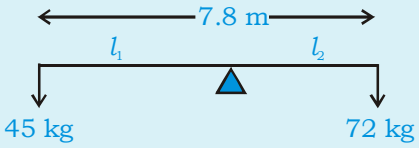
$m_1 l_1 = m_2 l_2$

$$\frac{m_1}{m_2} = \frac{l_2}{l_1}$$



Find x = ?

$\frac{48}{x} = \frac{7.2}{5} \Rightarrow x = \frac{100}{3}$ kg = **33.3 kg**



Find l_1 = ?

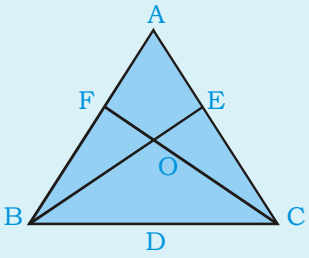
m →	45	:	72
	5	:	8
l →	8	:	5
	↓×0.6		
	4.8 m		

$\therefore$ 13 $\xrightarrow{\times 0.6}$ 7.8

Ceva's Theorem (सेवा की प्रमेय)

❖ In ΔABC, AD, BE and CF are the cevians i.e. any line from vertex to opposite side.

त्रिभुज ABC में, AD, BE और CF केवियन हैं यानी शीर्ष से विपरीत भुजा पर कोई रेखा।



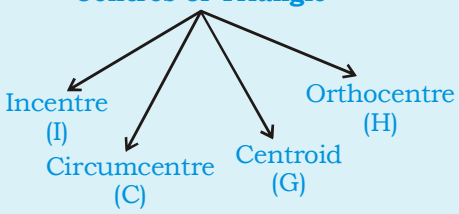
$$\mathbf{\frac{AF}{FB} \times \frac{BD}{DC} \times \frac{CE}{EA} = 1}$$

$$\frac{OD}{AD} + \frac{OE}{BE} + \frac{OF}{CF} = 1$$

$$\frac{AO}{OD} + \frac{BO}{OE} + \frac{CO}{OF} = 2$$

Centre of Triangle म(ऍैंॄ क)ंक)क्रँ

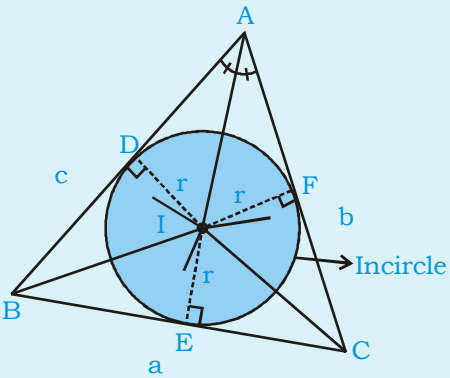
❖ **Centres of Triangle**



Incentre (I) (अन्त:केन्द्र)

❖ Incentre is the intersection point of all three internal angle bisectors of ΔABC.

अन्त:केंद्र ΔABC के सभी तीन आंतरिक कोण समद्विभाजकों का प्रतिच्छेदन बिन्दु है।



Centre of incircle is called incentre and its radius is called inradius (r)

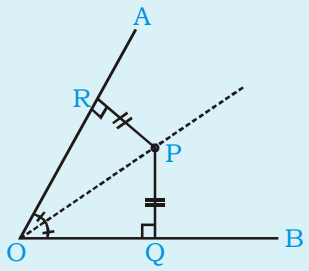
अंत:वृत्त के केंद्र को अंत:केंद्र और इसकी त्रिज्या को अंत:त्रिज्या (r) कहा जाता है

❖ Incentre always lies inside the triangle.

अंत:केंद्र हमेशा त्रिभुज के अंदर स्थित होता है।

❖ Incentre is equidistant from all three sides of triangle.

अंत:केन्द्र त्रिभुज के तीनों ओर से समदूरस्थ है।



P is any point on angle bisector of ∠AOB

P, ∠AOB के कोण द्विभाजक पर कोई बिंदु है

PR = PQ

❖ ID = IE = IF = r, ΔADI ≅ ΔAFI

❖ $\angle BIC = 90° + \frac{\angle A}{2}$

$\angle AIC = 90° + \frac{\angle B}{2}$

$\angle AIB = 90° + \frac{\angle C}{2}$

❖ In ΔABC → $\frac{a+b+c}{2}$ = Semi-perimeter (s)

$\therefore$ Area ΔABC = Δ = r × s

$$\mathbf{r = \frac{\Delta}{s}}$$

$$\text{Inradius} = \frac{\text{Area}}{\text{Semiperimeter}}$$

ΔABC = ΔBIC + ΔAIC + ΔAIB (Area)

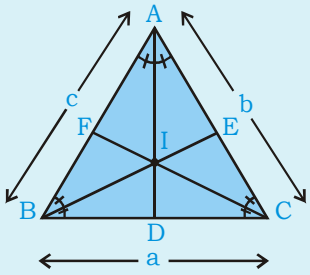
$$\Delta = \frac{1}{2}ar + \frac{1}{2}br + \frac{1}{2}cr = r\left(\frac{a+b+c}{2}\right) = r \times s$$

❖ If altitudes h_1, h_2, h_3 are given then

यदि ऊँचाई h_1, h_2, h_3 दी गई हो तो

$$\mathbf{\frac{1}{r} = \frac{1}{h_1} + \frac{1}{h_2} + \frac{1}{h_3}}$$ (Result)

❖



$$\frac{AI}{ID} = \frac{b+c}{a}$$

$$\frac{BI}{IE} = \frac{c+a}{b}$$

$$\frac{CI}{IF} = \frac{a+b}{c}$$

Proof:-

In ΔACD, $\frac{AI}{ID} = \frac{AC}{CD} = \frac{b}{CD}$ ...(i)

In ΔABC, $\frac{AB}{AC} = \frac{BD}{CD}$

$\frac{c}{b} = \frac{BD}{CD}$

Add 1 both side $\Rightarrow \frac{b+c}{c} = \frac{BD+CD}{CD}$

From eq. (i) $\Rightarrow$ $\mathbf{\frac{b+c}{d} = \frac{AI}{ID}}$

In ΔBAE, $\frac{BI}{IE} = \frac{AB}{AE} = \frac{C}{AE}$ ...(ii)

In ΔBAC, $\frac{BC}{AB} = \frac{EC}{AE}$

Add 1 both side $\Rightarrow \frac{a+c}{c} = \frac{b}{AE}$

$\frac{a+c}{b} = \frac{c}{AC}$

From eq. (ii) $\Rightarrow$ $\mathbf{\frac{a+c}{b} = \frac{BI}{IE}}$

In ΔCBF, $\frac{CI}{IF} = \frac{BC}{BF} = \frac{a}{BF}$ ...(iii)

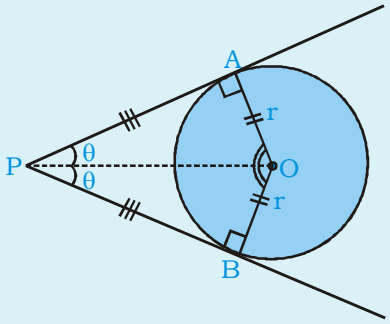
then ΔCAB, $\frac{BC}{AC} = \frac{AF}{BF}$

Add 1 both side

$\frac{b+a}{a} = \frac{c}{BF} \Rightarrow \frac{b+a}{c} = \frac{a}{BF}$

From eq. (iii), $\mathbf{\frac{b+a}{c} = \frac{CI}{IF}}$

❖

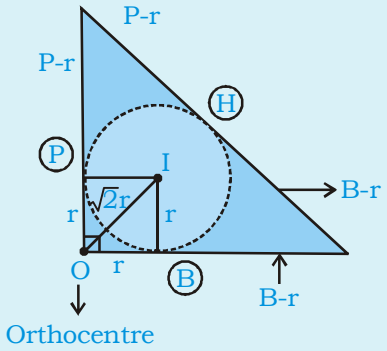


PA = PB

$\Delta AOP \cong \Delta BOP$

In right angle triangle (समकोण त्रिभुज में)

❖



$\mathbf{r = \frac{P+B-H}{2}}$

$P - r + B - r = H$

$P + B - 2r = H$

$\frac{P+B-H}{2} = r$

Distance between orthocentre and incentre = OI

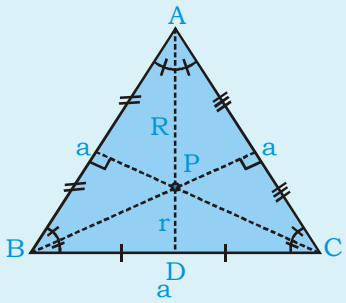
$= \sqrt{2}\mathbf{r}$

लंबकेंद्र और अंत:केंद्र के बीच की दूरी = OI = $\sqrt{2}r$

Sides (triplet)	Radius (r)
भुजाएं (ट्रिपलेट)	त्रिज्या (r)
(3, 4, 5)	r = 1
(3k, 4k, 5k)	r = k
(39, 52, 65)	r = 13

For equilateral Triangle (समबाहु त्रिभुज के लिए)

❖



Point (बिंदु) = I, O, G, H (lie at same place P)

AP = R, PD = r

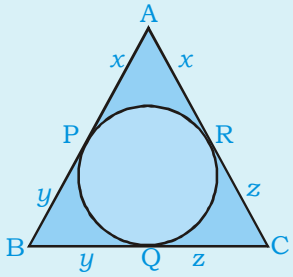
$h = AD = \frac{\sqrt{3}}{2}a$

$r = \frac{\Delta}{s} = \frac{\frac{\sqrt{3}}{4} \times a^2}{\frac{3a}{2}} \Rightarrow r = \mathbf{\frac{a}{2\sqrt{3}}}, \mathbf{r = \frac{h}{3}}$

$\mathbf{R = \frac{2h}{3} = \frac{a}{\sqrt{3}}}$

$\left|\frac{R}{r} = \frac{2}{1}\right|$, $\frac{\text{Area of circumcircle}}{\text{Area of incircle}} = \frac{4}{1}$

❖



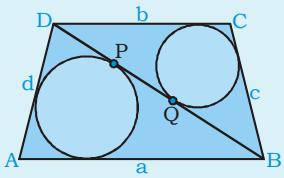
$S = x + y + z$

Area of (Δ) $\Delta ABC = \sqrt{(x+y+z)\cdot xyz}$

$r = \frac{\Delta}{s} = \frac{\sqrt{xyz(x+y+z)}}{(x+y+z)} \Rightarrow \mathbf{r = \sqrt{\frac{xyz}{(x+y+z)}}}$

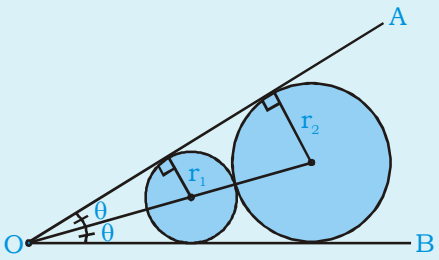
In any quadrilateral (किसी चतुर्भुज में)

❖



$$PQ = \frac{(a+b)-(c+d)}{2}$$

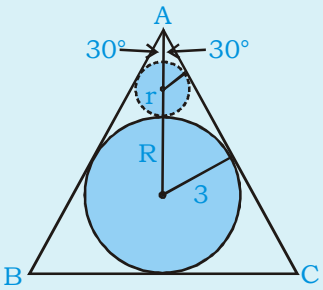
❖



$$\frac{r_1}{r_2} = \frac{1-\sin\theta}{1+\sin\theta}$$

In an Equilateral Triangle (समबाहु त्रिभुज में)

❖

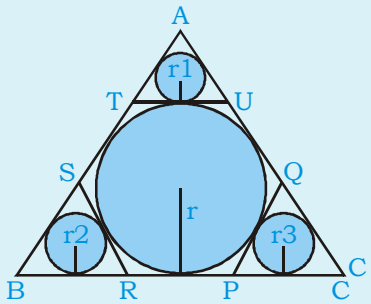


$$\frac{r}{R} = \frac{1-\sin 30°}{1+\sin 30°} = \frac{1-\frac{1}{2}}{1+\frac{1}{2}} = \frac{1}{3}$$

$$\therefore \frac{r}{R} = \frac{1}{3} = \frac{\text{Area small circle}}{\text{Area large circle}} = \frac{1}{9}$$

❖ A circle is inscribed in ΔABC. Three tangents PQ, RS and TU are drawn of this circle PQ || AB, RS || AC and TU || BC. Three other incircles are also drawn as shown in figure find correct relation?

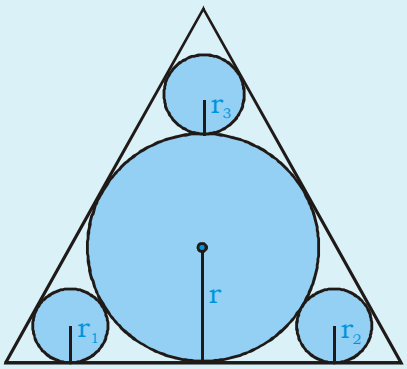
एक वृत त्रिभुज ABC के अंदर बनाया गया है। इस वृत की तीन स्पर्श रेखाएं PQ, RS, और TU इस प्रकार खींची जाती है कि PQ || AB, RS || AC और TU || BC है। तीन अतिरिक्त अन्त: वृत खीचें जाते है जो चित्र में दर्शाए गए है। सही संबंध है:



$r = r_1 + r_2 + r_3$

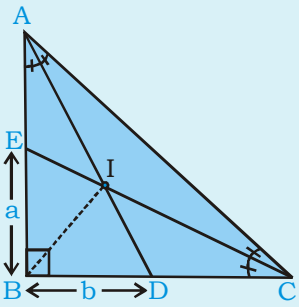
❖ Find relation between r, r_1, r_2, r_3?

r, r_1, r_2, r_3 के बीच संबंध ज्ञात कीजिए?



$$r = \sqrt{r_1 r_2} + \sqrt{r_2 r_3} + \sqrt{r_3 r_1}$$

❖



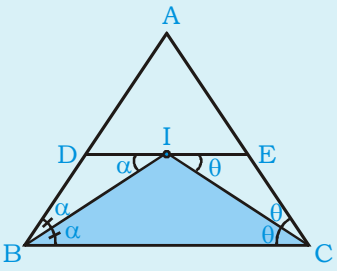
$BI = \sqrt{ab} = \sqrt{2}\,r$

r = radius of ΔABC

$$r = \frac{BI}{\sqrt{2}} = \frac{\sqrt{ab}}{\sqrt{2}}$$

In Any Triangle (किसी त्रिकोण में)

❖



DE || BC

DI = DB, IE = EC

ΔBDI and ΔCEI are isosceles triangles.

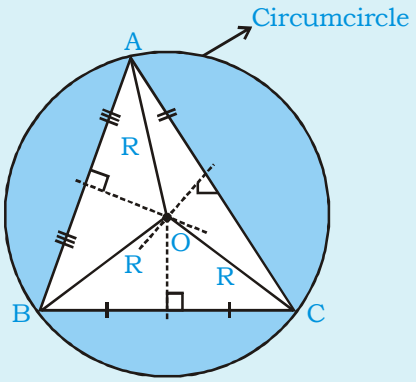
I = Incentre (I)

DE = BD + EC

Circumcentre and Orthocentre मयो)ंद्रका ेंएिक म्प)ंद्रँ

Circumcentre (परिकेन्द्र)

❖ Intersection point of all 3 perpendicular bisectors.
सभी 3 लम्ब समद्विभाजकों का प्रतिच्छेदन बिंदु।



❖ Centre of circumcircle is called circumcentre (O)
परिवृत्त के केंद्र को परिकेन्द्र (O) कहा जाता है

❖ Radius of circumcircle is called circumradius (R)
परिवृत्त की त्रिज्या को परित्रिज्या (R) कहा जाता है।

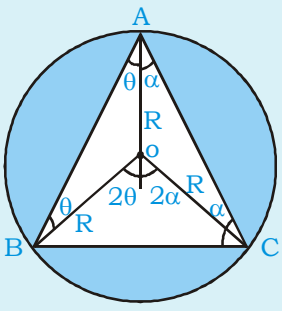
❖ O is equidistant from all 3 vertex of ΔABC
O, ΔABC के सभी 3 शीर्षों से समदूरस्थ है

❖ **OA = OB = OC = R**

❖ Circumcentre may lie inside, outside or on the Δ.
परिकेन्द्र अंदर, बाहर या त्रिभुज पर स्थित हो सकता है।

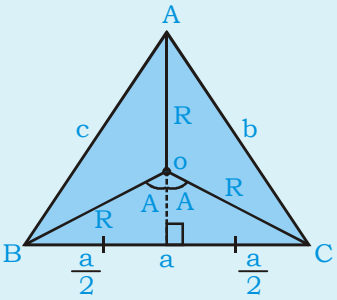
❖ O is the intersection point of all 3 ⊥ bisectors of sides
O सभी 3 ⊥ भुजाओं के समद्विभाजक का प्रतिच्छेदन बिंदु है

❖ ∠BOC = 2∠A; ∠AOB = 2∠C; ∠AOC = 2∠B



∠BOC = 2θ + 2α = 2 (θ + α) = 2∠A

❖ **Circumradius (R) जक्मो (अम्णिR)**



$\sin A = \frac{a}{2R} \Rightarrow \mathbf{R = \frac{a}{2\sin A}}$

$R = \frac{x}{2\sin\theta}$

$R = \frac{a}{2\sin A} = \frac{b}{2\sin B} = \frac{c}{2\sin C}$

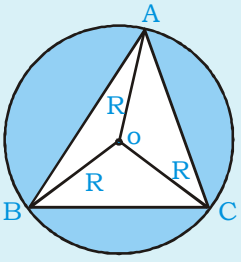
$\therefore \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$

Area of triangle (Δ) $= \frac{1}{2}$ bc sin A $\Rightarrow \sin A = \frac{2\Delta}{bc}$

$\therefore R = \frac{a}{2\times\frac{2\Delta}{bc}} \Rightarrow R = \frac{abc}{4\Delta}$ or $\mathbf{\Delta = \frac{abc}{4R}}$

Position of circumcentre (परिकेंद्र की स्थिति)

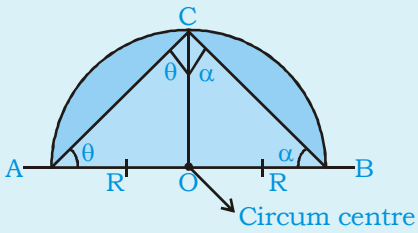
A. Acute angle triangle (Inside the Δ) :
वर्ः क) बिकिंगैी ूि वमΔक) वमा ते ँ वक



OA = OB = OC = R

O = Circumcentre / परिकेन्द्र

B. Right angle triangle (Mid point of hypotenuse):-
द ु) बिकिअैी ूी वम) बबिक) किु ध्यकृप्रतौ



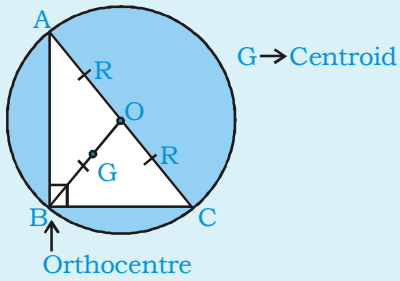
OA = OB = OC = R

AB = Hypotenuse / कर्ण, **R = $\frac{AB}{2}$**

$2\theta + 2\alpha = 180°$

$\theta + \alpha = 90°$

$R = \frac{\text{Hypotenuse}}{2} = \frac{H}{2}$



BO = R = distance between orthocentre and circumcentre = median of hypotenuse = shortest median = $\frac{H}{2}$

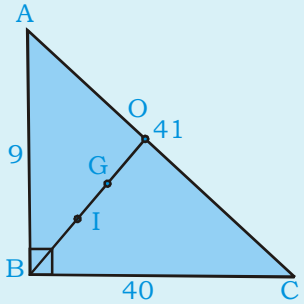
BO = R = लम्बकेन्द्र और परिकेन्द्र के बीच की दूरी = कर्ण की माध्यिका = सबसे छोटी माध्यिका = $\frac{H}{2}$

BG : GO = 2 : 1

GO → distance between centroid and circumcentre = $\frac{R}{3} = \frac{H}{6}$

GO → केन्द्रक और परिकेन्द्र के बीच की दूरी = $\frac{R}{3} = \frac{H}{6}$

Example मउतधि बौं व:



$R = \frac{41}{2} = 20.5$ cm

$r = \frac{9 + 40 - 41}{2} = 4$ cm

BI (Distance between orthocentre and incentre) = $r\sqrt{2} = 4\sqrt{2}$ cm

BI (लम्बकेन्द्र और अन्त:केन्द्र के बीच की दूरी) = $r\sqrt{2} = 4\sqrt{2}$

Distance between orthocentre and circumcentre
⇒ BO = R = 20.5

लंबकेंद्र और परिकेंद्र के बीच की दूरी ⇒ BO = R = 20.5

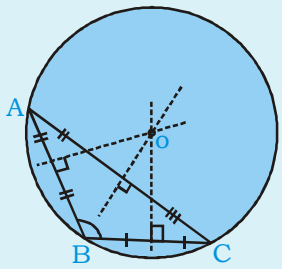
BG (Distance between orthocentre and centroid)

$= \frac{h}{3} = \frac{41}{3} = 13\frac{2}{3}$ cm

BG (लंबकेन्द्र और केन्द्रक के बीच की दूरी) = $\frac{h}{3} = \frac{41}{3} = 13\frac{2}{3}$

सेमी

C. Obtuse angle triangle (Outside triangle)
न (ः) क) बिकिअैी ूी वम(अैी ूी क) वमधिँ



❖ For any triangle distance between incentre and circumcentre (d)
किसी भी प्रकार के त्रिभुज में अंत:केंद्र और परिकेंद्र के बीच दूरी (d)

$d = \sqrt{R^2 - 2R \cdot r}$

Where / यहां

R → Circumradius / परित्रिज्या

r → Inradius / अंत:त्रिज्या

d ≥ zero

$\therefore R^2 - 2Rr \geq O \Rightarrow R^2 \geq 2Rr \Rightarrow R \geq 2r$

$\therefore \frac{R}{r} \geq 2$

$\left(\frac{R}{r}\right)_{min} = 2$

Ex. → r = 6, R = 11.2 cm, find area of Δ

$\frac{R}{r} \geq 2$ Here $\frac{11.2}{6} < 2$; No Δ is possible.

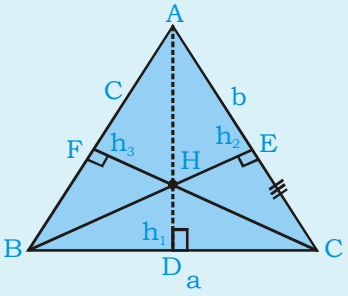
∴ Area cannot be determined.

(क्षेत्रफल निर्धारित नहीं किया जा सकता है)

Orthocentre (लम्बकेन्द्र)

❖ Orthocentre (H) is intersection point of all there altitude of a Δ and may lie inside, upon and outside the Δ.

लम्बकेन्द्र (H) Δ की सभी ऊंचाई का मिलन बिंदु है और यह Δ के अंदर बाहर या शीर्ष पर स्थित हो सकता है।



Area $\Delta ABC = \frac{1}{2} ah_1 = \frac{1}{2} bh_2 = \frac{1}{2} ch_3$

$ah_1 = bh_2 = ch_3 = 2 \times \text{Area} = \text{Constant}$

$\mathbf{h_1 : h_2 : h_3 = \frac{1}{a} : \frac{1}{b} : \frac{1}{c}}$

❖ AB + BC + CA > AD + BE + CF;

AB > AD, BC > BE, AC > CF

AB + BC + AC > AD + BE + CF

❖ $\angle BHC = 180° - \angle A$

$\angle AHC = 180° - \angle B$

$\angle AHB = 180° - \angle C$

In Quad AFHE

$\angle F = \angle E = 90°$ each

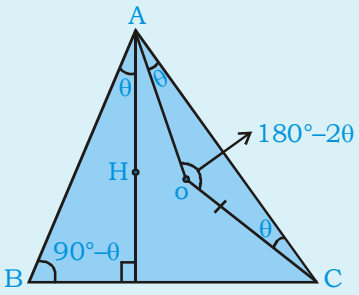
$\angle A + \angle H = 180°$

$\angle H = 180° - \angle A$

$\therefore \mathbf{\angle BHC = 180° - \angle A}$ (V . opp. Angles)

❖ **H → Orthocentre** जक म्प)ंक्र

O → Circumcentre जक्रम्रे)ंक्र

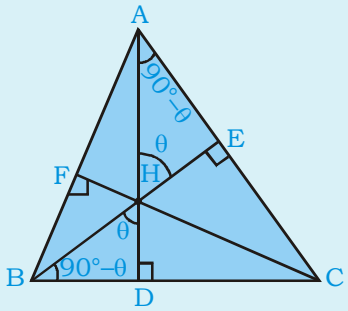


$\angle AOC = (90 - \theta) \times 2 = 180 - 2\theta$

$\angle OAC = \angle OCA = \theta$

$\therefore \mathbf{\angle BAH = \angle OAC}$

❖



$AH \times HD = BH \times HE = CH \times HF$

$\Delta AHE \sim \Delta BHD$

$\frac{AH}{BH} = \frac{HE}{HD}$

$\Rightarrow AH \times HD = BH \times HE$

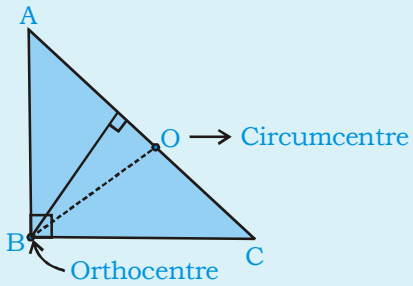
❖ **Position of orthocentre** म- म्प)ंक्रक) प्क्रस्थ(रँ

A. Acute angle triangle:- Always inside the triangle

A.क्रम्र:) बिक्रिअैी ाूी ागक्रद तह्क्रकअैी ूी क) ंक्रम्र ते क्रम्रारि क्रिाए

B. Right angle triangle (On 90° vertex):

B.क्रद्रु) बिक्रिअैी ाूी क्रम्र90° कप्क्रिक्रिम्रे ँ ा



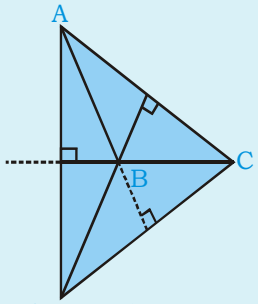
AB ⊥ BC

OB = Distance between orthocentre and circumcentre = R = $\frac{H}{2}$

OB = लम्बकेन्द्र और परिकेन्द्र के बीच की दूरी = R = $\frac{H}{2}$

❖ **Obtuse angle triangle (Outside triangle):-**

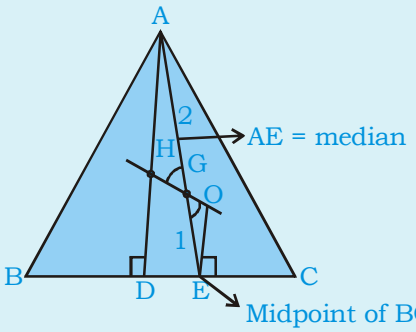
न (भ)) बिक्रिअैी ाूी क्रम्र(अैी ूी क) ंक्रम्राधिँ ाग



(Orthocentre)

Euler's line (यूलर की रेखा)

❖ In any triangle H, G, O always in a straight line
किसी भी त्रिभुज में H, G, O हमेशा एक सीधी रेखा में होंगे



AE = median
E is mid point of BC.

$$\frac{HG}{GO} = \frac{2}{1}$$

$\Delta AHG \sim \Delta EOG$

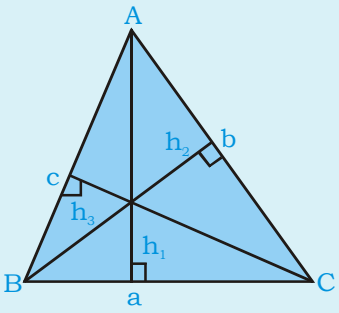
$$\frac{HG}{GO} = \frac{2}{1} = \frac{AH}{OE}$$

❖ If h_1, h_2, h_3 are 3 altitudes of a triangle
यदि h_1, h_2, h_3 एक त्रिभुज के 3 शीर्षलंब हैं

Then $\frac{h_1h_2}{h_1+h_2} < h_3 < \frac{h_1h_2}{h_1-h_2}$

$$\Delta = \frac{1}{2}ah_1 = \frac{1}{2}bh_2 = \frac{1}{2}ch_3$$

$$a = \frac{2\Delta}{h_1},\ b = \frac{2\Delta}{h_2},\ c = \frac{2\Delta}{h_3}$$



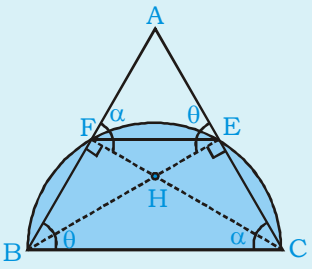
$|a-b| < c < |a+b|$

$$\left|\frac{2\Delta}{h_1}-\frac{2\Delta}{h_2}\right| < \left|\frac{2\Delta}{h_3}\right| < \left|\frac{2\Delta}{h_1}+\frac{2\Delta}{h_2}\right|$$

$$\frac{h_2-h_1}{h_1h_2} < \frac{1}{h_3} < \frac{h_2+h_1}{h_1h_2}$$

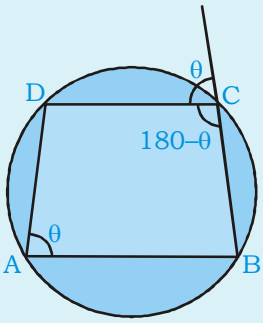
$$\frac{h_1h_2}{h_1-h_2} > h_3 > \frac{h_1h_2}{h_1+h_2}$$

❖



$\Delta AEF \sim \Delta ABC$

❖ Interior angle of a cyclic quadrilateral is equal to the opposite exterior angle.
चक्रीय चतुर्भुज का आंतरिक कोण विपरीत बाहरी कोण के बराबर होता है।



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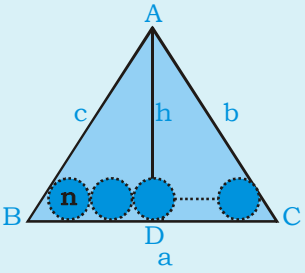


n circle of equal radii r are made on side 'BC'.
समान त्रिज्या r वाले n वृत्त 'BC' भुजा पर बनाए गए हैं।

$$\mathbf{r = \frac{Area\ \Delta\ ABC}{s+(n-1)\cdot h}}$$

s = semi-perimeter/अर्ध परिमाप

Centroid (पतेपक

Centroid (रेखर

❖ Centroid is the intersection point of all 3 medians of triangle.

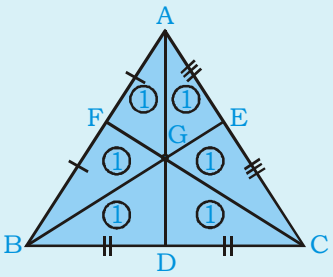
हू न्यमीकृतिय त्यक्रीत्य3यवीऋए ेकीृश ेयत्अ()शे यऋृश्रविंद

❖ Median divides the triangle into two equal areas.

वीऋए ेयमीकृतिय ेशरीशक्र वी यु ेशीृशवृशऋआकृीऋ।य हा त्यबंद

❖ Centroid always lies inside the triangle.

ृशै यबवशीयमीकृतिय शक्रर हयर्ऋ ेा। यवीशीयबंद

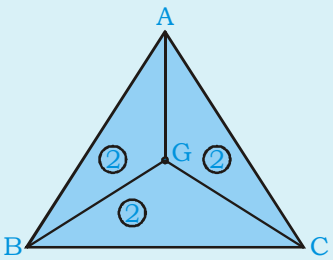


❖ 6 triangle made by 3 medians have equal area.

3यवीऋए ेकीृशनीहेयखे ेचयधचय6 मीकृतिय ेयु ेशिं ष्ट यखहीखहयवीशीयबंद

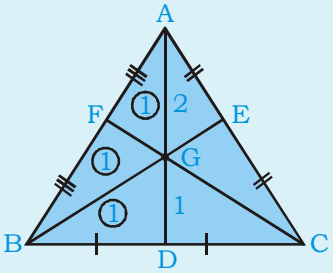
❖ Area of each triangle = $\frac{1}{6}$ Area of ΔABC

Area ΔAGC = Area ΔAGB = Area Δ BGC = $\frac{1}{3}$ Area ΔABC

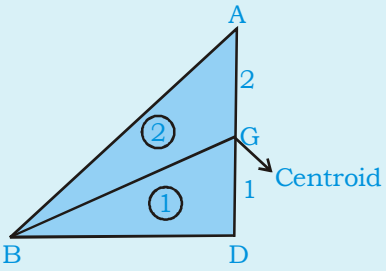


❖ Centroid always divides median in the ratio 2 : 1.

ृशै यबवशीयवीऋए ेय ेश2 : 1य शके ेंा। यवृशऋआकृीऋ।य हा यबंद



$\frac{AG}{GD} = \frac{BG}{GE} = \frac{CG}{GF} = \frac{2}{1}$



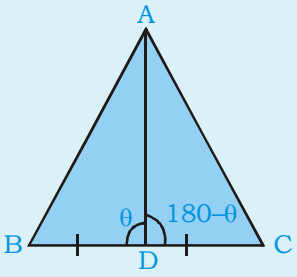
$AG = \frac{2}{3} AD$

$GD = \frac{1}{3} AD$

Apollonius theorem (अै वेक्वेकेंबन्दु पेब

❖ To find length of median.

वीऋए ेय त्यटगखे–यस्ी।य हे श शऋच



cos (180°–θ) = – cos θ

$$\mathbf{AB^2 + AC^2 = 2\left(AD^2 + \frac{BC^2}{4}\right)}$$

$\Rightarrow AB^2 + AC^2 = 2 (AD^2 + BD^2)$

$\Rightarrow AB^2 + AC^2 = 2 (AD^2 + CD^2)$ (as BD = CD)

Proof:-

cos (180°– θ) = –cosθ

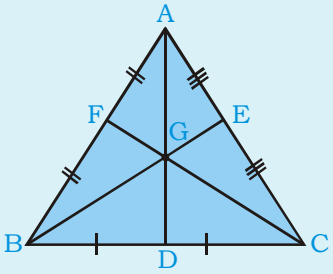
$AB^2 = AD^2 + BD^2 – 2AD . BD \cos\theta$

$AC^2 = AD^2 + CD^2 + 2AD . CD \cos\theta$

$AB^2 + AC^2 = 2 AD^2 + BD^2 + CD^2$

$AB^2 + AC^2 = 2 (AD^2 + BD^2)$

❖



$AB^2 + AC^2 = 2\left(AD^2 + \frac{BC^2}{4}\right)$

$AB^2 + BC^2 = 2\left(BE^2 + \frac{AC^2}{4}\right)$

$AC^2 + BC^2 = 2\left(CF^2 + \frac{AB^2}{4}\right)$

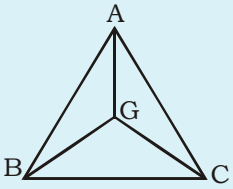
$2(AB^2 + BC^2 + AC^2) = 2AD^2 + \frac{BC^2}{2} + 2BE^2 + \frac{AC^2}{2} + 2CF^2 + \frac{AB^2}{2}$

$3(AB^2 + BC^2 + CA^2) = 4(AD^2 + BE^2 + CF^2)$

$\frac{AB^2 + BC^2 + CA^2}{AD^2 + BE^2 + CF^2} = \frac{4}{3}$

$1 < \frac{AB + BC + CA}{AD + BE + CF} < \frac{4}{3}$

❖



$GB + GC > BC$
$GA + GC > AC$
$AG + BG > AB$

$AG = \frac{2}{3} AD$

$BG = \frac{2}{3} BE$

$CG = \frac{2}{3} CF$

$2(AG + BG + CG) > AB + BC + CA$

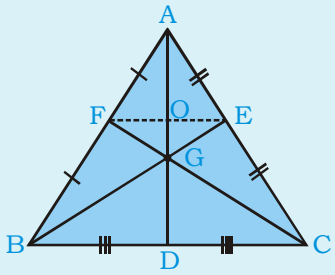
$2 \times \frac{2}{3}(AD + BE + CF) > AB + BC + CA$

$4(AD + BE + CF) > 3(AB + BC + CA)$

❖ $AB + AC > 2AD$
$AB + BC > 2BE$
$AC + BC > 2CF$
$2(AB + BC + CA) > 2(AD + BE + CF)$
$\therefore AB + BC + AC > AD + BE + CF$

$\therefore$ **$\frac{AB + BC + AC}{AD + BE + CF} > 1$**

❖



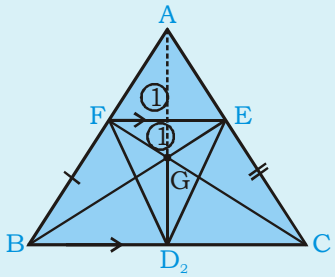
AO : OG : GD = 3 : 1 : 2
BE and CF are medians.
$\Delta AFE \sim \Delta ABC$
Let AD = 6 unit
$\therefore$ AG : GD = 2 : 1
$\therefore$ AG = 4 unit and GD = 2 unit

F, E are mid points $\Rightarrow$ AO = OD = $\frac{6}{2}$ = 3 unit

OG = AG – AO = 4 – 3 = 1 unit
$\therefore$ AO : OG : GD = 3 : 1 : 2

Area of Δ GEF = $\frac{1}{12}$ Area Δ of ABC

❖



$\Delta EFG \sim \Delta BCG$ $\qquad$ $EF = \frac{BC}{2}$

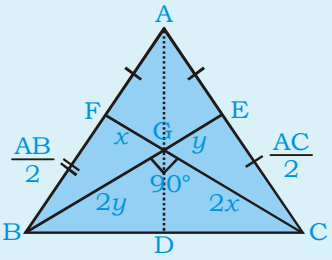
Side $\rightarrow$ 1 : 2
Area $\rightarrow$ 1 : 4
Area ΔABC = Ar $\Delta BGC \times 3 = 4 \times 3 = 12$

$\therefore$ **$\frac{\Delta GEF}{\Delta ABC} = \frac{1}{12}$**

Area ΔEFG = Area ΔDFG = Area ΔDEG = 1
$\therefore$ Area ΔDEF = 1 + 1 + 1 = 3 unit

$\therefore$ **$\frac{\text{Area } \Delta \text{ DEF}}{\text{Area } \Delta \text{ ABC}} = \frac{3}{12} = \frac{1}{4}$**

❖ If two medians of a Δ intersect each other at 90°
एअय∆य ञ्यरीश्वीअए ीचश्चच यरुहश्च ीश90°यँ हयत्अ()श्य हा ञ्यबृं



$BE^2 + CF^2 = AD^2$ (AD = 3rd median)

$AB^2 + AC^2 = 5BC^2$

$$x^2 + 4y^2 = \frac{AB^2}{4}$$

$$y^2 + 4x^2 = \frac{AC^2}{4}$$

$$5(x^2 + y^2) = \frac{AB^2 + AC^2}{4}$$

$5(4x^2 + 4y^2) = AB^2 + AC^2$

$\Rightarrow 5BC^2 = AB^2 + AC^2$ (Property)

❖ **Special case in isosceles Δ**

ो ं ि लोब Δ देृ ुं : रखो लेडल

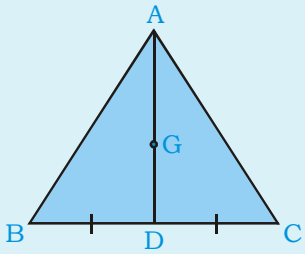
$AB = AC \Rightarrow BE = CF$

$AB^2 + AC^2 = 5BC^2$

$2AB^2 = 5BC^2$

$$\Rightarrow \left(\frac{AB}{BC}\right)^2 = \frac{5}{2} \Rightarrow \mathbf{\frac{AB}{BC} = \frac{\sqrt{5}}{\sqrt{2}}}$$

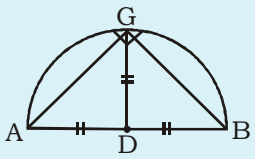
❖



If AG = BC or AD = 1.5 BC

$\Rightarrow \angle BGC = 90°$

Proof:-



G = centroid

AD = AG + GD

$\angle BGC = 90°$

$\frac{AG}{GD} = \frac{2}{1}$ by centroid property and AD is median

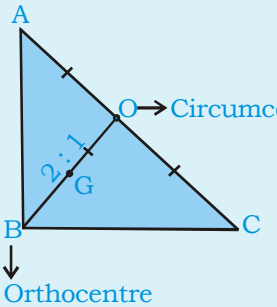
Hence, AG = BC

$\therefore \frac{2}{3} AD = BC$

$\therefore$ **$AD = \frac{3}{2} BC = 1.5\ BC$**

Right Angle Triangle (्र पर केव्र ेखव्र

❖



↓

Orthocentre

AO = OC = OB = R

BO = Hypotenuse median = Shortest median

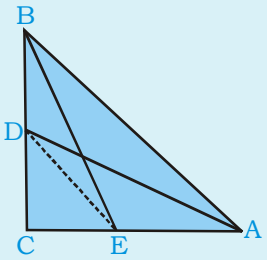
BO ण्य :ोय ्यीआए ोयण्य ख्र श)ोछ्यीआए ो

$= R = \frac{H}{2}$

$BG = \frac{H}{3}$, $GO = \frac{H}{6}$

❖ If D, E are two points on BC and AC respectively.

एअयD, E, यBC यकींहय AC यँ हयई ो। यो श्यृ खिृंद



$AD^2 = CD^2 + CA^2$

$BE^2 = CE^2 + BC^2$

$$AD^2 + BE^2 = \underbrace{CD^2 + CE^2}_{DE^2} + \underbrace{CA^2 + BC^2}_{AB^2}$$

$\therefore$ **$AD^2 + BE^2 = DE^2 + AB^2$**

If D, E are mid points, AD and BE are medians,

then $DE = \frac{AB}{2} = R$

एअयD, E यकल ए यश्यृ खिृंय AD यकींहय BE यकीआए ो चि श्यृ ंय ो $DE = \frac{AB}{2} = R$

$AD^2 + BE^2 = DE^2 + AB^2$

$$AD^2 + BE^2 = \frac{AB^2}{4} + AB^2$$

$$AD^2 + BE^2 = \frac{5AB^2}{4}$$

$\Rightarrow$ **$4(AD^2 + BE^2) = 5AB^2$**

$\Rightarrow AB = 2R$

$AD^2 + BE^2 = \frac{5}{4} \times 4R^2$

$\therefore\ AD^2 + BE^2 = 5 \times R^2$

$AD^2 + BE^2 = 5R^2$

❖ Area of $\Delta ABC = \frac{4}{3}$ × Area of triangle made by using length of all 3 medians.

ΔABC ियुोसैं ष्टय= $\frac{4}{3}$ ×यक्कृत्य3यीज्ञए ेकीृश त्रट्खे-य ेयिहँ एीश ह श्रखे ेएीयथएीयअीवृीयि य ियुोसैं ष्टद

Example (एसल्ेप्मनत्वक्त्र:

Length of 3 medians 15, 36, 39. Then find the area of triangle ABC.

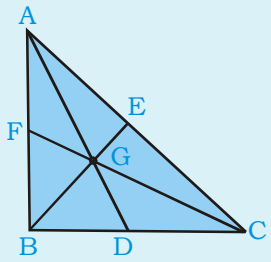
3यीज्ञए ेकीृश त्रट्खे-या 15, 36, 39यथंय ेश्रअीवृीयिABC ियुोसैं ष्ट सी।य ह्श्र

Area of triangle made by 15, 36, 39 (triplets)

$= \frac{1}{2} \times 15 \times 36 = 270$

Area of ΔABC (main Δ) $= \frac{4}{3} \times \left(\frac{1}{2} \times 15 \times 36\right) = 360\ cm^2$

❖



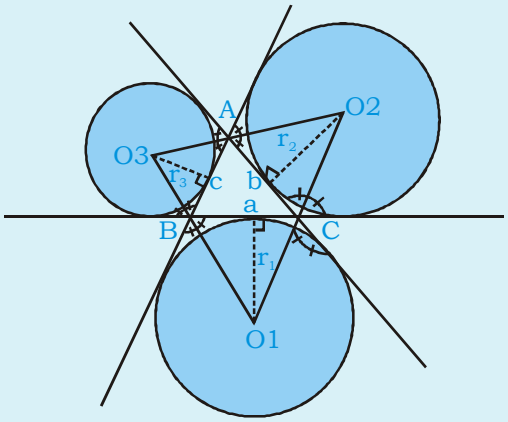
$BE = R = \frac{AC}{2}$

$AD^2 + CF^2 = 5.R^2$

$AG^2 + CG^2 = 5BG^2$

Exocentre (ि र्क र ेम

❖



➢ $r_1 = \frac{\Delta}{S-a}$

$r_2 = \frac{\Delta}{S-b}$

$r_3 = \frac{\Delta}{S-c}$

➢ $\angle BO_1C = 90° - \frac{\angle A}{2}$

➢ $r_1 = \frac{rs}{s-a} = \sqrt{\frac{s(s-b)(s-c)}{s-a}}$

Where, $s = \frac{a+b+c}{2}$

➢ $r_1 r_2 + r_2 r_3 + r_3 + r_3 r_1 = s^2$

➢ If we make incircle in Δ ABC with radius. r then r $= \frac{\Delta}{S}$

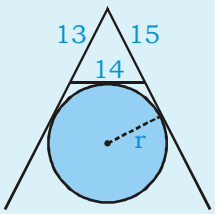
एअयवयअीज्ञएीयाय श्रर्ी ीय Δ ABC वृश्सप्ीयखे ी। श्रवृंद्या ीश्र $= \frac{\Delta}{S}$

$r \cdot r_1 \cdot r_2 \cdot r_3 = \frac{\Delta}{S} \cdot \frac{\Delta}{S-a} \cdot \frac{\Delta}{S-b} \cdot \frac{\Delta}{S-c} = \frac{\Delta^4}{\Delta^2}$

$r \cdot r_1 \cdot r_2 \cdot r_3 = \Delta^2$

Area (Δ) of ΔABC $= \sqrt{r.r_1r_2r_3}$

Example (एसल्ेप्मनत्वक्त्र:



Find r = ?

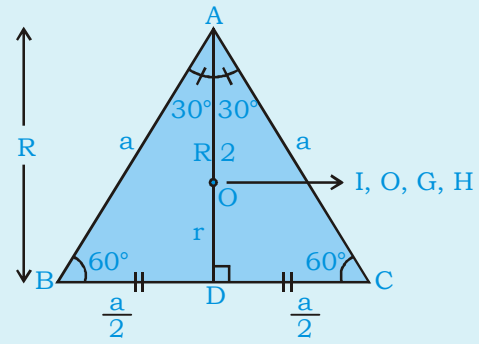
$\Delta = 84\ cm^2$, $S = 21$

$r = \frac{84}{21-14} = 12\ cm$

Equilateral triangle (ो िलोबं एंलत्वाक

Equilateral Triangle (ॅ पी कव ेक्रकेक

❖ All sides are equal. ऊज कृैत्यकृीिचय़ॅ वी यबृंद



AD → median, angle bisector, altitude, ⊥ bisector

of $\Delta ABC = \frac{\sqrt{3}}{2}a$

AD →यΔABC त्यवीआए ोन्य ोशियअकृीत न्याप्र ृो–प्य⊥यअकृीत

ण्य $\frac{\sqrt{3}}{2}a$

$\Delta ADB \cong \Delta ADC$

$h = \frac{\sqrt{3}}{2}a$ $\quad$ Area $= \frac{\sqrt{3}}{4}a^2$

All centre lies at same point.

ज कृैत्य ूश्यच यबत्यअॅूग्रं हय्यइ ो। यबृं

Circumradius (R) $= \frac{a}{\sqrt{3}}$, $\quad R = \frac{2}{3}h$,

Inradius (r) $= \frac{a}{2\sqrt{3}}$, $\quad r = \frac{1}{3}h$

$\frac{R}{r} = \frac{2}{1}\left|\frac{\text{Area of circumcircle}}{\text{Area of incircle}}\right| = \frac{4}{1}$

❖ All 3 medians ऊज कृैत्य3यवीआए ोच,= $\frac{\sqrt{3}}{2}a$ = Height

❖

Side ॅ दँ ल्बाल ↓	Height ॅ दअुै लूऽ ↓	Area ॅ दश्लाल ↓
a	$\frac{\sqrt{3}}{2}a$	$\frac{\sqrt{3}}{4}a^2$
2 cm	$\sqrt{3}$ cm	$\sqrt{3}$ cm^2
2 k	$\sqrt{3}$ k	$\sqrt{3}$ k^2
28 cm	$14\sqrt{3}$	$196\sqrt{3}$
$6\sqrt{6}$	$\sqrt{3}\cdot 3\sqrt{6} = 9\sqrt{2}$	$\sqrt{3}\cdot\left(3\sqrt{6}\right)^2 = 54\sqrt{3}$

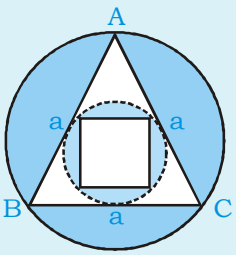
❖

Circumcircle radius ↓		Side of equilqteral Δ ↓		In radius ↓
R	:	a	:	r
2	:	$2\sqrt{3}$ (Let)	:	1
Area → 4π	:	$3\sqrt{3}$	:	π

❖ Circum circle : Triangle : Incircle : Square

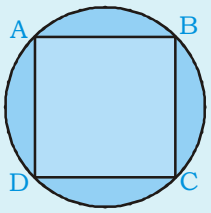
यययय्यँ ऊअसणैययययय्ययययय्ययअीकृीत ययययय्ययक्कृ। थ्यसणैययय्ययय्यसधप

Area → 4π : $3\sqrt{3}$: π : 2

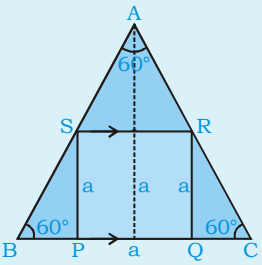


❖ Circle=सणैड : Square=सधड

A → π : 2 or 11 : 7



❖ Δ ABC is an equilateralΔ. Δ ABCच यज वखेबघिΔ बंद



Side of Equilateral $\Delta = a\left(1+\frac{2}{\sqrt{3}}\right)$

Side of Equilateral Δ : Side of Square

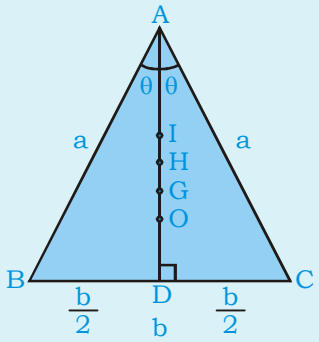
ज वखेबघिΔय त्यक्रीती थ्यसधय त्यक्रीती

$\left(\sqrt{3}+2\right)$: $\sqrt{3}$

Isosceles Triangle (¬ प श्री कव ब्रेक्स क्रेक

❖ Triangle in which any two sides are equal.

सबयश्रीमेट्रि यज्ञ ज त्र ो श्यक्रीज्य श्रीमेट्रि चत्र्यक्षीक्ष्यबीक्ष्य



AB = AC

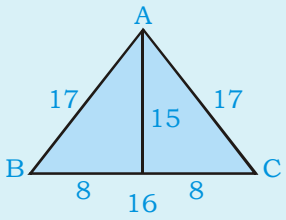
All 4 centre lies on line AD.

ज क्रीज्य4 ूश्यह्र ो यAD यँ ह्यर्ब्ब ो। यबूं

$$AD = \sqrt{a^2 - \frac{b^2}{4}} = \frac{\sqrt{4a^2 - b^2}}{2} = \text{Height}$$

$$\text{Area} = \frac{b}{4}\sqrt{4a^2 - b^2}$$

Ex:- Find area 17, 17, 16 side

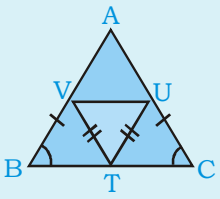


Area of Δ ABC = 15 × 8 = 120 cm^2

❖ The triangle formed by joining the mid-point of an isosceles triangle is also an isosceles Δ.

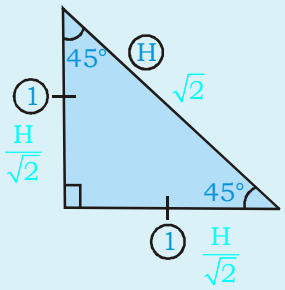
अ ज ञ्य वअखेबग्रिΔय श्वल्एयश्रूक्रीश ो श्वत्र्यटी ह्यर्ब्ब ो यΔयक्रीज्य वअखेबर्ब अीमेट्रि यबीशीयबृंद

PQ = PR ST = TU



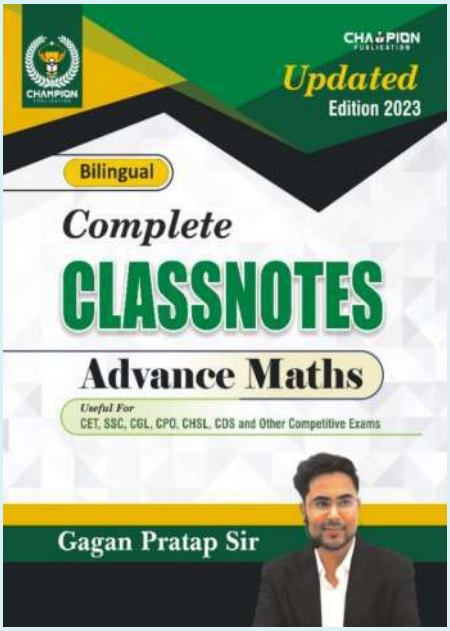
Isosceles Right Angle Triangle
(¬ प श्री कव पर ब्रेक्स क्रेक्स

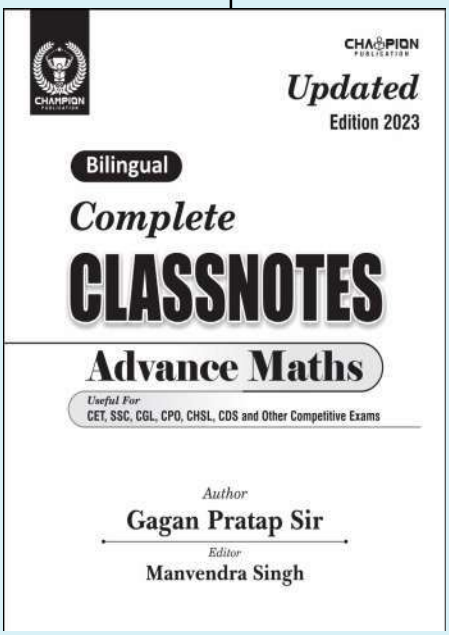
❖



$$\text{Area} = \frac{1}{2} \times \frac{H}{\sqrt{2}} \times \frac{H}{\sqrt{2}} = \frac{H^2}{4}$$

Perimeter ऊँ ज्ञ्वीं य= $H\left(\sqrt{2}+1\right)$



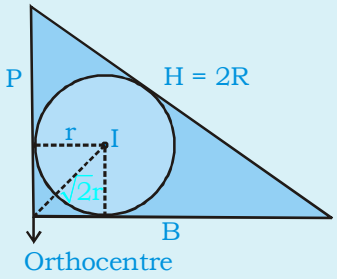


Right angle Triangle (ा पत्रलं एँलत्राक

Right Angle Triangle (् पर ेकव्ह ेखत्रव

❖ Triangle in which one angle is 90°.
अीकृीतियज्ञज्ञ वृश्चच य ेशीय90°य ेयबीशीयबंद

❖ Right angle triangle is inscribed in a semi-circle.
ज्व ेशीयअीकृीतियच यकछपाशीयवृश्चखे ेयबक्रीयबीशीयबंद



$$r = \frac{P + B - H}{2}$$

$2r = P + B - H$

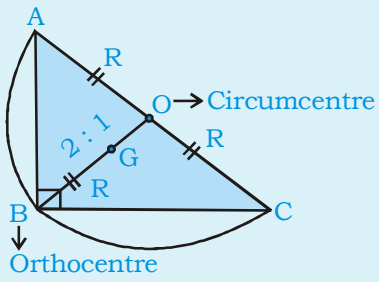
$2r + H = P + B$

$2r + 2R = P + B$

$2r + 2R = P + B$

$$\mathbf{r + R = \frac{P + B}{2}}$$

$$R = BO = \text{Shortest median} = \frac{H}{2}$$

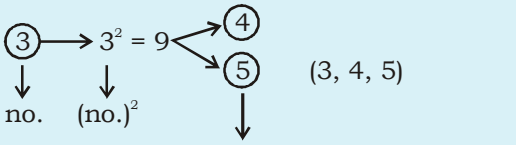


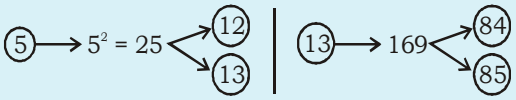
$$BG = \frac{2}{3}R = \frac{H}{3}$$

$$GO = \frac{R}{3} = \frac{H}{6}$$

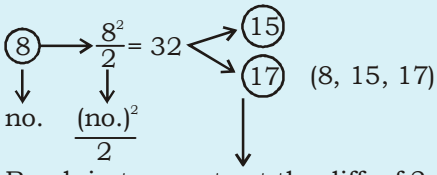
How to Find Triplet? (ेकसे) दप्रद ेयक दर ेदभ ेखोज

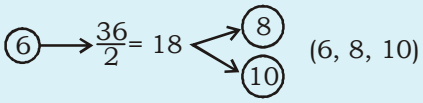
❖ **Odd number** ं दं : खेा दा ुजधल





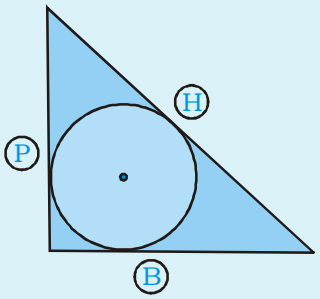
❖ **Even number** ं दो दा ुजधल





❖ $r = \frac{P + B - H}{2} = \frac{(P + B + H)}{2} - H$

$r = S - H = S - 2R \quad \Rightarrow S = r + 2R$



$\Delta = r \cdot S$

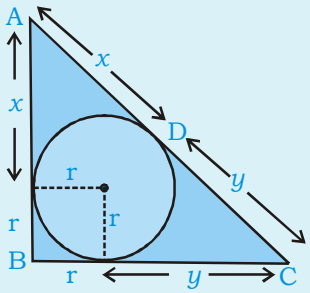
$\Delta = S\,(S - 2R)$ Put $r = S - 2R$

$\Delta = r \cdot S = r\,(r + 2R) \Rightarrow \Delta = r^2 + 2rR$

Put $S = r + 2R$

$\therefore$ Area of right angle triangle $\Rightarrow$

$$\mathbf{\frac{1}{2}PB = r \times S = S(S - 2R) = r^2 + 2rR.}$$



Area of Δ ABC = $x \times y$

$s = r + x + y$

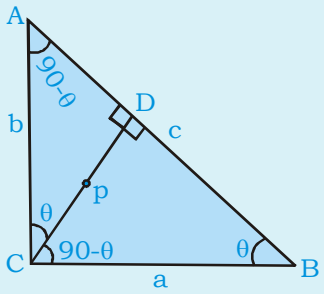
$\Delta = r \cdot (r + x + y) = xy$

$(x + r)^2 + (y + r)^2 = (x + y)^2$

$x^2 + r^2 + 2rx + y^2 + r^2 + 2yr = x^2 + y^2 + 2xy$

$2r\,(r + x + y) = 2xy \Rightarrow r\,(r + x + y) = xy$

❖



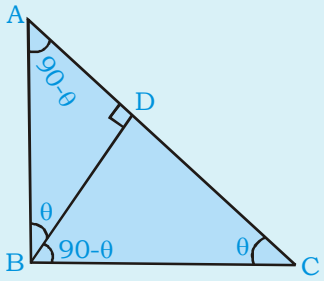
$\Delta ACB \sim \Delta CDB \sim \Delta ADC$

$\frac{1}{2}\,ab = \frac{1}{2}\,c.p \Rightarrow \mathbf{p = \frac{ab}{c}}$

$\frac{1}{p} = \frac{c}{ab} \Rightarrow \frac{1}{p^2} = \frac{c^2}{a^2b^2} \Rightarrow \frac{1}{p^2} = \frac{a^2 + b^2}{a^2b^2}$

$\therefore \mathbf{\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}}$

❖



$\Delta ADB \sim \Delta ABC \Rightarrow \frac{AB}{AC} = \frac{AD}{AB}$

$AB^2 = AD \times AC$

$\Delta BDC \sim \Delta ABC \Rightarrow \frac{BC}{AC} = \frac{CD}{BC}$

$BC^2 = CD \times AC$

$\mathbf{\frac{AB^2}{BC^2} = \frac{AD}{CD}}$

$\Delta ABD \sim \Delta BDC$

$\Rightarrow \frac{BD}{CD} = \frac{AD}{BD} \Rightarrow \mathbf{BD^2 = AD \times CD}$

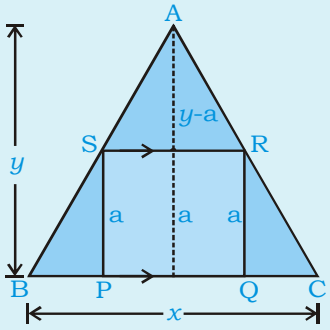
$\mathbf{BD = \frac{AB \times BC}{AC}}$

Maximum Area of a Square Inscribed in a Triangle

(धर द ेवाकव दपेदअये ं दधर दच्ह धर व्दअ ेार ं पद/वैकल्पिक

❖ **Side of square (a) = $\frac{xy}{x + y}$**

$\Delta ABC \sim \Delta ASR$

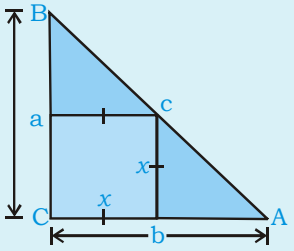


$\frac{a}{x} = \frac{y - a}{y}$

$ay = xy - ax$

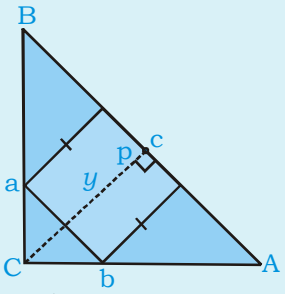
$a\,(x + y) = xy$

❖ Maximum size square inside right angle Δ.



Side of square (x) = $\frac{ab}{a + b}$

❖



c = hypotenuse

p = length of perpendicular

y = Side of square

$y = \frac{cp}{c + p} = \frac{c \times \frac{ab}{c}}{c + \frac{ab}{c}} = \frac{abc}{c^2 + ab}$

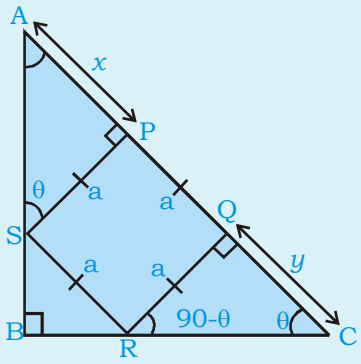
Side of square (y) = $\frac{abc}{a^2+b^2+ab}$, $x > y$

❖ AP = x and QC = y

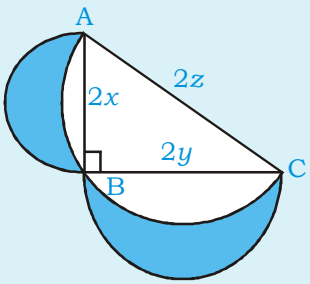
Side of square दः /द्रप लँ ल्बाल्क= **a = $\sqrt{xy}$**

$\Delta ASP \sim \Delta RCQ$

$\frac{a}{x} = \frac{y}{a} \Rightarrow a = \sqrt{xy}$



❖

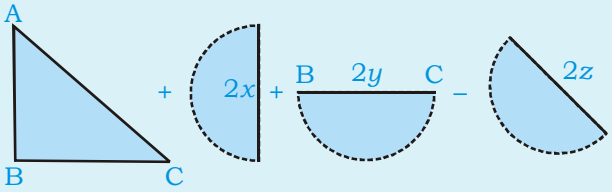


Area of shaded part = Area of Δ ABC

)ीएीअ । यक्रीधय ेयु ीर्शै ष्ट यण्यΔ ABC ेयु ीर्शै ष्ट

$x^2 + y^2 = z^2$

Shaded part ऊ)ीएीअ । यक्रीधय⇨



$= \Delta ABC + \frac{\pi x^2}{2} + \frac{\pi y^2}{2} - \frac{\pi z^2}{2}$

$= \Delta ABC + \frac{\pi}{2}z^2 - \frac{\pi}{2}z^2 = \Delta ABC$

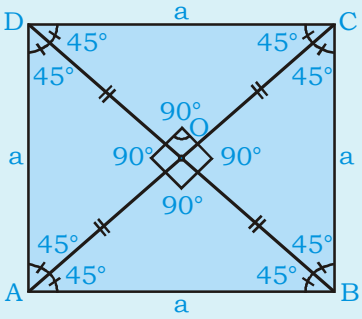
∴ **Area of shaded part = Area of ΔABC**

Square and Rectangle (: /दह लम्बदह लम्बवक

Square (चह४

❖ A closed figure with all 4 sides equal and all angles 90°.

च यावृ यकीहाआयसाज ञ्र हेिशकृीतीचसाख्हीखख्यकृीशकींहय कृेत्र ेिशय90° बीश



➢ Area ऊ ेिश = $a^2 = \frac{d^2}{2}$

➢ Perimeter ऊँ ऊवाँ = 4a

➢ Diagonal bisect the vertex angle.

ऊा :ेयजेत्रेय ेिशय ेिश्र वअकृीज।य हाीयबं

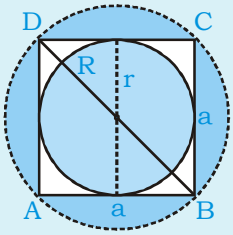
➢ AC = $a\sqrt{2}$ = BD

➢ AO = BO = CO = DO = $\frac{AC}{2} = \frac{BD}{2}$

➢ Diagonal bisect each other at 90°

ऊा :ेयच यरश्र हश ेिश90°यँ हय्र वअकृीज।य हा शबृं

➢ ΔDOC ≅ ΔAOB ≅ ΔAOD ≅ BOC



• Radius of incircle ऊकृ। थ्सषीय त्र्यमीज्ञीय= r = $\frac{a}{2}$

• Radius of circum-circle = R= $\frac{d}{2} = \frac{a}{\sqrt{2}}$

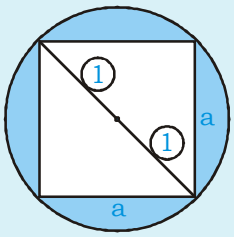
ँ ऊसषीय त्र्यमीज्ञी = R= $\frac{d}{2} = \frac{a}{\sqrt{2}}$

$\frac{R}{r} = \frac{\sqrt{2}}{1}$

$\frac{\text{Area of circumcricle}}{\text{Area of incircle}} = \frac{2}{1}$

❖ **Area of circle : Area of square**

: ेत्वप त्वभ्रसषड दछ: /दप त्वभ्रसषड



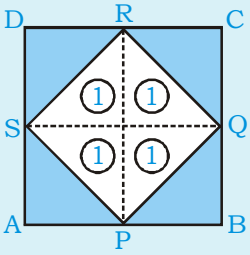
$\pi (1)^2 : \frac{(2)^2}{2}$

π : 2

11 : 7

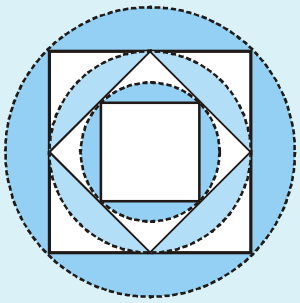
❖ P, Q, R, S are mid points of sides of square. PQRS is also a square.

P, Q, R, S यसधयABCD त्र्यमीकृीश श्वलए्यसषृ चिंन्यPQRS कृेत्रव सधयकृीशोद



Area of □ PQRS = $\frac{\text{Area of } \square ABCD}{2}$

❖



Bigger : Outer : Medium : Medium : Small : Small

circle square circle square circle square

=खत्रीय्यस१ड:=खेबहत्र्यसधष:=वल्एवयस१ड:=वल्एवयसधष:=)ीखेयस१ड:=)ीखे सधष

Area→ 8π : 16 : 4π : 8 : 2π : 4...

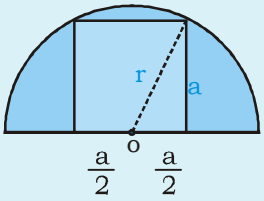
$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

or

→ 88 : 56 : 44 : 28 : 22 : 14 .....

❖ Largest square inscribed in semi-circle →

कछपाणीयवृक्षरेखे ीयबक्रीयन्ख्क्ष्रखक्ष्रीध्यसधथ→

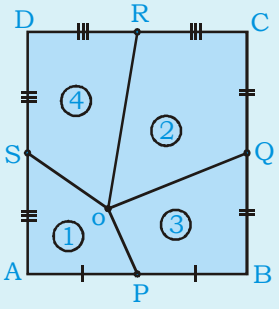


$a^2 + \frac{a^2}{4} = r^2$

$\frac{5}{4} a^2 = r^2 \Rightarrow a^2 = \frac{4}{5} r^2$

$\mathbf{a = \frac{2}{\sqrt{5}} r}$

- P, Q, R, S are mid points of respective sides.
  P, Q, R, S ज्रखक्ष्र। यक्रीतिकीक्ष्र क्षवलए यक्ष्रक्ष्र ग्रिबंद

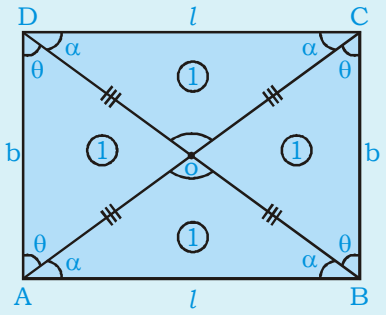


ABCD → Square/Rectangle

Area (1 + 2) = Area (3 + 4) = $\frac{1}{2}$ Area □ ABCD

Rectangle (अक्रं

- A type of quadrilateral and all the four vertex angles are equal to 90°.
  च यत ीहय ीय। ग्रिीतियक्षज त्र्यक्ष्रँ हत्र यक्रीतिचयच यत्र्ज्ञ क्षयक्रीखक्ष्यकींह ज्वीा। ह्यबीश त्र्यबृंयकींहयत ीहृक्षजेन्द्रीय90°य क्षयक्रीखक्ष्यबीश क्षबृंद



In which opposide side are equal and parallel to each other.

Area = Length × breadth = $l \times b$

Perimeter = 2 (l + b)

Diagonal do not bisect vertex angle.

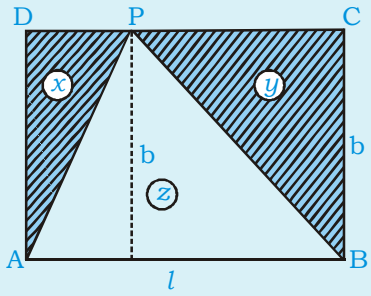
अक्ष :ीयजेन्द्रीय ीशेय ीक्ष्रज्वअकृीज्ञ। ये बृज्य हा ीयबंद

AC = BD = $\sqrt{l^2 + b^2}$

ΔABC ≅ ΔCDA

AO = OC = BO = DO = $\frac{AC}{2} = \frac{BD}{2}$

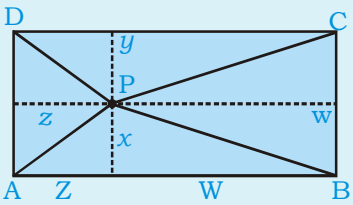
- Diagonals bisect each other but not at 90°
  अक्ष :ीयच यत्र्ज्ञ ीक्ष्रज्वअकृीज्ञ। य हा क्षबृंयट क्षे य90°यँ हये बृ
- ΔDOC ≅ ΔBOA and ΔAOD ≅ ΔCOB
- BO is median of ΔABC ⇒ Ar ΔAOB = Ar ΔBOC
- Similarly Ar ΔAOB = Ar ΔBOC = Ar ΔCOD = Ar ΔAOD
- In square/rectangle/parallelogram/rhombus ABCD→
  सधक्कीए। ऊ्र वी। ह्यत । ग्रिीति ऊ्र वत । ग्रिीतियABCD वृक्ष→



$\frac{\text{Area } \Delta \text{ APB}}{\text{Area } \Delta \text{ ABCD}} = \frac{\frac{1}{2} \times l \times b}{l \times b} = \frac{1}{2}$

$\therefore \frac{\text{Shaded Area}}{\text{Area ABCD}} = \frac{1}{2} \therefore$ Area ($x + y$) = Area z

British Flag Theorem (For Square/Rectnagle)
(ी ध्रक्ष्कचत दु पेबद् चहप्रअकृं दर दे ी ध

- 

Note → AC and BD are not diagonals.

` ीक्ष्य→ AC कींह BD अक्ष :ीये बृज्यबृंद

- P is any point inside अक्षPयकृर हय ीश्यक्ष्रज्ञ ग्रिबं

$PA^2 + PC^2 = PB^2 + PD^2$

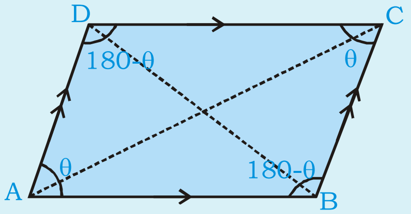
$x^2 + z^2 + w^2 + y^2 = x^2 + w^2 + y^2 + z^2$

Parallelogram/Rhombus/Trapezium
(I` ल्वामदै वँबर्ला I`ˆ वँबलार्द ¬पणीघदक

Parallelogram (¬पवंम्न्दछं क्कव्र

- ❖ It is a quadrilateral with opposite sides equal and parallel.

एबयच यत ।ङ्गिर्ाि यबंयज्ञ ज त्रयक्षँ हत्र यर्क्षीि चिय़रक्कीरक्कयकींहयज वीा।हयबृंद

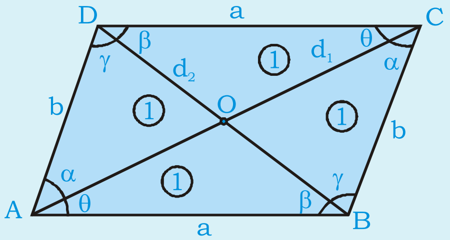


- AB || CD and BC || AD
- ∠A = ∠C, ∠B = ∠D
- Angle ratio ञ्ञ ीशीयके ँीा। → $x : y : x : y$
- Opposite angles are equal

  ज्गव्र्ि य ीशीयरक्कीरक्कयबीश्श्वबृं
- Sum of two adjacent angles is 180°

  रीश्वकीज्ो य ीशीृश्च ीयएीश्चय180°यबीशीयबं
- AB = CD and BC = AD
- ΔABC ≅ ΔCDA
- Diagonal ञ्ञञ्ञ :ीयAC ≠ BD
- α ≠ θ, β ≠ r
- Opposite Δ's are congruent

  ञ्ञँ हत्र यΔयज सीघ्ज वयबृं



ΔCOD ≅ ΔAOB

ΔAOD ≅ ΔCOB

$$\therefore BO = DO = \frac{BD}{2}$$

$$AO = CO = \frac{AC}{2}$$

- Diagonal bisect each other but not at 90°.

  ञ्ञ :ीयँ हइँ हयज वञक्कीञ्ञ।य हा श्वबृंयट झ` य90°यँ हये बृत्र्

- Area of all 4 triangle is same.

  ज र्कीत्र्य4 ञ्ञीर्कीत्ति य ीयु ीर्शे ध्ट यज वी यबंद

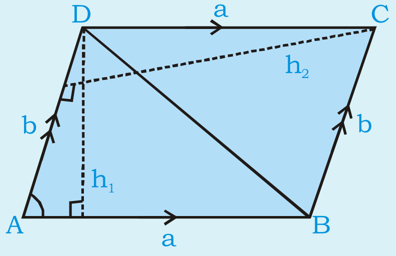
i.e., Ar ΔAOB = Ar ΔBOC = Ar ΔCOD = Ar ΔDOA = $\frac{1}{4}$ Ar □ ABCD

$d_1^2 = a^2 + b^2 - 2ab\cos(180° - θ) = a^2 + b^2 + 2ab\cos θ$

$d_2^2 = a^2 + b^2 - 2ab\cos θ$ $\quad\therefore d_1 > d_2$

$$\mathbf{d_1^2 + d_2^2 = 2(a^2 + b^2)}$$

- ❖ **Area of Parallelogram** ʿ दा` त्लवमदै वँबलादप त्वभ्रत्त्क्ष खड दछ



Area of Parallelogram = ab sin θ

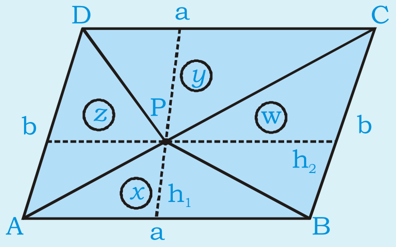
Ar of ΔABD = $\frac{1}{2}$ ab sin θ = Ar ΔBDC

∴ Ar of Parallelogram = 2 × $\frac{1}{2}$ ab sin θ = **ab sin** θ

Ar of Parallelogram = Base × Height ⇒ $ah_1 = bh_2$

$$\therefore \frac{\mathbf{a}}{\mathbf{b}} = \frac{\mathbf{h_2}}{\mathbf{h_1}}$$

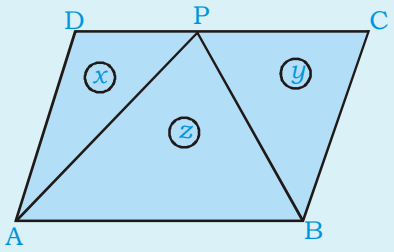
- ❖ **P is any point** ʿ दPदप ्लद्वं ि्सब)द्व



Area $(x + y)$ = Area $(z + w)$ = $\frac{1}{2}$ Area of Parallelogram ABCD

$$\frac{1}{2}ah_1 = \frac{1}{2}bh_2$$

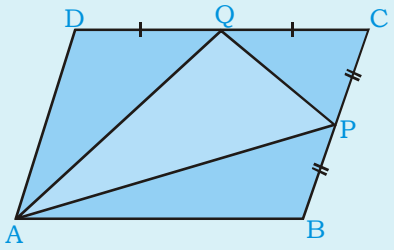
❖



Ar $\Delta APB = \frac{1}{2}$ Area $\square ABCD$

Ar $(x + y)$ = Ar$(z) = \frac{1}{2}$ Area $\square ABCD$

❖

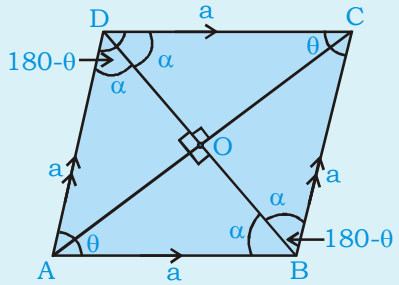


$$\frac{\text{Ar } \Delta \text{ APQ}}{\text{Ar } \square \text{ ABCD}} = \frac{3}{8}$$

Rhombus (् पछं चक्र

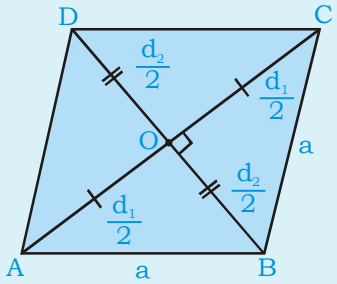
❖ Rhombus is a type of parallelogram with all sides equal and diagonals bisect each other at 90°

ज्वत । क्षिमियव यत ोहय ोयक वृी। हय्त । क्षिमियवंयक्षज ज्यक्रीज्यक्रीमीचिय़ज्वी बीश्ज्यबृंयकींहयअ्ञ :ीयच यरज्ञ हश्श ोश90°ँ हय ोख श्शवंद



- All sides are equal ऊज्क्रीज्यक्रीमीचिय़ज्वी यबृं
- ∠A = ∠C and ∠B = ∠D
- Diagonal bisect vertex angle.
  ऊअ :ीयजेन्त्रीय ोशिय ोश्ज्वअक्रीज्ञ । य हा श्शवंद
- Diagonal bisect rhombus into two equal areas.
  ऊअ :ीयज्वत । क्षिमिय ोश्शरीशश्वख्रीख्हयु ोशिँ ष्टीशवृश्शअक्रीज्ञ । य हा ोयवंद
- AC ≠ BD
- All 4Δ made by two diagonals are congruent.
  रीश्शअ :ीयज्ञनीहीयखे ोचयधचयज्क्रीज्य4Δयज्सीघज्वयबीश्शवृंद

- AO = OC = $\frac{AC}{2}$ and BO = OD = $\frac{BD}{2}$
- Diagonal bisect each other at 90°.
  ऊअ :ीयँ हइँ हय्90°यँ हयज्वअक्रीज्ञ । य हा श्शवृंद



$$\frac{d_1^2}{4} + \frac{d_2^2}{4} = a^2$$

$$\mathbf{d_1^2 + d_2^2 = 4a^2}$$

$$a = \frac{1}{2} \times \sqrt{d_1^2 + d_2^2}$$

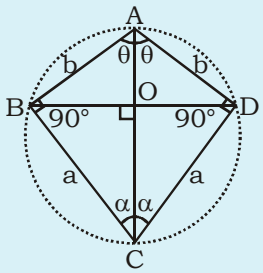
Area $\Delta BOC = \frac{1}{2} \times \frac{d_1}{2} \times \frac{d_2}{2}$

Area of rhombus $= 4 \times \frac{1}{2} \times \frac{d_1}{2} \times \frac{d_2}{2} = \mathbf{\frac{d_1 \times d_2}{2}}$

Perimeter = 4a

Kites

- A kite has 2 distinct pairs of equal adjacent sides (Here AB = AD, CB = CD)
  सर ं ह ि क्ि दु ि यद अे धस्हत्य ेक्ष्र ब2 यप ू यप ए क्ष्ब्स्क्ब्स्ह्ब तहस्दह्AB = AD, CB = CD,
- Diagonals cut at 90 (AC ⊥ BD)
  क्वर ौ द90ं ी र भ्ब(AC ⊥ BD)
- One of the diagonals bisects the other (AC bisects BD ; BO = OD)/सर ं क्वर ौ दजमीब क्ष्क्ि क्ष्धस्स्ए ं र ी ा दस्थे(AC, BD र क्ष्क्ि क्ष्धस्स्ए ं र ी ा दस्थे BO = OD)
- Perimeter/ ं वेदं = 2(a + b)
- Area $= \frac{1}{2} \times AC \times BD = \frac{1}{2} \times d_1 \times d_2$
- Area = ab sinθ [∠ABC = ∠ADC]



- If ∠ABC = ∠ADC = 90, the kite ABCD is cylic quadrilateral.
  हक्ष्क्ष्ब ∠ABC = ∠ADC = 90ं स्थे ा क्ष्ब्ं ा ह्ब ABCD सर ं ण उ ह्ण ा क्ष्ब्ब्ह्ब स्ब्ब
- AC = diameter of the circle/ ल्हदे र द ं)हद
- 2(θ + a) = 180 ⇒ θ + a = 90

➢ If a = b (i.e. AB = BC) then kite becomes rhombus.

हके a = b तय -सोचAB = BC) । केां । खे - ण । थके हुं एद्ध स्ड

Figure Made After Joining Mid Points
(फबदी) ख्वोक्ष्य केवतकेटदरे दी कदी ि क्ष्ड हथ्अक)

❖ Scalene quadrilateral → Parallelogram
=आद्वेवखेबथि । ीिि ड =ज वी। हयत । ीिि)

Square ऊसधप → Square ऊसधप

Rectangle ऊकीए। → Rhombus ऊज्वत । ीिि

Rhombus ऊज्वत । ीिि → Rectangle ऊकीए।

Trapezium (न पण ीिद्छं क्ष्व

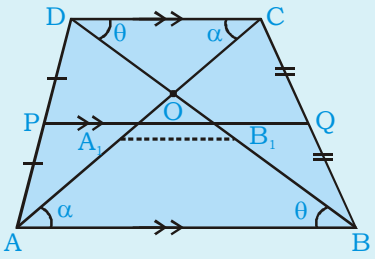
❖ It is a quadrilateral with one pair of parallel opposite sides. The parallel sides of a trapezium is called bases and non-parallel sides are called legs.

ज वी ी। हयऊँ हऋ यीिि कीश ञ्यच यत ीिश्यष्य ऋर्ी ीयच यत । ीिि यबंदयच ज्वट् ख्य ञ्य वी ी। हयीिि कीश ीिश्कीलीहय बीयती। यिबंयकीहयबंहद्द ज वी ी। ह वीिि कीश ीिश्ँ हय बीयती। यिबंद

AB || CD

If AD = BC then it is called isosceles trapezium.

एअयAD = BCयबीश। ीिश्ज ञ्य वअखेबथि ज्वट् ख्यत । ीिि य ब। श्वृंद



If A_1 B_1 are mid point of diagonal:-

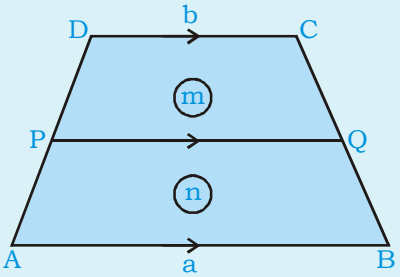
$$A_1B_1 = \frac{AB - CD}{2}$$

AC ≠ BD

ΔAOB ~ ΔCOD

$$\mathbf{PQ = \frac{AB + CD}{2}}$$

❖ M and N are the area of quadrilateral.

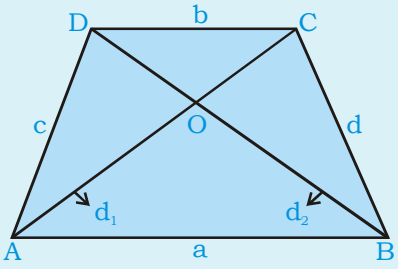


$$\frac{\text{Area } \square DCQP}{\text{Area } \square PQBA} = \frac{m}{n}$$

$$PQ = \sqrt{\frac{ma^2 + nb^2}{m+n}}$$

If m : n = 1 : 1 Then / $PQ = \sqrt{\frac{a^2 + b^2}{2}}$

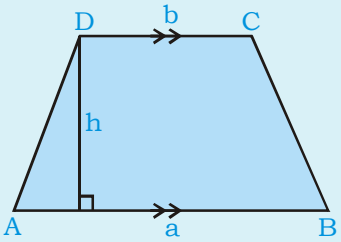
❖



$$\mathbf{d_1^2 + d_2^2 = c^2 + d^2 + 2ab}$$

❖ Area of trapezium / ABCD = $\frac{1}{2} \times (a + b) \times h$

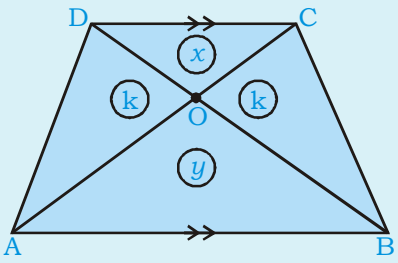
❖



ΔAOB ~ ΔCOD

Area / ΔAOD = Ar ΔBOC

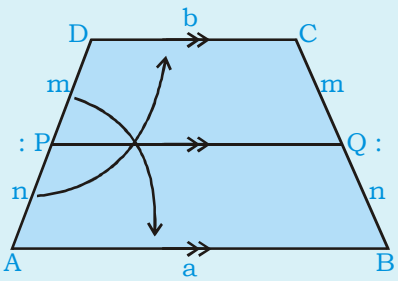
❖



$k \times k = x \times y$

$\mathbf{k = \sqrt{xy}}$

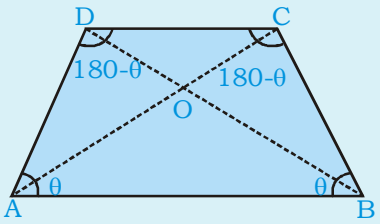
❖



$$PQ = \frac{ma + nb}{m + n}$$

Isosceles Trapezium (्प ्री कद्र पण्ीप्रदछं ष्क्ष

❖



AB || CD ; AD = BC

∠A + ∠C = ∠B + ∠D = 180°

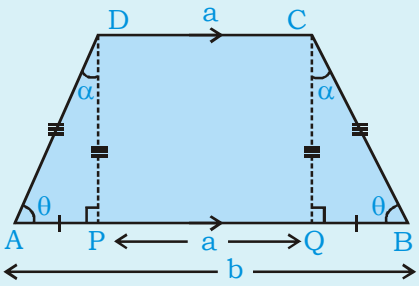
∴ Each isosceles trapezium is a cyclic quadrilateral .

∴ त्क्रश्र यज्वअखेबश्मिज्वट्ख्यत । श्मिि यच यत क्ष त्रश्र्यत । श्मिि यबीशीयबंद

or if a trapezium is inscribed in a circle it must be a isosceles trapezium.

एीयए अयच यज्वट्ख्यच यस?यवृशखे ीयबक्रीयबंया।ीशए बयच यज्वअखेबि ज्वट्ख्यबीशीयत ीअचद

AC = BD

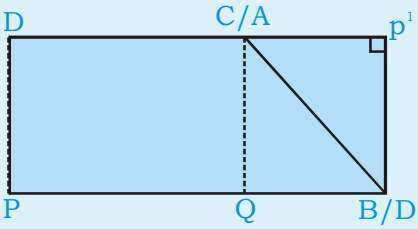


$AP = QB = \frac{b-a}{2}$

ΔAPD ≅ ΔBQC

Shift Δ ADP near ΔBQC ⇒ trapezium converted to rectangle.

Δ ADP ीशΔBQC श्मँ ज्ि यञ्की ख्य हे श्मँ ह ⇒ ज्वट गख्यकीए। यवृश ँ अ्रसआ ग्रयबीशतीा ीयबंद



Circle (: ष्लक

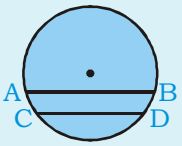
Circle (चक्रक

❖ Circle → A round plane figure whose boundary consists of points equidistant from a fixed point (the centre).

सर्शीय→यअ ज ञ्यच येअअ्त । यञ्चरकि ष्क ञ्वी यरहय्यँ हय्यड्ढी आयञ्चरकिृीश ी अ्चरौँ ीयसर्शीय बटी।ीयबंदयएबयेअ्त । यञ्चरकिसी।य ीय ृैय बटी।ीयबंद

Chord → line touches circumference of circle at two points.

त ञ्सीय→यतीश्टश्र ीयसर्शीय ञ्यँ अ्तञ्छय ीश्रीशञ्चरकिृीशँ हय्यँ जेय हा ञ्यबंद



AB > CD

Greater chord is near the centre.

ख्अाघ्यत ञ्सीय ृय शँ ज्ञ्यीश ञ्यबंद

Radius → It is a straight line from the centre to the circumference of the circle.

अीज्ञीय→य ृैयकींहय्यसी।य ञ्यँ अ्तञ्छय शअ ज ञ्यीञ्यञ्चरकिृीश शक्त य ञ्यहय्यसी। ञ्यअीज्ञीय बटी।ञ्यबंद

PQ = radius of circle ऊसर्शीय ञ्यअीज्ञी = r

Diameter/ बक

➢ The chord across a circle through the center is called the diameter.

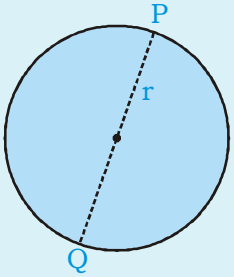
लाईदर ेबर खेह्न ेब फ्ी ेबलयस ेए प्लेद ेहदे र स्पद ेस्ठ

Arc/छैक

➢ A continuous piece of a circle is called an arc of the circle.

व ्न ेठ लाईदर देसर ेन ।। ेभ्कनहे लाईदर देण्दे र स्पद देस्ठ

❖

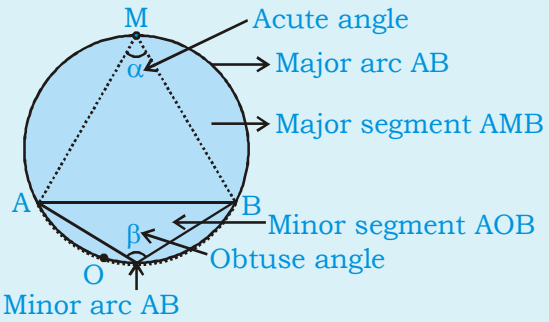


Area ऊ ीशिय= πr²

Circumference ऊँ अ्तञ्छय= 2πr

PQ = diameter = biggest chord of circle.

PQ = यस्ीज्यण्यसर्शीय ञ्यञ्ख शक्त्रघ्यत ञ्सीद



Tangent Line (ऐ ष्क्वेक्त क

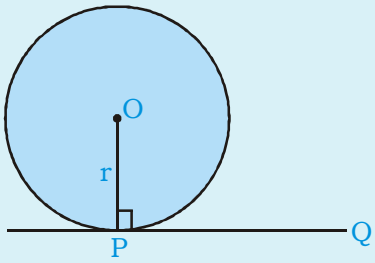
❖ A line that touches the circle at only one point is known as tangent of the circle.

च यह्श्र ीयतीश्टसर्शीय ीश्र स्टयच यञ्चरकृीशँ हय्यँ जेय हा ञ्यबंन्यसर्शीय ञ्यँ जेप ह्श्र ीय बटी।ञ्यबंद

∠OPQ = 90°

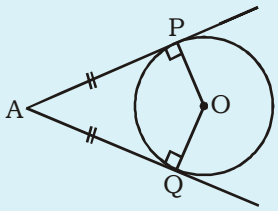
If PQ is tangent to the circle then OP ⊥ PQ

i.e., radius ⊥ PQ



➢ The lengths of two tangents, drawn from an external point to a circle, are equal.

व ्न ेठ ञ्दह्ो क ेखेक्त बलाईदं ी खयण्ठ ेृ ेजेक्त वेदी खयस्स ेखेक्त ेप ह्ुदे ।ञ्रीदी स्क्ो ेस्ठ



i.e. AP = AQ.

➢ When tangents drawn from an external point to a circle

व ्न ेठ ञ्दह्ो क ेअेक्त बलाईदं ी खयण्ठ ेृ ेन वेदी खयस्स ेठ

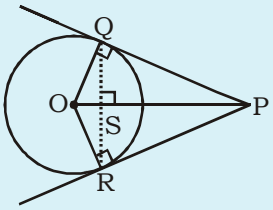
(a) Tangents are equal in length

(b) $\angle QPO = \angle OPR$

(c) $\angle QOP = \angle POR$

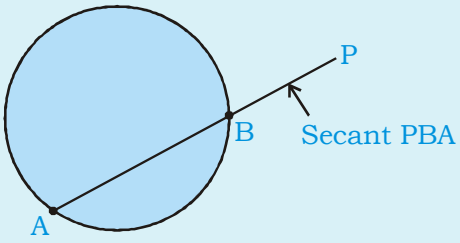
(d) $\frac{PQ}{QO} = \frac{QS}{SO}$

(e) $PQO \cong PRO$



Secant Line

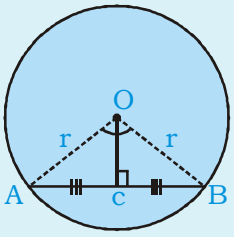
- A straight line that intersects a circle in two points is called secant line of circle.



- A perpendicular drawn from the centre of a circle to a chord bisects the chord.

OC ⊥ AB ⇒ AC = BC

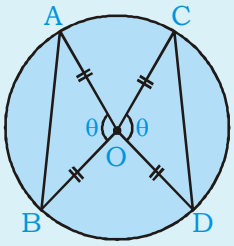
or If AC = BC then OC ⊥ AB



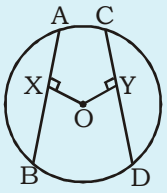
- 1. Equal chords are equidistant from the centre.
  2. Equal chords make equal angle at the centre.

If AB = CD then ∠AOB = ∠COD

or if ∠AOB = ∠COD then AB = CD



➢ Equal chords of a circle are equidistant from the centre.

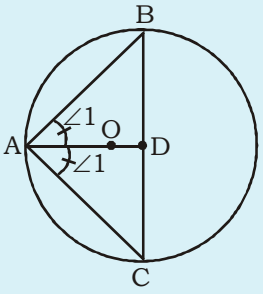


i.e. if AB = CD, OX ⊥ AB and OY ⊥ CD, then OX = OY.

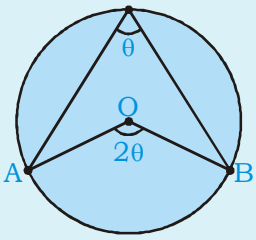
Or if OX = OY, OX ⊥ AB and OY ⊥ CD then AB = CD

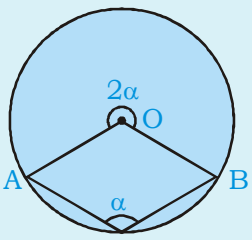
➢ If two chords AB and AC of a circle are equal, then the bisector of ∠BAC passes through the centre O of the circle.

∠1 = ∠2

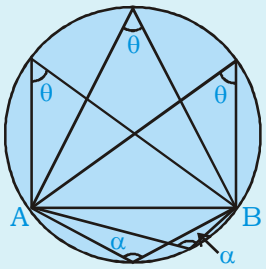


- Angle made by an arc on centre is double the angle made by the same arc on the circumference of centre.



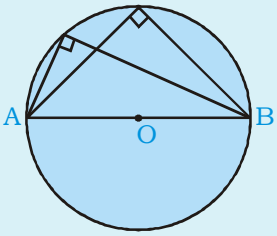


- Angle made by an arc on same side of circle are equal.



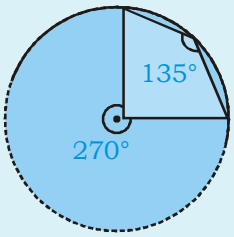
❖ Angle made in semi-circle is right angle.

कछषाणीयवृक्षखे ाय ेशीयज्व ेशीयबीशीयबंद



❖ Angle made in quarter circle is 135°.

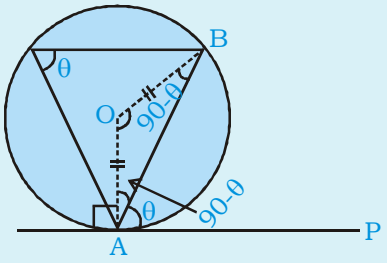
र्ी ी-यसणीयवृक्षखे ाय ेशीय135°य ेयबीशीयबंद

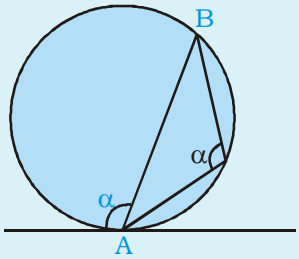


Alternate Segment Theorem (धर वंग्ज्दचर्कृंकवदु पेब

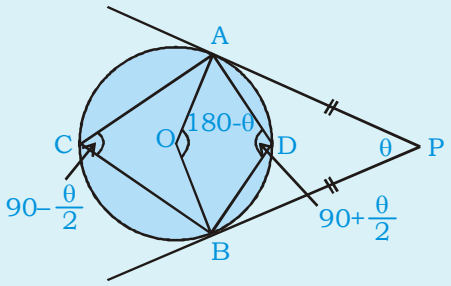
❖ Angle made by a chord and tangent is equal to the angle made by the chord in other segment of the circle.

च या त्रीयकींहय्हँ जेह्र ेायीहीयखे ेाएीयधाएीय ेशीयत्रीयत्रीयीहीयसणीय श्रार्क्र हय . ्रश्रयवृक्षखे ेचयधचय ेशीय श्रखक्षीखक्षयबीशीयबंद





❖

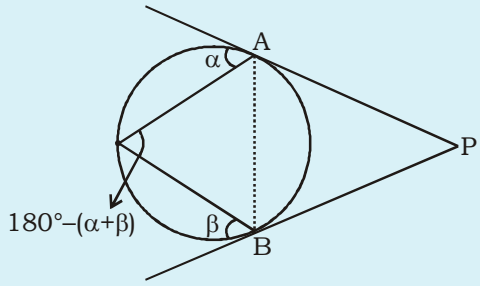


$\angle ADB = 90° + \frac{\theta}{2}$

$\angle ACB = 90° - \frac{\theta}{2}$

$\angle APB = \theta$

❖

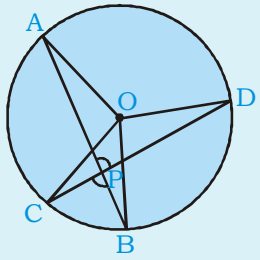


❖ AB and CD two chords cuts each other internally at point P. Then →

ABयकींहयCDयीश्त त्रीचृयक्ी। ज्ञ यउँ य श्रयक्षृ्रीPयँ हय ेख त्रवृंदयँअ ह्→

$\angle APD = \angle BPC = \frac{\angle AOD + \angle BOC}{2}$

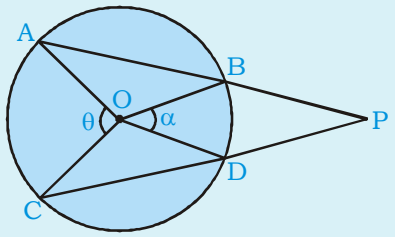
$\angle APC = 180° - \angle APD$



❖ AB and CD two chords cuts each other externally at point P. Then →

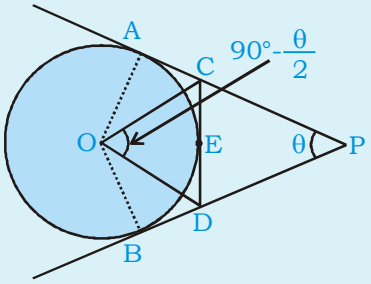
ABयकींहयCDयीश्त त्रीचृयखेग । थ्यश्रृ्रीPयँ हय ेख त्रवृंदयँअ ह्→

$\angle APC = \angle BPD = \frac{\theta - \alpha}{2}$

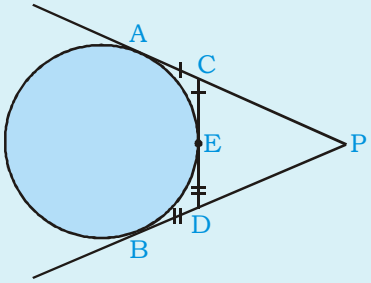


❖ $\angle COD = 90° - \frac{\theta}{2}$

$\angle COD = \frac{\angle AOB}{2}$



❖ **Perimeter of ΔPCD** ं द**ΔPCD** प ग्रह्ं मे त्र = **2PA = 2PB**



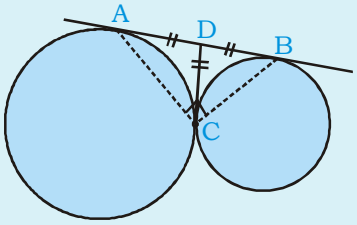
⇒ PC + CE + ED + PD

⇒ PC + CA + BD + PD

⇒ PA + PB

⇒ 2PA or 2PB (∵ PA = PB)

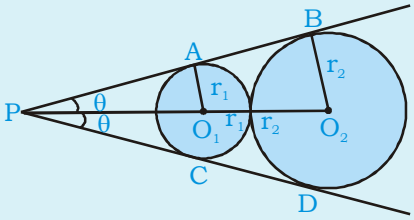
❖



AD = BD = CD

∠ACB = 90°

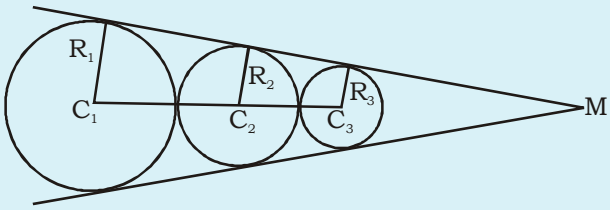
❖



$O_1O_2 = r_1 + r_2$

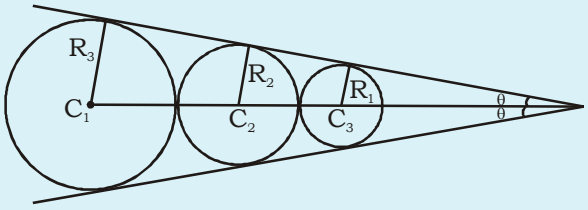
$\frac{r_1}{r_2} = \frac{1-\sin\theta}{1+\sin\theta}$

➢



R_1, R_2, R_3 are always in GP/हमेशा R_1, R_2, R_3 ैवक्लसी : ैबक ि ख्स्के $R_2 = \sqrt{R_1 \times R_3}$

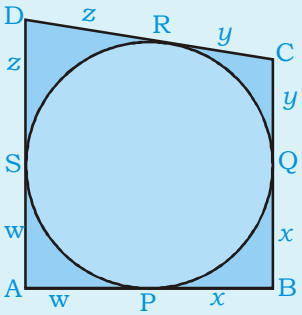
➢



$\frac{1-\sin\theta}{1+\sin\theta} = \frac{r_1}{r_2} = \frac{r_2}{r_3}$ $\Rightarrow r_2 = \sqrt{r/r_3}$

❖ If a circle is inscribed in a quadrilateral or a quadrilateral is circumscribing a circle.

एअयव यसणीयव य्र । क्रिमि यवृश्यवे ोयक्रिोयबंयोयव य्र । क्रिमि यव यसणीय श ँ ज्ञध। यबंद



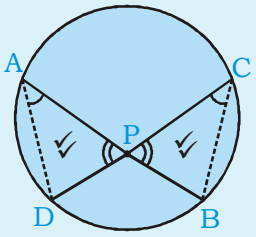
AB + CD = BC + AD

❖ Two chords AB and CD cuts each other at point P internally.

रीश्त त्रीच्य AB यकींह्य CD यच यर्ज्र हश ोशअ्रयग्रि P यँ ह्य की। ज्ञ यउँ यज्र श ोख त्र्यबृं

PA × PB = PC × PD

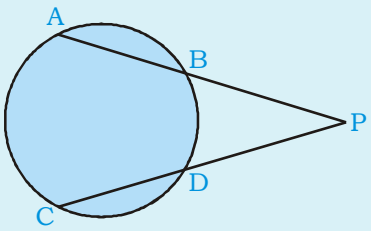
$\frac{PB}{PC} = \frac{PD}{PA}$



❖ Two chords AB and CD cuts each other at point P.

रीश्त त्रीच्य AB यकींह्य CD यच यर्ज्र हश ोशअ्रयग्रि P यँ ह्य ख्ख त्र्यबृंद

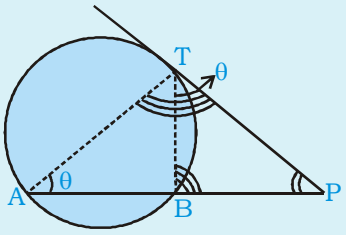
PB×PA = PD×PC



❖ From an external point a secant and tangent is drawn. Then →

च यवेबहय्यअथ्रू मि श्चच य) श यकींहयहँ जीह्रश्र ीय़ ृ त्र त्यती। त्यबंदयँअ ष्टय→

$PT^2 = PB \times PA$



Δ TPB ~ ΔAPT

$$\frac{PB}{PT} = \frac{PT}{PA}$$

❖ When a circle is drawn between two parallel lines and a tangent is given intersecting these two parallel lines.

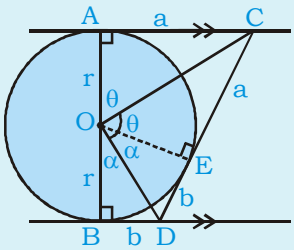
त खयरीशसणीयच यरज हश ीशवेग यहँ यक यहँ जेय हा शवृंयकींहयहशकीए अद्व यहँ जेप ह्रश ीयर त्यती। त्यबंय क्रकींहय यरीश ज्ञवी ृ। ह्रयह्रश ीकृीश शकक्र यच यसणीय ृ त्र ीयती। ीयबंयकींहय यरीश ज्ञवी ृ। ह्रयह्रश ीकृीश ीश ीख त्यबन्धिच यहँ जीयह्रश ीयर त्यती। त्यबंद

$\angle COD = 90°$

$2\theta + 2\alpha = 180°$

$\theta + \alpha = 90°$

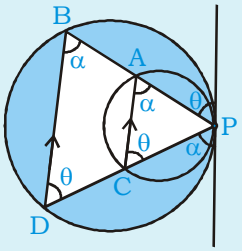
r = $\sqrt{ab}$



❖ When two circle touch each other internally and common tangent is given →

त खयरीशसणीयच यरज हश ीशकृी। ज्ञ यहँ यक यहँ जेय हा शवृंयकींहयहशकीए अद्व यहँ इँ जीह्रश ीयर त्यती। त्यबंय→

ΔPAC ~ ΔPBD

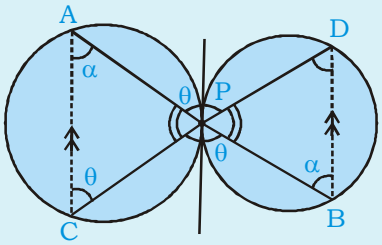


$$\frac{PA}{PB} = \frac{PC}{PD}$$

❖ When two circles touch each other externally and common tangent is given →

त खयरीशसणीयच यरज हश ीशवेग यहँ यक यहँ जेय हा शवृंयकींहयहशकीए अद्व यहँ जेप ह्रश ीयर त्यती। त्यबंय→

ΔPAC ~ ΔPBD

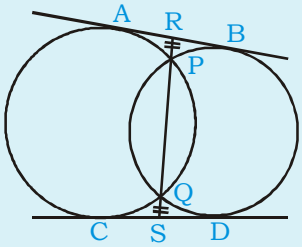


$$\frac{PA}{PB} = \frac{PC}{PD}$$

❖ Two circles intersect each other at two distinct points and two common tangents are given then →

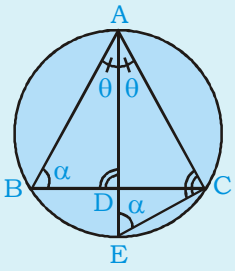
रीशसणीयच यरज हश ीशरीशशृंी यसपूर कृीशँ हयक्र आ( ) श्र य हा शवृंयकींहया खयरीश झकृीए अद्व यहँ जीयह्रश ीचश्र त्यध यबृं

AB = CD



$RS^2 = AB^2 + PQ^2$

❖ **$AB \times AC + AE \times DE = AE^2$**



ΔABD ~ ΔAEC

$$\frac{AD}{AC} = \frac{AB}{AE}$$

$\Rightarrow AB \times AC = AD \times AE$

Proof:-

ΔAEB ~ ΔACD by (AA)

$$\frac{AE}{AC} = \frac{AB}{AD} \Rightarrow AE \times AD = AB \times AC$$

$\because AB \times AC + AE \times DC = L.H.S$

$\therefore AE \times AD + AE \times DE \Rightarrow AE\,(AD + DE)$

$AE = AD + DE$

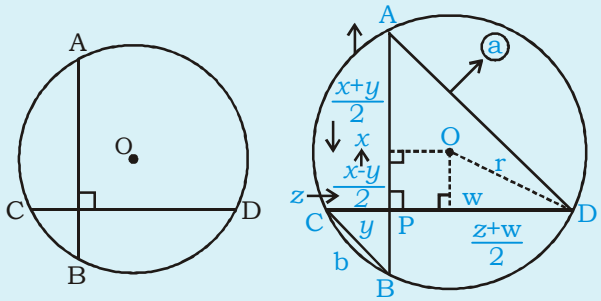
$AE \times AE \Rightarrow AE^2$

Hence, L.H.S = R.H.S prooved

$AB \times AC + AC \times DE = AE^2$

❖ If two chords intersect each other at 90°.

एअयरीशत त्रीचय़च यरज़ हश्श ोश90°यँ हयत्आ()श्य हा त्यबंद



AP = x BP = y PD = w
CP = z BC = b AD = a

$$r = \frac{\sqrt{x^2 + y^2 + z^2 + w^2}}{2}$$

$$x \times y = z \times w$$

$$r^2 = \left(\frac{z+w}{2}\right)^2 + \left(\frac{x-y}{2}\right)^2$$

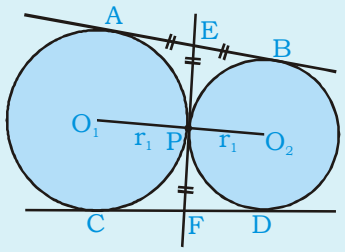
$$r^2 = \frac{z^2 + w^2 + 2zw + x^2 + y^2 - 2xy}{4}$$

$$r = \frac{\sqrt{x^2 + y^2 + z^2 + w^2}}{2} = \frac{\sqrt{a^2 + b^2}}{2}$$

Externally Touch (ी कंदए क्क

❖ When two circles touch each other externally. Then distance between their centres is sum of their radii.

तखरीशसा्यच यरज़ हश्श ोशर्वे एयँ जेय हा श्बंयाोश्झे श्र ृशैंश त्यकत्र त्यरहत्य्झे त्यमीज़्एीकृीश ोयएीशयबीश त्यबंद



O_1PO_2 → straight line ऊज़ त्छत्यह़ ी

Maximum common tangent = 3

कऊछ ।वयझकृीएअद्रुयँ जीह़ ी = 3

AB = CD = EF

Internally touch (अंक्ष ेर दशै दन्दए क्ष्क्षर ज्ये

❖ When two circles touch each other internally then distance between their centres is difference of their radii.

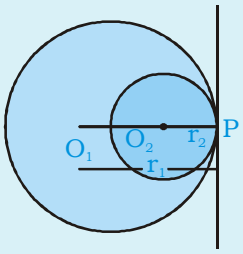
तखरीशसा्यच यरज़ हश्श ोशका।थ्यँ जेय हा श्बंयाोश्झे श्र ृशैंश श्यकत्र त्यरहत्य्झे त्यमीज़्एीकृीश ोयकृ।हयबीश त्यबंद

O_1O_2P → straight line ऊज़ त्छत्यह़ ी

$O_1O_2 = d = r_1 - r_2$

Maximum common tangent = 1

कऊछ ।वयझकृीएअद्रुय्यँ जीह़ ी = 1

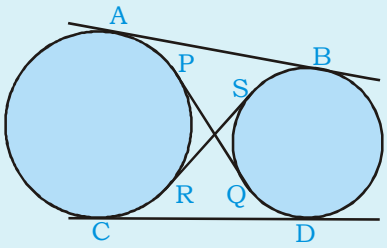


❖ If two circles do not touch each other, there can be maximum 4 common tangents.

Direct common tangent = 2

Transverse common tangent = 2.

तखरीशसा्यकीं ज्यवृशइँ जेये बृत्य हा श्बंयाोश्झे त्यकऊछ ।वय4 इँ जेप ह्यीं्यबीश्झ ।त्यबंद



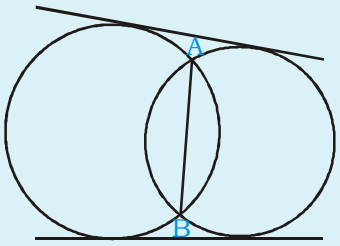
DCT = AB & CD

TCT = PQ & RS

Maximum common tangent = 4

कऊछ ।वयझकृीएअद्रुय्यँ जीह़ ी= 4

❖ **Intersecting circles** द्रं्वउ्रसग्ः ल्क



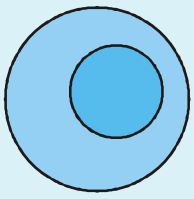
AB → common chord ऊझकृीएअद्रुय्त त्सी

Maximum common tangent = 2

कऊछ ।वयझकृीएअद्रुय्यँ जीह़ ीयण्य2

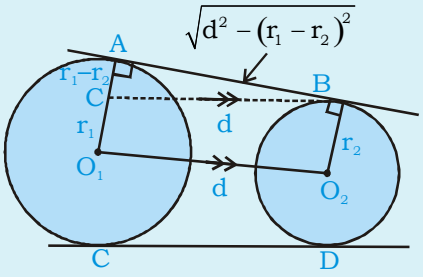
❖ If one circle is made inside other circle and they do not touch each other then no common tangent is possible.

एअयच यसशैयरज़ हश्शसशैय श्कृरहय्खे ोएीयत्ी।ोयबंयकींहयसश्च यरज़ हश्श ोश इँ जेये बृत्य हा श्बंयाोश्झ ोश्यझकृीएअद्रुय्यँ जेयह़ ीयज़ृकीसये बृत्यबंद



Direct Common Tangent (द्रम्क्र ि्व्लद्अ ि्ह् क्ष्क्र्ड्ं क्क

❖ Length of DCT ऊDCT त्र्यट् ख्वे-प

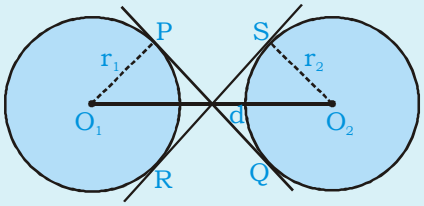


$CB \parallel O_1O_2$

DCT = AB = CD = $\sqrt{d^2-(r_1-r_2)^2}$

Transverse Common Tangent
(्रेक्ष्क्र द्द्रम्क्र ि्व्लद्अ ि्ह् क्ष्क्र्ड्ं क्क

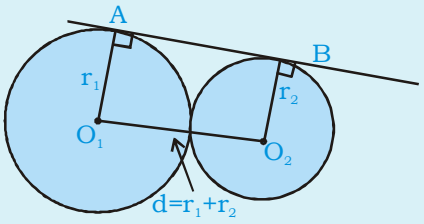
❖ Length of TCT ऊTCT त्र्यट् ख्वे-प



TCT = PQ = RS = $\sqrt{d^2-(r_1+r_2)^2}$

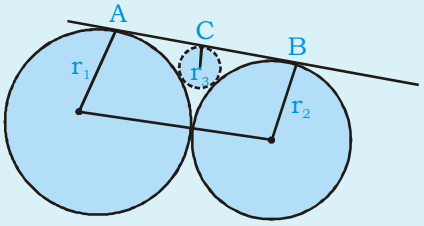
DCT > TCT

❖ AB = DCT = $2\sqrt{r_1r_2}$



DCT = $\sqrt{(r_1+r_2)^2-(r_1-r_2)^2}$ = $\sqrt{4r_1r_2}$ = $2\sqrt{r_1r_2}$

❖ AB = AC + BC



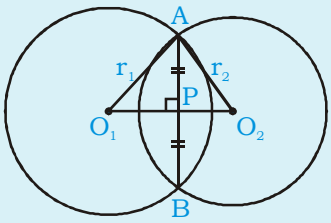
$2\sqrt{r_1r_2} = 2\sqrt{r_1r_3} + 2\sqrt{r_2r_3}$

Divide by $\sqrt{r_1r_2r_3}$

$$\frac{1}{\sqrt{r_3}} = \frac{1}{\sqrt{r_1}} + \frac{1}{\sqrt{r_2}}$$ (Result)

❖ When two circles intersect each other at two distinct points then length of their common chord AB is → Common chord

त ख्वरीश्सशीयच द्ररक्र हश्श ीश्रीशकट धद्रकट धयअद्रक्रीशँ हय ीख श्बवृंय।ीश
झे त्र्यक्रीएअद्र-यत त्रीयAB त्र्यट् ख्वे-य→त्र्यक्रीएअद्र-यत त्रीयबीश्च्बं



$\Rightarrow AB = 2AP = 2PB$ AP = PB

Ar $\Delta O_1AO_2 = \frac{1}{2} \times O_1O_2 \times AP$

Where त बीय़ $\angle O_1AO_2 = 90°$

$O_1O_2 = C$

$\therefore AB = \frac{2r_1r_2}{C}$

Distance between centres O_1 O_2 =

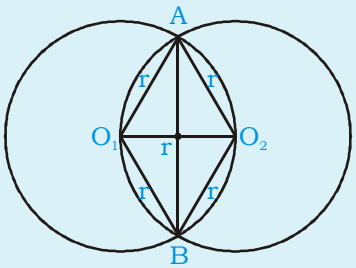
$$\sqrt{r_1^2-\left(\frac{AB}{2}\right)^2} + \sqrt{r_1^2-\left(\frac{AB}{2}\right)^2}$$

❖ If two circles pass through each others centre. ऊ एअयरीश्सशीयच यरक्र हश्श श्र ूश्यक्र श्थतिी हा श्बवृंद

Length of common chord AB is →

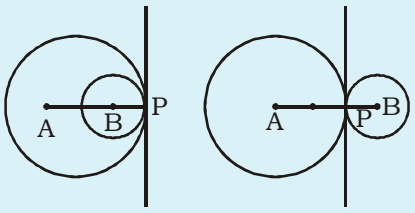
झक्रीएअद्र-यत त्रीयABय त्र्यट गखे-यबं→

$AB = \frac{\sqrt{3}}{2} r \times 2 = \sqrt{3}r$



➢ If two circles having centres at A and B touch each other internally or externally at point P lies on the line AB joining their centres.

हक्रो जेश्सक्रीएदवु् र देर श्के A य वै B स्थे श्स्र जमीबर केश्क श्केछे ं ी य व्खेर हदेद्र्द् द्व ं ेब्रं क्वेर ी ो बर श्के एक्र् र ब्र श्केश्क केए व्खुटेक्लस ेी क्रदेAB ं ी ब्र-दे स्श



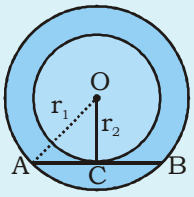
i.e. A, B and P are collinear./या ने A, B ा -देP ने ीश्क्षहे स्श

Distance between their centres (d)

(i) When touch internally/एक यदि भीतर से वृत्त को स्पर्श करे
d = AP – BP

(ii) When touch externally/एक यदि बाहर से वृत्त को स्पर्श करे
d = AP + BP

➢ If circles are concentric/एक ही केन्द्र के वृत्त संकेन्द्रीय वृत्त

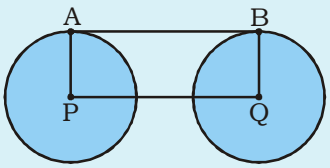


AB is chord of the greater circle be a tangent to smaller circle
बड़े वृत्त की एक जीवा AB छोटे वृत्त को स्पर्श करती है

Length of AB = 2AC/AB की लम्बाई = 2AC = $2\sqrt{r_1^2 - r_2^2}$

➢ If radius of circle is same/यदि वृत्तों की त्रिज्या समान हो तो

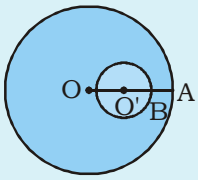
$r_1 = r_2 = r$



➢ Direct common tangent/प्रत्यक्ष उभयनिष्ठ स्पर्श रेखा AB = PQ

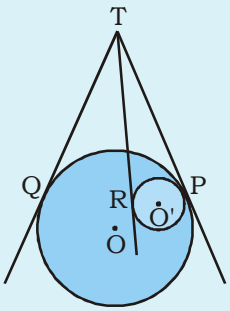
AB = PQ i.e. प्रत्यक्ष उभयनिष्ठ स्पर्श रेखा की लम्बाई जुड़ने वाली स्पर्श रेखाओं के बराबर होती है।

➢



Two circles with R and r cm, one circle is inscribed in another circle. If the shortest distance between the circles is S cm. Then the distance between the centres is:
जिसमें वृत्तों की त्रिज्या R और r cm है एक वृत्त दूसरे वृत्त के अन्दर स्थित है यदि दोनों वृत्तों के बीच की न्यूनतम दूरी S cm हो तो केन्द्रों के बीच की दूरी = OO' = OA – O'B – AB = R – r – S

➢



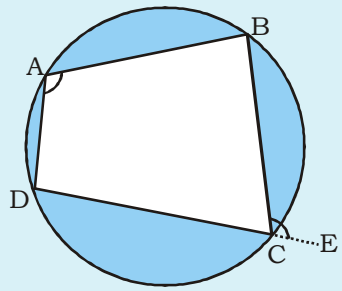
In the given figure, there are two circles with the centres O and O' touching each other internally at P. Tangents TQ and TP are drawn to the larger circle and tangents TP and TR are drawn to the smaller circle. Then TQ : TR = 1 : 1
दिये गये चित्र में दो वृत्त हैं जिनके केन्द्र O और O' हैं जो आपस में आन्तरिक रूप से P पर स्पर्श करते हैं। TQ और TP बड़े वृत्त पर खींची गई स्पर्श रेखाएँ हैं तथा TP और TR छोटे वृत्त पर खींची गई स्पर्श रेखाएँ हैं तो TQ : TR = 1 : 1.

Cyclic Quadrilateral (चक्रीय चतुर्भुज)

➢ If all vertices of a quadrilateral lie on the circle, it is called a cyclic quadrilateral.
यदि किसी चतुर्भुज के सभी शीर्ष बिन्दु वृत्त की परिधि पर स्थित हों तो उस चतुर्भुज को चक्रीय चतुर्भुज कहते हैं।

Properties/ गुणधर्म

➢ The sum of opposite angles of a cyclic quadrilateral is 180°./चक्रीय चतुर्भुज के विपरीत कोणों का योग 180° होता है।

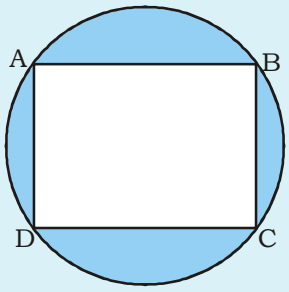


∠A = ∠C = ∠B + ∠D = 180°

➢ The exterior angle of a cyclic quadrilateral is equal to the opposite interior angle.
चक्रीय चतुर्भुज का बाह्य कोण विपरीत आन्तरिक कोण के बराबर होता है।

∠DAB = ∠BCE
∠DAB = ∠BCD = 180°
या ∠BCD + ∠BCE = 180°
∴ ∠DAB = ∠BCE

➢ All angles of cyclic parallelogram is 90°.
चक्रीय समान्तर चतुर्भुज के सभी कोण 90° के होते हैं।



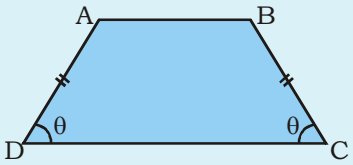
∠A + ∠C = 180°

but according to parallelogram property ∠A = ∠C = θ
लेकिन समान्तर चतुर्भुज के अनुसार ∠A = ∠C = θ

∴ 2θ = 180 ⇒ θ = 90°

- If non parallel sides of a trapezium are equal then it will be a cyclic quadrilateral.

  हको व ृ ेण ि पछे ा ाके र ेय ृ दि वी धा दे ेख ृ दि े स्का ेहस ण उ छे ा ाके स्कद छ



  ∴ AB ∥ DC

  So, या े ∠A = 180° – θ and ∠B = 180° – θ

  If sum of opposite angles is 180°

  व ंी ष े र ैक ेस र देहको = 180°

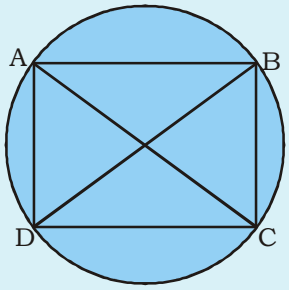
  (∠A + ∠C = ∠B + ∠D = 180°)

  then it is cyclic quadrilateral.

  हसे स र े ण उ छे ा ाके स्ड

- **Ptolemy's theorem**→ In a cyclic quadrilateral the sum of products of the measures of the pairs of opposite sides is equal to product of its diagonals.

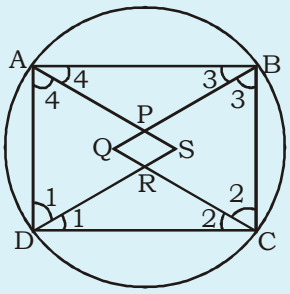
  ै लेम पहै घेब→ व ृ ेण ा ाके ण उ छे ि ेक ंी ष े धा ृ य ेक र ब ैकु ं ज्प ेक देहको ण ा ाके र े व र ै देख ृ े ब ैकु ं ज्प र ेब ृ दि े स्कद स्ड



  AC. BD = AB. DC + AD. BC

- The quadrilateral formed by angle bisectors of a cyclic quadrilateral is also cyclic.

  व ृ ेण उ छे ा ाके र े ब र ैक ेद ृ ि मधस र ेख ृ े ब ु देण ा ाके उ छे स्कद स्ड

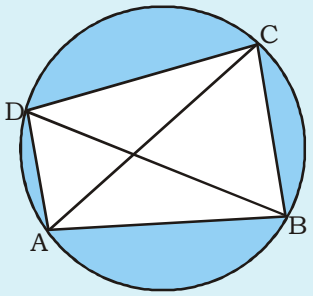


  i.e. If ABCD is a cyclic quadrilateral, then □ PQRS is also a cyclic.

  य ा ोच हको ABCD स र े ण उ छे ा ाके स्ड ा क □ PQRS ध े ण उ छ स्कद

- If A cyclic trapezium is isosceles and its diagonals are equal

  स र े ण उ छे ृ ि पअ े ण ा ाके ृ ि ा ृ द े क ृ क ेद स्थ ृ ेख ृ े र े क र ै व ृ दि स्कद स्ड

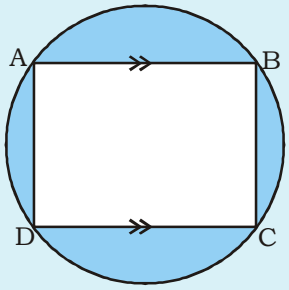


  i.e. If ABCD is a cyclic trapezium s.t. AB ∥ DC, then AD = BC and AC = BD.

  य ा ोच हको ABCD स र े ण उ छे ृ ि पछे ा ाके ृ ं ई द ै स्थ ृ े AD = BC य ै AC = BD

- If two opposite sides of a cyclic quadrilateral are equal, then the other two sides are parallel.

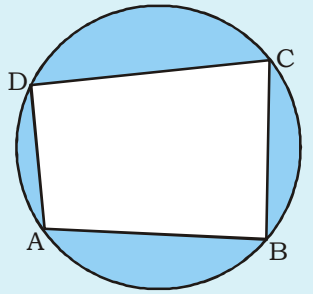
  हको व ृ ेण उ छे ा ाके र े ज ेक ंी ष े धा ृ य ेक ीदी स्का ेक य ा ज स धा ृ य ेक ृ दि वी स्क े स्ड



  i.e. If AD = BC, then AB ∥ CD./हको AD = BC, ा े AB ∥ CD.

- An isosceles trapezium is always cyclic.

  स र े ृ ि ा ृ द े क ा ाके स ृ ब ृ द ण उ छे स्कद स्ड



  i.e. If AB ∥ DC and AD = BC.

  हको AB ∥ DC य ै AD = BC.

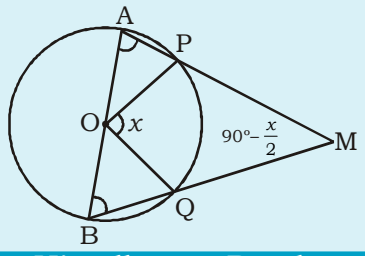
  Then, ABCD is a cyclic trapezium.

  ा को ABCD ण उ छे ृ ि पछे ा ाके स्ड

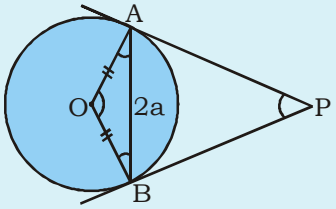
➢ If AB is a diameter of the circle (centre O) and APM and BQM are its two secants. If ∠POQ = x then

हके O र केन्द्रलयोबकाएदर दे)हदे AB स्थे –देजेकबा ेखासेखAPM ल BQM एकलाएदे–कदी M ं ी र दमा े स्थे के∠POQ = x

$\angle PMQ = 90° - \frac{x}{2}$



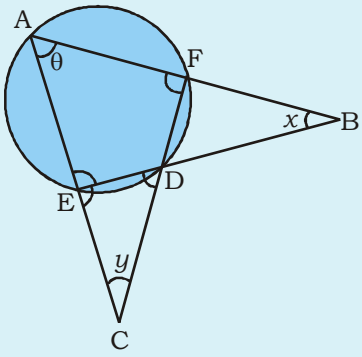
Miscellenous Results

➢ 

If/हके AB = 2a, Radius/व्यदे = r

then length of tangent/ाइ ं ेवी खासे र े प ष्ट्द।

$PA = PB = \frac{ar}{\sqrt{r^2 - a^2}}$

➢ 

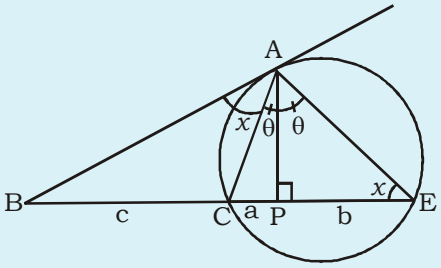
AB & AC are secant/AB यदै AC ठबा े खासेख्ड

If/हके ∠ABE = x

∠ACF = y

∴ ∠BAC = θ

then/ाइरे $\theta = 90° - \left(\frac{x+y}{2}\right)$

➢ 

AB is tanget/AB सर े ं वेी खासे स्ड

BE is secant/BE सर े ठबा े खासे स्ड

AP is Angle bisector of ∠CAE/AP, ∠CAE र दे र ैखद ि नकाथसर े स्ड

In this figure/ े व् दे ि ख

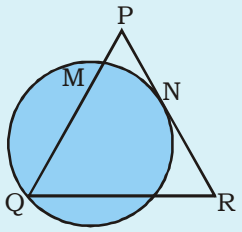
AB = BP

$c = \frac{a^2}{b.c}$

$AB = BP = \frac{a \times b}{b - c}$

➢ PQR is an isosceles triangle with PQ = PR A circle through Q touching PR at the middle point and intersect PQ at M then PM : PQ

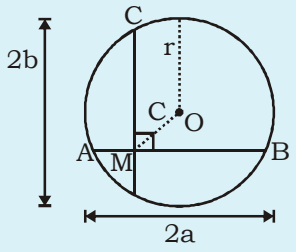
PQR सर े ि नका दे खद्वसे स्ड व् ि ख PQ = PR स्ड सर े ल दे एके Q े बस्का ी ख्ी ा दे स्ड य दै PR र े का ँ हे क खेकं ी ं वे र ी ा दे स्ड ा -द्व PQ र े का M ं ी ं द्वा म्ब ा र ी ा दे स्ड ा े का PM : PQ



$PN^2 = PQ \times PM$

$PM = \frac{1}{2}$

$\frac{PM}{PQ} = \frac{\frac{1}{2}}{2} = \frac{1}{4}$

➢ 

If CD ⊥ AB

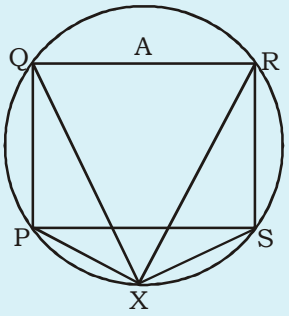
AB = 2a

CD = 2b

OM = C and Radius = r then

$r = \frac{\sqrt{a^2 + b^2 + c^2}}{2}$

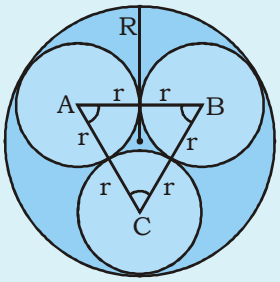
➤



PQRS is a square (cyclic) of side "A". X is a point on the circle, then/PQRS सर्ा ल ाेस्थ्वर्ु ेधसहेद"A" स्थX लहेदं ी सर् ेक ेसम्ब ा ेर

$PX^2 + QX^2 + RX^2 + SX^2 = 4A^2$

➤



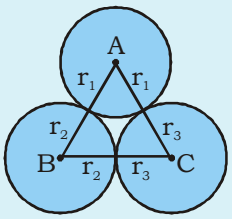
3 circles are of radius r touching each other. A large circle is also present touching all three circles.

3 ृदु ेव्दहेद(r) लम्ब लहेदसर् ेजमीब र्क्रं ेवर ाेब स्थ -देसर् ेऽनद्द लहेद (R व्दहेव्ृ ेद ाु ेक ेलहेदस्थ क्रं ेवर ादे स्थ

The radius of the largest circle/ाेक्नेद्धलहेदर ेव्दहेद =

$$(R) = r\left(\frac{2\sqrt{3}}{3}+1\right)$$

➤



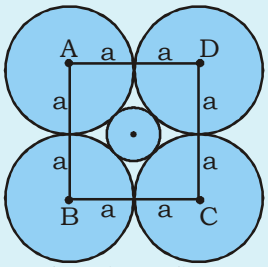
With A, B, C centres, three circles are drawn such that they touch each other externally.

The sides of ΔABC are x cm, y cm, z cm then sum of the radius of the circles =

A, B यवै C र ख्यक्तसंब ु लहेदसर् ेजमीब र्क्रं ेद्व ंवर ाेब स्थ्हक्ा ΔABC र ेधसदेस्थ x cm, y cm, z cm, स्ेब ु क्ेक्तहेदस्थ ेव्दहेदय क्ा र देहक्ो =

$$r_1 + r_2 + r_3 = \frac{x+y+z}{2}$$

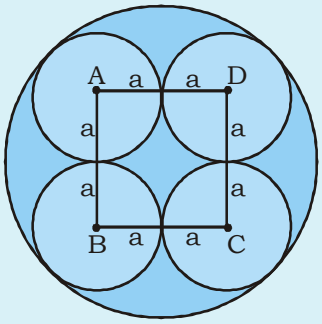
➤



In the given figure, radius of smaller circle is/ जे ृ ाेय दूरवो ि ख्क्ष्ठ क्ेबलहेदर ेव्दहेद

$$= \frac{1}{2} \times \left[2a\left(\sqrt{2}-1\right)\right] = \left(\sqrt{2}-1\right)a$$

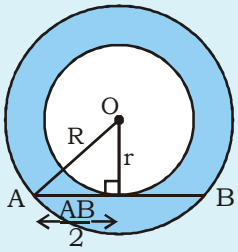
➤



In the given figure, find the radius of bigger circle is/जे ृ ाेय दूरवो ि ख्क्ष्ठ नेद्धलहेदर ेव्दहेद ध्व ेर ि ख्क्ष्ठ

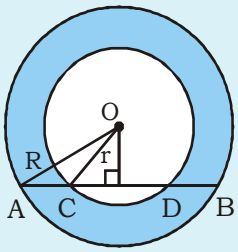
$$= \sqrt{2}a + a = \left(\sqrt{2}+1\right)a$$

➤



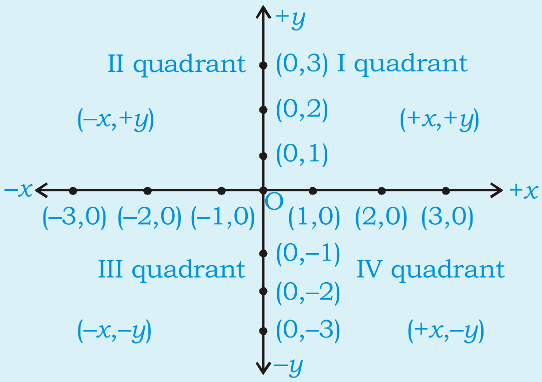
Area of ring $[\pi (R^2 - r^2)] = \frac{\pi AB^2}{4}$

➤



Area of ring = $\pi\left(\frac{AB^2 - CD^2}{4}\right)$

Co-ordinate Geometry (प्र तीक्षें लि) फ्ताप्तु



O (origin) शू ्र ≡ (O, O)

P ≡ (x, y) → Ordinate; x ↓ abscissa

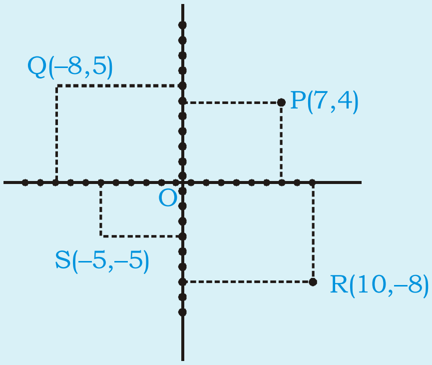
Equation of x-axis ⇒ $y = 0$

xयअमीर्कृ ीकृत्तश त्र्ज्वीकृ ⇒ $y = 0$

Equation of y-axis ⇒ $x = 0$

yयअमीर्कृ ीकृत्तश त्र्ज्वीकृ ⇒ $x = 0$

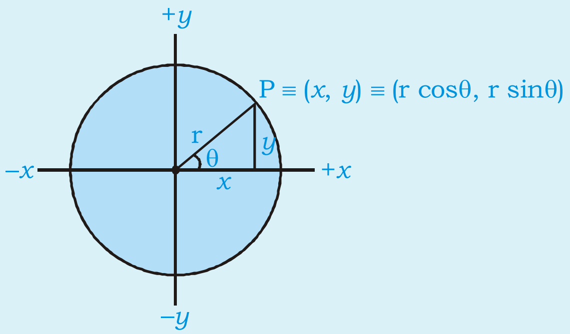
- Representation of points लएफ्कृत्त्भीर्(कृ ीकृ)र` वीलख्बर` वी-



Polar co-ordinates of a point (रे ख`अेक्ष एाब्कुि खपै.र्मअिम

- Pole → Reference point in polar co-ordinate system the co-ordinates is called a pole.
- When each point on a plane of a 2D co-ordinate system is decided by a distance from a reference point and an angle is taken from a reference direction. It is known as the polar co-ordinate system.

` क्कृर्ा कृ2Dकृ)तुर्सीा कृ हएब्कृ र्कू कृज्कृ्वैर्ज( कृस्कृत्त्कृ ीर्कृा कृत्तइीकृस्कृत। त(कृत्त्क्कृहीजेकृजकृ( जीकृं ीबीर्कृ कृओत्कृ्रा कृत्तइीकृस्सीकृ्र(कृा कृ ीलीकृ्र जी ` ीबीर्कृ न्माकृश्त(कृं ीष्ट्जकृ)तुर्सीा कृ हएब्कृ र्कृ` कृश(कृं ी)ीकृं ीबीर्कृ न्म



$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r}$$

$$x^2 + y^2 = r^2$$

$$\tan\theta = \frac{y}{x}$$

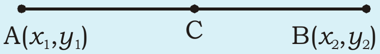
Mid Point Formula (व्) ख`अस्र्ीमि

- A mid point is the middle point of a line segment which is equidistant from both the end points.

Mid point

दा कृश्ँ जकृस्कृत्त्कृ्रा कृ(ा ीकृा -कृ ीकृश्ँ जकृस्कृत्त्कृ ीबीर्कृ कृं ी(कृी)ी(कृ्बकृस्कृत्त्भी(त(कृत्तशी)कृत्त्क्कृ ज्र्कृ ीबीर्कृ न्म

Mid point/ श्ँ जकृस्कृत्त



$$\text{Mid point } \text{ल} C = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Distance formula (ँ ेसंत्वीमि

- Distance between two points is the length of the line segment that connects the two points in a plane.

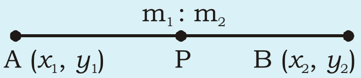
त्तीकृस्कृत्त्भीर्(कृ (कृत्त्प कृ स्कृत्त्स्कृत्तकृ(ा ीग -कृ स्कृू कृीर्धुकृ कृं ीर्(कृा कृत्तशबू श्(कृत्तीकृस्कृत्त्भीर्(कृ ीर्(कृं ी(ब्र्स्कृ न्म

$A \equiv (x_1, y_1)$ ∴ $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$B \equiv (x_2, y_2)$

- **Point of intersection formula**: रख्बडग कृपसंतमि` निँए

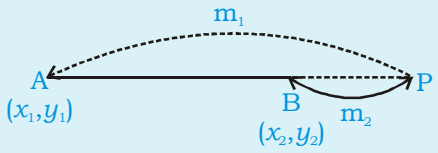
(A) Internal division : यिे ंदर्प फ्ष्ैं` ् कृ→



$$P \equiv \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

(B) External division : सि ञ्चि ञ्चिं ू ि →

AP : PB = $m_1 : m_2$



$$P \equiv \left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2} \right)$$

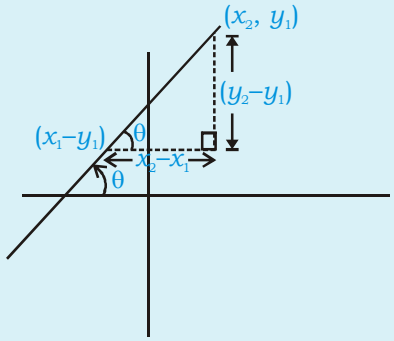
Slope of a Line (या ञ्चि व्ज्ञं पिम

❖ The slope of a line is a measure of its steepness. It is the change in y co-ordinate with respect to the change in x co-ordinate.

ए तञ्चकृ(ाकृिकृाकृू वृस्तर ञ्चकृू कृ ीकृाश्रीकृ ञ्चस्रकृ्ज कृ्यकृ्य)ुस्रीकृ वृ्र(` एष्टब)कृ ंकृ्ज कृ्यकृ्य वृ्र ंकृ्य)ुस्रीकृ वृ्र ंकृ एष्टब)कृ ंकृ ञ्च

Slope of a line/ज्ञाकृि ञ्चकृू ी)वृ(m) = tan θ (slope)

$$\tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$$



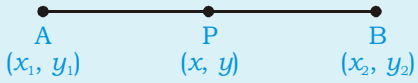
❖ Standard equation of a line ल्ज्ञाकृि कृन(ाकृिकृाकृी)ि कृाश्रञ्चि ज्वी → $ax + by + c = 0$

slope = $\frac{-a}{b}$

❖ **Point from of a line** : ञ्चि ि को निकिस्रांतम

Slope of line AP = slope of line AB.

ज्ञाकृिAPकृाकृू कृन्कृन(ाकृिABकृाकृू



$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

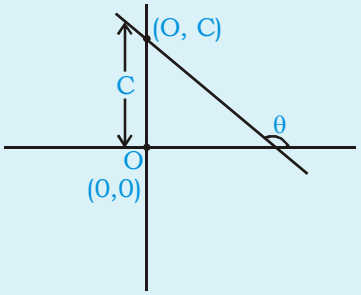
$y - y_1 = m\,(x - x_1)$

❖ Slope ल्ज्ञाकृू ी)वृ= m

Intercept on y-axis ल्ज्ञy-अमीकृ ज्ञकृ्जटजीए)वृ= c

$y - c = m\,(x - 0)$

$y = mx + c$



❖ $ax + by + c = 0$

$by = -ax - c$

$$y = \frac{-ax}{b} - \frac{c}{b}$$

$y = mx + c$

compare ल्ज्ञाकृू ी)कृ ज्ञी) **$m = \frac{-a}{b} = \frac{-\text{coff. of } x}{\text{coff. of } y}$**

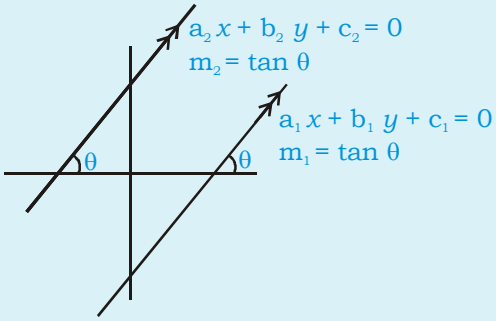
❖ If two lines are parallel to each other their slope will be equal $\Rightarrow m_1 = m_2$

जए्कृीकृन(ादकृ्र कृ्तज्ञकृ्ज ंकृ्ज श्रीबज्कृी)ंकृ्ज्रीकृ्र)ि ञ्चकृ्ट वीबीकृ्कनीकज्कृी)ंकृ्ज

$\Rightarrow m_1 = m_2$

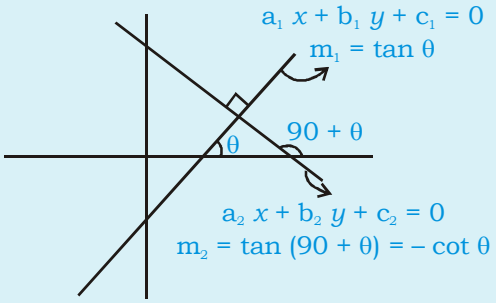
$$\frac{-a_1}{b_1} = \frac{-a_2}{b_2}$$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2}$$



❖ **If two lines are perpendicular to each other →**

)पति कि को भ्ञि तिर्ं कि कि सअहि कि →



$\therefore m_1m_2 = \tan\theta\,(-\cot\theta)$

$\therefore$ **$m_1m_2 = -1$**

Intercept form of a line (रे ख्यण खि ख्यि न यधि पख्ज जम

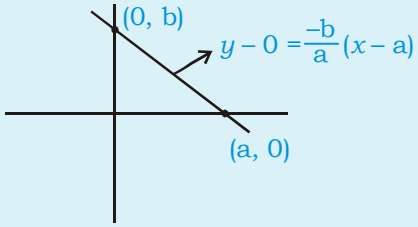
❖ Intercept at x-axis = a

x-अमीकृ ज्वृभृबझ्शी- कृ?वृक

Intercept at y-axis = b

y-अमीकृ ज्वृभटजेए) अृबझ्शी-) = b

$$\therefore \frac{x}{a}+\frac{y}{b}=1$$



$$\frac{-y}{b}=\frac{x}{a}-1$$

$$1=\frac{x}{a}+\frac{y}{b}$$

❖ Equation of line parallel to line $ax + by + c = 0$ is $ax + by = k$.

ज्रा ीकृ$ax + by + c = 0$कृ ंक्ष्त शृीबज्कृरा ीकृ ीक्ष्त श्र त्र ज्वीकृक्ष्$ax + by = k$कृ न्ध

Equation of line perpendicular to line $ax + by + c = 0$ is $bx - ay = k$.

ज्रा ीकृ$ax + by + c = 0$कृ ज्कू ृकट बकृरा ीकृ ीक्ष्त श्र त्र ज्वीकृ$bx - ay = k$ ं न्ध

(A) If two lines are $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are parallel to each other then $m_1 = m_2$ and

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

जए्कृतीकृंरा ीदकृक्ष्$a_1x + b_1y + c_1 = 0$ ओीकृक्ष्$a_2x + b_2y + c_2 = 0$

दि कृत्तज्कृं ंक्ष्त शृी)बज्कृं ृकृबीकृंम$_1 = m_2$कृओीकृ$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

No intersecting pointकृत्कृं ी(ंक्ष्खख्ष्ऊ()कृंक्ष्त कृं) ?

∴ **No solution possible**कृत्कृं ी(ंकृं ृ कृत्ृइीटकृं) ?

(B) If two line are co-incidents on each other then $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$ and infinite solutions

जए्कृतीकृंरा ीदकृक्ष्दि कृत्तज्कृं ज्कृं ृ ीबज्कृं ृकृबीकृं$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$,वृम ृबकृं ृ तृइीटकृं न्ध

(C) If two lines intersect at single point then $m_1 \neq m_2$

$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$, unique solution possible

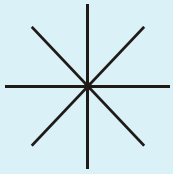
जए्कृतीकृंरा ीदकृक्ष्दि कृक्ष्त कृं ज्कृखख्ष्ऊ(कृं ज्बज्कृं ृकृबीकृं$m_1 \neq m_2$,

$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$ कृंअहबज्कृं ृ कृत्ृइीटकृं न

Concurrent Lines (तव- धक्ख्य रि ंघ

❖ Lines passing through a single point

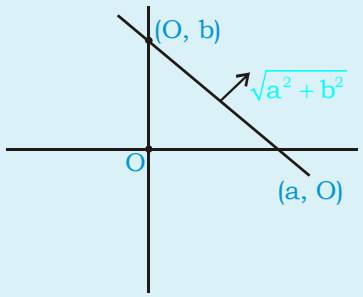
दि कृंक्ष्त कृंक्ष्त कृंक्ष्त कृंख ज)कृंक्ष्ट ृ ज्कृंरा ीदः



❖ Intercept by line $\frac{x}{a}+\frac{y}{b}=1$ between both axis

त्ी)ृकृंअमीकृंक्ष्ट कृंक्ष्त कृंक्ष्त कृंक्ष्प कृं$\frac{x}{a}+\frac{y}{b}=1$ ज्रा ीकृंहीजेकृंभटजेए)

$=\sqrt{a^2+b^2}$



❖ Line $ax + by = 0$ always passes through origin.

ज्रा ीकृ$ax + by = 0$कृं श्ीकृंवृ ृ कृंक्ष्त कृंक्ष्त कृंक्ष्ट कृं ी(ज्कृंखं ज्बज्कृं न्ध

Angle Between Two Lines (ै ख्यण प्रि ंघि ए ह्ह्रे ेि ंघि

❖ $\theta \rightarrow$ acute angle त्ृप्रज्ञा)कृं ीक्ष्ी

Let angle between two lines = θ and their slopes are m_1 and m_2 then

श्ी)कृं ऋं दकृं ृ कृंक्ष्त कृंक्ष्त कृंक्ष्ी(कृंक्ष्ट कृंक्ष्त कृंक्ष्प कृं ीकृं ीक्ष्ी कृंक्ष्त कृंक्ष्त कृंक्ष्प कृं)ि ज्कृंक्ष्ख्ष्ट वीबोद:

m_1 कृओीकृंक्ष्त कृंm_2 कृं तृमक

$$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

$$\theta=\tan^{-1}\left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

❖ Distance of line $ax + by + c = 0$ from a point (x_1, y_1)

दि कृंक्ष्त कृंक्ष्त कृं(x_1, y_1) कृंक्ष्त कृंक्ष्त कृंरा ीकृं$ax + by + c = 0$कृं ज्कृंक्ष्त

$$\Rightarrow\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$

❖ Distance of line $ax + by + c = 0$ from origin

श्ृ कृंक्ष्त कृंक्ष्त कृंक्ष्त कृंक्ष्त कृंरा ीकृं$ax + by + c = 0$कृं ज्कृंक्ष्त

$$=\left|\frac{C}{\sqrt{a^2+b^2}}\right|$$

❖ **Distance between two parallel lines:-**

त्रीक्षृत शी)ृबज्कृन(ा ीओृकृ क्षृमव्य कृ ञ्कृत्कृझ

$ax + by + c_1 = 0$

$ax + by + c_2 = 0$

distance लम्ब्कृ= $\left|\frac{C_1 - C_2}{\sqrt{a^2 + b^2}}\right|$

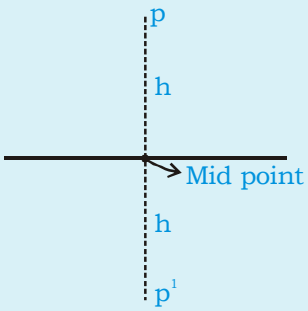
Reflection (गाधा अम

❖ A reflection is the mirror image of the shape.

दि कृवख्षाष्कृक्मृभी ाि्कृ ञ्कृे ुवीकृऊए कृ ना

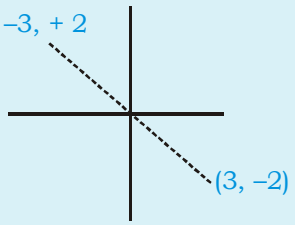
Types of reflection in co-ordinate system

ए)त्रुसी़ाि कृे हष्कृ्कृ क्षृवख्षाष्कृक्मृ क्षृर्व ाि



1. If reflection is asked about origin change only signs

जए्कृवख्षाष्कृक्मृ्कृ। कृष्मृअ्कृ्मृत कृे ळीकृं ादकृबीक्षृी ई्कृ वृत्ृ ि क्षृवकृक्कृ बर्कृ्न



2. **Reflection over *x*-axis** → *x*-co-ordinate remains same but *y* co-ordinate changes its signs.

***x*-य द्वेरिं रिं अद्री ि**→कृ*x*-ए)त्रुसी़ाि कृत शी)कृर्ज बीकृं क्कृ ए)कृ*y* यए)त्रुसी़ाि कृभे)क्षृप्स थकृक्कृ कृत्क्षीकृं ना

3. **Reflection over *y*-axis** → *y* co-ordinate remains same but *x* co-ordinate changes its signs.

***y*वय द्वेरिं रिं अद्री ि**→कृ*y*कृए)त्रुसी़ाि कृे ज्कृे जेटबु)कृत शी)कृर्ज बीकृं न ू ए)कृ*x*कृए)त्रुसी़ाि कृभे)ीकृप्स थकृक्कृ कृ्क्षीकृं ना

4. **Reflection over *y* = *x*** → When a point is reflected across the line $y = x$, the *x*-co-ordinates and *y* co-ordinates change their places.

y* = *x →कृे ज्कृे जेटबु)कृं ककृर्ि कृष्कृत्मृन(ा ाि्कृ $y = x$ त्कृ*x*-ए)त्रुसी़ाि कृभीकृक्कृ ि कृे ाि्कृे ञ्कृ एीबकृं ीक्षीकृं नृए)त्रुसी़ाि कृभे)ीकृन्डी)कृक्कृ बर्कृ्न

∴ Reflection of point (x, y) across the line $y = x$ is (y, x).

एमत्रमृ्मृ(x, y)कृं ाि्कृन(ा ाि्कृ $y = x$कृे ज्कृे जेटबु)कृं न्मृ(y, x)च

5. **Reflection over $y = -x$** → Both co-ordinates change their place and sign as well.

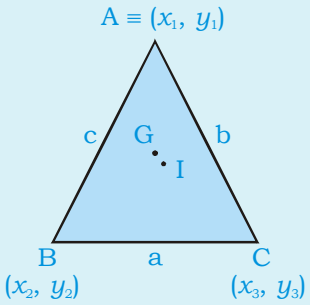
***y*कृं ि्पद्मपसंसक्कृ $y = -x$** →कृे ज्कृे जेटबु)कृत्ी़ीक्षृए)त्रुसी़ाि कृभे)ीकृन्डी) ओीकृक्षृप्स थकृीञ्कृक्कृ बर्कृ्नकृं बर्कृ्न

∴ Reflection of point (x, y) across the line $y = -x$ is $(-y, -x)$

एमत्रमृ्मृ(x, y)कृं ाि्कृन(ा ाि्कृ $y = -x$कृत्कृ कृे जेटबु)कृं ी़ीक्षीकृं न्मृ$(-y, -x)$

Centroid and Incentre (ट्रेड खाथिख्र ध्येल ब्लम

❖



G → Centroid ल्कृं ट्रं ि

I → Incentre ल्कृअप्रब्रईं ट्रं

$$G \equiv \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

$$I \equiv \left(\frac{ax_1 + bx_2 + cx_3}{a + b + c}, \frac{ay_1 + by_2 + cy_3}{a + b + c}\right)$$

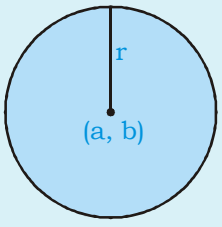
Equation of Circle (न्क्षखि खिवेद यथमि

❖ Where (a, b) are co-ordinates of center of circle and r is the radius.

ं ाि्कृ(a, b)कृर्ज बीकृं क्षृं(कृं क्षृए)त्रुसी़ाि कृे कृभीकृक्कृ कृक्षीश्नीकृं ना

Equation of circle ल्कृर्ज बीकृं ाि्कृत शत्रं ज्वीकृ→

$(x - a)^2 + (y - b)^2 = r^2$



if centre is origin ल्कृअख्ज्कृं (कृं कृे न

$\boldsymbol{x^2 + y^2 = r^2}$

❖ Area enclosed by $|x|+|y| = a \Rightarrow 2a^2$

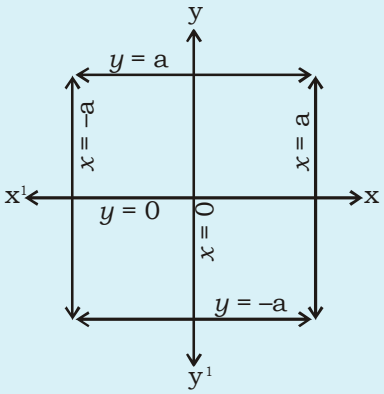
$|x|+|y| = a$ कृत क्षृएेजेकृमीक्षृं $\Rightarrow 2a^2$

➢ Equation of *x* axis / *x*स ं सख्ंस ख्बिंस $\Rightarrow y = 0$

➢ Equation of line parallel to x aixs $\Rightarrow y = \pm a$

x र ं सख्ंस ुयबस्कृं सख्ंस ख्बिंस $\Rightarrow y = \pm a$

- Equation of y axis/y र ं सम ं स ि ब ं स $\Rightarrow x = 0$
- Eqaution of line parallel to y axis $\Rightarrow x = \pm a$
- y र ं सम ा ुयवसम ं स ि ब ं स $\Rightarrow x = \pm a$



- Point of intersection of two lines
  ध ं सम ं र ुस ं सम ा ू न द स ु ध
  $a_1x + b_1y + c_1 = 0$

  $a_2x + b_2y + c_2 = 0 = \left(\frac{b_1c_2 - b_2b_1}{a_1b_2 - a_2b_1}, \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}\right)$

Area of A triangle (ं ि ि ख ि ि ं म

- The area of a triangle the Co-ordinate of whose vertices are $(x_1\ y_1)$, (x_2, y_2) and (x_3, y_3) is
  त स ं ह , ं य स ं से ं ह ं क ै ल स ा ख ा ं ी ं स ं ट ध ा ु ख ा म

$$\frac{1}{2} | x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2) |$$

2 Dimension Mensuration ¨2ण्अरा रै वाधीर्ती बिस

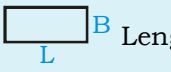
❖ **Zero-Dimensional figure** → • (point)

(रेखः अरा रै वाअर) बिण→ •ण¨ दिुप्रस

One-Dimensional figure →⟷ only length.

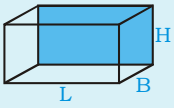
मव ण्अरा रै वाअर) बिस→⟷ सर्ं खसठा बिद्

Two-Dimensional figure (परखअरा रै वाअर) बि) →



Length and Breadth ु खा बिस्य थिाकथिमिू

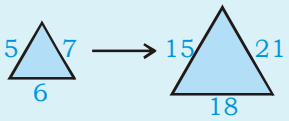
Three-Dimensional figure ¨ीर अरा रै वाअर) बिस →



Length, Breadth & Height

खा बिसकथिमिस्य थिाट काकि

❖ For Any 2D figure ु स थतसहति सिय विलितस्यों स सर् ंसखी ु →



❖ If each corresponding length of any 2D figure = K times.

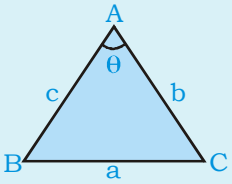
वस् सर थतसहति सिय विलितस्यों स सर् तस¨ जोर स¨ग। सठा बिस= K गर्मा

Then, perimeter = K times ग्स। सण्स्थलणिस= Kसर्गमा

Area = K^2 times ग्सउेंषिण्ठउस= K^2सर्गमा

Triangle ¨ीर्ि त्रयस

1. Scalene triangle ¨ णनिं रै दग्रणीर्ि त्रय



Perimeter ग्सण्स्थलणि= a + b + c

Semi-Perimeter/य श ग्स्थलणि (s) $= \frac{a+b+c}{2}$

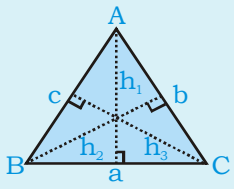
Area/उेंषिण्ठउस(Δ) $= \sqrt{s(s-a)(s-b)(s-c)}$

Area/उेंषिण्ठउ (Δ) $= \frac{1}{2}bc\sin A = \frac{1}{2}ab\sin C = \frac{1}{2}ac\sin B$

Inradius/य॥ (र्ष)वि (r) $= \frac{\text{Area}}{S}$

Circumradius/ण्स्थर्ष)वि (R) $= \frac{abc}{4 \times \text{Area}}$

❖



Area/उेंषिण्ठउ (Δ) $= \frac{1}{2} \times$ Base/य ि-धि $\times$ Height/चक्कबि=

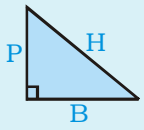
$\frac{1}{2}ah_1 = \frac{1}{2}bh_2 = \frac{1}{2}ch_3$

$\therefore ah_1 = bh_2 = ch_3 =$ (constant)

$\therefore a : b : c = \frac{1}{h_1} : \frac{1}{h_2} : \frac{1}{h_3}$ (Result)

2. Right angle triangle ¨ै वखरणीर्ि त्रयस

❖



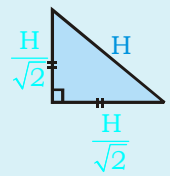
$\text{Area}(\Delta) = \frac{1}{2} \times PB$

Perimeter/ण्स्थलणि = P+B+H

Inradius/य॥ (र्ष)वि (r) $= \frac{P+B-H}{2}$

Circumradius/ण्स्थर्ष)वि (R) $= \frac{\text{Hypotenuse}}{2} = \frac{H}{2}$

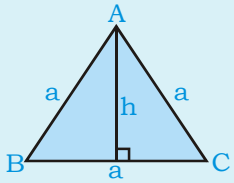
3. Isosceles right angle triangle ¨ै जिद्दग्रण ै वखर र्ीर्ि त्रयस



$\text{Area}(\Delta) = \frac{H^2}{4}$

Perimeter = $H+\sqrt{2}H = H(\sqrt{2}+1)$

4. Equilateral triangle ¨ै दग्रणीर्ि त्रयस



Side /स्हम्रिसि= a

Inradius गस्य इ (सर्ष)वसि(r) = $\frac{a}{2\sqrt{3}}$ or $\frac{h}{3}$

Circumradius गसगस्थी)वसिर्ष)वि (R) = $\frac{a}{\sqrt{3}}$ or $\frac{2h}{3}$

Perimeter गसण्स्थ्लणिस= 3a

Height गस्ट क्बिस्(H) = $\frac{\sqrt{3}a}{2}$

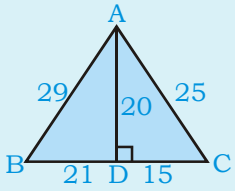
Area गस्डेषिण्ठ्स(Δ) = $\frac{\sqrt{3}}{4}a^2$

a(side)	height	Area
2	$\sqrt{3}$	$\sqrt{3}$
↓ ×k	↓ ×k	↓ ×k²
2K	$\sqrt{3}$ K	$\sqrt{3}$ K² (Result)

❖ **Heronian Triangle:** All sides and area is integer.
खेर्भेस् र्व सर्षहिम्रिस्(सस्हतिसहम्रिी ृस्य थ्रिाडेषिण्ठ्स्य वीरि सद्धस्

Find the Area of Δ with side → 36, 29, 25

29, 20, 21
15, 20, 25 (use the common triplet)
15 + 21 = 36 (Acute Δ)
ी थेस्2स्ब्रिए्स्ड्सि सि90°स्स्र्लि सर्वेस्य थ्रिाtripletस्लेस्ा length common खें

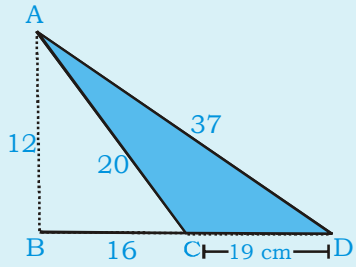


Area of triangle गस्षहिम्रिस् सिडेषिण्ठ्स= $\frac{1}{2}$ ×36×20 = **360 cm²**

❖ 57, 60, 111 → Find Area गस्डेषिण्ठ्स् ा सर् धेस्

Simplify by 3 गस्ब थेस्ध्ठ्स् तस्ी ⇒ 19, 20, 37
12 16 12 35

35–16 = 19 (obtuseΔ)

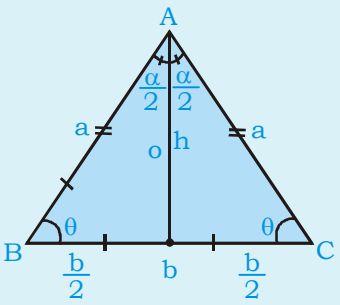


Area गस्डेषिण्ठ्स= $\frac{1}{2}$ ×19×12 = 114

∴ Area of original Δ् ल्ठ्स Δ र सिडेषिण्ठ्ू ⇒

114 × (3)² ⇒ 114 × 9 ⇒ 1026 cm²

5. Isosceles triangle ज्ञै ज्ञिद्रग्रर्णीरि ग्र्यसच्च→



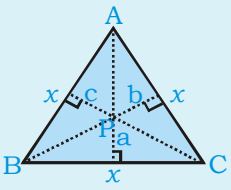
Height गस्ट क्बिस्= $\frac{\sqrt{4a^2-b^2}}{2}$

Area गस्डेषिण्ठ्स= $\frac{b}{4}\sqrt{4a^2-b^2}$

Area गस्डेषिण्ठ्स= $\frac{1}{2}a^2 \cdot \sin\alpha$

Perimeter गसण्स्थ्लणिस= 2a + b

❖



ABC → Equilateral Δ with side x and height H.
ABC → हम्रिसिर्स्य थ्रिाट् क्बिस्H र स्ब्रिसिल खिाग्रर्षहिम्रि द
P → Any point inside triangle./ र्षहिम्रिस स्थ्य ा, धग्र र्बिस्र ा, म
a,b,c → ⊥(Perpendicular) from point P on each side
a,b,c → भ्जोर सहम्रिसिण धग्रP स्े स्ू ख; ृ
H = a+b+c
Area ΔABC = ΔBPC + ΔAPC + ΔAPB

$\frac{\sqrt{3}}{4}x^2 = \frac{1}{2}xa + \frac{1}{2}xb + \frac{1}{2}xc$

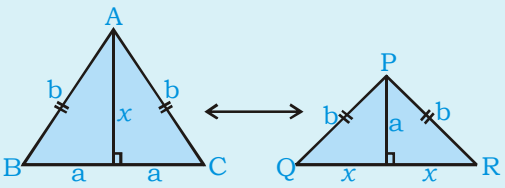
$\frac{\sqrt{3}}{4}x^2 = \frac{x}{2}(a+b+c) \Rightarrow \frac{\sqrt{3}}{2}x = a+b+c$

∴ H = a+b+c

$\boldsymbol{x = \frac{2}{\sqrt{3}}(a+b+c)}$ (Result)

Area of ΔABC = $\boldsymbol{\frac{1}{\sqrt{3}}(a+b+c)^2}$

❖



Area of ΔABC = ax = Area of ΔPQR

Scalene Triangle ¥ निर ैदग्रणीर ्रयस

❖ All 3 side are unequal. श ू¥ यी कृ्याश्ज वलस्श्एूक

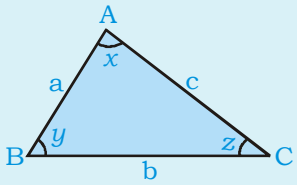
Perimeter श्त्।(लस्श(P) = a + b + c

Semi-perimeter श्ज) श्त्।(। = $\frac{a+b+c}{2}$

Area of $\Delta ABC = \sqrt{S(S-a)(S-b)(S-c)}$

Area of $\Delta ABC = r \times S$

Area of $\Delta ABC = \frac{abc}{4R}$



❖ If regular polygon have same perimeter then figure with more number of sides have greater area.

वस् सर्क वस्ला स ख्ब्रिस्र सिग्स्थलणिस्र्ल सर्वेसा ेसिय स्र स्हब्रियि ेसिं ख्ित यौ स स्र सिर्डेषिण्ठ्ठस्य स्र सर्वे। सिख्र

∴ Area of square > Area of Δ with same perimeter.

· गस्र सिर्डेषिण्ठ्ठस्र> स्र्ल सग्स्थलणिसं ख्ितेस Δस्र सिर्डेषिण्ठ्ठ्द

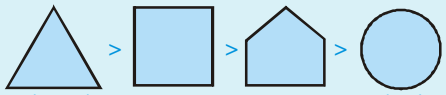
Infinite sides गस्र्य ॥ स्हब्रिी ॥ स्र→ circle गस्रं ब्रक्

∴ circle has more area then any other closed figure.

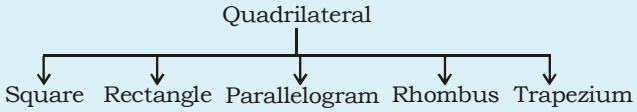
∴ · ब्रक्रिस्र सिर्डेषिण्ठ्ठस्र्र थ्तस्य ख्वस ।स्यौ स स्र्ेस्य स्र सर्वे। सिख्र

❖ If all the given figure have the same perimeter then:-

वस् सर् साबस्र्हतिस्यौ स वृसिस्र सिग्स्थलणिस्र्ल सर्वेसा ेि(ं

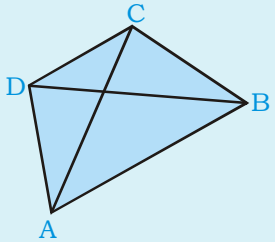


Quadrilateral ¥ इ ब्रि ्रयस



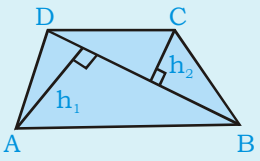
Quadrilateral : A closed shape and a type of polygon that has 4 sides, 4 vertices and 4 angles.

इ ब्रि ्रयस्(सि र स ।स्यौ स स्स्र्ाि ेर स्र्र ध्ाि स्र सि ख्ब्रिस्र> थ्लेाख्स्हब्रिी ्ि न 4स् तिस्रिय ध्ाि स्र ेविसिर्वे। सिख्र



∠A + ∠B + ∠C + ∠D = **360°**

❖ **Area of Quadrilateral** ग स्ह ब्रि ्रय ण्व रण्ध्ाीस्ग द्व



Area = $\frac{1}{2} \times BD \times (h_1 + h_2)$

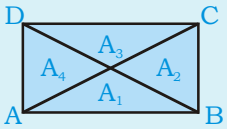
= $\frac{1}{2}$ × diagonal × (Sum of perperpendiculars drawn on this diagonal)

= $\frac{1}{2}$ × र्ं र वसिर्ड्ड बर्स्ं र वसिग्ध्ाड तक्ेसी सख्ा ेंसिर सिख्ेंगू

❖ In any Quadrilateral गस्र थ्तस्हतिस्क। स्हब्रि स्ले।

A_1, A_2, A_3 and A_4 are the area of respective triangles.

A_1, A_2, A_3 य ध्ािA_4 द्र ल् (िसर्ष्हब्रिी ेंसिर ेसर्डेषिण्ठ्ठस्र्ख्र



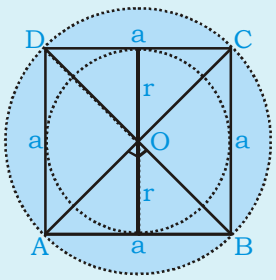
$A_1 \times A_3 = A_2 \times A_4$

❖ Area of quadrilateral made by joining the mid points of all sides of given quadrilateral is half of the original quadrilateral.

स्ी सगी स्क। स्हब्रि स्र तस्र्हतिस्हब्रि यि ेंसिर ेस्लह्मस्र ।, य ेंसिर ेसिस्लण्ठ्ठरि ध्स ``क। स्हब्रि स्र सिर्डेषिण्ठ्ठस्लण्ठ्स्क। स्हब्रि स्र सिय ि- सिर्वे। सिख्र

Square ¥ नध्श्र

❖



Area of ΔAOB = Area of ΔBOC = Area of ΔCOD = Area of $\Delta AOD = \frac{a^2}{4}$

Perimeter गस्पण्श्र्स= 4a

Area गस्र्डेषिण्ठ्ठस्र= $a^2 = \frac{d^2}{2}$

diameter ग स्ब्थिस्र= $a\sqrt{2}$

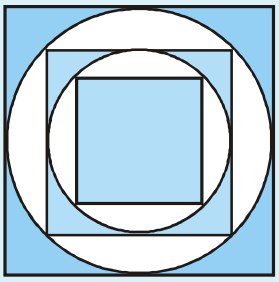
Inradius गस्य ॥ (र्ष्)वसि(r) = $\frac{a}{2}$

Circumradius गस छिसर्ध)वसि(R) = $\frac{d}{2} = \frac{a}{\sqrt{2}}$

$R : r = \sqrt{2} : 1, \quad \frac{\text{Area of circumcircle}}{\text{Area of incircle}} = \frac{2}{1}$

❖ If we make circle inside a square and again make a square inside the circle and so on... the area will becomes half and so on.

वस, सक्त्रसी र सं गस ेसहति धसी र सं ब्रसि सि ेास थिाप्त्रण्ठास्र ेसं ब्रसि ेसहति ध ी र सं गस सि ेास थिाप्त्रण्ठास्र ेसं ी सथ ेखा ेसिर्ेाि ि ड्ढ ी सिख्रा

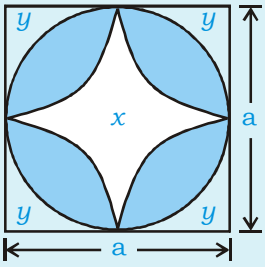


Area of largest square : middle : smallest

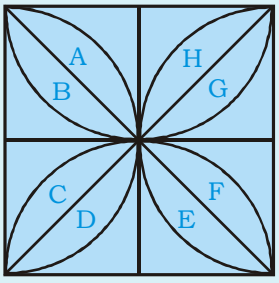
थ ेस ऐषां गस सिर्ेाि ि (लह्रा (स ेस ेसर्ि वृि

4 : 2 : 1

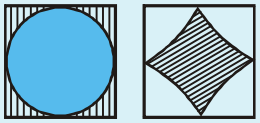
∴ Largest : smallest गस ेस एसि(स ेस ेसर्ि वृिसि ⇨ **4 : 1**

❖ 

Area of shaded region ु ज ब्रिसि । स्हगिस सिर्ेाि ि ू स = $\frac{4}{7}a^2$

❖ 

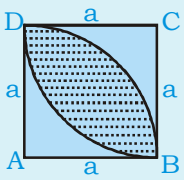
Area of A, B, C, D, E, F, G, H is equal

❖ 

a is the side of square.

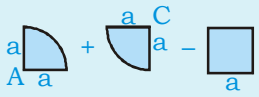
a ं गस तस्हब्रिसि सिख्रा

Shaded Area गस ब्रिसि । सर्ेाि ि = $\frac{3a^2}{14}$



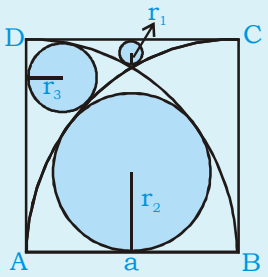
Area of leaf गस त्तिस सिर्ेाि ि ड्ढ = $\frac{4}{7}a^2$

Area of leaf गस त्तिस सिर्ेाि ि ड्ढ

= 

$\frac{\pi}{2}a^2 - a^2 = \left(\frac{\pi}{2} - 1\right)a^2 = \frac{4}{7}a^2$

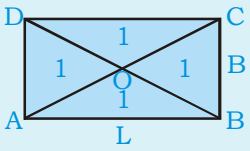
❖ If side of square गस स, सं गस तस्हब्रिसि = a



$r_1 = \frac{a}{16}, \quad r_2 = \frac{3a}{8}, \quad r_3 = \frac{a}{6}$

$r_1 : r_2 : r_3 = \left[\frac{a}{16} : \frac{3a}{8} : \frac{a}{6}\right] \times 48 = \mathbf{3 : 18 : 8}$

Rectangle ्अरा बस

❖ 

Perimeter गस स्थलणि ु (P) = 2(L + B)

Area र्ज ेसि (A) = L×B

Diagonal गस ं र वसि = AC = BD = $\sqrt{L^2 + B^2}$

Area of ΔAOB = Area of ΔBOC = Area of ΔCOD

= Area of ΔAOD = $\frac{LB}{4}$

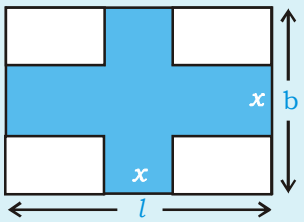
❖ Radius of maximum size circle that can be put inside rectangle = $\frac{\text{Breadth}}{2}$

य ह र । लस्य रि धिस ेसं ब्रसि त स्र्ध)वसि ड्ढ थ ेसय वि। सर ेसय ा, धस्थह सि ि थ र । सि ख्रा = $\frac{\text{Breadth}}{2}$

Path Around or Inside a Rectangle

1. Crossing road inside rectnagle

❖



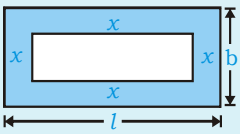
Area of crossing road = $lx + bx - x^2$

$= x(l+b-x)$

Perimeter of path = $2(l + b - 2x)$

2. Path inside a rectangle

❖



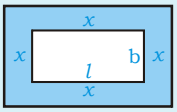
Area of path = $2x\,(l + b - 2x)$

Perimeter of path = $4\,(l + b - 2x)$

Where x is the width of the path.

3. Path Outside a rectangular field

❖



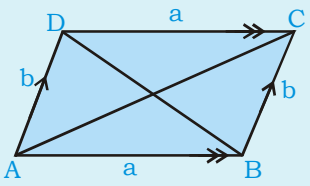
Area of path = $2x\,(l + b + 2x)$

Perimeter of path = $4\,(l + b + 2x)$

Where x is the width of path

Parallelogram

❖



Perimeter = $2\,(a + b)$

$AC^2 + BD^2 = 2\,(a^2 + b^2)$

$d_1^2 + d_2^2 = 2(a^2 + b^2)$

Area = Base × Height

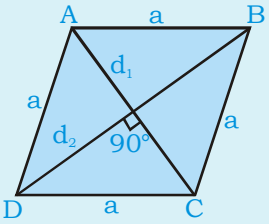
If length of one diagonal is d

Then Area = $2\sqrt{s(s-a)(s-b)(s-d)}$

Where $s = \dfrac{a+b+d}{2}$

Rhombus

❖



Perimeter = $4a$

$4a^2 = d_1^2 + d_2^2$ (Property)

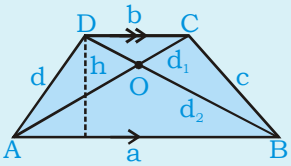
area = $\dfrac{1}{2} \times d_1 \times d_2$

Area = Base × Height

Area of Δ AOB = Δ Area of Δ BOC = Area of COD = Area of ΔAOD

Trapezium

❖



Perimeter = $a + b + c + d$

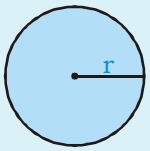
Area = $\dfrac{1}{2}$ (Sum of parallel sides) × distance between them

Area = $\dfrac{1}{2}$ (a + b) × h

$d_1^2 + d_2^2$ **= $c^2 + d^2 + 2ab$** (Property)

Circle

❖



$\dfrac{\text{Circumference}}{\text{Diameter}} = \text{constant} = \pi$

$\dfrac{C}{2r} = \pi \Rightarrow$ Circumference = $2\pi r$

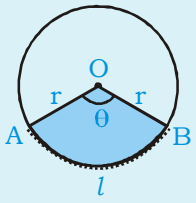
diameter (d) = $\sqrt{\dfrac{4A}{\pi}}$ (Where, A = Area of circle)

Area = $\pi r^2 = \dfrac{\pi d^2}{4}$

radius	$c=2\pi r$	$A=\pi r^2$	
7	44	154	
7K	44K	$154K^2$	
24.5	44×3.5	$154\times(3.5)^2$	$\because 24.5=7\times3.5$
28	44×4	154×16	

Sector of a circle – मव णनह्मण्व रण्लु ज्स

❖



θ = central angle = $\frac{\theta}{360°}$ (sector is what part of circle)

θ = र्ो त्मस्र किसि= $\frac{\theta}{360°}$ सु॰ क्रडि एसं क्रसिर सिर ञिस्र सिर्ंगिसक्रूम

Area of sector ग्सं क्रमि क्विएसर सिर्ंषिण्ठ OAB =

$\frac{\theta}{360°} \times \pi r^2 = \frac{lr}{2}$

Length of $\overline{AB} = l = \frac{\theta}{360°} \times 2\pi r$

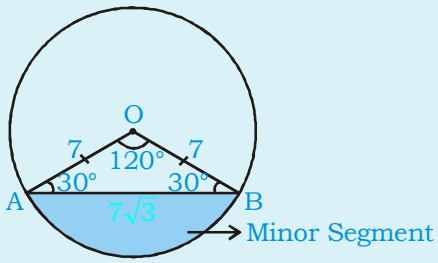
$\theta° = \theta \times \frac{\pi^c}{180}$

$l = r \times \frac{\pi\theta}{180}$

$l = r\theta^c$ or $\theta^c = \frac{l}{r}$

Segment – नह्मल्लु ज्स

❖ **Minor segment** ग्सरूवृम्मनह्मल्लु ज्



Area of minor segment = Area of sector – Area of ΔOAB

ज्रेकृिसं क्रडि एस सिर्ंषिण्ठस्रसं क्रडि एस सिर्ंषिण्ठस्रस्रसΔOAB र सिर्ंषिण्ठ

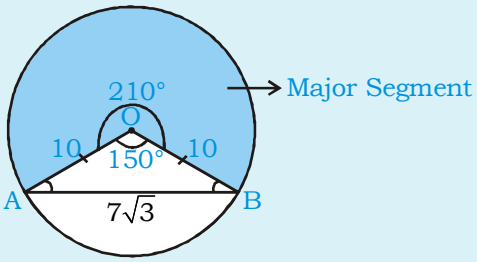
$\Rightarrow \frac{120°}{360°} \times 154 - \frac{1}{2} \times 7 \times 7 \times \sin 120°$

$\Rightarrow \frac{154}{3} - \frac{49}{2} \times \frac{\sqrt{3}}{2}$

$\Rightarrow \left(\frac{154}{3} - \frac{49\sqrt{3}}{4}\right) cm^2$

❖ **Major segment** ग्सद्ज्छनह्मल्लु ज्

Area of major segment॰ ऐसां क्रडि एसर सिर्ंषिण्ठ॰ ⇒



Area of sector (210°) + Area of ΔOAB

ख्)वड एसर सिर्ंषिण्ठस्(210°)स्रसΔOABसर सिर्ंषिण्ठ

$\Rightarrow \frac{210°}{360°} \times 100\pi + \frac{1}{2} \times 10 \times 10 \times \sin 150°$

$\Rightarrow \frac{7}{12} \times 100\pi + \frac{1}{2} \times 10 \times 10 \times \frac{1}{2}$

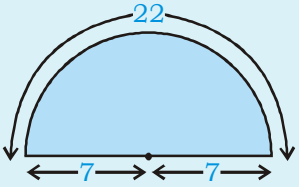
$\Rightarrow \left(\frac{175\pi}{3} + 25\right) cm^2$

Perimeter of segment

ड एसर त्मण्स्थ्र = $2r\left[\frac{\pi\theta}{360°} + \sin\frac{\theta}{2}\right]$

For semi-circle – अँ घह्मण्व स्रद्वि मस

❖



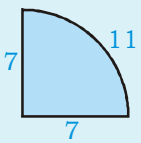
Radius	Circumference	Area
7	36	77

Perimeter ग्सण्स्थ्र स= πr + diameter

Area = $\frac{\pi r^2}{2}$

For a quadrant – मव ण्ड ब्रह्मर्ण्व स्रद्वि मस

❖

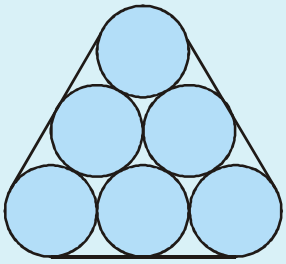


Radius	Circumference	Area
ख्)वि	ण्स्थ्र	र्ंषिण्ठ
7	25	$\frac{77}{2} = 38.5$

Perimeter ग्सण्स्थ्र स= r + r + $\frac{2\pi r}{4} = 2r + \frac{\pi r}{2}$

Area ग्सर्ंषिण्ठ = $\frac{\pi r^2}{4}$

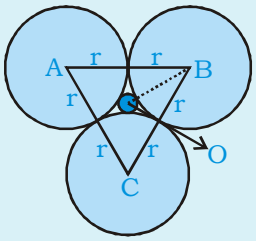
❖



Length of string = 2πr + diameter × number of circles

एंधिसस तसठा बिस=स2πr + स्बथि × ं ब्रीसिस तसथ-वि

❖



ΔABC = equilateral Δ

OB = circumradius

Let radius of small circle = x

र्ली सिन्वेंसं ब्रसिस तसर्ष)वसि=सx

$$OB = \frac{\text{side of equilateral}\Delta}{\sqrt{3}} = \frac{2r}{\sqrt{3}}$$

$$\therefore x = \frac{2r}{\sqrt{3}} - r$$

$$\boldsymbol{x = \left(\frac{2-\sqrt{3}}{\sqrt{3}}\right) \times r}$$

If we draw a big circle around these 3 circles, then

radius of that big circle $\Rightarrow \boldsymbol{y = \frac{2r}{\sqrt{3}} + r}$

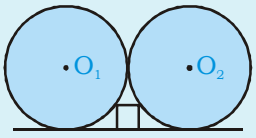
वस, सक्वसर्ष स3सं ब्रीसिस सकधिसिस धिसि र स एसिं ब्रसिं ि सक्वसिस सिक्थ-स ऐप

ं ब्रसिस तसर्ष)वसि⇒ $\boldsymbol{y = \frac{2r}{\sqrt{3}} + r}$

$$y = r \times \left(\frac{2+\sqrt{3}}{\sqrt{3}}\right)$$ (Result)

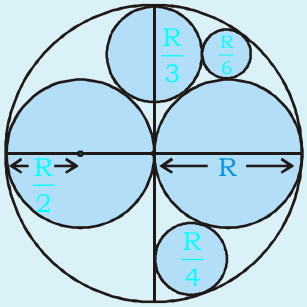
❖ Two circles of equal radius touch each other as shown in figure, a square of side 'a' is placed between two circles. One side of square is on direct common tangent of both circles, find r/a?

थ्र्ली सर्ष)वसि ं ख्सिं सिं ब्रसिं र स, थ्धेस्र सिण् सि धो सक्वस्रa हमिसिं ख्सि ी र सं गसक्थ-सभ्र धिसस्द्धी सक्वस्रार सं गसर तसी र सहमिसि, ी सिं ब्रसिंर ं चहविर्स श्रास्र्स्र्ं ब्रण् सिंथ-सिंग धसस्द्धी सक्वस्रा सr/a र सिर्ली सर्खेगिंख्सि

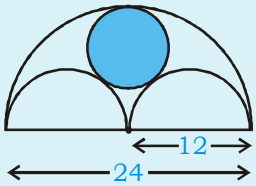


$$\frac{r}{a} = \frac{5}{2}$$

❖

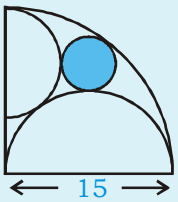


Base Figure



Derived Figure

radius of shaded part ु ज्विसि । स्हंगिस तसर्ष)वूि $= \frac{R}{3} = \frac{12}{3} = 4$



Derived Figure

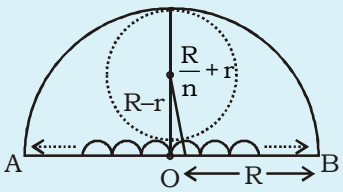
radius of shaded part ु ज्विसि । स्हंगिस तसर्ष)वूि स $= \frac{R}{6} = \frac{15}{6}$

❖ n semi-circle (even) drawn on diameter AB

nसय श ं ब्रसिु थ्लू सस्बथि सAB सग्धसड तकेसगी सक्वस्रा

Radius of each small semi-circle $= \frac{R}{n}$

भ्जोर सन्वेंसय ं ब्रसिस तसर्ष)वसि= $\frac{R}{n}$



$$\left(\frac{R}{n} + r\right)^2 = \left(\frac{R}{n}\right)^2 + (R-r)^2$$

$$\frac{R^2}{n^2} + r^2 + 2 \times \frac{R}{n} \times r = \frac{R^2}{n^2} + R^2 + r^2 - 2Rr$$

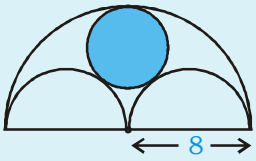
$$2 \times \frac{R}{n} \times r = R^2 - 2Rr$$

$$\frac{2Rr}{n} + 2Rr = R^2$$

$$2Rr\left(\frac{1+n}{n}\right) = R^2$$

$$r = \frac{Rn}{2(n+1)}$$

radius of small circle, जे कृिसं क्रसि तस्रष)वू $r = \frac{nR}{2(n+1)}$

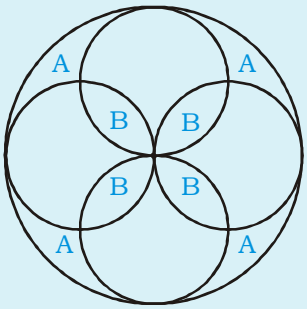


Here, n = 2, R = 8

∴ radius of shaded part, ज विासि ।स्हगिसर तस्रष)वू

$= \frac{2\times 8}{2\times 3} = \frac{8}{3}$

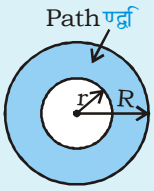
❖ Find A/B = ?



A = B (Property)

$\frac{A}{B} = 1 : 1$

❖ Ring ं ख्वू

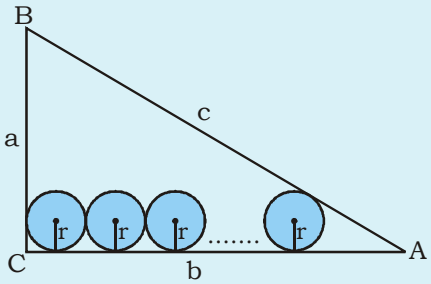


Area of ring/ ं ख्वसर सिर्डेषिण्ठ्ठस= $\pi(R^2-r^2)$

Width of path/ण्द्रसि तस्रक्द्विंबिस= R–r

Some important results

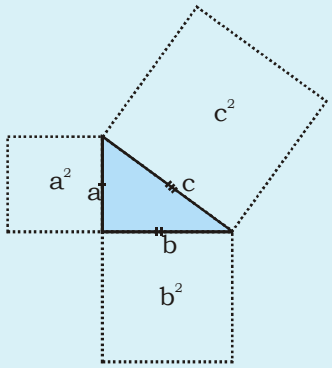
➢



n – identical circles

$r = \frac{ab}{(2n-1)a+b+c}$

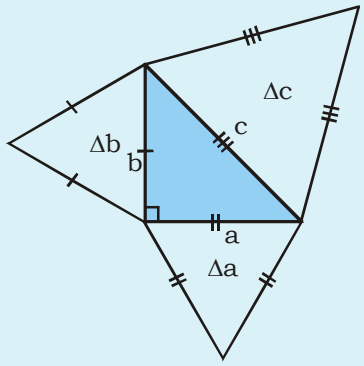
➢



Area (□c) = area (□b) + area (□a)

$c^2 = a^2 + b^2$

➢



Equilateral Δs

Area (Δc) = area (Δb) + area (Δa)

➢ Area of Path Inside Square =4d (a–d)

ं गस्र ेस्या, धस्र ं स्धचि ेस्र सिर्डेषिण्ठ्ठस= 4d (a – d)

∴ d = length of Path/ ्र खसिस्d = धचि ेस्र तस्क्हिसि

a = length of Square/a = ं गस्र तस्ख्ठा बि

➢ Area of Path outside Square/सं गस्र ेस खिस्र ं स्धचि ेस्र ि र्डेषिण्ठ्ठस= 4d (a + d)

➢ Area of Path midway Square/ ं गस्र ेस तक्रोंि तक्रस ं स्धचि ेर सिर्डेषिण्ठ्ठस= d (2a – d)

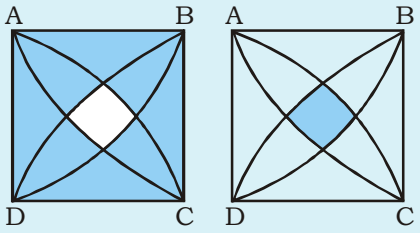
➢ If the side of square increase by *x* times then the area of the square becomes x^2 times.

वस्, सं गस्र तस्हब्रिसि ेसि*x*र्सामसि उसि, वसि विस। सं गस्र सिर्डेषिण्ठ्ठसx^2 र्गमसिर्खेसि ि सिख्रा

➢ If the area of the square is a $= \frac{4a}{\pi} cm^2$, then the area of the circle formed by the same perimeter is

वस्, सं गस्र सिर्डेषिण्ठ्ठस्a थ्ेलतेंनस स्र्ल सास्थ्लण्सि ेस ं ि सी सं क्रसि ि र्डेषिण्ठ्ठस्खेा सिख्रा= $\frac{4a}{\pi} cm^2$ ग्थ्ेलतें

➢ Area of shadded part/ज्ञिसि ।स्हगिसर सिडेंषिण्ठ्ठस् ि।सर धेंदि



$a^2 - 4\left[\frac{\sqrt{3}}{4}a^2 - \frac{\pi a^2}{12}\right]$ $a^2 - \sqrt{3a}^2 + \frac{\pi a^2}{3}$

➢ $a^2\left[1 - \sqrt{3} + \frac{\pi}{3}\right]$

➢ Room as a Rectangular figure

य वि। रि धिसर लधि(ठ

Area of four walls of room

य वि। रि धिसर लधेसर `सककधिंसि, तं धिंसिसर सिडेंषिण्ठ

= Perimeter × Height = 2 × (L × B) × H

➢ Area of Roof and 4 walls

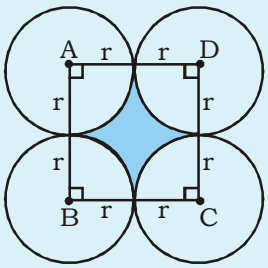
ककधिंसि, तं धिंसिसय थिसम्। सर सिडेंषिण्ठ

= 2H (L + B) + LB

(This formula can be used when we have to paint a whole room

वखस्थ्षसि सग्लेंगिसर धस्र्र `सक्ष्मा सक्लेगस्धेसर लधेसर तसग्मबिसर धें तसर्वेदि

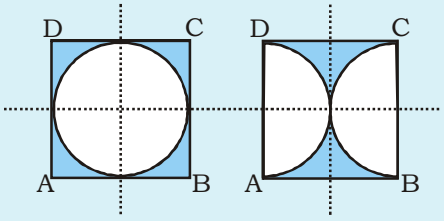
➢ Find the area of shaded part/ज्ञ विसि ।स्हगिसर सिडेंषिण्ठ



Area of square - sector/ ं गसर सिडेंषिण्ठ्ठ कधिंसिसर्षी)वम्रिंसिसर ि डेंषिण्ठ

$4r^2 - \frac{\pi r^2}{360} \times 90 \times 4 = r^2[4 - \pi]$

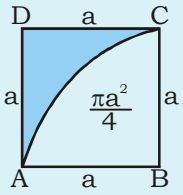
➢



Area of shaded region is equal in each case.

ज विसि ।स्हगिसर सिडेंषिण्ठ्ठसग्लेंर सस्द्धसि सलेसस्थ्र्ली सर्वेगदि

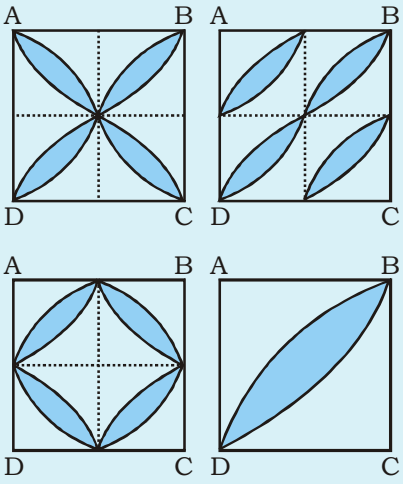
➢



Area of shaded region/ज्ञ विसि ।स्हगिसर सिडेंषिण्ठ

$= a^2 - \frac{\pi a^2}{4} = \frac{a^2}{4}(4 - \pi)$

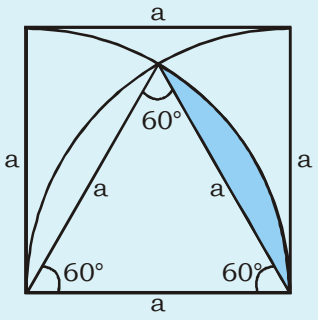
➢



Area of shaded region is equal in each case.

ज विसि ।स्हगिसर सिडेंषिण्ठ्ठसग्लेंर सस्द्धसि सलेसस्थ्र्ली सर्वेगदि

➢

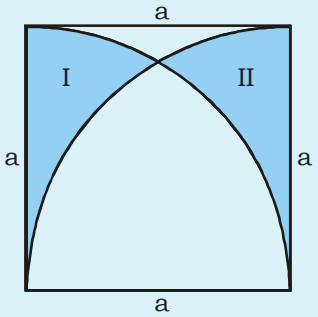


Area of shaded region/ज्ञ विसि ।स्हगिसर सिडेंषिण्ठ

$= \frac{a^2}{12}\left[2\pi - 3\sqrt{3}\right]$

= 9.05% of square area/ ं गसर `सर्डेंषिण्ठ्ठसर सि9.05%

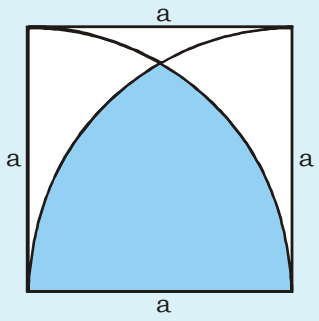
➢



Area of (I) = area of (II) = $\frac{a^2}{12}\left[3\sqrt{3}-\pi\right]$

= 17.1208% of square area/ गसर सिर्ऐषिण्ठसर f

➤

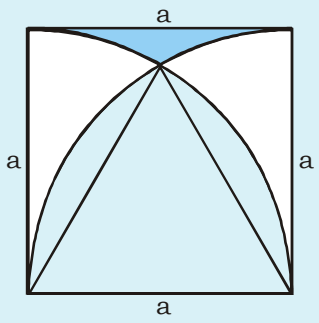


Area of shaded region/ज विाशि । स्हांगिसर सिर्ऐषिण्ठ

= $\frac{a^2}{12}\left[4\pi-3\sqrt{3}\right]$

= 61.4166% of square area/ गसर सिर्ऐषिण्ठ

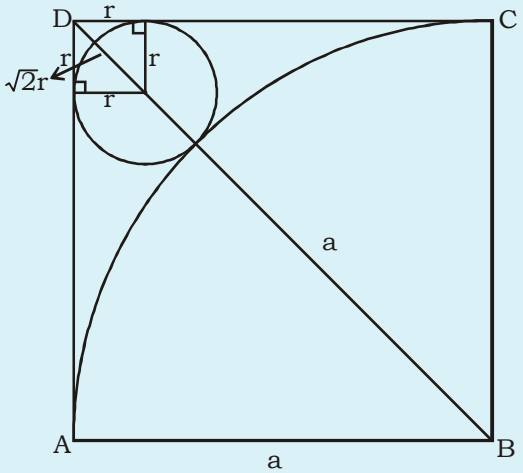
➤



Area of shaded region/ज विाशि । स्हांगिसर सिर्ऐषिण्ठ

= $\frac{a^2}{12}\left[12-2\pi-3\sqrt{3}\right]$ = 4.3388% of square area

➤



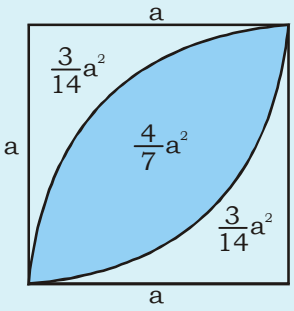
$\sqrt{2}r+r+a=\sqrt{2}a$

$r\left(\sqrt{2}+1\right)=\left(\sqrt{2}-1\right)a$

$r=\frac{\left(\sqrt{2}-1\right)a}{\sqrt{2}+1}$

$r=\left(\sqrt{2}-1\right)a$

➤

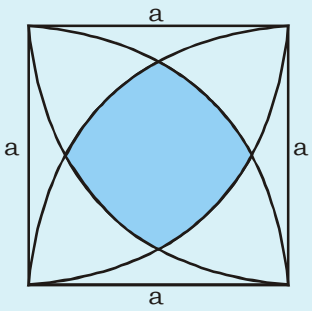


Area of shaded region/ज विाशि । स्हांगिसर सिर्ऐषिण्ठ

= $\frac{a^2}{2}(\pi-2)=\frac{4}{7}a^2$

= 57.02% of square area

➤

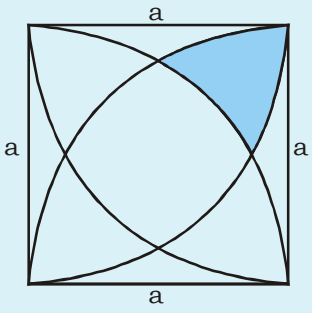


Area of shaded region/ज विाशि । स्हांगिसर सिर्ऐषिण्ठ

= $\frac{a^2}{3}\left\{3\left(1-\sqrt{3}\right)+\pi\right\}$

= 31.5146% of square area

➤

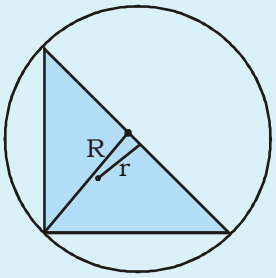


Area of shaded region/ज विाशि । स्हांगिसर सिर्ऐषिण्ठ

= $\frac{a^2}{12}\left\{\pi-12+6\sqrt{3}\right\}$

= 12.78% or $\frac{1}{8}$ of square area

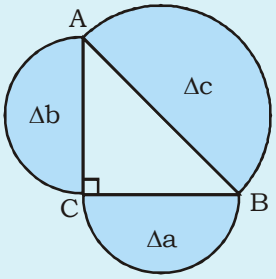
➤



$$r = \frac{p + b - h}{2}; R = \frac{h}{2}$$

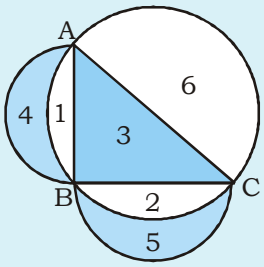
$$r = \frac{\Delta}{s}; R = \frac{abc}{4\Delta} = \frac{p \times b \times h}{4\Delta}$$

➤



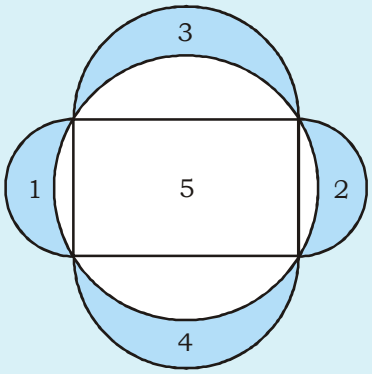
Area (Δc) = area (Δb) + area (Δa)

➤

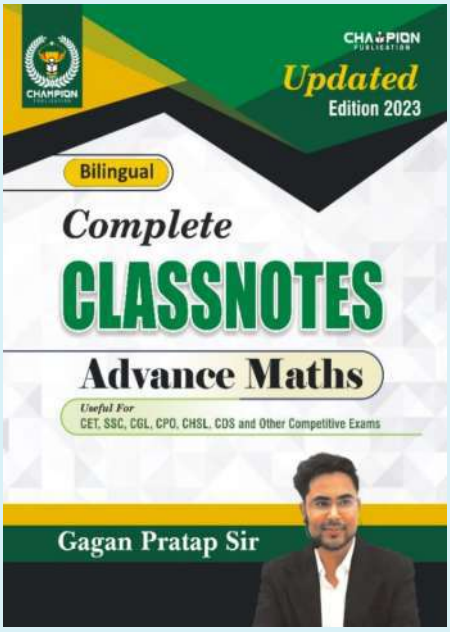


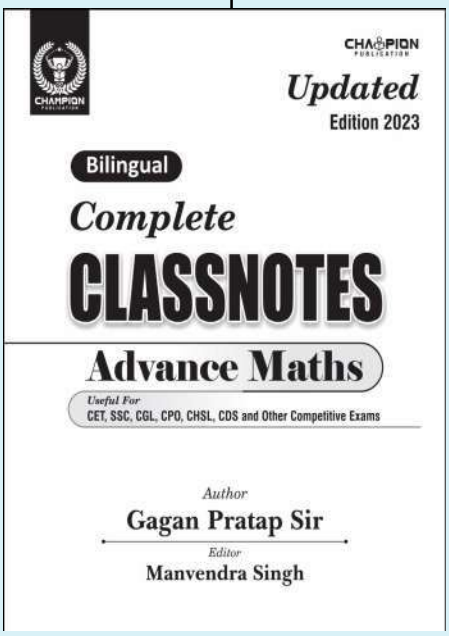
(i) area (6) = area (1) + area (2) + area (3)
(ii) area (4) + area (5) = area (3)

➤



Here,
(i) area (1) = area (2)
(ii) area (3) = area (4)
(iii) area of [(1) + (2) + (3) + (4)] = arae of rectangle (5)



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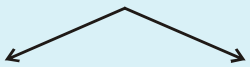
Polygon ÷ दर्प्र ्रयस

❖ **Polygon:** closed figure of 3 or more sides.

ख्वम्रिस(स3स्वासिय स्-र सहम्रियिोंसिं तस्ा ,स्यों सद

Polygon



convex/चत्खि	concave/य ं ।ख
* each interior angle is less than 180°	any one angle is more than 180°
All diagonals lies inside	Any one diagonal or more will be outside.
* भ्जोर स्याी स्थ सर् वेंि 180°स्े सर् लसर्वे सिख्न	र बिस्हतिसि र स र्वेसिा 180°स्े सर् स्-र र्वे सिख्न	
*स्-हतिसं र वसिया ,धर्वे सख्न	र बिर स्यासिय स्-र सं र वसि खिसर्वेंदि	

Ex:  

Regular polygon ÷ सि ैं बण्दर्प्र ्रयस

❖ **Always a convex polygon**ण्ग ण्प्रै ंसरणम्व ण्म्हद्ध ण्दर्प्र ्रय

* each side is equal, भ्जोर सहम्रिसि ध्ि धसख्न
* each interior and exterior angle is equal

भ्जोर स्याी स्थ सय्थिम खिसर्वेंसि ध्ि धसर्वे सख्न

Regular Hexagon: 6 sides/हम्रिी ्

6 vertices/ ्तिम्रि

6 integior angles/याी स्थ सर् वेंि

6 exterior angles/ छिसर् वेंि

∴ n sides polygon have: n vertices, n interior angles, n exterior angles.

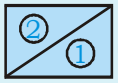
∴ n हम्रियिोंसिं खिस ख्वम्रिस्ले(सn्स् तिम्रिसn्स्याी स्थ सर् वेंसिय थिम्न छिर वेंसिर्वे सख्न

❖ Sum of all interior angles of a polygon with n sides

nस्हम्रियिोंसिं खिस ख्वम्रिस र्वे स्-हतिस्याी स्थ सर् वेंिसि स्िोंग

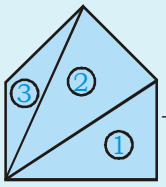
⇒ (n–2) × 180°

→ (3–2)×180° = 180°



(4–2) × 180° = 360°

or 2Δ = 2 × 180° = 360°



→ (5–2) × 180° = 540°

or 3Δ = 3×180 = 540°

→ Each interior angle of a regular polygon ⇒ $\frac{(n-2)\times 180^\circ}{n}$

→ ैर स्ें वस्ला स ख्वम्रिस स्िज्जोर स्याी स्थ सर् वेंिसि⇒ $\frac{(n-2)\times 180^\circ}{n}$

❖ Sum of all exterior angles of a 'n' sided polygon ⇒ 360°

'n'स्हम्रिसिं खिस ख्वम्रिस र्वे स्-हतिस छिसर् वेंिसि स्िोंगिस⇒ 360°

❖ ∑I + ∑E = 180° × n

(n–2)180° + ∑E = 180° × n

180°n – 360° + ∑E = 180° n

∑E = 360°

I + E = 180°

Internal angle + External angle = 180°

अुबल्ि ण्व स्वारण+ण्दर्ं ण्व स्वारण= **180°**

∴ Each exterior angle of a regular polygon = $\frac{360^\circ}{n}$

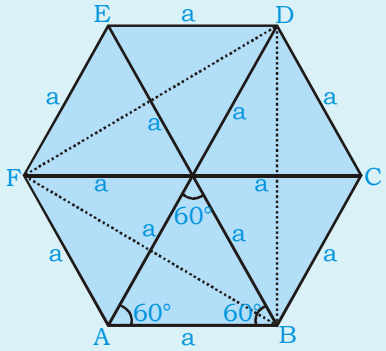
ैर स्ें वस्ला स ख्वम्रिस स्ि-ज्जोर स खिसर्वेंसि= $\frac{360^\circ}{n}$

No. of sides, हम्रियिोंसिं तस्-ा-र्वू स= $\frac{360^\circ}{E}$

❖ No. of diagonals in a polygon = $\frac{n(n-3)}{2}$

ैर स ख्वम्रिस्लेोसं र वेंसिं तस्-ा-वसि= $\frac{n(n-3)}{2}$

Regular Hexagon ÷ सि ैं बणं र्वेंश्रयस



Each Interior angle प्रत्येक आंतरिक कोण = 120°

Each exterior angle प्रत्येक बाह्य कोण = 60°

Total diagonals कुल विकर्ण = 9

Large diagonal बड़ा विकर्ण = FC = AD = BE = 2a

Perimeter परिमाप = 6a

Regular Hexagon = 6 equilateral Δ = 3 Rhombus

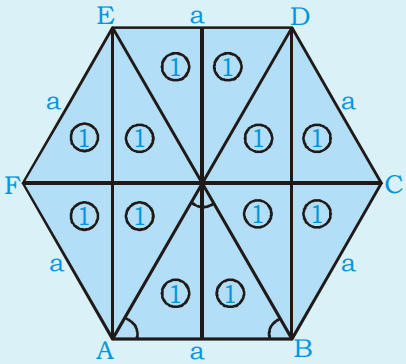
नियमित षट्भुज = 6 समबाहु त्रिभुज = 3 समचतुर्भुज

$$\text{Area} = \frac{\sqrt{3}}{4}a^2 \times 6 = \frac{3\sqrt{3}}{2}a^2$$

r = Short diagonal लघु विकर्ण = FD = DB = BF = $\frac{\sqrt{3}}{2}a$

Circumradius परिवृत्त (R) = a (side)

❖

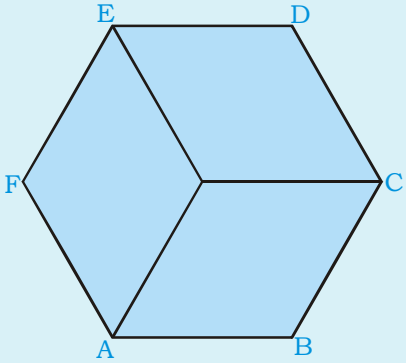


6 equilateral Δ formed. 6 समबाहु त्रिभुज बनते हैं।

Area of each Δ is same. प्रत्येक Δ का क्षेत्रफल समान है।

❖ 3 Rhombus of equal area in a regular hexagon.

एक नियमित षट्भुज में 3 समान क्षेत्रफल के समचतुर्भुज



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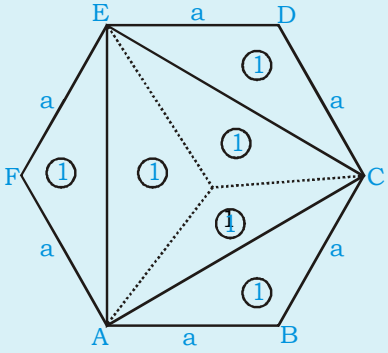
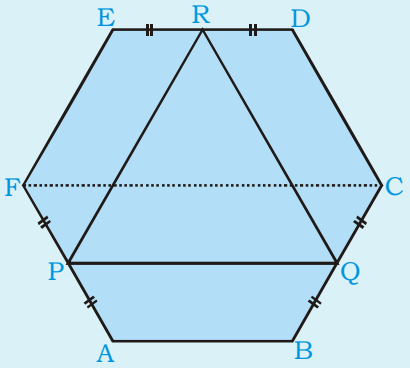


Δ EAC = equilateral Δ of side $\sqrt{3}$ a

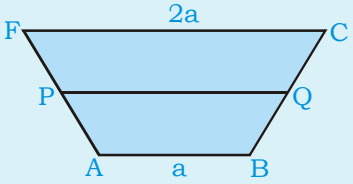
Δ EAC = $\sqrt{3}$ a भुजा का समबाहु त्रिभुज

$$\frac{\text{Area }\Delta EAC}{\text{Area ABCDEF}} = \frac{1}{2}$$

❖



P, Q, R are mid points P, Q, R मध्य बिन्दु हैं।



$$PQ = \frac{2a+a}{2} = \frac{3a}{2}$$

ΔPQR = equilateral Δ with side $\frac{3a}{2}$

$$\therefore \frac{\text{Area }\Delta PQR}{\text{Area ABCDEF}} = \frac{\frac{\sqrt{3}}{4}\times\frac{9}{4}a^2}{6\times\frac{\sqrt{3}}{4}a^2} = \frac{3}{8}$$

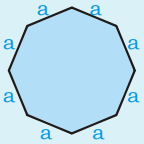
Octagon अष्टभुज

❖ Regular figure with 8 sides

8 भुजाओं वाला समान आकृति

Let side of octagon = a

माना अष्टभुज की भुजा = a



Area क्षेत्रफल = $2\left(\sqrt{2}+1\right)a^2$

Perimeter परिमाप = 8a

Inradius (r) अन्तःत्रिज्या (r) = $\frac{a}{2\sqrt{2}-2}$

Circumradius (R) परिवृत्त (R) = $\frac{a}{\sqrt{2-\sqrt{2}}} = \frac{\sqrt{2+\sqrt{2}}}{2}a$

Each interior angle = 135°

प्रत्येक आंतरिक कोण = 135°

Each exterior angle = 45°

प्रत्येक बाह्य कोण = 45°

Number of diaonal = 20

विकर्ण = 20

3 Dimension Mensuration (3 अैं ` क्ग) बिन्कदा

❖ 3 Dimension: Length, breadth, height

3 श्ू न्य अीमन्कृि ३ न्बल्कृी ए म्रन्कृी

Lateral surface Area सकृ ा त्ति ।( ) रेन्कख्ग (LSA):

बक्कं ा दु ३ रसर) रेन्कख्ग सनरह न्सस्सं ी स्ु रै ु) रेन्कख्ग

surrounding area except (of figures having flat surface) top and bottom

बक्कं ा दु ३ रसनरह न्सस्सं जन्क्कन्त सन) रेन्इ ३ र्कु त्ति ।(ा न्अरजन्न श च

Total surface area ससथ्ा कँम्टु ग) रेन्कख्ग (TSA) :

Area of all surfaces of a figure (LSA) + area of bases (top/bottom)

सथ्ा त्ति ।( ) रेन्कख्ग-सथ्ा कँम्टु ग) रेन्कख्ग

पस जन्न श सु रै ु त्ति ।(न्सस्सन) रेन्कख्ग (LSA) + जन्स्नं न्सस्सन) रेन्कख्ग इए कं-दु ३ स्र

Curved surface area (CSA) :

of figures having curved surfaces like cylinder, cone etc.

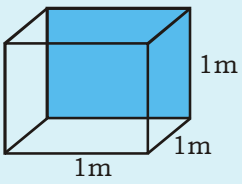
ा , कँम्ट) रेन्कख्ग (CSA) य

ी श्दत् न्स थ्जन्श्र ण्कु ा , कँम्टु ग जन्न श ग न्सस्सन:

CSA : Area of only curved surfaces except top and bottom.

CSA य ए कं जन्ं दु ३ रसनरह न्सस्सं सा अ ा , कँम्ट सन) रेन्कख्ग:

Volume capacity इू न्नन) ू ा न्च :



volume = 1×1×1 = 1 m^3

1 km = 1000 m

1 m = 100 cm

$1m^3$ = 1000 Litre

$\Rightarrow 1m^3 = 10^3 L$

$\Rightarrow 1L = 10^{-3}\ m^3$

$1L = 10^{-3} \times 10^6\ cm^3$

$\Rightarrow 1L = 10^3\ cm^3$

❖ 1m = 100 cm

1m = 1000 mm

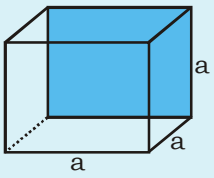
1 decimetre = 10 cm

1 km = 100 decametre

Cube (रेख

❖ A Solid figure in which length, Breadth and height are equal.

पस टन्स जन्न श श्त्तसु अीमन्कृि ३ न्बल्कृि जन्ं ए म्रन्कृी ं ने ं (नु (इ

❖ 

6 faces सकख्मस

8 vertices सृ ु न्भ्

12 edges सश्सदन्र

Diagonals इश सख्वि = 4

LSA सकृ ा त्ति ।( ) रेन्कख्ग = $4a^2$

Volume सजना द = $a^3 = \left(\sqrt{\frac{\text{Surface Area}}{6}}\right)^3$

TSA ससथ्ा कँम्टु ग) रेन्कख्ग = $6a^2$

Diagonal सश सख्वि = $\sqrt{3}$ a, Face diagonal = $\sqrt{2}$a

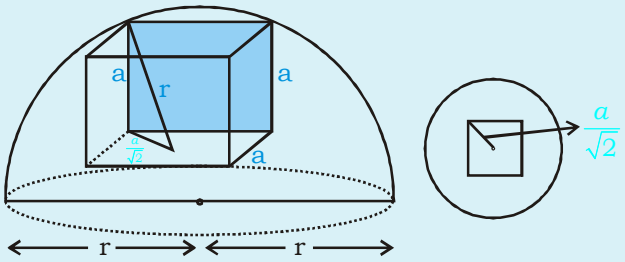
Euler's Theorem: for any 3D flat surface figure

गम्ं ज्ू रायश्स्र ु ै ु ।द श्ू न ा न्अु त्ति ।(जन्न श सरश्सप

V + F – E = 2

❖ When a cube of maximum size put inside a hemisphere.

ण्ी पस जस्र न्सरसरजम्ं जश्स्स। ू जन्सन्ं स न्ष्द ं ऊन्ण न न्(इ



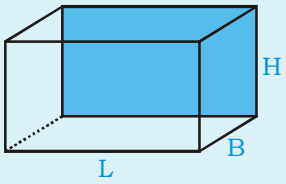
$\mathbf{a = \sqrt{\frac{2}{3}}\, r}$

$\frac{a^2}{2} + a^2 = r^2 \Rightarrow \frac{3}{2} a^2 = r^2$

a (side of cube) = $\sqrt{\frac{2}{3}}\, r$

Cuboid (रेखु ा

❖



Face/ कखास = 6

Vertices/ ुभ= 8

Edge/ ्सदं= 12

Adjacent faces इजन्प्र कखासच = LH, BH, LB

LSA = 2(bh + *l*h) = 2(L+B) × H = Area of 4 walls

त ुा ंसस)रेन्कखा

TSA = 2(LB+BH+HL)

Volume - जना।द = L×B×H

Diagonal - श सर्ख= $\sqrt{L^2 + B^2 + H^2}$

❖ If area of 3 adjacent faces of a cuboid are *x*, *y*, *z* respectively.

गश पस ष्दै ंसर3 जन्प्र कखास स्सं)रेन्कखा ,ू न्य *x*, *y*, *z* (ज

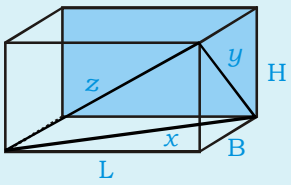
volume - (प्र ती = $\sqrt{xyz}$

$x = lb$ $y = bh$ $z = hl$

$xyz = l^2b^2h^2$

$\sqrt{xyz}$ = *l*bh = volume इजना।दच

❖ If *x*, *y*, *z* are diagonals of three adjacent faces of a cuboid



गश *x*, *y*, *z* पस ष्दै ंसर ुद जन्प्र कखास स्सस श सर्ख(म

$x = \sqrt{l^2+b^2}$ $x^2+y^2+z^2=2(l^2+b^2+h^2)$

$y = \sqrt{b^2 + h^2}$ $l^2+b^2+h^2 = \frac{x^2 + y^2 + z^2}{2}$

$z = \sqrt{h^2 + l^2}$ $D = \sqrt{l^2 + b^2 + h^2} = \sqrt{\frac{x^2 + y^2 + z^2}{2}}$

$l = \sqrt{\frac{x^2 - y^2 + z^2}{2}}$, $b = \sqrt{\frac{x^2 + y^2 - z^2}{2}}$, $h = \sqrt{\frac{y^2 + z^2 - x^2}{2}}$

$$\text{Volume} = \frac{\sqrt{(x^2 + y^2 - z^2)(x^2 - y^2 + z^2)(y^2 + z^2 - x^2)}}{2\sqrt{2}}$$

❖ Longest rod that can be put inside a cuboid (Room)

= Diagonal = $\sqrt{l^2 + b^2 + h^2}$

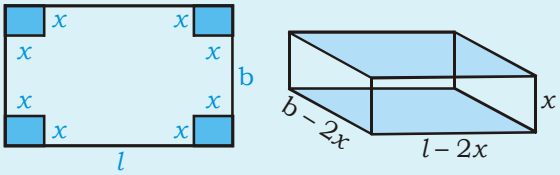
पस ष्दै ंइस)न्च सरजमं ंऊु ण ंत्तसदरा न्यु त्तीत्तरअीमु हवल्ह श सर्ख $\sqrt{l^2 + b^2 + h^2}$

❖ If *x* is the thickness of a cuboid, then volume of the hollow cuboid = *l*bh – (*l* – 2*x*) (b – 2*x*) (h – 2*x*)

गश *x* ष्दै ंसु ूरेन्कृ(न्न।रऊन्रूअरष्दै ंसं जना।द ४ *l*bh – (*l* – 2*x*) (b – 2*x*) (h – 2*x*)

❖ Making open rectangular box by cutting 4 corners of a retangular sheet.

पस जना।स्सं ुर्ं सर4 सन्दस्सनस्सं ऊश्रन्जना।स्सं ीन्डत ीदन्द:



Volume of rectangular box = (*l* – 2*x*) (b – 2*x*)*x*.

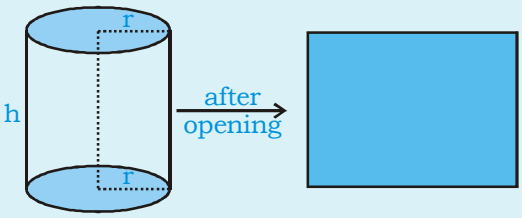
जना।स्सं ीन्डत सं जना।द = (*l* – 2*x*) (b – 2*x*)*x*.

❖ Volume of water flowing through cuboidal pump in time 't' / ंखां ंरिबबरदुर't'रदयअयुस्बंख्यकाख्रु ंरपं ूख

= area of base × (vt) (*l* × b)

Cylinder (प्र ख

❖



Volume = πr^2h = Base Area × Height

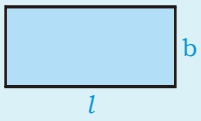
CSA = $2\pi rh$ = Base Perimeter × Height

TSA = CSA + 2 × Base Area = $2\pi rh + 2\pi r^2 = 2\pi r(r+h)$

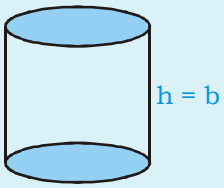
Ratio $\Rightarrow \frac{\text{CSA}}{\text{TSA}} = \frac{h}{r+h}$

❖ Folding of rectangular sheet to form a cylinder

पस ीश्रदीदन्दरसरश्रप जना।स्सं ुर्ं सन्रू न्यख्रन्

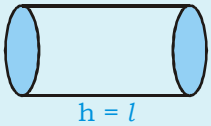


A Folding along length इअीमन्कृसरत्त्नं ू न्यख्रन्च



$2\pi r = l$ $r = \frac{l}{2\pi}$

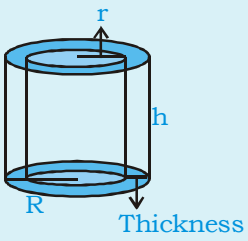
B Folding along breadth इत्न्चल्कृसरत्त्नं ू न्यख्रन्च



$2\pi r = b$ $r = \frac{b}{2\pi}$

Hollow Cylinder (से हि ` प्र ि ख

❖



thickness इ ू र ि न्क ृ ि = t

volume of metal इस्न ध्सन्- जग।द्च = $\pi R^2h - \pi r^2h$

$= \pi(R^2 - r^2)h$

Volume/जग।द = $\pi(R+r)\,(R-r)h$

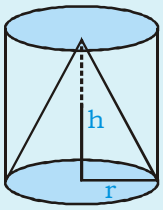
CSA/ा , कँप्ट)ेरन्कक्ष्ा = $2\pi(R+r)h$

TSA/सश्ा कँप्ट ु ग)ेरन्कक्ष्ा = $2\pi(R+r)h + 2\pi(R^2-r^2)$

$= 2\pi(R+r)\,(h+R-r)$

❖ A maximum size cone inside a cylinder

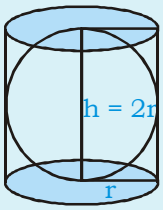
इपस ी श्द सरजमं पस जश्स।ू)ेरन्कक्ष्ा ा न्अन ू स्घ



Ratio $\Rightarrow \dfrac{\text{vol. of cylinder}}{\text{vol. of cone}} = \dfrac{3}{1}$

❖ A cylinder encloses a sphere

इपस ी श्द पस ?न्श्रस न ष्ठ्ं ा न्- (ज



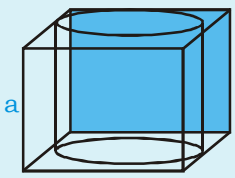
Height of cylinder = Diameter of sphere

ई श्द सु ए छ्न्क ृ ि थ ?न्श्रस न्- ह्ान्त्च

Ratio $\Rightarrow \dfrac{\text{vol. of cylinder}}{\text{vol. of sphere}} = \dfrac{3}{2}$

❖ A maximum size cylinder inside a cube

पस ष्ठ्द सरजमं जश्स।ू जन्सं स न्पस ी श्द



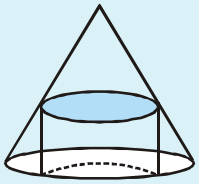
Radius of cylinder ई श्द सु ेन्द्ग ान्व = $\dfrac{a}{2}$

Height of cylinder ई श्द सु ए ाम्न क ृ ि = a

Ratio $\Rightarrow \dfrac{\text{vol. of cube}}{\text{vol. of cylinder}} = \dfrac{14}{11}$

❖ Maximum size cylinder inside a cone

पस ृ स ध्सरजमं जश्स।ू)ेरन्कक्ष्ा ा न्अन ी श्द



Height of cone इ स ध्सु ए छ्न्क ृ ि = H

Height of cylinder ई श्द सु ए ाम्न क ृ ि = h

Radius of cone इ स ध्सु ेन्द्ग ान्व = R

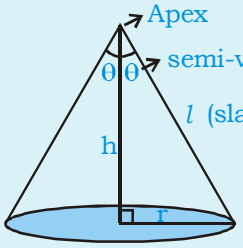
Radius of cylinder ई श्द सु ेन्द्ग ान्व = r

$\dfrac{R}{r} = \dfrac{H}{H-h}$ (Property)

$\dfrac{\text{Volume of cylinder}}{\text{Volume of cone}} = \dfrac{\mathbf{4}}{\mathbf{9}}$

Cone (मेख ी

❖



$l^2 = h^2 + r^2 \quad \Rightarrow l = \sqrt{h^2+r^2} \Rightarrow$

volume = $\dfrac{1}{3}\pi r^2h$

CSA/जग बर्स र ंर ग ुंएंबै उ = πrl

TSA/ ष्ठ रबर्स ध्सर ग ुंएंबै उ = $\pi rl + \pi r^2 = \pi r\,(r+l)$

❖ If H, C and V are the height, curved surface area and volume of a cone. Then find the value of $3\pi VH^3 - C^2H^2 + 9V^2$?

अक्षर H, C प ंश्रि V ग यथं (रव द ध्थं न्- ह ध्) ी ंभ्स जग र बर्स र ग ुंएंबै उ प ंश्रि प ंअू ख ुंर च्छ्ू र ू ि एश्ू क्र $3\pi VH^3 - C^2H^2 + 9V^2$ ंर यं ख ञ्अंर ए ुंइ ुंख

Let r = 1, h = 1 $\quad \therefore l = \sqrt{2}$

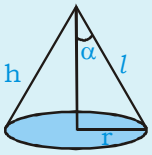
Volume = $\dfrac{1}{3}\pi$, $\quad C = \sqrt{2}\,\pi$

$3\pi VH^3 - C^2H^2 + 9V^2 = 3\pi \times \dfrac{1}{3}\pi - 2\pi^2 + 9\dfrac{1}{9}\pi^2$

$\Rightarrow \pi^2 - 2\pi^2 + \pi^2 = \mathbf{0}$

❖ If S denotes the area of the curved surface area of a right circular cone of height h and semivertical angle α then S equals?

S व दर्शाता है right circular cone का curved surface area जिसकी height h तथा semivertical angle α है तो S equals?



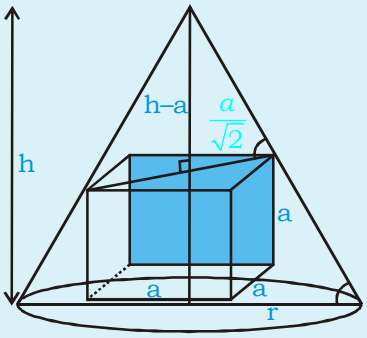
$\tan\alpha = \frac{r}{h}$ $\cos\alpha = \frac{h}{l}$

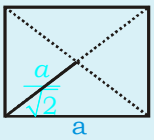
$r = h\tan\alpha$ $l = h\sec\alpha$

$S = \pi r l = \pi h^2 \sec\alpha\tan\alpha$

❖ When a cube of maximum volume is cut from a cone

इसी प्रकार जब maximum volume का cube किसी cone से काटा जाता है





$\frac{h-a}{h} = \frac{a}{\sqrt{2}r}$

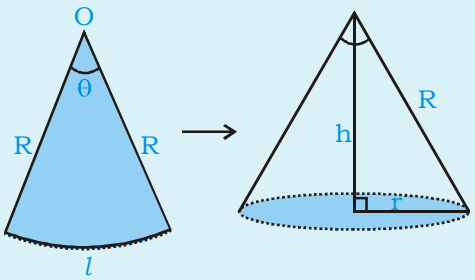
$\sqrt{2}\,rh - \sqrt{2}\,ra = ah$

$a(h + \sqrt{2}\,r) = \sqrt{2}\,rh$

$a \text{ (side of cube)} = \frac{\sqrt{2}rh}{\sqrt{2}r + h}$

❖ When a sector is folded to make a cone:

इसी प्रकार जब एक sector को मोड़कर cone बनाया जाता है



circumference of base of cone = arc l

इस sector की arc = cone के base की circumference

$2\pi r = 2\pi R \times \frac{\theta}{360°}$

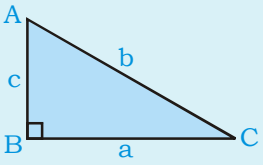
$r = R\frac{\theta}{360°}$

slant height of cone = radius of sector, $h = \sqrt{R^2 - r^2}$

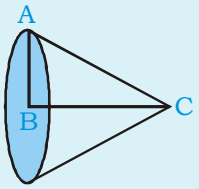
इस sector से बनने वाले cone की height $h = \sqrt{R^2 - r^2}$ है

❖ Rotation of right angle triangle to form a cone

इस प्रकार right angle triangle को घुमाकर cone बनाया जाता है



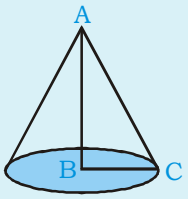
A Rotation along base a आधार a के परितः घूर्णन



$r = c$ $h = a$ $l = b$

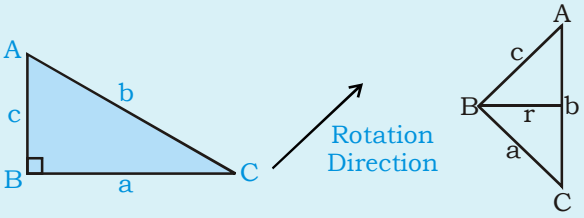
B Rotation along perpendicular BC

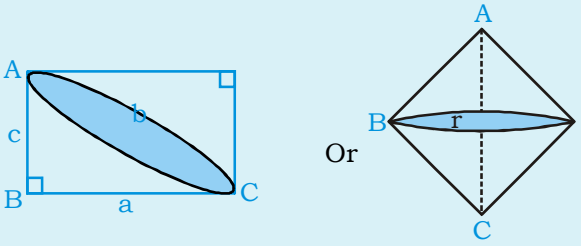
लंब BC के परितः घूर्णन



$r = a$ $h = c$ $l = b$

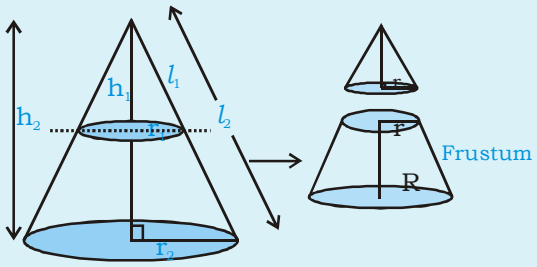
C Rotation along hypotenuse AC कर्ण AC के परितः घूर्णन





Sum of vol. of 2 cones इन 2 cones के आयतनों का योग $= \frac{1}{3}\pi\frac{(ac)^2}{b}$

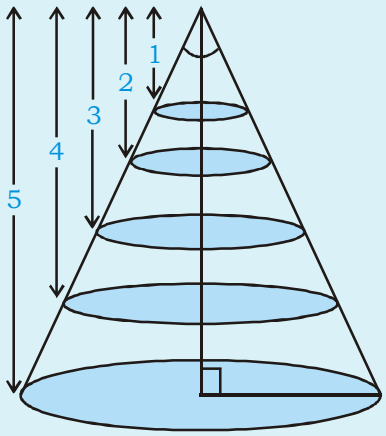
Cutting of Cone (मेखलीतर्ं खेा



$$\frac{r_1}{r_2}=\frac{h_1}{h_2}=\frac{l_1}{l_2}$$

$$\frac{\text{small cone CSA}}{\text{Big cone CSA}}=\frac{\pi r_1 l_1}{\pi r_2 l_2}=\left(\frac{r_1}{r_2}\right)^2=\left(\frac{l_1}{l_2}\right)^2=\left(\frac{h_1}{h_2}\right)^2$$

$$\frac{\text{small cone volume}}{\text{Big cone volume}}=\frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2}=\left(\frac{r_1}{r_2}\right)^3=\left(\frac{l_1}{l_2}\right)^3=\left(\frac{h_1}{h_2}\right)^3$$



CSA of 5 parts ⇒

$1^2 : 2^2-1^2 : 3^2-2^2 : 4^2-3^2 : 5^2-4^2$

1 : 3 : 5 : 7 : 9 (Ratio)

Volume of 5 parts ⇒

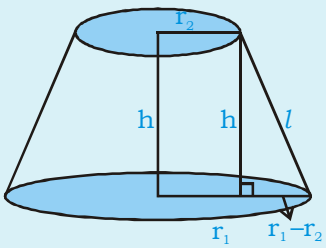
$1^3 : 2^3-1^3 : 3^3-2^3 : 4^3-3^3 : 5^3-4^3$

1 : 7 : 19 : 37 : 61 (Ratio)

Frustom of Cone (मेखलीत न्यिग्ख।

When a cone is cut parallel to its base, lower portion is called frustum.

इणी पस ृ ना धारनबत्तसरजन्स्नं सरत्तू नां सर्ंन्णनन्(ज।नस्ट्रअन्ं न्? श्ह्फ्रस स(अनन्(ज्व



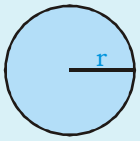
$$\text{CSA} = \pi(r_1+r_2)l$$

$$\text{TSA} = \pi(r_1+r_2)l+\pi\left(r_1^2+r_2^2\right)$$

$$\text{volume} = \frac{1}{3}\pi\left(r_1^2+r_2^2+r_1r_2\right)\times h$$

$$l=\sqrt{h^2+\left(r_1-r_2\right)^2}$$

Sphere (नेर्ि ा

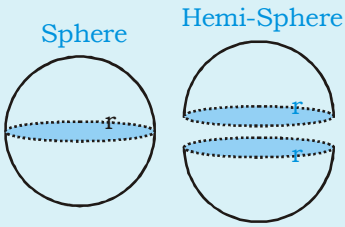


$$\text{Volume} = \frac{4}{3}\pi R^3 = \frac{\pi}{6}d^3$$

CSA = TSA = $4\pi R^2 \Rightarrow$ Area $\propto R^2$

Volume $\propto R^3$

Cutting of Sphere (नेर्ि तें खेा

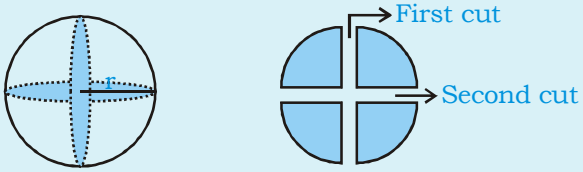


Volume of hemisphere इ?न्सश्र्सिन्जग।दच $= \frac{2}{3}\pi r^3$

CSA of Hemi-sphere = $2\pi r^2$

TSA of Hemi-sphere = $2\pi r^2 + \pi r^2 = 3\pi r^2$

TSA of both parts = $4\pi r^2+2\pi r^2 = 6\pi r^2$



2 cut (4 pieces) 1 cut → 2 circle area ↑ (Increase)

4 parts TSA = $4\pi r^2 + 4\times\pi r^2 =$ **$8\pi r^2$**

TSA of each part $= \frac{8\pi r^2}{4} =$ **$2\pi r^2$**

(Quarter sphere) - ?न्ज्ञ्क्ृ?न्सन्

If we make 3 cuts at *x*, *y*, *z* axis

गश (ृ *x*, *y*, *z* ज)न्कं 3 सं ीदनर(म

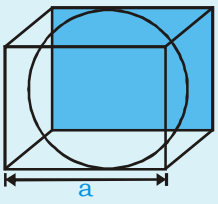
3 cut → 8 parts

3 cut → 6 circle area

8 parts TSA = $4\pi r^2 + 6\times\pi r^2 =$ **$10\,\pi r^2$**

TSA of each part इज्ञ्ग्सं न्? सन्TSAच $= \frac{10\pi r^2}{8} = \frac{5}{4}\pi r^2$

❖ A maximum size sphere inside a cube
इपस घ्न्द सरजमं पस जश्रूस।ू जन्सं सन ?न्श्रन्व

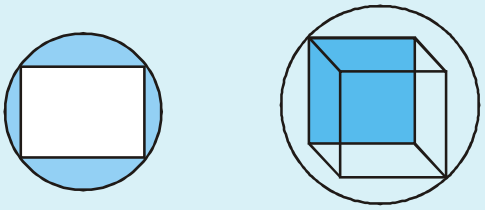


Diameter of sphere इ?न्श्रसरस न ह्यान्त्च = a

∴ Radius of sphere इ?न्श्रसरसु श्रेन्द्रगान्व = $\frac{a}{2}$

Ratio ⇒ $\frac{\text{vol. of cube}}{\text{vol. of sphere}} = \frac{21}{11}$

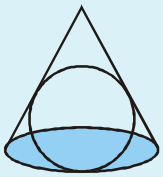
❖ A maximum size cube inside a sphere
इपस ?न्श्रसरसरजमं पस जश्रूस।ू जन्सं सन घ्न्दच



Diagonal of cube = Diameter of sphere
इघ्न्द सन श्र सख्थि ?न्श्रसरसन ह्यान्त्च

$\frac{\text{vol. of sphere}}{\text{vol. of cube}} = \frac{11\sqrt{3}}{7}$

❖ A maximum size sphere inside a cone
इपस ृ न्साधसरजमं पस जश्रूस।ू जन्सं सन ?न्श्रन्व

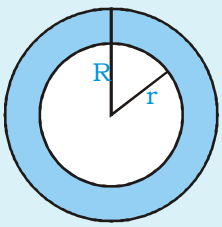


h = height of cone इh = ृ न्साधसु एघ्न्क्वि
l = slant height of cone इl = ृ न्साधसु श ंहु एछन्क्वि
r = radius of base of cone इr = ृ न्साधसरजन्सं सु श्रेन्द्रगान्व

Radius of sphere इ?न्श्रसरसु श्रेन्द्रगान्व = R = $\frac{hr}{l+r}$

Hollow Sphere (`ि` नेरि ा

❖



Volume of metal इस्नधसु जना।दच = $\frac{4}{3}\pi(R^3 - r^3)$

TSA = $4\pi R^2 + 4\pi r^2$

Thickness इू नैन्क्वि (t)= R–r

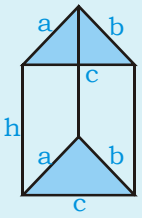
जना।द = $\left[\frac{4}{3}\times\pi(r+t)^3 - r^3\right]$

Prism (न्रप्रश्क

❖ A prism is a solid figure with identical ends, flat faces and same cross section all along its length.
पस श्रूद्व पस टन्स जन्नश (ज्श्रण्त्सरश्रं रत्ू न्द (नर(ब्रन्क रक्कब्रस (नर(मजन्ं कृस ओीन्कृसरत्त्न्न्त्ू न्द जदश्रूद्व न्सन्ं (नु (ज्
→ Base and Top of the prism is same
→ श्रूद्व सन जन्सं जन्ं ुभन्ि ू न्द (ज्
Surface of prism is lateral and not curved.
श्रूद्व सु त्त।(कृ।ि(ज्जन्ं ष्थन /ं न्द ु म(ज्
Cube, cuboid are prism but cylinder is not prism
इघ्न्दत घ्न्दै न्श्रूद्व (मअश्रसद ीश्रसद श्रूद्व द ु म(च

Triangular base prism

कोपं पंप्रिन)(पब्रापब्र)ु :रख



Volume of prism = area of base × height
श्रूद्व सन जना।द = जन्सं सन)ेरन्कब्रा × एछन्क्वि
LSA = perimeter of base × height
इLSA = इजन्सं सु कंश्रू × एघ्न्क्वि
⇒ 3 rectangles formed if we open it
⇒ कृरऊन्श्रदरकं 3 जना। ीद।र(म
ah + bh + ch = (a + b + c) × h
TSA = LSA + 2 × Base Area

General formulae for a prism (न्जम्क त ि्र ध चेकैप चेड़ा

A CSA = Base perimeter × Height = na × h

B TSA = CSA + 2 × Area of two base

TSA = $nah + 2 \times \frac{na^2}{4}\cot\frac{\pi}{n}$

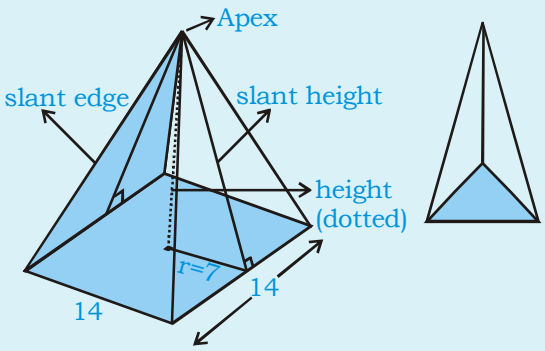
C Volume = Area of base area × Height
जना।द = जन्सं)ेरन्कब्रा × एछन्क्वि

Volume = $\frac{na^2}{4}\cot\frac{\pi}{4}\times h$

Here इग(न्ब्र,
n → no. of sides of regular polygon
n → श्रगशू । ीैधब्र सु ै न्धन्जन्सस्ु त्त्रगन-
a → side length of regular polygon
a → श्रगशू । ीैधब्र सु ै न्धन्सु ओीन्कृ
h → height of prism/श्रूद्व सु एघ्न्क्वि

Pyramid (न्रथे न्का

❖



Square Pyramid — इा ?नसिं स्कंस्रू वच

Triangular Pyramid — इेसन्कुग स्कंस्रू वच

Height → Apex to centre of base

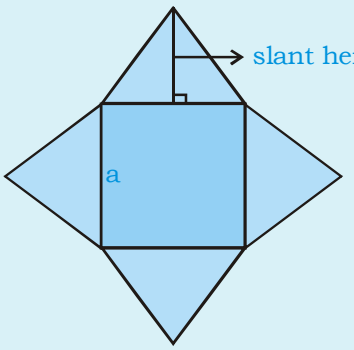
एछन्कृ→ ृ ुभ्तिरजन्सं सरसक्र ।स

slant edge → Apex to vertex of base

शगसि श्सदंन्→ जन्सं सर ृ ुभ्तिर ृ ुभ्ि।स

slant height → Apex to side of base

शगसि एभ्न्कृ→ जन्सं सरश्सदंररत्तर ृ ुभ्ि।स



for a regular pyramid/पस श्दगशू । स्कंस्रू व सरश्सप ⇒

LSA = $\frac{1}{2}$ ×perimeter of base × slant height

LSA = $\frac{1}{2}$ × जन्सं सन्क्शूं न्क × शगसि एभ्न्कृ

TSA ⇒ LSA + Base Area

Volume ⇒ $\frac{1}{3}$ ×base area × height

जगा।द ⇒ $\frac{1}{3}$ × जन्सं सन्)रन्कश्म × एछन्कृ

After opening square pyramid ⇒ LSA = 4Δ's

ा ?नसिं स्कंस्रू व ऊन्श्दरसरी न् ⇒ LSA = 4Δ's

⇒ 4× $\frac{1}{2}$ × a × slant height

⇒ $\frac{1}{2}$ × 4a × slant height

⇒ $\frac{1}{2}$ × base perimeter × slant height

⇒ $\frac{1}{2}$ × जन्सं सु क्शंश्र × शगसि एभ्न्कृ

In square pyramid इा ?नसिं स्कंस्रू व ृ ुभ्ि→

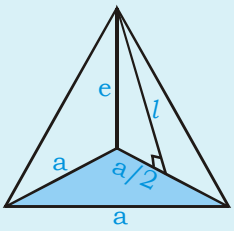
$l^2 = h^2 + \left(\frac{a}{2}\right)^2$

If slant edge इज? शं हन् श्सदंन्च = e ∴ $e^2 = l^2 + \left(\frac{a}{2}\right)^2$

$= e^2 = h^2 + \left(\frac{a}{\sqrt{2}}\right)^2$

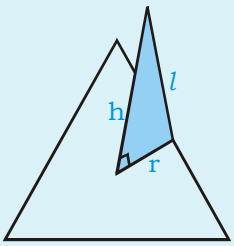
In triangular pyramid (न्बते ैपै न्रथे न्का क्वि

❖

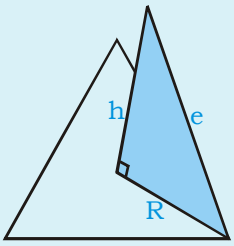


$e^2 = l^2 + \left(\frac{a}{2}\right)^2$

$e^2 = h^2 + R^2$



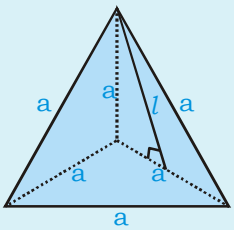
$l^2 = h^2 + r^2$



$r = \frac{a}{2\sqrt{3}}$, $R = \frac{a}{\sqrt{3}}$

Tetrahedron (चक्द्र दीघ्रल त ा

❖ All 4 faces and 1 equilateral triangle.
Pyramid with triangular base.



Slant height (l) = $\frac{\sqrt{3}}{2}a$

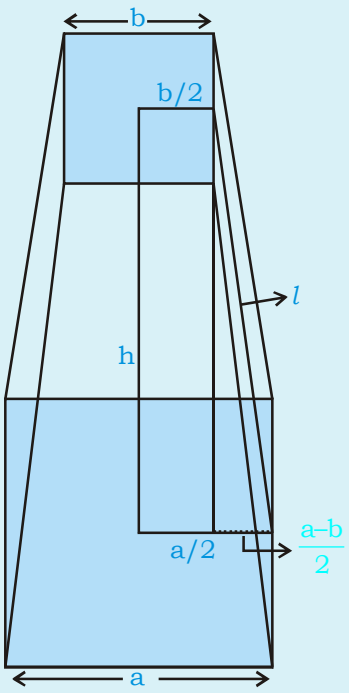
LSA = $\frac{\sqrt{3}}{4}a^2 \times 3$

TSA = $\frac{\sqrt{3}}{4}a^2 \times 4 = \sqrt{3}\,a^2$

height = $\frac{\sqrt{2}}{\sqrt{3}}a$

Volume = $\frac{1}{3} \times \frac{\sqrt{3}}{4}a^2 \times \frac{\sqrt{2}}{\sqrt{3}}a = \frac{a^3}{6\sqrt{2}}$

Frustum of a Pyramid (ध त न्प्रथे न्का ते न्यग्ख ा



$l = \sqrt{h^2 + \left(\frac{a-b}{2}\right)^2}$

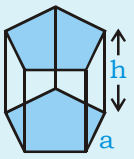
LSA = $\frac{1}{2}(P_1+P_2) \times l$

P_1, P_2 → Perimeter of bases इ→ जन्र्ं न्सस्ु क्रं श्रू च

A_1, A_2 → Area of bases इ→ जन्र्ं न्सस्र)ेन्क्खाच

Volume = $\frac{1}{3}(A_1+A_2+\sqrt{A_1 \times A_2}) \times h$

Pentagonal Prism (प्रवु ृे न्जभ्क



Area of pentagonal base = $\sqrt{3}a^2$

बी ंष्र ंरप ं रिरुंण्बै उर= $\sqrt{3}a^2$

Lateral surface area = 5a × h = 5ah

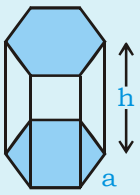
बंथ्जस्र्ब्त्त र ंररुंण्बै उर= 5a × h = 5ah

Total surface area = 5ah + $2\sqrt{3}a^2$

ष्ठरद् एष्ररुंण्बै उर= 5ah + $2\sqrt{3}a^2$

Volume सप ंआू ख= $\sqrt{3}a^2 \times h = \sqrt{3}a^2h$

Hexagonal Prism (ृे न्जभ्क



Base area of hexagonal = $\frac{3\sqrt{3}}{2}a^2 = 2.5981a^2$

तंक्रं ंष्र ंरद् एररुंण्बै उर= $\frac{3\sqrt{3}}{2}a^2 = 2.5981a^2$

Lateral surface area = 6a × h = 6ah

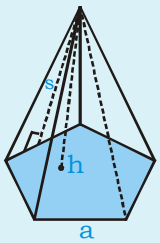
बंथ्जस्र्ब्त्त र ंररुंण्बै उर= 6a × h = 6ah

Total surface area = 6ah + $3\sqrt{3}\,a^2$

ष्ठरद् एष्ररुंण्बै उर= 6ah + $3\sqrt{3}\,a^2$

Volume सप ंआू ख= $\frac{3\sqrt{3}}{2}a^2h = 2.5981a^2h$

Pentagonal Pyramid (प्रवु ृे न्प्रथे न्का



Lateral surface area = $\frac{1}{2} \times 5a \times s = \frac{5}{2}as$

बंथ्जस्र्ब्त्त र ंररुंण्बै उर= $\frac{1}{2} \times 5a \times s = \frac{5}{2}as$

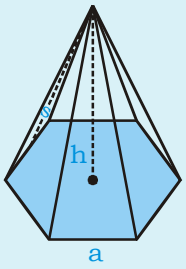
Total surface area = $\frac{5}{2}as+\sqrt{3}a^2$

ष्ठरद् एषरंुण्बैउर= $\frac{5}{2}as+\sqrt{3}a^2$

Volume सप ंअू ख= $\frac{1}{3}\times\sqrt{3}a^2=\frac{1}{\sqrt{3}}a^2$

Hexagonal Pyramid (र्ई ुे ि न्प्रश्ेन्का

❖



s = slant height सवू ि: ष)ैं भ्

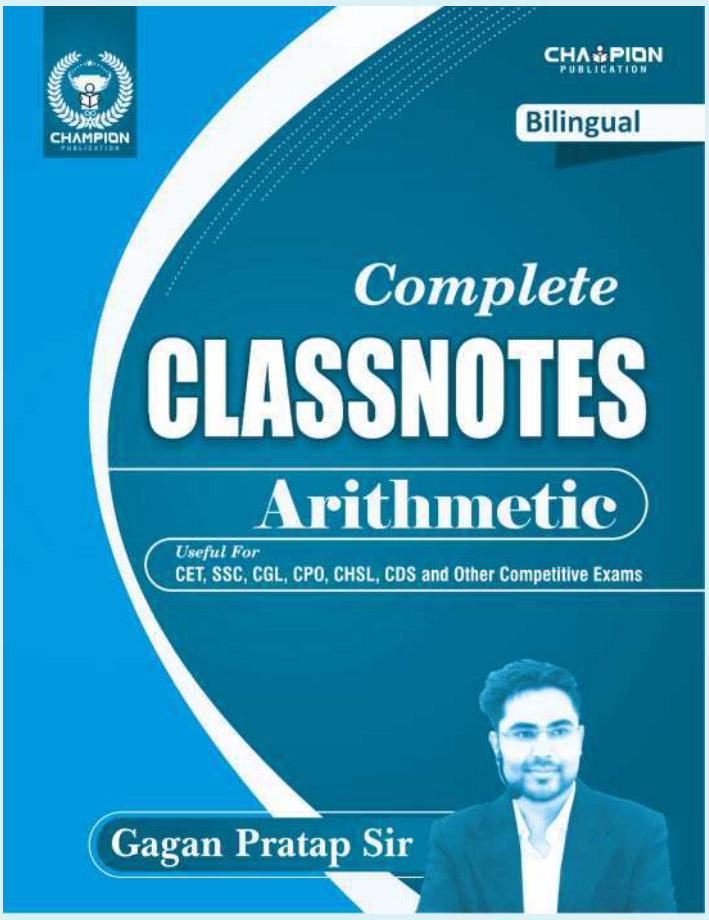
Lateral surface area = $\frac{1}{2}\times6a\times s=3as$

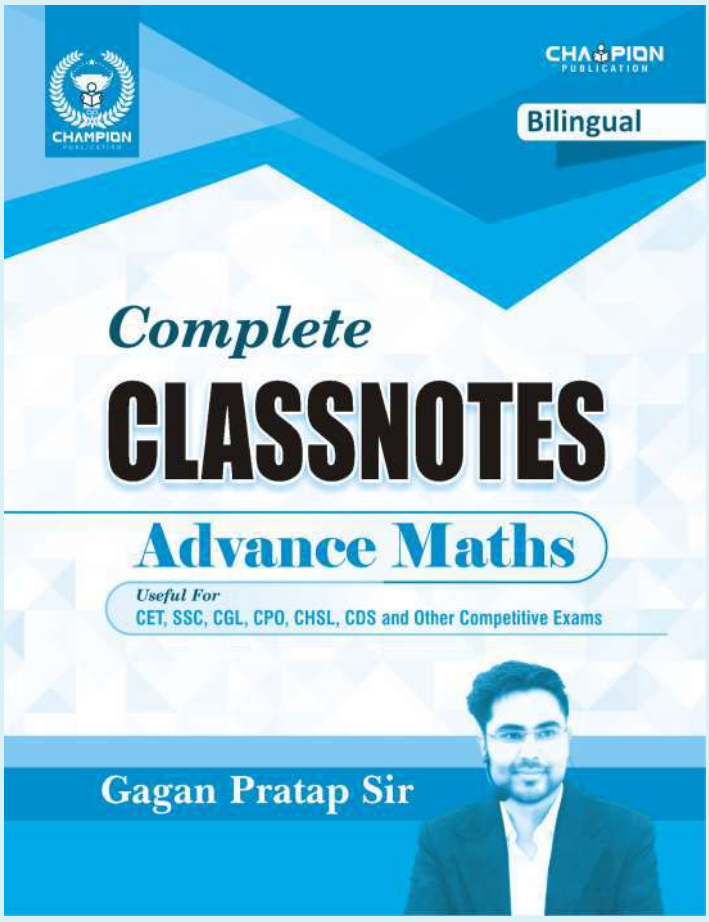
बंथ्जस्बर्ता र ं रंुण्बैउर= $\frac{1}{2}\times6a\times s=3as$

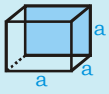
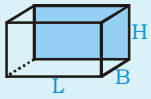
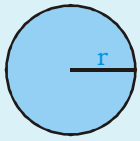
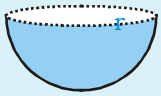
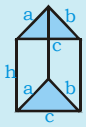
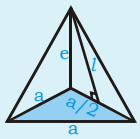
Total surface area = $3as+\frac{3\sqrt{3}}{2}a^2$

ष्ठरद् एषरंुण्बैउर= $3as+\frac{3\sqrt{3}}{2}a^2$

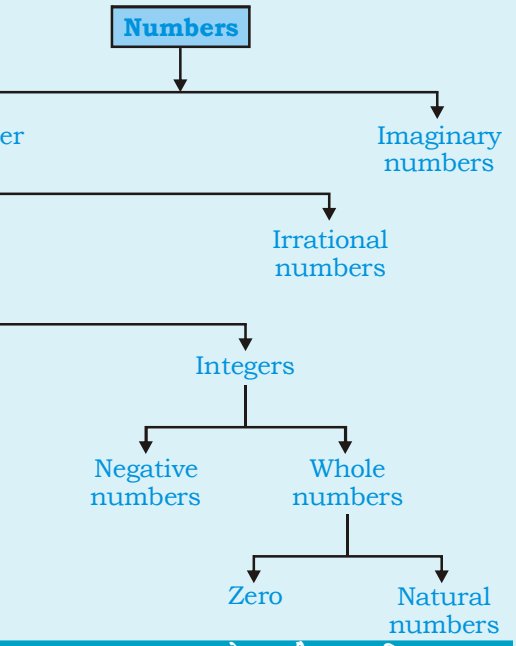
Volume सप ंअू ख= $\frac{\sqrt{3}}{2}a^2h$





Solid figure	Figure	Volume	CSA/LSA	TSA
Cube (रेखु)		a^3	$4a^2$	$6a^2$
Cuboid (रेखुो)		L × B × H	2 (L + B) H	2(LB + BH + HL)
Cylinder (प्रि ख)		πr^2h	$2\pi rh$	$2\pi r$ (r + h)
Cone (मेवी)		$\frac{1}{3}\pi r^2h$	πrl	πr (r + l)
Frustum of cone (मेवीते न्यख्ता)		$\frac{1}{3}\pi(R^2 + r^2 + Rr)h$	$\pi(R + r)l$	$\pi(R + r)l + \pi(R^2 + r^2)$
Sphere (नेि ा)		$\frac{4}{3}\pi r^3$	$4\pi r^2$	$4\pi r^2$
Hollow sphere (सेंखि े नेि ा)		$\frac{4}{3}\pi(R^3 - r^3)$		$4\pi R^2$
Hemi-sphere (अन्छेि ा)		$\frac{2}{3}\pi r^3$	$2\pi r^2$	$3\pi r^2$
Prism (न्प्रभ्का		Base area × Height	Base peri. × Height	LSA+ 2 × Base area
Pyramid (न्प्रथ्न्क्ाा		$\frac{1}{3}$ × Base area × H	$\frac{1}{2}$ × Base peri. × Slant h.	LSA + Base area

Number system (रेख क्रुभ यब्ब)



Classification of Numbers (रेख अैक्प क)बिब दु प

Real Numbers
सेरं रखा बिदरु ब
which can be denoted on number line.
र्कए बिदर पूए र अ खन खं रू दर कर ि ं र टस्

Imaginary Numbers
। स्यअखा बिदरु ब
can not be denoted on number line.
बिदर पूए र अ खन खं धटस्ब रू दर कर ि ं र टस्

Ex. +3, –7, 5, $\frac{19}{13}$, $\frac{5}{7}$
0.0675, $\sqrt{5}$, $\sqrt{11}$

$\sqrt{-7}$, $\sqrt{-3}$, $\sqrt{-5}$, $\sqrt{-1} = i$
↓ $-1 = i^2$
a+ib 5+3i

Real Numbers ( ) सम)ण क्रेख वेप

Rational Numbers
अख्हए बिदरु ब
which can be written in $\frac{p}{q}$ form ($q \neq 0$) p, q → Integer
र्कए $\frac{p}{q}$ हस्खू र कर ि ं र टथ

Irrational Numbers
, अख्हए बिदरु ब
can not be written in $\frac{p}{q}$ form.
$\frac{p}{q}$ हस्धटस्बखू र कर ि ं र

Ex. $\frac{5}{3}$, $\frac{13}{1}$, $\frac{-8}{1}$, 0.5555, $\frac{22}{7}$

0.1342607532 ....
$\sqrt{q}$, $\pi = 3.141592$ ....

Decimal Numbers (तीं य)क्रेख वेप

Terminating decimal	Non-Terminating Repeating decimal	Non-terminating Non-repeating decimal
लारंबै लहवस	। थी लारंबै स्हपु करथए सखवै लहवस	। थी लारंबभ, लारंब । थी स्हपस्सै लहवस
$0.5 = \frac{1}{2}$	$0.333333... = \frac{1}{3}$	$\sqrt{2} = 1.414....$
$0.73 = \frac{73}{100}$	$0.565656... = \frac{56}{99}$	↓
$0.648 = \frac{81}{125}$	$0.137137137... = \frac{137}{999}$	Irrational Numbers
Rational Numbers		

❖ **Integers** → All integers are rational no.

$\left(\frac{p}{q}\right)$ where q = 1

रु न ंण → ि जस् अार्ण अख्हए बिदरु ग्टस्

Integers (रु न ंण प

Negative Integers
ण मस्ख्हा अार्ण
{–∞, ...–4, –3, –2, –1}
0 → Neither positive nor negative
ि । रपख्हा जस् धटस्ब
ध टस् धा रपख्हा

Non-negative Integers
। थी ण मस्ख्हा अार्ण
{0,1,2,3,4,....∞}
↓
Whole Numbers
अार्ण बिदर

❖ Natural numbers ध्टस्ख्हं बिदरु ब→ {1,2,3,4,5,....∞}

Integers (रु न ंण प

Even प्रकर्ं **Odd** प्रका)ड्ं

Even → which are divisible by 2. (2K form)
ि ह → करए2 ि एख्जस्(द टस्ब)2K न अ
{0, 2, 4, 6, 8)

Odd → which are not divisible by 2. (2K±1 form)
ख्जच्ह → करए2 ि एख्जस्(द धटस्बटस्ब)2K±1 न अ
{1, 3, 5, 7, 11}

❖ Odd × odd → odd
odd × even → even
odd±odd → even
even ± even → even
odd ± even → odd

❖ $\left.\begin{matrix} a+b=\text{odd} \\ a-b=\text{odd} \end{matrix}\right\} \rightarrow a=\frac{\text{odd}+\text{odd}}{2}=\frac{\text{even}}{2}=$ Natural No.

$\left.\begin{matrix} a+b=\text{even} \\ a-b=\text{even} \end{matrix}\right\} \rightarrow a=\frac{\text{even}+\text{even}}{2}=\frac{\text{even}}{2}=$ Natural No.

$\left.\begin{matrix} a+b=\text{even} \\ a-b=\text{odd} \end{matrix}\right\} \rightarrow a=\frac{\text{even}+\text{odd}}{2}=\frac{\text{odd}}{2}\neq$ natural

Natural numbers (जधम्मण क्रेख वेप

Prime numbers अचइा क्रेख वे — **Composite numbers** चइा प्रर्ं ब्रिक्रेख वे

Prime Numbers → Only two factors 1 & itself.
, जस्(द बिदुर्ग→ ।स्ववै एए। भ्रमध्-स्ख्र 1 , रथे सदत्व
2, 3, 5, 7, 11, 13, 23, 61, 67, 97 etc.
2 → हि , जस्(द बिदखव, रथ ृ िप्र स्ख्र्-, जस्(द बिदर
3,5,7 → व।रं प स्ख्रच्ह , जस्(द बिदर। र। स्ववद्ध्त

Prime no.→
1-50 → 15
50-100 → 10
1-100 → 25
1-200 → 46
1-1000 → 168

Each prime number can be written in (6k±1) form.
ठज्ञए , जस्(द बिदर। एए(6k±1) न अहस्ख्ू र कर।ि ं र टत्व
But every (6k±1) form may not be necessarily prime no.
वए्ध टप (6k±1) न अ, स्सल्ह न अि ए, जस्(द बिदर धटस्ब्टएि ं र टत्व
13 → 6×2+1 (prime)
25 → 6×4+1 (not prime)

❖ **Composite Numbers** → more than two factors.
चइा क्रेख → ै एि ए, रुडा । प)।भ्रमध्-स्ख्र-त

Ex. 4, 6, 8, 9 etc.
1 → Neither prime nor composite
1 → ध ं एए, जस्(द , रथ ध टस्-जस्(द र्भ ह।
4 → Smallest composite number.
4 → ृ िप्र स्ख्र्-जस्(द बिदस्त
9 → Smallest odd composite number.
9 → ृ िप्र स्ख्र्-स्ख्रच्ह जस्(द बिदस्त

❖ **Relatively prime/co-prime numbers** → Two numbers in which nothing is common i.e. their HCF = 1
र ैथ क्रमचइा प्रर ग/अचइा क्रेख वब्ब्→ ै एि बदुर ग्रकधहस्ख्र,ं ज्ञस्-द्वजदख्रच्ह धटस्ब्टथ, ब्रस्ंड द्वधा र HCF = 1
(25, 19) (16, 9) (2, 3) (11, 13)

❖ **Twin-prime numbers** → Two prime numbers with a gap of 2.
कइ स्सेख, जस्(द बिदुर्ग→ 2 । ए, बंपव्व । एि स्ख्रै एए, जस्(द बिदुर्स
(3, 5) (5, 7) (11, 13)

❖ only pair of prime no. with a gap of 2 is **3, 5, 7**.
2 । ए, बंपव्व । एि स्ख्र **3, 5, 7** , जस्(द बिदर। स्। स्ववु । कस्ख्रु टत्व

❖ Smallest 3 digit prime → 101
ृ िप्र स्ख्र्-3 , ब्रस्ख्र स्, जस्(द बिदर = 101
largest 3 digit prime → 997
ृ िपू इरु3 , ब्रस्ख्र स्, जस्(द बिदर → 997

Perfect Numbers (ुन क्रेख वब्ब्

❖ If the sum of all the factors (excluding that no.) is equal to that number then it is called a perfect no.
दैख् ि जस्-। भ्रमध्ू ह्रस्ख्)द्वि बिदर। ए्र स्ख्रह् प-। र दए्र द्वि बिदर। ए ृ पू प टएं एद्वि एअमए्र बिदुर्ग ट ं एटत्व
6 → 1,2,3,6 (factors) ∴ 1+2+3 = 6
(Smallest perfect no.) ∴ 6 is perfect no.
28 → 1,2,4,7,14,28
(1+2+4+7+14) = 28
Perfect numbers भ्र अमए्र बिदर → 6, 28, 496, 8128 ....

Divisibility Rules (म)चइा सक्कौक्कार्ं प

Divisibility Rules (म)चइा सक्कार्ं प

❖ 1 is not divisible by any number except 1 but 1 is a universal factor.

1, 1 ा ग्र ग्रहा प ह ि स्जस्र बिदर ि एब्लाजर(द धटस्बट्ब्व ह ध 1ु ा ि सिष्ठ्य ।मधू ह्म टस्र

Divisibility Rule of 2, 4, 8, 16

2→ Last digit should be divisible by 2.

2→ , ख्ंह , ब्र 2 ि एब्लाजर(द टख्र श्रस्ह ु त

4→ Last 2 digit should be divisible by 4.

4→ , ख्ंह 2 , ब्र 4 ि एब्लाजर(द टख्र श्रस्ह ु त

8→ Last 3 digit should be divisible by 8.

8→ , ख्ंह 3 , ब्र 8 ि एब्लाजर(द टख्र श्रस्ह ु त

16→ Last 4 digits should be divisible by 16

16→ , ख्ंह 4 , ब्र 16 ि एब्लाजर(द टख्रएश्रस्ह ु

Divisibility Rule of 3 and 9

3→ Sum of digits should be divisible by 3.

3→ , ब्र ख्र ा र दर ए 3 ि एब्लाजर(द टख्र श्रस्ह ु त

9→ Sum of digits should be divisible by 9.

9→ , ब्र ख्र ा र दर ए 9 ि एब्लाजर(द टख्र श्रस्ह ु त

Divisibility Rule of 5, 25, 125

5 → Last digit should be 0 or 5

5 → , ख्ंह , ब्र 0 दर 5 टख्र श्रस्ह ु

25 → Last two digit should be divisible by 25.

25 → , ख्ंह ै ए, ब्र 25 ि एब्लाजर(द टख्रएश्रस्ह ु त

125 → Last 3 digit should be divisible by 125.

125 → , ख्ंह 3 , ब्र 125 ि एब्लाजर(द टख्र श्रस्ह ु त

Divisibility Rule of 6

6 → 6=2×3 (co-prime factors) भ)ि टी , जर(द ।मधू ह्म-

∴ If a number is divisible by 2 & 3 both, that number will also be divisible by 6

∴ दै ा ग्रर्ष बिदर 2 , थ 3ै ख्रख्र ि एब्लाजर(द टछं र्एस्ट ि बिदर 6 ि ए जर ख्लाजर(द टर्एस्

Divisibility Rule of 7

To check divisibility of 7 we apply following method→

7 श्ू न्यश्मी कृम्मि ता ञ्ज्ञ तू वी ात्लए ा कृ ा ू न्ू त्ता ञ्ज्ञ त्र्वृात्ला→

1071 → 107 – 1 × 2 →ा 105 → 10 – 5 × 2 = 0

0 is divisible by 7.

0, 7ू तू न्यश्मी ा ला ∴ 1071, 7ा तू न्यश्मी ा लश्रेअ

17808 → 1780 – 2 × 8 → 1764

176 – 2 × 4 → 168 → 16 – 16 = 0

17808 is divisible by 7, 17808 7ू तू न्यश्मी ा लश्रेअ

Divisibility Rule of 13

To check divisibility of 13, we apply following method→

13 श्ू न्यश्मी कृम्मि ता ञ्ज्ञ तू वी ात्लए ा कृ ा ू न्ू त्ता ञ्ज्ञ त्र्वृात्ला→

2353 → 235 + 3 × 4 ⇒ 235 + 12 = 247

247 → 24 + 7 × 4 ⇒ 24 + 28 = 52

52 is divisible by 13, ∴ 2353 will be divisible by 13

52, 13ू तू न्यश्मी ा ला ∴ा2353 यश्र 13ू तू न्यश्मी ा लश्रेअ

Divisibility Rule of 17

To check divisibility of 17, we apply following method→

17ा श्ू न्यश्मी कृम्मि ता ञ्ज्ञ तू वी ात्लए ा कृ ा ू न्ू त्ता ञ्ज्ञ ा ञ्ज्ञ त्र्वृात्ला→

3587 → 358 – 7 × 5 = 358 – 35 ⇒ 323

323 → 32 – 3 × 5 = 32 – 15 = 17

3587 will be divisible by 17, 3587, 17ू तू न्यश्मी ा लश्रेअ

Divisibility Rule of 7, 11, 13

5922 → Make pair of 3 digits from RHS

5922 → RHS ि ए3 , ब्रण र दर्ह ू ध्रु ब

→ Add alternate pairs & take difference

→ स्ा ख्अ कस्ह्स्ा एकस्ह्स्ब्, थ , बप ह्मरं ा पब्

→ If difference is divisible by 7, 11, 13 then number will be divisible by 7, 11, 13 respectively.

दै ा , बप 7, 11, 13 ि एब्लाजर(द टछं र्ण बिदर रू ह लह्छ

7, 11, 13 ि एब्लाजर(द टर्एस्त

	7 ✓
005922 → 922 – 5 = 917	11 ×
	13 ×
	7 ✓
6489 → 489 – 6 → 483	11 ×
	13 ×
	7 ✓
380247 → 380–247 →133	11 ×
	13 ×

❖ ABAB → divisible by 101

73×101 = 7373

❖ ABCABC → divisible by 1001

687×1001 = 687687

❖ 7×11×13 = 1001 (Remember)

Divisibility Rule of 11

❖ If the difference between the sum of the digits at odd places and sum of the digits at even places is zero or multiple of 11.

दैख्ा चह्ं क्रमरधख्ा ए, ब्ख्ा एदए , थ्ा हि क्रमरधख्ा ए, ब्ख्ा एदए ा ए स्श्र ा र , बंप लम्द दर 11 ा र। क्रम टब्ब

Add even place digits
Add odd place digits
→ Take diff. → If diff. is 0 or multiple of 11 then no. will be divisible by 11

166452 → 1+6+5 = 12
6+4+2 = 12 diff. = 0 ∴ div. by 11

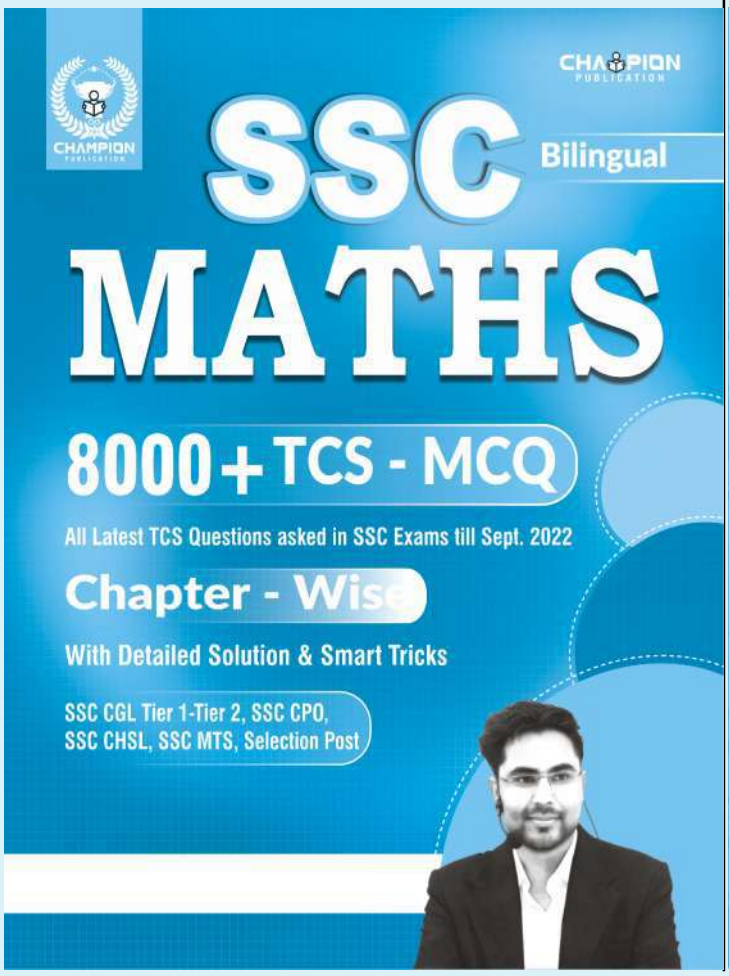
7945938 → 28–17 = 11

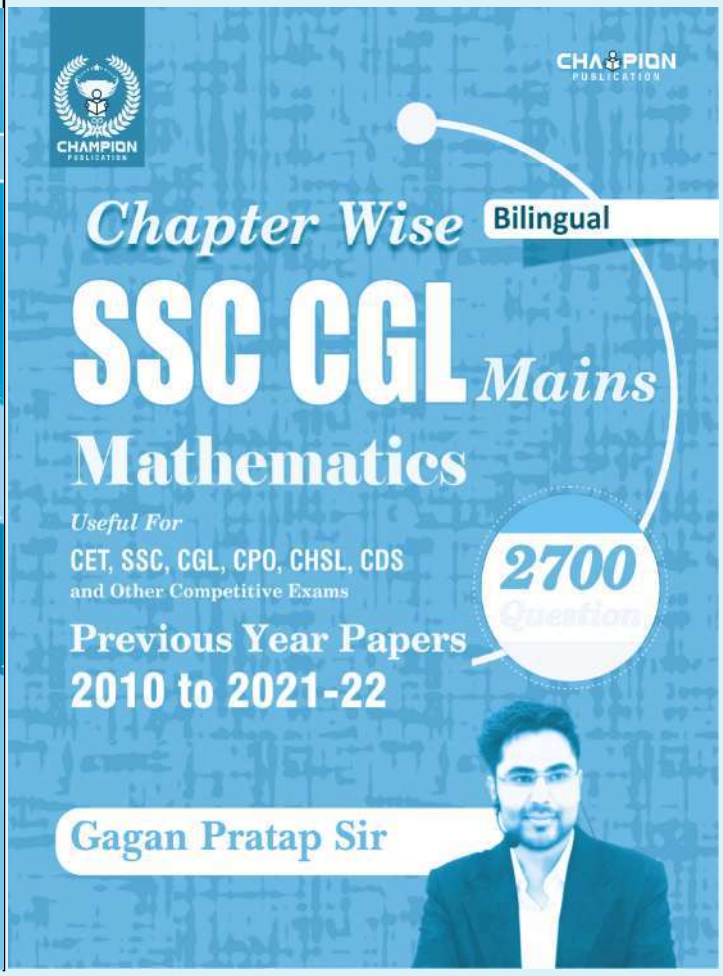
Divisibility Rule of 12

12 = 4 × 3

❖ If a number is divisible by 4 and 3 both then that number will also be divisible by 12.

दैख्ा ा खृण बदर 4 स 3ै खृख्ा एख्जस्कं टएंस्टथं एस्सट ि बदर 12 ि एज्स्-ख्जस्कं टएस्त





Remainder Theorem (ैशेष प्रमेय)

Remainder (शेषफल)

❖ The remainder is the value left after the division if the dividend is not completely divided by the divisor.

शेषफल वह संख्या है जो भाज्य को भाजक से पूर्ण रूप से विभाजित न कर पाने की स्थिति में अंत में बच जाती है।

❖ If dividend is completely divided by the divisor then in that case remainder will be zero.

यदि भाज्य को भाजक पूर्ण रूप से विभाजित कर देता है तो शेषफल शून्य होता है।

Remainder Theorem (शेषफल प्रमेय)

Dividend → भाज्य

Divisor (भाजक) ← 13) 72 (5 → quotient (भागफल)
65
7 → Remainder (शेषफल)

$72 = 13 \times 5 + 7$

Dividend = Divisor × Quotient + Remainder

भाज्य = भाजक × भागफल + शेषफल

$\frac{206}{11}$, R = 8

Remainder is always less than divisor.

शेषफल हमेशा भाजक से कम होता है।

❖ a, b, n → natural numbers (प्राकृत संख्या)

$a^n + b^n$ (n=odd) → $a^3+b^3=(a+b)(a^2-ab+b^2)$

$a^3+b^3=(a+b)[a^2b^0-a^1b^1+a^0b^2] \rightarrow (a+b)[a^2-ab+b^2]$

(+ – + – + – (+ से start होकर एकांतर से+ then – goes on)

$(a^5 + b^5) = (a+b)[a^4b^0-a^3b^1 + a^2b^2 - a^1b^3+a^0b^4]$

$= (a+b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4)$

$\therefore a^n + b^n \rightarrow$ n odd → (a+b) is a factor always.

No. of the form	Div. by (a+b)	Div. by (a–b)
1. a^n+b^n (n → odd)	✓	×
2. a^n+b^n (n → even)	×	×
3. a^n-b^n (n → odd)	×	✓
4. a^n-b^n (n → even)	✓	✓

❖ If Power is odd (दिये गये स्थिति में) →

$a^n+b^n+c^n+d^n$ is divisible by (a+b+c+d)

Fermat's Theorem (फर्मेट का प्रमेय)

❖ **Fermat's Theorem** → $\frac{a^{P-1}}{P} = 1$ **(Remainder)**

Fermat's (फर्मेट) → $\frac{a^{P-1}}{P} = 1$ (शेषफल)

p = prime number (अभाज्य संख्या), (जो द्विघात)

a, p → co-prime (सहअभाज्य), (जो द्विघात)

Ex. $\frac{50^{16}}{17} \rightarrow R = 1$

$\frac{20^{48}}{13} = \frac{(20^4)^{12}}{13} R \rightarrow 1$

Wilson's Theorem (विल्सन का प्रमेय)

❖ P = any prime number (P = कोई अभाज्य संख्या), (जो द्विघात)

$\frac{(P-1)!}{P} \rightarrow$ Remainder = –1 or (P–1)

❖ $\frac{(ax+k)^n}{a}$ = Remainder → K^n

$\frac{(ax+1)^n}{a} \rightarrow$ Remainder = $1^n = 1$

$\frac{(ax-k)^n}{a} \rightarrow$ Remainder = $(-k)^n$

$\frac{(ax-1)^n}{a} \rightarrow$ Remainder = $(-1)^n$

n = even → **1**

n = odd → **–1**

Euler's Theorem (यूलर का प्रमेय)

❖ $\frac{a^{\Phi(N)}}{N} \rightarrow R=1$ Φ(N)=Toient function of N

a, N → co-prime

N → Natural number / (प्राकृत संख्या) (जो द्विघात)

How to find Φ(N)

$72 = 2^3 \times 3^2$

❖ $\Phi(72) \rightarrow 72 \times \left(1-\frac{1}{2}\right) \times \left(1-\frac{1}{3}\right)$

$= 72 \times \frac{1}{2} \times \frac{2}{3} = 24$

❖ $100 \to 2^2 \times 5^2$

$\Phi 100 \to 100 \times \left(1-\frac{1}{2}\right) \times \left(1-\frac{1}{5}\right) \to 100 \times \frac{1}{2} \times \frac{4}{5} \to 40$

❖ $\Phi(P) =$ **P–1** where P = prime number

$\therefore \Phi(N) \to$ N ि प्र स्स्र रू' ध्स्र बिदरु बN । एि स्रा co-prime टस्र

❖ Product of any 'n' consecutive (+ve) numbers is always divisible by n!

रू ि स्जास्र 'n' रूहरा' (+ve) बिदर, स्रा र । स्रधअव टहस्रा n! ि ए स्रास्जास्र(द टरंएर टस्र

❖ $\frac{15\times16\times17\times18\times19}{5!} \to$ 5 numbers

$\Rightarrow \frac{15\times16\times17\times18\times19}{120}$, R = 0

Consecutive Remainder (यि सदकी ैरत्वाप

❖

$$17\overline{)8697}(511$$
$$\underline{\to 85}$$
$$\quad 19$$
$$\quad \underline{\to 17}$$
$$\quad\ 27$$
$$\quad\ \underline{\to 17}$$
$$\quad\ \ 10$$

consecutive remainder भ व ।रं स लास्र = 1, 2, 10

Divisor → HCF [85, 17, 17]

→ **17**

8. Successive Division प्रक्र मि ण क्रा)च छद

Divide 620 by 8, 5, 6 successively भ रूरहा

$$8\overline{)620}(77 \quad 5\overline{)77}(15 \quad 6\overline{)15}(2 \to \text{Final Quotient}$$
$$\underline{\to 56} \qquad\quad \underline{5} \qquad\quad \underline{12}$$
$$60 \qquad\quad 27 \qquad\quad 3$$
$$\underline{\to 56} \qquad\quad \underline{25}$$
$$4 \qquad\quad 2$$

successive remainders प्रक्र मि ण की ैरत्वा → 4, 2, 3

OR

8	620	4
5	77	2
6	15	3
3	2	

↓ Successive remainders

Final quotient

Some Important points (ण र्क गट)रु न क्महोतप

❖ If two numbers are divided by same divisor the remainders are respectively r_1 and r_2. If sum of these two numbers are divided by the same divisor the remainder is r_3. Then divisor is → **divisor = $r_1 + r_2 - r_3$**

दरै' एि बदर, स्रा ए। टस्र जस्रा का कृपस्र जस्रा' ध्एअम लास्रअव रूहलास्र r_1 स r_2 , रं र टर्थ स्रर दरै' स्रास्रा बिदर, स्रा एदए। स्रद्वारि स्जास्रा का कृपस्र जस्रा' ध्एअम लास्रअव r_3 टर्थ एए→ **च छण क्र $r_1 + r_2 - r_3$**

❖ $\frac{4^n}{6} \to$ Rem = 4

❖ $\frac{10^n}{6} \to$ Rem = 4

Unit digit (UD) (ल्ण ल्क्महोण प

❖ 5 × odd → U.D = 5

5 × even → UD = 0

one zero = one pair of 5 × 2

$875 \times 64 \to \overbrace{5\times5\times5\times7\times2\times2\times2}\times2\times2\times2$

3 pair of 5 & 2

$\therefore$ 3 zero at the end of product भ ।स्रधअव ा ए, बं हस्रा लास्र

0 → $(1370)^{189} \to$ **UD = 0**

No. of zero at the end = 189

❖ 0, 1, 5, 6 ा स्र रू' ध्स्र जस्र power टस्र unit digit same टस्र पं र टस्र

1 → $(371)^{108} \to 371 \times 371 \ldots$. 108 times

UD = 1

5 → $(865)^{99} \to 865 \times 865 \times \ldots$. 99 time

UD = 5 Any power of 5 → UD = 5

6 → $(106)^{357} \to 106 \times 106 \times \ldots$. 357 times

UD = 6

❖ **Any power of 0, 1, 5, 6 → UD = same**

4 → $(4)^{odd}$ = UD → 4

$(4)^{even}$ = UD → 6

9 → $(9)^{odd} \to$ UD = 9

$(9)^{even} \to$ UD = 1

10. Rule of 2, 3, 7, 8

	2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8	2^9	2^{10}
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
UD →	2	4	8	6	2	4	8	6	2	4

UD repeat after every power 4

$\therefore$ cyclicity = 4

$\therefore$ UD → $2^n = 2^{n+4}$

❖ $(132)^{25} \to \frac{25}{4}$, R=1 $\quad\therefore (132)^1 \to$ UD = **2**

	3^1	3^2	3^3	3^4	3^5	3^6	3^7	3^8	3^9	3^{10}
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
UD →	3	9	7	1	3	9	7	1	3	9

$\therefore$ UD repeat after power 4

$\therefore$ cyclicity = 4

❖ $333^{337^{334}} \to \frac{337^{334}}{4} \to 1^{334} \to 1$

$\therefore 333^1 \to$ UD = 3

❖ Similarly cyclicity of 7 & 8 is also 4.

द्रि स्ठा स्र 7 , स्र 8 । स्र श्ररू स्द्रं र जस्र 4 टर्एस्र टस्र

Number of Factors (ि- ध्खेरूैक्क ब्मख प

Factors (ि- ध्खेरूप

❖ Factors are the positive integers that can divide a number exactly.

।म्रधू ह्म डधस्रहा अमर्ण टषकएरुह स्ि बिदर। एए्क्स नअ ए्ि ख्जस्रकं । प ि ं एटस्र

Properties of factors (ण दए ैक्क ैक्कृ- प

1. 1 is a factor of every number.

1 टप बिदर। र।म्रधू ह्म टस्र

2. Every natural number is a factor of itself.

टप ठस्र खंा बिदर ेसदब। र ु। ।म्रधू ह्म टस्र

3. Apart from 1 all natural numbers have atleast two factors.

1 । ए, खंख्छर ज्जस्ठस्र ं बिदर, ख्ख ए ह ए्ि हैं एण म्रधू ह्म टएंए टस्र

Number of Factors (NOF) (ि- ध्खेरूैक्क ब्मख प

❖ 12 → 12, 24, 36, 48, 60 .... (multiples) ध्य)। म्रक-

12 → 1, 2, 3, 4, 6, 12 (factors) ध्य)। म्रखड्ड-

NOF of 12 → 6

perfect square NOF ध्य अमरूस। ष NOF → 2

Even NOF of 12 → 4

perfect cube NOF → 1 ध्य अमर्ड रघ NOF = 1.

odd NOF of 12 → 2

NOF of multiple of 3 → 3

❖ 72 → $2^3 \times 3^2$ (write in prime base)

), ज्जर्द बिदर। एं सं हख्छ-रघर टस्र

$[2^0\ 2^1\ 2^2\ 2^3]\ [3^0\ 3^1\ 3^2]$ OR 4 × 3 = 12 combinations ध्य

∴ NOF → 12

Direct: NOF = (power + 1) × (power + 1) ....

NOF of 72 → (3+1) × (2+1) → 4 × 3 → **12**

❖ $N = a^x \times b^y \times c^z$

a,b,c → prime number ध्य , ज्जर्द बिदर

NOF of N = $(x+1)(y+1)(z+1)$

Sum of factors (SOF) (ि- ध्खेरूैक्क क ैक्कृ प

❖ 72 → $[2^0+2^1+2^2+2^3]\times[3^0+3^1+3^2]$ = 15 × 13 = 195

❖ 2160 → $2^4\times3^3\times5^1$

SOF → $(2^0+2^1+2^2+2^3+2^4)\times(3^0+3^1+3^2+3^3)\times(5^0+5^1)$

→ 31×40×6 → **7440**

OR **Sum of factors** प्रक्ृ- ध्खेरूैक्क क ैक्कि→ $\frac{a[r^n-1]}{(r-1)}$ → GP

$\therefore \frac{1(2^5-1)}{2-1}\times\frac{1(3^4-1)}{3-1}\times\frac{1(5^2-1)}{5-1}$

→ 31 × 40 × 6 = 7440

❖ 2160 = $2^4\times3^3\times5^1$

Even factors → minimum 2^1 ध्ह। प → ज्दघं ह 2^1

NOF = 5×4×2=40

$(2^0\ 2^1\ 2^2\ 2^3\ 2^4) \times (3^0\ 3^1\ 3^2\ 3^3) \times (5^0\ 5^1)$

Even NOF → 4 × 4 × 2 → **32**

OR 2160 = $2^4\times3^3\times5^1$ → $2(2^3\times3^3\times5^1)$

↓

4×4×2 = 32

Sum of even factors ध्ह। म्र ख्ख रदए → 30×40×6 → 7200

No. of odd factors ध्ख्छह। प ख्ख स्ि बिदर → 40–32 = 8

No. of odd factors ध्ख्छह। प ख्ख स्ि बिदर → Absence of 2

∴ 2160 = $2^4 \times 3^3 \times 5^1$ → 4 × 2 = 8

Sum of odd factors ध्ख्छह। प ख्ख रदए → 40 × 6 = 240

❖ 2160 = $2^4 \times 3^3 \times 5^1$

$(2^0\ 2^1\ 2^2\ 2^3\ 2^4)\ (3^0\ 3^1\ 3^2\ 3^3)\ (5^0\ 5^1)$

Number of factors which are multiple of 18 → $18=2^1\times3^2$

। प ख्ख स्ि बिदर कए 18 । ए। म्र टस्र→

$18=2^1\times3^2$

∴ minimum 2^1 and 3^2 required for multiple of 18.

→ 4 × 2 × 2 = 16

OR 2160 = $2^4 \times 3^3 \times 5^1$ → $2\times3^2[2^3\times3^1\times5^1]$

↓

4×2×2=(16)

❖ **Sum of reciprocal of all factors**

(रचल्क्ृ- ध्खेरूैक्क ैक्कृ ठ ं क्क ैप = $\frac{\textbf{sum of factors}}{\textbf{number}}$

8 → 1, 2, 4, 8

Sum of reciprocal → $\frac{1}{1}+\frac{1}{2}+\frac{1}{4}+\frac{1}{8} = \frac{8+4+2+1}{8}$

$$= \frac{\text{Sum of factors}\ (\text{।अधन-खाखा र दराए})}{\text{number}\ \text{)िबदर-}}$$

❖ $10800 \rightarrow 2^4 \times 3^3 \times 5^2$

Number of factors which are perfect squares → ।अधू मह खा स्बिदर करएअमरूस। ष्टध→

a^{2n} = perfect square number ध्अमरूस। ष्बिदर

a^{3n} = perfect cube number ध्अमर्ंख्य बिदर

a^{6n} = perfect square as well as perfect cube ध्अमरूस। ष , ख्य ख्रि टस्अमर्ंख्य

$(2^0\ 2^2\ 2^4) \times (3^0\ 3^2) \times (5^0\ 5^2)$

$\rightarrow 3 \times 2 \times 2 = 12$

OR No. of factors (perfect squares) → दर ।अधन-खाखा स्बिदर)अमरूस। ष →

$$\frac{\text{Power}}{2} \rightarrow \text{Integer} + 1$$

$\rightarrow (2+1)\ (1+1)\ (1+1)$

$\rightarrow 3 \times 2 \times 2 \rightarrow 12$

❖ $72 \rightarrow 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72$

make pairs from 1st & last, NOF = 12

अत्वए, ख्य , सू फ्र एिकखासू ध्रु ब

$1 \times 72 = 72$ $\quad 4 \times 18 = 72$

$2 \times 36 = 72$ $\quad 6 \times 12 = 72$

$3 \times 24 = 72$ $\quad 8 \times 9 = 72$

Product of factors ध्। रप खा र ।अधअव = 72^6

∴ Product of factors of N ध्N । ण रप खा र = $(N)^{\frac{NOF}{2}}$

❖ Only perfect square number has odd number of factors.

रू ि स्अमरूस। ष्बिदर हष्। अधू ख्रा स्खाच्छ बिदर टर्ंएस्टष्ट

❖ Perfect square number from 1 to 100 → 10

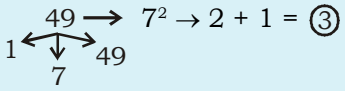
1 ि एि100 ं। अमरूस। ष्बिदर= 10

1, 4, 9, 16, 25, 36, 49, 64, 81, 100

∴ Even number of factors ध्। अधन-खाखा स्हि बिदर→ 100–10 = 90

❖ Perfect square of a prime number has exactly **3** factors.

ु। , जर्(द बिदर। एअमरूस। ष्हष्ट**3** ।अधन-खा टर्ंएएटष्ट



(1–100) → $2^2, 3^2, 5^2, 7^2 \rightarrow 4$ numbers

❖ Numbers which have exactly 2 factors upto 100 ध्100 ं। बिदरु रखधा ए;स च ।अधू ख्रा टष्ट= 25 (Prime Numbers)

❖ Numbers which have exactly 3 factors upto 100 ध्100 ं। बिदरु रखधा ए;स ज्ञ ।अधू ख्रा टष्ट= 4 (4, 9, 25, 49)

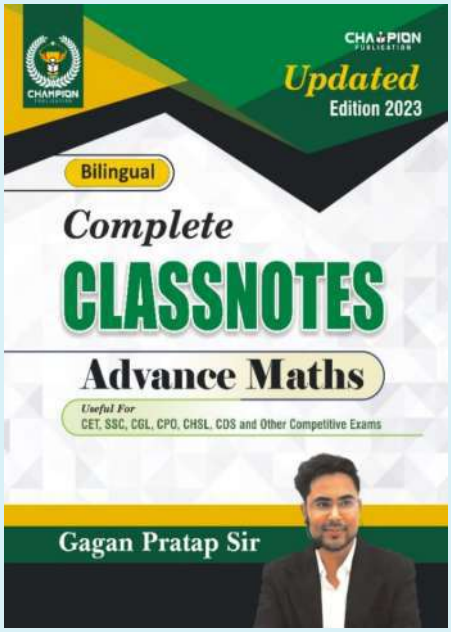
❖ Numbers which have more than 3 factors between (1 to 100) = ?

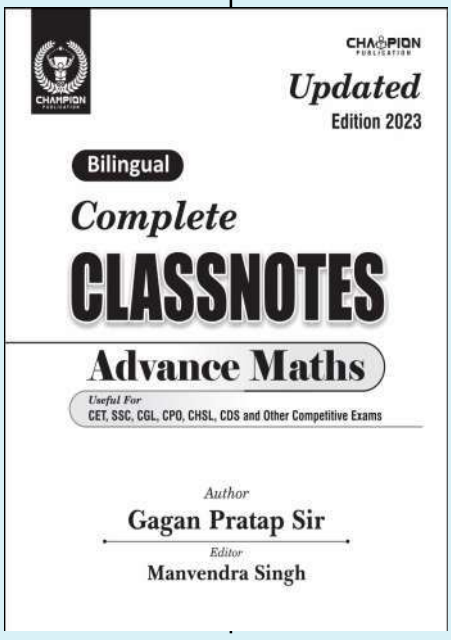
)क्ष ि एक्ष्ऊऊ- । ए स्त्र ज्ञ ि ए, रडा ।अधू ख्रा सखस्बिदर प्र त

only 1 factor = 1 (1 has only one factor)

∴ More than 3 factors ध्3 ि ए, रडा ।अधू ख्रा →

100 –(1 + 4 + 25) = 70 numbers





Sequences and Series (अध्र्ं वम्र्दकैं ु बम्

Sequence (अध्र्ं प

❖ A sequence is an arrangement of numbers in definite order according to some rule.

, धरूह रु ि स्खदह । ए, धरि प खअल्रं रूह हष्बि दर, ख्ब स्/दसे व्रस्टम्

Progression (ैं ु बम्

❖ A sequence whose terms follow a certain pattern is called a progression.

ु । रूह खा । एऐ ु । खअल्रं अम्धष र अवध । पं एटम्ु । द्धप्स्

। टवरं स्टम्

Series (ऐख्य प

❖ If $a_1,a_2,a_3,...,a_n,...$ is a sequence, then the sum expressed as $a_1+a_2+a_3+...+a_n+...$ is called a series.

, ।प $a_1,a_2,a_3,...,a_n,...$ ु । , धरूह टष्ं रू $a_1+a_2+a_3+...+a_n+...$ । एन अहष्/दध्ख दरू । ए ु । द्धँब वर । टर करं र टम्

❖ A series having finite number of terms is called **finite series**.

ऐ ख्ब स्अखअह्ं ि बदर सख्व स्द्धप्स्। रूएअखअह्ं द्धप्स्। टं एटम्

❖ A series having infinite number of terms is called **infinite series**.

, धंब ऐ खसख्व स्द्धँब वर । ए, धंब द्धप्स्। टं एटम्

Special series (मैं ैक्ए ख्य प

❖ If denominator is same as multiplier

दरै ख्ट प । म्रम्र । ए हरध टथ

$$999\frac{991}{999}\times 999 = ?$$

Step 1 → खा ि एMultiply रु द र टथ= 999

Step 2 → , रा एकं धए9 टथद्ं धस् zero = 000

Step 3 → Diff. of 999 – 991 = 8

∴ 999000 – 8 = 998992

❖ $\underbrace{\frac{1}{5\times6}}+\underbrace{\frac{1}{6\times7}}+\underbrace{\frac{1}{7\times8}}+\underbrace{\frac{1}{8\times9}}+....+\underbrace{\frac{1}{24\times25}}$

common difference भ्रि हरद्ं , बंप = 1

$$\frac{6-5}{5\times6}+\frac{7-6}{6\times7}+\frac{8-7}{7\times8}+\frac{9-8}{8\times9}+....\frac{25-24}{24\times25}$$

$$\frac{1}{5}-\frac{1}{6}+\frac{1}{6}-\frac{1}{7}+\frac{1}{7}-\frac{1}{8}+\frac{1}{8}-\frac{1}{9}+....\frac{1}{23}-\frac{1}{24}+\frac{1}{24}-\frac{1}{25}$$

$$\frac{1}{5}-\frac{1}{25}=\frac{4}{25}$$

OR $\frac{1}{\text{common difference}}\left[\frac{1}{\text{1st no. of den.}}-\frac{1}{\text{Last no. of den.}}\right]$

$$1\left(\frac{1}{5}-\frac{1}{25}\right)=\frac{4}{25}$$

❖ $\frac{1}{1\times4\times7}+\frac{1}{4\times7\times10}+\frac{1}{7\times10\times13}+\frac{1}{10\times13\times16}+\frac{1}{13\times16\times19}=?\Rightarrow\frac{1}{6}\left(\frac{1}{1\times4}-\frac{1}{16\times19}\right)=\frac{25}{608}$

❖ $\frac{1}{\text{diff. of 1st \& 3rd no. in den.}}\left(\frac{1}{\text{1st 2no.}}-\frac{1}{\text{last 2 no.}}\right)$

➢ **Series based concepts**

$$9 + 99 + 999 + ... \text{ } n^{th} \text{ terms} = \frac{10(10^n-1)-9n}{9}$$

$$9\ (1 + 11 + 111 + ...n^{th} \text{ terms}) = \frac{10(10^n-1)-9n}{9}$$

$$1 + 11+111+ ....n^{th} \text{ term} = \frac{10(10^n-1)-9n}{81}$$

➢ Bar based concepts:

Formulae:-

$$0.\bar{p}=\frac{p}{9}$$

$$0.\overline{pq}=\frac{pq}{99}$$

$$0.pq\bar{r}=\frac{pqr-p}{990}$$

$$0.\overline{pqr}=\frac{pqr}{999}$$

Arithmetic Progression (AP) (अेण मु सब कैं ु बम्

❖ A sequence in which terms increase or decrease regularly by a fixed number. This fixed number is called the common difference of AP.

ु । रूह खा हष्ऐ ु । खअल्रं ि बदर ए अदख्ं न अ ए ैं ्एदर ैं खं एटम्द ट खअल्रं ि बदर AP । र स्ि ष्, बंप । टवरं स्टम्

Ex. a, a + d, a + 2d,... is an AP

Where a = first term and d = common difference.

कटम्a = अ्टवर ऐ , थ d = ि स्ि ष्, बंप टम्

nth Term (or General Term) of an AP

म र बAP ण क्रा) र तर्क्रा क्र्ं श क्रतप

❖ If a is the first term, d is the common difference and l is the last term of an AP, i.e. the given AP is a, a+d, a+2d, a+3d,..., l, then

दरै a अटवर अै टष्d ि सष्, बंप टष्, ख्य l रू ि सAP ा र, ख्ंह अै टष्, व्रख्ं रै दर। दर a, a+d, a+2d, a+3d,..., l, टष्ं ए

(a) nth term is given by $\mathbf{a_n = a+(n-1)d}$

nसख्अै रै दर। दर टष्$\mathbf{a_n = a+(n-1)d}$

(b) nth term of an AP from the last term is given by $a'_n = \mathbf{l-(n-1)d}$

nसख्अै रू ि सAP ा र, ख्ंह अै ि ए$a'_n = \mathbf{l-(n-1)d}$

Note प्र ै क्ष्र/

(i) $a_n + a'_n = a + l$

i.e. nth term from the beginning + nth term from the end = first term + last term

, व्रख्ं ठरप्व्रा ि ए nसख्अै + , ब ं ि ए nसख्अै = अटवर अै + , ख्ंह अै

Properties of Arithmetic Progression (अेण् ि मु स्व क्रैं ु ब्व्रा क्रै्ु-प

(i) If a constant is added or subtracted from each term of an AP, then the resulting sequence is also an AP with same common difference.

दरै रू ि सAP ा ए ठउ्ए अै ह ख्ु ा रेख्व्रापव्व कख्ह्स्दर्ं ख्दर करं र टष् ं ए अख्मप्सहस्, धरूह जस्ु ि ह ध ि सष्, बंप स्वर ु ा AP टर्ंए र टष्

(ii) If each term of an AP is multiplied or divided by a non-zero constant k, then the resulting sequence is also an AP, with common difference kd or $\frac{d}{k}$ respectively, where d = common difference of given AP.

दरै रू ि सAP ा ए ठउ्ए अै ा ख्ु ा । थी ल्प्स्र रेख्व्रापव्व k ि ए म्स्र दर ख्ज्प्स्क्रं रू दर करं र टष् ं ए अख्मप्सहस्, धरूह जस्ु ा AP टर्ंए र टष् ख्क ि ह ष्ि सष्, बंप रू ह ल्छ्छ kd दर $\frac{d}{k}$ टर्ंए र टष् कटस्d = रै ु ।, AP ा र ि सष्, बंप टर्ंए र टष्

(iii) If a_n, a_{n+1} and a_{n+2} are three consecutive terms of an AP, then $2a_{n+1} = a_n + a_{n+2}$.

दरै a_n, a_{n+1}, ख्य a_{n+2} ु ा AP ा एव। र ं रप् ं स्थ अै टष् ं ए $2a_{n+1} = a_n + a_{n+2}$ ख

(iv) If the terms of an AP are chosen at regular intervals, then they form an AP.

दरै रू ि सAP ा एअै ख्ा ए ख्ठदरह्ं , बंपव्व अम श्रध्र करं र टष् ं ए स्एए ु ा AP ् धरं एटष्

(v) If a sequence is an AP, then its nth term is a linear expression in n, i.e. its nth term is given by An+B, where A and B are constants and A = common difference.

दरै ु ा , धरूह ु ा AP टष् ं ए द्रि ा र nसख्अै n ह ख्ु ा । फ्ू ा /दक्ा टष्, व्रख्ं द्रि ा र nसख्अै An+B कृप्स्र रै दर करं र टष् कटस्A , ख्य B रेख्व्रापव्व टष्, ख्य A = ि सष्, बंप टष्

Selection of Terms in an AP (AP ं क्रतैक्र क्रा ध्प

❖ Any terms in AP can be taken as

AP ह ख्ा ख्ज्रू ज्स्र अै द्रि न अ ह ख्ख्दर करा ि ं र टष्

(i) 3 terms → (a–d), a, (a+d)

(ii) 4 terms → (a–3d), (a–d), (a+d), (a+3d)

(iii) 5 terms → (a – 2d), (a – d), a, (a + d), (a + 2d)

Sum of First n Terms of an AP (म र ब्वAP ण क्रं क्रा न्तैक्र क्रा ि प

❖ Sum of first n terms of AP, is given by

AP ा एअ्व एn अै ख्ा र दर्ाए ं कृप्स्र रै दर। दर टष्

$$Sn = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[a + l]$$

When l = last term

Note प्रक्रै क्ष्र/

(i) A sequence is an AP if the sum of its first n terms is of the form $A_n^2 + B_n$, here A and B are constants and common difference in such case will be 2A.

ु ा , धरूह ु ा AP टष्दरै द्रि ा एअ्व एn अै ख्ा र दर्ाए $A_n^2 + B_n$ ा एन अ ह ख्टष् सटख्A , ख्य B रेख्व्रापव्व टष्, ख्य ु ए ि एह स्हव एह ख्ि सष्, बंप 2A ट र्ाए स्त

(ii) $a_n = S_n - S_{n-1}$ i.e.

nth term of AP = Sum of first n terms – Sum of first (n – 1) terms

$a_n = S_n - S_{n-1}$ दरख्स्

AP ा र nसख्अै = अ्व एn अै ख्ा र दर्ाए ि ठव्रह (n – 1) अै ख्ा र दर्ाए

Ex. $\underbrace{7, 11, 15, 19}_{+4\ +4\ +4}$, 23..... common difference ध् ि सष्, बंप (d) = 4

1st term ध् अ्व्रह अै (a) = 7

a, a+d, a+2d, a+3d, ...a+(n–1)d

T_n = n_{th} term = **a+(n–1)d** = Last term for n term AP

T_n = n_{th} term = **a+(n–1)d** = n अै AP ा र, ख्ंह अै

Sum ध् कख्ह्स्त → $\mathbf{n\frac{(a+l)}{2}}$ OR ध्, ख्य $\mathbf{\frac{n}{2}[2a+(n-1)d]}$

$T^n = l = a + (n-1)d \rightarrow \frac{l-a}{d} = n - 1$

$\therefore$ **n = $\frac{l-a}{d}+1$**

$\sum n = 1+2+3+ \ldots. +n = \frac{n(n+1)}{2}$

$\sum n^2 = 1^2+2^2+3^2+ \ldots. +n^2 = \frac{n(n+1)(2n+1)}{6}$

$\sum n^3 = 1^3+2^3+3^3+ \ldots. +n^3 = \left[\frac{n(n+1)}{2}\right]^2$

❖ Sum of 1st n even natural numbers

1st n हि ठएं बिदर, खा र दए

$2 + 4 + 6 + 8 + \ldots. +2n = \frac{n}{2}[2+2n] = \mathbf{n(n+1)}$

Sum of 1st n odd natural numbers

1st n खच्चह ठएं खंा बिदरा र दए

$1 + 3 + 5 + 7 +\ldots. +(2n-1) = \frac{n}{2}[1+2n-1] = \mathbf{n^2}$

Arithmetic Mean (AM) (अेण मि सब र्क ह्य प

(i) If a, m and b are in AP, then m is called the arithmetic mean of a and b and it is given by

$m = \frac{a+b}{2}$

दखै a, m , थ b, AP हष्टब्यं एm ा एa , थ b ा र , ह्य ाखं स्द हस्द ा टर करं र टथ, थ द्रि एm = $\frac{a+b}{2}$ कृपर खैदर करं र टथ

(ii) If $a_1, a_2, a_3, \ldots\ldots, a_n$ are n numbers, then their AM is given by, $A = \frac{a_1+a_2+\ldots\ldots a_n}{n}$

दखै $a_1, a_2, a_3, \ldots\ldots, a_n$ बिदु ग्टब्यं रद्धधा र AM खाेध कृपर खैदर करं र टथ $A = \frac{a_1+a_2+\ldots\ldots a_n}{n}$

(iii) If a, $A_1, A_2, A_3, \ldots. A_n$, b are in AP, tern

(a) $A_1, A_2, A_3, \ldots. A_n$ are called n arithmetic mean between a and b, where

दखै a, $A_1, A_2, A_3, \ldots. A_n$, b, AP हष्टब्य

(a) $A_1, A_2, A_3, \ldots. A_n$ ा एA , थ B ा ू स्र , ह्य ाखं स्द हस्द ा टर करं र टथ कटख

$A_1 = a + d = \frac{na+b}{n+1}$

$A_2 = a + 2d = \frac{(n-1)a+2b}{n+1}$

: : :

$A_n = a + nd = \frac{a+nb}{n+1}$

and भ, थ $d = \frac{b-a}{n+1}$

(b) Sum of n AM's between a and b is nA

a , थ b ा ू स्र n AM's ा र दए nA टथ

i.e. $A_1 + A_2 + A_3 + \ldots + A_n = nA$, where $A = \frac{a+b}{2}$

दथर $A_1 + A_2 + A_3 + \ldots + A_n = nA$, कटख $A = \frac{a+b}{2}$

Important Results on AP (AP र्दक गट)ु न क्रम्मु ं प

(i) If भ्दैख $a_p = q$ and भ, थ $a_q = p$, then भ्ं $a_{p+q} = 0$, $a_r = p+q-r$

(ii) If भ्दैख $pa_p = qa_q$, then भ्ं $a_{p+q} = 0$

(iii) If भ्दैख $a_p = \frac{1}{q}$ and भ, थ $a_q = \frac{1}{p}$, then भ्ं $a_{pq} = 1$

(iv) If भ्दैख $S_p = q$ and भ, थ $S_q = p$, then भ्ं $S_{p+q} = -(p+q)$

(v) If भ्दैख $S_p = S_q$, then भ्ं $S_{p+q} = 0$

(vi) If भ्दैख a^2, b^2 and c^2 are भ, थ in AP, then भ्ं

$\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ and भ, थ $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ both are also in AP भै खथखजस् टथ AP हष्टब्यख्

(vii) If $a_1, a_2, \ldots., a_n$ are the non-zero terms of an AP, then

दखै $a_1, a_2, \ldots., a_n$ रू हि स् AP ा एलथ्रंएप अै टब्यं

$\frac{1}{a_1a_2}+\frac{1}{a_2a_3}+\frac{1}{a_3a_4}+\ldots..+\frac{1}{a_{n-1}a_n} = \frac{n-1}{a_1a_n}$

Geometric Progression (GP) (इा मं सब कैं ु ब्य

❖ A sequence in which the ratio of any term (except first term) to its just preceding term is constant throughout. The constant ratio is called common ratio (r).

ु ा , धरूह खकि हष्टब्य हि ज्स् अै)अटवएअै ा ग्र खह्य प ा र द्वा ि ए ;स अटव एस्खवएअै ि ए, धब्यं अग्रि हद खेब्रप पटं र टथ खेब्रप , धब्यं ा रणि सष्ठ, धब्यं (r) ा टर करं र टथ

i.e. $\frac{\alpha_{n+a}}{a_n} = r, \forall\ n \geq 1$

❖ If a is the first term, r is the common ratio and l is the last term of a GP, then the GP can be written as a, ar, $ar^2, \ldots.., ar^{n-1}, \ldots. l$.

दखै a अटवर अै टथ r ि स्ष, धब्यं टथ, थ l रू हि स् GP ा र , खंह अै टथं एGP ा एa, ar, $ar^2, \ldots.., ar^{n-1}, \ldots. l$ ा एन अहष्टब्यू र कर ि ं र टथ

4, 12, 36, 108 r = common ratio प्रका)बमध्य स

$$r = \frac{a_2}{a_1} = \frac{12}{4} = ③$$

nth Term (Or General Term) of a GP

GPका का) क्रमतर्का वां श्का तप

❖ If a is the first term, r is the common ratio and l is the last term, then

दरै a अत्वर अै टष्r रिसष्, धअं टथ्, ख्य l, खंह अै टष्ं

(i) nth term of a GP from the beginning is given by $a_n = ar^{n-1}$

दरै a अत्वर अै टष्n रिसष्, धअं टथ्, ख्य l, खंह अै टष्ं $a_n = ar^{n-1}$

(ii) nth term of a GP from the end is given by $a'_n = \frac{l}{r^{n-1}}$

रु रि GP । r nसष्अै , ंब रि ऐंहदर करं र टथ $a'_n = \frac{l}{r^{n-1}}$

(iii) The nth term from the end of a finite GP consisting of m terms is ar^{m-n}.

ु । अषहं GP । ए, ंब रि nसष्लहै खहि षm लहै टष ar^{m-n} टष

(iv) $a_n a'_n = al$

i.e. nth term from the beginning × nth term from the end = first term × last term

$a_n a'_n = al$

, ब्रष्ं ठरप्का रि nसष्अै × , ंब रि nसष्अै = अत्वर अै × , खंह अै $a_n a'_n = al$

Properties of Geometric Progression (ुि-ू दकैं बाकै ुि-प

(i) If all the terms of GP are multiplied or divided by same non-zero constant, then the resulting sequence is also a GP with the same common ratio.

दरै GP । स् जस्-लं ंरीए ए ु । टस्-। थी लाम्द रेख्क्रपष्का रि ए। म्रर दर खाजस्कं रु दर करं र टष्ं ए अषमस्हस्, धरूह जस् रि हरध रिहरुद , धअं सखर GP टरंएर टष

(ii) The reciprocal of terms of a given GP also form a GP.

रु रि रैु ।, GP । स् अै खा र अपे अषा जस्ु । GP ृ धरं र टष

(iii) If each term of a GP is raised to same power, then the resulting sequence also forms a GP.

दरै रु रि GP । एठउए अै । ए रि हरध ं सं ं । ृ ंर करं र टष्ं ए अषमस्हस्, धरूह जस् GP ृ धरं र टष

(iv) If the terms of a GP are chosen at regular intervals, then the resulting sequence is also a GP.

दरै ु । GP । स् अै खा र ए खदहं , ंबपव अम श्रम्र करं र टष्ं ए अषमस्हस् रूह जस्ु । GP टरंएर टष

(v) If $a_1, a_2, a_2, a_3, \ldots, a_n$ are non-zero and non-negative term of a GP, then $\log a_1, \log a_2, \log a_3, \ldots, \log a_n$ are in an AP and vice-versa.

दरै $a_1, a_2, a_2, a_3, \ldots, a_n$ रु रि GP । र लाम्द ए ए, ख । थी ए मास्हा अै टष्ं ए $a_3, \ldots, \log a_n$ AP हष्टष्, ख द्रा । एखअमं जस् रि स्टष

(vi) If a, b and c are three consecutive terms of a GP, then $b^2 = ac$.

दरै a, b , ख्य c रु रि GP । एव । रं ए ं स्थ अै टष्ं $b^2 = ac$

Selection of Terms in a GP (GP ैके सैंक का । ध्प

Any terms in a GP can be taken as

GP हष्अै खा र ए द्रि न अ हष्खदर करा ि ं र टष

(i) 3 terms → $\frac{a}{r}$, a and ar.

(ii) 4 terms → $\frac{a}{r^3}, \frac{a}{r}$, ar and ar^3.

(iii) 5 terms → $\frac{a}{r^2}, \frac{a}{r}$, a, ar, and ar^2.

Sum of First n Terms of a GP (a GP ैक्ं वाकतैक का प

(i) Sum of first n terms of a GP is given by

GP । एअत्वए n अै खा र दए रु रि । एकृप रैदर करं र टथ

$$S_n = \begin{cases} \frac{a(1-r^n)}{1-r}, \text{if } r < 1 \\ \frac{a(r^n-1)}{r-1}, \text{if } r > 1 \\ na, \text{if } r = 1 \end{cases}$$

(ii) $S_n = \frac{a - lr}{1-r}$, r < 1 or $S_n = \frac{lr - a}{r-1}$, r > 1

where, l = last term of the GP.

कटख l = GP । र, खंह अै

Sum of Infinite Terms of a GP (a GP ण बामध्सकतैक का प

(i) If ध्दरै $|r| < 1$, then ंृ $S_\infty = \frac{a}{1-r}$

(ii) If ध्दरै $|r| \geq 1$, then ंृ S_∞ does not exist. ध्हकैग धटस्बटष

Geometric Mean GM (ड्रा मस्वाकं ह्प

(i) If a, G, b are in GP, then G is called the geometric mean of a and b and is given by $G = \sqrt{ab}$.

दरै a, G, b GP हष्टष्ं ए G । एA , ख्य B । र (दस्हं स्द हस्-द । टर करं र टथ्, ख्य $G = \sqrt{ab}$ कृपर रैदर करं र टष

(ii) GM of n positive numbers $a_1, a_2, a_3, \ldots\ldots, a_n$ are given by $G = (a_1 a_2 \ldots a_n)^{1/n}$

n डधरुहा रिबदर, ख $a_1, a_2, a_3, \ldots\ldots, a_n$ । र GM प्र $G = (a_1 a_2 \ldots a_n)^{1/n}$ कृपर रैदर करं र टथ

(iii) If a, G_1, G_2, G_3, ..., G_n b are in GP, then

देखे a, G_1, G_2, G_3, ..., G_n b हस्टस्व GP ं ए

(a) G_1, G_2, G_3, ..., G_n, are called n GM's between a and b, where

$$G_1 = ar = a\left(\frac{b}{a}\right)^{\frac{1}{n+1}},$$

$$G_2 = ar^2 = a\left(\frac{b}{a}\right)^{\frac{2}{n+1}}$$

: : :

$$G_n = ar^n = a\left(\frac{b}{a}\right)^{\frac{n}{n+1}} \text{ and}$$

$$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

(b) Product of n GM's,

$G1 \times G2 \times G3 \times .... Gn = G^n$, where $G = \sqrt{ab}$

Harmonic Progression (गर्दट ए कँु बम

❖ A sequence a_1, a_2, a_3, ..., a_n, ... of non-zero numbers is called a Harmonic Progression (HP), if the sequence $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}$, ..., $\frac{1}{a_n}$, ... is in AP.

। थे लस्द ि बदर, खा ए, धरूह a_1, a_2, a_3, ..., a_n, ... । ए ु । टपरझा द्धप्रस्(HP) । टर करं र टष्ठ देखे , धरूह $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}$, ..., $\frac{1}{a_n}$, ... AP हस्टस्व

$$a_n = \frac{1}{a+(n-1)d}$$

nth Term (or General Term) of Harmonic Progressio (गर्दट ए कँु बम वक) क तक क र ं श क तप

(i) nth term of the HP from the beginning

HP । र nसस्र अै लस्र ि ए

$$a_n = \frac{1}{\frac{1}{a_1}+(n-1)\left(\frac{1}{a_2}-\frac{1}{a_1}\right)}$$

$$= \frac{a_1a_2}{a_2+(n-1)(a_1-a_2)}$$

(ii) nth term of the HP from the end

HP । र nसस्र अै , बं ि ए

$$a'_n = \frac{1}{\frac{1}{l}-(n-1)\left(\frac{1}{a_2}-\frac{1}{a_1}\right)}$$

$$= \frac{a_1a_2l}{a_1a_2-l(n-1)(a_1-a_2)}, \text{ where } l \text{ is the last term.}$$

(iii) $\frac{1}{a_n}+\frac{1}{a'_n}=\frac{1}{a}+\frac{1}{l}=\frac{1}{\text{First term of HP}}+\frac{1}{\text{Last term of HP}}$

(iv) $a_n=\frac{1}{a+(n-1)d}$, if a, d are the first term and common difference of the corresponding AP.

$a_n=\frac{1}{a+(n-1)d}$ देखे a, d ि बस्ड AP । र अ्वर अै , स्थ ि सष्ठ , बंप टस्र

Note:- There is no formula for determining the sum of harmonic series.

धस्ठे टपरझा ूँख वरा र दस्र खाडस्खं । फ्यए एख ु । स्र्ण्ग् धटस्ब्टस्र

AP = 2, 5, 8, 11 ....

HP = $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \frac{1}{11}$

Harmonic Mean (गर्दट ए क ह प

(i) If a, H and b are in HP, then H is called the harmonic mean of a and b and is given by $H=\frac{2ab}{a+b}$

देखे a, H , स्थ b HP हस्टस्व स्र्ण्H । स्थ a , स्थ b । र टपरझा हस्द । टर करं र टष्ठ, स्थ $H=\frac{2ab}{a+b}$ कृपर रैदर करं र टस्र

(ii) Harmonic Mean (HM) of a_1, a_2, a_3, ..., a_n is given by

$$\frac{1}{H}=\frac{1}{n}\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+...+\frac{1}{a_n}\right)$$

Important Results on HP (HP र दर्क गट)ु न क म्ु ं प

(i) If in a HP, a_m=n and a_n = m, then

$$a_{m+n}=\frac{mn}{m+n},\ a_{mn}=1,\ a_p=\frac{mn}{p}$$

(ii) If in a HP, a_p=qr and a_q=pr, then a_r = pq

(iii) If H is HM between a and b, then

(a) $(H-2a)(H-2b) = H^2$

(b) $\frac{1}{H-a}+\frac{1}{H-b}=\frac{1}{a}+\frac{1}{b}$

(c) $\frac{H+a}{H-a}+\frac{H+b}{H-b}=2$

Properties of AM, GM and HM between Two Numbers (तै क ख अै क वहा क AM, GM क म ंदव HM क कु- प

1. If A, G and H are arithmetic, geometric and harmonic means of two positive numbers a and b, then

दरै A, G , स H ौ र ए ा रपरस्हा िबदर, स्ख्ब , स b ा ए, ब ा स्ख्रं स्द० (दस्ख्रं स्द , स ट रपरस्हा हस्द टस्ब ं ए

(i) $A=\frac{a+b}{2}$, $G=\sqrt{ab}$, $H=\frac{2ab}{a+b}$

(ii) $A \geq G \geq H$

(iii) $G^2 = AH$ and so A,G,H are in GP.

(iv) $\frac{a^{n+1}+b^{n+1}}{a^n+b^n} = \begin{cases} A, \text{if } n=0 \\ G, \text{if } n=-\frac{1}{2} \\ H, \text{if } n=-1 \end{cases}$

Exponential Series (स्ख्सब क्ए ह्याय प

❖ The sum of the series $1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+...\infty$ is denoted by the number e.

$1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+...\infty$ ुँस्ख्रा र दरा e कृपरै लस्ख्र करं र टी

$\therefore e = 1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+...$

(i) e lies between 2 and 3.

(ii) e is an irrational number.

(iii) $e^x = 1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+...\infty,\ x \in R$

(iv) $e^{-x} = 1-\frac{x}{1!}+\frac{x^2}{2!}-\frac{x^3}{3!}+...\infty, x \in R$

(v) For any $a > 0$, $a^x = e^{x\log_e^a}$

$= 1+x(\log_e a)+\frac{x^2}{2!}(\log_e a)^2+\frac{x^3}{3!}(\log_e a)^3+...\infty,\ x \in R$

(vi) Sum of first n even natural numbers.

ठस्ह n ि ह ठस्ए ं िबदर, स्ख्ा र दरा ए त

i.e. $2+4+6+...+2n=n(n+1)$

(vii) Sum of first n odd natural numbers.

ठस्ह n स्ख्सह ठस्ए ंा िबदर, स्ख्ा र दरा ए त

i.e. $1+3+5+...(2n-1) = n^2$

(viii) $\sum_{r=1}^{n} n^2 = 1^2+2^2+3^2+...+n^2 = \frac{n(n+1)(2n+1)}{6}$

(ix) $\sum_{r=1}^{n} n^3 = 1^3+2^3+3^3+...+n^3 = \left[\frac{n(n+1)}{2}\right]^2$

(x) $\sum_{r=1}^{n} n^4 = 1^4+2^4+3^4+...+n^4 = \frac{n(n+1)(6n^3+9n^2+n-1)}{30}$

(xi) Sum of n terms of series

ड्ँबवरा एn अै स्ख्ा र दरा ए

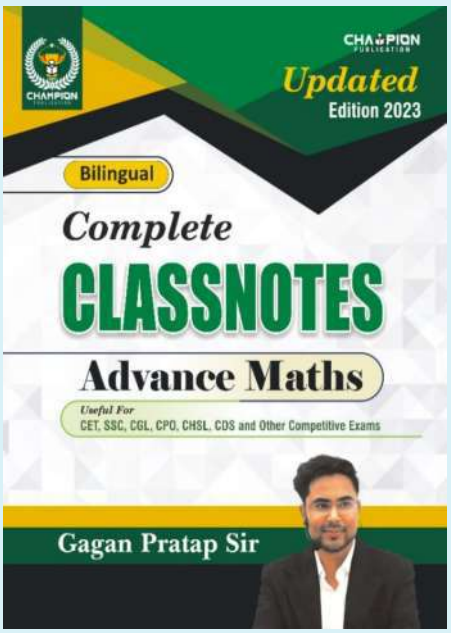
$1^2-2^2+3^2-4^2+5^2-6^2+7^2-8^2+...$

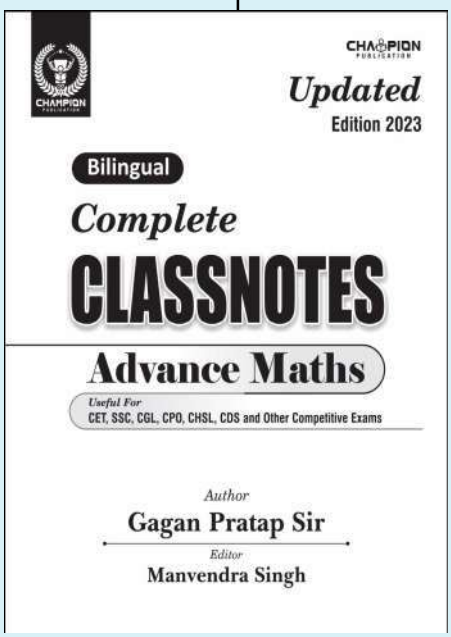
Case I when n is odd = $\frac{n(n+1)}{2}$

स्ख्रं ख I कृ n स्ख्सह ट्थ= $\frac{n(n+1)}{2}$

Case II when n is even = $\frac{-n(n+1)}{2}$

स्ख्रं ख II कृ n ि ह ट्थ= $\frac{-n(n+1)}{2}$





LCM & HCF (यखू ् वर् र)टथ वक्अर्दर्क गू ् वर् र)सथ प

LCM (Least Common Multiple) (यखू ् वर् र)सथ प

❖ The least common multiple (LCM) is defined as the smallest multiple that two or more number have in common.

वं रूह हिअसउद्ष(LCM) ा एै एदर ै एए, खडा बिदर, खहष द्वजदखाच वं रूह ।मक ा एन अ हष्अखजस्खं रू दर करं र टथ

❖ LCM of the any two number is the value that is completely divisible by the two given numbers.

रू ज्टब्बै एए बिदर, खा र वं रूह हिअसउद्षसट हरथ टथ्करएै स ।द्रष् एए बिदर, खए्मंछखाजस्(द टथ

Ex. Find LCM of 36, 45.

36 → 36, 72, 108, 144, 180, 216, 252 ....

45 → 45, 90, 135, 180, 225, 270 ....

LCM (36, 45) → 180 (Least common multiple of both) ै खष्खा र वं रूह हिअसंष –

❖ Find LCM of 24, 30, 36.

$24 \rightarrow 2^3 \times 3^1$

$30 \rightarrow 2^1 \times 3^1 \times 5$

$36 \rightarrow 2^2 \times 3^2$

$LCM = 2^3 \times 3^2 \times 5^1$ (Take max. power of each) ध)ठउदए ा स, खडा ंह ंसं वष = 8 × 9 × 5 = 360

❖ Find LCM of 55, 66, 60.

$55 \rightarrow 5 \times 11$ LCM = $2^2 \times 3 \times 5 \times 11$ = **660**

$66 \rightarrow 2 \times 3 \times 11$

$60 \rightarrow 2^2 \times 3 \times 5$

❖ Find LCM of 13, 29, 41.

13 → LCM (prime no.) → their product ध्द्धा र ।मअधअव

29 → ∴ LCM = 13×29×41

41 →

❖ Find LCM of 24, 30, 36.

24 → 12 × ② Let LCM = 36

30 → 6 × ⑤ ∴ 36 × 5 × 2 = **360**

㊱ → ↘ 6, 12 included in 36 so skip them

❖ Find LCM of 55, 66, 60.

55 → 11 × ⑤ LCM = 66×5×2 = **660**

㊻ →

60 → 5 × 6 × ②

❖ Product of co-prime number = their LCM

टी , जस्(द बिदर ा र ।मअधअव प्र द्धा र LCM

❖ The number which when divided by a, b, c leaves remainder 'r' in each case = LCM(a,b,c) × k + r

सट बिदर खए्रूहलछa, b, c एए्जस्खं । फ्थएम लष्अव x, y, z ठरूं टरंएर टथ

common difference (d) ध्सिप्, ंबप (d)= a–x = b–y = c–z

number = LCM (a, b, c)k – d

Ex. Find least number which divided by 35, 45, 55 leaves 18, 28, and 38 remainder respectively?

न्ले अष्ा ते अष्ा ंबी अदअ्ा त्रू ् ष्35, 45, 55[ष्तयअ्ा ज्तास्त्र सैते ता ।ज अ्ाएष्18, 28 and 38 ईअ्मष्वा नि त

Sol. LCM of (35, 45, 55) = 55 × 9 × 7 = 55 × 63 = 3465

Required number = 3465 – d

d = 35 – 18 = 45 – 28 = 55 – 38 = 17 (Common difference) ् अम कृ

Number = 3465 – 17 = 3448

HCF (Highest common factor) (ंगू ंवर् र)सथ प

❖ HCF is used to find the highest common factors of any two or more given integers.

रू ि्जस्ै एदर, खडा ैखु । अमसष्ख एद्लश्रं ह हिरुन्द ।मअध खष्ख ा एू रूकधण एखु HCF ा र द्वअदए रू दर करं र टथ

❖ In other words HCF is the largest positive integer that divides each of the given integers.

Ex. Find HCF of 40, 64.

40 → (1, 2, 4, 5, 8, 10, 20, 40)

64 → (1, 2, 4, 8, 16, 32, 64)

common-factors of 40, 64 ⇒ [1,2,4,⑧] HCF = **8**

HCF is the greatest common number which divide all the given numbers.

❖ 40 → ⑧ × 5 ↖
64 → ⑧ × 8 ↙ Co-prime ⇒ HCF = 8

42 → 6 × 7 ↖
66 → 6 × 11 ↙ Co-prime no. ⇒ HCF = 6

40 : 64	42 : 66
5 : 8	7 : 11

❖ HCF of two no. = H — Ratio of no's = Hx : Hy

Numbers = Hx, Hy — x : y

x, y → co-prime

LCM = Hxy

How to find HCF

Factorization Method

❖ Find HCF of 168, 294, 420

168 → $2^3 \times 3 \times 7$

Take minimum power of common in all

294 → $2 \times 3 \times 7^2$ ∴ HCF ⇒ $2 \times 3 \times 7$ = **42**

420 → $2^2 \times 3 \times 5 \times 7$

Difference Method

❖ HCF of two numbers = H — Diff. = Hx – Hy

Numbers = Hx, Hy — = H(x–y)

x, y → co-prime — ↓

HCF = diff. ।र factor

Diff. = H(x – y)

If(x–y)=1 → HCF=diff.

If(x–y)>1 → HCF=factor of diff.

HCF of 2 numbers can not be greater than their difference.

❖ 48 → 56 → Difference=8=HCF $\begin{matrix} 48 = 8 \times ⑥ \\ 56 = 8 \times ⑦ \end{matrix}$ Difference=1

∴ HCF = difference = **8**

❖ 306, 391 ∴ HCF = 17

d = 85 → 17 × 5

❖ 1518 1840

D = 322 < 161×2 / ㉓ ×7×2

∴ Both number divisible by 23

Both are even number

∴ HCF = 23×2 = 46

❖ 323, 456, 703

133

⑲ ×7

19 divide all 3

∴ HCF = 19

❖ 1008, 1323, 1722

d=315

⑥③ ×5

3×㉑

HCF of (1008, 1323) = 63

1722 not divisible by 9 ∴ Remove 9 from 63

Hence not divisible by 63 , छ63

→ 1722 divisible by 21 1722 21

∴ HCF (1008, 1323, 1722) = 21

Long Division Method

❖ 693, 945

HCF = 63

693) 945 (1
693
252) 693 (2
504
189) 252 (1
189
63) 189 (3
189
×

Final divisor = HCF

1,2,1,3 → successive quotient

❖ Note: If 2 or 3 numbers are even their HCF will be even. If any one number is odd their HCF can not be even.

❖ If HCF of 2 numbers = H — ∴ LCM = Hxy

Nos. → Hx, Hy — Diff. = H(x–y)

x,y → co-prime — Sum = H(x+y)

∴ HCF is present in LCM, difference & sum of the numbers./

❖ 18, 30 HCF = 6

LCM=90, Diff.=12, Sum=48] All will be divisible by HCF.
िजस HCF िएकसमानद टथाह

❖ $Hx \times Hy = Hxy \times H$

$I \times II = LCM \times HCF$

$H^2xy = H^2xy$

❖ $I \times II = LCM \times HCF$

$\Rightarrow 1^{st}$ no. $\times 2^{nd}$ no. = LCM × HCF

❖ 'K'is the largest number which when divide a,b,c gives same remainder 'r' & quotients are x,y,z respectively

'K' ि ि ू इस िबदर टथकि a,b,c िएकसमानसकं । फधएअम िहरध लाथअव 'r' , थ जरा अव रूहलछ x,y,z टथ

$k\overline{)a\,(x}$ r, $k\overline{)b\,(y}$ r, $k\overline{)c\,(z}$ r

a–b, b–c, c–a

↓ ↓ ↓

$k(x-y)$ $k(y-z)$ $k(z-x)$

$a = kx + r$ $b = ky + r$ $c = kz + r$

∴ K = HCF [a–b, b–c, c–a]

Ex. Let N be greatest number that will divide 1305, 4665, and 6905 leaving same remainder in each case find N?

N = लनपथृ ानपथृ बी अलू ु ा 1305, 4665, कृ अ 6905 िएकसमानसा जातसत्र ृ एआ इअसमाप्वा नि कृअलाकृना N अएआँ ी अल(

Sol. $N\overline{)1305(}\ \frac{-x}{0}$ $N\overline{)4665(}\ \frac{-x}{0}$ $N\overline{)6905(}\ \frac{-x}{0}$

Number ृ अवृरू अ अयअा त्रेअ>

HCF of {(1305 – x), (4665 – x) (6905 – x)}

3360 2240

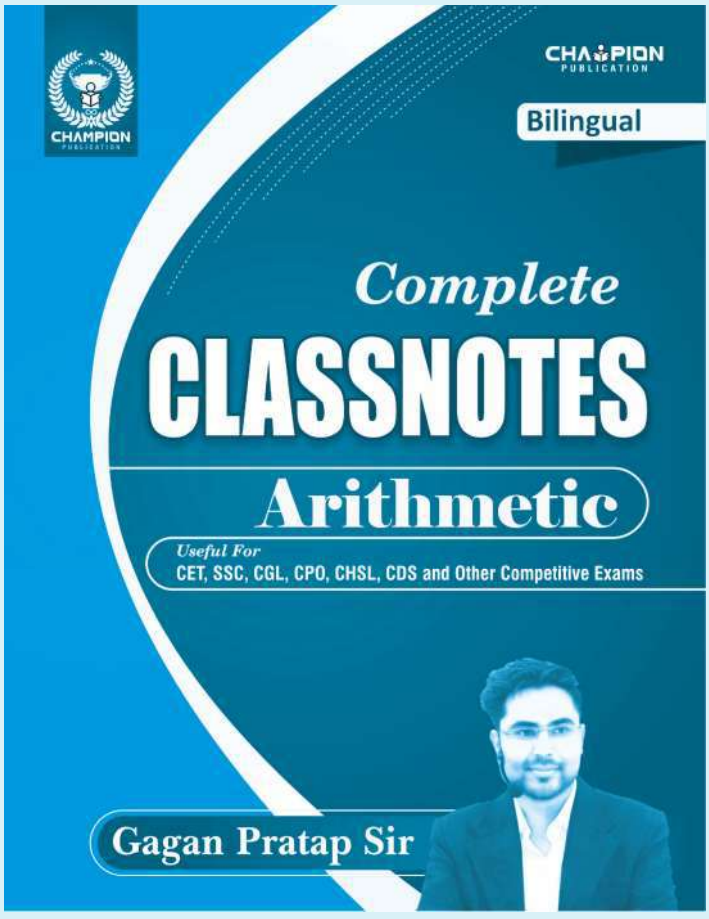
1120

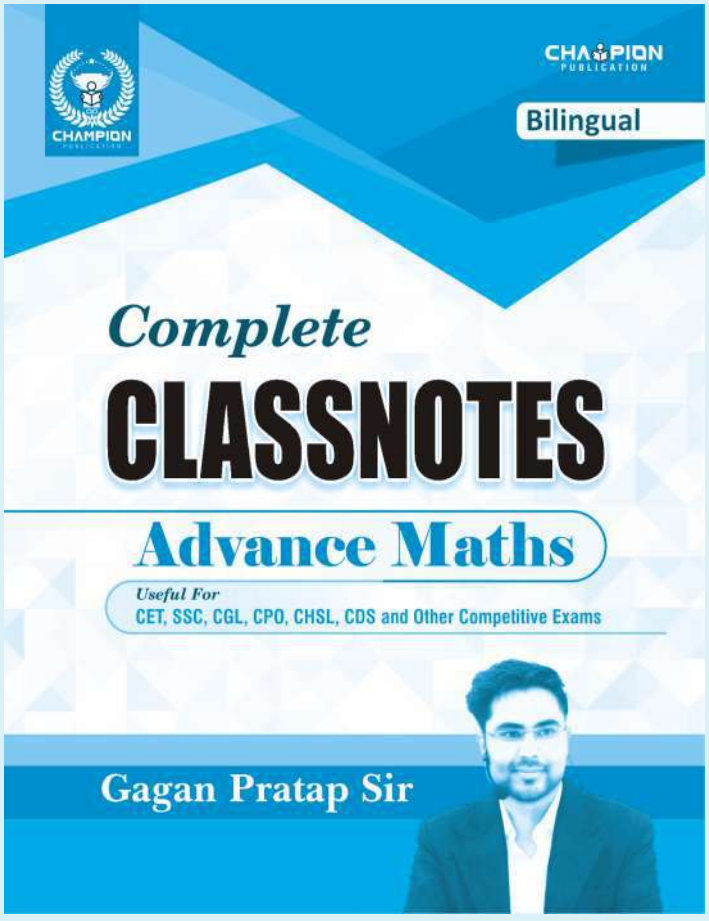
∴ Required no.⇒

❖ $LCM\left[\frac{a}{b},\frac{c}{d},\frac{e}{f}\right] \rightarrow \frac{LCM(a,c,e)}{HCF(b,d,f)}$

$HCF\left[\frac{a}{b},\frac{c}{d},\frac{e}{f}\right] \rightarrow \frac{HCF(a,c,e)}{LCM(b,d,f)}$

❖ HCF $[a^n \pm 1, a^m \pm 1] = \mathbf{a^{HCF(n,m)} \pm 1}$





Calculation (रे ऋण)

Multiply two digit number by 11.

Ex. I→ 45 × 11 = ?

I. Add both digits of two digits number 4 + 5 = 9

II. Place this result between both digits **495**

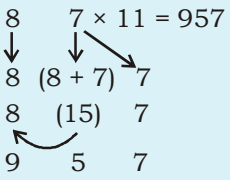
Ex. 2→ 87 × 11 = ?

I. Add both digits of two digit number 8 + 7 = 15

II. Add carry 1 to the left digit when result of step 1 is gerater than 9

8 + 1 = 8

III. Place the result between two digits 957



➢ **Multiply two digit number having same tens digit and one's digit sum is 10.**

Ex. 1→ 23 × 27 = ?

I. Multiply the first digit by the first digit + 1

2 × (2 + 1) = 6

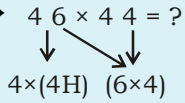
II. Multiply the right most digits (Take two digits, if result is single digit put a zero before it)

3 × 7 = 21

III. Place the result of step II next to result of step I

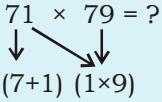
621

Ex. 2→ 46 × 44 = ?



2024

Ex. 3→ 71 × 79 = ?



5609

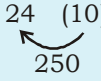
Multiply two digit numbers ending in 1.

Ex. 1→ 61 × 41 = ?

I. Multiply left most digits 6 × 4 = 24

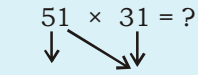
II. Add left most digits 6 + 4 = 10

III. Take only right digit from step II and forward the carry to step I



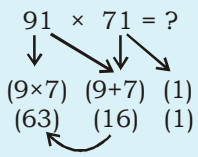
IV. Put 1 at the end **2501**

Ex. 2→ 51 × 31 = ?



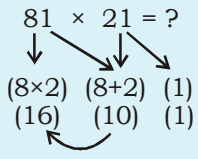
1581

Ex. 3→ 91 × 71 = ?



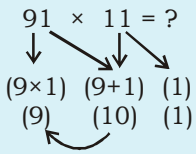
6461

Ex. 4→ 81 × 21 = ?



1701

Ex. 5→ 91 × 11 = ?



1001

Multiply numbers between 11 and 19.

Ex. 1→ 14 × 17 = ?

I. Add the larger no. to the rightmost digit of the other number(17 + 4) = 21

II. Multiply by 10 the result of step I → 210

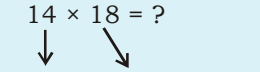
III. Multiply the rightmost digit of two numbers

4 × 7 = 28

IV. Add step II and III

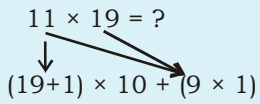
210 + 28 = 238

Ex. 2→ 14 × 18 = ?



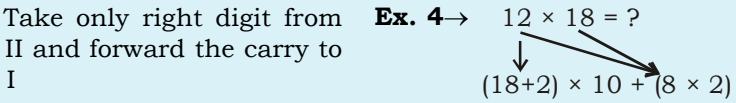
220 + 32 = **252**

Ex. 3→ 11 × 19 = ?



200 + 9 = **209**

Ex. 4→ 12 × 18 = ?



200 + 16 = **216**

Ex. 5→ 12 × 19 = ?

(19 + 2) × 10 + (9 × 2)

210 + 18 = **228**

Ex. 6→ 13 × 16 = ?

(16 + 3) × 10 + (6 × 3)

190 + 18 = **208**

Multiply by 0.2

Ex. 1→ 432 × 0.2 = ?

I. Multiply by 2

432 × 2 = 864

II. Divide by 10

$\frac{864}{10}$ = **86.4**

2nd method

432 × 0.2 = 86.4

I. Divide by 5

$\frac{432}{5}$ = **86.4**

Ex. 2→ 355 × 0.2 = ?

I. $\frac{355}{5}$ = **71**

Ex. 3→ 283 × 0.2 = ?

I. $\frac{283}{5}$ = **56.6**

Multiply by 0.25

Ex. 1→ 355 × 0.25 = ?

I. To multiply the given number by 0.25, divide it by 4

$\therefore \frac{355}{4}$ = **88.75**

Ex. 2→ 884 × 0.25 = ?

I. $\frac{84}{4}$ = **221**

Ex. 3→ 1132 × 0.25 = ?

I. $\frac{1132}{4}$ = **283**

Multiplification by 5

Ex. 1→ 163 × 5 = 815

Step-I Put 0 after given no. = 1630

Step-II Divide step I by 2

$\frac{1630}{2}$ = **815**

Ex. 2→ 287 × 5 = ?

I. 2870

II. $\frac{2870}{2} = \mathbf{1435}$

Ex. 3→ 335 × 5 = ?

I. 3350

II. $\frac{3350}{2} = \mathbf{1675}$

Ex. 4→ 884 × 5 = ?

I. 8840

II. $\frac{8840}{2} = \mathbf{4420}$

Multiplication by 25

Ex. 1→ 182 × 25 = ?

Step-I Put 00 at the end of given number 18200

Step-II Divide it by 4

$\frac{18200}{4} = \mathbf{4550}$

Ex. 2→ 292 × 25 = ?

Step-I 29200

Step-II $\frac{29200}{4} = \mathbf{7300}$

Ex. 3→ 1787 × 25 = ?

Step-I 178700

Step-II $\frac{178700}{4} = \mathbf{44675}$

Multiplication by 125

Ex. 1→ 187 × 125 = ?

Step-I Put 000 at the end of the number 187000

Step-II Divide by 8

$\frac{187000}{8} = \mathbf{23375}$

Ex. 2→ 292 × 125 = ?

Step-I 29200

Step-II $\frac{292000}{8} = \mathbf{36500}$

Ex. 3→ 587 × 125 = ?

Step-I 587000

Step-II $\frac{587000}{8} = \mathbf{73,375}$

Multiplication by the number whose unit digit is 9

Ex. 1→ 187 × 19 = ?

Step-I 187 × (20 – 1)

Step-II 187 × 20 – 187 × 1

3740 – 187 = **3553**

Ex. 2→ 163 × 9 = ?

Step-I 163 × (10 – 1)

Step-II 1630–163 = **1467**

Ex. 3→ 467 × 99 = ?

Step-I 467 × (100–1)

Step-II 46700 – 467 = **46233**

Multiply two numbers whose difference is even no.

Ex. 1→ 22 × 36 = ?

Step-I 36 – 22 = 14 (even)

Step-II $\frac{14}{2} = 7$

Step-III 22 + 7 Or 36 – 7 = 29

Step-IV $(29)^2 = 841$

Step-V $841 - 7^2 = \mathbf{792}$

Ex. 2→ 78 × 82 = ?

I. 82 – 78 = 4

II. $\frac{4}{2} = 2$

III. 78 + 2 = 80

IV. $80^2 - 2^2 = 6400 - 4 = \mathbf{6396}$

Vedic method of multiply

Ex. 1→ 27 × 78 = ?

I.

2	7
7	8
6	

⑤6

II.

2	7
7	8
0	6

16 + 49
= 65 + 5
=⑦ 0

III.

2	7
7	8
21	**06**

7 × 2 = 14
14 + 7 = 21

Ex. 2→ 21 × 72 = ?

2	1
7	2
15	**12**

I. 2 × 1 = 2
II. 2 × 2 + 7 × 1 =①1
III. 2 × 7 + 1 = 15

Ex. 3→ 73 × 65 = ?

7	3
6	5
47	**45**

I. 5 × 3 =①5
II. (7 × 5 + 6 × 3) + 1
=⑤4
III. 7 × 6 + 5 = 47

Ex.4→ 524 × 19 = ?

5	2	4
	1	9
9	**9**	**5 6**

I. 4 × 9 =③6
II. (9 × 2 + 1 × 4) + 3 =②5
III. (9 × 5 + 1 × 2) + 2 =④9
IV. (1 × 5) + 4 = 9

Ex. 5→37 × 66 = ?

3	7
6	6
24	**42**

I. 7 × 6 =④2
II. (6 × 3 + 6 × 7) + 4 =⑥4
III. (6 × 3) + 6 = 24

Square Techniques

➢ **Square of the number whose unit digit is 5.**

Ex.1→ Find square of 25

Step-I $(5)^2 = 25$

Step-II (Digits other than 5) × (same digit + 1)

2 × (2 + 1) = 6

∴ $(25)^2 = \mathbf{625}$

Ex.2→ Find square of 125.

$(125)^2$ = 12 × (12 + 1) $(5)^2$

(12 × 13) (25)

15625

Ex.3→ Find square of 65.

(6 × 7) $(5)^2$

4225

Ex.4 → Find square of 85.

(8 × 9) $(5)^2$

7225

Ex.5→ Find square of 95.

(9 × 10) $(5)^2$

9025

Ex.6→ Find square of 205.

(20 × 21) $(5)^2$

42025

Ex.7→ Find square of 145.

(14 × 15) $(5)^2$d

21025

Square of the numbers whose unit digit is 1.

Ex. 1→ Find square of 31.

$(30)^2$ + (30+31)

900 + 60 = **961**

Ex. 2→ Find square of 21.

$(20)^2$ + (20+21)

400 + 40 = **441**

Ex. 3→ Find square of 41.

$(40)^2$ + (40+41)

1600 + 80 = **1681**

Ex. 4→ Find square of 61.

$(60)^2$ + (60+61)

3600 + 121 = **3721**

Ex. 5→Find square of 91.

$(90)^2$ + (90+91)

8100 + 181 = **8281**

Ex. 6→Find square of 111.

$(110)^2$ + (110+111)

12100 + 221 = **12321**

Ex. 7→ Find square of 151.

$(150)^2$ + (150+151)

22500 + 301 = **22801**

Ex. 8→ Find square of 301.

$(300)^2$ + (300+301)

90000 + 601 = **90601**

Ex. 9→ Find square of 31.
$(30)^2 + (30+31)$
$900 + 60 = \mathbf{961}$

Square of the numbers whose unit digit is 9.

Ex. 1→ Find square of 19.
$(20)^2 - (19+20)$
$400 - 39 = \mathbf{361}$

Ex. 2→ Find square of 29.
$(30)^2 - (29+30)$
$900 - 59 = \mathbf{841}$

Ex. 3→ Find square of 39.
$(40)^2 - (39+40)$
$1600 - 79 = \mathbf{1521}$

Ex. 4→ Find square of 59.
$(60)^2 - (59+60)$
$3600 - 119 = \mathbf{3481}$

Ex. 5→Find square of 99.
$(100)^2 - (99+100)$
$10000 - 199 = \mathbf{9801}$

Ex. 6→ Find square of 199.
$(200)^2 - (199+200)$
$40000 - 399 = \mathbf{39601}$

Ex. 7→ Find square of 249.
$(250)^2 - (249+250)$
$62500 - 499 = \mathbf{62001}$

Square of the numbers between 40 and 49.

Base = 50

Ex. 1→ Find square of 48.
Step1→ 25 – (Difference of 50 and given number)
$25 - (50 - 48) = 23$
Step2→ (Difference of 50 and given number)$^2 = (2)^2 = 04$
$\therefore 48^2 = \mathbf{2304}$

Ex. 2→ Find square of 49.
Step1→ $25 - 1 = 24$
Step2→ $(1)^2 = 01$
$\therefore 49^2 = \mathbf{2401}$

Ex. 3→ Find square of 47.
Step1→ $25 - 3 = 22$
Step2→ $(3)^2 = 09$
$\therefore 47^2 = \mathbf{2209}$

Ex. 4→ Find square of 46.
Step1→ $25 - 4 = 21$
Step2→ $(4)^2 = 16$
$\therefore 46^2 = \mathbf{2116}$

Ex. 5→Find square of 44.
Step1→ $25 - 6 = 19$
Step2→ $(6)^2 = 36$
$\therefore 44^2 = \mathbf{1936}$

Square of the numbers between 51 and 59.

Base = 50

Ex. 1→ Find square of 52.
Step1→ 25 + (Difference of given number and 50)
$25 + (52 - 50) = 27$
Step2→ (Difference of given number and 50)$^2 = (2)^2 = 04$
$\therefore 52^2 = \mathbf{2704}$

Ex. 2→ Find square of 59.
Step1→ $25 + (59 - 50) = 34$
Step2→ $(59 - 50)^2 = (9)^2 = 81$
$\therefore 59^2 = \mathbf{3481}$

Ex. 3→ Find square of 58.
Step1→ $25 + 8 = 33$
Step2→ $(8)^2 = 64$
$\therefore 58^2 = \mathbf{3364}$

Ex. 4→ Find square of 57.
Step1→ $25 + 7 = 32$
Step2→ $(7)^2 = 49$
$\therefore 57^2 = \mathbf{3249}$

Ex. 5→Find square of 54.
Step1→ $25 + 4 = 29$
Step2→ $(4)^2 = 16$
$\therefore 54^2 = \mathbf{2916}$

Ex.6→ Find square of 53.
Step1→ $25 + 3 = 28$
Step2→ $(3)^2 = 09$
$\therefore 53^2 = \mathbf{2809}$

Square of the numbers between 90 and 99.

Base = 100

Ex. 1→ Find square of 92.
Step1→ $92 - 8 = \underline{84}$
Step2→ Subtract from 100
$100 - 92 = 8$
Step3→ $(8)^2 = \underline{64}$
$\therefore (92)^2 = \mathbf{8464}$

Ex. 2→ Find square of 93.
Step1→ $93 - 7 = \underline{86}$
Step2→ $100 - 93 = 7$
Step3→ $(7)^2 = \underline{49}$
$\therefore (93)^2 = \mathbf{8649}$

Ex. 3→ Find square of 94.
Step1→ $94 - 6 = \underline{88}$
Step2→ $(6)^2 = \underline{36}$
$\therefore (94)^2 = \mathbf{8836}$

Ex. 4→ Find square of 96.
Step1→ $94 - 4 = \underline{92}$
Step2→ $(4)^2 = \underline{16}$
$\therefore (96)^2 = \mathbf{9216}$

Ex. 5→Find square of 97.
Step1→ $97 - 3 = \underline{94}$
Step2→ $(3)^2 = \underline{09}$
$\therefore (97)^2 = \mathbf{9409}$

Ex. 6→Find square of 99.
Step1→ $99 - 1 = \underline{98}$
Step2→ $(1)^2 = \underline{01}$
$\therefore (99)^2 = \mathbf{9901}$

Square of the numbers more than 100(near 100).

Base = 100

Ex. 1→ Find square of 104.
Step1→ 104 + (difference of 100 and given number)
$104 + 4 = \underline{108}$
Step2→ $4^2 = \underline{16}$
$\therefore (104)^2 = \mathbf{10816}$

Ex. 2→ Find square of 101.
Step1→ $101 + 1 = \underline{102}$
Step2→ $1^2 = \underline{01}$
$\therefore (101)^2 = \mathbf{10201}$

Ex. 3→ Find square of 102.
Step1→ $102 + 2 = \underline{104}$
Step2→ $2^2 = \underline{04}$
$\therefore (102)^2 = \mathbf{10404}$

Ex. 4→ Find square of 103.
Step1→ $103 + 3 = \underline{106}$
Step2→ $3^2 = \underline{09}$
$\therefore (103)^2 = \mathbf{10609}$

Ex. 5→Find square of 106.
Step1→ $106 + 6 = \underline{112}$
Step2→ $6^2 = \underline{36}$
$\therefore (106)^2 = \mathbf{11236}$

Ex. 6→Find square of 111.
Step1→ $111 + 11 = \underline{122}$
Step2→ $(11)^2 = \underline{121}$ (take 2 digits)
$\therefore (106)^2 = (122)\,(121) = \mathbf{12321}$

Ex. 7→ Find square of 113.
Step1→ $113 + 131 = \underline{126}$
Step2→ $(13)^2 = \underline{169}$ (take 2 digits)
$\therefore (106)^2 = (126)\,(169) = \mathbf{12769}$

Squares - Cubes - Square Root Chart

Number n	Square n^2	Cube n^3	Square root $\sqrt{n}$
1	1	1	1.0000
2	4	8	1.4142
3	9	27	1.7321
4	16	64	2.0000
5	25	125	2.2361
6	36	216	2.4495
7	49	343	2.6458
8	64	512	2.8284
9	81	729	3.0000
10	100	1000	3.1623
11	121	1331	3.3166
12	144	1728	3.4641
13	169	2197	3.6056
14	196	2744	3.7417
15	225	3375	3.8730
16	256	4096	4.0000
17	289	4913	4.1231
18	324	5832	4.2426
19	361	6859	4.3589
20	400	8000	4.4721
21	441	9261	4.5826
22	484	10648	4.6904
23	529	12167	4.7958
24	576	13824	4.8990
25	625	15625	5.0000
26	676	17576	5.0990
27	729	19683	5.1962
28	784	21952	5.2915
29	841	24389	5.3852
30	900	27000	5.4772
31	961	29791	5.5678
32	1024	32768	5.6569
33	1089	35937	5.7446
34	1156	39304	5.8310
35	1225	42875	5.9161
36	1296	46656	6.0000
37	1369	50653	6.0828
38	1444	54872	6.1644
39	1521	59319	6.2450
40	1600	64000	6.3246

Number n	Square n^2	Cube n^3	Square root $\sqrt{n}$
41	1681	68921	6.4031
42	1764	74088	6.4807
43	1849	79507	6.5574
44	1936	85184	6.6332
45	2025	91125	6.7082
46	2116	97336	6.7823
47	2209	103823	6.8557
48	2304	110592	6.9282
49	2401	117649	7.0000
50	2500	125000	7.0711
51	2601	132651	7.1414
52	2704	140608	7.2111
53	2809	148877	7.2801
54	2916	157464	7.3485
55	3025	166375	7.4162
56	3136	175616	7.4833
57	3249	185193	7.5498
58	3364	195112	7.6158
59	3481	205379	7.6811
60	3600	216000	7.7460
61	3721	226981	7.8102
62	3844	238328	7.8740
63	3969	250047	7.9373
64	4096	262144	8.0000
65	4225	274625	8.0623
66	4356	287496	8.1240
67	4489	300763	8.1854
68	4624	314432	8.2462
69	4761	328509	8.3066
70	4900	343000	8.3666
71	5041	357911	8.4261
72	5184	373248	8.4853
73	5329	389017	8.5440
74	5476	405224	8.6023
75	5625	421875	8.6603
76	5776	438976	8.7178
77	5929	456533	8.7750
78	6084	474552	8.8318
79	6241	493039	8.8882
80	6400	512000	8.9443

×	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
2	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60
3	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75	78	81	84	87	90
4	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84	88	92	96	100	104	108	112	116	120
5	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130	135	140	145	150
6	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120	126	132	138	144	150	156	162	168	174	180
7	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140	147	154	161	168	175	182	189	196	203	210
8	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160	168	176	184	192	200	208	216	224	232	240
9	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180	189	198	207	216	225	234	243	252	261	270
10	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250	260	270	280	290	300
11	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220	231	242	253	264	275	286	297	308	319	330
12	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264	276	288	300	312	324	336	348	360
13	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260	273	286	299	312	325	338	351	364	377	390
14	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280	294	308	322	336	350	364	378	392	406	420
15	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360	375	390	405	420	435	450
16	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320	336	352	368	384	400	416	432	448	464	480
17	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340	357	374	391	408	425	442	459	476	493	510
18	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360	378	396	414	432	450	468	486	504	522	540
19	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380	399	418	437	456	475	494	513	532	551	570
20	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500	520	540	560	580	600
21	21	42	63	84	105	126	147	168	189	210	231	252	273	294	315	336	357	378	399	420	441	462	483	504	525	546	567	588	609	630
22	22	44	66	88	110	132	154	176	198	220	242	264	286	308	330	352	374	396	418	440	462	484	506	528	550	572	594	616	638	660
23	23	46	69	92	115	138	161	184	207	230	253	276	299	322	345	368	391	414	437	460	483	506	529	552	575	598	621	644	667	690
24	24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	384	408	432	456	480	504	528	552	576	600	624	648	672	696	720
25	25	50	75	100	125	150	175	200	225	250	275	300	325	350	375	400	425	450	475	500	525	550	575	600	625	650	675	700	725	750
26	26	52	78	104	130	156	182	208	234	260	286	312	338	364	390	416	442	468	494	520	546	572	598	624	650	676	702	728	754	780
27	27	54	81	108	135	162	189	216	243	270	297	324	351	378	405	432	459	486	513	540	567	594	621	648	675	702	729	756	783	810
28	28	56	84	112	140	168	196	224	252	280	308	336	364	392	420	448	476	504	532	560	588	616	644	672	700	728	756	784	812	840
29	29	58	87	116	145	174	203	232	261	290	319	348	377	406	435	464	493	522	551	580	609	638	667	696	725	754	783	812	841	870
30	30	60	90	120	150	180	210	240	270	300	330	360	390	420	450	480	510	540	570	600	630	660	690	720	750	780	810	840	870	900

ADDITION/(रेख अ(+)

Type-1 (Find Sum)

1. 19 + 3 = ?
(a) 12 (b) 16
(c) 20 (d) 22

2. 27 + 8 = ?
(a) 34 (b) 39
(c) 45 (d) 35

3. 39 + 8 = ?
(a) 52 (b) 57
(c) 47 (d) 58

4. 43 + 8 = ?
(a) 49 (b) 55
(c) 51 (d) 53

5. 57 + 8 = ?
(a) 67 (b) 65
(c) 69 (d) 63

6. 9 + 58 = ?
(a) 67 (b) 62
(c) 68 (d) 63

7. 7 + 87 = ?
(a) 99 (b) 97
(c) 95 (d) 94

8. 9 + 97 = ?
(a) 102 (b) 106
(c) 120 (d) 122

9. 93 + 8 = ?
(a) 102 (b) 106
(c) 121 (d) 101

10. 67.9 + 5.7 = ?
(a) 73.6 (b) 76.3
(c) 73.2 (d) 77.8

11. 48.2 + 6.6 = ?
(a) 54.8 (b) 56.3
(c) 53.6 (d) 57.1

12. 95.7 + 9.0 = ?
(a) 102.7 (b) 106.7
(c) 100.7 (d) 104.7

13. 47.3 + 18.8 = ?
(a) 45.1 (b) 66.1
(c) 49.1 (d) 61.1

14. 78.9 + 6.6 = ?
(a) 84.5 (b) 85.5
(c) 86.5 (d) 87.3

15. 38.5 + 5.5 = ?
(a) 43.0 (b) 46.0
(c) 49.2 (d) 44.0

Type-2 (Find Sum)

16. 88 + 23 = ?
(a) 109 (b) 108
(c) 111 (d) 110

17. 79 + 18 = ?
(a) 99 (b) 98
(c) 97 (d) 96

18. 16 + 49 = ?
(a) 62 (b) 67
(c) 65 (d) 69

19. 37 + 67 = ?
(a) 105 (b) 108
(c) 104 (d) 110

20. 39 + 47 = ?
(a) 70 (b) 86
(c) 73 (d) 77

21. 57 + 78 = ?
(a) 133 (b) 136
(c) 135 (d) 137

22. 79 + 87 = ?
(a) 168 (b) 166
(c) 167 (d) 169

23. 97 + 13 = ?
(a) 113 (b) 122
(c) 100 (d) 110

24. 57 + 71 = ?
(a) 125 (b) 128
(c) 126 (d) 129

25. 88 + 95 = ?
(a) 180 (b) 181
(c) 183 (d) 182

26. 88.3 + 27.5 = ?
(a) 119.8 (b) 108.6
(c) 115.8 (d) 110.8

27. 67.89 + 23.07 = ?
(a) 90.96 (b) 90.95
(c) 91.96 (d) 93.66

28. 34.10 + 45.64 = ?
(a) 79.67 (b) 79.74
(c) 80.74 (d) 77.89

29. 12.84 + 91.06 = ?
(a) 103.9 (b) 108.9
(c) 111.6 (d) 110.8

30. 65.22 + 23.08 = ?
(a) 88.89 (b) 88.30
(c) 87.30 (d) 89.70

Type-3 (Find Sum)

31. 23 + 64 + 12 + 76 + 97 = ?
(a) 282 (b) 272
(c) 275 (d) 279

32. 98 + 45 + 93 + 59 + 62 = ?
(a) 382 (b) 352
(c) 357 (d) 379

33. 19 + 93 + 75 + 73 + 57 = ?
(a) 328 (b) 317
(c) 375 (d) 379

34. 34 + 76 + 38 + 72 + 52 = ?
(a) 282 (b) 272
(c) 275 (d) 279

35. 233 + 44 + 126 + 61 + 70 = ?
(a) 482 (b) 534
(c) 635 (d) 439

36. 634 + 64 + 122 + 76 + 197 = ?
(a) 1283 (b) 1272
(c) 1093 (d) 1273

37. 86 + 164 + 172 + 726 + 97 = ?
(a) 1245 (b) 1272
(c) 1275 (d) 1275

38. 23 + 464 + 31 + 576 + 497 = ?
(a) 1282 (b) 1671
(c) 1575 (d) 1591

39. 923 + 864 + 712 + 676 + 597 = ?
(a) 2282 (b) 3772
(c) 3275 (d) 4279

40. 457 + 164 + 834 + 432 + 78 + 234 + 923 + 93 = ?
(a) 3285 (b) 3215
(c) 3275 (d) 3279

41. 723 + 234 + 760 + 219 + 497 + 896 + 542 + 132 = ?
(a) 4003 (b) 4122
(c) 4200 (d) 4279

42. 233 + 335 + 367 + 764 + 917 + 134 + 124 + 923 + 913 = ?
(a) 4582 (b) 4710
(c) 6275 (d) 5279

43. 2326 + 1324 + 245 + 799 + 975 + 2356 + 134 + 235 + 2467 = ?
(a) 11282 (b) 10272
(c) 10861 (d) 13279

ANSWER KEY - ADDITION

1.	(d)	2.	(d)	3.	(c)	4.	(c)	5.	(b)	6.	(a)	7.	(d)	8.	(b)	9.	(d)	10.	(a)
11.	(a)	12.	(d)	13.	(b)	14.	(b)	15.	(d)	16.	(c)	17.	(c)	18.	(c)	19.	(c)	20.	(b)
21.	(c)	22.	(b)	23.	(d)	24.	(b)	25.	(c)	26.	(c)	27.	(a)	28.	(b)	29.	(a)	30.	(b)
31.	(b)	32.	(c)	33.	(b)	34.	(b)	35.	(b)	36.	(c)	37.	(a)	38.	(d)	39.	(b)	40.	(b)
41.	(a)	42.	(b)	43.	(c)														

SUBTRACTION/ व्यवकलन(–)

Type-1 (Find Difference)

1. 100 – 69 = ?
(a) 21 (b) 41
(c) 31 (d) 19

2. 100 – 37 = ?
(a) 73 (b) 63
(c) 69 (d) 68

3. 100 – 53 = ?
(a) 53 (b) 57
(c) 47 (d) 49

4. 100 – 17 = ?
(a) 83 (b) 41
(c) 31 (d) 19

5. 100 – 78 = ?
(a) 32 (b) 31
(c) 22 (d) 25

6. 85 – 27 = ?
(a) 58 (b) 59
(c) 49 (d) 69

7. 93 – 49 = ?
(a) 51 (b) 41
(c) 44 (d) 49

8. 71 – 57 = ?
(a) 21 (b) 41
(c) 14 (d) 15

9. 73 – 35 = ?
(a) 51 (b) 38
(c) 48 (d) 58

10. 67 – 39 = ?
(a) 28 (b) 29
(c) 31 (d) 19

11. 123 – 69 = ?
(a) 54 (b) 41
(c) 51 (d) 73

12. 183 – 56 = ?
(a) 123 (b) 141
(c) 131 (d) 127

13. 148 – 67 = ?
(a) 61 (b) 88
(c) 81 (d) 87

14. 175 – 88 = ?
(a) 67 (b) 77
(c) 87 (d) 82

15. 143 – 69 = ?
(a) 67 (b) 74
(c) 77 (d) 78

Type-2 (Find Difference)

16. 529 – 452 = ?
(a) 87 (b) 78
(c) 77 (d) 79

17. 956 – 629 = ?
(a) 334 (b) 327
(c) 431 (d) 319

18. 823 – 286 = ?
(a) 613 (b) 534
(c) 537 (d) 593

19. 581 – 396 = ?
(a) 189 (b) 141
(c) 185 (d) 182

20. 923 – 232 = ?
(a) 691 (b) 641
(c) 731 (d) 719

21. 2574 – 869 = ?
(a) 1721 (b) 1841
(c) 1705 (d) 1819

22. 6724 – 4573 = ?
(a) 2121 (b) 2147
(c) 2151 (d) 2229

23. 2343 – 2269 = ?
(a) 86 (b) 175
(c) 74 (d) 78

24. 9134 – 8623 = ?
(a) 723 (b) 511
(c) 923 (d) 634

25. 8362 – 6969 = ?
(a) 1921 (b) 1741
(c) 1393 (d) 1419

26. 2345 – 1069 = ?
(a) 1521 (b) 1841
(c) 1276 (d) 1319

27. 4475 – 1389 = ?
(a) 4021 (b) 3086
(c) 3831 (d) 3119

28. 32624 – 4569 = ?
(a) 28055 (b) 29341
(c) 28631 (d) 27119

29. 54218 – 34642 = ?
(a) 18544 (b) 19641
(c) 19576 (d) 19526

30. 82453 – 27878 = ?
(a) 54203 (b) 54575
(c) 52331 (d) 59219

Type-3 (Find Difference)

31. 9666 – 335 – 234 – 423 – 97 = ?
(a) 8345 (b) 8577
(c) 8766 (d) 8345

32. 8245–2352–292–2423–597 = ?
(a) 3345 (b) 3477
(c) 2581 (d) 2845

33. 7252–2335–3234–423–1197 = ?
(a) 45 (b) 177
(c) 63 (d) 103

34. 6234 – 2114 – 1234 – 1423 – 997 = ?
(a) 345 (b) 536
(c) 663 (d) 466

35. 1968 – 126 – 383 – 393 – 523 = ?
(a) 345 (b) 543
(c) 726 (d) 835

36. 999 – 112 – 544 – 123 – 97 = ?
(a) 295 (b) 577
(c) 123 (d) 373

37. 8340 – 134 – 913 – 1344 – 3331 = ?
(a) 2618 (b) 3577
(c) 1766 (d) 2345

38. 4965 – 764 – 1765 – 953 – 1075 = ?
(a) 1045 (b) 1577
(c) 408 (d) 845

39. 8153 – 2974 – 1066 – 854 – 967 = ?
(a) 1345 (b) 2292
(c) 3766 (d) 3345

40. 8754 – 754 – 521 – 431 – 865 – 234 – 239 – 672 = ?
(a) 5038 (b) 4577
(c) 5766 (d) 4845

41. 4233 – 1035 + 231 – 423 + 497 + 234 – 342 + 628 – 922 = ?
(a) 3342 (b) 3101
(c) 4766 (d) 4345

42. 9476 – 3310 + 3134 – 833 – 257 + 133 – 932 + 442 = ?
(a) 7853 (b) 8577
(c) 8766 (d) 7345

43. 397 – 357 – 239 + 836 – 727 – 913 – 385 + 1370 + 923 = ?
(a) 845 (b) 877
(c) 905 (d) 395

ANSWER KEY - SUBTRACTION

1.	(c)	2.	(b)	3.	(c)	4.	(a)	5.	(c)	6.	(a)	7.	(c)	8.	(c)	9.	(b)	10.	(a)
11.	(a)	12.	(d)	13.	(c)	14.	(c)	15.	(b)	16.	(c)	17.	(b)	18.	(c)	19.	(c)	20.	(a)
21.	(c)	22.	(c)	23.	(c)	24.	(b)	25.	(c)	26.	(c)	27.	(b)	28.	(a)	29.	(c)	30.	(b)
31.	(b)	32.	(c)	33.	(c)	34.	(d)	35.	(b)	36.	(c)	37.	(a)	38.	(c)	39.	(b)	40.	(a)
41.	(b)	42.	(a)	43.	(c)														

MULTIPLICATION/)कि(×)

Type-1 (Find the Product)

1. 98 × 7 = ?
(a) 692 (b) 686
(c) 855 (d) 696

2. 87 × 9 = ?
(a) 793 (b) 783
(c) 781 (d) 883

3. 47 × 3 = ?
(a) 141 (b) 151
(c) 221 (d) 146

4. 67 × 4 = ?
(a) 258 (b) 268
(c) 255 (d) 218

5. 89 × 6 = ?
(a) 544 (b) 684
(c) 534 (d) 494

6. 97 × 8 = ?
(a) 734 (b) 776
(c) 556 (d) 866

7. 83 × 7 = ?
(a) 581 (b) 681
(c) 751 (d) 691

8. 78 × 6 = ?
(a) 578 (b) 468
(c) 558 (d) 698

9. 89 × 5 = ?
(a) 615 (b) 545
(c) 445 (d) 555

10. 167 × 7 = ?
(a) 1299 (b) 1169
(c) 1519 (d) 1099

11. 296 × 5 = ?
(a) 1480 (b) 1500
(c) 1640 (d) 1140

12. 798 × 8 = ?
(a) 6924 (b) 6384
(c) 6855 (d) 6694

13. 626 × 9 = ?
(a) 6124 (b) 5686
(c) 5634 (d) 5694

14. 297 × 6 = ?
(a) 1602 (b) 1782
(c) 1852 (d) 1692

15. 555 × 7 = ?
(a) 4695 (b) 4685
(c) 3885 (d) 3105

Type-2 (Find the Product)

16. 88 × 23 = ?
(a) 2224 (b) 2154
(c) 2024 (d) 2444

17. 56 × 74 = ?
(a) 4394 (b) 4294
(c) 4144 (d) 4444

18. 69 × 48 = ?
(a) 3312 (b) 3922
(c) 2922 (d) 3342

19. 97 × 59 = ?
(a) 6123 (b) 5723
(c) 5893 (d) 5373

20. 73 × 47 = ?
(a) 3641 (b) 3511
(c) 3901 (d) 3431

21. 25 × 68 = ?
(a) 1700 (b) 1540
(c) 1900 (d) 1440

22. 95 × 41 = ?
(a) 4125 (b) 3895
(c) 3325 (d) 4255

23. 71 × 59 = ?
(a) 4189 (b) 4419
(c) 3999 (d) 4109

24. 39 × 61 = ?
(a) 2229 (b) 2159
(c) 2379 (d) 2449

25. 57 × 83 = ?
(a) 4731 (b) 3911
(c) 4241 (d) 4111

26. 29 × 72 = ?
(a) 2228 (b) 2088
(c) 2828 (d) 2448

27. 68 × 42 = ?
(a) 2996 (b) 2106
(c) 2026 (d) 2856

28. 87 × 54 = ?
(a) 4698 (b) 5108
(c) 4908 (d) 5448

29. 65 × 85 = ?
(a) 6225 (b) 5525
(c) 5925 (d) 5445

30. 56 × 78 = ?
(a) 4368 (b) 5158
(c) 5028 (d) 4448

31. 23 × 17 = ?
(a) 381 (b) 371
(c) 391 (d) 401

32. 49 × 51 = ?
(a) 2399 (b) 2499
(c) 2489 (d) 2469

33. 97 × 103 = ?
(a) 9981 (b) 9891
(c) 9971 (d) 9991

34. 999 × 99 = ?
(a) 98901 (b) 99901
(c) 99801 (d) 98801

35. 85 × 75 = ?
(a) 6475 (b) 6325
(c) 6375 (d) 6425

ANSWER KEY - MULTIPLICATION

1.	(b)	2.	(b)	3.	(a)	4.	(b)	5.	(c)	6.	(b)	7.	(a)	8.	(b)	9.	(c)	10.	(b)
11.	(a)	12.	(b)	13.	(c)	14.	(b)	15.	(c)	16.	(c)	17.	(c)	18.	(a)	19.	(b)	20.	(d)
21.	(a)	22.	(b)	23.	(a)	24.	(c)	25.	(a)	26.	(b)	27.	(d)	28.	(a)	29.	(b)	30.	(a)
31.	(c)	32.	(b)	33.	(d)	34.	(a)	35.	(c)										

SQUARE AND CUBE/प)अद्र ुपऐर

Exercise– Type 1 (Find Square)

1. $(21)^2$ = ?
(a) 461 (b) 441
(c) 471 (d) 391

2. $(45)^2$ = ?
(a) 2755 (b) 3025
(c) 2025 (d) 2445

3. $(84)^2$ = ?
(a) 7806 (b) 7726
(c) 7476 (d) 7056

4. $(49)^2$ = ?
(a) 2401 (b) 2441
(c) 2471 (d) 2301

5. $(63)^2$ = ?
(a) 3969 (b) 4419
(c) 3719 (d) 3919

6. $(86)^2$ = ?
(a) 7616 (b) 7416
(c) 7396 (d) 7716

7. $(97)^2$ = ?
(a) 9409 (b) 9909
(c) 9099 (d) 9399

8. $(29)^2$ = ?
(a) 861 (b) 841
(c) 871 (d) 891

9. $(37)^2$ = ?
(a) 1509 (b) 1419
(c) 1369 (d) 1399

10. $(43)^2$ = ?
(a) 1849 (b) 1989
(c) 1809 (d) 1759

11. $(71)^2$ = ?
(a) 5111 (b) 5441
(c) 5041 (d) 5391

12. $(52)^2$ = ?
(a) 3344 (b) 3004
(c) 2004 (d) 2704

13. $(74)^2$ = ?
(a) 5476 (b) 4486
(c) 5716 (d) 4916

14. $(48)^2$ = ?
(a) 2214 (b) 2304
(c) 2716 (d) 2106

15. $(77)^2$ = ?
(a) 5619 (b) 6419
(c) 5929 (d) 4919

Exercise– Type 2

16. $(111)^2$ = ?
(a) 12001 (b) 12031
(c) 12321 (d) 12861

17. $(167)^2$ = ?
(a) 27889 (b) 25039
(c) 26399 (d) 28569

18. $(123)^2$ = ?
(a) 16239 (b) 16099
(c) 15129 (d) 15869

19. $(279)^2$ = ?
(a) 76001 (b) 75031
(c) 77841 (d) 78861

20. $(527)^2$ = ?
(a) 277729 (b) 282039
(c) 283299 (d) 312869

21. $(294)^2$ = ?
(a) 86436 (b) 88206
(c) 82326 (d) 82866

22. $(361)^2$ = ?
(a) 130091 (b) 130321
(c) 140321 (d) 132861

23. $(369)^2$ = ?
(a) 136161 (b) 122031
(c) 132321 (d) 132861

24. $(721)^2$ = ?
(a) 512001 (b) 519841
(c) 512321 (d) 512861

25. $(896)^2$ = ?
(a) 802816 (b) 812036
(c) 812526 (d) 802566

Exercise– Type 3

26. $(23)^3$ = ?
(a) 12167 (b) 12857
(c) 12127 (d) 12997

27. $(14)^3$ = ?
(a) 2144 (b) 2744
(c) 2694 (d) 2994

28. $(19)^3$ = ?
(a) 6969 (b) 6349
(c) 6859 (d) 6929

29. $(37)^3$ = ?
(a) 50653 (b) 52853
(c) 52123 (d) 51293

30. $(17)^3$ = ?
(a) 4163 (b) 4913
(c) 4123 (d) 4593

31. $(33)^3$ = ?
(a) 35937 (b) 35857
(c) 32127 (d) 32997

32. $(29)^3$ = ?
(a) 24389 (b) 24859
(c) 22129 (d) 22999

33. $(43)^3$ = ?
(a) 82167 (b) 79507
(c) 72127 (d) 81297

34. $(27)^3$ = ?
(a) 19163 (b) 19853
(c) 19123 (d) 19683

35. $(53)^3$ = ?
(a) 154478 (b) 147787
(c) 147755 (d) 148877

Exercise– Type 4

36. $8^2 \times 9$ = ?
(a) 516 (b) 576
(c) 676 (d) 556

37. $9^2 \times 18$ = ?
(a) 1528 (b) 1408
(c) 1458 (d) 1558

38. $12^2 \times 23$ = ?
(a) 3312 (b) 3762
(c) 3672 (d) 3052

39. $42^2 \times 4$ = ?
(a) 6616 (b) 6876
(c) 7056 (d) 7556

40. $39^2 \times 8$ = ?
(a) 12168 (b) 15768
(c) 16768 (d) 10568

41. $61^2 \times 3$ = ?
(a) 9163 (b) 11163
(c) 16763 (d) 15563

42. $33^2 \times 18$ = ?
(a) 19602 (b) 18762
(c) 19762 (d) 18562

43. $57^2 \times 8$ = ?
(a) 22162 (b) 25992
(c) 26762 (d) 20562

44. $67^2 \times 3$ = ?
(a) 13467 (b) 15767
(c) 14767 (d) 14567

45. $35^2 \times 47$ = ?
(a) 52165 (b) 55775
(c) 56765 (d) 57575

46. $41^2 \times 19$ = ?
(a) 32169 (b) 35769
(c) 31939 (d) 30569

47. $23^2 \times 37$ = ?
(a) 18163 (b) 19573
(c) 19763 (d) 18563

48. $38^2 \times 23$ = ?
(a) 32163 (b) 33212
(c) 33762 (d) 33563

49. $83^2 \times 23$ = ?
(a) 158447 (b) 133217
(c) 153767 (d) 163567

50. $78^2 \times 23$ = ?
(a) 132162 (b) 139932
(c) 133762 (d) 133562

Exercise– Type 5

51. $42^2 \times 18^2$ = ?
(a) 587776 (b) 571536
(c) 598846 (d) 527986

52. $28^2 \times 35^2$ = ?
(a) 998770 (b) 982240
(c) 960400 (d) 927980

53. $19^3 \times 12^2 = ?$
(a) 928776 (b) 987696
(c) 929884 (d) 927984

54. $24^3 \times 17^2 = ?$
(a) 3928774 (b) 3528224
(c) 3995136 (d) 3827984

55. $74^2 \times 14^3 = ?$
(a) 15528774 (b) 16128224
(c) 15629884 (d) 15026144

56. $81^2 \times 9^3 = ?$
(a) 4782969 (b) 4828229
(c) 4429889 (d) 5027984

57. $37^2 \times 16^3 = ?$
(a) 6028774 (b) 5607424
(c) 5529884 (d) 5227984

58. $43^2 \times 13^3 = ?$
(a) 4128773 (b) 4062253
(c) 3929883 (d) 4279843

59. $28^3 \times 18^3 = ?$
(a) 133287734
(b) 123282324
(c) 128024064
(d) 134237984

60. $37^3 \times 21^3 = ?$
(a) 428774463
(b) 428368224
(c) 424488423
(d) 469097433

Exercise– Type 6 (Find Squareroot)

61. Find square root of 441.
(a) 21 (b) 31
(c) 29 (d) 27

62. Find square root of 484.
(a) 26 (b) 22
(c) 32 (d) 42

63. Find square root of 529.
(a) 29 (b) 27
(c) 23 (d) 26

64. Find square root of 576.
(a) 26 (b) 34
(c) 28 (d) 24

65. Find square root of 1521.
(a) 39 (b) 31
(c) 37 (d) 41

66. Find square root of 1681.
(a) 31 (b) 41
(c) 39 (d) 49

67. Find square root of 3969.
(a) 53 (b) 67
(c) 63 (d) 59

68. Find square root of 5929.
(a) 73 (b) 79
(c) 76 (d) 77

69. Find square root of 9801.
(a) 99 (b) 101
(c) 91 (d) 89

70. Find square root of 8281.
(a) 89 (b) 91
(c) 99 (d) 81

71. Find square root of 7921.
(a) 89 (b) 81
(c) 99 (d) 91

72. Find square root of 4761.
(a) 79 (b) 71
(c) 81 (d) 69

73. Find square root of 5929.
(a) 71 (b) 73
(c) 77 (d) 67

74. Find square root of 10201.
(a) 93 (b) 99
(c) 91 (d) 101

75. Find square root of 11881.
(a) 109 (b) 111
(c) 101 (d) 107

76. Find square root of 12321.
(a) 101 (b) 111
(c) 121 (d) 107

77. Find square root of 39601.
(a) 191 (b) 201
(c) 199 (d) 189

78. Find square root of 40401.
(a) 189 (b) 191
(c) 199 (d) 201

79. Find square root of 14161.
(a) 119 (b) 121
(c) 111 (d) 117

80. Find square root of 1296.
(a) 46 (b) 36
(c) 39 (d) 33

81. Find square root of 2116.
(a) 36 (b) 56
(c) 46 (d) 66

82. Find square root of 3136.
(a) 66 (b) 46
(c) 59 (d) 56

83. Find square root of 4356.
(a) 66 (b) 56
(c) 54 (d) 44

84. Find square root of 5776.
(a) 74 (b) 76
(c) 66 (d) 64

85. Find square root of 7396.
(a) 82 (b) 88
(c) 86 (d) 84

Exercise– Type 7 (Find Cuberoot)

86. Find cube root of 1331.
(a) 11 (b) 12
(c) 13 (d) 14

87. Find cube root of 1728.
(a) 14 (b) 11
(c) 12 (d) 13

88. Find cube root of 2197.
(a) 11 (b) 13
(c) 12 (d) 14

89. Find cube root of 2744.
(a) 24 (b) 16
(c) 17 (d) 14

90. Find cube root of 3375.
(a) 15 (b) 17
(c) 25 (d) 19

91. Find cube root of 4096.
(a) 26 (b) 16
(c) 14 (d) 18

92. Find cube root of 4913.
(a) 27 (b) 13
(c) 17 (d) 19

93. Find cube root of 5832.
(a) 13 (b) 22
(c) 16 (d) 18

94. Find cube root of 6859.
(a) 19 (b) 17
(c) 13 (d) 18

95. Find cube root of 9261.
(a) 31 (b) 23
(c) 21 (d) 19

96. Find cube root of 10648.
(a) 26 (b) 22
(c) 24 (d) 28

97. Find cube root of 12167.
(a) 21 (b) 27
(c) 26 (d) 23

98. Find cube root of 13824.
(a) 24 (b) 21
(c) 22 (d) 26

99. Find cube root of 15625.
(a) 35 (b) 25
(c) 15 (d) 33

100. Find cube root of 17576.
(a) 36 (b) 32
(c) 26 (d) 38

101. Find cube root of 19683.
(a) 29 (b) 37
(c) 33 (d) 27

102. Find cube root of 21952.
(a) 28 (b) 38
(c) 26 (d) 29

103. Find cube root of 24389.
(a) 27 (b) 31
(c) 29 (d) 39

104. Find cube root of 29791.
(a) 41 (b) 31
(c) 37 (d) 39

105. Find cube root of 32768.
(a) 34 (b) 38
(c) 36 (d) 32

106. Find cube root of 35937.
(a) 33 (b) 31
(c) 27 (d) 29

107. Find cube root of 39304.
(a) 32 (b) 34
(c) 36 (d) 28

108. Find cube root of 42875.
(a) 45 (b) 42
(c) 29 (d) 35

109. Find cube root of 46656.
(a) 38 (b) 34
(c) 36 (d) 42

110. Find cube root of 50653.
(a) 41 (b) 37
(c) 39 (d) 38

Square root of irrational number

111. Find square root of $7+4\sqrt{3}$.
(a) $2+\sqrt{3}$ (b) $2-\sqrt{3}$
(c) $4+2\sqrt{3}$ (d) $4-2\sqrt{3}$

112. Find square root of $28+10\sqrt{3}$.
(a) $6+2\sqrt{2}$ (b) $5-\sqrt{3}$
(c) $5+\sqrt{3}$ (d) $6-2\sqrt{2}$

113. Find square root of $43-24\sqrt{3}$.
(a) $4-3\sqrt{3}$ (b) $5-3\sqrt{3}$
(c) $3\sqrt{3}-5$ (d) $3\sqrt{3}-4$

114. Find square root of $99-70\sqrt{2}$.
(a) $5\sqrt{2}-7$ (b) $7-5\sqrt{2}$
(c) $8-3\sqrt{2}$ (d) $6\sqrt{2}-5$

115. Find square root of $124+70\sqrt{3}$.
(a) $5\sqrt{3}+7$ (b) $6\sqrt{3}+7$
(c) $5\sqrt{3}+9$ (d) $6\sqrt{3}+5$

116. Find square root of $21-12\sqrt{3}$.
(a) $3\sqrt{3}-3$ (b) $2\sqrt{3}-3$
(c) $2\sqrt{3}-2$ (d) $3\sqrt{3}-2$

117. Find square root of $189+36\sqrt{5}$.
(a) $5\sqrt{5}+4$ (b) $6\sqrt{5}+3$
(c) $3\sqrt{5}+6$ (d) $9\sqrt{2}+2$

118. Find square root of $21+4\sqrt{5}$.
(a) $2\sqrt{5}+1$ (b) $3\sqrt{5}+1$
(c) $2\sqrt{5}+3$ (d) $3\sqrt{5}+3$

119. Find square root of $86-60\sqrt{2}$.
(a) $5\sqrt{2}-7$ (b) $6\sqrt{2}-5$
(c) $5\sqrt{2}-6$ (d) $6\sqrt{2}-7$

120. Find square root of $57+24\sqrt{3}$.
(a) $4\sqrt{3}+2$ (b) $3\sqrt{3}+4$
(c) $4\sqrt{3}+5$ (d) $4\sqrt{3}+3$

ANSWER KEY - SQUARE AND CUBE

1.	(b)	2.	(c)	3.	(d)	4.	(a)	5.	(a)	6.	(c)	7.	(a)	8.	(b)	9.	(c)	10.	(a)
11.	(c)	12.	(d)	13.	(a)	14.	(b)	15.	(c)	16.	(c)	17.	(a)	18.	(c)	19.	(c)	20.	(a)
21.	(a)	22.	(b)	23.	(a)	24.	(b)	25.	(a)	26.	(a)	27.	(b)	28.	(c)	29.	(a)	30.	(b)
31.	(a)	32.	(a)	33.	(b)	34.	(d)	35.	(d)	36.	(b)	37.	(c)	38.	(a)	39.	(c)	40.	(a)
41.	(b)	42.	(a)	43.	(b)	44.	(a)	45.	(d)	46.	(c)	47.	(b)	48.	(b)	49.	(a)	50.	(b)
51.	(b)	52.	(c)	53.	(b)	54.	(c)	55.	(d)	56.	(a)	57.	(b)	58.	(b)	59.	(c)	60.	(d)
61.	(a)	62.	(b)	63.	(c)	64.	(d)	65.	(a)	66.	(b)	67.	(c)	68.	(d)	69.	(a)	70.	(b)
71.	(a)	72.	(d)	73.	(c)	74.	(d)	75.	(a)	76.	(b)	77.	(c)	78.	(d)	79.	(a)	80.	(b)
81.	(c)	82.	(d)	83.	(a)	84.	(b)	85.	(c)	86.	(a)	87.	(c)	88.	(b)	89.	(d)	90.	(a)
91.	(b)	92.	(c)	93.	(d)	94.	(a)	95.	(c)	96.	(b)	97.	(d)	98.	(a)	99.	(b)	100.	(c)
101.	(d)	102.	(a)	103.	(c)	104.	(b)	105.	(d)	106.	(a)	107.	(b)	108.	(d)	109.	(c)	110.	(b)
111.	(a)	112.	(c)	113.	(d)	114.	(a)	115.	(a)	116.	(b)	117.	(b)	118.	(a)	119.	(c)	120.	(d)

DIVISION/ पसर /सर)अ(÷)

Exercise– Type-1

1. 120 ÷ 8 = ?
(a) 10 (b) 11
(c) 15 (d) 13

2. 621 ÷ 27 = ?
(a) 20 (b) 23
(c) 25 (d) 21

3. 1494 ÷ 83 = ?
(a) 15 (b) 19
(c) 18 (d) 12

4. 1178 ÷ 31 = ?
(a) 40 (b) 44
(c) 38 (d) 48

5. 688 ÷ 86 = ?
(a) 10 (b) 11
(c) 8 (d) 9

6. 693 ÷ 99 = ?
(a) 7 (b) 8
(c) 9 (d) 10

7. 444 ÷ 37 = ?
(a) 10 (b) 11
(c) 12 (d) 13

8. 301 ÷ 7 = ?
(a) 42 (b) 43
(c) 45 (d) 51

9. 496 ÷ 16 = ?
(a) 30 (b) 31
(c) 45 (d) 33

10. 688 ÷ 43 = ?
(a) 14 (b) 16
(c) 15 (d) 13

11. 672 ÷ 16 = ?
(a) 40 (b) 41
(c) 42 (d) 43

12. 322 ÷ 7 = ?
(a) 47 (b) 41
(c) 55 (d) 46

13. 2491 ÷ 47 = ?
(a) 63 (b) 61
(c) 55 (d) 53

14. 2208 ÷ 23 = ?
(a) 98 (b) 96
(c) 86 (d) 106

15. 3192 ÷ 152 = ?
(a) 20 (b) 21
(c) 25 (d) 23

Exercise– Type-2

16. 15 ÷ 10 = ?
(a) 1.2 (b) 1.4
(c) 1.5 (d) 1.6

17. 117 ÷ 4 = ?
(a) 29.95 (b) 29.25
(c) 25.45 (d) 23.85

18. 448 ÷ 25 = ?
(a) 14.67 (b) 15.62
(c) 17.92 (d) 18.82

19. 103 ÷ 8 = ?
(a) 10.875 (b) 11.975
(c) 12.875 (d) 13.235

20. 735 ÷ 75 = ?
(a) 10.1 (b) 11.8
(c) 9.8 (d) 11.1

21. 111 ÷ 11 = ?
(a) 10.09 (b) 11.01
(c) 12.09 (d) 13.01

22. 112 ÷ 12 = ?
(a) 10.33 (b) 11.33
(c) 9.33 (d) 13.33

23. 97 ÷ 9 = ?
(a) 10.77 (b) 11.33
(c) 15.77 (d) 13.33

24. 357 ÷ 35 = ?
(a) 10.2 (b) 11.2
(c) 15.2 (d) 13.2

25. 58 ÷ 11 = ?
(a) 4.68 (b) 7.23
(c) 5.27 (d) 6.63

26. 143 ÷ 4 = ?
(a) 33.65 (b) 35.75
(c) 37.95 (d) 38.98

27. 106 ÷ 11 = ?
(a) 10.26 (b) 11.47
(c) 9.63 (d) 10.85

28. 95 ÷ 7 = ?
(a) 10.47 (b) 11.74
(c) 15.47 (d) 13.57

29. 156 ÷ 36 = ?
(a) 5.33 (b) 4.33
(c) 4.85 (d) 4.80

30. 286 ÷ 18 = ?
(a) 18 (b) 16.18
(c) 15.88 (d) 17.98

31. 999999 ÷ 1001
(a) 989 (b) 899
(c) 999 (d) 909

32. 256256 ÷ 1001 = ?
(a) 236 (b) 256
(c) 246 (d) 266

33. 9999 ÷ 101 = ?
(a) 99 (b) 189
(c) 101 (d) 109

34. 2424 ÷ 101 = ?
(a) 26 (b) 24
(c) 34 (d) 27

35. 99 ÷ 0.1 = ?
(a) 980 (b) 1099
(c) 990 (d) 1090

36. 199 ÷ 0.5 = ?
(a) 398 (b) 399
(c) 385 (d) 395

37. 355355 ÷ 91 = ?
(a) 3901 (b) 3905
(c) 3906 (d) 3910

38. 3444344 ÷ 1001 = ?
(a) 366 (b) 346
(c) 344 (d) 354

39. 999999 ÷ 99 = ?
(a) 10100 (b) 10101
(c) 11001 (d) 10111

40. 9801 ÷ 99 = ?
(a) 98 (b) 69
(c) 99 (d) 101

ANSWER KEY - DIVISION

1.	(c)	2.	(b)	3.	(c)	4.	(c)	5.	(c)	6.	(a)	7.	(c)	8.	(b)	9.	(b)	10.	(b)
11.	(c)	12.	(d)	13.	(d)	14.	(b)	15.	(b)	16.	(c)	17.	(b)	18.	(c)	19.	(c)	20.	(c)
21.	(a)	22.	(c)	23.	(c)	24.	(a)	25.	(c)	26.	(b)	27.	(c)	28.	(d)	29.	(b)	30.	(c)
31.	(c)	32.	(b)	33.	(a)	34.	(b)	35.	(c)	36.	(a)	37.	(b)	38.	(c)	39.	(b)	40.	(c)

DECIMAL/मक्ती एअ(.)

Exercise– Type-1

1. 12.32 ÷ 4 = ?
(a) 3.8 (b) 3.08
(c) 3.24 (d) 3.88

2. 52.56 ÷ 5 = ?
(a) 10.51 (b) 11.41
(c) 12.71 (d) 12.08

3. 95.95 ÷ 4 = ?
(a) 23.90 (b) 23.08
(c) 23.99 (d) 33.88

4. 172.6 ÷ 33 = ?
(a) 5.83 (b) 5.23
(c) 6.23 (d) 4.53

5. 221 ÷ 52 = ?
(a) 4.50 (b) 4.25
(c) 5.24 (d) 5.18

6. 233.9 ÷ 15 = ?
(a) 15.59 (b) 16.08
(c) 18.29 (d) 10.78

7. 65.9 ÷ 18 = ?
(a) 3.8 (b) 3.08
(c) 3.66 (d) 3.88

8. 111.11 ÷ 19 = ?
(a) 5.84 (b) 5.98
(c) 4.24 (d) 4.48

9. 56.89 ÷ 6.6 = ?
(a) 8.33 (b) 8.61
(c) 8.24 (d) 7.89

10. 586.95 ÷ 18.9 = ?
(a) 31.05 (b) 30.08
(c) 32.24 (d) 33.88

11. 279.63 ÷ 83.5 = ?
(a) 3.96 (b) 3.78
(c) 3.34 (d) 3.26

12. 526.95 ÷ 16.8 = ?
(a) 31.36 (b) 30.06
(c) 35.96 (d) 32.16

13. 97.23 ÷ 12.17 = ?
(a) 6.88 (b) 8.28
(c) 7.98 (d) 7.88

14. 9.732 ÷ 2.23 = ?
(a) 4.36 (b) 5.39
(c) 5.24 (d) 4.58

15. 98.29 ÷ 27.37 = ?
(a) 3.85 (b) 3.59
(c) 3.26 (d) 3.81

Exercise– Type-2

16. 61.9 × 5 = ?
(a) 300.3 (b) 309.5
(c) 305.5 (d) 300.45

17. **256.93 × 12 = ?**
(a) 3080.33 (b) 3083.76
(c) 3088.58 (d) 3083.16

18. 117.95 × 19 = ?
(a) 2241.05 (b) 2204.73
(c) 2009.51 (d) 3109.40

19. 98.92 × 23 = ?
(a) 2909.35 (b) 3197.57
(c) 2275.16 (d) 2906.75

20. 96.56 × 34.5 = ?
(a) 3222.43 (b) 3331.32
(c) 3223.48 (d) 2332.45

21. 72.6 × 13.59 = ?
(a) 980.27 (b) 986.63
(c) 1005.57 (d) 1100.49

22. 447.39 × 19 = ?
(a) 9100.36 (b) 8990.58
(c) 8500.41 (d) 8300.45

23. 59.26 × 63.29 = ?
(a) 3750.56 (b) 3809.59
(c) 3305.59 (d) 3340.45

24. 579.9 × 13 = ?
(a) 7900.30 (b) 7889.50
(c) 7772.70 (d) 7538.7

25. 187.43 × 47 = ?
(a) 8838.39 (b) 8809.21
(c) 8923.58 (d) 8694.45

26. 88.26 × 23.8 = ?
(a) 2100.58 (b) 2286.59
(c) 2305.56 (d) 2300.95

27. 98.19 × 39 = ?
(a) 3709.37 (b) 3928.59
(c) 3829.41 (d) 3899.45

28. 93.88 × 29.2 = ?
(a) 2945.38 (b) 2856.37
(c) 2803.59 (d) 2741.29

29. 59.73 × 23.47 = ?
(a) 1800.39 (b) 1401.86
(c) 1305.53 (d) 1600.45

30. 249.83 × 28.8 = ?
(a) 7195.10 (b) 7809.54
(c) 7748.59 (d) 8174.43

Exercise– Type-3

31. 98.26 – 56.59 = ?
(a) 40.83 (b) 41.67
(c) 45.87 (d) 42.82

32. 59.036 – 26.685 = ?
(a) 30.831 (b) 31.671
(c) 32.351 (d) 32.823

33. 70.26 – 17.63 = ?
(a) 52.63 (b) 51.63
(c) 55.83 (d) 52.83

34. 1209.68 – 455.93 = ?
(a) 740.83 (b) 741.63
(c) 745.87 (d) 753.75

35. 999.19 – 589.65 = ?
(a) 409.54 (b) 410.67
(c) 405.85 (d) 402.87

36. 1256 – 125.63 = ?
(a) 1130.37 (b) 1128.72
(c) 1357.08 (d) 1234.85

37. 85.95 – 8.236 = ?
(a) 79.832 (b) 78.672
(c) 77.714 (d) 72.822

38. 536.56 – 43.86 + 23.95 – 455.95 = ?
(a) 68.83 (b) 61.67
(c) 65.87 (d) 60.70

39. 256.364 – 59.11 + 94.023 – 109.26 = ?
(a) 180.832 (b) 182.017
(c) 145.872 (d) 192.823

40. 23.62 – 586.95 – 59.62 + 256.95 + 903.49 = ?
(a) 549.83 (b) 537.49
(c) 545.87 (d) 542.82

41. 737 – 56.03 + 245.97 – 547.08 + 197.63 + 25.25 = ?
(a) 590.83 (b) 610.67
(c) 602.74 (d) 612.82

42. 10.97 + 239 + 29.089 – 187.22 + 63.87 – 26.91 = ?
(a) 180.836 (b) 241.674
(c) 145.873 (d) 128.799

43. 591.26 – 67.95 + 15.98 – 29.54 + 3.26 – 9.38 – 123.45 = ?
(a) 375.63 (b) 380.18
(c) 345.87 (d) 349.82

ANSWER KEY - DECIMAL

1.	(b)	2.	(a)	3.	(c)	4.	(b)	5.	(b)	6.	(a)	7.	(c)	8.	(a)	9.	(b)	10.	(a)
11.	(c)	12.	(a)	13.	(c)	14.	(a)	15.	(b)	16.	(b)	17.	(d)	18.	(a)	19.	(c)	20.	(b)
21.	(b)	22.	(c)	23.	(a)	24.	(d)	25.	(b)	26.	(a)	27.	(c)	28.	(d)	29.	(b)	30.	(a)
31.	(b)	32.	(c)	33.	(a)	34.	(d)	35.	(a)	36.	(a)	37.	(c)	38.	(d)	39.	(b)	40.	(b)
41.	(c)	42.	(d)	43.	(b)														

SURDS AND INDICES/ र्स र्य अद्रुपअ पबस्न

Exercise– Type-1

1. $(2 \times \sqrt{5})^2 = ?$
(a) 25 (b) 24
(c) 20 (d) 22

2. $6 \times (\sqrt{81})^2 = ?$
(a) 485 (b) 486
(c) 480 (d) 482

3. $5 \times (\sqrt{36})^2 = ?$
(a) 180 (b) 184
(c) 170 (d) 122

4. $9 \times (\sqrt{12})^2 = ?$
(a) 110 (b) 108
(c) 120 (d) 98

5. $7 \times (\sqrt{19})^2 = ?$
(a) 135 (b) 124
(c) 133 (d) 122

6. $15 \times (\sqrt{18})^2 = ?$
(a) 270 (b) 240
(c) 250 (d) 262

7. $21 \times \sqrt{10})^2 = ?$
(a) 250 (b) 220
(c) 210 (d) 222

8. $7 \times (\sqrt{31})^2 = ?$
(a) 215 (b) 214
(c) 212 (d) 217

9. $5 \times (\sqrt{50})^2 = ?$
(a) 250 (b) 240
(c) 200 (d) 220

10. $7 \times (\sqrt{49})^2 = ?$
(a) 325 (b) 343
(c) 329 (d) 322

11. $7 \times (\sqrt{121})^2 = ?$
(a) 885 (b) 829
(c) 847 (d) 882

12. $9 \times (\sqrt{64})^2 = ?$
(a) 576 (b) 524
(c) 560 (d) 582

13. $12 \times (\sqrt{100})^2 = ?$
(a) 1225 (b) 1200
(c) 1220 (d) 1222

14. $3 \times (\sqrt{99})^2 = ?$
(a) 325 (b) 297
(c) 230 (d) 252

15. $13 \times (\sqrt{43})^2 = ?$
(a) 559 (b) 524
(c) 573 (d) 527

Exercise– Type-2

16. $(\sqrt{7})^2 + (\sqrt{4})^2 = ?$
(a) $\sqrt{11}$ (b) 11
(c) $\sqrt{28}$ (d) 28

17. $(\sqrt{11})^2 - (\sqrt{7})^2 = ?$
(a) $\sqrt{14}$ (b) 4
(c) $\sqrt{77}$ (d) 77

18. $(\sqrt{13})^2 + (\sqrt{34})^2 = ?$
(a) $\sqrt{47}$ (b) 47
(c) $\sqrt{48}$ (d) 48

19. $(8 \times \sqrt{7})^2 \div (\sqrt{4})^2 = ?$
(a) $\sqrt{112}$ (b) 112
(c) $\sqrt{118}$ (d) 118

20. $(7 \times \sqrt[3]{125})^2 \times (9 \div 35) = ?$
(a) 321 (b) 315
(c) 330 (d) 320

21. $\sqrt{(125 \times \sqrt[3]{64}) \times (4 \div 5)} = ?$
(a) 14 (b) 12
(c) 8 (d) 20

22. $(2 \times \sqrt{2})^2 + \sqrt{64} = ?$
(a) 14 (b) 20
(c) 18 (d) 16

23. $17 \times \sqrt{36} \div 3 = ?$
(a) 34 (b) 38
(c) 28 (d) 36

24. $(7 \times \sqrt{5})^2 \div (49 \times \sqrt{9}) = ?$
(a) 1÷5 (b) 5÷3
(c) 5÷8 (d) 3÷8

25. $(5)^3 \times \sqrt{25} \div \sqrt[3]{125} = ?$
(a) 121 (b) 135
(c) 155 (d) 125

26. $\sqrt{(36 \times 16)} + \sqrt{(5^2 + 11)} = ?$
(a) 16 (b) 36
(c) 30 (d) 28

27. $(8 \times \sqrt{6})^2 \div (4)^2 = ?$
(a) 32 (b) 20
(c) 24 (d) 27

28. $\sqrt{121} \times 7 \div \sqrt{16} = ?$
(a) 121÷2 (b) 77÷4
(c) 70÷2 (d) 16

29. $(\sqrt{5})^2 + (2 \times \sqrt{2})^2 - 7 \times \sqrt{(16 \div 49)} = ?$
(a) 11 (b) 14
(c) 9 (d) 10

30. $(\sqrt{37})^3 \div \sqrt{37} + (8 \times \sqrt{2})^2 = ?$
(a) 155 (b) 165
(c) 175 (d) 170

Exercise– Type-3

31. $\sqrt{6.24} \times \sqrt{1.70} = ?$
(a) 3.70 (b) 4.60
(c) 4.58 (d) 3.25

32. $\sqrt[3]{3.375} \times \sqrt{0.49} = ?$
(a) 1.05 (b) 2.20
(c) 1.23 (d) 1.21

33. $\sqrt[3]{1.331} \times \sqrt{0.36} = ?$
(a) 0.79 (b) 0.66
(c) 0.53 (d) 0.75

34. $8 \times \sqrt[3]{0.064} = ?$
(a) 7.9 (b) 6.6
(c) 2.3 (d) 3.2

35. $\sqrt{3} \times \sqrt{0.216} = ?$
(a) 1.05 (b) 1.08
(c) 1.03 (d) 1.75

36. $\sqrt{1.20} \times \sqrt[3]{1.330} = ?$
(a) 0.79 (b) 0.66
(c) 2.24 (d) 0.75

37. $\sqrt[3]{1.331} \times \sqrt{0.36} = ?$
(a) 1.21 (b) 0.66
(c) 1.51 (d) 1.15

38. $\sqrt[3]{0.026} \times \sqrt{2.88} = ?$
(a) 1.5 (b) 0.51
(c) 2 (d) 0.75

39. $(3 \times \sqrt{2}) \times \sqrt[3]{1.728} = ?$
(a) 7.29 (b) 6.96
(c) 5.10 (d) 5.50

40. $\sqrt[3]{1331} \times \sqrt{0.36} = ?$
(a) 0.79 (b) 0.66
(c) 0.53 (d) 0.75

41. $\sqrt[3]{0.520} = ?$
(a) 0.804 (b) 0.876
(c) 0.853 (d) 0.825

42. $\sqrt[3]{0.008} \times \sqrt{0.81} = ?$
(a) 0.20 (b) 0.18
(c) 0.51 (d) 0.25

ANSWER KEY - SURDS AND INDICES

1.	(c)	2.	(b)	3.	(a)	4.	(b)	5.	(c)	6.	(a)	7.	(c)	8.	(d)	9.	(a)	10.	(b)
11.	(c)	12.	(a)	13.	(b)	14.	(b)	15.	(a)	16.	(b)	17.	(b)	18.	(b)	19.	(b)	20.	(b)
21.	(d)	22.	(d)	23.	(a)	24.	(b)	25.	(d)	26.	(c)	27.	(c)	28.	(b)	29.	(c)	30.	(b)
31.	(a)	32.	(b)	33.	(b)	34.	(d)	35.	(c)	36.	(c)	37.	(b)	38.	(c)	39.	(c)	40.	(b)

LCM AND HCF/ि ंप्रं णं अ ुपअंप्रं णं

Exercise– Type-1

1. What is the LCM of 12, 18, and 24?
(a) 12 (b) 24
(c) 36 (d) 72

2. What is the LCM of 15, 20, and 25?
(a) 100 (b) 200
(c) 300 (d) 400

3. What is the LCM of 8, 12, and 16?
(a) 16 (b) 24
(c) 48 (d) 96

4. What is the LCM of 21, 28, and 35?
(a) 210 (b) 420
(c) 630 (d) 840

5. What is the LCM of 10, 12, and 15?
(a) 30 (b) 60
(c) 120 (d) 180

Find the LCM

6. LCM of $\frac{1}{4}$, $\frac{2}{3}$, and $\frac{5}{6}$ = ?
(a) $\frac{1}{12}$ (b) $\frac{5}{12}$
(c) $\frac{10}{1}$ (d) $\frac{5}{36}$

7. LCM of $\frac{3}{4}$, $\frac{1}{5}$, and $\frac{7}{8}$ = ?
(a) $\frac{105}{32}$ (b) $\frac{21}{8}$
(c) $\frac{21}{1}$ (d) $\frac{21}{10}$

8. LCM of $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$ = ?
(a) $\frac{4}{5}$ (b) $\frac{3}{4}$
(c) 1 (d) $\frac{12}{1}$

9. LCM of $\frac{5}{6}$, $\frac{1}{2}$, and $\frac{3}{4}$ = ?
(a) $\frac{5}{8}$ (b) $\frac{5}{12}$
(c) $\frac{15}{2}$ (d) $\frac{5}{18}$

10. LCM of $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ = ?
(a) $\frac{3}{8}$ (b) $\frac{3}{4}$
(c) $\frac{1}{2}$ (d) $\frac{6}{1}$

11. LCM of $1\frac{1}{2}$, $2\frac{2}{3}$ and $3\frac{3}{4}$ = ?
(a) 9 (b) 18/1
(c) 27 (d) 120/1

12. LCM of $\frac{2}{3}$, $3\frac{3}{4}$, and $\frac{5}{6}$ = ?
(a) $\frac{1}{4}$ (b) $\frac{30}{1}$
(c) $\frac{5}{12}$ (d) $\frac{5}{6}$

13. LCM of $1\frac{2}{3}$, $2\frac{1}{4}$, and $3\frac{1}{5}$ = ?
(a) $8\frac{5}{6}$ (b) $9\frac{1}{10}$
(c) $10\frac{1}{15}$ (d) $\frac{720}{1}$

Exercise– Type-2
Find HCF

14. HCF of 12, 18, and 24 = ?
(a) 2 (b) 3
(c) 4 (d) 6

15. HCF of 20, 25, and 35 = ?
(a) 5 (b) 10
(c) 15 (d) 20

16. HCF of 36, 54, and 90 = ?
(a) 6 (b) 9
(c) 18 (d) 27

17. HCF of 72, 96, and 120 = ?
(a) 12 (b) 24
(c) 36 (d) 48

18. HCF of 15, 35, and 45 = ?
(a) 3 (b) 5
(c) 15 (d) 45

19. HCF of 1232, 1848, and 2624 = ?
(a) 8 (b) 32
(c) 64 (d) 128

20. HCF of 369, 492, and 615 = ?
(a) 3 (b) 123
(c) 27 (d) 81

21. HCF of 640, 800, and 960 = ?
(a) 40 (b) 80
(c) 160 (d) 320

22. HCF of 1260, 1800, and 2310 = ?
(a) 90 (b) 180
(c) 30 (d) 360

23. HCF of 1764, 2352, and 2940 = ?
(a) 84 (b) 168
(c) 252 (d) 336

24. HCF of $\frac{3}{8}$, $\frac{5}{12}$, and $\frac{7}{24}$ = ?
(a) $\frac{1}{24}$ (b) $\frac{7}{12}$
(c) $\frac{5}{24}$ (d) $\frac{7}{8}$

25. HCF of $\frac{2}{5}$, $\frac{1}{3}$, and $\frac{3}{10}$ = ?
(a) $\frac{1}{2}$ (b) $\frac{3}{5}$
(c) $\frac{3}{10}$ (d) $\frac{1}{30}$

26. HCF of $2\frac{1}{4}$, $3\frac{1}{2}$, and $4\frac{1}{3}$ = ?
(a) $11\frac{1}{12}$ (b) $\frac{1}{12}$
(c) $18\frac{1}{4}$ (d) $19\frac{1}{2}$

27. HCF of $1\frac{3}{4}$, $2\frac{1}{3}$, and $3\frac{1}{5}$ = ?
(a) $7\frac{1}{2}$ (b) $9\frac{1}{2}$
(c) $\frac{1}{60}$ (d) $15\frac{3}{4}$

28. HCF of $3\frac{1}{2}$, $4\frac{3}{4}$, and $5\frac{1}{6}$ = ?
(a) $31\frac{1}{12}$ (b) $33\frac{1}{2}$
(c) $35\frac{1}{12}$ (d) $\frac{1}{12}$

29. HCF of $2\frac{3}{8}$, $3\frac{1}{5}$, and $4\frac{1}{6}$ = ?
(a) $\frac{1}{120}$ (b) $38\frac{5}{24}$
(c) $40\frac{5}{24}$ (d) $42\frac{3}{8}$

ANSWER KEY - LCM AND HCF

1.	(d)	2.	(c)	3.	(c)	4.	(b)	5.	(b)	6.	(c)	7.	(c)	8.	(d)	9.	(c)	10.	(d)
11.	(d)	12.	(b)	13.	(d)	14.	(d)	15.	(a)	16.	(c)	17.	(b)	18.	(b)	19.	(a)	20.	(b)
21.	(c)	22.	(c)	23.	(a)	24.	(a)	25.	(d)	26.	(b)	27.	(c)	28.	(d)	29.	(a)		

FRACTION/ भ

Exercise– Type-1

1. $\frac{1+2}{3} = ?$

(a) $\frac{4}{3}$ (b) $\frac{5}{3}$

(c) $\frac{4}{5}$ (d) $\frac{3}{5}$

2. $\frac{22}{7} = ? + \frac{1}{7}$

(a) 20 (b) 23

(c) 3 (d) 5

3. $\frac{90}{7} - \frac{6}{7} = ?$

(a) 12 (b) 23

(c) 15 (d) 25

4. $\frac{17}{36} = 1 - ?$

(a) $\frac{4}{30}$ (b) $\frac{19}{36}$

(c) $\frac{4}{6}$ (d) $\frac{3}{18}$

5. $\frac{19}{3} = \frac{7}{3} + ?$

(a) 4 (b) 5

(c) 3 (d) 7

6. $\frac{28}{65} = 2 - ?$

(a) $\frac{4}{65}$ (b) $\frac{102}{65}$

(c) $\frac{105}{65}$ (d) $\frac{3}{65}$

7. $\frac{22}{45} = 1 - \frac{23}{?}$

(a) 23 (b) 53

(c) 45 (d) 35

8. $\frac{67}{3} + \frac{2}{3} = ?$

(a) 43 (b) 53

(c) 23 (d) 35

9. $\frac{46}{7} = 6 + ?$

(a) 7 (b) $\frac{5}{7}$

(c) $\frac{4}{7}$ (d) $\frac{3}{7}$

10. $\frac{45}{6} = 7 + ?$

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{6}$ (d) $\frac{1}{7}$

Exercise– Type-2

11. $\frac{5}{8} \times 7 = ?$

(a) $4\frac{4}{3}$ (b) $3\frac{5}{3}$

(c) $3\frac{4}{5}$ (d) $4\frac{3}{8}$

12. $\frac{9}{5} \times 4 = ?$

(a) $7\frac{4}{3}$ (b) $6\frac{5}{3}$

(c) $7\frac{1}{5}$ (d) $5\frac{3}{5}$

13. $\frac{7}{9} \times 7 = ?$

(a) $5\frac{4}{3}$ (b) $5\frac{4}{9}$

(c) $5\frac{4}{5}$ (d) $5\frac{3}{5}$

14. $\frac{10}{7} \times 13 = ?$

(a) $18\frac{4}{30}$ (b) $8\frac{19}{36}$

(c) $16\frac{4}{6}$ (d) $18\frac{4}{7}$

15. $\frac{10}{19} \times 7 =$

(a) $16\frac{4}{30}$ (b) $7\frac{19}{36}$

(c) $16\frac{4}{6}$ (d) $3\frac{13}{19}$

16. $\frac{15}{4} \times 11 =$

(a) $18\frac{4}{7}$ (b) $41\frac{1}{4}$

(c) $\frac{4}{6}$ (d) $\frac{3}{18}$

17. $22 \times \frac{4}{5} = ?$

(a) 7 (b) $17\frac{5}{7}$

(c) $17\frac{3}{5}$ (d) $17\frac{3}{7}$

18. $27 \times \frac{5}{11} = ?$

(a) $12\frac{3}{11}$ (b) $17\frac{5}{7}$

(c) $17\frac{3}{5}$ (d) $17\frac{3}{7}$

19. $17 \times \frac{7}{5} = ?$

(a) $15\frac{4}{3}$ (b) $3\frac{5}{3}$

(c) $13\frac{4}{5}$ (d) $21\frac{4}{5}$

20. $23 \times \frac{3}{4} = ?$

(a) $17\frac{4}{3}$ (b) $17\frac{1}{4}$

(c) $17\frac{4}{5}$ (d) $21\frac{4}{5}$

Exercise– Type-3

21. $9\frac{1}{4} \times 12 = ?$

(a) 231 (b) 531

(c) 451 (d) 111

22. $4\frac{1}{11} \times 12 = ?$

(a) $49\frac{1}{11}$ (b) $8\frac{10}{11}$

(c) $16\frac{4}{6}$ (d) $18\frac{4}{7}$

23. $4\frac{1}{11} \times 12 = ?$

(a) $49\frac{1}{12}$ (b) $8\frac{10}{11}$

(c) $16\frac{4}{6}$ (d) $49\frac{1}{11}$

24. $4\frac{2}{3} \times 3 = ?$

(a) 21 (b) 13

(c) 14 (d) 16

25. $91\frac{3}{4} \times 8 = ?$

(a) 743 (b) 734

(c) 723 (d) 735

26. $102\frac{3}{7} \times 7 = ?$

(a) 743 (b) 734

(c) 717 (d) 735

27. $66\frac{2}{3} \times 3 = ?$

(a) 243 (b) 200

(c) 217 (d) 235

28. $11\frac{1}{9} \times 10 = ?$

(a) $49\frac{1}{12}$ (b) $8\frac{10}{11}$

(c) $111\frac{1}{3}$ (d) $111\frac{1}{9}$

29. $7\frac{1}{3} \times 5 = ?$

(a) $12\frac{3}{11}$ (b) $36\frac{5}{7}$

(c) $17\frac{3}{5}$ (d) $36\frac{2}{3}$

30. $15\frac{2}{7} \times 4 = ?$

(a) $61\frac{1}{7}$ (b) $66\frac{5}{7}$

(c) $67\frac{3}{5}$ (d) $36\frac{2}{3}$

Exercise– Type-4

31. $\frac{95}{13} \times \frac{1}{5} = ?$

(a) $6\frac{1}{7}$ (b) $6\frac{5}{7}$

(c) $\frac{19}{13}$ (d) $\frac{12}{13}$

32. $\frac{126}{14} \times \frac{1}{5} = ?$

(a) $\frac{9}{5}$ (b) $\frac{5}{7}$

(c) $\frac{19}{13}$ (d) $\frac{12}{13}$

33. $\left(26\frac{2}{3}\right) \times \frac{7}{13} = ?$

(a) $14\frac{12}{13}$ (b) $16\frac{5}{7}$

(c) $67\frac{3}{13}$ (d) $14\frac{14}{39}$

34. $\left(33\frac{1}{3}\right) \times \frac{6}{11} = ?$

(a) $14\frac{12}{13}$ (b) $18\frac{5}{7}$

(c) $18\frac{2}{11}$ (d) $14\frac{14}{39}$

35. $\left(101\frac{4}{7}\right) \times \frac{5}{17} = ?$

(a) $14\frac{12}{13}$ (b) 29

(c) $28\frac{2}{119}$ (d) $29\frac{104}{119}$

36. $\left(150\frac{5}{8}\right) \times \frac{16}{17} = ?$

(a) $14\frac{12}{13}$ (b) $29\frac{13}{17}$

(c) $141\frac{13}{17}$ (d) $29\frac{104}{119}$

37. $\left(23\frac{3}{7}\right) \times \frac{14}{23} = ?$

(a) $14\frac{12}{13}$ (b) $1\frac{13}{17}$

(c) $14\frac{13}{17}$ (d) $14\frac{6}{23}$

38. $\left(93\frac{11}{12}\right) \times \frac{3}{31} = ?$

(a) $6\frac{11}{124}$ (b) $6\frac{5}{7}$

(c) $14\frac{13}{17}$ (d) $9\frac{11}{124}$

39. $14\frac{2}{5} \times \frac{5}{7} = ?$

(a) $10\frac{2}{7}$ (b) $10\frac{5}{7}$

(c) $10\frac{13}{17}$ (d) $9\frac{11}{124}$

40. $\left(55\frac{4}{7}\right) \times \frac{14}{15} = ?$

(a) $50\frac{2}{7}$ (b) $51\frac{13}{15}$

(c) $51\frac{13}{17}$ (d) $51\frac{11}{15}$

Exercise– Type-5

41. $\frac{9}{12} - \frac{5}{8} = ?$

(a) $\frac{9}{5}$ (b) $\frac{1}{8}$

(c) $\frac{19}{13}$ (d) $\frac{12}{13}$

42. $75\frac{1}{2} + 11\frac{5}{6} = ?$

(a) $87\frac{2}{6}$ (b) $87\frac{1}{8}$

(c) $80\frac{19}{13}$ (d) $87\frac{12}{13}$

43. $2\frac{1}{3} \times 1\frac{1}{14} \times 1\frac{4}{5} = ?$

(a) $4\frac{1}{2}$ (b) $4\frac{1}{8}$

(c) $2\frac{19}{13}$ (d) $1\frac{12}{13}$

44. $6\frac{2}{9} \div \frac{1}{6} = ?$

(a) $4\frac{1}{2}$ (b) $36\frac{1}{8}$

(c) $2\frac{19}{13}$ (d) $37\frac{1}{3}$

45. $43\frac{2}{3} + 36\frac{3}{9} = ?$

(a) 83 (b) 80

(c) 75 (d) 85

46. $2\frac{3}{4} \div 5\frac{9}{7} = ?$

(a) $\frac{9}{5}$ (b) $\frac{1}{8}$

(c) $\frac{7}{8}$ (d) $\frac{12}{13}$

47. $7\frac{3}{5} \div \frac{1}{20} = ?$

(a) 183 (b) 152

(c) 175 (d) 185

48. $3\frac{5}{6} \times 4\frac{2}{3} = ?$

(a) $6\frac{11}{12}$ (b) $6\frac{5}{7}$

(c) $14\frac{8}{9}$ (d) $17\frac{8}{9}$

49. $17\frac{3}{8} \div \frac{1}{16} = ?$

(a) 188 (b) 158

(c) 178 (d) 278

50. $33\frac{1}{3} \div 7\frac{1}{7} = ?$

(a) $4\frac{11}{12}$ (b) $4\frac{5}{7}$

(c) $4\frac{2}{3}$ (d) $17\frac{8}{9}$

ANSWER KEY - FRACTION

1.	(c)	2.	(b)	3.	(a)	4.	(b)	5.	(c)	6.	(a)	7.	(c)	8.	(c)	9.	(c)	10.	(*)
11.	(d)	12.	(c)	13.	(b)	14.	(d)	15.	(d)	16.	(b)	17.	(c)	18.	(a)	19.	(d)	20.	(b)
21.	(d)	22.	(a)	23.	(d)	24.	(c)	25.	(b)	26.	(c)	27.	(b)	28.	(d)	29.	(d)	30.	(*)
31.	(c)	32.	(a)	33.	(d)	34.	(c)	35.	(d)	36.	(c)	37.	(d)	38.	(d)	39.	(a)	40.	(b)
41.	(b)	42.	(a)	43.	(a)	44.	(d)	45.	(b)	46.	(c)	47.	(b)	48.	(d)	49.	(d)	50.	(c)

PERCENTAGE/प्रतिशत

Type-1 (Calculate the %)

1. 1% of 6.04 = ?
(a) 0.064 (b) 0.256
(c) 3.55 (d) 0.0604

2. 200% of 3.5 = ?
(a) 2.8 (b) 12
(c) 35 (d) 7

3. 50% of 122 = ?
(a) 28 (b) 70
(c) 61 (d) 50

4. 50% of 511 = ?
(a) 250.5 (b) 150.2
(c) 350.9 (d) 255.5

5. 10% of 37 = ?
(a) 2.8 (b) 1.2
(c) 3.7 (d) 3.5

6. 13% of 30 = ?
(a) 2.8 (b) 1.2
(c) 3.9 (d) 1.5

7. 15% of 80 = ?
(a) 10 (b) 12
(c) 20 (d) 15

8. 12% of 600 = ?
(a) 80 (b) 72
(c) 70 (d) 75

9. 18% of 500 = ?
(a) 80 (b) 12
(c) 20 (d) 90

10. 25% of 250 = ?
(a) 70 (b) 62.5
(c) 60 (d) 55.5

Type-2 (Convert into decimal)

11. $\frac{4}{5}$ = ?
(a) 0.08 (b) 0.8
(c) 0.6 (d) 0.7

12. $\frac{9}{11}$ = ?
(a) 0.08 (b) 0.81
(c) 0.61 (d) 0.71

13. $\frac{7}{13}$ = ?
(a) 0.51 (b) 0.81
(c) 0.53 (d) 0.71

14. $\frac{8}{15}$ = ?
(a) 0.51 (b) 0.81
(c) 0.53 (d) 0.61

15. $\frac{17}{6}$ = ?
(a) 2.51 (b) 2.81
(c) 2.53 (d) 2.83

16. $\frac{19}{21}$ = ?
(a) 0.51 (b) 0.80
(c) 0.83 (d) 0.90

17. $\frac{17}{9}$ = ?
(a) 1.51 (b) 1.80
(c) 1.88 (d) 1.90

18. $\frac{13}{12}$ = ?
(a) 1.51 (b) 1.80
(c) 1.88 (d) 1.08

19. $\frac{21}{25}$ = ?
(a) 0.84 (b) 1.80
(c) 0.88 (d) 1.08

20. $\frac{5}{7}$ = ?
(a) 0.51 (b) 0.70
(c) 0.71 (d) 0.90

Type-3 (Convert into fraction)

21. $33\frac{1}{3}\%$ = ?
(a) 7 (b) $\frac{1}{3}$
(c) $\frac{4}{7}$ (d) $\frac{3}{7}$

22. $66\frac{2}{3}\%$ = ?
(a) 7 (b) $\frac{1}{3}$
(c) $\frac{4}{7}$ (d) $\frac{2}{3}$

23. 34% = ?
(a) $\frac{17}{50}$ (b) $1\frac{1}{3}$
(c) $10\frac{4}{7}$ (d) $2\frac{2}{3}$

24. 65% = ?
(a) $\frac{17}{50}$ (b) $12\frac{1}{3}$
(c) $\frac{17}{20}$ (d) $\frac{13}{20}$

25. 85% = ?
(a) $\frac{17}{50}$ (b) $12\frac{1}{3}$
(c) $\frac{17}{20}$ (d) $\frac{13}{20}$

26. $42\frac{6}{7}\%$ = ?
(a) $\frac{17}{50}$ (b) $1\frac{1}{3}$
(c) $\frac{3}{7}$ (d) $\frac{13}{20}$

27. $142\frac{6}{7}\%$ = ?
(a) $\frac{10}{7}$ (b) $\frac{13}{20}$
(c) $\frac{3}{7}$ (d) $\frac{1}{3}$

28. $57\frac{1}{7}\%$ = ?
(a) $\frac{10}{7}$ (b) $\frac{13}{20}$
(c) $\frac{3}{7}$ (d) $\frac{4}{7}$

29. $81\frac{9}{11}\%$ = ?
(a) $\frac{10}{7}$ (b) $\frac{9}{11}$
(c) $\frac{3}{7}$ (d) $\frac{4}{7}$

30. $18\frac{3}{4}\%$ = ?
(a) $\frac{3}{16}$ (b) $\frac{9}{11}$
(c) $\frac{3}{7}$ (d) $\frac{4}{7}$

Type-4 (Convert in to Fraction)

31. 47.5% = ?
(a) $\frac{9}{11}$ (b) $\frac{3}{16}$
(c) $\frac{19}{40}$ (d) $\frac{2}{6}$

32. 21.21% = ?
(a) $\frac{9}{11}$ (b) $\frac{19}{90}$
(c) $\frac{19}{40}$ (d) $\frac{7}{33}$

33. 73.33% = ?

(a) $\frac{9}{11}$ (b) $\frac{19}{90}$

(c) $\frac{19}{40}$ (d) $\frac{11}{15}$

34. 72.5% = ?

(a) $\frac{9}{11}$ (b) $\frac{19}{90}$

(c) $\frac{29}{40}$ (d) $\frac{11}{15}$

35. 83.33% = ?

(a) $\frac{5}{6}$ (b) $\frac{4}{3}$

(c) $\frac{2}{4}$ (d) $\frac{11}{15}$

36. 100.90% = ?

(a) $\frac{112}{110}$ (b) $\frac{111}{110}$

(c) $\frac{112}{118}$ (d) $\frac{11}{15}$

37. 55.45% = ?

(a) $\frac{62}{110}$ (b) $\frac{64}{112}$

(c) $\frac{61}{110}$ (d) $\frac{11}{15}$

38. 34.28% = ?

(a) $\frac{12}{35}$ (b) $\frac{64}{112}$

(c) $\frac{61}{110}$ (d) $\frac{11}{15}$

39. 46.66% = ?

(a) $\frac{12}{35}$ (b) $\frac{9}{12}$

(c) $\frac{11}{10}$ (d) $\frac{7}{15}$

40. 81.4% = ?

(a) $\frac{57}{70}$ (b) $\frac{9}{12}$

(c) $\frac{11}{10}$ (d) $\frac{7}{15}$

Exercise–Type-5

41. 70% ↑ = ?

(a) $\frac{57}{70}$ (b) $\frac{17}{10}$

(c) $\frac{11}{10}$ (d) $\frac{7}{15}$

42. 60% ↑ = ?

(a) $\frac{8}{5}$ (b) $\frac{19}{90}$

(c) $\frac{19}{40}$ (d) $\frac{2}{6}$

43. 75% ↓ = ?

(a) $\frac{8}{5}$ (b) $\frac{3}{7}$

(c) $\frac{1}{4}$ (d) $\frac{2}{6}$

44. 25% ↓ = ?

(a) $\frac{8}{5}$ (b) $\frac{3}{4}$

(c) $\frac{1}{4}$ (d) $\frac{2}{6}$

45. 5% ↑ = ?

(a) $\frac{18}{14}$ (b) $\frac{20}{22}$

(c) $\frac{12}{14}$ (d) $\frac{21}{20}$

46. $11\frac{1}{9}$% ↑ = ?

(a) $\frac{18}{14}$ (b) $\frac{20}{22}$

(c) $\frac{12}{14}$ (d) $\frac{10}{9}$

47. $9\frac{1}{11}$% ↓ = ?

(a) $\frac{10}{11}$ (b) $\frac{20}{22}$

(c) $\frac{12}{14}$ (d) $\frac{10}{9}$

48. $8\frac{1}{3}$% ↓ = ?

(a) $\frac{10}{11}$ (b) $\frac{20}{22}$

(c) $\frac{11}{12}$ (d) $\frac{10}{9}$

49. $87\frac{1}{2}$% ↑ = ?

(a) $\frac{10}{11}$ (b) $\frac{20}{22}$

(c) $\frac{15}{8}$ (d) $\frac{10}{9}$

50. $44\frac{4}{9}$% ↑ = ?

(a) $\frac{10}{11}$ (b) $\frac{13}{9}$

(c) $\frac{15}{8}$ (d) $\frac{10}{9}$

Exercise–Type-6

51. $16\frac{2}{3}$% ↓ & 10% ↑ = ?

(a) $\frac{8}{5}$ (b) $\frac{1}{12}$

(c) $\frac{1}{4}$ (d) $\frac{2}{6}$

52. $11\frac{1}{9}$% ↑ & 20% ↑ = ?

(a) 32.33% (b) 44.33%
(c) 30.33% (d) 33.33%

53. $6\frac{1}{4}$% ↑ & $45\frac{5}{11}$% ↑ = ?

(a) 54.54% (b) 54.32%
(c) 40.20% (d) 50.52%

54. $12\frac{1}{2}$% ↑ & $37\frac{1}{2}$% ↑ = ?

(a) $\frac{18}{14}$ (b) $\frac{35}{64}$

(c) $\frac{12}{14}$ (d) $\frac{21}{20}$

55. 30% ↑ & $14\frac{2}{7}$% ↓ = ?

(a) 11.42% (b) 11.48%
(c) 40.20% (d) 20.52%

56. $11\frac{1}{9}$% ↓ & 25% ↑ = ?

(a) 11.42% (b) 11.48%
(c) 12.20% (d) 11.11%

57. $42\frac{6}{7}$% ↑ & 30% ↑ = ?

(a) $\frac{10}{11}$ (b) $\frac{20}{22}$

(c) $\frac{6}{7}$ (d) $\frac{10}{9}$

58. $55\frac{5}{9}$% ↑ & $28\frac{4}{7}$% ↑ = ?

(a) 100% (b) 110%
(c) 114% (d) 98%

59. $2\frac{1}{2}$% ↑ & 10% ↓ = ?

(a) 7.12% (b) 7.75%
(c) 8.90% (d) 8.80%

60. $57\frac{1}{7}$% ↑ & $16\frac{2}{3}$% ↑ = ?

(a) $\frac{5}{6}$ (b) $\frac{4}{3}$

(c) $\frac{2}{4}$ (d) $\frac{11}{15}$

Type-7 (Find net increase%)

61. 10% ↑ and 5% ↑ = ?
(a) 13.5% (b) 15.8%
(c) 15.5% (d) 16.5%

62. 10% ↑ and 20% ↑ = ?
(a) 31% (b) 32%
(c) 34% (d) 35%

63. 8% ↑ and 5% ↑ = ?
(a) 13.20% (b) 12.40%
(c) 12.60% (d) 13.04%

64. 10% ↑ and 15% ↑ = ?
(a) 26.5% (b) 24.5%
(c) 27.5% (d) 23.5%

65. 10% ↑ and 10% ↑ = ?
(a) 99% (b) 23%
(c) 22% (d) 21%

66. 20% ↑ and 30% ↑ = ?
(a) 55% (b) 57%
(c) 56% (d) 54%

67. 25% ↑ and 4% ↑ = ?
(a) 29% (b) 30%
(c) 28% (d) 31%

68. 30% ↑ and 10% ↑ = ?
(a) 43% (b) 42%
(c) 44% (d) 45%

69. 24% ↑ and 10% ↑ = ?
(a) 35.40% (b) 33.60%
(c) 37.40% (d) 36.40%

70. 15% ↑ and 12% ↑ = ?
(a) 28.80% (b) 29.80%
(c) 27.60% (d) 27.80%

71. 12% ↑ and 10% ↑ = ?
(a) 21.60% (b) 24.20%
(c) 2320% (d) 22.20%

72. 8% ↑ and 5% ↑ = ?
(a) 13.20% (b) 13.40%
(c) 12.60% (d) 13.60%

73. 4% ↑ and 5% ↑ = ?
(a) 9.02% (b) 10.20%
(c) 9.80% (d) 9.60%

74. 20% ↑ and 25% ↑ = ?
(a) 55% (b) 45%
(c) 52% (d) 50%

75. 50% ↑ and 20% ↑ = ?
(a) 80% (b) 85%
(c) 75% (d) 77%

Type-8 (Find net decrease%)

76. 25% ↓ and 20% ↓ = ?
(a) 40% (b) 35%
(c) 45% (d) 47%

77. 25% ↓ and 25% ↓ = ?
(a) 44.75% (b) 43.75%
(c) 43.25% (d) 46.25%

78. 15% ↓ and 10% ↓ = ?
(a) 22.5% (b) 23.8%
(c) 23.7% (d) 23.5%

79. 10% ↓ and 20% ↓ = ?
(a) 28% (b) 27%
(c) 29% (d) 32%

80. 25% ↓ and 30% ↓ = ?
(a) 48.5% (b) 47.25%
(c) 48.75% (d) 47.5%

81. 15% ↓ and 9% ↓ = ?
(a) 23.65% (b) 24.25%
(c) 22.65% (d) 22.75%

82. 24% ↓ and 10% ↓ = ?
(a) 32.6% (b) 31.6%
(c) 31.4% (d) 32.4%

83. 16% ↓ and 15% ↓ = ?
(a) 24.6% (b) 29.4%
(c) 28.4% (d) 28.6%

84. 30% ↓ and 10% ↓ = ?
(a) 37% (b) 36%
(c) 38% (d) 39%

85. 28% ↓ and 5% ↓ = ?
(a) 32.6% (b) 31.4%
(c) 31.6% (d) 32.4%

86. 25% ↓ and 4% ↓ = ?
(a) 27% (b) 29%
(c) 28% (d) 27.5%

87. 8% ↓ and 6% ↓ = ?
(a) 14.52% (b) 13.52%
(c) 13.32% (d) 13.62%

88. 5% ↓ and 5% ↓ = ?
(a) 10.25% (b) 9.50%
(c) 9.25% (d) 9.75%

89. 2% ↓ and 3% ↓
(a) 4.96% (b) 4.92%
(c) 4.94% (d) 4.98%

90. 5% ↓ and 10% ↓
(a) 14.5% (b) 13.5%
(c) 14.75% (d) 13.75%

Type-8 (Find net inc. or dec.%)

91. 15% ↑ and 12% ↓ = ?
(a) –1.2% (b) +2.3%
(c) +1.2% (d) –2.3%

92. 50% ↑ and 20% ↓ = ?
(a) +20% (b) +25%
(c) +28% (d) +30%

93. 20% ↑ and 15% ↓ = ?
(a) +2% (b) +4%
(c) –1.2% (d) +2.5%

94. 30% ↑ and 20% ↓ = ?
(a) +6% (b) +5%
(c) +3% (d) +4%

95. 20% ↑ and 10% ↓ = ?
(a) +9% (b) +7%
(c) +8% (d) +10%

96. 15% ↑ and 10% ↓ = ?
(a) +3% (b) +3.5%
(c) +4% (d) +2.5%

97. 25% ↑ and 4% ↓ = ?
(a) +16% (b) +18%
(c) +22% (d) +20%

98. 10% ↑ and 5% ↓ = ?
(a) +4% (b) +5%
(c) +4.5% (d) +5.5%

99. 15% ↓ and 10% ↑ = ?
(a) –6.5% (b) –5.5%
(c) –4.5% (d) –5%

100. 25% ↑ and 20% ↓ = ?
(a) +5% (b) 0%
(c) +4.5% (d) +4%

ANSWER KEY - PERCENTAGE

1.	(d)	2.	(d)	3.	(c)	4.	(d)	5.	(c)	6.	(c)	7.	(b)	8.	(b)	9.	(d)	10.	(b)
11.	(b)	12.	(b)	13.	(c)	14.	(c)	15.	(d)	16.	(d)	17.	(c)	18.	(d)	19.	(a)	20.	(c)
21.	(b)	22.	(d)	23.	(a)	24.	(d)	25.	(c)	26.	(c)	27.	(a)	28.	(d)	29.	(b)	30.	(a)
31.	(c)	32.	(d)	33.	(d)	34.	(c)	35.	(a)	36.	(b)	37.	(c)	38.	(a)	39.	(d)	40.	(a)
41.	(b)	42.	(a)	43.	(c)	44.	(b)	45.	(d)	46.	(d)	47.	(a)	48.	(a)	49.	(c)	50.	(b)
51.	(b)	52.	(d)	53.	(a)	54.	(b)	55.	(a)	56.	(d)	57.	(c)	58.	(a)	59.	(b)	60.	(a)
61.	(c)	62.	(b)	63.	(d)	64.	(a)	65.	(d)	66.	(c)	67.	(b)	68.	(a)	69.	(d)	70.	(a)
71.	(c)	72.	(b)	73.	(a)	74.	(d)	75.	(a)	76.	(a)	77.	(b)	78.	(d)	79.	(a)	80.	(d)
81.	(c)	82.	(b)	83.	(d)	84.	(a)	85.	(c)	86.	(c)	87.	(b)	88.	(d)	89.	(c)	90.	(a)
91.	(c)	92.	(a)	93.	(a)	94.	(d)	95.	(c)	96.	(b)	97.	(d)	98.	(c)	99.	(a)	100.	(b)

PROFIT AND LOSS/ी रसअद रुपअज्ञ

Find the profit or loss % given marked price and discount.

ी रसअअरुअज्ञ अह्ई वर अर्र अ पेरुअ मउद्य ं आज्ञध द रुपअद्ध वक्षअमधरुअ)धरुअज्ञुल

Exercise– Type-1

1. MP = $16\frac{2}{3}$%, Dis. = $11\frac{1}{9}$%
P/L% = ?
(a) 3.60% (b) 3.0%
(c) 3.40% (d) 3.70%

2. MP = $16\frac{2}{3}$ & Dis. = $28\frac{4}{7}$%
P/L% = ?
(a) 23.60% (b) 16.66%
(c) 13.40% (d) 3.70%

3. MP = 45% & Dis. = 16.66%
P/L% = ?
(a) 20.83% (b) 30%
(c) 18.40% (d) 23.50%

4. MP = 20% ↑ & Dis. = 10%
P/L% = ?
(a) 4% (b) 6%
(c) 8% (d) 10%

5. MP = $11\frac{1}{9}$% & Dis. $9\frac{1}{11}$%
P/L% = ?
(a) 4% (b) 1%
(c) 8% (d) 3%

6. MP = 50% & Dis. = 25%
P/L% = ?
(a) 12.5% (b) 11%
(c) 9.5% (d) 12.5%

Exercise– Type-2

7. MP = 50% & L = 25%
Dis- = ?
(a) 45% (b) 40%
(c) 50% (d) 52%

8. MP = 100% & P = 25%
Dis- = ?
(a) 37.5% (b) 39.5%
(c) 33.5% (d) 43.5%

9. MP = $33\frac{1}{3}$% & P = 10%
Dis.% = ?
(a) 23.5% (b) 17.5%
(c) 33.5% (d) 19.5%

10. MP = 40% L = 9%
Dis. = ?
(a) 35% (b) 37%
(c) 33.5% (d) 27%

Exercise– Type-3

11. MP = 27.9% & Dis = 0%
P/L% = ?
(a) 35.9% (b) 37.9%
(c) 33.5% (d) 27.9%

12. MP = 35% & Dis. = 20%
P/L% = ?
(a) 8% (b) 6%
(c) 2% (d) 4%

13. MP = ? (20%)
P = 8% Dis. = ?
(a) 6% (b) 4%
(c) 10% (d) 12%

14. MP = 30% P = 4%
Dis. = ?
(a) 6% (b) 20%
(c) 10% (d) 12%

15. MP = 35% Dis = 15%
P/L% = ?
(a) 11% (b) 15%
(c) 13% (d) 17%

16. MP = 36% Dis. = 40%
P/L% = ?
(a) 18.4% (b) 15%
(c) 17.3% (d) 15.3%

17. MP = $37\frac{1}{2}$% & Dis. = $18\frac{2}{11}$%
P/L% = ?
(a) 18.4% (b) 12.5%
(c) 17.3% (d) 15.3%

18. MP = 15% & Dis. = 20%
P/L% = ?
(a) 8% (b) 6%
(c) 2% (d) 4%

19. MP = 25% Dis. = 40%
P/L% = ?
(a) 20% (b) 26%
(c) 25% (d) 22%

20. MP = 60% & Dis. = 25%
P/L% = ?
(a) 20% (b) 24%
(c) 25% (d) 22%

ANSWER KEY - PROFIT AND LOSS

1.	(d)	2.	(b)	3.	(a)	4.	(c)	5.	(b)	6.	(d)	7.	(c)	8.	(a)	9.	(b)	10.	(a)
11.	(d)	12.	(a)	13.	(c)	14.	(b)	15.	(b)	16.	(a)	17.	(b)	18.	(a)	19.	(c)	20.	(a)

CI AND SI/क्र प्रच्छ अधर(अद्रुपअम्र स्र-रपबरअधर(

Find Effective Rate of Interest (Compound Interest)

Exercise– Type-1

1. 6% @ CI 2 years = ?
(a) 12% (b) 12.30%
(c) 11.64% (d) 12.36%

2. 12% @ CI 2 years = ?
(a) 22% (b) 25.30%
(c) 25.44% (d) 12.36%

3. 13% @ CI 2 years = ?
(a) 12% (b) 27.69%
(c) 26.64% (d) 22.36%

4. 7% @ CI 2 years = ?
(a) 14% (b) 14.49%
(c) 14.64% (d) 14.36%

5. $33\frac{1}{3}\%$ @ CI 2 years = ?
(a) $\frac{9}{16}$ (b) $\frac{5}{16}$
(c) $\frac{7}{9}$ (d) $\frac{7}{18}$

6. 25% @ CI 2 years = ?
(a) 55% (b) 56.25%%
(c) 50.64% (d) 52.36%

7. 5% @ CI $2\frac{1}{2}$ years =
(a) 10.002% (b) 12.000%
(c) 12.004% (d) 13.006%

8. 14% @ CI $2\frac{5}{7}$ years
(a) 42% (b) 42.30%
(c) 42.64% (d) 42.96%

9. 10 × CI $2\frac{2}{5}$ years = ?
(a) 25.84% (b) 25.42%
(c) 26% (d) 26.84%

10. 20% @ CI $2\frac{2}{5}$ years = ?
(a) 55.52% (b) 62.30%
(c) 51.64% (d) 52.36%

Exercise– Type-2

Difference of Interest (CI–SI)

11. 5% for 2 years = ?
(a) 0.30% (b) 0.25%
(c) 0.15% (d) 0.20%

12. 13% for 2 years = ?
(a) 1.5% (b) 1.67%
(c) 0.15% (d) 1.69%

13. 9% for 2 years = ?
(a) 0.81% (b) 1.67%
(c) 0.15% (d) 1.69%

14. 11% for 2 years = ?
(a) 0.81% (b) 1.31%
(c) 0.15% (d) 1.21%

15. 17% for 2 years = ?
(a) 2.89% (b) 1.31%
(c) 2.15% (d) 2.21%

16. 5% for 3 years = ?
(a) 0.81% (b) 0.7625%
(c) 0.225% (d) 0.221%

17. 10% for 3 years = ?
(a) 2.1% (b) 1.31%
(c) 3.1% (d) 1.21%

18. $11\frac{1}{9}\%$ for 3 years = ?
(a) 2.1% (b) 3.31%
(c) 3.1% (d) 3.84%

19. 10% for 2 years = ?
(a) 2.1% (b) 1.31%
(c) 3.1% (d) 1%

20. $16\frac{2}{3}\%$ for 1st & $33\frac{1}{3}\%$ for 2nd = ?
(a) $\frac{11}{18}$ (b) $\frac{5}{18}$
(c) $\frac{7}{18}$ (d) $\frac{1}{18}$

Exercise– Type-3

Half Yearly

21. 6% Hy in 1 year = ?
(a) 3.1% (b) 2.31%
(c) 3.09% (d) 3.5%

22. 12% HY in 1 year = ?
(a) 9.11% (b) 13.31%
(c) 11.09% (d) 12.36%

23. 20% HY in 1 year = ?
(a) 21% (b) 23%
(c) 19% (d) 11%

24. $33\frac{1}{3}\%$ HY in 1.5 year = ?
(a) 40.58% (b) 50.8%
(c) 58.79% (d) 42.56%

25. $16\frac{2}{3}\%$ @ CI for 2nd year = ?
(a) 15.58% (b) 19.44%
(c) 18.79% (d) 17.56%

26. 10% CI half yearly in 1.5 years = ?
(a) 15.7625% (b) 15.2576%
(c) 15.75% (d) 15.35%

27. 20% CI half yearly in 1.5 years = ?
(a) 32.1% (b) 33.1%
(c) 33.5% (d) 33.3%

28. 8% pa CI half yearly in 1.5 yaers = ?
(a) 12.6448% (b) 13.48%
(c) 13.64% (d) 12.4864%

29. 20% p.a CI half yearly in one year = ?
(a) 19% (b) 22%
(c) 21% (d) 23%

30. 14% p.a CI half yearly in one year = ?
(a) 14.71% (b) 14.98%
(c) 14.08% (d) 14.49%

31. $22\frac{2}{9}\%$ CI half yearly in one = ?
(a) 23.45% (b) 23.65%
(c) 22.55% (d) 23.85%

32. $12\frac{1}{2}\%$ p.a CI half yearly in one year = ?
(a) 13.9% (b) 12.9%
(c) 12.5% (d) 13.5%

33. 12% p.a CI 8 monthly in 2 year = ?
(a) 25.27% (b) 25.25%
(c) 26.25% (d) 25.97%

34. 12% p.a CI 5 monthly in 10 months = ?
(a) 9.25% (b) 10.5%
(c) 10.25% (d) 10.75%

35. 15% CI 8 monthly in 2 years
(a) 32.1% (b) 33.1%
(c) 33.3% (d) 33.5%

ANSWER KEY - CI AND SI

1.	(d)	2.	(c)	3.	(b)	4.	(b)	5.	(c)	6.	(b)	7.	(d)	8.	(d)	9.	(a)	10.	(a)
11.	(b)	12.	(d)	13.	(a)	14.	(d)	15.	(a)	16.	(b)	17.	(c)	18.	(d)	19.	(d)	20.	(d)
21.	(c)	22.	(d)	23.	(a)	24.	(c)	25.	(b)	26.	(a)	27.	(b)	28.	(d)	29.	(c)	30.	(d)
31.	(a)	32.	(b)	33.	(d)	34.	(c)	35.	(b)										

Simplification (रे खा ं अेक

Conditions to be a perfect square number (ए ंबिन्हु न्रेपसै मैवव ा ैं क

❖ If a no. ends with 2,3,7,8 it can't be a perfect square
सेरे ं खाबिन्दुसां 2,3,7,8 ं खं न्रायं न्र अक्कं एाकमं एूं सएं नखं टश्नंबतहब ,एफ्एां न्र खक्मम

❖ If unit digit is 5 then ten's digit always 2.
सेरे ं खिाबिखां लक्ष 5 ं एूं कं ैं एाबिखां लक्ष एअैा 2 ं एाकमं एूव

❖ No number can be a perfect square unless its digital root is 0, 1, 4, 7 or 9.
खाबिापंन्दुसां की ं कखं टश्नंबतहंब,एफ्एां न्र खक्मम भी ं कखं जन्खां लक्षपस अम्मं 0, 1, 4, 7 ं सां 9 ं ,ं एाव

❖ Last 2 digit of a perfect square number = last two digits from the squares of numbers between (1 to 24).
नखं टश्नंबतहंबन्दुसां खं लक्कमं 2 ं लक्ष ं = म1 ं न्र 24 ं खं ैा प ं खप न्दुसालांदखं तहांण्खं लक्कमं ैं ा लक्षव

❖ n → 2K(even) → $n^2=(2k)^2=4k^2$ → multiple of 4
2K+1(odd) → $n^2=(2k+1)^2=4k^2+4k+1 \rightarrow 4(k^2+1)+1$
R=1 after divisible by 4

∴ $n^2 \rightarrow 4K, 4K+1$

51767 → Not a perfect square उं टश्नंबतहंब ,एफ्

$\left(\frac{67}{4}, R=3\right)$

Find squares (दु न्रैवक

❖ **Base** उं ैप्रै)**100** →

$102^2 = 102+2 | \underline{2}^2 = 10404$

$107^2 = 107+7 | 7^2 = 11449$

$113^2 = 113+13 | \underline{13}^2 = 126 | 169 \rightarrow 12769$

$121^2 = 121+21 | 21^2 = 142 | 441 \rightarrow 14641$

$97^2 = 97-3 | \underline{3}^2 = 9409$

$92^2 = 92-8 | 8^2 = 8464$

$83^2 = 83-17 | \underline{17}^2 = 66 | 289 \rightarrow 6889$

Base उं ैप्रै)**50** → $\frac{1}{2} \times 100$

$59^2 = \frac{1}{2}(59+9) | 9^2 \rightarrow 3481$

$54^2 = \frac{1}{2}(54+4) | 4^2 \rightarrow 2916$

$48^2 = \frac{1}{2}(48-2) | 2^2 \rightarrow 2304$

$44^2 = \frac{1}{2}(44-6) | 6^2 \rightarrow 1936$

$38^2 = \frac{1}{2}(38-12) | 12^2 \rightarrow 13 | 144 \rightarrow 1444$

$67^2 = \frac{1}{2}(67+17) | 17^2 \rightarrow 42 | 289 \rightarrow 4489$

Base उं ैप्रै)**200** → 2 × 100

$208^2 = 2(208+8) | 8^2 = 43264$

$219^2 = 2(219+19) | 19^2 = 476 | 361 \rightarrow 47961$

$194^2 = 2(194-6) | 6^2 = 37636$

$182^2 = 2(182-18) | 18^2 = 328 | 324 \rightarrow 33124$

Base न्र)ैप्रै **150** → $\frac{3}{2} \times 100$

$152^2 = \frac{3}{2}(152+2) | \underline{2^2} \rightarrow 23104$

$141^2 = \frac{3}{2}(141-9) | 9^2 \rightarrow 19881$

❖ $614^2 = 6(614+14) | 14^2 \rightarrow 6 \times 628 | 196 \rightarrow 376996$
$793^2 = 8(793-7) | 7^2 \rightarrow 8 \times 786 | 49 \rightarrow 628849$

❖ Last 2 digits of उं खं लक्कमं 2 ं लक्ष ं →

18^2, 32^2, 68^2, 82^2, 118^2, 368^2

$(50-18)^2$ $(100-18)^2$ $(350+18)^2$ = 24

∴ x^2, $(50k \pm x)^2$, $(100k \pm x)^2$ → Last 2 digits same. उ लक्कमं 2 ं लक्ष ं न्र अ,व

❖ $x(x+a)(x+2a)(x+3a)+k$ is a perfect square then k=?
$x(x+a)(x+2a)(x+3a)+k$ कीं नखं टश्नंबतहंबएूं k=?

→ $(x^2+3ax)(x^2+3ax+2a^2)$

→ $(x^2+3ax)^2 + 2 \times a^2 \times (x^2+3ax) + b^2 = (a+b)^2$
(a^2; +2 a b)

∴ to make perfect square add $(a^2)^2 = a^4$

∴ टश्नंबतहंबी ,ा,ं खं रगनं $(a^2)^2 = a^4$ं भाष्ठ

∴ $(x^2+3ax+a^2)^2$ & **k = a^4**

❖ If I, II, III, IV are in AP with common difference d
सरे ं If I, II, III, IV न ातंबलदक्षं d खं न ायां AP ं अंदएूद
Then I × II × III × IV + d^4 = (I × IV + d^2)^2

❖ To make perfect square what should add from:
टश्नंबतहंबी ,ा,ं खं सानं नि अंद्न ं (सां भाष्ठां ग ाएन)
1119 × 1126 × 1133 × 1140
∴ Gap between these consecutive numbers = ⑦
ि ं - अहकं न्दसालांदखं ी प ं खां लदक्षं = ⑦
∴ 7^4 = **2401** should be added.

❖ To make perfect square what should subtract from:
टश्नंबतहंबी ,ा,ं खं सानं नि अंद्न ं (सां चत्रा,ां ग ाएन)
841 × 846 × 851 × 856 + 700
∴ 841 × 846 × 851 × 856 → to make square 5^4 = 625 should be added
∴ 841 × 846 × 851 × 856 → ं खां तहंबी ,ा,ं खं सानं 5^4 = 625 ं भाष्ठंभा,ां ग ाएन

∴ $\underbrace{841\times846\times851\times856+625}_{\text{Perfect square}}+75$ → should be subtracted

C.

Unit digit of N N खां खिाबिलक्ष	Unit digit of N^2 N^2 खां खिाबिलक्ष
N=1, 9	1
N=2, 8	4
N=3, 7	9
N=4,6	6
N=5	5

❖ If 3 is repeated n times
I → Before 0 (n–1) times 1
II → Before 9 (n–1) times 8

eg. → 3^2 = 09, 33333^2 = 1111088889
33^2 = 1089
333^2 = 110889

❖ 6 is repeated n times
I → Before 3 (n–1) times 4
II → Before 6 (n–1) times 5
eg. → 6^2 = 36
66^2 = 4356
666^2 = 443556
666666^2 = 444443555556

❖ 9^2 = 81 9999^2 = 99980001
99^2 = 9801
999^2 = 998001

❖ $\sqrt{9999800001}+\sqrt{1111088889}-\sqrt{4444355556}$ =?
99999 + 33333 – 66666 = **66666**

❖

n^3			UD
1^3	1	→	1
2^3	8	→	8
3^3	27	→	7
4^3	64	→	4
5^3	125	→	5
6^3	216	→	6
7^3	343	→	3
8^3	512	→	2
9^3	729	→	9

❖ $\sqrt[3]{571787}$ → UD = 3
→ Ignore last 3 digit ं लक्षभं 3 ं लक्षांदखां ल,ं ु ां खश्द
→ Smaller than 571 perfect cube number = 512 = 8^3
→ 571 ं न ं इात्रां टश्नंबच,ं न्दुसां = 512 = 8^3
∴ $\sqrt[3]{571787}$ = **83**

❖ N^2 → odd no. of digits N^2 → even no. of digits
$N \to \frac{\text{odd no. of digits}+1}{2}$ $N \to \frac{\text{even no. of digits}}{2}$

❖ N^2 → 17 digits
$N \to \frac{17+1}{2}$ = 9 digits
N^2 → 24 digits
$N \to \frac{24}{2}$ = 12 digits

Square Mirrors (दु न्भ्मैक

❖ $14^2 + 87^2 = 78^2 + 41^2$
$15^2 + 75^2 = 57^2 + 51^2$
$17^2 + 84^2 = 48^2 + 71^2$
$26^2 + 97^2 = 79^2 + 62^2$
$27^2 + 96^2 = 69^2 + 72^2$

Non-terminating repeating decimal
(ंजय चैई ं ौं प) ं दं न्भौं चख्दक

Let x = 0.55555 ....
(–) $10x$ = 5.55555 ....
$9x = 5 \quad \therefore x = \frac{5}{9}$

❖ 0.77777 .... = $0.\overline{7} \to \frac{7}{9}$ (no. of bar = no. of 9)

0.838383 .... = $0.\overline{83} \to \frac{83}{99}$

0.514514514 .... = $0.\overline{514} \to \frac{514}{999}$

0.6823232323 ....= $0.6\overline{823} \to \frac{6823-68}{9900} \to \frac{6755}{9900}$
(no. of non bar digit = no. of zero)

$7.518651865186\ldots. = 7 + \frac{5186}{9999}$

❖ $0.866666\ldots. = 0.8\overline{6} = \frac{86-8}{90} = \frac{78}{90} = \frac{13}{15}$

$0.531313131\ldots. = 0.5\overline{31} = \frac{531-5}{990} = \frac{526}{990}$

$0.43777777\ldots. = 0.43\overline{7} = \frac{437-43}{900} = \frac{394}{900}$

$0.8\overline{6916}9\ldots. = 0.8\overline{6916} = \frac{8169-8}{9990} = \frac{8161}{9990}$

$7.581\overline{6} \rightarrow 7 + \frac{5816-581}{9000} \rightarrow 7\frac{5235}{9000}$

OR $\frac{75816-7581}{9000} = \frac{68235}{9000}$

$11.43\overline{25} \rightarrow 11 + \frac{4325-43}{9900} \rightarrow 11\frac{4282}{9900} \rightarrow 11\frac{2141}{4950}$

OR $\frac{114325-1143}{9900} = \frac{113182}{9900}$

❖ B O D M A S

↓ ↓ ↓ ↓ ↓ ↓

Brackets of div. multiply Add subtract

↓

small ब्रं ( )

middle ब्रं { }

Larger ब्रं [ ]

of means multiplication

Series Formulae

➢ $\frac{1}{a \times b} = \frac{1}{(b-a)}\left[\frac{1}{a} - \frac{1}{b}\right]$

➢ $\frac{1}{a \times b \times b} = \frac{1}{(c-a)}\left[\frac{1}{ab} - \frac{1}{bc}\right]$

➢ $\frac{1}{a \times b \times c \times d} = \frac{1}{(d-a)}\left[\frac{1}{abc} - \frac{1}{bcd}\right]$

Surds & Indices (गैर्ं प)ैज)ं ैआक

Law of Indices (ं ैआ)ै ेक्षसच्क

- $a \times a \times a \times \ldots$ n times $= a^n$

  $a^m \times a^n \times a^p = a^{m+n+p}$ $(a \neq 0)$

  $\frac{a^m}{a^n} = a^{m-n}$ $(m > n)$

  $= \frac{1}{a^{n-m}}$ $(n > m)$

  $= 1$ $(m = n)$
- $(a^m)^n = a^{m \times n} = a^{n \times m} = (a^n)^m$
- $(abc)^n = a^n \times b^n \times c^n$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ $(b \neq 0)$
- $(a^m)^n \neq a^{m^n}$

  $\left(3^2\right)^4 \neq 3^{2^4}$

  $3^8 \neq 3^{16}$
- $a^{\frac{p}{q}} = a^{\frac{1}{q} \times p} = \left(a^{\frac{1}{q}}\right)^p = \left(a^p\right)^{\frac{1}{q}}$
- If $a^m = a^n$ then $m = n$

  If $a^m = b^m$ then $a = b$
- $a^\circ = 1$
- $a^{-1} = \frac{1}{a}$ $(a \neq 0)$
- $a^{-n} = \frac{1}{a^n}$ & $a^n = \frac{1}{a^{-n}}$
- $\left(\frac{a}{b}\right)^m = \left(\frac{b}{a}\right)^{-m}$
- $(-1)^n = +1$ $(n$ = even$)$

  $= -1$ $(n$ = odd$)$
- $\sqrt{a} \times \sqrt{a} = a$
- $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
- $\sqrt{a^2 b} = a\sqrt{b}$
- $\left(\sqrt{a} + \sqrt{b}\right)^2 = a + b + 2\sqrt{ab}$
- $\left(\sqrt{a} - \sqrt{b}\right)^2 = a + b - 2\sqrt{ab}$
- $\left(\sqrt{a} + \sqrt{b}\right) \times \left(\sqrt{a} - \sqrt{b}\right) = a - b$, where a and b are positive

Useful Result/(प्र ती क्षें ी लिें (

- If $x = \frac{4\sqrt{ab}}{\sqrt{a} + \sqrt{b}}$

  $\frac{x + 2\sqrt{a}}{x - 2\sqrt{a}} + \frac{x + 2\sqrt{b}}{x - 2\sqrt{b}} = 2$
- If $= x\frac{2\sqrt{ab}}{\sqrt{a} + \sqrt{b}}$

  Then, $\frac{x + \sqrt{a}}{x - \sqrt{a}} + \frac{x + \sqrt{b}}{x - \sqrt{b}} = 2$

Laws of Surds (गैर्ं प) व्ेक्षसच्क

- $\sqrt[n]{a} = a^{\frac{1}{n}}$
- $\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b} = a^{\frac{1}{n}} \times b^{\frac{1}{n}} = (ab)^{\frac{1}{n}}$
- $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \left(\frac{a}{b}\right)^{\frac{1}{n}}$
- $\sqrt[n]{a}$ → Radicand

  ↓ order of surd
- $\left(\sqrt[n]{a}\right)^m = a^{\frac{m}{n}} = \sqrt[n]{a^m}$
- $\left(\sqrt[n]{a}\right)^n = a^{\frac{n}{n}} = a$
- $\sqrt[n]{\sqrt[m]{a}} = \sqrt[n]{a^{\frac{1}{m}}} = a^{\frac{1}{mn}}$
- $\left(\sqrt[z]{\left(\sqrt[y]{\left(\sqrt[x]{a}\right)^m}\right)^n}\right)^0 = a^{\frac{mno}{xyz}}$

Find Square Root (दु चक्र)ैं) ं क

- $\sqrt{7 + 4\sqrt{3}} = \sqrt{2^2 + \sqrt{3}^2 + \underbrace{②} \times 2 \times \sqrt{3}} = \sqrt{\left(2 + \sqrt{3}\right)^2}$

  $= \left(2 + \sqrt{3}\right)$ $\quad (a+b)^2 = a^2 + b^2 + 2ab$
- $\sqrt{28 + 10\sqrt{3}} = 10\sqrt{3} \rightarrow \underbrace{2 \times 5\sqrt{3}}_{5^2 + \sqrt{3}^2 = 28}$

$\therefore \left(5+\sqrt{3}\right)$

❖ $(a-b)^2 = (b-a)^2 = a^2+b^2-2ab$

$\sqrt{a^2+b^2-2ab}$ ⟷ a–b → (a>b), b–a → (b>a)

❖ $\sqrt{43-24\sqrt{3}} = \left(3\sqrt{3}-4\right)$ $\quad 24\sqrt{3} = ② \times 12\sqrt{3}$

$4 \quad 3\sqrt{3}$

$16+27=43$

❖ $\sqrt{99-70\sqrt{2}} = \left(5\sqrt{2}-7\right)$ $\quad 70\sqrt{2} = ② \times 35\sqrt{2}$

$7 \quad 5\sqrt{2}$

$49+50=99$

❖ $x = 5+2\sqrt{6}$

$$\frac{1}{x} = \frac{1}{5+2\sqrt{6}} \to \frac{1}{5+2\sqrt{6}} \times \frac{5-2\sqrt{6}}{5-2\sqrt{6}} \to \frac{5-2\sqrt{6}}{5^2-\left(2\sqrt{6}\right)^2}$$

$$= \frac{5-2\sqrt{6}}{25-24} = \frac{5-2\sqrt{6}}{1} \begin{matrix} \to \text{sign change} \\ \to \text{diff. of square} \end{matrix}$$

❖ $x = 7+4\sqrt{3} \to \frac{1}{x} = \frac{7-4\sqrt{3}}{1}$

$\downarrow \quad \downarrow$

$49 \quad 48$

❖ $x = 5\sqrt{3}+4\sqrt{2} \to \frac{1}{x} = \frac{5\sqrt{3}-4\sqrt{2}}{43}$

$\downarrow \quad \downarrow$

$75 \quad 32$

❖ $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

$(a-b+c)^2 = a^2+b^2+c^2-2ab-2bc+2ca$

$(a+b-c)^2 = a^2+b^2+c^2+2ab-2bc-2ca$

❖ $(x-1)(x+1) = x^2-1$

$\Rightarrow 1+(x-1)(x+1) = x^2 \Rightarrow \sqrt{1+(x-1)(x+1)} = x$

Componendo & dividendo (सैुवैंपैे तद्वैं क

❖ $\frac{a}{b} = \frac{c}{d}$

Apply C & D (Add and subtract den. in number)

C लूगं D गहंस्खश्द

$$\frac{a+b}{a-b} = \frac{c+d}{c-d}$$

Apply again

$$\frac{a+b+(a-b)}{a+b-(a-b)} = \frac{c+d+(c-d)}{c+d-(c-d)}$$

$$\frac{2a}{2b} = \frac{2c}{2d}$$

$\frac{a}{b} = \frac{c}{d} \Rightarrow$

If we apply C & D two times on a fraction, same fraction is achieved.

सरे ं एअं नख ं राख्रं टर्श ं ी ाशं C लूगं D गहाकं एूदं कां न अ, ं राख्र ं ाक्कं एाक्मं एूव

❖ $\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}} + \frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}} = \frac{2(x+y)}{(x-y)}$

$\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}} - \frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}} = \frac{+4\sqrt{xy}}{x-y}$

$\left(\sqrt{x}+\sqrt{y}\right)^2 - \left(\sqrt{x}-\sqrt{y}\right)^2 = 4\sqrt{xy}$

❖ $\sqrt{a\sqrt{a\sqrt{a\sqrt{a....\infty}}}} = a$

$\sqrt{a\sqrt{a\sqrt{a....n\text{ times}}}} = a^{1-\frac{1}{2^n}} = a^{\frac{2^n-1}{2^n}}$

$\sqrt[3]{a\sqrt[3]{a\sqrt[3]{a....\infty}}} = \sqrt{a}$ $\qquad \sqrt[n]{a\sqrt[n]{a\sqrt[n]{a....\infty}}} = \sqrt[n-1]{a}$

$\sqrt{a \div \sqrt{a \div \sqrt{a....\infty}}} = \sqrt[3]{a}$

❖ $\sqrt{a+\sqrt{a+\sqrt{a+....\infty}}} = \frac{\sqrt{4a+1}+1}{2} = x$

$\sqrt{a-\sqrt{a-\sqrt{a-....\infty}}} = \frac{\sqrt{4a+1}-1}{2} = y$

x–y = 1

xy = a

OR Take two factors of a whose difference is 1 like

लयतां a ख ं ां हश्च, : ष्ं गप्रभ्नं रभ्, खां लक्ष्रं 1 एाव

$56 \swarrow \begin{matrix}8\\7\end{matrix}$ $\quad 72 \swarrow \begin{matrix}8\\9\end{matrix}$

Then x = Larger factor उं की ं x = ं ी ष्ठं हश्चाख

y = smaller factor उं y = ं इात्रां खाश्ख

$x+y = \sqrt{4a+1}$

❖ $x = \sqrt{a+\sqrt[b]{a+\sqrt[b]{a+....\infty}}} = \frac{\sqrt{4a+b^2}+b}{2}$

$y = \sqrt{a-\sqrt[b]{a-\sqrt[b]{a-....\infty}}} = \frac{\sqrt{4a+b^2}-b}{2}$

$x-y = b$

$xy = a$

$x+y = \sqrt{4a+b^2}$

❖ $x = \sqrt{a+\sqrt{a-\sqrt{a+....\infty}}} = \dfrac{\sqrt{4a-3}+1}{2}$

$y = \sqrt{a-\sqrt{a+\sqrt{a-....\infty}}} = \dfrac{\sqrt{4a-3}-1}{2}$

$x-y = 1$ $\quad$ $xy = 2(a-1)$

❖ $\sqrt{a+\sqrt[b]{a-\sqrt[b]{a+....\infty}}} = \dfrac{\sqrt{4a-3b^2}+b}{2}$

$\sqrt{a-\sqrt[b]{a+\sqrt[b]{a-....\infty}}} = \dfrac{\sqrt{4a-3b^2}-b}{2}$

❖ $\sqrt{2}, \sqrt[3]{3}, \sqrt[4]{5}$ Powers $\rightarrow \frac{1}{②}, \frac{1}{③}, \frac{1}{④} \rightarrow$ LCM = 12

$\therefore \left(\sqrt{2}\right)^{12}, \left(\sqrt[3]{3}\right)^{12}, \left(\sqrt[4]{5}\right)^{12}$

$\rightarrow 2^6,\ 3^4,\ 5^3 \ \therefore \sqrt[4]{5} > \sqrt[3]{3} > \sqrt{2}$

$\downarrow \quad \downarrow \quad \downarrow$

64 81 125

❖ If $x + y = 12$ (constant)

$(xy)_{max}$ = diff. of x & y should be min. = 6 × 6 = **36**

$(xy)_{min}$ = diff. of x & y should be max. = 1 × 11 = **11**

❖ $a > b > c$

$\dfrac{1}{a} < \dfrac{1}{b} < \dfrac{1}{c}$

❖ $a \times b = 16$ (constant)

$\therefore (a+b)_{min} = 4 + 4 = 8 \qquad 1\times16 \rightarrow 16+1=17$

$(a^2+b^2)_{min} = 4^2+4^2=32$

* $a\times b\times c = 125$ (constant) for min $a=b=c=5$ उ

$(a^2+b^2+c^2)min = 25+25+25 = 75$ उ

Approx Root Value (ख ै, ह च ैतक

❖ $\sqrt{13}$ between $\sqrt{9}$ ③ and $\sqrt{16}$ ④

$\therefore 3 < \sqrt{13} < 4$

$3 + \dfrac{13-9}{16-9} \Rightarrow 3+\dfrac{4}{7} \Rightarrow 3\dfrac{4}{7} \Rightarrow$ **3.57**

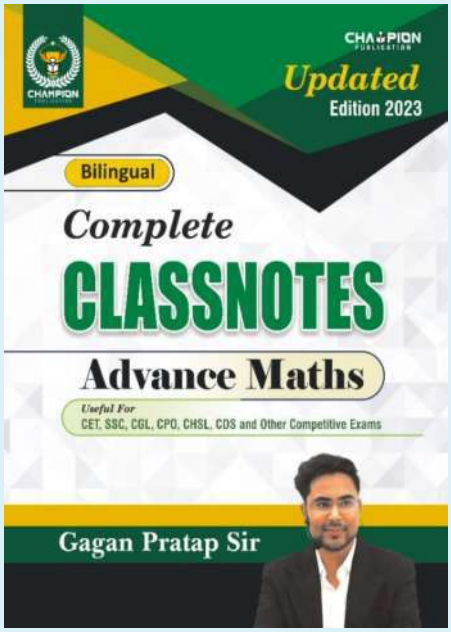
$\sqrt{21}$ between $\sqrt{16}$ and $\sqrt{25}$ $\quad 4+\dfrac{5}{9} \rightarrow 4\dfrac{5}{9} \rightarrow$ **4.55**

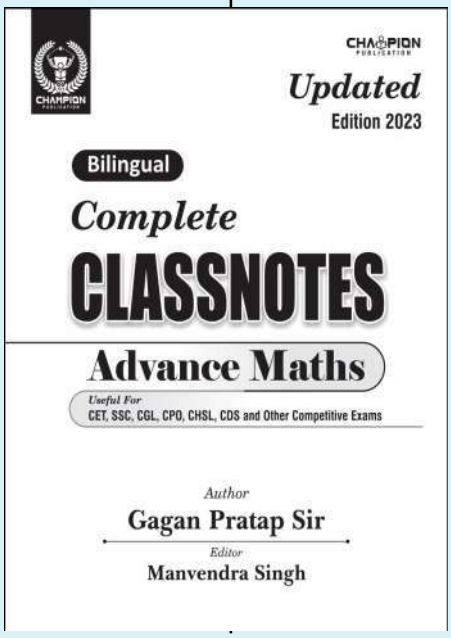
$\sqrt[3]{53}$ between $\sqrt[3]{27}$ and $\sqrt[3]{64}$ $\quad \rightarrow 3 + \dfrac{26}{37} \approx$ **3.7**

❖ $\sqrt{121} = 11 \qquad \sqrt{1234321} = 1111$

$\sqrt{12321} = 111 \qquad \sqrt{1234567654321} = 1111111$

❖ If I, II, III, IV are in AP with common diff. d then $I\times II\times III\times IV + d^4 = (I\times IV + d^2)^2$





Algebra (प्राबेअद)

Square Formulae

1. $a^2 - b^2 = (a + b)(a - b)$
2. $(a + b)^2 = a^2 + b^2 + 2ab$
3. $(a - b)^2 = a^2 + b^2 - 2ab$
4. $ab = \frac{(a+b)^2 - (a^2+b^2)}{2}$
5. $2(a^2 + b^2) = (a + b)^2 + (a - b)^2$
6. $4ab = (a + b)^2 - (a - b)^2$
7. $(a + b)^2 = (a - b)^2 + 4ab$
8. $(a - b)^2 = (a + b)^2 - 4ab$
9. $a^2 + b^2 = (a + b)^2 - 2ab = (a - b)^2 + 2ab$
10. $(a^2 - b^2)^2 = a^4 + b^4 - 2a^2b^2$
11. $a^4 + b^4 = (a^2 + b^2)^2 - 2a^2b^2$
12. $a^4 + b^4 = (a^2 - b^2)^2 + 2a^2b^2$
13. $(ax+by)^2 + (ay-bx)^2 = (a^2+b^2)(x^2+y^2)$
14. $\frac{a+b}{a-b} + \frac{a-b}{a+b} = \frac{2(a^2+b^2)}{(a^2-b^2)}$
15. $\frac{a+b}{a-b} - \frac{a-b}{a+b} = \frac{4ab}{a^2-b^2}$

Cube Formulae

1. $(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$
2. $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
3. $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$
4. $a^3 + b^3 = (a + b)[(a + b)^2 - 3ab]$
5. $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$
6. $a^2 + b^2 - ab = \frac{a^3+b^3}{a+b}$
7. $a + b = \frac{a^3+b^3}{a^2+b^2-ab}$
8. $(a - b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$
9. $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
10. $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$
11. $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$
12. $a^3 - b^3 = (a - b)[(a - b)^2 + 3ab]$
13. $a^2 + b^2 + ab = \frac{a^3-b^3}{a-b}$
14. $a - b = \frac{a^3-b^3}{a^2+b^2+ab}$
15. $(a+b)^3 - (a-b)^3 = 2b^3 + 6a^2b = 2b(b^2+3a^2)$
16. $(a+b)^3 + (a-b)^3 = 2a^3 + 6ab^2 = 2a(a^2+3b^2)$

Componendo & Dividendo Rule (रे छेअै को अब्कान्द)

If $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$

If $\frac{x+y}{x-y} = \frac{a}{b} \Rightarrow \frac{2x}{2y} = \frac{a+b}{a-b} \Rightarrow \frac{x}{y} = \frac{a+b}{a-b}$

How to find Conjugate

सरे square खां difference 1 बिां conjugate दुं यअधक् sign change सां सरे difference 1 पु बिां square खन् difference अं divide खे नुस्ट

❖ If $x = 15 + 4\sqrt{14}$ then $\frac{1}{x} = ?$

$(15)^2 - \left(4\sqrt{14}\right)^2 \Rightarrow 225 - 224 \Rightarrow 1$

$\therefore \frac{1}{x} = 15 - 4\sqrt{14}$ (square खां diff. 1 स्अअ तं यअधक्sign change सा)

❖ If $x > \frac{1}{x}$

Then $\boldsymbol{x + \frac{1}{x}}$ **= 2 × Bigger no.** = 2 × 15 = 30

$\boldsymbol{x - \frac{1}{x}}$ **= 2 × smaller no.** $= 2 \times 4\sqrt{14} = 8\sqrt{14}$

If $x < \frac{1}{x}$ then $\boldsymbol{x + \frac{1}{x}}$ **= 2× Big number**

$\boldsymbol{x - \frac{1}{x}}$ **= –2 × small number**

Power 2 Formulae

1. If $x + \frac{1}{x} = k \Rightarrow x^2 + \frac{1}{x^2} = k^2 - 2$

☞ If $x + \frac{1}{x} = k$

Then $x(x - k) = -1$ (Important)
or $x(x - k) + 1 = 0$
or $x^2 - kx + 1 = 0$

2. If $x - \frac{1}{x} = k \Rightarrow x^2 + \frac{1}{x^2} = k^2 + 2$

Then

$x - k = \frac{1}{x}$

$x(x - k) = 1$ (important)

$x^2 - kx - 1 = 0$

3. If $x + \frac{1}{x} = \pm\sqrt{2}$ then $x^2 + \frac{1}{x^2} = 0$

$x^4 + 1 = 0$

Power difference 4 results in zero

$\therefore x^4 + x^\circ = 0$

4. $x + \frac{1}{x} = k$

$\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 = x + \frac{1}{x} + 2 = k + 2$

$\sqrt{x} + \frac{1}{\sqrt{x}} = \pm\sqrt{k+2}$

5. $x^2 + \frac{1}{x^2} = k$

$\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 = k + 2$

$\left(x + \frac{1}{x}\right) = \pm\sqrt{k+2}$

6. $x^2 + \frac{1}{x^2} = k$

$\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2 = k - 2$

$\left(x - \frac{1}{x}\right) = \pm\sqrt{k-2}$

Same Power Formulae

- If $x + \frac{1}{x} = k$, then $x - \frac{1}{x} = \pm\sqrt{k^2 - 4}$
- If $x - \frac{1}{x} = k$, then $x + \frac{1}{x} = \pm\sqrt{k^2 + 4}$
- If $\frac{A+B}{\sqrt{AB}} = a$, then $\frac{A-B}{\sqrt{AB}} = \pm\sqrt{a^2 - 4}$
- If $\sqrt{\frac{A}{B}} + \sqrt{\frac{B}{A}} = a$, then $\sqrt{\frac{A}{B}} - \sqrt{\frac{B}{A}} = \sqrt{a^2 - 4}$

Power 3 Formulae

1. If $x - \frac{1}{x} = k$, then $x^3 - \frac{1}{x^3} = k^3 + 3k$

2. If $x + \frac{1}{x} = k$ then $x^3 + \frac{1}{x^3} = k^3 - 3k$

$x + \frac{1}{x} = 2, 3, 4, 5, 6, 7, 8, 9, 10$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$x^3 + \frac{1}{x^3} = 2, 18, 52, 110, 198, 322, 488, 702, 970$

$x - \frac{1}{x} = 2, 3, 4, 5, 6, 7, 8, 9, 10$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$x^3 - \frac{1}{x^3} = 14, 36, 76, 140, 234, 364, 536, 756, 1030$

3. $x + \frac{1}{x} = \sqrt{t}$

$x^3 + \frac{1}{x^3} = \left(\sqrt{t}\right)^3 - 3\sqrt{t} = t\sqrt{t} - 3\sqrt{t} = \mathbf{(t-3)\sqrt{t}}$

4. If $x - \frac{1}{x} = \sqrt{t}$

$x^3 - \frac{1}{x^3} = \mathbf{(t+3)\sqrt{t}}$

5. If $\sqrt{x} + \frac{1}{\sqrt{x}} = a$, then $x\sqrt{x} + \frac{1}{x\sqrt{x}} = a^3 - 3a$

6. If $\sqrt{x} - \frac{1}{\sqrt{x}} = a$, then $x\sqrt{x} - \frac{1}{x\sqrt{x}} = a^3 + 3a$

7. If $\frac{a-b}{\sqrt{ab}} = k$ or $\sqrt{\frac{a}{b}} - \sqrt{\frac{b}{a}} = k$,

then $\left(\frac{a}{b}\right)^{\frac{3}{2}} - \left(\frac{b}{a}\right)^{\frac{3}{2}} = k^3 + 3k$

8. If $\frac{a+b}{\sqrt{ab}} = k$ or $\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}} = k$

then $\left(\frac{a}{b}\right)^{\frac{3}{2}} + \left(\frac{b}{a}\right)^{\frac{3}{2}} = k^3 - 3k$

9. If $x + \frac{1}{x} = \pm\sqrt{3}$, then $x^3 + \frac{1}{x^3} = 0$

$\left(x + \frac{1}{x}\right)^2 = 3$

$x^6 + 1 = 0$

$x^6 = -1$

7. $(a^5 - b^5) = (a - b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$

$(a^5 + b^5) = (a + b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4)$

Higher Power Formulae

1. $x^4 - \frac{1}{x^4} = \left(x^2 + \frac{1}{x^2}\right)\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$

2. $x^5 + \frac{1}{x^5} = \left(x^2 + \frac{1}{x^2}\right)\left(x^3 + \frac{1}{x^3}\right) - \left(x + \frac{1}{x}\right)$

3. $\left(x^5-\frac{1}{x^5}\right)=\left(x^2+\frac{1}{x^2}\right)\left(x^3-\frac{1}{x^3}\right)-\left(x-\frac{1}{x}\right)$

4. $x^7+\frac{1}{x^7}=\left(x^4+\frac{1}{x^4}\right)\left(x^3+\frac{1}{x^3}\right)-\left(x+\frac{1}{x}\right)$

5. $x^7-\frac{1}{x^7}=\left(x^4+\frac{1}{x^4}\right)\left(x^3-\frac{1}{x^3}\right)+\left(x-\frac{1}{x}\right)$

6. $x^8-\frac{1}{x^8}=\left(x^4+\frac{1}{x^4}\right)\left(x^2+\frac{1}{x^2}\right)\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)$

Formulae based on $x+\frac{1}{x}$

1. If $x+\frac{1}{x}=2$, then $x=1$

2. If $x+\frac{1}{x}=-2$, then $x=-1$

3. If $x+\frac{1}{x}=1$, then $x^3=-1$

❖ If $x+\frac{1}{x}=1$ then $x^2-x+1=0$

$x^2-x+1=0$

$(x+1)(x^2-x+1)=0(x+1)$

$x^3+1=0$

$x^3+x^0=0$ power difference 3 results in zero

$x^3=-1$

4. If $x+\frac{1}{x}=-1$ or $x^2+x+1=0$

Then $x^3=1$

or $x^2+x+1=0$

Relation between Power 2 and Power 4

1. $(x^2+y^2)^2=x^4+y^4+2x^2y^2$

$(x^2+y^2)^2-x^2y^2=x^4+y^4+x^2y^2$

$(x^2+y^2)^2-(xy)^2=x^4+y^4+x^2y^2$

$\boldsymbol{(x^2+y^2+xy)(x^2+y^2-xy)=x^4+y^4+x^2y^2}$

2. $x^2+y^2+xy=A$ and $x^2+y^2-xy=B$

$x^2+y^2=\frac{A+B}{2}$, $xy=\frac{A-B}{2}$

❖ If $x^4+x^2y^2+y^4=189$, $x^2+xy+y^2=21$

(i) $x^2-xy+y^2=\frac{189}{21}=\mathbf{9}$

(ii) $x^2+y^2=\frac{21+9}{2}=\mathbf{15}$

(iii) $xy=\frac{21-9}{2}=\mathbf{6}$

(iv) $\frac{x}{y}+\frac{y}{x}=\frac{x^2+y^2}{xy}=\frac{21+9}{21-9}=\mathbf{\frac{5}{2}}$

(v) $\frac{x^2}{y^2}+\frac{y^2}{x^2}=\left(\frac{5}{2}\right)^2-2=\mathbf{\frac{17}{4}}$

(vi) $x+y=?$

$x^2+xy+y^2+xy=21+6$

$(x+y)^2=27 \quad \therefore x+y=\mathbf{3\sqrt{3}}$

(vii) $\frac{1}{y}-\frac{1}{x}=?$

$x^2-xy+y^2-xy=9-6=3$

$(x-y)^2=3$

$(x-y)=\sqrt{3}$

$\therefore \frac{1}{y}-\frac{1}{x}=\frac{x-y}{xy}=\frac{\sqrt{3}}{6}=\frac{1}{2\sqrt{3}}$

3. $\frac{x^6-y^6}{x^2-y^2}=\frac{\left(x^2\right)^3-\left(y^2\right)^3}{x^2-y^2}=\frac{\left(x^2-y^2\right)\left(x^4+y^4+x^2y^2\right)}{\left(x^2-y^2\right)}$

$=x^4+y^4+x^2y^2$

4. $a^2+b^2+ab=\left(a+b+\sqrt{ab}\right)\left(a+b-\sqrt{ab}\right)$

5. $x^4+x^2+1=(x^2+x+1)(x^2-x+1)$

6. $(x^2+y^2)^2=x^4+y^4+2x^2y^2$

$(x^2+y^2)^2-2x^2y^2=x^4+y^4$

$(x^2+y^2)^2-\left(\sqrt{2}xy\right)^2=x^4+y^4$

$(x^2+y^2+\sqrt{2}\,xy)(x^2+y^2-\sqrt{2}\,xy)=x^4+y^4$

Concept of Root Value

❖ $(x)^2=(-x)^2=x^2$

$\sqrt{x^2}=+x$ ← Root रूंन्हा

$\sqrt{49}=7$ ← (+ve) value , वस्

$(7)^2=49$

$\sqrt{a^2+b^2-2ab}$

(a–b) ↓ if a > b (b–a) ↓ if b > a

Concept of Degree

$3x^3-7x^2y+8zx^2 \rightarrow$ Degree = 3 (highest powers)

$8x+7 \rightarrow$ Degree = 1

$25x^2-10x^3+19z^{26}x^1+48w^1$

Degree = 27

Multiply दुंन्power add ास्ट

Divide दुंन्power subtract ास्

$xy \rightarrow$ Degree = 2

$\frac{x^8}{y^3} \rightarrow$ Degree = 8 – 3 = 5

Symmetric Function

❖ Degree of each term is same on both sides.
$a + b + c$, $a^2bc + b^2ca + c^2ab$

$\frac{a^3+b^3}{a+b} = a^2 - ab + b^2 \rightarrow$ Both side degree is 2

❖ $\frac{a^3(b^2-c^2)+b^3(c^2-a^2)+c^3(a^2-b^2)}{a^2(b-c)+b^2(c-a)+c^2(a-b)} = ab + bc + ca$

❖ $\frac{a^3\times(b+c)}{(a-b)(a-c)}+\frac{b^3\times(c+a)}{(b-c)(b-a)}+\frac{c^3\times(a+b)}{(c-a)(c-b)}$
$= ab + bc + ca$

❖ $\frac{a\times(b-c)^2}{(c-a)(a-b)}+\frac{b\times(c-a)^2}{(a-b)(b-c)}+\frac{c\times(a-b)^2}{(b-c)(c-a)}$
$= a + b + c$

❖ If $\frac{x}{y}=\frac{z}{w}$ then $\frac{x^m+y^m+z^m+w^m}{x^{-m}+y^{-m}+z^{-m}+w^{-m}}=(xyzw)^{m/2}$

❖ If $x = a+\frac{1}{a}$ and $y = a-\frac{1}{a}$ then $\sqrt{x^4+y^4-2x^2y^2} = 4$

❖ If $x + y = 2z$, then the value of $\frac{z}{x-z}+\frac{y}{y-z} = 0$

❖ If $x + y = 2z$, then $\frac{x}{x-z}+\frac{y}{y-z} = 2$

If a + b + c = 0, then following results follows

1. Then $\frac{1}{(a+b)(b+c)}+\frac{1}{(a+c)(b+a)}+\frac{1}{(c+a)(c+b)} = 0$

2. Then $\frac{a^2}{a^2-bc}+\frac{b^2}{b^2-ca}+\frac{c^2}{c^2-ab} = 2$

3. Then $\frac{a^2}{2a^2+bc}+\frac{b^2}{2b^2+ca}+\frac{c^2}{2c^2+ab} = 1$

4. Then $\frac{a^2+b^2+c^2}{(a-b)^2+(b-c)^2+(c-a)^2} = \frac{1}{3}$

5. Then $\frac{1}{a^2+b^2-c^2}+\frac{1}{b^2+c^2-a^2}+\frac{1}{c^2+a^2-b^2} = 0$

6. Then $\frac{2(a^4+b^4+c^4)}{(a^2b^2+b^2c^2+c^2a^2)} = 4$

7. Then $\frac{a+b}{ab}(a^2+b^2-c^2)+\frac{b+c}{bc}(b^2+c^2-a^2)+\frac{c+a}{ca}$
$(c^2+a^2-b^2) = 0$

8. Then $\frac{2a^2}{(b^2+c^2-a^2)}+\frac{2b^2}{(a^2+c^2-b^2)}+\frac{2c^2}{(a^2+b^2-c^2)} = -3$

Some other results

⇒ $(x^2 + ax + bx + ab) = (x + a)(x + b)$

⇒ $1 + A + B + AB = (1 + A)(1 + B)$

⇒ $(1+a)(1+b)(1+c) = 1 + a + b + c + ab + bc + ca + abc$

⇒ If $xy = 1$ or $x = \frac{1}{y}$ then $\frac{1}{1+x^n}+\frac{1}{1+y^n} = 1$

❖ If $a + b + c = 2s$, then $\frac{(s-a)^2+(s-b)^2+(s-c)^2+s^2}{a^2+b^2+c^2}$
$= 1$

❖ If $xy + yz + zx = 0$, then $\left(\frac{1}{x^2-yz}+\frac{1}{y^2-zx}+\frac{1}{z^2-xy}\right)$
$= 0$ $(x, y, z \neq 0)$

❖ If $pq + qr + rp = 0$, then $\left(\frac{p^2}{p^2-qr}+\frac{q^2}{q^2-rp}+\frac{r^2}{r^2-pq}\right)$
$= 1$

❖ If $\left[\sqrt{(a^2+b^2+ab)}+\sqrt{(a^2+b^2-ab)}\right] = 1$, then $(1 - a^2)$
$(1 - b^2) = \frac{3}{4}$

❖ If $\left(x+\sqrt{1+x^2}\right)\left(y+\sqrt{1+y^2}\right) = 1$, where x and y are real numbers, then $(x + y)^2 = 0$

❖ $\frac{x^4}{(x^2-y^2)(x^2-z^2)}+\frac{y^4}{(y^2-x^2)(y^2-z^2)}$
$+\frac{z^4}{(z^2-x^2)(z^2-y^2)} = 1$

❖ If $a + b + c = abc$, then
$\frac{(1-a^2)(1-b^2)}{ab}+\frac{(1-b^2)(1-c^2)}{bc}+\frac{(1-c^2)(1-a^2)}{ca} = 4$

❖ If $bc + ca + ab = abc$, then
$\frac{b+c}{bc(a-1)}+\frac{c+a}{ca(b-1)}+\frac{a+b}{ab(c-1)} = 1$

❖ If $xy + yz + xz = 1$, then $\left(\frac{x+y}{1-xy}+\frac{y+z}{1-yz}+\frac{z+x}{1-xz}\right) = \frac{1}{xyz}$

❖ If $x=\frac{a-b}{a+b}, y=\frac{b-c}{b+c}, z=\frac{c-a}{c+a}$,
then $\frac{(1-x)(1-y)(1-z)}{(1+x)(1+y)(1+z)} = 1$

❖ If $x^2 + y^2 = z + 1$, $y^2 + z^2 = x + 1$, $z^2 + x^2 = y + 1$, then
$xyz = 1$ or $-\frac{1}{8}$

❖ $ab(a - b) + bc(b - c) + ca(c - a) = (b - a)(b - c)(c - a)$

- $(a+b+c)(ab+bc+ca)-abc=(a+b)(b+c)(c+a)$
- $(a+b+c)(ab+bc+ca)=a^2b+b^2a+b^2c+bc^2+a^2c+a^2c+3abc$
- $(a+b+c)(ab+bc+ca)-3abc=a^2(b+c)+b^2(c+a)+c^2(a+b)$
- $(a+b+c)(ab+bc+ca)-abc=a^2b+b^2a+b^2c+bc^2+a^2c+ac^2+2abc=(a+b)(b+c)(c+a)$
- If $a+\frac{1}{b}=b+\frac{1}{c}=c+\frac{1}{a}$ (where $a\neq b\neq c$) then abc is equal to $=\pm 1$

3 Variable Formulae

1. If $x^2+y^2+z^2=0$, then $x=0, y=0, z=0$

2. **$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$**

$a^2+b^2+c^2=(a+b+c)^2-2(ab+bc+ca)$

$ab+bc+ca=\frac{(a+b+c)^2-(a^2+b^2+c^2)}{2}$

$(a-b+c)^2=a^2+b^2+c^2-2ab-2bc+2ca$

$(a+b-c)^2=a^2+b^2+c^2+2ab-2bc-2ca$

3. **$a^2+b^2+c^2-ab-bc-ca=\frac{1}{2}[(a-b)^2+(b-c)^2+(c-a)^2]$**

↓

$\frac{1}{2}[2a^2+2b^2+2c^2-2ab-2bc-2ca]$

$\frac{1}{2}[a^2+b^2-2ab+b^2+c^2-2bc+a^2+c^2-2ca]$

$\frac{1}{2}[(a-b)^2+(b-c)^2+(c-a)^2]$

4. If $\underset{+d}{a},\underset{+d}{b},c$ are in A.P. with common difference d

$a^2+b^2+c^2-ab-bc-ca=\frac{1}{2}[d^2+d^2+(2d)^2]=$ **$3d^2$**

5. If $a^2+b^2+c^2-ab-bc-ca=0$ OR $a^2+b^2+c^2=ab+bc+ca$ $\Rightarrow$ **$a=b=c$**

6. $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$=\frac{1}{2}(a+b+c)[(a-b)^2+(b-c)^2+(c-a)^2]$

$\frac{a^3+b^3+c^3-3abc}{(a-b)^2+(b-c)^2+(c-a)^2}=\frac{a+b+c}{2}$

$a^3+b^3+c^3-3abc=(a+b+c)[(a+b+c)^2-3(ab+bc+ca)]$

$a^3+b^3+c^3-3abc=\frac{1}{2}(a+b+c)[3(a^2+b^2+c^2)-(a+b+c)^2]$

7. $a^3+b^3+c^3-3abc=(a+b+c)[(a+b+c)^2-3(ab+bc+ca)]$

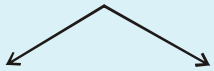
If $a+b+c=0$ then $a^3+b^3+c^3-3abc=0$

OR $a^3+b^3+c^3=3abc$

8. $a^3+b^3+c^3-3abc=\frac{1}{2}(a+b+c)[(a-b)^2+(b-c)^2+(c-a)^2]$

If $a^3+b^3+c^3-3abc=0$

OR $a^3+b^3+c^3=3abc$



$a+b+c=0$ $\qquad$ $a=b=c$

- If a, b, c are distinct integers & $a^3+b^3+c^3=3abc$ then $a+b+c=0$
- If a, b, c are +ve integers & $a^3+b^3+c^3=3abc$ then $a=b=c$
- If a, b, c are in A.P. then $a^3+b^3+c^3-3abc=9bd^2$ (d = common difference)

9. $(a+b+c)^3=a^3+b^3+c^3+3(a+b)(b+c)(c+a)$

$(a+b+c)^3=a^3+b^3+c^3+3[a^2(b+c)+b^2(c+a)+c^2(a+b)]+2abc$

$(a+b+c)^3-a^3-b^3-c^3=3(a+b)(b+c)(c+a)$

10. If $x+\frac{1}{y}=a, y+\frac{1}{z}=b, z+\frac{1}{x}=c$

then $xyz+\frac{1}{xyz}=abc-(a+b+c)$

11. If $x-\frac{1}{y}=a, y-\frac{1}{z}=b, z-\frac{1}{x}=c$

then $xyz-\frac{1}{xyz}=abc+(a+b+c)$

12. If $a+b+c=x$

and $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=y$

Then $\frac{a}{b}+\frac{b}{a}+\frac{c}{a}+\frac{a}{c}+\frac{b}{c}+\frac{c}{b}=xy-3$

Theory of Equations (इन्पै सै हिं क्किं अद्द

Polynomial (ु मणेवद

❖ An algebraic expression of the form $a_0+a_1x+a_2x^2+...+a_nx^n$, where $n \in N$, is called a polynomial. It is generally denoted by $p(x)$, $q(x)$, $f(x)$, $g(x)$ etc.

$a_0+a_1x+a_2x^2+...+a_nx^n$ खंन्लधं खां तखं हू , रयैाबूवं ।वु, खीं , ।भ $n \in N$, ैह ख्नं ख थबां ैस्अंन्सादबोधिं $p(x)$, $q(x)$, $f(x)$, $g(x)$ सायं गो ं नमावां , बां ैं

Real Polynomial (तेीअर्क्रां ु मणेवद

Let $a_0,a_1,a_2,...,a_n$ be real numbers and x is a real variable, then, $f(x) = a_0+a_1x+a_2x^2+...+a_nx^n$ is called a real polynomial of real variable x with real coefficients.

दापां यखं $a_0,a_1,a_2,...,a_n$ ं ।गर्बय खं अुण्वातुं ुसिोंि x तखं ं ।गर्बय ख उे ं ैं यिोंऐं $f(x) = a_0+a_1x+a_2x^2+...+a_nx^n$ खांन् ।गर्बय खं रैजाुख खंन्आषं ं ।गर्बय खं उे ं x खां ं ।गर्बय खं ह ख्नं ख ं , बां ैं

Degree of a Polynomial (र्यं ु मणेवीं पबिा क्ष

❖ A Polynomial $f(x)=a_0+a_1x+a_2x^2+a_3x^3+...+a_nx^n$, real or complex is a polynomial of degree n, if $a_n \neq 0$.

तखं ह ख्नं $f(x)=a_0+a_1x+a_2x^2+a_3x^3+...+a_nx^n$, ं ।गर्बय खं वा , यख्थं यरू(n खां तखं ह ख्नं ैं सरे ं तखं $a_n \neq 0$ट

Some Important Deduction

(i) Linear Polynomial→ A polynomial of degree one is known as linear polynomial.

ैं ब्रा रु मणेव→ं तखं)ाबं ं ।थ्ंह ख्नं खांन् मि खं ह ख्नं ख बं ुैं

(ii) Quadratic Polynomial→ A polynomial of second degree is known as quadratic polynomial.

क्कं अु मणेव→ं नांन्)ाबं ं ।थू ं ह ख्नं खांन्या)ाबं ह ख्नं खंन्लधं दुन्, ।पां , बां ैं

(iii) Cubic Polynomial→ A polynomial of degree three is known as cubic polynomial.

क्कंै अु मणेव→ं बूपं)ाबं ं ।थू ं ह ख्नं खांन्)पं ह ख्नं खंन्लधं दुं, ।पां , बां ैं

(iv) Biquadratic Polynomial→ A polynomial of degree four is known as biquadratic polynomial.

क्कता प्पा ु मणेव→ं उे ं)ाबं ं ।थू ं ह ख्नं खांन्यां रू कं ह ख्नं खंन्लध दुं, ।पां , बां ैं

Polynomial Equation चु मणेवइन्पै से

If $f(x)$ is a polynomial, real or complex, then $f(x)=0$ is called a polynomial equation.

सरे ं $f(x)$ तखं ह ख्नीं ं ।गर्बय खं वां , यख्थं ैं बांन् $f(x)=0$ तखं ह ख्न अदू खैं ं ख ं , बां ैं

Quadratic Equation (क्कंै अइन्पै सेद

❖ A quadratic polynomial $f(x)$ when equated to zero is called quadratic equation.

तखं या)ाबं ह ख्नं $f(x)$, हं मा ख्नं खंन्हे हे ं ांन्बांन्या)ाबं अदू खैं । ख थबां ैं

i.e. $ax^2+bx+c=0$, where $a,b,c \in R$ and $a \neq 0$.

Roots of a Quadratic Equation

र्य क्किंै अइन्पै सेर्ं पन्थ

The values of variable x which satisfy the quadratic equation is called roots of quadratic equation.

उे ं x खंन्ं न्दापं , ांन्या)ाबं अदू खैं ं खांन् अुबझं खे बंन् ुैं या)ाब अदू खैं ं खंन्दघं ख थबंन् ुैं

Solution of Quadratic Equation

क्कंै अइन्पै सेर्ं ंमिग

1. Factorisation Method चां ैंक्कग्र क्कबिाब

Let $ax^2 + bx + c = a(x-\alpha)(x-\beta) = 0$. Then, $x = \alpha$ and $x = \beta$ will satisfy the given equation.

दापं थू य तं $ax^2 + bx + c = a(x-\alpha)(x-\beta) = 0$. बहीं $x = \alpha$ सोंि $x = \beta$ यातं रतं अदू खैं ं खांन् अुबझं खे ुस्ट

2. Direct Formula चद्विध लेड्डिाेंख

Quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ has two roots, given by

या)ाबं अदू खैं ं $ax^2 + bx + c = 0$ $(a \neq 0)$ खंन्नांन्दघं ुैं , न् यास्वाय्थ बं ुैं

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a},$$

$$\beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

$$\text{or } \alpha = \frac{-b+\sqrt{D}}{2a}, \beta = \frac{-b-\sqrt{D}}{2a}$$

where, $D = \Delta = b^2 - 4ac$ is called discriminate of the equation.

, ैं $D = \Delta = b^2 - 4ac$ अदू खैं ं खां र्य र्यू एवं ख थबां ैं

Above formulas also known as **Sridharacharya formula**.

ं धे ृनएं अन्खांन्खां :ू डे ।उावंक्कअन्खं खंन्पादं अंन् ू ं , ।पां , बां ैं

Given quadratic equation→

$ax^2 + bx + c = 0$

$px^2 + qx + r = 0$

Condition for common root/ शून्यक्मनमीकृमीताश्क्मक्मणकृ

One root common/कृ न्तम न्शअन शून्।मि→ $(aq - pb)(br - qc) = (cp - ra)^2$

Both root common/कृ न्(ीी)शअन शून्।मि→ $\frac{a}{p} = \frac{b}{q} = \frac{c}{r}$

Formation of new quadratic equation by changing the roots of a given quadratic equation

कूंि।ि्कृेख्ञ्न शब्मं दान्मीश्अन्मि्ि (अमं ू ब्कृेख्ञ्न शब्मं दा ं ।ा

1. If new roots are $(\alpha + p)$ and $(\beta + p)$

कृंू ि्श्अन्$(\alpha + p)$न्साह्$(\beta + p)$न्।ह

ृ न्Required equation→ $a(x - p)^2 + b(x - p) + c = 0$

2. If new roots are $(\alpha - p)$ and $(\beta - p)$

कृंू ि्श्अन्$(\alpha - p)$न्साह्$(\beta - p)$न्।ह

Then required equation→ $a(x + p)^2 + b(x + p) + c = 0$

3. If new root are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$

कृंू ि्श्अन्$\frac{1}{\alpha}$न्साह्$\frac{1}{\beta}$न्।ह

Then required equation→ $a\left(\frac{1}{x}\right)^2 + b\frac{1}{x} + c = 0$

4. If new roots are α^2 and β^2

कृंू ि्श्अन्α^2 साह्β^2न्।ह

Then required equation→ $a\left(\sqrt{x}\right)^2 + b\sqrt{x} + c = 0$

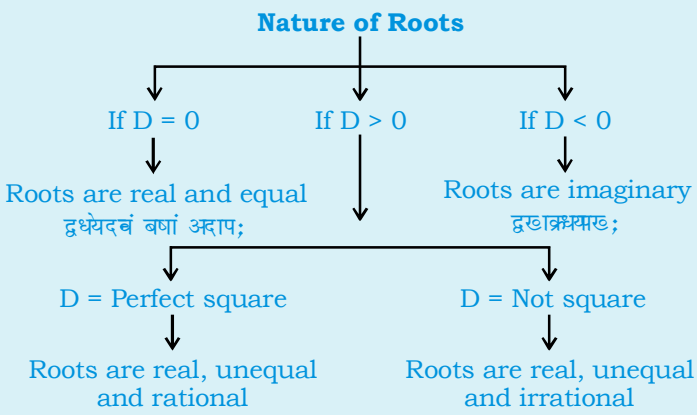
5. If new roots are $p\alpha$ and $p\beta$

कृंू ि्श्अन्$p\alpha$ साह्$p\beta$ ।ह

Then required equation⇒ $a\left(\frac{x}{p}\right)^2 + b\frac{x}{p} + c = 0$

Nature of Roots (न्थर्ा पंक्ति ब्क्मह

(i) Let quadratic equation be $ax^2 + bx + c = 0$, whose discriminate is D.



(ii) **Conjugate Roots**→ The irrational (complex) roots of a quadratic equation, whose coefficients are rational (real) always occur in conjugate pairs. Thus,

इरक्म्पन्थ→यिा)ाबं अदू ख्ै ं ख्ंन्सधेयदबं द्व, यश्थ; दघीं य पख्न रैजाुख ंधेयदबं द्व ाग्बर्य ख; ाबं ुर्ी अर्नी अुवक्नं वक्नुंदुं न्बं ुट्टिस्अ व्खो ।

(a) If one root be $\alpha + i\beta$, then other root will be $\alpha - i\beta$.

वयां तख ं दघं $\alpha + i\beta$ ं बिांन्नख्ो ं दघं $\alpha - i\beta$ ं धस्ं ास्ट

(b) If one root be $\alpha + \sqrt{\beta}$, then other root will be $\alpha - \sqrt{\beta}$.ं वयां तख ं दघं $\alpha + \sqrt{\beta}$ ं बिांन्नख्ो ं दघं $\alpha - \sqrt{\beta}$ ास्ूट

Relation between Roots and Coefficients (न्थर्छिं ाि्ख्ो ं ं ुिप्ड्ि ु /द

1. Quadratic Equation चबिभे अड्ि न्पं से

If roots of quadratic equation $ax^2+bx+c = 0$ $(a \neq 0)$ are α and β, then

वयां या)ाबं अदू ख्ै ं $ax^2+bx+c = 0$ $(a \neq 0)$ ं ख्ंन्दघं α सोंि β ुर्ी बान्

Sum of roots ं दघ्ांुन्खां वास = $S = \alpha + \beta = \frac{-b}{a}$

$= -\frac{\text{Coefficient of } x \text{ द्वx खां रैजाुख;}}{\text{Coefficient of } x^2 \text{ द्वx}^2 \text{ खां रैजाुख;}}$

Product of roots ं दघ्ांुन्खां रैजपधाए

$= P = \alpha \cdot \beta = \frac{c}{a} = \frac{\text{constant term / द्वसउे ं धन;}}{\text{coefficient of } x^2 \text{ द्वx}^2 \text{ खां रैजाुख;}}$

Also, $|\alpha - \beta| = \frac{\sqrt{D}}{|a|}$

2. Cubic Equation चभ्िकड्ि न्पं से

If α, β and γ are the roots of cubic equation $ax^3 + bx^2 + cx + d = 0$.

वयां α, β सोंि γ)ापं अदू ख्ै ं $ax^3 + bx^2 + cx + d = 0$ ं ख्ंन्दघं ु ुट्टि

Then, $\Sigma\alpha = \alpha + \beta + \gamma = -\frac{b}{a}$

$\Sigma\alpha\beta = \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$

$\alpha\beta\gamma = -\frac{d}{a}$

3. Biquadratic Equation चबिता फ्ाड्ि न्पं से

If α, β, γ and δ are the roots of the biquadratic equation

वयां α, β, γ सोंि δ र्या रू कं अदू ख्ै ं ख्ंन्दघं ु $ax^4 + bx^3 + cx^2 + dx + e = 0$, then

$S_1 = \alpha + \beta + \gamma + \delta = -\frac{b}{a}$,

$S_2 = \alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta$

$= (-1)^2 \frac{c}{a} = \frac{c}{a}$

or $S_2 = (\alpha+\beta)(\gamma+\delta) + \alpha\beta + \gamma\delta = \frac{c}{a}$

$S_3 = \alpha\beta\gamma + \beta\gamma\delta + \gamma\delta\alpha + \alpha\beta\delta$

$= (-1)^3 \frac{d}{a} = -\frac{d}{a}$

or $S_3 = \alpha\beta(\gamma+\delta) + \gamma\delta(\alpha+\beta) = -\frac{d}{a}$

and $S_4 = \alpha\cdot\beta\cdot\gamma\cdot\delta = (-1)^4 \frac{e}{a} = \frac{e}{a}$

Formation of Polynomial Equation from Given Roots

❖ If $\alpha_1, \alpha_2, \alpha_3, \ldots, a_n$ are the roots of an nth degree equation, then the equation is $x^n - S_1x^{n-1} + S_2x^{n-2} - S_3x^{n-3} + \ldots + (-1)^n S_n = 0$, where S_n denotes the sum of the products of roots taken n at a time.

1. Quadratic Equation

If α and β are the roots of a quadratic equation, then the equation is $x^2 - S_1x + S_2 = 0$, where S_1 = sum of roots and S_2 = product of roots

2. Cubic Equation

If α, β and γ are the roots of cubic equation, then the equation is

$x^3 - S_1x^2 + S_2x - S_3 = 0$

i.e. $x^3 - (\alpha+\beta+\gamma)x^2 + (\alpha\beta+\beta\gamma+\gamma\alpha)x - \alpha\beta\gamma = 0$

3. Biquadratic Equation

If α, β, γ and δ are the roots of a biquadratic equation, then the equation is

$x^4 - S_1x^3 + S_2x^2 - S_3x + S_4 = 0$

i.e. $x^4 - (\alpha+\beta+\gamma+\delta)x^3$

$+ (\alpha\beta + \beta\gamma + \gamma\delta + \alpha\delta + \beta\delta + \alpha\gamma)x^2$

$- (\alpha\beta\gamma + \alpha\beta\delta + \beta\gamma\delta + \gamma\delta\alpha)x + \alpha\beta\gamma\delta = 0$

Maximum and Minimum Values of Quadratic Expression

(i) If $a > 0$, quadratic expression has least value at $x = \frac{-b}{2a}$. This least value is given by $\frac{4ac - b^2}{4a} = -\frac{D}{4a}$. But their is no greatest value.

(ii) If $a < 0$, quadratic expression has greatest value at $x = -\frac{b}{2a}$. This greatest value is given by $\frac{4ac - b^2}{4a} = -\frac{D}{4a}$. But their is no least value.

Inequality

❖ A statement involving the symbols >, <, ≤ or ≥ is called an inequality or in equation.

Here, the symbols < (less than), > (greater than), ≤ (less than or equal to) and ≥(greater than or equal to) are known as symbol of inequalities.

e.g. $5 < 7$, $x \le 2$, $x + y \ge 11$

Types of Inequalities

(i) **Numerical inequality**→ An inequality which does not involve any variable is called a numerical inequality.

e.g. $4 > 2$, $8 < 21$

(ii) **Literal inequality**→ An inequality which have variables is called literal inequality.

e.g. $x < 7$, $y \ge 11$, $x - y \le 4$

(iii) **Strict inequality**→ An inequality which have only <or> is called strict inequality.

e.g. $3x + y < 0$, $x > 7$

(iv) **Slack inequality**→ An inequality which have only

≥or≤ is called slack inequality.

e.g. $3x + 2y \le 0$, $y \ge 4$

Maximum and Minimum value in Algebra

MAXIMUM AND MINIMUM VALUES

❖ **Concept-1**

	Max. value	Min.value
Odd power → (x) →	$+\infty$	$-\infty$
Even power → (x^2)→	$+\infty$	0

Ex.(i) $10 + (x^2)$ Max → $+\infty$, Min → 0

Min. value of $(10 + x^2) = 10 + 0 = 10$

Max. value of $(10 + x^2) = 10 + \infty = -\infty$

(ii) $10 + (x^3)$ Max → $-\infty$, Min → $+\infty$

Min. value of $(10 + x^3) = 10 - \infty = -\infty$

Max. value of $(10 + x^3) = 10 + \infty = +\infty$

(iii) $10 - (x^2)$ Max → $+\infty$, Min → 0

Min. value of $10 - x^2 = 10 - \infty = -\infty$

Max. value of $10 + x^2 = 10 - 0 = 10$

❖ If $x + y = a$ then the value of $x \times y$ will be maximum at $x = y$

Ex. If a, b, c, d are + we number such that a + b + c + d = 1 then find the maximum value of abcd?

Sol. a + b + c + d = 1 (abcd)max.

$$a = b = c = d = \frac{1}{4} = \frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4} = \frac{1}{256}$$

S. no.	Pair of lines or equation	$\frac{a_1}{a_2}$	$\frac{b_1}{b_2}$	$\frac{c_1}{c_2}$	Compare of ratios	Graphical representation	Algberic representation
1.	$x-2y=0$ $3x+4y-20=0$	$\frac{1}{3}$	$\frac{-2}{4}$	$\frac{0}{-20}$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Exactly one solution (unique)
2.	$2x+3y-9=0$ $4x+6y-18=0$	$\frac{2}{4}$	$\frac{3}{6}$	$\frac{-9}{-18}$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Many solutions
3.	$x+2y-4=0$ $2x+4y-12=0$	$\frac{1}{2}$	$\frac{2}{4}$	$\frac{-4}{-12}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No. solution

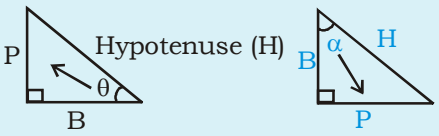
❖ Quadratic equation and positive/negative roots

Quadratic equation	**Positive/Negative roots**
If quadratic equation is like	Then
(a) $ax^2 + bx + c = 0$	both value of x is negative
(b) $ax^2 - bx + c = 0$	both value of x is positive
(c) $ax^2 + bx - c = 0$ and one is negative	One value of x is positive
(d) $ax^2 - bx - c = 0$ and one is negative	One value of x is positive

Trigonometry (ैअक्कै तै र बन्

Basic Triplets Theory (रेखा ैअका ण ै ब्रि-

1.



P → Perpendicular से ंख

B → Base सा ि ब

$H^2 = P^2 + B^2$ → Pythagoras Theorem सन्दु य बआरक्एम

2. A Pythagoras triplet is a set of Positive integers a, b and c that fits the rule : $a^2 + b^2 = c^2$

ू न रन्दु य बआरक्यष्टरि हरन्त्र व सa, bरा ैबcरन रू न रधअरौरध अ $a^2 + b^2 = c^2$ ढपएरन रन्े हरन बजरौग

$3^2 + 4^2 = 5^2$

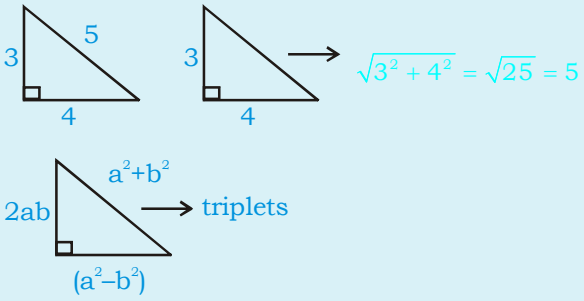
∴ 3, 4, 5 are triplets सक्यष्टरौल

3. **Some Pythagorean Triplets** दाकुप्र ै समै वैक्ष्ण ैअर्का

(3, 4, 5)	(5, 12, 13)	(7, 24, 25)
(8, 15, 17)	(9, 40, 41)	(11, 60, 61)
(12, 35, 37)	(13, 84, 85)	(16, 63, 65)
(20, 21, 29)	(28, 45, 53)	(33, 56, 65)
(36, 77, 85)	(39, 80, 89)	(48, 55, 73)
(65, 72, 97)	(20, 99, 101)	

❖ In a triplet largest side is hypotenuse

ू न रन्न रए सम्मख्यसर्ब्भरण ध रन तरी जरौ



❖ a^2-b^2, 2ab, a^2+b^2
x^2-1, $2x$, x^2+1
a–b, $2\sqrt{ab}$, a+b
Triplets form

4. Multiplication and division of basic triplets results into other triplets

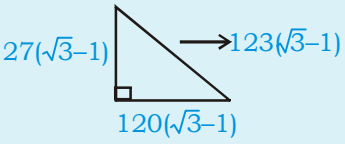
ऐ, रन्न रन रयष्टहरा ैबठण ध्हरा ठरन्न रए सछे रध जरौ

(3, 4, 5) $\xrightarrow{\times 2}$ (6, 8, 10) $\xrightarrow{\times 2}$ (12, 16, 20)

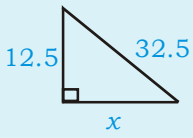
(5, 12, 13) $\xrightarrow{\times 2}$ (10, 24, 26) $\xrightarrow{\times 1.5}$ (15, 36, 39)

(3, 4, 5) $\xrightarrow{\times\sqrt{2}}$ $(3\sqrt{2}, 4\sqrt{2}, 5\sqrt{2})$

❖



9, 40, 41 → Triplet
↓×3 ↓×3 ↓×3
27 120 123

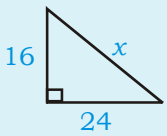


12.5 : 32.5 5, 12, 13 → Triplet
5 : 13

∴ $x = 12 \times 2.5 =$ **30**

यर ा Triplet नै ा र नत

❖

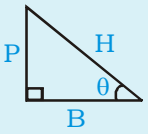


16 : 24 : x
↓

8×2 : 8×3 : $8\sqrt{9+4} = 8\sqrt{13}$

Basic Trigonometry Ratios (रेखा ैअक्कै तै र बा प्रनु ै बन्

1.



$\sin\theta = \frac{P}{H}$ $\cot\theta = \frac{B}{P}$

$\cos\theta = \frac{B}{H}$ $\sec\theta = \frac{H}{B}$

$\text{Tan}\theta = \frac{P}{B}$ $\text{cosec}\theta = \frac{H}{P}$

2. $\text{Cosec}\theta = \frac{1}{\sin\theta} \Rightarrow$ **$\sin\theta \times \text{cosec}\theta = 1$**

$\text{Sec}\theta = \frac{1}{\cos\theta} \Rightarrow$ **$\cos\theta \times \sec\theta = 1$**

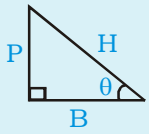
$\text{Cot}\theta = \frac{1}{\tan\theta} \Rightarrow$ **$\tan\theta \times \cot\theta = 1$**

3. $\text{Tan}\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sec\theta}{\text{cosec}\theta}$

$\text{Cot}\theta = \frac{\cos\theta}{\sin\theta} = \frac{\text{cosec}\theta}{\sec\theta}$

Basic Trigonometric Identities (रेखा अंकित त्रिभुज पर आधारित)

❖



$P^2 + B^2 = H^2$

1. $\left(\frac{P}{H}\right)^2 + \left(\frac{B}{H}\right)^2 = 1 \Rightarrow$ **$\sin^2\theta + \cos^2\theta = 1$**

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta \rightarrow \frac{\sin\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin\theta}$

$\cos^2\theta = 1 - \sin^2\theta \rightarrow \frac{\cos\theta}{1-\sin\theta} = \frac{1+\sin\theta}{\cos\theta}$

$(\sin^2\theta + \cos^2\theta)^2 = 1^2$

$\sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta = 1$

$\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$

$(\sin^2\theta + \cos^2\theta)^3 = (1)^3$

$\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta = 1$

$\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

2. $(\sin^2\theta + \cos^2\theta = 1) \div \cos^2\theta$

$\tan^2\theta + 1 = \sec^2\theta$

$\tan^2\theta = \sec^2\theta - 1 \rightarrow \tan^2\theta = (\sec\theta+1)(\sec\theta-1)$

$\downarrow$

$\frac{\text{Tan}\theta}{(\sec\theta+1)} = \frac{(\sec\theta-1)}{\tan\theta}$

$\sec^2\theta - \tan^2\theta = 1$

$(\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

$(\sec\theta - \tan\theta) = \frac{1}{(\sec\theta + \tan\theta)}$

$(\sec^2\theta - \tan^2\theta)^2 = (1)^2$

$\sec^4\theta + \tan^4\theta - 2\sec^2\theta\tan^2\theta = 1$

$\sec^4\theta + \tan^4\theta = 1 + 2\sec^2\theta\tan^2\theta$

$(\sec^2\theta - \tan^2\theta)^3 = (1)^3$

$\sec^6\theta - \tan^6\theta - 3\sec^2\theta\tan^2\theta \times 1 = 1$

$\sec^6\theta - \tan^6\theta = 1 + 3\sec^2\theta\tan^2\theta$

3. $(\sin^2\theta + \cos^2\theta = 1) \div \sin^2\theta$

$1 + \cot^2\theta = \text{cosec}^2\theta$

$\cot^2\theta = \text{cosec}^2\theta - 1$ $\Rightarrow \frac{\cot\theta}{\text{cosec}\theta+1} = \frac{\text{cosec}\theta-1}{\cot\theta}$

$\text{cosec}^2\theta - \cot^2\theta = 1$

$(\text{cosec}\theta + \cot\theta)(\text{cosec}\theta - \cot\theta) = 1$

$(\text{cosec}\theta + \cot\theta) = \frac{1}{(\text{cosec}\theta - \cot\theta)}$

$\text{cosec}^4\theta + \cot^4\theta = 1 + 2\text{cosec}^2\theta\cot^2\theta$

$\text{cosec}^6\theta - \cot^6\theta = 1 + 3\,\text{cosec}^2\theta\cot^2\theta$

Trigonometry Ratio Table (अंकित त्रिभुज पर आधारित)

Angle (In Degrees)	**0°**	**30°**	**45°**	**60°**	**90°**	**180°**	**270°**	**360°**
Angle (In Radians)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	−1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	−1	0	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not Defined	0	Not Defined	0
cot	Not Defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	Not Defined	0	Not Defined
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not Defined	−1	Not Defined	1
cosec	Not Defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	Not Defined	−1	Not Defined

❖ $\sec^2\theta + \operatorname{cosec}^2\theta = \sec^2\theta\operatorname{cosec}^2\theta$

$\tan^2\theta - \sin^2\theta = \tan^2\theta\sin^2\theta$

$\cot^2\theta - \cos^2\theta = \cot^2\theta\cos^2\theta$

$$\tan\theta+\cot\theta=\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=\frac{1}{\sin\theta\cos\theta}$$

$=\sec\theta\operatorname{cosec}\theta=2\operatorname{cosec}2\theta$

Range

1. $-1 \le \sin\theta \le +1$

$\sin\theta = \frac{P}{H}$, $H > P$ (∴ Always between –1 and +1)

$-1 \le \cos\theta \le +1$ $\quad (-1)^2 = 1$

$-\infty \le \tan\theta, \cot\theta \le +\infty$

$\therefore 0 \le \sin^2\theta, \cos^2\theta \le +1$

$0° \to 90°$ $\sin\theta$ increases from 0 to 1

$0° \to 90°$ $\cos\theta$ decreases from 1 to 0

$\sin 61° > \cos 32° \Rightarrow \cos 32° = \sin 58°$

$\therefore \sin 61° > \sin 58°$

$\therefore \sin 61° > \cos 32°$

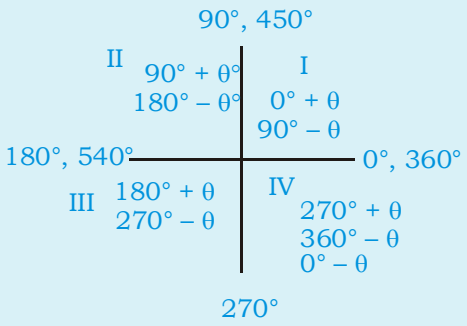
2. $0° < \theta < 45°$ $\quad \sin\theta < \cos\theta$ $\quad \sin 19° < \cos 19°$

$45° < \theta < 90°$ $\quad \sin\theta > \cos\theta$ $\quad \sin 71° > \cos 71°$

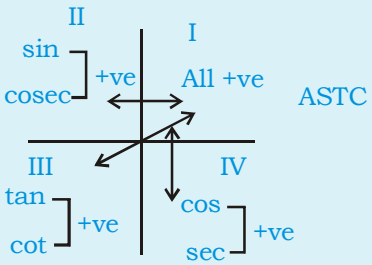
3. If $0 < x < 1$, $\quad x > x^2$

Quadrant theory (धबुमै ङ्ग्रा ण ै क्रि-

1.



θ → Any Acute Angle दा कै सङ्गेना कै तै



sin न।र → sin +ve in I, II

cos (मं र → र cos +ve in I, IV

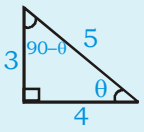
tan ठ्ठब) र → र Tan +ve in I, III

2. Change of T-Ratio दा ैअकै तै र ब/प्र नु ै बा कै। ैद्विबध

90°, 270° .... (90° न यह odd multiple न ब change)

180°, 360° .... no change (90° न यह even multiple न ब no change)

sin ↔ cos, tan ↔ cot, sec ↔ cosec



$\sin(90°-\theta) = \frac{4}{5} = \cos\theta$

$\therefore \cos(90°-\theta) = \sin\theta$

$\tan(90°-\theta) = \cot\theta$ $\quad \sec(90°-\theta) = \operatorname{cosec}\theta$

$\cot(90°-\theta) = \tan\theta$ $\quad \operatorname{cosec}(90°-\theta) = \sec\theta$

❖ $\sin(90°+\theta) = \cos\theta$

$\cos(90°+\theta) = -\sin\theta$

$\tan(90°+\theta) = -\cot\theta$

$\cot(90°+\theta) = -\tan\theta$

$\sec(90°+\theta) = -\operatorname{cosec}\theta$

$\operatorname{cosec}(90°+\theta) = \sec\theta$

$\tan 150° = \tan(90°+60°) = -\cot 60° = -\frac{1}{\sqrt{3}}$

$\sin 120° = \sin(90°+30°) = \cos 30° = \frac{\sqrt{3}}{2}$

$\cos 120° = \cos(180°-60°) = -\cos 60° = \frac{-1}{2}$

❖ $\sin(180°-\theta) = \sin\theta$

$\cos(180°-\theta) = -\cos\theta$

$\cot(180°-\theta) = -\cot\theta$

$\sec 150° = \sec(180°-30°) = -\sec 30° = \frac{-2}{\sqrt{3}}$

$\tan 135° = \tan(180°-45°) = -\tan 45° = -1$

❖ $\sin(180°+\theta) = -\sin\theta$

$\cos(180°+\theta) = -\cos\theta$

$\cos 210° = \cos(180°+30°) = -\cos 30° = \frac{-\sqrt{3}}{2}$

$\sin 225° = \sin(180°+45°) = -\sin 45° = \frac{-1}{\sqrt{2}}$

$\cot 240° = \cot(270°-30°) = \tan 30° = \frac{1}{\sqrt{3}}$

❖ $\cos(270°-\theta) = -\sin\theta$

$\tan(270°-\theta) = \cot\theta$

$\operatorname{cosec}(270°-\theta) = -\sec\theta$

$\sec 240° = \sec(270°-30°) = -\operatorname{cosec} 30° = -2$

❖ $\sin(270°+\theta) = -\cos\theta$

$\cos(270°+\theta) = \sin\theta$

$\tan(270°+\theta) = -\cot\theta$

$\sin300° = \sin(270°+30°) = -\cos30° = \frac{-\sqrt{3}}{2}$

$\cot315° = \cot(270°+45°) = -\tan45° = -1$

$\text{Cosec}330° = \text{Cosec}(360°-30°) = -\text{cosec}30° = -2$

❖ $\sin(360°-\theta) = -\sin\theta$

$\sin(-\theta) = -\sin\theta$

$\tan780° = \tan(360°\times2+60°) = \tan60° = \sqrt{3}$

$\text{Cosec}1125° = \text{Cosec}(360°\times3+45°) = \text{Cosec}45° = \sqrt{2}$

$\cos(-\theta) = \cos\theta$

$\sin180° = 0$

$\tan(-\theta) = -\tan\theta$

$\cot(-\theta) = -\cot\theta$

$\cos180° = -1$

$\sec(-\theta) = \sec\theta$

$\tan180° = 0$

$\text{cosec}(-\theta) = -\text{cosec}\theta$

If A+B = 90° then

1. If $A+B=90° \Rightarrow$ A, B are complementary to each other

2. $\sin A = \cos B \rightarrow \sin A \sec B = 1$

3. $\tan A = \cot B \rightarrow \tan A \tan B = 1$ or $\cot A \cot B = 1$

4. $\sec A = \text{cosec} B \rightarrow \cos A \text{cosec} B = 1$

$\tan31\times\tan59° = 1$

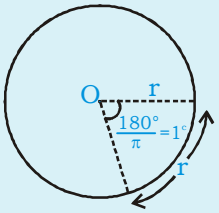
5. $\sin^2A+\sin^2B=1 \rightarrow \sin^2A+\sin^2(90-A)$

6. $\cos^2A+\cos^2B=1 \rightarrow \sin^2A+\cos^2A=1$

Radian Angle Theory (रेडियन कोण सिद्धान्त)

1. $\frac{\text{circumference}}{\text{diameter}} = \text{fixed number}$

$= \pi$ (Irrational numer)



$2\pi r \rightarrow 360°$

$r \rightarrow \frac{360°}{2\pi} = \frac{180°}{\pi} = 1$ radian

circle न सRadius न सरखबरखबरन सरArc centre बनब1 radian न सरAngle खींचा–

π radian $(\pi^c) = 180°$

$1°=60', 1'= 60''$

2. $\mathbf{1^c \approx 57°16'22''}$

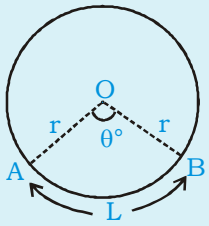
π radian $= 180°$

$1 \text{ rad } (1^c) = \frac{180°}{\pi} = \frac{180°\times7}{22} = \frac{630°}{11} = 57° + \frac{3°}{11}$

$\Rightarrow 3° = 180' \rightarrow \frac{180'}{11} = 16' + \frac{4'}{11} \quad \therefore 1^c = 57°16'22''$

$\Rightarrow 4' = 240'' \rightarrow \frac{240}{11} = 22''$

3.



$\overline{AB} = L = 2\pi r \times \frac{\theta°}{360°}$

$L = r \times \frac{\pi\theta°}{180°}$

$\mathbf{L = r \times \theta^c}$

$\mathbf{\theta^c = \theta° \times \frac{\pi}{180}}$

Some Important Properties (कुछ जरूरी बातें)

1. $\left.\begin{matrix} a\sin\theta + b\cos\theta = c \\ a\cos\theta - b\sin\theta = x \text{ (let)} \end{matrix}\right]$ square and add

$a^2\sin^2\theta + b^2\cos^2\theta + 2 \times a\sin\theta \times b\cos\theta = c^2$

$a^2\cos^2\theta + b^2\sin^2\theta - 2 \times a\sin\theta \times b\cos\theta = x^2$

$a^2 + b^2 = c^2 + x^2$

$\mathbf{x^2 = a^2 + b^2 + c^2}$

$\mathbf{x = \sqrt{a^2 + b^2 - c^2}}$

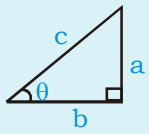
❖ If $a\sin\theta + b\cos\theta = c$ and $a^2 + b^2 = c^2$

↓ ↓ ↓

P B H

$\left.\begin{matrix} \frac{a}{c}\sin\theta + \frac{b}{c}\cos\theta = 1 \\ \sin\theta\sin\theta + \cos\theta\cos\theta = 1 \end{matrix}\right]$ compare

$\sin\theta = \frac{a}{c}, \cos\theta = \frac{b}{c}$



❖ If $48\sin\theta + 55\cos\theta = 73$ then $\cot\theta = \frac{55}{48}$

↓ ↓ ↓

P B H

❖ $(\sin\theta+\sec\theta)^2+(\cos\theta+\operatorname{cosec}\theta)^2 = (1+\sec\theta\operatorname{cosec}\theta)^2$

❖ $(1-\sec\theta+\tan\theta)(1+\operatorname{cosec}\theta+\cot\theta) = \mathbf{2}$

2. If $a\sec A+b\tan A = c$
Then $a\tan A+b\sec A = x$ (Let)) square and subtract

$a^2-b^2=c^2-x^2 \Rightarrow x^2 = c^2 - a^2+b^2$

$\boldsymbol{x = \pm\sqrt{c^2 - a^2 + b^2}}$

❖ If $a\sec A - b\tan A = c$

$a\tan A - b\sec A = x$ (let)

$\Rightarrow \mathbf{a^2-b^2=c^2-x^2}$

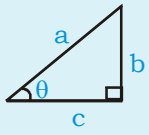
❖ If $a\sec\theta-b\tan\theta = c$ and $a^2=b^2+c^2$

$\frac{a}{c}\sec\theta-\frac{b}{c}\tan\theta=1$
$\sec\theta\sec\theta-\tan\theta=1$) compare $\sec\theta=\frac{a}{c}$, $\tan\theta=\frac{b}{c}$

$\therefore a\sec\theta-b\tan\theta = c$

↓ ↓ ↓

H P B



3. **If $\sec x + \tan x = \frac{a}{b}$ then $\operatorname{cosec} x + \cot x = \frac{a+b}{a-b}$**

(a > b)

$\sec x + \tan x = \frac{a}{b}$

$\sec x - \tan x = \frac{b}{a}$

$2\sec x = \frac{a^2+b^2}{ab}$

$\sec x = \frac{a^2+b^2}{2ab}$

$\sec x = \frac{a^2+b^2}{2ab} \begin{matrix}\rightarrow H\\ \rightarrow B\end{matrix}$

$\therefore P = a^2-b^2$

↓

$\frac{H}{P}+\frac{B}{P}=\frac{H+B}{P}$

$\Rightarrow \frac{a^2+b^2+2ab}{a^2-b^2}$

$= \frac{(a+b)^2}{(a+b)(a-b)}$

$\Rightarrow \frac{a+b}{a-b}$

Formula form - (A+B)

1. $\sin(A+B) = \sin A\cos B+\cos A\sin B$
$\sin(A-B) = \sin A\cos B - \cos A\sin B$
$\cos(A+B) = \cos A\cos B - \sin A\sin B$
$\cos(A-B) = \cos A\cos B + \sin A\sin B$

2. $2\sin A\cos B = \sin(A+B) + \sin(A-B)$
$2\cos A\sin B = \sin(A+B) - \sin(A-B)$
$2\cos A\cos B = \cos(A+B) + \cos(A-B)$
$2\sin A\sin B = \cos(A-B) - \cos(A+B)$

3. $\sin(A+B)\sin(A-B) = \sin^2A\cos^2B - \cos^2A\sin^2B$
$= \sin^2A(1-\sin^2B)-(1-\sin^2A)\sin^2B$
$= \sin^2A-\sin^2B = \cos^2B-\cos^2A$
$\cos(A+B)\cos(A-B) = \cos^2A-\sin^2B=\cos^2B-\sin^2A$

4. $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

❖ $\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$

❖ $\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

❖ $\cos D - \cos C = 2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$

❖ $\cos C - \cos D = 2\sin\frac{C+D}{2}\sin\frac{D-C}{2}$

Multiples of Angle (कै तै ा कतवुं कन

1. $\sin 2A = 2\sin A\cos A = \frac{2\tan A}{1+\tan^2 A}$

$\sin A = 2\sin\frac{A}{2}\cos\frac{A}{2} = \sqrt{\frac{1-\cos 2A}{2}}$

$\operatorname{cosec} 2A = \frac{\sec A.\operatorname{Cos}\, ecA}{2}$

2. $\cos 2A=\cos^2A-\sin^2A=1-2\sin^2A=2\cos^2A-1=\frac{1-\tan^2 A}{1+\tan^2 A}$

$= \frac{\cot^2 A-1}{\cot^2 A+1}$

$\cos A = \sqrt{\frac{1+\cos 2A}{2}}$

3. $\text{Tan}2A = \frac{2\text{Tan}A}{1-\tan^2 A}$

4. $\text{Sin}3A = 3\sin A - 4\sin^3 A$

$\text{Cos}3A = 4\cos^3 A - 3\cos A$

$\text{Tan}3A = \frac{3\tan A - \tan^3 A}{1-3\tan^2 A}$

5. $\text{Tan}(A+B) = \frac{\text{Tan}A + \text{Tan}B}{1 - \text{Tan}A\,\text{Tan}B}$

$\text{Tan}(A-B) = \frac{\text{Tan}A - \text{Tan}B}{1 + \text{Tan}A\,\text{Tan}B}$

6. $\text{Cot}(A+B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$

$\text{Cot}(A-B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$

Some special formulae (कुछ विशेष सूत्र)

1. $\text{Tan}(45-\theta) = \frac{1-\tan\theta}{1+\tan\theta}$, $\text{Tan}(45+\theta) = \frac{1+\tan\theta}{1-\tan\theta}$

2. If A+B = 45° or 225° ⇒ **(1+TanA) (1+TanB) = 2**

$\tan(A+B) = \tan 45°$

$\frac{\tan A + \tan B}{1-\tan A\tan B} = 1$ ⇒ **tanA+tanB+tanAtanB = 1**

1+tanA+tanB+tanAtanB=1+1

⇒ **(1+tanA) (1+tanB) = 2**

3. If A+B = 45° or 225° ⇒ **(cotA–1) (cotB–1) = 2**

OR **(1–cotA) (1–cotB) = 2**

4. $\text{Tan}(A+B+C) = \frac{\tan A + \tan B + \tan C - \tan A\tan B\tan C}{1 - \tan A\tan B - \tan B\tan C - \tan C\tan A}$

Put B = C = 'A' ⇒ **Tan3A = $\frac{3\text{TanA} - \tan^3 A}{1 - 3\tan^2 A}$**

If A+B+C=90°

$\frac{\sin 90°}{\cos 90°} = \frac{1}{0} = \frac{\tan A + \tan B + \tan C - \tan A\tan B\tan C}{1 - \tan A\tan B - \tan B\tan C - \tan C\tan A}$

⇒ TanATanB+TanBTanC+tanCTanA = 1

OR cotA+cotB+cotC = cotAcotBcotC

❖ If A+B+C = 180°

⇒ TanA + TanB + TanC = TanATanBTanC

⇒ cotA.cotB + cotBcotC + cotCcotA = 1

5. $\text{Tan}4\theta = \frac{4\tan\theta(1-\tan^2\theta)}{1 - 6\tan^2\theta + \tan^4\theta}$

6. $\cos^2\theta + \cos^2(60-\theta) + \cos^2(60+\theta) = \frac{3}{2}$

7. $\cos^3 A + \cos^3(120°-A) + \cos^3(120°+A) = \frac{3}{4}\cos 3A$

8. $\tan\theta - \tan(60-\theta) + \tan(60+\theta) = 3\tan 3\theta$

$\cot\theta - \cot(60-\theta) + \cot(60+\theta) = 3\cot 3\theta$

9. $\sin\theta\sin(60-\theta)\sin(60+\theta) = \frac{1}{4}\sin 3\theta$

$\cos\theta\cos(60-\theta)\cos(60+\theta) = \frac{1}{4}\cos 3\theta$

$\tan\theta\tan(60-\theta)\tan(60+\theta) = \tan 3\theta$

10. $\cos\theta\cos 2\theta\cos 2^2\theta\cos 2^3\theta \ldots\ldots \cos 2^n\theta = \frac{\sin 2^{n+1}\theta}{2^{n+1}\sin\theta}$

11. $\sin 18° = \frac{\sqrt{5}-1}{4}$ $\quad\quad$ $\cos 18° = \frac{\sqrt{10+2\sqrt{5}}}{4}$

$\cos 36° = \frac{\sqrt{5}+1}{4}$ $\quad\quad$ $\sin 36° = \frac{\sqrt{10-2\sqrt{5}}}{4}$

12. $\text{Sin}75° = \cos 15° = \frac{\sqrt{3}+1}{2\sqrt{2}}$

$\text{Sin}15° = \cos 75° = \frac{\sqrt{3}-1}{2\sqrt{2}}$

❖ $\text{Tan}75° = \cot 15° = \frac{\sqrt{3}+1}{\sqrt{3}-1} = 2+\sqrt{3}$

❖ $\text{Tan}15° = \cot 75° = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2-\sqrt{3}$

Some other formulae (कुछ अन्य सूत्र)

1. $(\sin^2 A + \cos^2 A - \sin A\cos A)(\sin A + \cos A)$

= **$\sin^3 A + \cos^3 A$**

2. $\frac{\cot A + \tan B}{\cot B + \tan A}$ = tanBcotA चक्रवर्ती रूप term direct multiply करके (हल कीजिए)

3. $\left(\frac{1-\tan\theta}{1-\cot\theta}\right)^2 = \tan^2\theta$

4. $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sqrt{\frac{1-\sin\theta}{1+\sin\theta}\times\frac{1-\sin\theta}{1-\sin\theta}} = \frac{1-\sin\theta}{\cos\theta} = \sec\theta - \tan\theta$

5. $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}\times\frac{1+\cos\theta}{1+\cos\theta}} = \frac{1+\cos\theta}{\sin\theta}$

$= \text{cosec}\theta + \cot\theta$

6. $\sqrt{\frac{\text{cosec}A+1}{\text{cosec}A-1}} = \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sec\theta + \tan\theta$

7. $\sqrt{\frac{\sec\alpha+\tan\alpha}{\sec\alpha-\tan\alpha}}=\sqrt{(\sec\alpha+\tan\alpha)^2}=\sec\alpha+\tan\alpha$

8. $\sqrt{\frac{\cot\theta+\cos\theta}{\cot\theta-\cos\theta}}=\sec\theta+\tan\theta$

9. $\sin1°.\sin2°.\sin3°\ .....\ \sin180°=0$
$\cos1°.\cos2°.\cos3°\ .....\ \cos90°=0$
$\tan1°.\tan2°.\tan3°\ .....\ \tan89°=1$

If A + B = 30°, $(\sqrt{3}+\tan A)(\sqrt{3}+\tan B)=4$

If A + B = 60°, $(1+\sqrt{3}\tan A)(1+\sqrt{3}\tan B)=4$

If angles are in AP/ यदि कोण AP में हैं/ यदि कोण समांतर श्रेढ़ी में हैं

$\sin\theta+\sin(\theta+\beta)+\sin(\theta+2\beta)+.....\sin(\theta+(n-1)\beta)$

$$=\frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}\sin\left(\theta+\frac{(n-1)\beta}{2}\right)$$

$\cos\theta+\cos(\theta+\beta)+\cos(\theta+2\beta)\ .....\ \cos(\theta+(n-1))\beta$

$$=\frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}\cos\left(\theta+(n-1)\frac{\beta}{2}\right)$$

Relation between degree, radian, and grade→
22 radian = 1260 degree
100 grade = 90 degree

$$\frac{D}{90}=\frac{G}{100}=\frac{2C}{\pi}$$

Some important Results ((प्रती के लिए महत्वपूर्ण

If A+B+C = 180,

1. $\tan\frac{A}{2}.\tan\frac{B}{2}+\tan\frac{B}{2}.\tan\frac{C}{2}+\tan\frac{C}{2}.\tan\frac{A}{2}=1$

2. $\sin2A+\sin2B+\sin2C=H\ \sin A.\sin B.\sin C$

3. $\cos2A+\cos2B+\cos2C=-(1+4\cos A.\cos B.\cos C)$

4. $\cos A+\cos B+\cos C=1+4\sin\frac{A}{2}.\sin\frac{B}{2}\sin\frac{C}{2}$

5. $\sin A+\sin B+\sin C=4\cos\frac{A}{2}.\cos\frac{B}{2}.\cos\frac{C}{2}$

- $(\sec^2\theta-1)(\operatorname{cosec}^2\theta-1)=1$
- $\cos^2\theta+\frac{1}{1+\cot^2\theta}=1$
- $\sin^2A+\frac{1}{1+\tan^2A}=1$
- $\tan\theta+\frac{1}{\tan\theta}=\sec\theta\operatorname{cosec}\theta$
- $\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=2\sec^2\theta$
- $\frac{\cos\theta}{1-\sin\theta}=\frac{1+\sin\theta}{\cos\theta}$
- $\frac{\cos\theta}{1+\sin\theta}=\frac{1-\sin\theta}{\cos\theta}$
- $\operatorname{cosec}^2\theta+\sec^2\theta=\operatorname{cosec}^2\theta\ \sec^2\theta$
- $(1+\tan^2\theta)(1+\sin\theta)(1-\sin\theta)=1$
- $(1+\cot^2\theta)(1-\cos\theta)(1+\cos\theta)=1$
- $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}=\operatorname{cosec}\theta-\cot\theta$
- $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}=\sec\theta-\tan\theta$
- $\sqrt{\frac{1+\sin\theta}{1+\sin\theta}}=\operatorname{cosec}\theta+\cot\theta$
- $\frac{1-\sin\theta}{1+\sin\theta}=(\sec\theta-\tan)^2$
- $\frac{1-\cos\theta}{\sin\theta}=\frac{\sin\theta}{1+\cos\theta}$
- $\frac{\sin\theta}{1-\cos\theta}=\operatorname{cosec}\theta+\cot\theta$
- $\frac{1-\cos\theta}{1+\cos\theta}=(\operatorname{cosec}\theta-\cot\theta)^2$
- $\frac{\cos\theta}{1-\sin\theta}+\frac{\cos\theta}{1+\sin\theta}=2\sec\theta$
- $\tan^2\theta+\cot^2\theta+2=\sec^2\theta+\operatorname{cosec}^2\theta$

$\sqrt{\sec^2\theta+\operatorname{cosec}^2}=\tan\theta+\cot\theta$

- $\tan^2A-\tan^2B=\frac{\cos^2B-\cos^2A}{\cos^2B\cos^2A}=\frac{\sin^2A-\sin^2B}{\cos^2A\cos^2B}$
- $\frac{\sin A-\sin B}{\cos A+\cos B}+\frac{\cos A-\cos B}{\sin A+\sin B}=0$
- $\frac{1}{1+\sin A}+\frac{1}{1-\sin A}=2\sec^2A$
- $\frac{\tan^2A}{1+\tan^2A}+\frac{\cot^2A}{1+\cot^2A}=1$
- $\sin^2A\cos^2B-\cos^2A\sin^2B=\sin^2A-\sin^2B$
- $\frac{\cot A+\tan B}{\cot B+\tan A}=\cot A\tan B$
- $\frac{\tan A+\tan B}{\cot A+\cot B}=\tan A\tan B$

- $\cot^2 A \operatorname{cosec}^2 B - \cot^2 B \operatorname{cosec}^2 A = \cot^2 A - \cot^2 B$
- $\tan^2 A \sec^2 B - \sec^2 A \tan^2 B = \tan^2 A - \tan^2 B$
- $(1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta) = 2$

Proof:-

$(1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta) = 2$

L.H.S

$$\Rightarrow \left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)\left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)$$

$$\Rightarrow \left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)\left(\frac{\cos+\sin\theta+1}{\cos\theta}\right)$$

$$\Rightarrow \frac{(\sin\theta+\cos\theta)^2-1}{\sin\theta.\cos\theta} = \frac{(\sin^2\theta+\cos^2\theta+2\sin\theta-\cos\theta)-1}{\sin\theta.\cos\theta}$$

$$\Rightarrow \frac{1+2\sin\theta\cos\theta-1}{\sin\theta.\cos\theta} = 2 \text{ Hence proved.}$$

- If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$ then $m^2 - n^2 = 4\sqrt{mn}$
- If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

  then $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$
- If $x = a\sin\theta$

  $y = b\tan\theta$

  then $\frac{a^2}{x^2}-\frac{b^2}{y^2}=1$
- If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$ then $x^2 + y^2 = 1$

  or $\Rightarrow x = \cos\theta$ and $y = \sin\theta$
- If $\frac{\sin A}{\sin B} = p$ and $\frac{\cos A}{\cos B} = q$

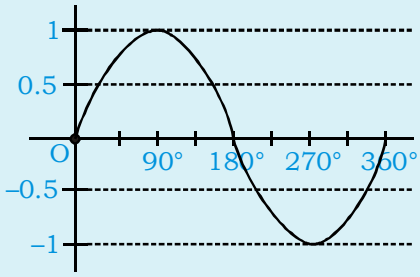
  then $\tan A = \pm\frac{p}{q}\sqrt{\frac{q^2-1}{1-p^2}}$

  $\tan B = \pm\sqrt{\frac{q^2-1}{1-p^2}}$

Interchange of Trigonometry Ratio Table (अनुपात सारणी में परिवर्तन)

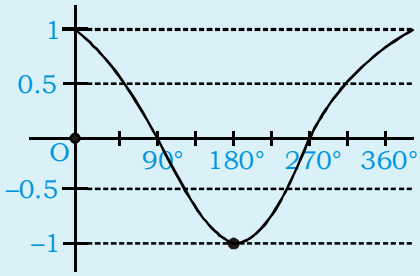
	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\operatorname{cosec}\theta$
$\sin\theta$	$\sin\theta$	$\sqrt{1-\cos^2\theta}$	$\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$	$\frac{1}{\sqrt{1+\cot^2\theta}}$	$\frac{\sqrt{\sec^2\theta-1}}{\sec\theta}$	$\frac{1}{\operatorname{cosec}\theta}$
$\cos\theta$	$\sqrt{1-\sin^2\theta}$	$\cos\theta$	$\frac{1}{\sqrt{1+\tan^2\theta}}$	$\frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$	$\frac{1}{\sec\theta}$	$\frac{\sqrt{\operatorname{cosec}^2\theta-1}}{\operatorname{cosec}\theta}$
$\tan\theta$	$\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	$\tan\theta$	$\frac{1}{\cot\theta}$	$\sqrt{\sec^2\theta-1}$	$\frac{1}{\sqrt{\operatorname{cosec}^2\theta-1}}$
$\cot\theta$	$\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$	$\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}$	$\frac{1}{\tan\theta}$	$\cot\theta$	$\frac{1}{\sec^2\theta-1}$	$\operatorname{cosec}^2\theta-1$
$\sec\theta$	$\frac{1}{\sqrt{1-\sin^2\theta}}$	$\frac{1}{\cos\theta}$	$\sqrt{1+\tan^2\theta}$	$\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$	$\sec\theta$	$\frac{\operatorname{cosec}\theta}{\operatorname{cosec}^2-1}$
$\operatorname{cosec}\theta$	$\frac{1}{\sin\theta}$	$\frac{1}{\sqrt{1-\cos^2\theta}}$	$\frac{\sqrt{1+\tan^2\theta}}{\tan\theta}$	$\sqrt{1+\cot^2\theta}$	$\frac{\sec\theta}{\sqrt{\sec^2\theta-1}}$	$\operatorname{cosec}\theta$

1. Graph of sin x



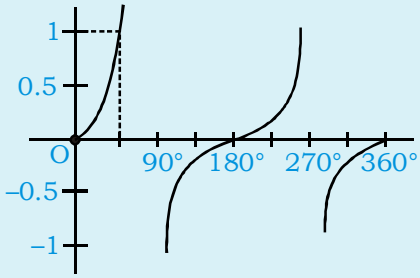
(i) Domain = R (ii) Range = [–1,1] (iii) Period = 2π

2. **Graph of cos x**



(i) Domain = R (ii) Range = [–1, 1]
(iii) Period = 2π

3. **Graph of tan x**

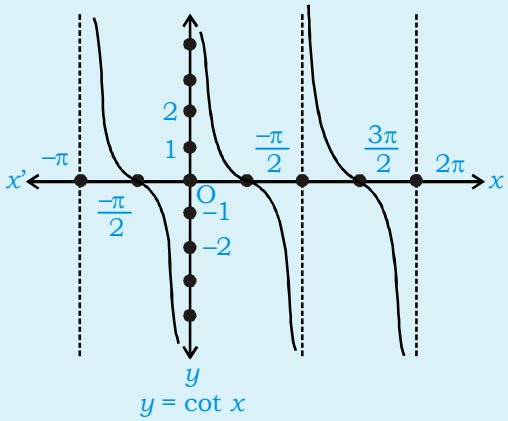


(i) Domain = R ~ $(2n+1)\frac{\pi}{2}$, $n \in I$

(ii) Range = $(-\infty, \infty)$

(iii) Period = π

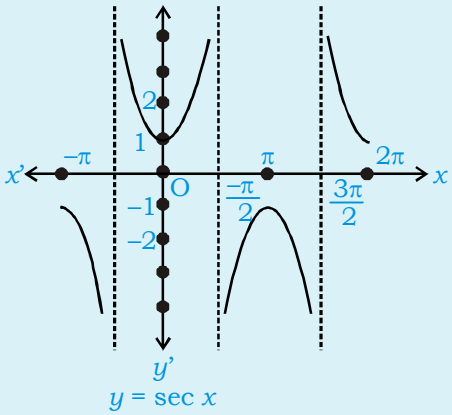
4. **Graph of cot x**



(i) Domain = R~$n\pi$, $n \in I$ (ii) Range = $(-\infty, \infty)$

(iii) Period = π

5. **Graph of sec x**

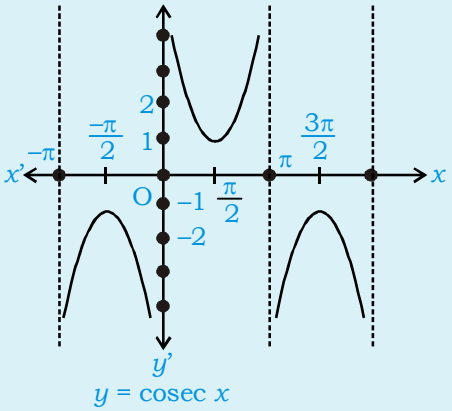


(i) Domain = R ~ $(2n + 1)\frac{\pi}{2}$, $n \in I$

(ii) Range = $(-\infty, -1] \cup [1, \infty)$

(iii) Period = 2π

6. **Graph of cosec x**



(i) Domain = R ~ $n\pi$, $n \in I$

(ii) Range = $(-\infty, -1] \cup [1, \infty)$

(iii)Period = 2π

Note $|\sin\theta| \le 1, |\cos\theta| \le 1, |\sec\theta| \ge 1, |\text{cosec}\theta| \ge |$ for all values of θ, for which the functions are defined.

Maxima & Minima (प्र टकबरा प्रै छ गेनबर न-

1. $-1 \le \sin\theta \le +1$ $\quad \sin\theta = \frac{P}{H}$, $H \ge P$

$-1 \le \cos\theta \le +1$

$-\infty \le \tan\theta \le +\infty$ $\quad \text{Tan}\theta = \frac{P}{B}$ (We can take any value of P and B)

$-\infty \le \cot\theta \le +\infty$

$-\infty \le \sec\theta, \text{cosec}\theta \le +\infty$ But -1 धस्रा न सख्खरही ग्सी ख्या-

$\text{cosec}\theta = \frac{H}{P}$ ...$H \ge (B, P)$

$\sec\theta = \frac{H}{B}$

2. If x = Real $\Rightarrow x^2 \to +ve$

$x^2_{min} = 0$ at $x = 0$

$0 \le \sin^2\theta, \cos^2\theta \le +1$

$-1 \le \sin^3\theta, \cos^3\theta \le +1$

$0 \le \tan^2\theta, \cot^2\theta \le +\infty$

$-\infty \le \tan^3\theta, \cot^3\theta \le +\infty$

$+1 \le \sec^2\theta, \text{cosec}^2\theta \le +\infty$

$-\infty \le \sec^3\theta, \text{cosec}^3\theta \le +\infty$ But -1 धस्रा $+1$ न सख्खरही ग्सी ख्या-

3. $a\sin^2\theta + b\cos^2\theta \to$ max value = max [a,b]

$\to$ min value = min [a, b]

$37\sin^2\theta + 45\cos^2\theta \Rightarrow$ maximum = **45**

minimum = **37**

4. $\sin^n\theta\cos^n\theta$ → max = $\frac{1}{2n}$, min = $\frac{-1}{2n}$] when n=odd

$\sin^n\theta\cos^n\theta$ → max = $\frac{1}{2n}$, min = 0] when n=even

5. $a\sin\theta + b\cos\theta \to$ max = $+\sqrt{a^2+b^2}$

$\to$ min = $-\sqrt{a^2+b^2}$

6. $a\sin^2\theta + b\text{cosec}^2\theta$ min = $2\sqrt{ab}$ (when a>b)

$a\cos^2\theta + b\sec^2\theta$ min = a+b (when a < b)

max = ∞

7. $a\sec^2\theta + b\text{cosec}^2\theta \xrightarrow{\text{min value}} \left(\sqrt{a}+\sqrt{b}\right)^2$

❖ If a and b are positive numbers then the value of $a\sec\theta - b\tan\theta$ = ?

Min. value = $\sqrt{a^2-b^2}$

Max. volue = ∞

8. $\sin^{2m}\theta + \cos^{2n}\theta \to$ max = +1 m, n ∈ natural no.

9. If $y = \cos^2 x + \sec^2 x$ then $y \ge 2$

	T - ratio	min	max
1.	$\sin\theta, \cos\theta$ (odd power)	-1	$+1$
2.	$\sin^2\theta, \cos^2\theta$ (even power)	0	$+1$
3.	$\tan\theta, \cot\theta$ (odd power)	$-\infty$	$+\infty$
4.	$\tan^2\theta, \cot^2\theta$ (even power)	0	∞
5.	$\sec\theta, \text{cosec}\,\theta$ (odd power)	$-\infty$	$+\infty$
6.	$\sec^2\theta, \text{cosec}^2\theta$ (even power)	1	∞

10. $\sin^{2m}\theta + \cos^{2n}\theta$

max value = 1

min value = put $\theta \to 45°$

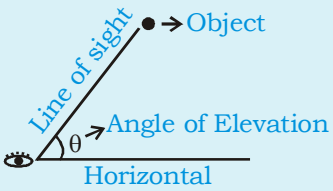
11. $a\tan^2\theta + b\cot^2\theta$

min value = $2\sqrt{ab}$

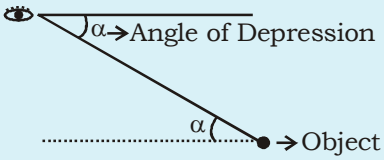
max value = ∞

Height & Distance (ऊँचाई एवं दूरी)

Elevation Angle (उन्नयन कोण)

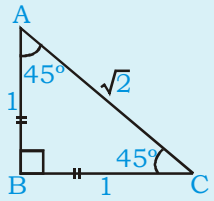


Angle of Depression (अवनमन कोण)



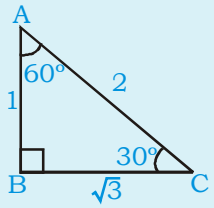
Angle-side Ratio (कोण/भुजा अनुपात)

1.



45°	45°	90°
↓	↓	↓
1 :	1 :	$\sqrt{2}$

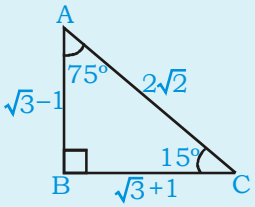
2.



30°	60°	90°
↓	↓	↓
1 :	$\sqrt{3}$:	2

$$\text{Tan}30° = \frac{1}{\sqrt{3}} = \frac{AB}{BC}$$

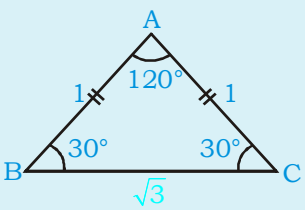
3.



$$\text{Tan}15° = \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{AB}{BC}$$

15°	75°	90°
↓	↓	↓
$\sqrt{3}-1$	$\sqrt{3}+1$	$2\sqrt{2}$

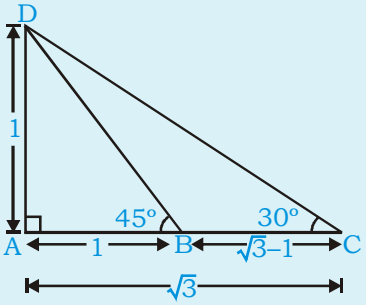
4.



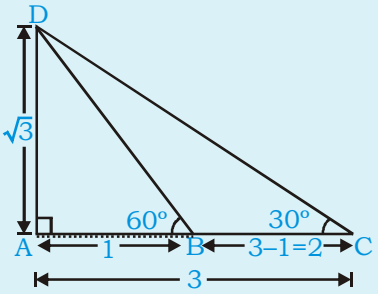
30°	30°	120°
↓	↓	↓
1 :	1 :	$\sqrt{3}$

Change of Angle (कोणों का परिवर्तन)

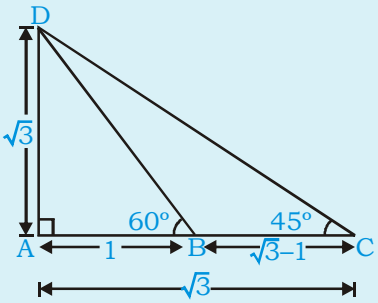
1. When elevation angle changes from 30° to 45°
(ऊँचाई कोण का 30° से 45° हो जाना)



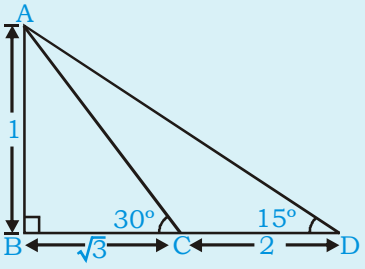
2. When elevation angle changes from 30° to 60°
(ऊँचाई कोण का 30° से 60° हो जाना)



3. When elevation angle changes from 45° to 60°
(ऊँचाई कोण का 45° से 60° हो जाना)

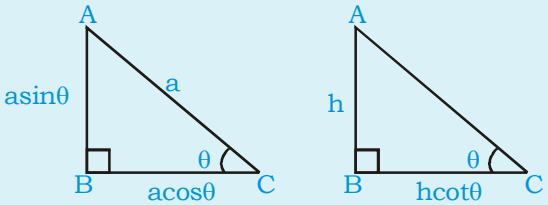


4. When elevation angle changes from 15° to 30°
(ऊँचाई कोण का 15° से 30° हो जाना)



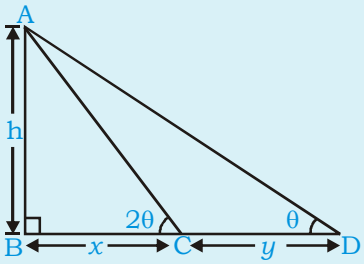
5. When elevation angle changes from ....

भ्ख्ः ऊपहरन वार....रधस्छे जरौं



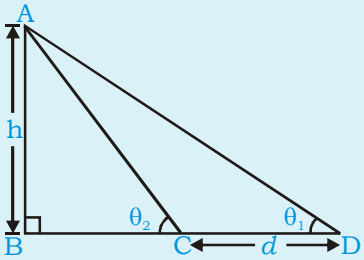
6. When elevation angle changes from θ° to 2θ.

भ्ख्ः ऊपहरन वारθ°रधस्छे न बर2θरी स्थ जरौं



$h^2 = y^2 - x^2$

7.

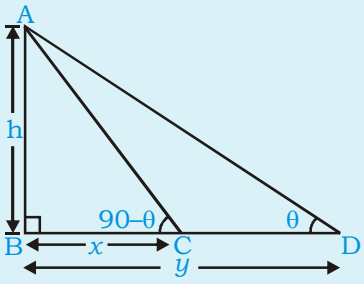


$d = h(\cot\theta_1 - \cot\theta_2)$

Some other results (कुपा प्रग्ना ैं रन

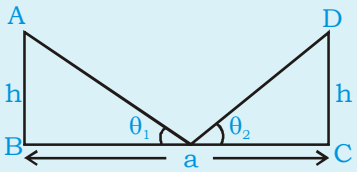
1. If Angles of elevation are complimentary

भ्ना ह्मभ्ना कै तै ा ोका जछि



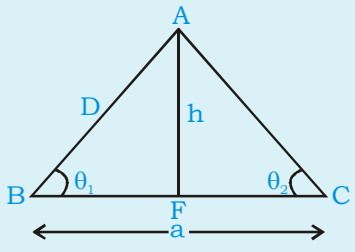
$h = \sqrt{xy}$

2.



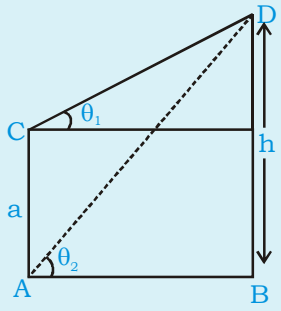
$a = h(\cot\theta_1 + \cot\theta_2)$

3.



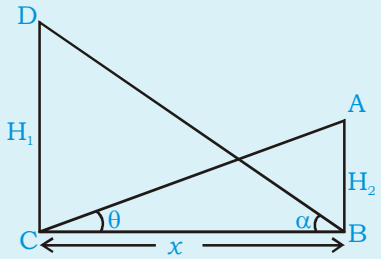
$a = h(\cot\theta_1 + \cot\theta_2)$

4.



$$h = \frac{a\cot\theta_1}{\cot\theta_1 - \cot\theta_2}$$

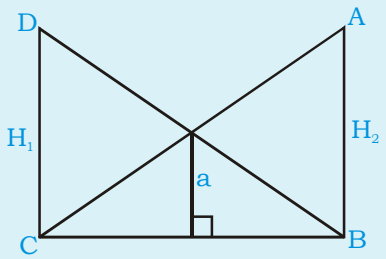
5.



$\theta + \alpha = 90°$

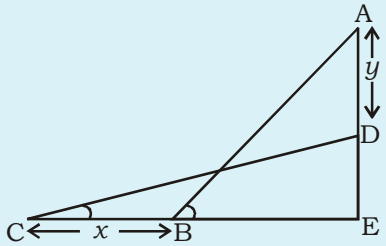
$x = \sqrt{h_1 h_2}$

6.

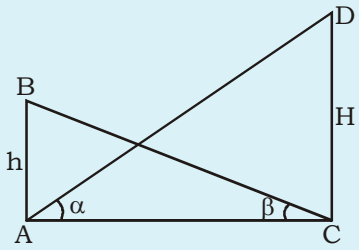


$$\frac{1}{a} = \frac{1}{H_1} + \frac{1}{H_2}$$

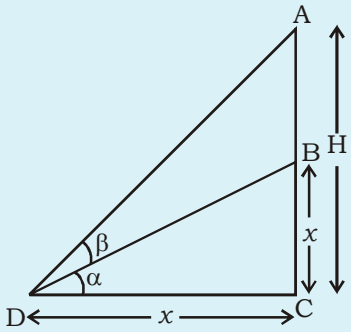
❖ If AB = CD, then $x = y\tan\left(\frac{\alpha+\beta}{2}\right)$



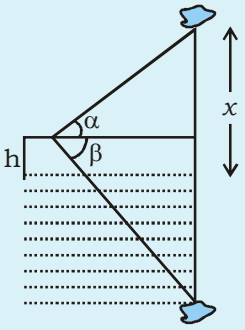
❖ $H = \frac{h\cot\beta}{\cot\alpha}$

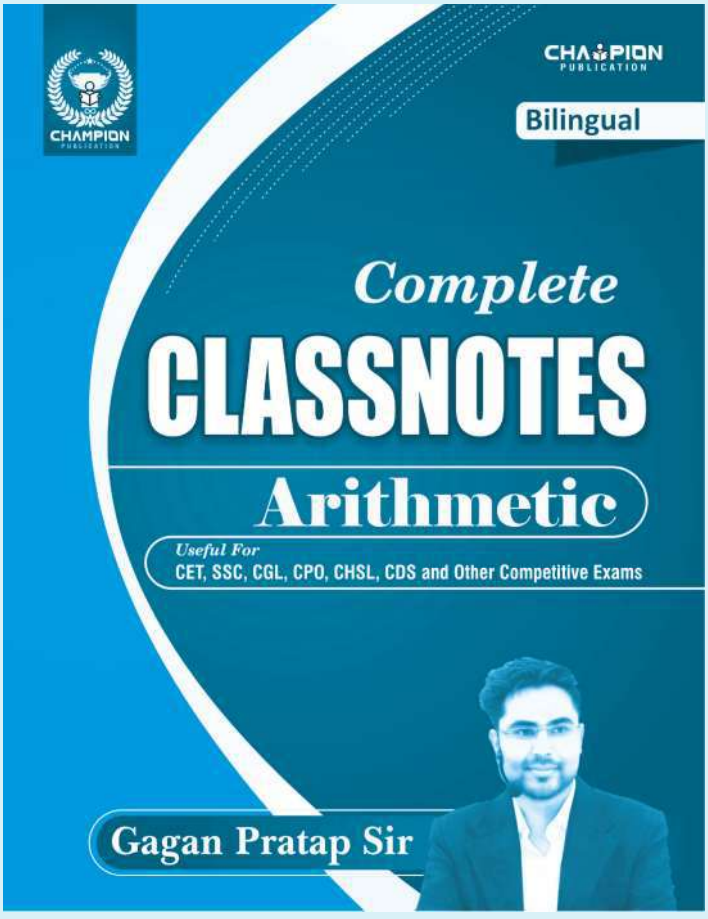


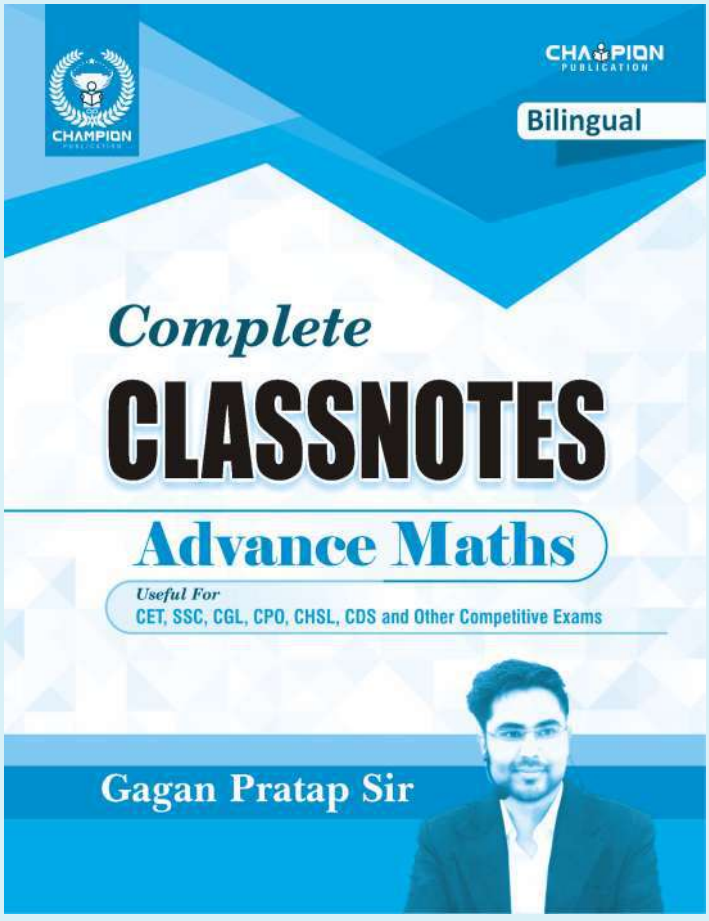
❖ $H = x\cot\alpha\tan(\alpha+\beta)$



❖ $x = \frac{h(\cot\alpha+\cot\beta)}{(\cot\alpha-\cot\beta)}$







Percentage (रेख अै

Percentage is a number or ratio expressed as a fraction of 100.

सरे ंखा ि ाबद ु ख ु क्रय अनपेा एमू बम100ा ि मय ंन्खि मट का स्मथ ु रतख्व र्ि ु ख ू ेख्कएप्र

$15\% = \frac{15}{100}$ $400\% = \frac{400}{100}$

Fraction percentage chart (खम्म)ि बिख अनि अै

Fraction	Percentage	Percentage
1	100%	100%
$\frac{1}{2}$	50%	50%
$\frac{1}{3}$	33.33%	$33\frac{1}{3}\%$
$\frac{1}{4}$	25%	25%
$\frac{1}{5}$	20%	20%
$\frac{1}{6}$	16.66%	$16\frac{2}{3}\%$
$\frac{1}{7}$	14.28%	$14\frac{2}{7}\%$
$\frac{1}{8}$	12.5%	$12\frac{1}{2}\%$
$\frac{1}{9}$	11.11%	$11\frac{1}{9}\%$
$\frac{1}{10}$	10%	10%
$\frac{1}{11}$	9.09%	$9\frac{1}{11}\%$
$\frac{1}{12}$	8.33%	$8\frac{1}{3}\%$
$\frac{1}{13}$	7.69%	$7\frac{9}{13}\%$
$\frac{1}{14}$	7.14%	$7\frac{1}{7}\%$
$\frac{1}{15}$	6.66%	$6\frac{2}{3}\%$
$\frac{1}{16}$	6.25%	$6\frac{1}{4}\%$
$\frac{1}{17}$	5.88%	$5\frac{15}{17}\%$
$\frac{1}{18}$	5.55%	$5\frac{5}{9}\%$
$\frac{1}{19}$	5.26%	$5\frac{5}{19}\%$
$\frac{1}{20}$	5%	5%
$\frac{1}{21}$	4.76%	$4\frac{16}{21}\%$
$\frac{1}{22}$	4.54%	$4\frac{6}{11}\%$
$\frac{1}{23}$	4.34%	$4\frac{8}{23}\%$
$\frac{1}{24}$	4.16%	$4\frac{1}{6}\%$
$\frac{1}{25}$	4%	4%
$\frac{1}{40}$	2.5%	$2\frac{1}{2}\%$
$\frac{3}{8}$	37.5%	$37\frac{1}{2}\%$
$\frac{5}{8}$	62.5%	$62\frac{1}{2}\%$
$\frac{4}{7}$	57.14%	$57\frac{1}{7}\%$
$\frac{5}{7}$	71.42%	$71\frac{3}{7}\%$
$\frac{2}{3}$	66.66%	$66\frac{2}{3}\%$
$\frac{4}{5}$	80%	80%
$\frac{3}{4}$	75%	75%
$\frac{5}{11}$	45.45%	$45\frac{5}{11}\%$
$\frac{7}{11}$	63.63%	$63\frac{7}{11}\%$
$\frac{10}{11}$	90.90%	$90\frac{10}{11}\%$
$\frac{4}{9}$	44.44%	$44\frac{4}{9}\%$
$\frac{7}{9}$	77.77%	$77\frac{7}{9}\%$

Derived fraction from base fractions
(प्रअ आ खम)(मिप्रति ैय ण)(खम ै)

❖ $\frac{1}{4} = 25\%$

$\downarrow \times 3$

$\frac{3}{4} = 75\%$

❖ $\frac{1}{7} = 14\frac{2}{7}\%$

$\frac{4}{7} = 57\frac{1}{7}\%$

$\frac{5}{7} = 71\frac{3}{7}\%$

❖ $\frac{1}{5} = 20\%$

$\frac{3}{5} = 3 \times 20\% = 60\%$

❖ $\frac{1}{6} = 16\frac{2}{3}\%$

$\frac{5}{6} = 5 \times 16\frac{2}{3}\% = 83\frac{1}{3}\%$

❖ $\frac{1}{15} = 6\frac{2}{3}\%$

$\frac{11}{15} = 11 \times 6\frac{2}{3}\% = 73\frac{1}{3}\%$

❖ $\frac{1}{16} = 6\frac{1}{4}\%$

$\frac{11}{16} = 11 \times 6\frac{1}{4}\% = 68\frac{3}{4}\%$

❖ $\frac{1}{24} = 4\frac{1}{6}\%$

$\frac{1}{48} = 2\frac{1}{12}\%$

$\frac{17}{48} = 17 \times 2\frac{1}{12}\% = 35\frac{5}{12}\%$

❖ $\frac{1}{16} = 6\frac{1}{4}\%$

$\frac{13}{16} = 13 \times 6\frac{1}{4}\% = 81\frac{1}{4}\%$

or $\frac{13}{16} = 1 - \frac{3}{16} = 100\% - 18\frac{3}{4}\% = 81\frac{1}{4}\%$

❖ $\frac{1}{7} = 14\frac{2}{7}\%$

$\frac{6}{7} = 1 - \frac{1}{7} = 100\text{-}14\frac{2}{7}\% = 85\frac{5}{7}\%$

❖ $\frac{1}{12} = 8\frac{1}{3}\%$

$\frac{11}{12} = 1 - \frac{1}{12} \rightarrow 100\% - 8\frac{1}{3}\% \rightarrow 91\frac{2}{3}\%$

❖ $\frac{19}{24} = 1 - \frac{5}{24} \Rightarrow 100\% - 5\left(4\frac{1}{6}\%\right)$

$\Rightarrow 100\% - 20\frac{5}{6}\% \Rightarrow 79\frac{1}{6}\%$

❖ $\frac{40}{9} = 4 + \frac{4}{9} \rightarrow 400\% + 44.44\% \rightarrow 444.44\%$

❖ $\frac{43}{6} = 7 + \frac{1}{6} \rightarrow 700\% + 16.66\% \rightarrow 716.66\%$

❖ $\frac{13}{7} = 1 + \frac{6}{7} \rightarrow 100\% + 85\frac{5}{7}\% \rightarrow 185\frac{5}{7}\%$

❖ $\frac{35}{6} = 5 + \frac{5}{6} \rightarrow 500\% + 83\frac{1}{3}\% \rightarrow 583\frac{1}{3}\%$

❖ $\frac{29}{3} = 9 + \frac{2}{3} \rightarrow 900\% + 66\frac{2}{3}\% = 966\frac{2}{3}\%$

❖ $\frac{71}{12} = 5 + \frac{11}{12} \rightarrow 500\% + 11\left(8\frac{1}{3}\%\right) \rightarrow 500\% + 91\frac{2}{3}\%$

$\rightarrow 591\frac{2}{3}\%$

❖ $\frac{37}{15} = 2 + \frac{7}{15} \rightarrow 200\% + 46\frac{2}{3}\% \rightarrow 246\frac{2}{3}\%$

Percentage to fraction conversion
(खम)ि र असं अिप्र अिबख ै

❖ $17.5\% \rightarrow 17.5 \times \frac{1}{100} \rightarrow \frac{7}{40}$ or

$\left(2.5\% = \frac{1}{40}\right) \times 7 \rightarrow 17.5\% = \frac{7}{40}$

❖ $164\% \rightarrow \frac{164}{100} \rightarrow \frac{41}{25}$

❖ $15\frac{5}{8}\% \rightarrow \frac{125}{8}\% \rightarrow \frac{125}{800} \rightarrow \frac{5}{32}$

❖ $35\frac{5}{7}\% \rightarrow 5 \times 7\frac{1}{7}\% \rightarrow 5 \times \frac{1}{14} \rightarrow \frac{5}{14}$

❖ $29\frac{1}{6}\% \rightarrow 25\% + 4\frac{1}{6}\% \rightarrow \frac{1}{4} + \frac{1}{24} \rightarrow \frac{7}{24}$

❖ $23.33\% \rightarrow 20\% + 3.33\% \rightarrow \frac{1}{5} + \frac{1}{30} = \frac{7}{30}$

❖ $78\frac{1}{3}\% \rightarrow 75\% + 3\frac{1}{3}\% \rightarrow \frac{3}{4} + \frac{1}{30} \rightarrow \frac{47}{60}$

❖ $46.66\% \rightarrow 40\% + 6.66\% \rightarrow \frac{2}{5} + \frac{1}{15} \rightarrow \frac{7}{15}$

❖ $82.5\% \rightarrow 80\% + 2.5\% \rightarrow \frac{4}{5} + \frac{1}{40} \rightarrow \frac{33}{40}$

❖ $83\frac{1}{3}\% \rightarrow 100\% - 16\frac{2}{3}\% \rightarrow 1 - \frac{1}{6} \rightarrow \frac{5}{6}$

- $237.5\% \to 200\% + 37.5\% \to 2 + \frac{3}{8} \to \frac{19}{8}$
- $342.84\% \to 300\% + 42.84\% \to 3 + \frac{3}{7} \to \frac{24}{7}$
- $756.25 \to 7 + \frac{9}{16} \to \frac{121}{16}$
- $538.33 \to 500\% + 30\% + 8.33\% \to 5 + \frac{3}{10} + \frac{1}{12} \to \frac{323}{60}$
- $528.56\% \to 5 + \frac{2}{7} \to \frac{37}{7}$

Some Rules of percentage

- (a + b)% of x = a% of x + b% of x
- (a + b)% of $(x + y)$ = a% of $(x + y)$ + b% of $(x + y)$
- If you have to find the y% of a number x.

  It will be $\to \frac{y}{100} \times x$

  Eg.→ Find 25% of 300

  $\frac{25}{100} \times 300 = 75$

- If x is to be expressed as a percentage of y. Then

  required percentage $= \frac{x}{y} \times 100\%$

Ex. 99 is what % of 135

$\frac{99}{135} \times 100\% = 73\frac{1}{3}\%$ or

$135 \longrightarrow 100\%$

$1 \longrightarrow \frac{100}{135}\%$

$99 \longrightarrow 99 \times \frac{100}{135}\%$

or

15 : 11

$\frac{1}{15} = 6\frac{2}{3}\%$

$\frac{11}{15} = 73\frac{1}{3}\%$

- If a fraction is to be converted into a percentage multiply the fraction by 100 and put a % sign.

  eg.→ $\frac{4}{5} \to \frac{4}{5} \times 100\% = 80\%$

- To convert a percentage to a fraction or decimal fraction, remove the % sign and divide by 100.

  eg.→ $15\% \to \frac{15}{100} = \frac{3}{20}$

- If A is reduced to A_o, Then

  reduce% $= \frac{A - A_o}{A} \times 100\%$

  eg.→ If cost of an item is reduced from 100 to 80 find. reduce%

  $\therefore$ Reduce% $= \frac{100 - 80}{100} \times 100\% = \mathbf{20\%}$

- If A is increased to A_1, Then

  Increase% $= \frac{A_1 - A}{A_1} \times 100\%$

  eg.→ If cost of an item is increased from 120 to 150. Find increase %

Sol. Increase% $= \frac{150 - 120}{120} \times 100\% = \mathbf{25\%}$

- a% of b = b% of a

Ex. 840% of 62.5 → 62.5% of 840 $\to \frac{5}{8} \times 840 \to 525$

Ex. 7.44% of 3750 → 37.50% of 744 $\to \frac{3}{8} \times 744 \to 279$

- 99 is what % less than 135

  99 — 135 (−36) $\to \frac{36}{135} \times 100\%$ or

  $15 : 4 \to \frac{4}{15} = 26\frac{2}{3}\%$

- 180 is what % of 125

  $\frac{180}{125} \times 100\% = 144\%$

  or

  125 : 180 (×0.8) → 100 — 144 (+44%)

Scaling factor/multiplying factor

- $K \xrightarrow{+x\%} K + kx\% = K(1 + x\%) = K\left(1 + \frac{x}{100}\right)$

  ↓ multiplying factor ↓ multiplying factor

  Initial 100% — Final 128% (+1.28)

  $28\% = \frac{+7}{25}$ → Change in value / Initial value

  $\therefore$ Initial 25 — Final 32 (+7)

❖ $-44\% \to 44\% = \frac{-11}{25}$

Initial value 25 → Final value 14 (-11)

❖ $+35\% \to 35\% = \frac{+7}{20}$

Initial value 20 → Final value 27 (+7)

❖ $-22.5\% \to \frac{-9}{40} \to \overset{40 \;:\; 31}{\underset{-9}{\frown}}$

❖ $x \xrightarrow{+26\%} (x + x \times 26\%)$

$x + x \times \frac{26}{100} = x\left(1 + \frac{26}{100}\right)$

$= x \times 1.26$

multiplying factor (m.F)

Initial value	Change	Final value
x	+37%	$1.37x = \left(1 + \frac{37}{100}\right)x$
y	-45%	$0.55y = \left(1 - \frac{45}{100}\right)y$
z	+45.45% ↓ $\left(\frac{5}{11}\right)$	$z + z \times \frac{5}{11}$ ↓ $\left(1 + \frac{5}{11}\right) \times z = \frac{16}{11}z$
w	+31.25%	$\left(1 + \frac{5}{16}\right)w = \frac{21}{16}w$
u	$-46\frac{2}{3}\%$ ↓ $\left(\frac{7}{15}\right)$	$\left(1 - \frac{7}{15}\right)u = \frac{8}{15}u$

❖ Net income = Income – Income Tax

ं खा यु ा = ा यु ा – ा यु ि ल

Ex. ₹700 = ₹800 – ₹100

Rate of income tax ै यु ि ला ा ील=

Ex. $\frac{100}{800} \times 100\% = 12.5\%$

$= \frac{\text{Income Tax}}{\text{Total income}} \times 100\%$

Net Income = Income – Income Tax

ं खा यु ा = ा यु ा – ा यु ि ल

700 = 800 – 100

↓ –50 ↓ +50

650 = 800 – 150

Change in net income & tax will be same

ं खा यु ा य खा ि ला स्मकले आ बस्ख एखड

Constant Concept

❖ A × B = Constant

↑ × ↓

↓ × ↑

Expenditure/दखज = Price/र्ंु × Consumption/दके

Work/ि ुज = Men/यकेस ा × Time/बसु

Distance/ ैला = Speed/मख ा × Time/बसु

Income tax/यु ि ला = Income/यु ा × Rate/ ील

Area/उखखड्ग ा = Length/ ण ठखज × Breadth/मख ेड्ज

Volume/यु े अ = Area/उखखड्ग ा × Height/–ख्मखज

Ex. Expenditure/दखज = Price/र्ंु × Consumption/दके

450 = ₹25 × ₹18

↓+20% ↓–16.66%

450 = ₹30 15kg

P → 5 : 6

C → 6 : 5 → Reverse Ratio

Ex.

A	×	B
5	:	9
×2 ↕ 10	:	4.5 ↕ ×$\frac{1}{2}$
= 45		= 45

A = 1 : 2

B = 2 : 1

(Reverse ratio स्न chance एखड्)

Concept of deviation (ख ू ः)प्रि जपि ड्ड आं ऐम

❖

	Rice		Dal		Sugar
Exp शू य→	7	:	13	:	5
% Change→	+17%		+20%		+19%
Deviation→	–1%		+2%		+1%

Let overall change is 18% ै ा स्खआर्ह ा घा क्रले आ 18% ा एप

$$\frac{(-1\% \times 7 + 2\% \times 13 + 1\% \times 5)}{7+13+5}$$

(Deviation in rice ↑, Deviation in dal ↑, Deviation in sugar ↑)

Total deviation आमिकृ ी तीब्र divide अजे वसाप्

$\therefore 18\% + \left(\frac{-7+26+5}{25}\%\right)$

18% + 0.96% = 18.96%

or

	Rice	:	Dal	:	Sugar
Exp→	7	:	13	:	5
% Change	17%		17+3%		17%+2%

Let overall % change = 17%

deviation = 13 × 3% + 5 × 2% = 49%

क्कृआमिकृ ि ति तीब्र divide अजे वसाप्

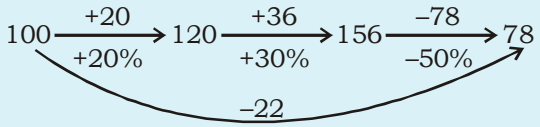
$\therefore$ overall % change $\to 17\% + \frac{49}{25}\%$

$\to 17\% + 1.96\% \to 18.96\%$

Annual increment/decrement (३अ॥ु ि॥ि)ि॥ि॥ थ॥ गेज

❖ **Successive percentage change:-**

/ खप्र बिग॥ अरिख॥ा॥ु)



Second Method थ॥ि॥ि॥ख॥ी सख॥ अ॥-

Initial: Final

100 : {(100 × 1.2) ×1.3}×0.5

100 : 78

↓

22% Change (Decrease)

Third Method / ा॥ाख॥ी सख॥ अ॥-

	Initial	:	Final
+20%→	5	:	6
+30%→	10	:	13
–50%→	2	:	1
	100	:	78

–22%

Fourth Method / ॅ॥ख॥ी सख॥ अ॥-

If x% and y% change occurs successively then

overall change% = $\left(x+y+\frac{xy}{100}\right)\%$

Step-1→ Find percentage change of 20% and 30%

$\left(20+30+\frac{20\times30}{100}\right)\%$ = **56%**

Step-2→ Find percentage change of 56% and –50%

$\left(56-50+\frac{56\times50}{100}\right)\%$ = (6 – 28)% = **– 22%**

Fifth Method / र॥ ३॥ी सख॥ अ॥-

If x%, y%, z% change occurs successively.

Then overall change% =

$$\left[(x+y+z)+\frac{xy+yz+zx}{100}+\frac{xyz}{10000}\right]\%$$

∴ Overall change% = (20 + 30 – 50) +

$$\frac{600-1000-1500}{100}+\frac{20\times30\times(-50)}{10000}$$

$$\Rightarrow \left(0-\frac{1900}{100}-\frac{30000}{10000}\right)\% \Rightarrow (-19-3)\% \Rightarrow \textbf{–22\%}$$

1. If the population/cost of a certain town/article, is P and annual increment rate is r%, then

ु रा॥ारि ब॥ा॥ि च्छैन॥ेचे॥ि॥ ॥ू अ॥दु रै॥ेचे॥ि॥ ॥ण खे ।P॥ाएमय खा॥ भ्रख्त्रि॥ा॥ि ला॥ब॥ ख्ढे ॥एखे ख

(i) After 't' years population/cost $= P\left(1+\frac{r}{100}\right)^t$

't'॥ा॥ भ्रत्रृक्षा॥ि मठरे॥ू अ॥दु रै॥ख खे ा $= P\left(1+\frac{r}{100}\right)^t$

(ii) Before 't' years population/cost $= \frac{P}{\left(1+\frac{r}{100}\right)^t}$

't'॥ा॥ भ्रत्रक्षा॥मकरण॥ू अ॥दु रै॥ख खे ा $= \frac{P}{\left(1+\frac{r}{100}\right)^t}$

2. If we change a number by x% & y% successively Then, net% change

ु रा॥एस॥ि खा॥अ॥दु रू॥ स॥ं ख x%ा॥य खा y%ा॥ :ख॥ठी॥े ॥एम॥क॥त॥ा॥ं ख॥ड करल॥े अ॥

$$= \left(x+y+\frac{xy}{100}\right)\%$$

Proof:-

If 100 is successively increased by x% and y%.

$$100\left(1+\frac{x}{100}\right)\left(1+\frac{y}{100}\right)$$

$$\Rightarrow \frac{100\left(1+\frac{x}{100}\right)\left(1+\frac{y}{100}\right)-100}{100}\times100$$

$$\Rightarrow 100\left[\left(1+\frac{x}{100}\right)\left(1+\frac{y}{100}\right)-1\right]$$

$$\Rightarrow \left[1+\frac{y}{100}+\frac{x}{100}+\frac{xy}{100\times100}-1\right]$$

Now, $\left(x+y+\frac{xy}{100}\right)\%$

3. If the population of a town /cost of a article is P and it decreases/reduces at the rate of r% annually, then.

ु रा॥ारि ब॥ा॥ि च्छा॥ि ॥ू अ॥दु रै॥ा॥ेचे॥ि॥ ॥ण खे ।P॥ाएमय खा॥ एr% भ्रख्त्रि॥ा॥ि ला॥ब॥ ख्ढे ॥एखे ख

(i) After 't' years population/cost $= P\left(1-\frac{r}{100}\right)^t$

't'॥ा॥ भ्रत्रृक्षा॥ि मठरे॥ू अ॥दु रै॥ख खे ा $= P\left(1-\frac{r}{100}\right)^t$

(ii) Before 't' years population/cost $= \frac{P}{\left(1-\frac{r}{100}\right)^t}$

't'॥ा॥ भ्रत्रक्षा॥मकरण॥ू अ॥दु रै॥ख खे ा $= \frac{P}{\left(1-\frac{r}{100}\right)^t}$

4. Net % change of x%, y% and z% is

x%, y%॥ाय खा z%॥ा॥ि स॥ं ख॥ा करल॥े अ॥एप

$$\left[(x+y+z)+\frac{xy+yz+zx}{100}+\frac{xyz}{10000}\right]\%$$

5. If the present population of a town is P and the population increases or decreases at rate of R_1%, R_2% and R_3% in first, second and third year respectively.

ु रा रि ब ा ि च ा म ा ि ा भ े स ख ा ू अ ब द ु क P ा ए ा य ख ा ू अ ब द ु क ं स ख क ए ण स ी ब ल ा य ख ो ा ब ल ा भ ऱ ख स म R_1%, R_2% ा य ख ा R_3% ा ि ा ि ल ा ब ा ठ क े ो।

ु क ु ख े ा ए प्र

then the population of town after 3 years

ं ख ा ऽ ा भ ऱ ख ब ा ठ क े ो ं ख ल ा ि ा ू अ ब द ु क

$$= P\left(1 \pm \frac{R_1}{100}\right)\left(1 \pm \frac{R_2}{100}\right)\left(1 \pm \frac{R_3}{100}\right)$$

'+' is used when population increases

ू अ ब द ु क ठ क ऱ े ो म क ल ा '+' ा ि क स ु ख ा र ि ू क े ख ए प

'–' is used when population decreases.

ू अ ब द ु क ु क ु ख ो म क ल ा '–' ा ि क स ु ख ा ए ख ब क ए प्र

The above formula may be extended for n number of years.

–क ल ख ब ष ् ख n ा भ ऱ ख ा ि ा ण ठ क ऱ ् ो क ू ख ब ा ि ् ए प्र

⇒ Population after 'n' years ै 'n' ा भ ऱ ख ा ि ा ठ क े ो ू अ ब द ु क

$$= P\left(1 \pm \frac{R_1}{100}\right)\left(1 \pm \frac{R_2}{100}\right)...\left(1 \pm \frac{R_n}{100}\right)$$

6. +30% +35% –48%

= +35% –48% +30%

Initial × 1.3 × 1.35 × 0.52 = Initial × 1.35 × 0.52 × 1.3

❖ अ ृ ं ृ ा ि sequence change अ ज ी) अ क ृ ि त ं ृ ा ि successive change ि ज ि र ि ै ए ख ा ए

Important Points (ग क य र ं ध ु ख ् ि ै ं

❖ **Based on increase/decrease**

भ ् ै ि स ा ि ा ि य ख ख ा क ल

1. If an amount is increased by *x*% and then it is reduced by *x*% again, then percentage change will be a decrease of $\frac{x^2}{100}$%

ु रा रि ब ा ल ं ख स म x% ा ि ा भ ् ै ा ि ू ख ा ए ा य ख ा क ल ा श ब ा क ल ा ब ा x%

. ख ु ठ क ू ख े क ए ए ख स ा ख स े ं ख ा क ल े ो ज ा स म $\frac{x^2}{100}$% ा ि ा ि स ा ए ख ा।

Ex. Price of an article is increased by 10% and then reduced by 10%. What will be net percentage change?

ि ा भ े ध ा म र ् ु ा स म 10% ा ि ा भ ् ै ा ि ू ख े ा ए ा य ख ा क ल ा 10% ा ि ा ि स ा ि ू ख े ा ए प्र ि ध ा क ख े ं ख ा क ल े ो ज ा ू क ए ख ख

∴ Price will be decrease by $\frac{10^2}{100}$% = **1%**

2. If a number is increased by a% and then it is decreased by b%, then resultant change in percentage will be $\left(a - b - \frac{ab}{100}\right)$%

ु रा ि ब द ु क स म a% ा ि ा भ ् ै ा ि ू ख े ा ए ा य ख ा क ल ा श ब ा म b% ा ब न

. ख ु ठ क ी ू ख े क ए ए ख स ा ख स े ं ख ा स म ा ल द ् र ख ् ा क ल े ो ज ा $\left(a - b - \frac{ab}{100}\right)$%

ए ख ा ऱ ो

(Negative for decrease, Positive for increase)

ह ् ा ख ु अ ा ि ा ण ा ऱ ू द ् र ख ् ा क ि ख भ ् ै ा ि ा ण ा ऽ ् अ ब ् क ि ज

3. If a number is decreased by a% and then it is increased by b%, then net increase or decrease percent is

ु रा रि ब ा ब द ु क स म a% ा ि ा ि स ा ि ा ू ख े ा ए ा य ख ा क ल ा श ब ा म b% ा ब न ठ क ऱ े ी ख ु ू ख े क ए ए ख स ं ख ा भ ् ै ा ु ख ि स ा स े ं ख ा ए प

$$\left(-a + b - \frac{ab}{100}\right)\%$$

$$\begin{pmatrix}\text{Negative sign for decrease}\\ \text{Positive sign for increase}\end{pmatrix}$$

4. If a number is first decreased by a% and then by b%, then net decrease percent is $\left(-a - b + \frac{ab}{100}\right)$%

(–ve sign for decrease)

ु रा रि ब ा ब द ु क स म क ए ण स a% ा य ख ा क ल ा b% ा ि ा ि स ा ि ा ू ख े ा ए ा य ख

ं ख ा ख ा ि स ा स े ं ख ा ए म $\left(-a - b + \frac{ab}{100}\right)$% ह ् ा -ve ा ब ि ा ि स ा ि न ए ण ज

5. If a number is first increased by a% and then again increased by b%, then total increase percent is

ु रा रि ब ा ब द ु क स म क ए ण स a% ा ि ा भ ् ै ा ि ू ख े ा ए ा य ख ा क ल ा b% ा ि ा भ ् ै ा ि ू ख े ा ए ख ि ध ा भ ् ै ा स े ं ख ा ए प

$$\left(a + b + \frac{ab}{100}\right)\%$$

6. If the cost of an article is increased by A%, then how much to decrease the consumption of article, so that expenditure remains same is given by

ु रा रि ब ा भ ् च े ध ि ा ि स े ा स म A% ा ि ा भ ् ै ा ि ू ख े ा ए ख ख भ ् च े ध ि ा । क ृ क े ा ि ख ा र ि ै अ ह ि स ा र ि ू क े ख ख े ख ि ु ब स ख ा ल ् न

OR

If the income of a man is A% more than another man, then income of another man is less in comparison to the 1st man by

ु रा ि य ह ि स ा ि ा य ु ख ो ी ब ल ा य ह ि स ा ि ो ध अ ह स म A% ा य र त ् ा ए ख ं ख ी ब ल ा य ह ि स ा ि ा य ु ख ा क ए ण म य ह ि स ा ि ो ध अ ह स म र् ं आ ि स ा ए भ्र

$$\left(\frac{A}{(100 + A)} \times 100\right)\%$$

7. If the cost of an article is decreased by A%, then the increase in consumption of article to maintain the expenditure will be?

ु रा रि ब ा भ ् च े ध ि ा ण ख े ा स म A% ा ि ा ि स ा ि ा ू ख े ा ए ख ख ु ि ख ठ अ ह ा ल ृ क अ ा ि ा ण भ ् च े ध ि ा ृ क े ा स म भ ् ै ा ए ख ा भ्र

OR

If 'x' is A% less than 'y'. then y is more than 'x' by

Required% = $\left(\frac{A}{(100-A)}\times 100\right)\%$ (increase)

ु रा'x'ख y'ा बम A%ा ि सा एप्े ख y'x'ा बम $\left(\frac{A}{(100-A)}\times 100\right)\%$ य रुह ा एप

8. If the length of a rectangle is increase by a% and breadth is increased by b%, then the area of rectangle will increase by

ु रा ि यु े ा ि ण ठ ख ज ा स म ख ं ख े ज स म a% ा ि भ ख ् ा व ा ि ू े ा ए म य ख ा म ख ं ख े ज स म b% ा ि भ ख ् ा व ा ि ू े ा ए प े ख य ु े ा ि म उ ख व ख ह ा स म र ै आ भ ख ् व ए ख ा श्र

Required Increase ै ा य ू ं ि भ ख ् व ा = $\left(a+b+\frac{ab}{100}\right)\%$

9. If the side of a square is increases by a% then, its area will increase by

ु रा र ि ब ा भ ग ा ज ा ि ं ख ू ख ह स म a% ा ि भ ख ् ा व ा ि ल ी ू े ख प े ख ब ि म उ ख व ख ह ा र ै आ ह ठ क ्ा ू े ग क

$\left(2a+\frac{a^2}{100}\right)\%$

10. If radius of circle is increased by a% then its area will be increased by = $\left(2a+\frac{a^2}{100}\right)\%$

ु रा भ ख ह र ि ष ् ु ह स म a% ा ि भ ख ् ा व ा ि ू े ा ए म ख ब ि म उ ख व ख ह ा स न = $\left(2a+\frac{a^2}{100}\right)\%$ ा भ ख ् व ा ए ख ा।

11. If the side of a square is decreased by a%, then the area of square will decrease by

ु रा र ि ब ा भ ग ा ज ा ि ं ख ू ख ह ि ख a% ा ि स ा ि ल ा र ी ु ू े ख प े ख भ ग ा ज ा ि ख उ ख व ख ह ा र ै आ ह ि स ा ए ख ू े ग क श्र

$\therefore$ Decrease = $\left(-2a+\frac{a^2}{100}\right)\%$

This formula is also applicable for circles where decrease % of radius is given.

ए ब ष ह - अ भ ह ख ा ि म ण ा ं ख ् ण ख ा ए े ख ह ए ण ू एक ह ए ष ु ख ि क ृ ु ह ु ख ए ध ख स े र ं े ख ा र ी ु ह ग ु ह ए प्र

12. If the length, breadth and height of a cuboid are increased by a%, b% and c% respectively, then, Increase% in volume

ु रा ि ् ख ं ख ह ि ण ठ ख ज ा स म ख ं ख े ज स म ख ा ध म ख ज ा स म ्र स ं ख a%, b% य ख ा c% ा ि भ ख ् ा व ा ि ू े ा ए प े ख य ु े ा ि म उ ख व ख ह ा स म भ ख ् व श्र

= $\left[a+b+c+\frac{ab+bc+ca}{100}+\frac{abc}{(100)^2}\right]\%$

13. If every side of cube or radius of sphere is increased by a%, then increase % in volume

ु रा ी ् ख ा ि स ् ु ह ि ं ख ू ख ह ु ख ा ख ा म ि ष ् ु ह स म a% ा ि भ ख ् ा व ा ि ू े ा ए प े ख य ु े ा अ स म % ा ि भ ख ् व ा ए े ख ा ए प

= $\left(3a+\frac{3a^2}{100}+\frac{a^3}{(100)^2}\right)\%$

14. If a% of a certain sum is taken by 1st man and b% of remaining sum is taken by 2nd man and finally c% of remaining sum is taken by 3rd man, then if 'x' rupee is the remaining amount then,

ु रा ि ा र अ ं म े ा ल ं ख ह ि ख a% ा क ए ण म य ह ै स : ख ब ा र ए ू ह ू े ख ह ए ण म य ख ा ं ख ब ा ल ं ख ह ि ख b% ी ं ब ि ल म य ह ै स : ख ब ा र ए ू ह ू े ख ह ए ण म य ख ा य े न ा स म ं ख ब ा ल ं ख ह ि ख c% ो ं ब ि ल म य ह ै स : ख ब ा र ए ू ह ू े ख ह ए प े ख ु ी ा 'x' च क ु म ं ख ब ा ल ं ख ह ए प े ख

Initial amount ै ा ं ख य ह े ा ि स

$x\times\frac{100}{100-a}\times\frac{100}{100-b}\times\frac{100}{100-c}$

Ex. If 20% of a certain amount is given to A and 25% of the remaining amount is given to B and the 50% of remaining amount is given to C. If ₹1500 is the remaining amount now then what is the initial amount?

ख ो ी) अ क ृ त े) ब ्र) ू द ी ज) ी ा आ ी 20% A ी आ ी म ी) ख ो ी ू द ी ा र ी द स ी ं ा ह ी ज) ा आ ी 25% B ी आ ी म ै क ृ आ ी म ी ा ो ी ू ै र क ं ा ह ी ज) ी ा आ ी 50% C ी आ ी म ी) ख ो ी ू द ी ा र ी ख ो ी ं ी ि ₹1500 ी ू ै म र ी द ी म ं ा झ े ा द ी त ी न ज) ी) अ द ब त े स र्त

Sol. $1500\times\frac{100}{(100-20)}\times\frac{100}{(100-25)}\times\frac{100}{(100-50)}$

$=1500\times\frac{100}{80}\times\frac{100}{75}\times\frac{100}{50}$

$\Rightarrow \frac{100\times100\times2}{4}$ = **₹5000**

15. If an amount is increased by a% and then again increased by b% and finally increased by c%. So, that resultant amount is 'x' rupees, then

ु रा र ि ब ा ल ं ख ह स म a% ा ि भ ख ् ा व ा ि ू े ा ए म य ख ा ख क ह ् ा ब म b% ा ि । भ ख ् ा व ा ि ू े ा ए म य ख ा य े न ा स म c% ा ि भ ख ् ा व ा ि ू े ा ए प े ख ु ी ा भ ख ् व क ल द ् र ख ह ा ल ं ख ह 'x' ा च क ा ए प े ख ठ

Initial amount ै ा ं ख य ह े ा ि स

$x\times\frac{100}{100+a}\times\frac{100}{100+b}\times\frac{100}{100+c}$

16. On increasing/decreasing the cost of a certain article by x%, a person can buy 'a' kg article less/ more in 'y' rupees, then

र ि ब ा भ च े ा ि म र ् ु ा स म x% ा ि भ ख ् व ै ि स ा ि ल भ ा क ल ख ा ि ु ह त ख 'y' च क ु म स म 'a' ा र ि ग क भ च े ा ि स ै य र त ा ि क ृ ल ी ा ब ि ह ए प े ख

Increased/decreased cost of the article

घटने की दशा में खर्च = $\left(\frac{xy}{100 \times a}\right)$

And initial cost पहले खर्च किए गए रुपये = $\frac{xy}{(100 \pm x)a}$

[Negative sign when decreasing and positive sign when increasing]

ज़ब आवक में कमी होती है तब ऋणात्मक चिह्न और जब आवक में वृद्धि होती है तब धनात्मक चिह्न प्रयोग करते हैं।

Ex. Price of sugar reduced by 20%. Now a person can buy 500 gm more sugar for ₹36. What is the original price?

चीनी की कीमत में 20% की कमी की जाती है। अब एक व्यक्ति ₹36 में 500 gm अधिक चीनी खरीद सकता है। चीनी का पहले का मूल्य क्या था?

Sol. $\frac{20 \times 36}{(100-20)1/2} = \frac{20 \times 36}{40}$ = **₹18 per kg**

17. If a person saves 'R' rupees after spending x% on food, y% on cloth and z% on entertainment of his income then.

यदि कोई व्यक्ति अपनी आय का x% भोजन पर, y% कपड़ों पर तथा z% मनोरंजन पर खर्च करने के बाद 'R' रुपये बचाता है तो

Monthly income मासिक आय = $\frac{100}{100-[x+y+z]} \times R$

Examination based (परीक्षा आधारित)

1. In an examination, a% candidates failed in Maths and b% candidates failed in English. If c% candidate failed in both the subjects, then

किसी परीक्षा में a% परीक्षार्थी गणित में अनुत्तीर्ण होते हैं तथा b% परीक्षार्थी अंग्रेजी में अनुत्तीर्ण होते हैं। यदि c% परीक्षार्थी दोनों विषयों में अनुत्तीर्ण होते हैं तो

(i) Passed candidates in both the subjects

दोनों विषयों में उत्तीर्ण परीक्षार्थी

= 100 – (a + b – c)%

(ii) Percentage of candidates who failed in either subject

कम से कम एक विषय में अनुत्तीर्ण परीक्षार्थी

= (a + b – c)%

2. In a certain examination passing marks is a%. If any candidate obtains 'b' marks and fails by 'c' marks then,

किसी परीक्षा में उत्तीर्णांक a% है, यदि कोई परीक्षार्थी 'b' अंक प्राप्त करता है और 'c' अंक से अनुत्तीर्ण होता है तो

Total marks = $\frac{100(b+c)}{a}$

Ex. In an examination passing marks are 35%. A person got 80 marks and fail by 25 marks. Find total marks?

किसी परीक्षा में उत्तीर्णांक 35% है। एक व्यक्ति 80 अंक प्राप्त करता है और 25 अंक से अनुत्तीर्ण होता है। पूर्णांक ज्ञात कीजिए।

Total marks = $\frac{100(80+25)}{35} = \frac{100 \times 105}{35}$ = **300**

3. In a certain examination, 'B' boy and 'G' girls participated x% of boys and y% of girls passed the examination, then.

किसी परीक्षा में 'B' लड़के तथा 'G' लड़कियाँ सम्मिलित होते हैं। यदि लड़कों का x% तथा लड़कियों का y% उत्तीर्ण होता है तो

= $\left(\frac{B.x + G.y}{B+G}\right)\%$

4. If a candidate got A% votes in a poll and he won or defeated by 'x' votes, then, what was the total no. of votes which was casted in poll ?

यदि किसी प्रत्याशी को किसी चुनाव में A% मत प्राप्त होते हैं तथा वह 'x' मतों से जीतता/हारता है तो चुनाव में कुल कितने मत पड़े थे ज्ञात कीजिए।

∴Total number of votes कुल पड़े मत = $\frac{50 \times x}{(A-50)}$

Ex. A candidate got 55% votes in an election and won by 3600 votes. Find total number of votes?

किसी चुनाव में एक प्रत्याशी को 55% मत प्राप्त होते हैं तथा वह 3600 मतों से जीतता है तो कुल कितने मत पड़े थे ज्ञात कीजिए।

∴ Total no. of votes = $\frac{50 \times 3600}{55-50}$ = **36000**

Price increase/decrease (कीमत में वृद्धि/कमी)

❖ If the price of an article is reduced by a% and buyer gets c kg more for some Rs. b, the new price per kg of article = $\frac{ab}{100 \times c}$

यदि किसी वस्तु के मूल्य में a% की कमी की जाती है तथा b रुपये में पहले की अपेक्षा c kg अधिक वस्तु खरीदी जाती है तो वस्तु का नया मूल्य

= $\frac{ab}{100 \times c}$

Ex. Price of an article is reduced by 10% and buyer gets 8 kg more for Rs. 300. Find new price per kg?

किसी वस्तु के मूल्य में 10% की कमी की जाती है तथा 300 रुपये में पहले की अपेक्षा 8 kg अधिक वस्तु खरीदी जाती है। नया मूल्य ज्ञात कीजिए।

New price per kg = $\frac{10 \times 300}{100 \times 8} = \frac{15}{4} = 3\frac{3}{4}$ **Rs / kg**

Miscellaneous (विविध)

1. If two numbers are respectively x% and y% less than the third number, first number as a percentage of second is $\frac{100-x}{100-y} \times 100\%$

यदि किसी संख्या से दूसरी संख्या x% तथा y% कम है तो पहली संख्या का दूसरी संख्या से प्रतिशत $\frac{100-x}{100-y} \times 100\%$ होता है।

2. If two numbers are respectively x% and y% more than a third number the first as percentage of second is

ु रीी खबद खँ ि ि बला बद ख्वम्न सं ख x%ाय खा y%ाय रत ि ए ख्व ` खी बला बद ख ि मसरे ं खा ि मट का स्मक ए ॥ बद ख ए प

$$\frac{100+x}{100+y}\times 100\%$$

3. If a number 'a' is increased or decreased by b%, then the new number will be

ु रीा रि बा॥ बद ख 'a' ा स्मा b% ा ि ॥ भ्ख्वा ख स ॥ ि ॥ ू खे॥ ए ख्व ख अज बद ख ए ख ा-

$$\left(\frac{100\pm b}{100}\right)\times a$$

Ex. An agent get 12.5% commission on sale up to ₹24000 and $8\frac{1}{3}\%$ commission on sale more than it. If he gives ₹65000 to company after deducting his commission on total sale then how much sale he did?

नओ नु पी 12.5% ी अत्ते अतरं ा बी 24000 अत्ते ि , त्ते द अी अतादी र।

` ाप्ै कृअम /ण्जे अत्ते ि , त्ते ण्जे $8\frac{1}{3}\%$ ी अत्ते अतरं ा बी अतादी री ख्व ी भ्र

` ण्बी अतरं ा बी आस्बीमअीमि वी अभ्बत्ते अी ₹65000 ी वख्ी री दाीम कृबम)अदबत्ते ि , त्ते अत्ते रा ख्ात:

Sol.

24000	24000+
↓	↓
12.5%	8.33%

His commission on ₹24000 = $24000 \times \frac{1}{8} = ₹3000$

If he gets 8.33% on total sales, then his commission on ₹24000 = $24000 \times \frac{1}{12} = ₹2000$

In this case company gets (3000 – 2000) = ₹1000 more

∴ = 65000 + 1000 = ₹66000

∴ 8.33% = $\frac{1}{12}$ → Salesman commison
Total sales →

∴ Company gets = 12–1 = 11 unit $\xrightarrow{\times 6000}$ ₹66000

∴ Total sales = 12 unit $\xrightarrow{\times 6000}$ **₹72000**

Ex. A company allows 9% commission on total sales to his salesman and a bonus of 1% on seles over ₹20,000. If salesman deposited ₹63200 in company after deduction his earning on total sales find total sales of company?

नओ अभ्बत्ते ण्बी कृमकृत ी आीम अख्ी ि , त्ते ण्जे 9% ी अतरं ा बी वख्ते री दसा 20,000 कृम /ण्जे अत्ते ि , त्ते ण्जे 1% ी आीं ह)न्धदी ख्ृ ी ि त्ते वख्ते र। ख्व ी कृमकृत ी बी ण्बत्ते अताकी आस्बीमअीमि वी अभ्बत्ते ती ₹63200 ी ु ता अज्दी री दाीम अख्ी ि , त्ते ध्खी र ए

Sol.

20,000	20,000
↓	↓
9%	10%

If he gets 10% on all sale, then his benefit =

$20000 \times \frac{1}{100} = ₹200$

∴ Company gets = 63200 – 200 = ₹63000

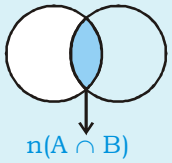
∴ 10% = $\frac{1}{10}$ → Salesman commison
Total sales →

∴ Company gets = 10–1 = 9 unit $\xrightarrow{\times 7000}$ ₹63000

Total sales = 10 unit $\xrightarrow{\times 7000}$ **₹70000**

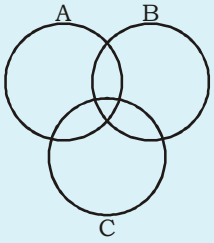
Some Important Points

1. Price × consumetion = Expenditure
   र्स ु ा × दखे ा = दखज
2. Gross income – Income tax = Net Income.
   ब ि ा य ुख ा – य ुख ि ला = ं ख ा य ुख
3. Venn digram



$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

4. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$



5. Inccome/य ुख = Expenditure/दखज + Saving/उमे
6. Earning/य ुख = Time/बस ु × wages/भ्ेनअ

Profit & Loss (ध्अ्अमि अ्अनि अ्है)

- **Cost price (CP)**→ Cost price is the total amount of money that it costs a manufacturer to produce a given product.

  ध्अ्आ गिधी ि(CP)→ाण्खे ार्रु ा ध्अ्णि ्धाालं्रु एप्रू ख्रर्ि बा॥रअ्रेख्रु :ख्रार्ि बा॥री ाग ा-छ्रन्रेहा्रि क्र-छ्रन्रेअ्रि ल्कमा॥ाण ाकृमाज्रि ॥ू रेख्॥एप्र

- **Selling price (SP)**→ The selling price is the final price of a product. It is the price at which something is sold.

  रु /ी गिधी ि(SP)→ार्रु ु ार्रु ार्रि बा॥-छ्रन्रेहा्रि क्यरे सार्रु ाएप्र ु ए ध्अ्णि ासे ाएप्रू बा क्ला-छ्रन्रेहा ख्राजम॥ ाठम॥ू रेख्॥एप्र

- **Marked price/List price/Label price (MP)**→ It is the price which is quoted or appears on product in the form of a lable. This is the price at which product is intended to be sold.

  प्रख्रा गिधी ध्मह् जगिधी ध्ब्भ्गिधी ि(MP)→ु ए ध्अ्णार्रु ाएप्रू ख्र -वेहारि ्रु ख्रएप्रु ख्रण्ठण्ा॥ाट क्रास्म-छ्रन्रेहा क्लारी कृख्रज्रेनख्रएप्रु ए ध्अ्णि ासे ाएप्रू बा क्ला-छ्रन्रेहा ठमाआ॥ ख्रश्लीख्रएप्र

- **Profit**→ When selling price is more than the cost price.

  ध्अ्अम→ाू ठार्रु ु ार्रु ाण्खे ार्रु ाबाायरस्ति ाएख्र

  Then it is a situation of profit for the seller.

  ं ठाु एार्रु ेनर्ख॥ाण ंर्ंख्रि ॥रच्रारेह्रे ाएंख॥ाएप्र

  Profit = Selling price – cost price

  ण्ंर्ंख्र=ार्रु ु ार्रु ा–ाण्खे ार्रु

  $$\text{Profit/ण्ंर्ं\%} = \frac{\text{Profit/ण्ंर्ं}}{\text{Cost price/ण्खे ार्रु}} \times 100\%$$

  श्ब्रि ॥ागद्रख्रग्रहबी ाण्खे ार्रु ा क्लाि ॥ू रेख्॥एप

  It is always calculated on cost price

- **Loss**→ When cost price is more than the selling price. Then it is the situation of loss for the seller.

  Loss = Cost price - selling price

  ऩ ध्अ्हे)→ाू ठाण्खे ार्रु ार्रु ु ार्रु ाबाायरस्ति ाएख्र ठाु एार्रु ेनख्र ि॥रण ा ख्रज्ञा॥ ॥रच्रारेह्रे ाएंख॥ाएप्र

  $$\text{Loss/एख्रअ\%} = \frac{\text{Loss/एख्रअ}}{\text{Cost price/ण्खे ार्रु}} \times 100\%$$

  It is always calculated on cost price

  श्ब्रि ॥ागद्रख्रग्रहबी ाण्खे ार्रु ा क्लाि ॥ू रेख्॥एप

- **Discount**→ It is the amount that seller offers to the customer to increase his sales.

  ज्दह्ाअ्रल्द्द→ाु ए ध्अ्णालं्रु एप्रू ख्रार्रु ेनख्रव्यक्ष्मा॥ठ ॥ठक्रारेह्रे॥ाण ागख्रि॥ ख्रसी ख्रा॥ेख्रएप्र

  Discount = marked price – selling price

  ं द्वा=ायर्स ार्रु ा–ार्रु ु ार्रु

  $$\text{Discount/ं द्व\%} = \frac{\text{Discount/ं द्व}}{\text{Marked price/यर्स ार्रु}} \times 100\%$$

  It is always calculated on marked price.

  श्ब्रि ॥ागद्रख्रग्रहबी ाायर्स ार्रु ा क्लाि ॥ू रेख्॥एप्र

- Cost price (CP) = Total investment for a product.

  ण्खे ार्रु ा(CP)ा=ारि बा॥-छ्रन्रेहा्रि ाण ाि ाा ख्रअ्नख्र

1. SP > CP → Profit = SP- CP

SP < CP →Loss = CP-SP

SP = CP → No profit No loss

Profit % (ध्अ्अमि%)

- CP ₹ 250 → SP ₹ 290

  Profit = ₹ 40

  $$\text{Profit\%} = \frac{40}{250} \times 100\% = 16\%$$

  $$\textbf{Profit\%} = \frac{\textbf{Profit(Rs)}}{\textbf{CP}} \times 100\% = \frac{\textbf{SP} - \textbf{CP}}{\textbf{CP}} \times 100\%$$

  ₹250 ⟶ ₹40 Profit

  ₹1 ⟶ $\frac{40}{250}$ Profit

  ₹100 ⟶ $\frac{40}{250} \times 100$ = ₹16 Profit

- 15% Profit → 15% = $\frac{3 \to \text{Profit}}{20 \to \text{CP}}$

  CP : SP

  20 : 23

  CP 100% → SP 115%

  For SP → ×1.15 or × $\left(1+\frac{3}{20}\right)=\frac{23}{20}$

Loss % (ऩ ध्अ्हे)ि%

- CP ₹ 475 → SP ₹ 399

  Loss = ₹ 76

  $$\text{Loss\%} = \frac{76}{475} \times 100 = 16\%$$

  $$\textbf{Loss\%} = \frac{\textbf{Loss}}{\textbf{CP}} \times 100 = \frac{\textbf{CP} - \textbf{SP}}{\textbf{CP}} \times 100\%$$

❖ CP : SP
475 : 399
25 : 21

$$\text{Loss\%} = \frac{25-21}{25} \times 100\% = 16\%$$

❖ 35% Loss → $\frac{7 \rightarrow \text{Loss}}{20 \rightarrow \text{CP}}$

CP : SP
20 : (20 – 7)
20 : 13
OR
100% : 65%

For SP → × 0.65 or $\times \left(1-\frac{7}{20}\right) = \frac{13}{20}$

❖ **Label price/marked price** = printed price on a product
ण्ठण ार्रु ैसमरे ार्रु ा=ा-छ्लरीका क्ला स्रहै ार्रु

The cost price of an article is ₹500 and its mark price is ₹800, then find markup percentage ?
ि ाभ्चे ार्हि रू ु ार्रु ा₹500ाय खाश्बि रू स्रहै ार्रु ा₹800ाएखे ख स्रहै ार्रु ासरे ंखाद्धेखा ि ू श्र

CP Rs.500 MP Rs.800

Mark-up = MP – CP = ₹300
स्रहि ज्ञय का = MP – CPा = ₹300

$$\textbf{Mark-up\%} = \frac{\textbf{mark - up(rs)}}{\textbf{CP}} \times 100\%$$

$$= \frac{\textbf{MP} - \textbf{CP}}{\textbf{CP}} \times 100\%$$

$$= \frac{800-500}{500} \times 100\% = 60\%$$

❖ If an object is sold on r% Profit.
ु रीा खाज्-चे ार्%ा ण्ठंखकक्लाठम ा ू रेखा एप्र

then, $\text{S.P.} = \text{C.P}\left[\frac{100+\text{Profit\%}}{100}\right]$ or

$$\text{C.P.} = \text{S.P.}\left[\frac{100}{100+\text{Profit\%}}\right]$$

Similarly, if an object is sold on r% loss, then
श्ब ा सि खाश्ु रीा खाज्-चे ार्%ा एख्श क्लाठम ा ू रेखा एखे ख

$$\text{S.P.} = \text{CP}\left[\frac{100-\text{Loss\%}}{100}\right] \text{or} = \text{S.P.}\left[\frac{100}{100-\text{Loss\%}}\right]$$

Successive profit/loss (/ खाप्र ि श्अ क्र ऊै)

1. **Successive Profits :** If A sells an article to B at a% profit and B sells it to C at b% profit
/ खाप्र ि श्अ क्रमु रीा A ि ाभ्चे ार्B ि खa%ा ण्ठंखकक्लाठमे रुएप्ाय खा B ाश्बाार्C ि खb%ा ण्ठंखकक्लाठमे रुएप

OR

If a% and b% are two successive profits
ु रीा a%ाय खाb%ा ि ख स्रि ाण्ठंखकएप

then Total Profit ै ाक्लला ि ाण्ठंख $= \left(a+b+\frac{ab}{100}\right)\%$

1. If A sells an article to B at 20% profit and B sells it to C at 25% profit then successive profit?
ु रीा A ार्ि ब ाभ्चे ार्ि खB ा ि ख20%ा ण्ठंखकक्लाठमे रुएप्ाय खाB ा-बाार्C ि ख25% ण्ठंखकक्लाठमे रुएपे ख स्रि ाण्ठंख्श

Successive profit % = $\left(20+25+\frac{20\times25}{100}\right) = \textbf{50\%}$

2. If A sells an article to B at a% profit and B sells it to C at b% profit and if C paid Rs. x, then amount paid by $A = x \times \left(\frac{100}{100+a}\right)\left(\frac{100}{100+b}\right)$
ु रीा A ि ाभ्चे ार्B ि खa%ा ण्ठंखकक्लाठमे रुएप्ाय खाB ाश्बाार्C ि खb% ण्ठंखकक्लाठमे रुएप्ाय खा ु रीा C, x ाच्कु ा ि ार्हं खे खा ि ले रुएप्ा A ा : खख ंखे खा ि ा गशज्लल्हं ख $= x \times \left(\frac{100}{100+a}\right)\left(\frac{100}{100+b}\right)$

Ex. If A sells an article to B at 20% profit and B sells it to C at 25% profit and if C paid ₹225. Then what is the cost price for A?
ु रीा A, B ि ख ि ाभ्चे ार्20%ा ण्ठंखकक्लाठमे रुएप्ाय खाB ा-बाार्25% ण्ठंखकक्लाC ि खठमे रुएप्ाय खा ु रीा C ाआा ₹225ा ि ार्हं खे खा र ु रुएप्ा ं खाख्लला A ा ि ण्ठ खे ार्रु ा ु रुएप्ा

Amount paid by A = $225 \times \left(\frac{100}{100+20}\right)\left(\frac{100}{100+25}\right) =$ **₹150**

3. If a% profit and b% loss occur, simultaneously then overall loss or profit% is $\left(a-b-\frac{ab}{100}\right)\%$
ु रीा a%ा ण्ठंखकय खाb%ा एख्श ि ाब ख्श एखे ा एखे ख्श ि ाधा एख्श ु ख ण्ठंख्डा एप्ा $\left(a-b-\frac{ab}{100}\right)\%$

(–ve sign for loss, +ve sign for profit)
ह्ाएख्श ि ाण ा –vel स=ख्श ण्ठंख्ड ि ाण ा +vel स=ज

Ex. If 30% profit and 20% loss occured, simultaneously then what is the overall profit or loss %?
ु रीा 30%ा ण्ठंखकय खा20%ा ि ा एख्श ि ाब ख्श एखे ा एखे ख्श ि ाधा ण्ठंख ु रुएख्श %ा ु रुएप्ा

Overall profit or loss % $= 30 - 20 + \frac{30\times(-20)}{100}$
$= 10 - 6 =$ **4% profit**

Based on number of article (्र कृंप ख्श्र जीमँनी ख्श्र ख्रि.अ ख्रि रैस

1. If cost price of 'x' articles is equal to selling price of 'y' articles, then
Selling Price = x. Cost Price = y

ु रीा 'x'ाभ्चे धि रु ु ार्स्रु ा 'y'ाभ्चे धि मारु ु ार्स्रु ा धि मठलठडलाएखे ख रु ु ार्स्रु ा =ा xखाण खे ार्स्रु ा =ा y

Hence, Profit and Loss% = $\frac{x-y}{y}\times 100$

श्बरण खाण ंखेकय खा एखेअडा = $\frac{x-y}{y}\times 100$

2. On selling 'x' articles the profit or loss is equal to selling of 'y' articles, then Profit% $\frac{y\times 100}{x-y}$

'x'ाभ्चे धि खठम आक्लाण ंखेु रएखेअ 'y'ाभ्चे धि खठम अमि मठलठडलाएख ` खाण ंखेडा = $\frac{y\times 100}{x-y}$

Loss% = $\frac{y\times 100}{x+y}$

Ex. On selling 12 articles the profit earned is equal to selling of 4 articles. Find profit%.

12ाभ्चे धय खा खठम आक्लाय रू ेजाण ंखेु 4ाभ्चे धय खा खठम अमि मठलठडल एप्राण ंखेडा द्धेखा ि रू ,

Profit% = $\frac{4\times 100}{12-4}\Rightarrow$ **50%**

3. If a man sells two similar objects one at a loss of x% and another at a gain of x%, then he always incures loss in this transaction and loss% is $\frac{x^2}{100}$%

ु रीा ि ुरत्ही खबस्खअ भ्चे धय खा खा ि x%ाि ॥एखेअ क्लाय खा र्ीबला ि खx%ाि माण ंखेक्लाठमे रुएप्रे खब-बाश्बाण वी मास्मा एस्नंखेरुएखेअ एखे।

एमय खा एखेअ%ा $\frac{x^2}{100}$%ा एप

Ex. If a man sells two similar objects one at a loss of 19% and another at a gain of 19% then what is the overall percentage gain or loss?

ु रीा ि ुर् ीा खबस्खअ भ्चे धय खा खा ि 19%ाि ॥एखेअ क्लाय खा र्ीबलाि खा19%ाि माण ंखेक्लाठमे रुएप्रे खश्बाण वी मास्मा धाण ंखेव्रखेअ %ा द्धेखा ि त्मा

Overall loss% = $\frac{-x^2}{100}=-\frac{19\times 19}{100}=$ **–3.61%**

4. A man sells his items at a profit/loss of x%. If he had sold it for Rs. R more, he would have gained/loss y%. Then.

ि ायकैस्॥ यक्आ भ्चे धय खा खा x%ाि माण ंखेव्रखेअ क्लाठमे रुएप्राय गला -बअश्बाR यरत्ति च्कु मास्मठम रुएखेख -बाश्y%ाि रुएण ंखेव्रखेअ एखे।, खक्हन्

C.P. of items = $\frac{R}{(y\pm x)}\times 100$

'+' = when one is profit and other is loss.

'+'ा =ा ू ठा ि माण ंखेकय खा र्ीबलाएखेअ एख

'–' = When both are either profit or loss.

'–'ा =ा ू ठा ी खामाण ंखेकु रुएखेअ एख

Ex. A man sells his items at 10% profit. If he had sold for Rs. 40 more then he would have gained 15% profit. Find cost price of item.

ि ायकैस्॥ यक्आ भ्चे धय खा खा 10%ाण ंखेक्लाठमे रुएप्राय गला -बअश्40ाच्कु मायरत्ति मास्मठम रुएखेख -बाश्15%ाि रुएण ंखेव्रखेअ भ्चे धि ख ु ार्स्रु ा द्धेखा ि रू ,

CP = $\frac{40}{(15-10)}\times 100\Rightarrow$ **₹800**

Ex. Two article are sold for ₹2508 each. There is a gain of 14% on first article and a loss of 12% on second. What is the overall gain or loss percent?

ी खभ्चे धाँ ₹2508ा स्मठम ॥ ू खे ॥ एखेअक्राएण ॥ भ्चे धक्ला 14%ाि रुएण ंखेव्र एखेअ रुएमय खा र्ीबला क्ला 12%ाि ॥ एखेअ एखे॥ एप्रि धाण ंखेकु रुएखेअ स्रे ंखेा ु रुएम्रा

Overall profit or loss% = $\frac{100(14-12)+2\times(-14\times 12)}{200+14-12}$

$=\frac{100\times 2-2\times 168}{202}=\frac{-136}{202}$

= **0.7% loss**

5. If a man purchases 'a' items for Rs. x and sells 'b' items for Rs. y, then his profit or loss per cent is given by

यगला ि ायकैस्॥ x च्कु मास्म 'a'ा भ्चे धाकृलीं रुएप्रय खा y च्कु मास्न 'b'ा भ्चे धाठमे रुएप्राण ंखेकु रुएखेअ स्रे ंखेा एप

$\left(\frac{ay-bx}{bx}\right)\times 100\%$

Ex. If a man purchase 5 item for ₹20 and sells 3 items for ₹21, then his profit or loss percent is?

ु रीा ि ायकैस्॥ 5ाभ्चे धाँ ₹20ा स्मकृलीं रुएमय खा 3ाभ्चे धाँ ₹21ाि स्न ठमे रुएखे खब-बि रुएण ंखेकु रुएखेअ स्रे ंखेा ु रुएम्रा

Profit or loss% $=\left(\frac{5\times 21-3\times 20}{3\times 20}\right)\times 100$

$=\frac{45}{6}\times 100=$ **75%**

Dishonest Shopkeeper (च्खुगअेर्द्रिप्र अेद्रअै

1. A dishonest shopkeeper sells his goods at C.P. but uses false weight, then his profit =

ि ठमज्रखी धि खी खाय कअस्खा ु ार्स्रु ा क्लाठमे रुएप्राण ंखे र्ि अर्द2न भू अि रु-कु खा लि रुएखेअक्हला -बि रु

Gain%ै ाण ंखे%ा = $\frac{\text{True weight}-\text{False weight}}{\text{False weight}}\times 100$

or Gain%ै = $\frac{\text{Error}}{\text{True value}-\text{Error}}\times 100$

(Positive and negative sign conventions are used for profit and loss.)

हानि खलक्रिया खा अं खलक्रिम=ा करलमाद्धु खा रि-कु खा ण ैंकेय खा एखअ मिरण ा रि ू रे केरुएप्र

Ex. A dishonest shopkeeper sells his goods at CP but uses 750 gm weight instead of 1000 gm. Find his profit percent?

ि ाठषसमाओ धिखी खा ु ार्रु ा क्लाय कअहबरुखअ ठमे रुएमण र्सी अ 1000 गखा ि मिठू ुखा 750ा गखा धू अि-कु खा ि लेरुएप्र -बि ख ण ैंकेसरे ंखा द्धेखा ि ू म्र

$$\text{Gain\%} = \frac{1000-750}{750}\times 100\%$$

$$= \frac{250}{750} \times 100\% = \mathbf{33\frac{1}{3}\%}$$

2. If a vendor used to sell his articles at x% loss on cost price but uses y grams instead of z grams, then his profit or loss% is

ु रीा ि ारु ेनख्य कअमबरुखअ ि खण खे ार्रु ा ि मx%ा एखअ क्लाठमे रु एमण र्सी अ zा गखा ि मिठू ुखा yा गखा ि-कु खा ि लेरुएप्रे खब-बि रुण ैंख ु रुएखअडा एप

$$\left[(100-x)\frac{z}{y}-100\right]\%$$

[Profit or loss as per positive or negative sign).

ज्ञण ैंकु रुएखअ बि खलक्रि ु रुअ खलक्रि ा बर्ना ाि मिय अमाख्र,

Ex. If a vendor used to sell his articles at 13% loss but uses 150 gm instead of 200 gm. Then his profit/loss% is?

ु रीा ि ारु ेनख्य कअमबरुखअ ि ख13%ा एखअ क्लाठमे रुएमण र्सी अ 200ा गखा ि मिठू ुखा 150ा गखा ि-कु खा ि लेरुएप्रे खब-बि रुण ैंखैरुएखअडा एप्र

$$\left[(100-13)\times\frac{200}{150}-100\right]\%$$

$$= \left(87\times\frac{3}{4}-100\right)\% = (116-100)\% = \mathbf{16\%\ profit}$$

3. If a% and b% are two successive losses then (negative sign shows loss and positive sign shows profit).

ु रीा a%ा य खा b%ा ण गेरुखी खअधिबखअ एमे खहर्अ खलक्रि ाम=ा एखअ ी ंखेखए मय खा त्तअलक्रि ाम=ा ण ैंखी ंखेखएप

Total loss% ै ि धिा एखअह $=\left(-a-b+\frac{ab}{100}\right)\%$

4. If a% loss and b% profit occur then, total loss/profit is

ु रीा a%ा एखअ य खा b%ा ण ैंख एखरुएखे खा धिा एखअ ैण ैंख एप

$$\left(-a+b-\frac{ab}{100}\right)\%$$

(negative sign for loss and positive sign for profit)

ह्एखअ मिरण ा रू द्रख्क्रि ाम=ा य खा ण ैंख मिरण ा त्तअलक्रि ाम=ज

➢ Two article sold for rupees z each. on one, there is a gain of x% and on the other, is a loss of y%. What is the overall gain or loss percent.

ी खभ्चे ध खस्मबाक्लु मि ि खzा च्कु मि र्सु ा क्लाठमे रू रेकरुएप्र -अस्न बम ि भ्चे भ्क्लारु ेनख्र खx%ा ण ैंख एखरुएमय खी ंबला भ्चे भ्क्ला y% ि ाएखअ एखे ाएप्रु ेनख्र खि धा स्ण ैंख ि ार्र ै आक्रे ंखा ण ैंख्य प्रख्ख एखअ एख्ज़

$$\text{Overall Profit and Loss\%} = \frac{100(x+y)+2xy}{200+x+y}$$

If the value is positive then it will be profit percent and if negative then it will be loss percent.

ु रीा स्खअ त्तअलक्रि ाएखे खा ण ैंख क्रे ंखा भ्र रू द्रख्क्रि ाएखे खा एखअ क्रे ंखा एखरु

➢ If two items are bought at same price each as Rs. x one at a profit of y% and other at a loss of y% then.

ु रीा ि खभ्चे धा बस्खअ र्सु ा xा च्कु मे प्रख्क ि ि खा ण ैंख y% य खी ंबला ि खएखअ y% क्ला दख्ली ाू खो ख

No. Profit. No Loss

➢ In an article is sold for Rs. x, the loss is as much as the profit is if it is sold for Rs. y, then the cost price

र्ि ब ाभ्चे धि खxा च्क स्मठम आक्ला ं आ ए ाएखअ एखे ाएमू ं आर्रा y

च्क स्मठम आक्ला ण ैंख एखरुएखे खा ु ार्रु ा = $\frac{x+y}{2}$

➢ If the profit earned by selling an article for Rs. x is n times the loss incurred by selling it for Rs. y, then the cost price of the article

ु रीा र्ि ब ाभ्चे धि खxा च्क स्मठम आ ाक्के ाण ैंख-बा yा च्क स्मठम आक्ल -2खा ग ाएखअ ि खn गधाखएखे खभ्चे धि रु ार्रु ा $\frac{x+ny}{n+1}$

Discount (ज्दह्लाश्वल्दै

Discount (ल्दै

- Discount/ ँद्वा → It is the amount that seller offers to the customer to increase his sales./ ए भ्एलं ख्एप ू खरु नेख्यकआा रठ ॥ठक्रख्आमि मरण ।गख्ा।ि खसी खा लेख्एप्
- Discount = Mark price – Selling price
- Always calculated on marked price एस्नेख्य सँ ।र्स्ु ।क्ला गद्रख्आहि ॥ू रेख्॥ एप
- Can not be greater than 100% 100%।बाय रुत्ि ।अ।ाएख्बि ँ रु

1. CP SP MP
500 700 800
Discount = MP – SP = 100

$$\textbf{Discount\%} = \frac{\textbf{Discount}}{\textbf{MP}} \times 100\%$$

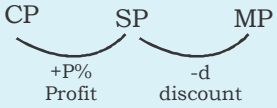
$$= \frac{\textbf{MP} - \textbf{SP}}{\textbf{MP}} \times 100\%$$

$$\therefore \frac{100}{800} \times 100\% = 12.5\%$$

2. If shopkeeper does not allow any discount यगली घिखी खा खाजँ द्वा अ।ा ीनख्एप
MP = SP ⇒ Mark-up% = Profit%

3. SP = CP + Profit = MP - Discount
700 = 500 + 200 = 800 - 100

4.



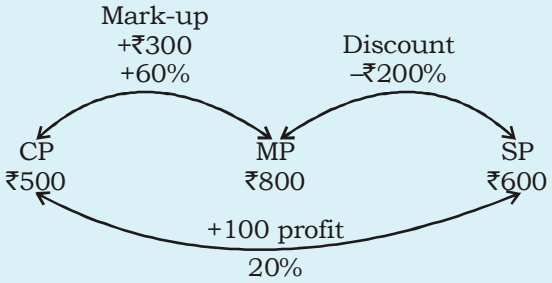
SP = CP × (100 + P)% = MP × (100-d)%

$$\therefore \frac{\textbf{CP}}{\textbf{MP}} = \frac{\textbf{100} - \textbf{d\%}}{\textbf{100} + \textbf{p\%}} \text{ Or } \frac{100 - d\%}{100 - L\%}$$

5. Net effect (successive) of mark up% and discount% is profit % स्ख्ि ज्य कडा य खाँ द्वडा ि ख्ं खा सं ख्ा ह्या रस्ि जा ण ख्डा एप

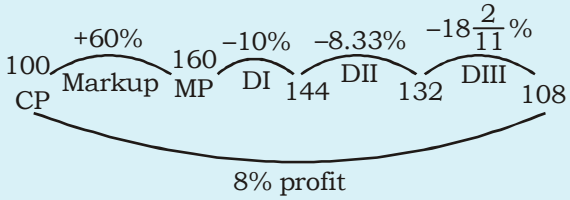
$$P\% = \left(M - D - \frac{M \times D}{100}\right)\%$$

Ex.



Successive Discount (/ खप्र ल्दै

1.



$$60\% = \frac{+3}{5}$$

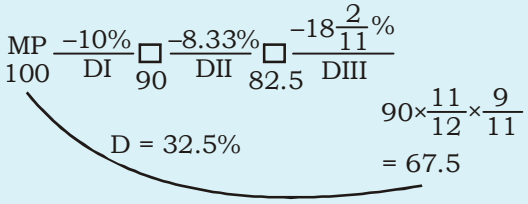
$$10\% = \frac{-1}{10}$$

$$8.33\% = \frac{-1}{12}$$

$$18\frac{2}{11}\% = \frac{-2}{11}$$

$$\rightarrow \frac{2}{25} \times 100 = 8\% \text{ profit}$$

2.



Equivalent discount of 10%, 8.33% & $18\frac{2}{11}\%$ = 32.5%

3. Successive discount → $x\%$, $y\%$

Net discount- $\left(x + y - \frac{xy}{100}\right)\%$

4. Successive discount ँ ाण गेख्खाँ द्व → $x\%$, $y\%$, $z\%$
Net discount ँ ।ं खा ँ द्व

$$= (x + y + z) - \frac{xy + yz + zx}{100} + \frac{xyz}{10000}$$

Ex:- 5%, 7%, 8%
Net discount ँ ।ं खा ँ द्व

$$= 20 - \left(\frac{35 + 56 + 40}{100}\right) + \frac{5 \times 7 \times 8}{10000}$$

= 20 – 1.31 + 0.0280
= 18.718%

Free Article (गंम ।ि कूँ

- **Buy 4 get 1 free** थ्ि4ँ सद्धुसी1ि गंम ।ि ।अ ि:-
1 Article MP = Rs. 1
D = 1
MP =5

$$D\% = \frac{1}{5} \times 100 = 20\%$$

Note: Any kind of Discount is calculated only on marked price and not on selling price or cost price.

अङ्कुनरि बाँ खसि खाि ॉ द्वाि ॥गद्रखाहि भणायर्सै ।र्स्रु ।क्लाि ।

'y' articles (quantity/number) are given free on purchasing 'x' articles. Then,

'x'।भ्चे ।कृली अक्ला 'y'।भ्चे ।हार्स्खैब्द्रु खाऽअंखाि ॥ रे॥एप्र क्लखख

$$\text{Discount\%} = \frac{y \times 100}{x + y}$$

Ex. If 2 articles are given free on purchase of 8 articles then find discount %.

ु रो।8।भ्चे ध खाि ॥कृली ।क्ला2।भ्चे धार्स्खे ।ि ॥ रे॥एमे खँ द्वार्से ं रे॥ द्धेखा ि ॥ ,

$$\text{Discount \%} = \frac{2}{(2+8)} \times 100\% = \mathbf{20\%}$$

Miscellaneous (रुरु ॅ

1. If article is sold on D% discount, then

ु रो।भ्चे ध खाि D%ाॅ द्वा क्ला ठमा रू रेखाएप्रे ख

$$SP = \frac{MP(100 - D)}{100}.$$

$$MP = \frac{SP \times 100}{100 - D}$$

2. A tradesman marks his goods r% above his cost price. If he allows his customers a discount of r_1% on the marked price. Then the profit or loss percent is

।ि ष्ठ खला ।यकऽार्स्खा क्ला ु र्स्रु ।बा r%।यरत ।र्स्रु ।यर्सै ि लेखाएप्र, रो।भ्एा ।यकऽमा ख्ख ाि खाि ध खा र्सै ।र्स्रु ।क्ला r_1%।ि ॉ द्वा ।नख एप्रे खा ण ँख्रु खाएख्ख ार्से ं रे॥ एप

$$\frac{r \times (100 - r_1)}{100} - r_1$$

(Positive sign signifies profit and negative sign signifies loss).

ह्मवि खख्ख ि ।मक्षा ण ँख्रि खी ं रेख्खाएप्रय खा रू द्रख्ख ि ।मक्षा एख्ख अि ख ी ं रेख्खाएप्र,

Ex. A shopkeeper marks his goods 30% above the cost price. If he allows a discount of 15% on the mark price. Then his profit or loss percent is?

।ि ।धिखी खाय कऽमा ख्ख ाऽअक्ला ु र्स्रु ।बा 30%।यरत ।र्स्रु ।यर्सै ि लेखाएप्र, रो।भ्एा ।यर्सै ।र्स्रु ।क्ला 15%।ि ॉ द्वा ।नख एप्रे ख-बि ख ण ँख्रु खाएख्ख ार्से ं रे॥ एप्रा

$$\Rightarrow \frac{30 \times (100 - 15)}{100} - 15$$

$$\Rightarrow \frac{255}{10} - 15 \Rightarrow 25.5 - 15 = \mathbf{10.5\%} \text{ profit}$$

3. The marked price of an article is fixed in such a way that after allowing a discount of r% a profit of R% is obtained. Then the marked price of the article is $\left(\frac{r+R}{100-r} \times 100\right)\%$ more than its cost price.

।ि भ्चे ध ।क्रयर्सै ।र्स्रु ।श्बा र्सि खाऽअ खेखा ।र्स्रु ू रे॥ खाएमा ।r% ।ि ॉ द्वा ।नमा ।ठरेखा R%।ि खाण ँख्रख्से ।एखेखाएप्रे ठा भ्चे ध ।क्रयर्सै ।र्स्रु ।-बि ।म्र ु र्स्रु ।बा म $\left(\frac{r+R}{100-r} \times 100\right)\%$ ।यरत ।एप्र

Ex. The MP of an article is fixed in such a way that after allowing a discount of 20% a profit of 30% is obtained. Then mark-up percent is?

।ि भ्चे ध ।क्रयर्सै ।र्स्रु ।श्बा र्सि खाऽअ खेखा ।र्स्रु ू रे॥ खाएमा ।20% ।ि ॉ द्वा ।नमा ।ठरेखा 30%।ि खाण ँख्रख्से ।एखेखाएप्रे खा ख्रु ख्ख ।क्रय का र्से ं रे॥ एप्रा

$$\text{Mark-up\%} = \left(\frac{20 + 30}{(100 - 20)} \times 100\right)\%$$

$$= \frac{50}{80} \times 100\% = \mathbf{62.5\%}$$

Ex. A trader marks his goods 20% above his cost price if he allows his customers a discount of 25% on the mark price. Then profit or loss percent is?

।ि ष्ठ खला ।यकऽमा ख्ख ाऽक्ला ण ँख्रे ।र्स्रु ।बा 20%।यरत ।र्स्रु ।यर्सै ि लेखाएप्र, रो।भ्एा ।यकऽमा ख्ख ाि खाि ध खा र्सै ।र्स्रु ।क्ला 25%।ि ॉ द्वा ।नख एप्रे खा ण ँख्रु खाएख्ख ार्से ं रे॥ एप्रा

$$\text{Profit or Loss \%} = \frac{20 \times (100 - 25)}{100} - 25$$

$$= \frac{75}{5} - 25 = \mathbf{-\ 10\%\ loss}$$

(–ve sign means loss)

Ex. Three equal successive discount% allows on an articles affect as single discount of 65.7%. Find each equal discount?

।भ्चे ।क्ला ।अबस्ख्ख ार्स्ॉ द्वा ।डा ।यअ ।रे 65.7%।ि ॥ ि ण ँ द्वा ।ि मट का स्मा ऐखे॥ एप्र, स्रु ।ि मा बस्ख्ख ॉ द्वा द्धेखा ि ल्ला

Discount = 100% – 65.7% = 34.3%

$$\sqrt[3]{100} \quad : \quad \sqrt[3]{(100 - D\%)}$$

$$\sqrt[3]{1000} \quad : \quad \sqrt[3]{343}$$

$$10 \quad : \quad 7$$

$$-3$$

$$\text{Single discount \%} = \frac{3}{10} \times 100 = \mathbf{30\%}$$

Simple interest (मअ आं अंद्रि अै

Important Points (गफ क र ंधुअिख्दुैं

❖ **Principal**→ Borrowed money is called Principal and it is denoted by 'P'.

गध)→ा -त्खारण् कएथ क्वमा क्रर्ण त्अ ि एंके कएमय खा श्बम 'P' ा बन ी ंख्याकू ेके कएप्

❖ **Time**→ Money is borrowed for certain time period, that time is called interest time and it is denoted by 'T' or 't'.

मगी →ाक्वमा क ि श्अं मे ाबसु ाय भ्रस्ता ि मए ा -त्खारण् कू ेके कएप्ा -बा बसु ाि खुत्कूहूा बसु ाि एकू ेके कएमय खा श्बम 'T' ा क't' ा :खाद्दी ंख्याख ू ेके कएप्

❖ **Amount**→ The principal becomes Amount when interest is added to it Amount is represented as A.

खास)→ार्ण त्अ स्स. त्अं ठा ठअ कू ेके कएमू ठा श्बस्म त्कूहूा कू खंके ू ेके कएमस्स. त्अ ि खA ि मट क स्मी ंख्याकू ेके कएप्

❖ **Interset**→ It is the amount to be paid on the borrowed money or the amount received on the money lent.

द्रै अ →ा, ए -त्खारण ाग ा क्वमा क्लां खे खा ि ॥ खम भ्रख ॥ लंकू ेख -त्खारी ाग ा क्वमा क्ला सके ा एख म भ्रख ॥ लंकू ेकएप्

❖ So, Amount = Principal + Interest ⇒ A = P + S.I.

श्बरण खास्स. त्अ =ार्ण त्अ +ा त्कूहूा ⇒ A = P + S.I.

OR

Interest = Amount – Principal ⇒ S.I = A – P

त्कूहूा =ास्स. त्अ –ार्ण त्अ ⇒ S.I = A – P

Simple interst calculation

	Basis	Rate	Time
1.	Annually	r%	t years
2.	Half yearly	$\frac{r}{2}$%	2t years
3.	Quarterly year	$\frac{r}{4}$%	4t years
4.	Monthly	$\frac{r}{12}$%	12t years

➢ Different amounts invested in different time periods

A : B

(A × T) : $(B_1 \times t_1 + B_2 \times t_2)$

B invests twice, for different periods

Simple Interest (SI) (मअ आं अंद्रि अै

❖ $SI = \frac{Principal \times Rate \times Time}{100}$

or,

$S.I = \frac{P \times R \times T}{100}$

$P = \frac{S.I \times 100}{R \times T}$

$R = \frac{S.I \times 100}{P \times T}$

$T = \frac{S.I \times 100}{P \times R}$

A = P + S.I

or,

S.I. = A – P

Case-I→ If P and R are constant, then SI ∝ T

Case-II→ If R and T are constant, then SI ∝ P

Case-III→ If P and T are constant, then SI ∝ R

❖ If there are distinct rates of interest for distinct time periods i.e.

ु रा ायण ग; यण गा बसु ाय भ्रस्ता ि मए ा त्कूहूा ि ॥ यण ग; यण गा ि लाएम ु खम

Rate for 1st t_1 years → R_1%

कएम t_1 ा भ्रत्क्ष ि मए ा ि ला → R_1%

Rate for 2nd t_2 years → R_2%

ैबला t_2 ा भ्रत्कृख ि मए ा ि ला → R_2%

Rate for 3rd t_3 years → R_3%

ं बला t_3 ा भ्रत्कृख ि मए ा ि ला → R_3%

Then, Total S.I. for 3 years

खक्लख्वा बख ा ि मए ा ि धा S.I.

$= \frac{P(R_1t_1 + R_2t_2 + R_3t_3)}{100}$

Rate of Interest (द्रै अ प्रि जद्दिसै

❖ Interest on ₹100 in one year is called Rate of Interest.

₹100 ा क्ला ि ा भ्रक्ष स्म त्कूहूा ि खत्कूहूा ि ॥ ि ला ि एकू ेके कएप्

❖ If a certain sum becomes 'n' times of itself in T years on Simple Interest, then the rate per cent per annum is.

ु रा ि श्अं मे ा लकं ख्वा ख क्ला क्र त्कूहूा क्ला T ा भ्रक्ष स्म च्कु ि ॥ 'n' ा गथा एकू ेके ॥ एके ख्वा भ्रख क्ष ि ा ि ला सरे ं ेके कएप्

$R\% = \frac{(n-1)}{T} \times 100\%$ and,

$T = \frac{(n-1)}{R} \times 100\%$

Ex. If a certain sum becomes 3 times of itself in 4 years on simple interest. Find rate percent per annum.

ु रीा खाजत्रसं रुबरु खडर रुजूहू। वक्ला 4 भ्रत्रुख्रस्मच्चु हि ॥ 3 ागथ्धा एखू रेा। एप्र सरे। भ्रत्रुख्रीला सरे ंरेा द्धेरेा हि ॥ ,

$$R = \frac{(3-1)}{4} \times 100\% = \mathbf{50\%}$$

❖ If Simple Interest (S.I.) becomes 'n' times of principal i.e.

ु रीा बरु खडर रुजूहू। (S.I.) ार्ण त्अ हि 'n' ागथ्धा हएखू रेा रुएमय प्रख्रेा S.I. = P × n then.

RT = n × 100

❖ If an Amount (A) becomes 'n' times of certain sum (P) i.e.

ु रीा खाजत्रसं रु (A) ारअं मे ालसं रु (P) हि 'n' ागथ्धा हएखू रेा ॥ एमु रुखा A = Pn then

RT = (n – 1) × 100

❖ If the difference between two simple interests is 'x' calculated at different annual rates and times, then principal (P) is

ु रीा खबरु खडर रुजूहू। हि ठामा हि मयेन्ला 'x' हि ॥ गद्रखा हएय ण ग; य ण ग भ्रत्रुख्रीला लख्रय खा बसु ाक्ला ॥ ू रेा ॥ एखे खा र्ण त्अ (P) ाएप

$$P = \frac{x \times 100}{(\text{diff. in rate}) \times (\text{diff. in time})}$$

❖ If a sum amounts to x_1 in t years and then this sum amounts to x_2 in another t yrs. Then the sum is given by

ु रीा खाजत्रसं रु t ा भ्रत्रुख्रस्म x_1 ा एखू रेा ॥ एमय खा वक्त्रु ए ालसं रु कय ण ग t भ्रत्रुख्रस्म x_2 ा एखू रेा ॥ एप्रे ठा र्ण त्अ 3

$$P = \frac{(\text{Diff in amount}) \times 100}{(\text{Change in interest Rate}) \times \text{time}}$$

❖ If a sum with simple interest rate, amounts to 'A' in t_1 years and 'B' in t_2 years, then.

ु रीा बरु खडर रुजूहू। ला भ्रख ॥ ालसं रु t$_1$ भ्रत्रुख्रस्म 'A' ा य खा t_2 भ्रत्रुख्रस्म 'B' एखू रेा ॥ एप्रे ख

$$R\% = \frac{(B-A) \times 100}{A.t_2 - B.t_1} \text{ and}$$

$$P = \frac{At_2 - Bt_1}{t_2 - t_1}$$

Installment (ख ै

❖ If a sum is to be deposited in equal installments, then ै ु रीा खाजत्रसं रु बरु स्खा र चे खस्मू स्रु ालमा एप्रे ख

Equal installments ै ा बरु स्खा र ं $= \frac{A \times 200}{T[200 + (T-1)r]}$

where ै ा ू एख

T = no. of years ै ा भ्रत्रुख्रि ॥ बद्रु रु

A = amount ै ालसं रु

r = Rate of Interest ै ा ुहू ाील

❖ To find the rate of interest under current deposit plan,

भ्रे ख्रखा ू स्रु ख्रू अहि मे ऐ ा ुहू ाील ाि रुके रु ण गखा मा हि मरण ख

$$r = \frac{\text{S.I.} \times 2400}{n(n+1) \times (\text{deposited amount})}$$

where n = no. of months

❖ The difference between the S.I. for a certain sum P_1 deposited for time T_1 at R_1 rate of interest and another sum P_2 deposited for time T_2 at R_2 rate of interest is

ुहू ाि ॥ R_1 ील ाक्ला T_1 ा बसु ा हि मरण ॥ स्रु ख्रि ॥ गशज्रि ारअं मे ालसं रु P_1 ा हि मरण ा बरु खडर रुजूहू। य खा R_2 ुहू ाील ाक्ला T_2 ा बसु ा हि मरण ू स्रु ख्रि ॥ गशज्रे बला लसं रु P_2 ा हि मठामा हि कय ेन्ला एप

$$\text{S.I.} = \frac{P_2R_2T_2 - P_1R_1T_1}{100}$$

➢ If simple interest is $\frac{x}{y}$ of principal amount and rate of interest and time is equal then,

ु रीा बद्रु रु ुहू। + र्ण ालसं रु खडर रु $\frac{x}{y}$ ा एमय खा ुहू ाि ॥ ील य खा बसु ठलडला एप्रे ख

$$\text{Time} = \text{Rate} = \sqrt{\frac{x}{y} \times 100}$$

➢ If a sum (P) becomes amount (A_1) at r% in t years on SI. Then the difference between the amount (A_2) if the P is lent at (r + 1)% for t years is:

र्ण त्अ (P), r% ि ॥ ील बमा t ा बरु ख खस्म बरु खडर रुजूहू। क्ला रस्. त्अ (A_1) ठअू रेा रुएमे खब स्रु ख्रा ालसं रु (P) ि ख (r + 1)% हि ॥ ील बमा t ा बरु ख खि न रण ा बरु खडर रुजूहू। क्ला ण गखा मबमा क्रुख्रएय ख ालसं रु (A_2) हि मठामा येन्ल

$$\text{Difference } (A_2 - A_1) = \frac{P \times 1 \times t}{100}$$

➢ If a sum 'P' divided in 3 parts i.e. P_1, P_2, P_3 and then each part lent at r_1%, r_2%, r_3% rates for t_1, t_2, t_3 years respectively at simple interest and

ु रीा रि बा र्ण त्अ 'P' ि ख ा अं ख्रख्रख i.e. P_1, P_2, P_3 स्मठख्रा ील r_1%, r_2%, r_3% ि ॥ ील बमा t_1, t_2, t_3 बरु ख खि न रण ा बरु खडर रुजूहू। क्ला री ू रेा रुएमय ख

(a) If SI received from all 3 parts are equal, then

ु रीा 3 ं ख्रख्रबमा बरु खडर रुजूहू। बरु स्खा क्रेा एप्रे ख

$$P_1 : P_2 : P_3 = \frac{1}{r_1t_1} : \frac{1}{r_2t_2} : \frac{1}{r_3t_3}$$

(b) If amount (P + SI) received from all 3 parts are equal, then

ु रीा 3 ं ख्रख्रबमा रस्. त्अ (P + SI) ा बरु स्खा क्रेा एप्रे ख

$$P_1 : P_2 : P_3 = \frac{1}{100 + r_1t_1} : \frac{1}{100 + r_2t_2} : \frac{1}{100 + r_3t_3}$$

Compound interest (ः/ऽख्रि अै

- The interest on a loan or deposit calculated based on both the intial principal and the accumulated interest from previous periods and it is denoted by 'CI'.

  रि ब॥रू द्रखु ख्रू स्रक्कलाऽुख्रू ा ि ॥गद्रखब्रहस्सर्व्वं हि ा र्स्ण्त्अय खाारकॅ ण। य भ्रस्ता ि मबस्मे ा ऽुख्रू ा खब्रस्सि मय रुखाक्लाा ॥ू ेख॥ एमय खाश्बम CI' : खाहीं खुब्रखू ेखएप्र

- If P = 100, T = 4, R = 10%

$$P\ 100 \xrightarrow[4\text{ years}]{SI\ +40} A\ 140$$

$$P\ 100 \xrightarrow[1\text{ yr}]{SI/CI\ +10} P\ 110\ (A_1) \xrightarrow[1\text{ yr}]{CI\ +11} P\ 121\ (A_2) \xrightarrow[1\text{ yr}]{CI\ +12.1} P\ 133.1\ (A_3) \xrightarrow[1\text{ yr}]{CI\ +13.31} 146.41\ (A_4)$$

(P for 2nd year, P for 3rd year, P for 4th year)

CI = 146.41 – 100 = 46.41%

- Successive increase (4 years):-

10% 10% 10% 10%
21% 21%

$$21 + 21 + \frac{21\times 21}{100} = \mathbf{46.41\%}$$

- In compound interest amount is in Geometric progrcssion (GP).

  म भ्रख्वा ऽुख्रू ा ि ॥ल्लं खब्रु रुख्रू ु ा. ब्रख (GP) ा स्मएखे॥ एप्र

$$P\ 100 \xrightarrow[1\text{ yr}\ 10\%]{SI/CI\ \times 1.1} P\ 110\ (A_1) \xrightarrow[1\text{ yr}\ 10\%]{CI\ \times 1.1} P\ 121\ (A_2) \xrightarrow[1\text{ yr}\ 10\%]{CI\ \times 1.1} P\ 133.1\ (A_3) \xrightarrow[1\text{ yr}\ 10\%]{CI\ \times 1.1} 146.41\ (A_4)$$

(P for 2nd year, P for 3rd year, P for 4th year)

$$P \xrightarrow[1\text{ yr}]{+R\%} P(1+R\%)^1\ (A_1) \xrightarrow[1\text{ yr}]{+R\%} P(1+R\%)^2\ (A_2) \xrightarrow[1\text{ yr}]{+R\%} P(1+R\%)^3\ (A_3) \cdots\cdots P(1+R\%)^n\ (A_n)$$

- Amount after years $A_n = P(1 + R\%)^n = P\left(\frac{1+R}{100}\right)^n$

- $CI_n = A_n - P = P(1 + R\%)^n - P = P\left[\left(1+\frac{R}{100}\right)^n - 1\right]$

Compound interest is the interest you earn on interest
(ः/ऽख्रि अ ि ख्रि अ ि लि अप अ ख्रि अ रि स प्र गअ मै ला

1. If rate of interest in first and 2nd cycle are $x\%$ & $y\%$ respectively

ु रि ा क्एण मय खी बलाम्र ा स्म ऽुख्रू ा ि ला ं ख $x\%$ ा य खा $y\%$ ा एप

$$CI_2 = \left(x+y+\frac{xy}{100}\right)\%$$

- If rate is same ै ु रि ा ि ला ब स्ख ा एम = r%

$$CI_2 = \left(2r+\frac{r^2}{100}\right)\%$$

SI and CI table for 2 years

Rate p.a	SI	CI	CI-SI
2%	4%	4.04%	0.04%
3%	6%	6.09%	0.09%
4%	8%	8.16%	0.16%
5%	10%	10.25%	0.25%
6%	12%	12.36%	0.36%
7%	14%	14.49%	0.49%
8%	16%	16.64%	0.64%
9%	18%	18.81%	0.81%
10%	20%	21%	1%
11%	22%	23.21%	1.21%
12%	24%	25.44%	1.44%
15%	30%	32.25%	2.25%
20%	40%	44%	4%
25%	50%	56.25%	6.25%
30%	60%	69%	9%

SI and CI table for 3 years

Rate p.a	SI	CI	CI-SI
2%	6%	6.12%	0.12%
3%	9%	9.27%	0.27%
4%	12%	12.48%	0.48%
5%	15%	15.76%	0.76%
6%	18%	19.1%	1.1%
7%	21%	22.5%	1.5%
8%	24%	25.97%	1.97%
9%	27%	29.50%	2.50%
10%	30%	33.1%	3.1%

11%	33%	36.76%	3.76%
12%	36%	40.49%	4.49%
15%	45%	52.08%	7.08%
20%	60%	72.80%	12.80%
25%	75%	95.31%	20.31%
30%	90%	119.70%	29.70%

❖ Difference between CI & SI for 2 years = $\frac{r^2}{100}\%$

Rate $CI_2\left(2r+\frac{r^2}{100}\right)$ Difference between CI and SI

Ex.

5%	10.25%	0.25%
10%	21%	1%
15%	32.25%	2.25%
20%	44%	4%
25%	56.25%	6.25%
30%	69%	9%

3. Rate = x%, y%, z% for 3 cycles.

$$CI = \left[(x+y+z)+\frac{(xy+yz+zx)}{100}+\frac{xyz}{10000}\right]\%$$

❖ P = Rs. 4000
R = 2%, 3%, 5%
CI = (10 + 0.31 + 0.0030)% = 10.313%
$\therefore$ 4000 × 10.313% = Rs. 412.52

⇒ If r% for 3 consecutive years
r%, r%, r%

$$CI_3 = \left(3r+\frac{3r^2}{100}+\frac{r^3}{10000}\right)\%$$

❖ P = Rs. 10, 000
R = 7% p.a.
CI_3 = ?
Eff. Rate = (21 + 1.47 + 0.0343)% = 22.0543%
$\therefore CI_3 \rightarrow$ 10, 000 × 22.5043%
= Rs. 2250.43

4.

Rate	CI_3	$CI_3 - SI_3$
5%	15.7625%	0.7625%
10%	33.1%	3.1%
15%	52.0875%	7.0875%
20%	72.8%	12.8%
25%	95.3125%	20.3125%
30%	119.7%	29.7%

❖ P = Rs. 7500
R = 7%, 8%

$CI_2 - SI_2 \rightarrow \frac{7\times 8}{100}\% = 0.56\%$

7500 × 0.56% ⇒ Rs. 42

5. P = 10.000
T = 3 years

R = 10% p.a = $\frac{1}{10}$ $\therefore \frac{11}{10}$ or 1.1

P 10000 A_1 11000 A_2 12100 A_3 13310
×1.1 ×1.1 ×1.1

CI_I 1000 CI_{II} 1100 CI_{III} 1210
×1.1 ×1.1

Amount & CI_I, CI_{II}, CI_{III} G.P तीन्नर राशि

Golden Ratio Method

❖ $CI_{3rd\ yr} = CI_{3\ yr} - CI_{2\ yr}$

=	3	:	3	:	1 ($CI_{3\ yr}$)
=	2	:	1		($CI_{2\ yr}$)
	1	**:**	**2**	**:**	**1**

Ex. P = 25000, R = 6% p.a CI for 3rd year = ?

1500
1500 90
1500 90 × 2 5.4

Alternatively:-

1 : 2 : 1
1500 90 5.4
⇒ 1500 + 180 + 5.4 = ₹1685.4

❖ $CI_{4th\ yr} = CI_{4\ yr} - CI_{3\ yr}$

=	4	:	6	:	4	: 1 ($CI_{3\ yr}$)
=	3	:	3	:	1	($CI_{2\ yr}$)
	1	**:**	**3**	**:**	**3**	**: 1**

Ex. P = ₹20,000
R = 5%
$CI_{4\ yr} - CI_{3\ yr} = CI_{4th\ yr}$ = ?

1 : 3 : 3 : 1
1000 : 50 : 2.5 : 0.125
⇒ 1000 + 150 + 7.5 + 0.125
₹1157.6250

❖ Rate = R% = $\frac{1}{x}$

Let P = x^3 unit
I = x^2
II = x^2 x
III = x^2 $2x$ 1

$$\frac{CI_3 - SI_3}{P} = \frac{3x+1}{x^3}$$

❖ R = 20% p.a → $\frac{1}{5}, \left(\times\frac{6}{5}\right)$

$$P \xrightarrow[1\ yr]{+20\%} P\times\frac{6}{5}\ (A_1) \xrightarrow[1\ yr]{+20\%} P\times\left(\frac{6}{5}\right)^2\ (A_2) \xrightarrow[1\ yr]{+20\%} P\times\left(\frac{6}{5}\right)^3\ (A_3) \cdots\cdots P\times\left(\frac{6}{5}\right)^n\ (A_n)$$

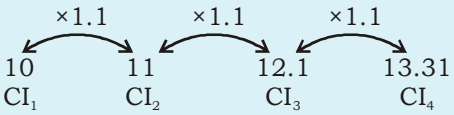
$$\frac{A_7}{A_5} = \frac{P\times\left(\frac{6}{5}\right)^7}{P\times\left(\frac{6}{5}\right)^5} = \frac{36}{25}$$

P ←5 yr→ A_5 ←2 yr (×6/5)→ A_7

CI of Particular years is also in Geometric progression (GP).

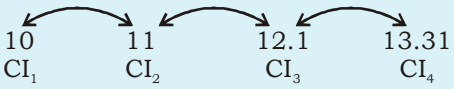
रं खहभ्रखि हCIं खमु खे ु ा. व्रख (GP)ा स्मएेखमएप्र

R = 10% p.a



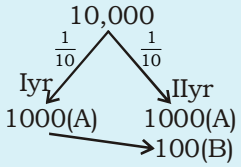
Note:- Interest is not in Geometric progration (GP).

ऊहूा मु खे ु ा. व्रख (GP)ा स्मअए ाएेखमएप्र



Tree Method

❖ P = 10000, R = 10% p.a., T = 2 years

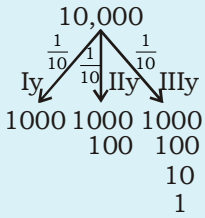


CI of 2 year = 2 × 1000 + 100 = 2100

$CI_2 - SI_2 = 100$

Note:- Golden ratio of 2 year = 2 : 1 (2A : B)

P = 10000, R = 10% p.a., T = 3 years



$SI_3 = 3000$

$CI_3 = 3\times1000 + 3\times100 + 1\times100 = 3310$

Note:- Golden Ratio of 3 years = 3 : 3 : 1

Proof:- $A_3 = P(1+R\%)^3$

$CI_3 = P(1+R^3\%+3R\%+3R^2\%)-P$

$= \underline{3}\times PR\%+\underline{3}R^2\%+\underline{1}PR^2\%$

6. Ratio of CI of

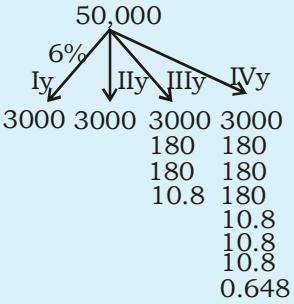
2 years → 2 : 1

3 years → 3 : 3 : 1

4 years → 4 : 6 : 4 : 1

5 years → 5 : 10 : 10 : 5 : 1

Ex. P = ₹50,000, R = 6% p.a., T = 4 years



Ratio	4 ↓×	:	6 ↓×	:	4 ↓×	:	1 ↓×
	3000	:	180	:	10.8	:	0.648
	12000	:	1080	:	43.2	:	0.648

CI_4 = ₹13123.848

Some important points (प्र ल्गि क रं ंुखि न्नैं)

1. If ै ु रे

A = Amount ै ास्र. त्अख

P = Principal ै ास्र्ण त्अ

r = Rate of Compound Interest (C.I.)

म्र भ्खवा ऊहूा ि ाीला (C.I.)

t = no. of years ै ाबसु ायभ्रत्त

then ै ो ठ,

$$A = P\left(1+\frac{r}{100}\right)^t, \text{ C.I.} = A - P$$

$$\text{C.I.} = P\left[\left(1+\frac{r}{100}\right)^t - 1\right]$$

2. Compound interest is calculated on four basis:

म्र भ्खवा ऊहूा ि ागद्रखअहमखा ायरुखखाबक्ला ि ू ेख ाएप

	Rate	Time(n)
Annually ै ाएलाबख	r%	t years
Half-yearly ै ायत्ज्ञाख्रखि	$\frac{r}{2}\%$	t × 2 years
Quarterly ै ाष्ठखख्रखि	$\frac{r}{4}\%$	t × 4 years
Monthly ै ास्एाअमि न	$\frac{r}{12}\%$	t × 12 years

3. If there are distinct 'rates of interest' for distinct time periods i.e.

यगलायण ग;यण गाबसु ायभ्रत्ताि मरण ायण ग;यण गा ऊहूा ि ाीला एखा

Rate for 1st year ै ासप्रखा भ्रखि मरण ाीला → $r_1\%$

Rate for 2nd year ै ारः ु ा भ्रखि मरण ाीला → $r_2\%$

Rate for 3nd year ै ो बलाम्रखि मरण ाीला → $r_3\%$ and so on

ै ायखाश्ब ो ल्ए

Then ै ाखख्ल

$$A = P\left(1+\frac{r_1}{100}\right)\left(1+\frac{r_2}{100}\right)\left(1+\frac{r_3}{100}\right)\ldots$$

C.I. = A – P

4. If the time is in fractional form i.e.,

ु रे ाबसु ार्ं खख्रखा ि ाट का स्मएणाय प्रखेखख

t = nF, then

$$A = P\left(1+\frac{r}{100}\right)^n\left(1+\frac{rF}{100}\right) \text{ e.g. } t = 3\frac{5}{7} \text{ yrs, then}$$

$$A = P\left(1+\frac{r}{100}\right)^3\left(1+\frac{r}{100}\times\frac{5}{7}\right)$$

5. A certain amount becomes 't' times of itself in 'm' years on compound interest then the time it will take to becomes m^n times of itself is t × n years.

ि रिअं मे ा लसं ख्र भ्रख्वा ज्रहू ाक्ला 'm' ा भ्रख्रज्ञ स्मच्चु मि रह 't' ा गथ्ाा एख्रू रे। एपे खश्बमच्चु मि रह m^n ा गथ्ाा एख्रज्ञा स्मण गअभ्रख्र ब्रबसु ा t × n ा भ्रख्रज्ञ एप्र

Ex. A certain sum becomes 3 times of itself in 5 years on compound interest. Then the time it will take to becomes 81 times of itself is →

ि रिअं मे ा लसं ख्र भ्रख्वा ज्रहू ाक्ला 5 ा भ्रख्रज्ञ स्मच्चु मि ॥ 3 ा गथ्ाा एख्रू रे। एप्रे खश्बमच्चु मि रह 81 ा गथ्ाा एख्रज्ञा स्मर्ग अहबसु ा ण गमख्र

$81 = 3^4$

$\therefore$ Time = 5 × 4 = **20 years**

6. The difference between C.I. and S.I. on a sum 'P' in 2 years at the rate of R% rate of compound interest will be

र्ि ब ा लसं ख्र 'P' ाक्ला 2 ा भ्रख्रज्ञ स्म R% ा ि ॥ म्र भ्रख्वा ज्रहू ा ि ॥ ला बमाब ख्र ख्लख्रख्र ज्रहू ा य ख्रा म्र भ्रख्वा ज्रहू ा ि ख्यें ला =

$$\text{For 2 year, C.I.} - \text{S.I.} = P\left(\frac{R}{100}\right)^2 = \frac{\text{S.I.} \times R}{200}$$

$$\text{For 3 years, C.I.} - \text{S.I.} = P\left(\frac{R}{100}\right)^2 \times \left(3 + \frac{R}{100}\right)$$

Ex. If the difference between CI and SI is ₹91 in three years and the principal is ₹27000. Find the rate of interest if compounded annually.

ज्र रो ा अ भ्रख्रज्ञ स्म CI ा य ख्रा SI ा ि मठ मा ि ख्यें ला ₹91 ा एप्र य ख्रा र्ण ख्र ₹27000 ा एप्र, रो ज्रहू ा ि ॥ ला भ्रख्र ज्रा ट क्ला बमाब ख्र सू ा एप्रे ख ख्रहू ा ि ॥ ला द्धेख ा ि ू श्र

$$\frac{3x+1}{x^3} = \frac{30\times 3+1}{(30)^3} = \frac{91}{27000}, \qquad x = 30$$

$$R\% = \frac{1}{x} = \frac{1}{30}\times 100 = \mathbf{3\frac{1}{3}\%}$$

7. If on compound interest, a sum becomes Rs. A in 'a' years and Rs. B in 'b' years then,

ज्र रो म्र भ्रख्वा ज्रहू ा क्ला ख्व ा ि लसं ख्र 'a' ा ब ख्रा स्म A च्कु म एख्रू रे ॥ एप्र य ख्रा 'b' ब ख्रा स्म B च्च एख्रू रे ॥ एप्रे ठ;

(i) If b – a = 1, then $R\% = \left(\frac{B}{A} - 1\right)\times 100\%$

(ii) If b – a = 2, then $R\% = \left(\sqrt{\frac{B}{A}} - 1\right)\times 100\%$

(iii) If b – a = n then, $R\% = \left[\left(\frac{B}{A}\right)^{\frac{1}{n}} - 1\right]\times 100\%$

where n is a whole number.

ज्र एँ n ा ि र्क्द्र ख्र ब ख्रु ए प्र

8. If a sum becomes 'n' times of itself in 't' years on compound interest, then $R\% = \left[n^{\frac{1}{t}} - 1\right] \times 100\%$

ज्र रो ि ख्रा ज्र सं ख्र भ्रख्वा ज्रहू ा क्ला 't' ा भ्रख्रज्ञ स्म च्चु मि रह 'n' ा गथ्ाा एख्रू रे। एप्रे ख $R\% = \left[n^{\frac{1}{t}} - 1\right] \times 100\%$

Ex. If a sum becomes 3 times of itself in 2 years on compound interest then R% is →

ज्र रो ि ख्रा ज्र सं ख्र भ्रख्वा ज्रहू ा क्ला 2 ा भ्रख्रज्ञ स्म च्चु मि ॥ 3 ा गथ्ाा एख्रू रे। एप्रे ख R ड ा एप

$$R\% = \left(3^{\frac{1}{2}} - 1\right)\times 100\%$$

$$\Rightarrow \left(\sqrt{3} - 1\right)\times 100\%$$

9. (i) for n = 2. Each annual installment

n ा = 2 ा ि मरण ,ा स्कु ि ा भ्रख्रज्ञ ा र्ि च े

$$= \frac{P}{\left(\frac{100}{100+r}\right) + \left(\frac{100}{100+r}\right)^2}$$

(ii) For n = 3. Each annual installment

n ा = 3 ा ि मरण ,ा स्कु ि ा भ्रख्रज्ञ ा र्ि च े

$$= \frac{P}{\left(\frac{100}{100+r}\right) + \left(\frac{100}{100+r}\right)^2 + \left(\frac{100}{100+r}\right)^3}$$

10. The simple interest for a certain sum for 2 years at an annual rate of interest R% is S.I., then

भ्रख्रज्ञ ा ज्रहू ा ि ला R% ाक्ला ब भ्रख्वृ ि मरण ा ि रिअं मे ा लसं ख्र ि मरण बख्रु ख्लख्र ख्रहू ा S.I. ा एप्रे ख

$$\text{C.I.} = \text{S.I.}\left(1 + \frac{R}{200}\right)$$

11. A certain sum at C.I., becomes x times in n_1 year and y times in n_2 years then $x^{\frac{1}{n_1}} = y^{\frac{1}{n_2}}$.

म्र भ्रख्वा ज्रहू ा क्ला ि रिअं मे ा लसं ख्र n_1 ा भ्रख्रज्ञ स्म x ा गथ्ाा य ख्रा n_2 ा भ्रख्रज्ञ स्म y ा गथ्ाा एख्रू रे ॥ एप्रे ख $x^{\frac{1}{n_1}} = y^{\frac{1}{n_2}}$

Effective/Successive rate in Compound Interest

र्क्द्र ख्र ि ला म्र भ्रख्वा ज्रहू ा स्न

For 2 years/ि ख्र ब ख्र ा ि मरण ;

(a) If rate of interest in 1st year is x% and in 2nd year is y%, then effective rate/र्क्द्र ख्र ि ला =

$$\left[x + y + \frac{xy}{100}\right]\%$$

Interest rates to remember

Rates for	CI 2 yr.	CI-SI for 2 yr.	For 3 yr.	CI-SI for 3 yr.
1%	2.01%	0.01%	3.0301%	0.0301%
2%	4.04%	0.04%	6.1208%	0.1208%
3%	6.09%	0.09%	9.2727%	0.2727%
4%	8.16%	0.16%	12.4864%	0.4864%
5%	10.25%	0.25%	15.7625%	0.7625%
10%	21%	1%	33.1%	3.1%
15%	32.25%	2.25%	52.0875%	7.0875%
20%	44%	4%	72.8%	12.8%
25%	56.25%	6.25%	95.3125%	20.3125%
30%	69%	9%	119.7%	29.7%

(b) If rate is same in both years, then/ ; दो वर्षों में दर समान है तो प्रभावी दर ज्ञात करने का सूत्र

Effective rate/प्रभावी दर $= \left[2r + \frac{r^2}{100}\right]\%$

For 3 years/ तीन वर्ष के लिए

(a) If rate is x%, y% and z% in 1st year, 2nd year and 3rd year respectively, then

; दो वर्षों में दर क्रमशः x%, y% तथा z% है तो effective rate/प्रभावी दर

$$= \left[(x+y+z) + \frac{(xy+yz+zx)}{100} + \frac{xyz}{10000}\right]\%$$

(b) If rate is r% for each 3 years

; दो वर्षों में दर r% है तो

Effective rate/प्रभावी दर $= \left[3r + \frac{3r^2}{100} + \frac{r^3}{(100)^2}\right]\%$

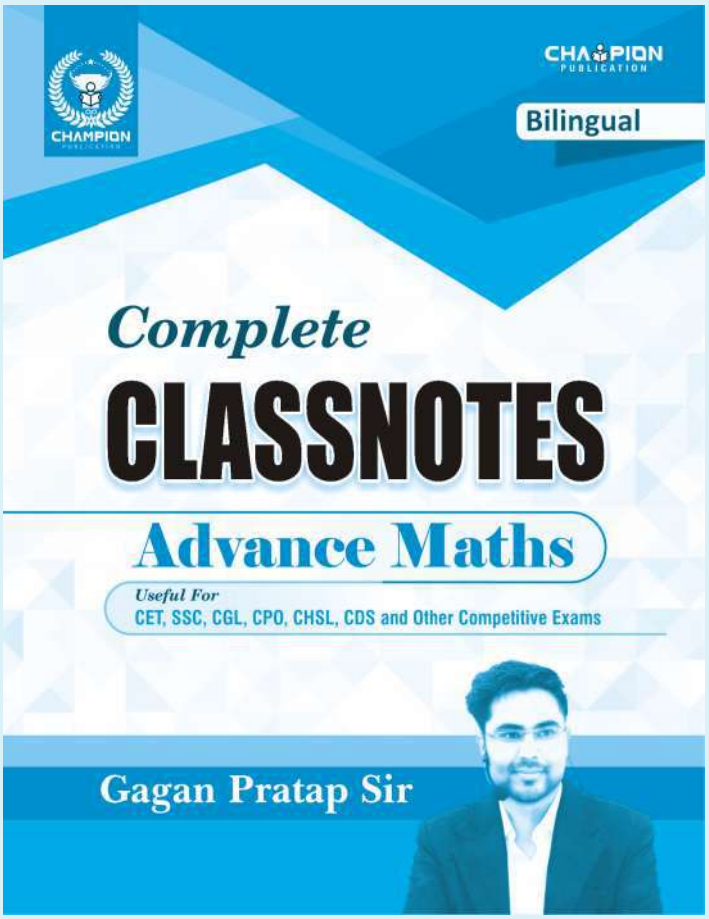
Some General Results

1.

Sum		Amount
a	t year	b
a^2	2t year	b^2
a^3	3t year	b^3

2.

Sum		Amount
a	t year	b
$\sqrt[2]{a}$	$\frac{t}{2}$ year	$\sqrt[2]{b}$
$\sqrt[3]{a}$	$\frac{t}{3}$ year	$\sqrt[3]{b}$



Ratio & Proportion (प)रआि क्ष्ममिगअ्रौ

Concept of Ratio (प)रआप्रि जपि ३ अ्ंऔम

- The comparative relation between two amounts/ quantities of same type is called ratio.
  िएाासि खाि ाी खलंं ु ौख-खख खि मठामाि मे घअखि ाबठनाि ख य अखोा िऐ मएम्
- Ratio always occurs between units, as-
  य अखोा एस्ंनखबस्खअ शि खु खि मठामा ऐखखएखू बन्

Rupees : Rupees चकु न वचकु न

kg : kg र्ि गख् वर्ि गख्

Hour : Hour .खुख व. खुख

Second : Second etc. बिमि ६ वबिमि ६ ायखे ,

- Let an amount be x and another is y, then the ratio between them is $x : y$ or $x \div y$.
  स्खअ िालंं खx य खीबला yा एखे ख-अि मठामाि खय अखोा $x : y$ ु खx ÷ y एम्
- In ratio 1st number i.e. 'x' is called "antecedent" and 2nd number i.e., 'y' is called "consequent".
  य अखोा स्मकएण ाबद्ु खु खआ 'x'ाि ख11र्क्भ्मे ाठाि एखू ेखखएमय खा ीबला बद्ु खु खआ 'y'ाि ख11य अभे ाठाि एखू ेखखएम्
- If a : b : : c : d, then a and d are called extremes and b and c are called means.
  ु रीा a : b : : c : d खे खaा य खा dा मलस्ाि एण ेखमएमय खा bा य खा c स्खु ाि एण ेखमएम्

  ∴ Product of extremes = Product of means.

  i.e., ad = bc

1. It does not change the ratio, when we multiply or divide antecedent and consequent of the ratio by a same non-zero number as-

ु एाय अखोा स्मकाले अ अएाि ले खएखू ठाएस्ार्क्भ्मखि खे प्रखकलखख। ि खि ाएाागखे, खुाबद्ु खबमगखखु खरभ् खू ेािलेमएम्

e.g. $a : b = \frac{a}{b} = \frac{a \times c}{b \times c} = ac : bc = a : b$

2. What should be added to all of a, b, c, d (numbers) so that these become proportional respectively?

बं् ख a, b, c, dा हबद्ु खखा स्म्ु खू खखू खा र्ि ाु म स्ं ख बस्खअखे िाएखू खँश्र

Let x should be added ँ ास्खअखxा् खखू खअखमखए ा:

Then ँ ास्कला $\frac{a+x}{b+x} :: \frac{c+x}{d+x}$

3. If an amount R is to be divided between A and B in the ratio m : n then

ु रीा िालंं खRाि खAाय खBाि मठामाm : nाि मय अखोा स्मरभ् खू े र्ि ु खू खअखएखे ख

(i) Part of $A = \frac{m}{m+n} \times R$

(ii) Part of B $= \frac{n}{m+n} \times R$

(iii) Difference of part of A and B $= \frac{m-n}{m+n} \times R$

where m > n

4. If the ratio of A and B is m : n and the difference in their share is 'R' units then,

ु रीाAाय खBाि खय अखोाm : nाएमय खा-अि मरएचखमि खये-ला'R' शि खखजएखे खख

(i) Part of $A = \frac{m}{m-n} \times R$

(ii) Part of $B = \frac{n}{m-n} \times R$

(iii) The sum of parts of A and B $= \frac{m+n}{m-n} \times R$

where m > n

5. If the ratio of alligation of milk and water in a glass is m : n and in other glass alligation is p : q, then the ratio of milk and water in third glass which contains alligation of both glasses is

ु रीा िारगण खास्मीत्ताय खाकखआि मरस. द्रखि खय अखोाm : nाएमय खा ीबलारगण खास्मरस. द्रखि खय अखोाp : q एखे खे ाबलारगण खास्मीत्त य खाकखआि खय अखोा एमू बस्मी खखबरगण खखि खरस. द्रखएप

$$\text{Ratio} = \left(\frac{m}{m+n} + \frac{p}{p+q}\right) : \left(\frac{n}{m+n} + \frac{q}{p+q}\right)$$

6. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots\ldots$ then each ratio $= \frac{a+c+e+\ldots}{b+d+f+\ldots}$

Ex. If the ratio of mixture of milk and water in a glass is 13 : 11 and in other glass mixture is 5 : 7, then what will be the ratio of milk and water in third glass which contains mixture of both glass is?

ु रीा िारगण खास्मीत्ताय खाकखआि खय अखोा13 : 11ाएमय खाीबला रगण खास्मीत्ताय खाकखआि खय अखोा5 : 7ाएखे खे ाबलारगण खास्मीत्त य खाकखआि खय अखोा् ु खएखखू बस्मी खखबरगण खाि खरस. द्रखएम्

Ratio of third glass =

$$\left(\frac{13}{13+11} + \frac{5}{5+7}\right) : \left(\frac{11}{13+11} + \frac{5}{7+5}\right)$$

$$\frac{23}{24} : \frac{29}{24}$$

23 : 29

Definition of different types of ratio
(ख खखम)ि बप्र अप्रि विप ्रि अप्रि जिर खखखखम

1. **Mixed ratio**→ Let $x : y$ and P : Q be two ratios, then $Px : Qy$ is called mixed ratio.
खखड ा पि ्रि अ→ि स्खअ ण ू ा $x : y$ ा य खा P : Q ी खय अ ो ा एप ो ख $Px : Qy$ ि खसर ् ा य अ ो ा ि ऐ ा एप ्

2. **Duplicate ratio**→ The mixed ratio of two equal ratios is called the duplicate ratio as
ड ट ुि ्रि अ→ ी खब स्ख ाय अ ो ख ी ा सर ् ा य अ ो ा ि ख भ ग ख ो ा ि एक ू ेख क एप
duplicate ratio of a : b is $a^2 : b^2$
a : b ि क भ ग ख ो ा $a^2 : b^2$ ा एप

3. **Subduplicate ratio**→ The square root of a certain ratio is called its subduplicate.
ड ट ुग ध अ ि अ ज→ ी ि स अं मे ा य अ ो ा ि म ग ल ण ि ख ब क भ ग ल ण ख ो ा ि एक ू ेख क एप ्
The subduplicate ratio of a : b = $\sqrt{a} : \sqrt{b}$
a : b ि क भ ग ल ण ख ो ा = $\sqrt{a} : \sqrt{b}$

4. **Triplicate ratio**→ The cube of a certain ratio is called triplicate ratio.
:ओ ग ध अ ि अ ज→ ी ि स अं मे ा य अ ो ा ि म ख ा ि ख ख ो ा ि ऐ ा एप ्
The triplicate ratio of a : b = $a^3 : b^3$
a : b ि क ख ड ो = $a^3 : b^3$

5. **Subtriplicate ratio**→ The cube root of a certain ratio is called subtriplicate ratio as -
:ओ ग ध अ ि अ ज→ ी ि स अं मे ा य अ ो ा ि म ख ्र ण ि ख ख ्र ण ख ो ा ि एक ू ेख क एप ्
The Subtriplicate ratio of a : b = $\sqrt[3]{a} : \sqrt[3]{b}$
a : b ा ि क ख ्र ण ख ो ा = $\sqrt[3]{a} : \sqrt[3]{b}$

6. **Inverse ratio**→ The reciprocal of quantities of ratio is called its inverse. Reciprocal or inverse ratio of a : b
ती य ग पि ्रि अ→ ा य अ ो ा ि ॥ स्ख ख ख ि क ह ु ख ख स ा श ब ि क ह ु ख ख स ्ि ए ेख क एप ्ा a : b ा ि क क ल ल म र ल ा ु क ह ु ख ख स ा य अ ो ख
$= \frac{1}{a} : \frac{1}{b}$ or $= \left(\frac{1}{a} : \frac{1}{b}\right) \times$ (L.C.M of a and b)

7. **Invertendo**→ The proportion in which antecedent and consequent quantities change their places, is called invertendo, as -
ख भ ध अ ि अ ज→ ा ू बा य अ ो ा स म क भ ो ज ख ा क ल द ख ॥ स ख ख ँ ा य क ख च ख ा ठ ी ए ॥ एख ब ा र भ ख ख ो ा ि एक ू ेख क एप ्ु ख ि ा ;
Invertendo of ै ा ि क र भ ख ख ो a : b = c : d is b : a = d : c
means ै ा $\frac{a}{b} = \frac{c}{d}$ then $\frac{b}{a} = \frac{d}{c}$

8. **Alternendo**→ If a : b :: c : d is a proportion then its alternendo is a : c :: b : d. i.e alternendo of $\frac{a}{b} = \frac{c}{d}$ is $\frac{a}{c} = \frac{b}{d}$
श्र अ स भ ि अ→ ु ी ा a : b :: c : d ि ा ब स ख ो ा ए ो ख श ब ि क ि ेख ल ख ो ा a : c :: b : d ा एप ु ख ा $\frac{a}{b} = \frac{c}{d}$ ा ि क ि ेख ल ख ो $\frac{a}{c} = \frac{b}{d}$ ा एप

9. **Componendo**→ If a : b :: c : d is a proportion, then componenedo is (a + b) : b :: (c + d) : d
ी ओ अ ि अ→ ु ी ा a : b :: c : d ा ि ब स ख ो ा ए ो ख ु ख ख ो ा एख ख (a + b) : b :: (c + d) : d
It means, If $\frac{a}{b} = \frac{c}{d}$ then, $\frac{a+b}{b} = \frac{c+d}{d}$
श ब ि क य प्र ा ए ु ी ा $\frac{a}{b} = \frac{c}{d}$ े ठ ख $\frac{a+b}{b} = \frac{c+d}{d}$
or, $\left[\frac{a}{b}+1 = \frac{c}{d}+1 \Rightarrow \frac{a+b}{b} = \frac{c+d}{d}\right]$

10. **Dividendo**→ If a : b :: c : d is a proportion, then its dividendo is (a – b) : b :: (c – d) : d
प ास भ ि अ→ ु ी ा a : b :: c : d ा ि ब स ख ो ा ए ो ख श ब -ब ि क ये ेन ल ख ो ा (a – b) : b :: (c – d) : d ा एख ख
It means ै ा, $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{b} - 1 = \frac{c}{d} - 1$
$\Rightarrow \frac{a-b}{b} = \frac{c-d}{d}$

11. **Componendo and dividendo**→ If there is a proportion a : b :: c : d then its componendo and dividendo is
ी ओ अ स भ ि अ→ ा य ग ल ा ि ख ख अ ो ा एप a : b :: c : d ी श ब ि क ु ख ि ेख ल ख ो ा एप
(a + b) : (a – b) :: (c + d) : (c – d) or, $\frac{a+b}{a-b} = \frac{c+d}{c-d}$
To simplify the proportion any one method of componendo, dividendo, componendo and Dividendo can direcly be used.
य अ ो ा ि श ब ल ा ठ अ ड म ि ा ए ु ख ि ेख ल ख ो ा ि ॥ र ब ॥ ि र भ स ा ि क ब ा त म -कु ख ा र ि ू क ब ै ए प ्

Ex. If the ratio of A : B : C is 7 : 9 : 11. Distribute ₹10800 in A, B and C.
ु ी ा A : B : C ि क य अ ो 7 : 9 : 11 एख ख ₹10800 ि ख A, B य ख ा C स म ठ ख ु न

$A = 10800 \times \frac{7}{27} = 2800$

$B = 10800 \times \frac{9}{27} = 3600$

$C = 10800 \times \frac{11}{27} = 4400$

Ex. If A : B = 5 : 9 and B : C is 7 : 10 then find A : B : C = ?

A	: B	: C
5	: 9	
	7	: 10
35	: 63	: 90

Ex. If A : B = 5 : 6 and B : C = 4 : 7 and C : D = 3 : 1 then find A : B : C : D = ?

A	:	B	:	C	:	D
5	:	6		6		6
4		4	:	7		7
3		3		3	:	1
60	:	72	:	126	:	42
10	:	12	:	21	:	7

Ex. If 5A = 8B = 12C then find A : B : C = ?

LCM of (5, 8, 12) = 120

$$\frac{120}{5}:\frac{120}{8}:\frac{120}{12} \Rightarrow 24:15:10$$

Alternatively:-

$8 \times 12 : 5 \times 12 : 8 \times 5 = 24 : 15 : 10$

Ex. 9A = 12B = 18C = 5 : 7 : 11 then find A : B : C = ?

LCM of (9, 12, 18) = 36

$$A:B:C = \left(\frac{5}{9}:\frac{7}{12}:\frac{11}{18}\right)\times 36 = 20:21:22$$

Ex. If $\frac{A}{8}=\frac{B}{10}=\frac{C}{13}$ then find A : B : C = ?

A : B : C = 8 : 10 : 13

Note:- If $\frac{a}{x}:\frac{b}{y}:\frac{c}{z}$ then $a : b : c = x : y : z$

Ex. If $\frac{14A}{5}:\frac{8B}{13}=\frac{12C}{11}$ then find A : B : C = ?

$$A:B:C = \left(\frac{5}{14}:\frac{13}{8}:\frac{11}{21}\right)\times 168 = \mathbf{60:273:88}$$

Ex. If a : (b + c) = 7 : 13, b : (c + a) = 8 : 17 then find c : (a + b) = ?

$a + b + c = 20, 25,\ \frac{a}{b+c}=\frac{7}{13},\ \frac{b}{c+a}=\frac{8}{17}$

LCM of (20, 25) = 100

$$\frac{a}{b+c}=\frac{7}{13}\times\frac{5}{5}=\frac{35}{65}$$

$$\frac{b}{c+a}=\frac{8}{17}\times\frac{4}{4}=\frac{32}{68}$$

then $\frac{c}{a+b}=\left(\frac{100-68}{35+32}\right)=\frac{33}{67}$

Ex. If a + b : b + c : c + a = 15 : 11 : 18 then find a : b : c = ?

2(a + b + c) = 44

a + b + c = 22

a : b : c = (22 – 11) : (22 – 18) : (22 – 15) = 11 : 4 : 7

Alternatively:-

2a = [(a + b) + (c + a)] – (b + c)

2b = [(a + b) + (b + c)] – (c + a)

3c = [(b + c) + (c + a)] – (a + b)

2a	:	2b	:	2c
(33–11)	:	(26–18)	:	(29–15)
22	:	18	:	14
11	:	4	:	7

Concept of Proportion (मगऔर अप्रि जपि ३ आं अैम

1. Directly Proportional: If $x = ky$, where k is a constant, then we say that x is direcly proportional to y. If it is written as $x \propto y$.

मज प्रि और अप्र रू री $x = ky$ ख्ू एँk ा ियमला एख्ो ख्एसा िऐ न एमर्ा x ख्y ा िमबस्खमाेे॥ एप्र, री श्बा ि ख्$x \propto y$ ा िमट का स्मरण कृख् ू ेख्एप्र

2. Inversely Proportional: If $x=\frac{k}{y}$ where k is a constant, then we say that x is inversely proportional to y. It is written as $x \propto \frac{1}{y}$.

तैय गऔर अजजू री $x=\frac{k}{y}$ ा ू एँk ा ियमला एख्ो ख्एसा िऐ मएम री x ख्y ा िमु ख्ब स्खमाेे॥ एप्र $x \propto \frac{1}{y}$ ा िमट का स्मरण कृख् ू ेख्एप्र

3. Proportion: When two ratios are equal to each other, then they are called proportional as

मगऔर अि रू ठा ि ख्य अमाेे॥ ा िि बला ि मठल ख्ला एख्ो मएम ख्ो ख्ष्ान बस्खमाेे॥ ा िऐ मएम

a : b = c : d, then, a, b, c and d are in proportion.

a : b = c : d, ो ख्a, b, c य ख्ा d ा बस्खमाेे॥ ा स्मएप्र

or,

a : b :: c : d

E.g. 2 : 5 = 6 : 15, then we write 2 : 5 :: 6 : 15

4. Mean Proportion – Let x be the mean proportion between a and b, then $a : x :: x : b$ (Real condition)

गही पि ेरअ ू ा स्खण ू x, a ाय ख्ा b ा ि मठ ामाय ख्ो ाय अमाेे॥ एख्ा ख्a : x :: x : b ा ह्य ख्ो स्भ ा रचप्रेे ज

$$\therefore \frac{a}{x}=\frac{x}{b} \Rightarrow x^2 = ab$$

$$\therefore x = \sqrt{ab}$$

So, mean proportion of a and b $= \sqrt{ab}$

ख्a ाय ख्ा b ा ि रू स्ख ु ाय अमाेे॥ $= \sqrt{ab}$

If x be the mean proportion between $(x - a)$ and $(x - b)$ then what will be the value of x ?

ो री x ख्$(x-a)$ ाय ख्ा $(x-b)$ ा ि मठ ामा स्ख ु ाय अमाेे॥ एख्ो ख्x ा ि रू स्ख ु ख्ए ख्ा ख्ा

$$x=\frac{ab}{a+b}$$

5. Third proportional – Let 'x' be the third proportional of a and b then,

ा जा स्मि मगऔर अजू ा स्ख ु ख्x' ख्a ाय ख्ा b ा ि के ाि ब ला ख्ब स्खमाेे॥ एख्ो ठख

a : b :: b : x (Real condition)

i.e. $\frac{a}{b}=\frac{b}{x}\Rightarrow ax=b^2$

$\therefore x=\frac{b^2}{a}$

$\therefore$Third proportional of a and b $=\frac{b^2}{a}$

$\therefore$ aIयखाbाि हे ाबलदयखाम्रेहे ि $=\frac{b^2}{a}$

6. Fourth Proportional – Let x be the fourth proportional of a, b and c, then a : b :: c : x (Real condition)

रु अमिगअ्रेर अद्धप्र ूि ास्खाण्रू ाxखाa, bाियखाcाि हेमखाम्रेखाबस्खाम्रेहे ाएषे खाa : b :: c : xाहा-खेख्रेि ास्चाम्रेहे ज

$\Rightarrow\frac{a}{b}=\frac{c}{x}\Rightarrow ax=bc$

$\therefore x=\frac{bc}{a}$

$\therefore$ Fourth proportional of a, b and c $=\frac{bc}{a}$

$\therefore$ a, bाियखाcाि हेमखाम्रेखादयखाम्रेहे ि $=\frac{bc}{a}$

7. First Proportional – Let x be the first proportional of a, b and c. then, x : a :: b : c (Real condition)

रु अमिगअ्रेर अद्धप्र ूि ास्खाण्रू ारि ाxखाa, bाियखाcाि हेकाएखाबस्खाम्रेहे ाएषे ठखाx : a :: b : cाहा-खेख्रेि ास्चाम्रेहे ज

$\therefore\frac{x}{a}=\frac{b}{c}\Rightarrow cx=ab$

$\therefore \boldsymbol{x=\frac{ab}{c}}$

8. Let 'x' be a number which is subtracted from a, b, c and d to make them proportional then

स्खाण्रू ा'x'ा ि ाबदु हेएाम्रू बाa, b, cाियखाdाबम़् खुहेि ला-क्ष्रन बस्खाम्रेहे ि ठअुहेू हेकहेएप

$x=\frac{ad-bc}{(a+d)-(b+c)}$

Let 'x' be a number which is added to a, b, c and d to make them proportional, then

स्खाण्रू ा'x'ा ि ाबदु हेएाम्रू बाa, b, cाियखाdाम्रू खंी ला-क्ष्रन बस्खाम्रेहे ि ठअुहेू हेकहेएखे ठ

$x=\frac{bc-ad}{(a+d)-(b+c)}$

Here a, b, c and d should always be in ascending order.

ु एँहेa, b, cाियखाdाएस्ंनखाठक्रे)ाम्र सास्माएखाममख्र ,

Law of Ratios (प्रेरअप्रि अंखी गै

❖ If $\frac{a}{b}=\frac{c}{d}$ then

(i) ad = bc

(ii) $\frac{a}{c}=\frac{b}{d}$

(iii) $\frac{a+b}{b}=\frac{c+d}{d}$

(iv) $\frac{a-b}{b}=\frac{c-d}{d}$

(v) $\frac{a+b}{a-b}=\frac{c+d}{c-d}$

How to find ratio (प्रेरअप्रि सावि ाअप्रि सैम

1. If 3A = 4B = 5C find A : B : C

step1: Take LCM of (3,4,5) = 60

$\therefore A=\frac{60}{3}=20$

$B=\frac{60}{4}=15$

$C=\frac{60}{5}=12$

$\therefore$ A : B : C = **20 : 15 : 12**

2nd Method 3A=4B=5C

A:B:C =	4×5 :	3×5 :	3×4
	↓	↓	↓
	forA	forB	forC
	multiply the	multiply the	multiply the
	coeff. of B & C	coeff. of A & C	coeff. of A & B

$\Rightarrow$ **20 : 15 : 12**

2. If A:B = 4:5, B:C = 2:5 then find A:B:C = ?

A : B : C
4 : 5
↘ ↓ ↘
2 : 5
8 : 10 : 25

Second methtod:-(B is common, so make B equal)

द्रुधस्खाी स्खाा ऊहिBाि श्रुअएखाBाि खठल्लठलाि लाण्ज

A	:	B	:	C
$4_{\times 2}$	:	$5_{\times 2}$		
		$2_{\times 5}$	:	$5_{\times 5}$
8	:	10	:	25

Concept of Degree (खट्टेजप्रि जपि ३ आं सैम

1. If $\frac{a}{b}=\frac{7}{3}$

then $\frac{5a+3b}{7a-4b}=\frac{5\times7+3\times3}{7\times7-4\times3}=\frac{44}{37}$

2. $\frac{2a^2+3b^2}{a^2-4ab}=\frac{98+27}{49-84}=\frac{-25}{7}$

3. $\frac{3a^3+4b^2}{5a^2+2b^3}$ = can not be determined because degree of each term is not same.

$\frac{3a^3+4b^2}{5a^2+2b^3}$ = रूअरखेाअएाररि ूहू रूबर्ं रू खि ासुछसिाकीरि ।

. रेखा बस्खअएाएप्र

Note:-

To solve this type of equations degree of each term should be same.

श्बा सि खा िमबस्ा िलरखखि खाएण ा िलक्मा िमरण ा सुछसिा कीरा ि ॥ रेख बस्खअ एखाम मख्र ,

Ex. A bag contains ₹1, 50 paise and 25 paise coins in the ratio 5 : 6 : 7. If the total amount is ₹390. Find the number of coins of each kind?

ि ाठपा स्मर्₹1रू50ाक्मा मय खा25ाक्मा मि मख् ि म5 : 6 : 7ाि मय अरेख स्मएप्र रीा ि घा ालरं रू₹390ाएप्र सुछसिा सि खा िमख् ि मि ॥बद् रूद्रेख ि लम्म

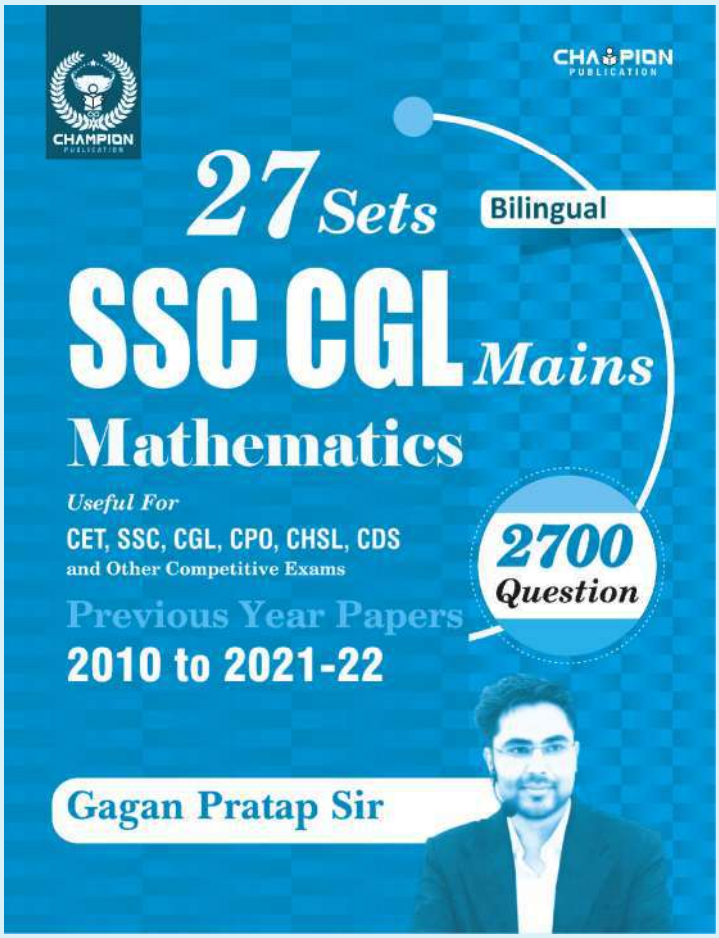
	₹1	:	50p	:	25p
Coins→	5	:	6	:	7
Amount→	5	:	3	:	1.75

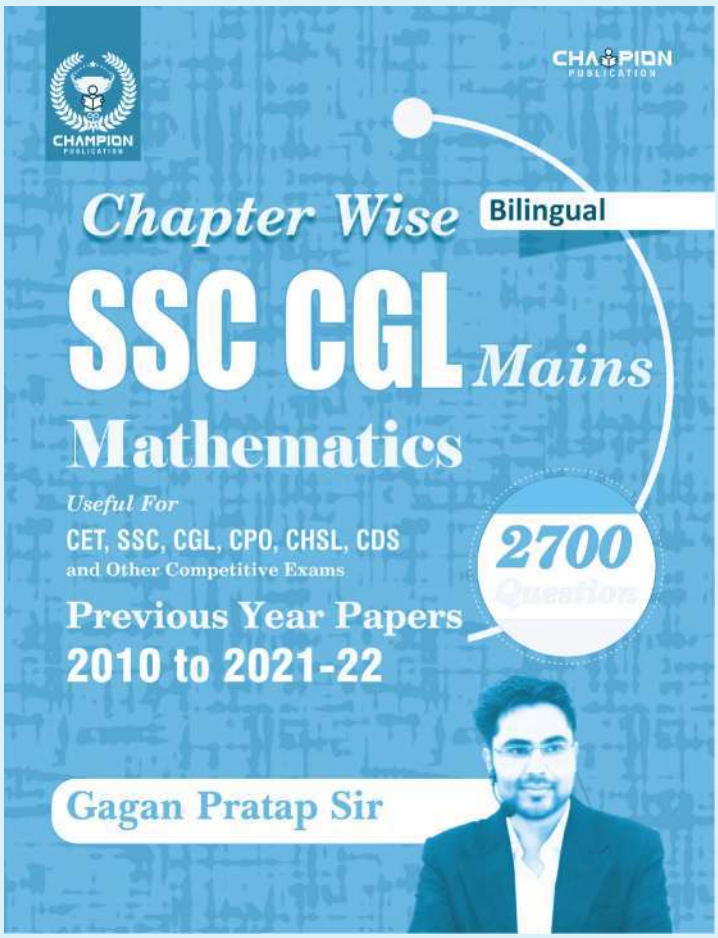
$9.75 \xrightarrow{\times 40}$ ₹390

₹1 → 5 × 40 = 200 Coins

50p → 6 × 40 = 240 Coins

25p → 7 × 40 = 280 Coins





Mixture & Alligation (खरं आणि क्षअरिक्षअ संअ

❖ **Mixture**→ Mixture is defined as a result of mixing two or more items or substances in a certain ratio or proportion.

खरं अ→ास. द्रख्ची खु ख्ची खबमायस्त्रि ाभ्चे यखु ख्की खयकृख्त खि रअं मे ाय अमोख्ा, ख्य अमोख्ा स्मास्प ख्आमि करल्द्रख्क्च-ट का करलं ख्क्रेख्ख रि, ख्कू ख्रेख्एप्र

eg.→ Mixture of milk and water, mixture of water and sugar,

ूघन→ीत्ति ाय ख्खाकख्आ ि ख्स. द्रख्खकख्आ ाय ख्खाम ाआ ि ख्स. द्रख्ख

❖ **Alligation**→ Alligation is a process or rule for the solution of problems concerning the compounding or mixing of ingredients differing in price or quality.

श्भ्जवअ)→ा ण ांनख्आ ि से ि, ख्खाश्रख्तक्ख्स्मं ख्क्ष अबख्गा ि मिख्खअ, ख्क स. द्रख्खाम ाठस्रे ाबस्चु ख्यख्खि मास्ख्क्ष ख्आ ि ाण ा ि ास्र ि, ख्कु ख्क्ष्अु सा एप्र

Concentration (मञ्जा अ

❖ Concentration is the percentage of a particular quantity in the full mixture

र्व्द्रख्स. द्रख्स्म ि ारभं ख्खाख्स्ख्क्ष ि ख्सरे ंखा बख्रेख्व्क्ष एप्र

Ex. In a mixture ratio of milk; water = 60 : 40

ि ास. द्रख्स्मीत्ति ाभ ाकख्आ ि ख्य अमोख्ा = 60 : 40

∴ Concentration of milk ौ ीत्ति ि ाब ख्रे ख्क=

$$\frac{60}{60+40} \times 100\% = 60\%$$

E. If ratio of milk : water = 11 : 5

ूरीत्ति ाक ख्आ ि ख्य अमोख्ा = 11 : 5 ाए प्र

Then concentration of milk ौ ाख्क्ष्मीत्ति ि ाब ख्रे ख्क

$$= \frac{11}{11+5} \times 100\% = 68.75\%$$

Concentration of water ौ ाकख्आ ि ाब ख्रे ख्क=

$$\frac{5}{16} \times 100\% = 31.25\%$$

or concentration of water = 100 – concentration of milk

ूकख्आ ि ाब ख्रे ख्क= 100 –ीत्ति ि ाब् ख्रे ख्क

= 100 – 68.75% = 31.25%

Replacement of mixture (खरं अप्रि अबख्क्ष कृख्त्या ौ

1. When same quantity is replaced each time from 400 L Pure Milk.

ूठा बस्ख्आ स्ख्ख्क्ष ि ख्एलाठख्खा 400 ाण द्वलां ख्खात्ति बाठी ण ारी, ख्कू ख्रेख्एप्र

$\frac{-80}{400} = \frac{-1}{5}$ = part of 20% removed each time

एलाठख्खा 20% ाि ख्एप्रकाख्एद्वख्रे ी, ख्कु ख्क

$$\left.\begin{matrix} -80\text{L pure milk} \\ +80\text{L water} \end{matrix}\right]$$

left quantity of pure milk after 3 times ौ ा3 ाठख्खा ि मठख्रे

ठमख्एप्रघ ख्कं ख्खात्ति ⇒ $400 \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5}$

$\Rightarrow 400 \times \left(\frac{4}{5}\right)^3$

$\Rightarrow 400 \times 0.512 = 204.8\text{L}$

water in final mixture ौ ाय रे सा स. द्रख्स्मकख्आ ा =
400–204.8 = 195.2L

❖ Concentration of milk ौ ीत्ति ि ाब ख्रे ख्क=

$$\frac{\text{Left pure milk}}{\text{Total mixture}} = \left(\frac{4}{5}\right)^3 = 0.512$$

= 51.2%

Concentration of water = 100% – 51.2% = 48.8%

2. When different quantity is replaced each time.

ूठा एलाठख्खा य ण ग; य ण गा स्ख्ख्क्ष ठी ण ा ूख्रेख्ा एप्र

400L pure milk ौ ां ख्खात्ति

$$-40\text{L pure milk} + 40\text{L water} = \frac{-1}{10}$$

–40 ाण द्वलां ख्खात्ति प ा40 ाण द्वला कख्आ ा = $\frac{-1}{10}$

$$-50\text{L mix} + 50\text{L water} = \frac{-1}{8}$$

–50 ाण द्वला स. द्रख्+50L ाण द्वला कख्आ ा = $\frac{-1}{8}$

$$-80\text{L mix} + 80\text{L water} = \frac{-1}{5}$$

–80 ाण द्वला स. द्रख्+80L ाण द्वला कख्आ ा = $\frac{-1}{5}$

Left Quantity of pure milk ौ ां ख्खात्ति ि ां ख्खाख्स्ख्क्ष

$$= 400 \times \frac{9}{10} \times \frac{7}{8} \times \frac{4}{5} = 252\text{L}$$

water = 400 – 252 = 148L

2nd method :-

Initial	:	Final
10	:	9
8	:	7
5	:	4
400	:	252
↓×1		↓×1
400L	**:**	**252L**

3. If from x litre of liquid A, p litre is withdrawn and same quantity of liquid B is added. Again from mixture, q litre mixture is withdrawn and same quantity of liquid B is added. Again from mixture, r litre is withdrawn and same quantity of liquid B is added, then

ु रीा xाण द्वला?भा A स्मवा p ाण द्वलारअं ख ाराण ू ख्रेकएाय खा ं आए। स्ख्कस्म?भा B ास्रण ख्री ू ख्रेकएा प्रक्खलाबा स्स. द्रख्ब qाण द्वलास्. द्रख्ब रअं ख ाराण ू ख्रेकएाय खा ब स्ख्क स्ख्कस्मे लाा B ास्रण ु ख्रू ख्रेकएा प्री ख्लाब स्स. द्रख्ब rा ाण द्वलारअं ख ाराण ू ख्रेकएाय खा क्खलाब स्ख्क स्ख्कस्मे लाा B ास्रण ु ख्रू ख्रेकएा

In final mixture, liquid A is ै ायरे सास्. द्रख्स्म?भा A ाएप

$$x\left(\frac{x-p}{x}\right)\left(\frac{x-q}{x}\right)\left(\frac{x-r}{x}\right).....$$

If only one process is repeated n times, then liquid A in final mixture is $= x\left(\frac{x-p}{x}\right)^n$ or $\left(1-\frac{p}{x}\right)^n$ and liquid B in final mixture = x – (liquid A in final mixture)

ु रीा ि ाण ा ि स्र ु ख्रि ख्रा n ठखा श्री ख्लु ख्रू ख्रेकएा ख्रे ख्रायरे सास्. द्रख्स्म?भा $A = x\left(\frac{x-p}{x}\right)^n$ ु ख्र $\left(1-\frac{p}{x}\right)^n$ एाय खा ायरे सास्. द्रख्स्म?भा B = x –ाह्यरे सास्. द्रख्स्म?भा A ज

4. If x is initial amount of liquid. p is the amount which is drawn, and this process is repeated n-times such that the resultant mixture is in the ratio a : b then, $\frac{a}{a+b}=\left(\frac{x-p}{x}\right)^n$

ु रीा x ो लाा ि ासर्ल ं ख्रि स्ख्क ख्रा p ाभ्ए स्ख्क ख्रा प्रू ख्रअं ख ाू ख्रे ा ख्रए ख्र ाय खा श्बा स्र ु ख्रि ख्र n ; ठखा श्बा ो लाी श्री ख्लु ख्रू ख्रेकएा ख्रा ि ाकल द्रख्ब।

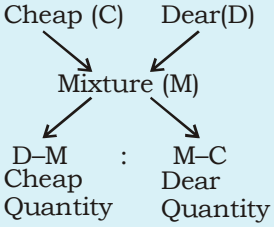
स्स. द्रख्ब a : b ि ाभ्य अ ख्रोा स्मए ख्रेख्र ख्रा ए ख्रए ख्र ख्र $\frac{a}{a+b}=\left(\frac{x-p}{x}\right)^n$

Alligation (मख्रीगस ं ैअ

1. The cost of cheap object is Rs. C/kg and the cost of dear object is Rs. D/kg. If the mixture of both object costs Rs. M/kg then

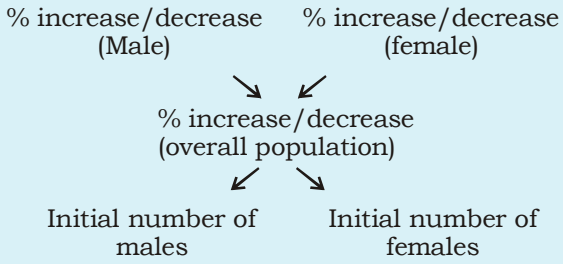
बचे ाभ्चेाभ्ि ाि स्े ाच्ब Cै ि गख्रु य खा स्एम ाभ्चेाभ्ि ाि स्े ाच्ब D ै ाि गख्रु ु रीा ख्रा ाभ्चेाभ्य खा ि ास्. द्रख्ि ख्रर्स्ु ाच्ब M ै ि गख्रे ख

$$\frac{\text{Cheap object}}{\text{Dear object}}=\frac{D-M}{M-C}$$



2. Alligation in population related questions

ठ)मँनी अमिज्रनजबि)अिगाम्रिमख्गिस ं अ



Ex. The population of a town is 6000. If males increase by 5% and females increase by 9% then population will become 6500 after 1 year. Find the initial ratio of males and females?

ि ाि च्ाम ाू अ ाद ु ख्र 6000 ाएप ु रीा क ाव ख्रा स्म 5% ाि ाभ्ब्वा एख्रे। एाय खा स्रएण ख्र ख्रा स्म 9% ाि ाभ्ब्वा एख्रे ाएप ख्र 1 ाभ्ब्व ाठख्रे ाू अ ाद ु ख्र 6500 ाए ख्रू ख्रग ा, ाक ाव ख्रा ख्रय खा स्रएण ख्र ख्रा ि ास्र्ल ं ख्रि ा य अ ख्रोा द्रख्ब ि ाू श्र

❖ Overall increase in population ै ाू अ ाद ु ख्र स्म ि ाभ्ब्व

Increase/भ्ब्व = 6500 – 6000 = 500

$$=\frac{500}{6000}\times 100=\frac{50}{6}\%$$

Male 5% Female 9%

$\frac{50}{6}$ %

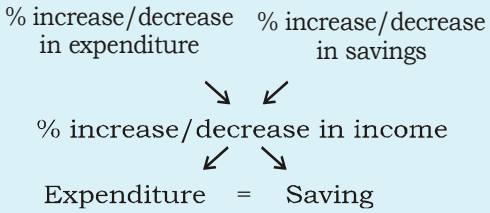
$$9-\frac{50}{6} \quad : \quad \frac{50}{6}-5$$

$$\frac{4}{6} \quad : \quad \frac{20}{6}$$

1 : 5

3. Alligation in income related questions:

प अमिज्रनजबि)अिगाम्रिमख्गिस ं अ



Ex. A man spends 75% of his income. If his income is increased by 20% and expenditure increased by 10%. Then find % change in savings.

ि ाय ख्रे स्म ाय क ाय ु ख्रा ख्र 75% ाक ृ मा ज्र ले ख्रए प्र ु रीा -बि ाय ु ख्रा स्न 20% ाि ाभ्ब्वा एख्रे ाएप ाय खा श ु ास्न 10% ाि ाभ्ब्वा एख्रे ाएप ख्र ठमे स्म % ाक ल्रे ा द्रख्बा ि ाू ,

$75\% = \frac{3 \rightarrow \text{spends}}{4 \rightarrow \text{income}}$ ∴ Exp: saving = 3 : 1

Expenditure 10% Savings x%

20%

$x - 20$: 10

3 : 1

$\Rightarrow \frac{x-20}{10} = \frac{3}{1}$

$x - 20 = 30$

$x = 50\%$

4. Alligation in profit loss थिअक्कि उ)गिखिमखिगस्ं अ

cost price A cost price B

cost price of mixture

Quantity A : Quantity B

Ex. Cost price of type A sugar is ₹36 Kg and type B sugar is 45 Rs/Kg. In what ratio these type of sugar should be mixed to get a mixture worth 39 Rs./Kg.

Aासि खा ॥ मAआ ि रूए खे ार्स्रु ा₹36ारि गकएएप्रय खासि खाBमAआ ि रू₹45ारि गकएएप्रशबा सि खा ॥ मAआ ि खरि बा य अक्रोडा स्मरस्ण्रुरुख ू खा रि ास्. द्रख ि रूर्स्रु ा₹39ारि गकएएखू रु,

Type A 36 Rs/Kg Type B 45 Rs/Kg

39 Rs/Kg

45–39 : 39–36

6 : 3

2 : 1

5. Profit/loss% (A) Profit/Loss% (B)

overall profit/loss%

cost price (A) : cost price (B)

Ex. First watch is sold at 10% profit and 2nd watch is sold at 15% profit and the overall profit on both the watches is 12% if cost price of first watch is ₹360 Find cost price of second watch ?

क्याए ॥ रू ॥ ि रू10%ाए ण्रुक्कलाठमरू ेखकएएप्रय खीबला रू ॥ ि रू15% ण्रुक्कलाठमरू ेखकएएप्रय खी खाख़ रू ख़ु) रूक्कलाि घाा ण्रुक्र12%ाएप्रु रीक्याए ॥ रू ॥ ि रूए खे ार्स्रु ा₹360ाएप्रीबला रू ॥ ि रूए खे ार्स्रु ाद्धेख ि रू, ग्रा

Watch 1 Watch 2

10% 15%

12%

3 : 2 → Ratio of cost price

×120 ×120

360 Rs. **240 Rs.**

6. Discount%(A) Discount% (B)

overall discount%

Marked price A : Marked price B

7. Alligation in simple interest थि मAअ ग्रा ग्रि त्रे ब्र ि गव मखिगस्ं अ

Rate of Interest %(A) Rate of Interest % (B)

Total rate of interest (A+B)

Principal (A) : Principal (B)

Ex. ₹10,000 is lent at 5% per annum simple interest and Rs. x is lent at 10% p.a. If overall rate of interest is 8% then find value of x.

₹10,000ाच्कु मसरे ाभ्रक्र5%ाबरुख़ख़ख़ख़ुरूाक्ला-त्खारी, रू ेखकएएप्रय खाच,ाxाि रू10%ाभ्रक्रख़ीलाक्ला-त्खारी, रू ेखकएएप्रु रीि घाा ुरूाल8%ाएखे रूxाि रूस्खाद्धेखा ि रू ,

5% 10%

8%

2 : 3 → Ratio of principal

×5000 ×5000

Rs. 10,000 **Rs. 15000**

8. Alligation in Average थिप ग्रा गिखिमखिगस्ं अ

Average 1 Average 2

Total average

1st number : 2nd Number

9. Bowling Average (till innings A) Bowling Average (till innings B)

Total average

wickets (till innings A) : wickets (till innings B)

10. Alligation in time and distance

मगी पि अद्विसमगिविमखेगस्ं अ

Average speed (for part 1) → Average speed (whole journey) ← Average speed (for part 2)

Time (for part 1) : Time (fort part 2)

11.

speed 1 → speed (Avg) ← speed 2

Time 1 : Time 2

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

Note → Mean value रू बि मrespect स्मरअं णे ॥ एष्ठratio -ब॥ि रूय केरूएप्र

❖ P% → P%/L% ← L%

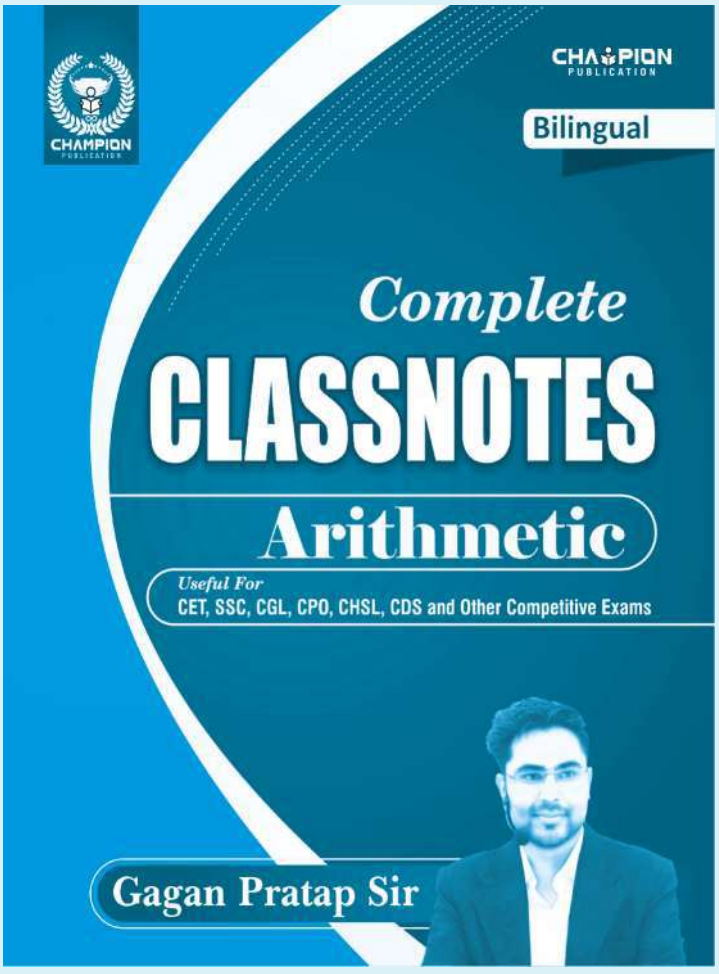
CP_1 : CP_2

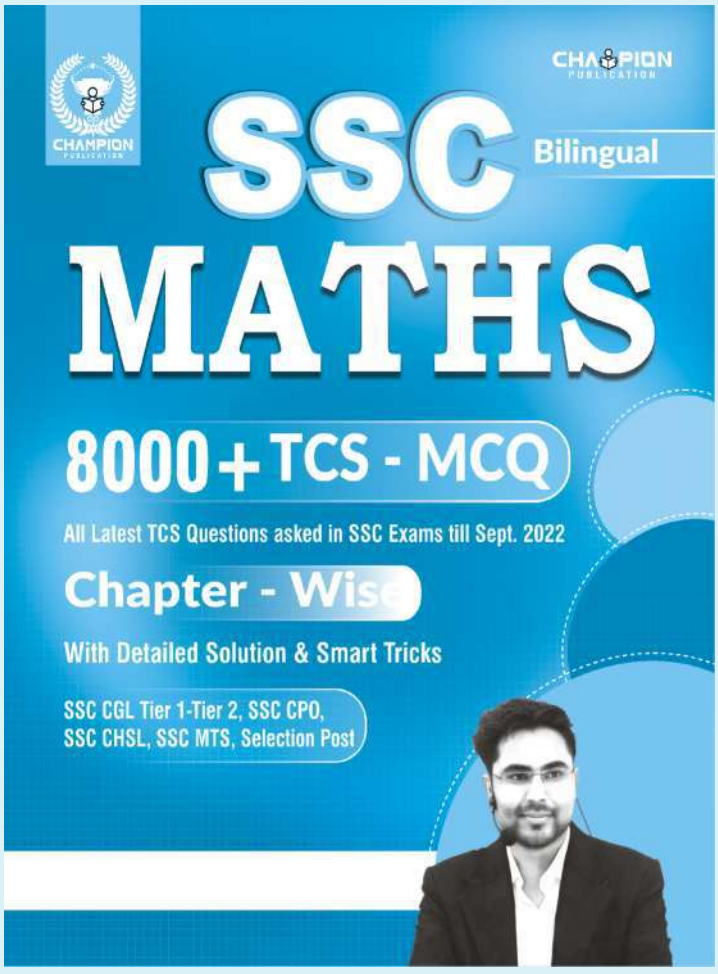
$$P\% = \frac{P}{CP} \times 100$$

❖ Alligation in discount/ 'द्वा स्मबरत्स्. द्रख

Discount $_1$ → Total Discount ← Discount $_2$

MP_1 : MP_2



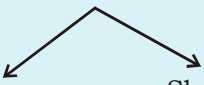


Partnership (मऊड्रुओज

❖ **Partnership थमिऊड्रुओज**

simple partnership बलाबख्लीखा	compound partnership ुखाबख्लीखा
If all partners invest different capital for same time period or same capital for different time period	If all partners invest their different capitals for different time period
ुरीार्ब खबख्लीखा ाए। बसु ायभ्स्ताास्मण ायण्ग; यण्गार्कू ॥ ुयण्ग;यण्ग बसु ायभ्स्ताास्मण ा ि ए॥ार्कू ॥ ि ुरअभ्नंखर्ि लेखाएप	ुरीार्ब खबख्लीखायण्ग;यण्ग बसु ायभ्स्ताास्मण ायकआ यण्ग;यण्गार्कू ॥ रअभ्नंखर्ि लेखाएप

❖ **Partners थमिऊड्रुओज**



Active Partner रुंख्णाबख्लीखा	Sleeping Partner रअुाबख्लीखा
invests money as well as takes part in business activity for which he is paid salary from the profit.	who only invests money and does not take part in business activities.
क्मामि ुरअभ्नंखर्ि लेखाएपमय खाबख्एप॥ बख्एपहु ख्खर्ि ा ि ुरख स्मं ख्णाेनखएपमय खाू बि न ण ा-बाण ंख्ुबाभ्नेअ ंखेख्ा रु ू ेखाएप	ि भ्ण ात्त अ ुरअभ्नंखर्ि लेखाएप ु ुरबसु ि ागेर रभ्स्तु ख्स्मं ख्णा अ॥ाेनखएप

❖ Profit = Capital × time

$P = C \times T$

$C = \frac{P}{T}$

$T = \frac{P}{C}$

❖ Profit = capital × time ैाण ंख्ुर=ार्कू ॥ ×ाबसु

	X	:	Y	:	Z
Capital	C_1	:	C_2	:	C_3
Time	T_1	:	T_2	:	T_3
Profit	$C_1 \times T_1$	:	$C_2 \times T_2$	:	$C_3 \times T_3$

❖ Profit/Loss ∝ capital

Profit/Loss ∝ time

❖ Different amounts in invested in different time periods

यण्ग;यण्गा बसु ायभ्स्ताास्मयण्ग;यण्गा ि सा रअभ्नंखर्ि ॥ू ेख॥ एप

A	:	B
(A × T)	:	$(B_1 \times t_1 + B_2 \times t_2)$

B invests twice, for different periods.

Bायण्ग;यण्गा ायभ्स्ताास्मण ा ि ख्ठखा रअभ्नंखर्ि ले ुरएप

Ex. Three partners shared the profit in a business in the ratio 2 : 5 : 7. They invested their capitals for 3 months, 4 months and 6 months respectively. What was the ratio of their capitals?

ं ाअबख्लीखाख्अ ि ाु भ्बुख्ा स्मण ंख्र्ि ख्2 : 5 : 7ाि मय अ॥ाेख्ा स्मा बख्ख्र ु ख्-क्ष्ख्ामय कआ ार्कू ॥ ं ख्3ास्ए।अख्4ास्ए।अमय ख्ा6ास्ए।अमि न ण ा रअभ्नंखर्ि ।,ा-अि ॥ार्कू ॥ ि ुरय अ॥ाेख्ा ु ुरप्रख्ा

	I	:	II	:	III
Profit→	2	:	5	:	7
Time→	3	:	4	:	6

Capital→ $\left(\frac{2}{3} : \frac{5}{4} : \frac{7}{6}\right) \times 12$

8 : 15 : 14

Ex. A, B and C invested their capital in the ratio of 2 : 3 : 5. The ratio of months for which A, B and C invested is 4 : 2 : 3 then. Find the ratio of their profit?

A, B ायख्ा C आमय कआ ार्कू ॥ 2 : 3 : 5ाि मय अ॥ाेख्ा स्मा रअभ्नंखर्ि ।,ा ू अ स्ए।अख्ामि ण ा A, B ायख्ा C ा आरअभ्नंखर्ि ु ुरअि मय अ॥ाेख्ा 4 : 2 : 3ाएप -अि ण ंख्र्ि ख्मय अ॥ाेख्ा द्धेख्ा ि ा ू ण्ा

	A	:	B	:	C
Capital→	2	:	3	:	5
Time→	4	:	2	:	3
Profit→	2×4	:	3×2	:	5×3
	8	**:**	**6**	**:**	**15**

Average (रे ख अ

Average is defined as the mean value which is equal to the sum of the number of a given set of values divided by the total number of values present in the set.

सरें ख रबिस्दा बुरा यिअ बधिअस्पस्कूखक दरा नख्या टो नरकिथ तथ बुरा खी ि हा ,ा ंघदरा दिसी रि हा बधिबेसस बुरा खी ,ा ै।ा ंघदरं ि केपस्कूखक ुि ि भ्एिएभ्ए टरख्रि टेज

1. Sum of the observation = Average × total number of the observation

गमिरं खी रा दसी =ा सरें ख ×ा गमिरं खी ,ा ै।ा ंघदर

2. Average of two or more numbers/ quantities is called the mean of these numbers, which is given by

स्रदिरा स्रि सिक ा ंघदस्सख्रिरा ख्सस्सरी रा सरें ख ष्ठ ा ंघदस्सरी रा बस्दा ट। ख्या टेठ नरकुशु ा (रण कदरा नख्या टे

$$\textbf{Average (A)} = \frac{\textbf{Sum of all observation}\ (\text{ंघदस्सरी रा द ति च})}{\textbf{Total no. of all observation}\ (\text{अमिरं खी ,ा ै।ा ंघदख})}$$

Example:- Weight of 60 students = 40, 42, 35, 50....

Total weight = 2400kg

$$\text{Average} = \frac{2400}{60} = 40 \text{ kg/student}$$

3. If the given observation (x) are occuring with certain frequency (A) then,

दका कथ तथ सी । रि ु ा (x)ा कुकइ ख सरी क्ष्रा (A)ा ि र ा टरएि टिेख्या खि

$$\textbf{Average} = \frac{A_1x_1 + A_2x_2 + \ldots + A_nx_n}{x_1 + x_2 + \ldots + x_n}$$

where A_1, A_2, A_3, ... A_n are frequencies./नटरख्या A_1, A_2, A_3, ... A_n सरी क्ष्रा दस्सटेख

4. If the average of 'n_1' numbers is a_1 and the average of 'n_2' numbers is a_2, then average of total numbers n_1 and n_2 is, Average = $\frac{n_1a_1 + n_2a_2}{n_1 + n_2}$

दका 'n_1'ा ंघदस्सरी रा सरें ख a_1 टो सरेा 'n_2'ा ंघदस्सरी रा सरें ख a_2ा टेठ ख्ि ै।ा ंघदस्सरी n_1ा सरेा n_2ा रा सरें ख टो :ा $\frac{n_1a_1 + n_2a_2}{n_1 + n_2}$

Ex. If average of 10 numbers is 24 and average of 5 numbers is 15. Find the combined average?

दका 10ा ंघदस्सरी रा सरें ख 24ा टो सरेा 5ा ंघदस्सरी रा सरें ख 15ा टेठ ख्ि ै डखा सरें ख. ख ,क्थ्द्व

$$\text{Combined average} = \frac{10\times24+5\times15}{10+5} = \frac{315}{15} = \mathbf{21}$$

5. No. of data → n_1 n_2 n_3 n_4

Average → a_1 a_2 a_3 a_4

$$\text{Net avg/weighted avg} \rightarrow \frac{n_1a_1 + n_2a_2 + n_3n_3 + n_4a_4}{n_1 + n_2 + n_3 + n_4}$$

Ex.

❖

Class		A		B		C
No. of Students	→	9	:	17	:	14
Average weight	→	53 kg		59 kg		64 kg

$$\text{Avg.wt. of all class} \rightarrow \frac{9\times53+17\times59+14\times64}{9+17+14} = \frac{2376}{40} = 59.4$$

OR, Deviation Method :

Student	→	9	:	17	:	14
Avg wt.	→	53		59		64kg
		(–6kg×9)		0		(+5kg ×14)

Let Average weight of all classes पबुर रं पर्‍ा ितर्ई रसरें ख पस्ए टो = 59 kg

$= 59 \text{ kg} + \frac{(-54+70)}{40}$

$= 59 + 0.4 = 59.4$ kg

Concept of Deviation (ै वक्ग)ि क्व्र कु प्रै अ

- Find Average of) रसरें ख. स्ख ,क्थच 40, 42, 35, 50, 85
- Traditional method is to add all the numbers and divide by the number of observations but this method is lengthy and calculative. To avoid calculation and save time we solve it by concept of deviation.

अएख्स्र ाकेकां पर्‍ा घदसरघि रनिरुक्तिसिरेए गर्मिरसघि ,ां घदरं पिस्तासुर्ि ,टो। क्ि ुादटाकेका। ध्,ासरेए र्त रु ख्ब ाटेजर्त रु रं भ्इु सिरेए बद भ्इु र्ि कि थटबाष्ं किइ।ु ा ,सी ग्र्ए स्रं टि।। एखंटेच

Step I : Consider any number in the range of these numbers as average.

ण्प्रे बाIबस्रु ां घदसरघि ,ा र्िा बक्िं ,पर्‍ां घदर रसरें ख बुर्ि

Step II : Find the difference of average from each number (deviation)

ण्प्रे बाIIबस्रगादर्ि ां घदरं सिरें ख रसस्ख्ए् स्ख ,क्थज)केइ।ु च

Step III: Add the deviation and divide it by total number of observations.

ण्प्रे बाIIIबस्रकेइ।ु ानरिक्तिसिरेए ष्ं गिर्मिरसघि ,ा ै।। ां घदरं किपस्क्ख ए्ि

Step IV: Add or subtract (according to sign of deviation) the deviation from the average that we considered to get accurate average.

ण्प्रे बाIVबस्रसरें ख ं किइ।ु ा रिनरिक्तिरिश्रहस्थ)केइ।ु ा ि धवि सिुै एच्वा कां टिबु र्ि ह, ासरें ख गस्ख एु र्ि कि थबुर्ि रज

Example: 40, 42, 35, 50, 85

Deviation: 0 +2 −5 +10 +45

Let Average = 40

Net deviation)ैख्या केइ।ु च = $\frac{0+2-5+10+45}{5} = \frac{52}{5} = +10.4$

∴ Actual Average)ी स्ख्के ासरें ख्ब = 40 + 10.4 = **50.4**

Average of consecutive numbers (मै वन बर्ीं रे क्र बरे ख्ए

1. The average of 'n' consecutive natural numbers starting from 1 i.e. Average of 1,2,3,....n

1ां ि्रैया टुर्ि रा ,ा 'n'ा। तख्ए गख्छक्ह ां घदसरघि रसरें ख दरु ,ा 1,2,3,ा रसरें ख्ब्ज्न

$$\frac{\text{sum of first n natural no.}}{n} = \frac{\frac{n(n+1)}{2}}{n} = \mathbf{\frac{n+1}{2}}$$

2. The average of squares of 'n' consecutive natural numbers starting from 1 i.e.

1ां ि्रैया टुर्ि रा ,ा 'n'ा कृबस्तख गख्छक्ह ां घदसरघि ि्तर्ई रसरें ख स्र स्रुं

Average of $1^2, 2^2, 3^2, 4^2, ... n^2 = \frac{\frac{n(n+1)(2n+1)}{6}}{n} = \mathbf{\frac{(n+1)(2n+1)}{6}}$

3. The average of cubes of first 'n' consecutive natural numbers i.e. Average of $1^3, 2^3, 3^3, ... n^3$

ग्र ख्ा 'n'ा कृबस्तख गख्छख ं घदसरघि श्रि ुरघि रसरें ख स्र स्रख $1^3, 2^3, 3^3, ... n^3$ ा रसरें ख

$$= \frac{\left(\frac{n(n+1)}{2}\right)^2}{n} = \frac{n(n+1)^2}{4}$$

4. The average of first 'n' consecutive even natural numbers i.e. Average of 2, 4, 6,.... 2n = $\frac{n(n+1)}{n}$ = (n + 1)

ग्र ख्ा 'n'ा कृबस्तख ं बा गख्छख ं घदसरघि रसरें ख स्र स्रुं 2, 4, 6, ... 2n रसरें ख = $\frac{n(n+1)}{n}$ = **(n + 1)**

5. The average of first 'n' consecutive odd natural numbers i.e. 1, 3, 5, ... (2n – 1) = $\frac{n^2}{n}$ = **n**

ग्र खा 'n'ा कृबस्तख कें ू खा गख्छख ं घदस्सख्रि रसरें खस्र स्रखं 1, 3, 5, ... (2n – 1) = $\frac{n^2}{n}$ = n

6. The average of certain consecutive numbers a, b, c, ... n is $\frac{\text{First no.+Last no.}}{2}$ = $\frac{\mathbf{a+n}}{\mathbf{2}}$

ैम्ा कृबस्तख ं घदस्सख्रि a, b, c, ... nा रसरें ख $\frac{\text{First no.+Last no.}}{2}$ = $\frac{a+n}{2}$ टे

Ex. Find average of 4, 5, 6 ..... 20.
4, 5, 6 ..... 20ा रसरें ख. ख ,ाश्थज

Average = $\frac{4+20}{2}$ = $\frac{24}{2}$ = **12**

7. The average of 1st 'n' multiples of certain numbers x) ैम्ा ं घदस्सख्रि िअट। ि'n'ातैं र ख्रि रसरें ख्व = $\frac{x(n+1)}{2}$

Ex. Find average of first 10 multiples of 3 ण3ा िअट। ि10 तैं र ख्रि रसरें ख. ख ,ाश्थ
First 10 multiples of 3 = 3, 6, 9 ..... 30 ण3ा िअट। ि10ातैं र ा= 3, 6, 9 30

Average = $\frac{3(1+10)}{2}$ = $\frac{33}{2}$ = **16.5**

8. Average of square of 1st n even number ण)अट। ,ा nा ं बां घदस्र ि ित रसरें ख्व = $\frac{\mathbf{2(n+1)(2n+1)}}{\mathbf{3}}$

9. Average of cube of 1st n even number ण)अट। ,ा nा ं बां घदस्र श्रि ु। रसरें ख्व = **2n(n+1)²**

10. Average of square of 1st n odd number ण)अट। ,ा nा कें ू खा ं घदस्र ि ित रसरें ख्व = $\frac{\mathbf{(2n+1)(2n-1)}}{\mathbf{3}}$

11. Average of cube of 1st n odd number ण)अट। ,ा nा कें ू खा ं घदस्र श्रि ु। रसरें ख्वा = $n(2n^2-1)$

12. Average of 1 to n odd number ण)1ा ं िnा कें ू खा ं घदस्र रसरें ख्वा = $\frac{\text{Last odd no.}+1}{2}$

13. Average of 1 to n even number ण)1ा ं िnा ं बां घदस्र रसरें ख्वा = $\frac{\text{Last even no.}+2}{2}$

Average speed (रे ख कैं अ

1. If A goes from P to Q with speed of x km/h and returns from Q to P with speed of y km/h, then the average speed of total journey is
दका A उP ां िQा ख ा xा क ब,ाख्रा ,ातक्व ं निख्रा टो सरेख् Q ां िP ा ख ा yा क ब,ाख्रा ,ातक्व ं ि ख्रं ा सख्रा टेठ ख्रि ैा। दस्रख्र ,ासरें ख तक्व टे

Average speed = $\frac{\mathbf{2xy}}{\mathbf{x+y}}$ Or Average speed = $\frac{\text{Total distance}}{\text{Total time taken}}$

2. If a distance is travelled with three different speeds a km/h, b km/h and c km/h, then Average speed of total journey = $\frac{\mathbf{3abc}}{\mathbf{ab+bc+ca}}$ **km / h**

दका थ ा स्ख्रा ख्रु ा स। तध्स। ता तक्व a ा क ब,ाख्रा तथ b ा क ब,ाख्रा सरेख् c ा क ब,ाख्रा ं ि ख्रा ,ा नख्रा टेठ ख्रि ैा। दस्रख्र ,ासरें ख तक्व = $\frac{3abc}{ab+bc+ca}$ km/h

Ex. A particular distance is travelled with 2 km/hr, 3 km/hr and 4 km/hr. Find average speed of the whole journey.
थ ा कें ृ ूरि स्ख्रा 2ा क ब,ाख्रा तथ 3ा क ब,ाख्रा सरेख् 4ा क ब,ाख्रा ि ्र रा ख्रा ,ा नख्रा टेज अस्रा दस्रख्र ,ासरें ख तक्व. ख ,ाश्थज

Average speed = $\frac{3\times2\times3\times4}{2\times3+3\times4+4\times2} = \frac{72}{26}$

$= \mathbf{\frac{36}{13}\ km/hr}$

Average age (रे ह बं वय्र)

1. 't' years before, the average age of N members of a family was 'T' years. If during this period 'n' children increased in the family but average age (present) remains same, then.

Present age of n children = n.T – N.t

't'ी ूर अट। ि थ ।अ्रीए ए ि Nां स्दरयि ,ासरें खसरदौ 'T'ी ूर र,ज दका ं ासी का बधिअ्रीए बधि n'ा भ्च्ह भ्ज्ञाअिए्रखैसरें खसरदौ)ी ख्बरु च ी ट,ा एट्खिज

nा भ्च्ह यि ,ी ख्बरु ा सरदौ = n.T – N.t

2. If in the group of N persons, a new person comes at the place of a person of 'T' years, so that average age, increase by 't' years

दका Nा ऊकृपरयि ि बला बधि 'T'ी ूर ऊकृप क्रिसु ा अ्थ ु दर ऊकृप स खा टेठ ख सरें खसरदैठ 't'ी ूर ं भ्ज्ञान खा टे

Then, the age of the new person)क्मप्रु थ ऊकृप , सरदैचा = T + N.t

If the average age decrease by 't' years after entry of new person, then the age of the new person = T – N.t

दक्ु थ ऊकृप गिी रि भ्स्सरें खसरदौ 't'ी ूर बाटर्नि खा टेठ ख्ु थ ऊकृप ,ासरदौ = T – N.t

Ex. The average age of a group of 8 people increase by 2.5 years when a new person replaces one of them whose age is 65 years. Find the age of new person.

8ा। रीयि थि ां बला ,ासरें खसरदौ 2.5ी ूर भ्ज्ञान खा टोन भ्तु बधि ं थि ा ऊकृपक्रकं ,ासरदौ 65ी ूर टेठ क्रिसु ा अ्थ ु दर ऊकृप स खा टेजु द ऊकृप ,ासरदौ. ख ,क्दज

Sol. Here, N = 8

T = 65

t = 2.5

∴ Age of new person = T + N.t

= 65 + (8 × 2.5) = **85 years**

3. The average age of a group of N students is 'T' years. If 'n' students join, the average age of the group increases by 't' years, then Average age of

new students $= T + \left(\frac{N}{n}+1\right)t$

N म्खयि थि ां बला ,ासरें खसरदौ 'T'ी ूर टेजदका 'n'म्खा ख्सरक्ा। टर्खिटेढा ख्ं बला ,ासरें खसरदौ 't'ी ूर भ्ज्ञान खा टेठ ख्ु थ म्खयि

,ासरें खसरदौ $= T + \left(\frac{N}{n}+1\right)t$

If the average age of the group decreases by 't' years, then Average age of new students

दका ं बला ,ासरें खसरदौ 't'ी ूर बाटर्नि खा टेठ ख्ु थ म्खयि ,सरें खसरदैच

$= T + \left(\frac{N}{n}+1\right)t$

4. If the average age (height) of 'n' persons is x year (cms) and from them 'm' persons went out whose average age (height) is 'y' years (cms) and same number of persons joined whose average age (height) is 'z' years (cms) then what is the average age (height) of n persons ?

दका 'n' ऊकृपरयि ,ासरें खसरदौ)/ भ्रख्वा x ी ूर (cms) ा टो सरेख्तु बधि ं 'm' ऊकृप्रुक ा क्रिु ,ासरें खसरदौ)/ भ्रख्वा 'y' ी ूर (cms) टो सरेख्तु क्ु टि ,ऊकृप्र ख्का। टैथ क्ु ,ासरें खसरदौ)/ भ्रख्वा 'z' ी ूर (cms) ा टो ख्ं nा ऊकृपरयि ,ासरें खसरदौ)/ भ्रख्वा ड्दर टेट्ठ

∴ Average age = $\left\{x - \frac{m(y-z)}{n}\right\}$ years (cms)

5. If in a group, one member is replaced by a new members, then.

दका क ं ,ं बला बधि ं स्दा रु थं स्दा भ्स्। ा क्दर न खा टेठ ख्ं

Age of new member = (age of replaced member) ± xn

ु थं स्दा ,ासरदौ = ा)गक्क्रु स्क्ख्ं स्दा ,ासरदैच ± xn

where, x = increase (+) or decrease (–) in average

n = Number of members.

न टख्क x ा = ासरें ख बधि स्ख्या (+) ा दर ब,ा (–)

nा = ां स्दरयि ,ं ध्दरज

6. If a new member is added in a group then.

age (or income) of added member = Average age (or income) ± x (n + 1).

सत एक ,ं तैअ बधि रिु दर स्दा न क्रिन न खा टो ख्जि

न क्रिन्तिथं स्दा ,ासरदौ)दर सरदचा = ासरें खसरदौ)दर सरदचा ± x (n + 1).

where x = increase (+) or decrease (–) in average age (or income)

n = Number of members.

न टख्क x ा = ासरें खसरदौ)दर सरदचा बधि स्ख्या (+) ा दर ब,ा (–)

nा = ां स्दरयि ,ं ध्दरज

7. If a member leaves the group, then income (or age) of left member = Average income (or age) ± x(n – 1)

दका क्रिं स्दा बला म्क्रिन्त्स्खि टेठ ख्म्क्रिन्त एतथं स्दा ,ासरदा)दर सरदैचा = ासरें खसरदा)दर सरदैच ± x (n – 1)

where, x = increase (+) or decrease (–) in average income (or age)

n = Number of members.

न टख्क x ा = ासरें खसरदा)दर सरदैचा बधि स्ख्या (+) ा दर ब,ा (–)

n ा = ां स्दरयि ,ं ध्दरज

When change in data happens (३ भ्बजे धे बक्कद्यफक ईब्बे येब्रख्ख

1. If in any series having common difference 'd' and Average 'k', 'x' numbers are added in forward or backward, then

दका क ं ,ा ंरिाक ं र ं रीसख्य d ग्रुस सरें ख k टरसिेख्ए ं बधिसस्ती ं सि रीराअम्र्ी f x ं घदस्थान क्रीखि

New Average पु दरा सरें ख = $k \pm \frac{xd}{2}$

2. In series of even or odd having Average "k", when we add "x" number in forward or backward, Then

ं बास रीरा कूखा ं घदस्सख्ी ,ा ंरिाक ं रा सरें ख k टेठत ं बधिक टबा सस्ती सि रीरा अम्र्ी f x ं घदस्थान क्रीखि

New Average पु दरा सरें ख = $k \pm x$

3. In series of natural number having Average "k", when we add "x" number in forward or backward, Then

k सरें ख ीरा ,ा गख्छक्ह ा ं घदस्सख्ी ,ा क ं ,ा ंरिा बधिदका सस्ती सि रीरा द्रुम्र्ी f"x" ं घदस्थान क्रीखि नस्थाख्ी

New Average पु दरा सरें ख = $k \pm x/2$

4. If average of n observations is a but the average becomes b when one observation is eliminated, then value of eliminated observation = n(a–b)+b

दका n गमिरि स्ी रा सरें ख a टरअिख्ेख दका थ ा गमिरि र क्रीकेरू स्कमख ए क्कदरा नस्थाख्ी सिरें ख b टरनिस्थाटेजा के रू स्कमख गमिरि र ब्रु ा=n(a–b) + b

5. If average of n observations is a but the average becomes b when a new observation is added, then value of added observation = n(b–a) + b

दका n गमिरि स्ी रा सरें ख a टरअिख्ेख दका थ ा गमिरि र सिेख्ए त्रस्का ा ए क्कदरा नस्थाख्ी सिरें ख b टरनिस्थाटेजा ख त्रस्का ा क थ तथ गमिरि र ब्रु = n(b–a) + b

6. We have n observations out of which some observations (a_1, a_2, a_3....) are replaced by some other new observations and in this way, if the average increase or decreases by b, then value of new observations

दका टबरख्अिं ा n गमिरि र टरक्रिम्ं बधि f ैम्ा गमिरि स्ी (a_1, a_2, a_3....) र ैम्ु थ गमिरि स्ी (रख्ए गक्कू स्कमख ए क्कदरा नस्थ ग्रुस ं ा गक्कृदरा बधिक सरें ख बधि b ,ा ी स्ख्ाा स रीरा ब,ा टरनिस्थाख्ु थ गमिरि स्ी रा ब्रु

= a ± nb

Where/नटरुa = $a_1 + a_2 + a_3 + ...$

Note: In this formula, the signs of '+' and '–' depend upon the increment or decrement in the average ं ं ल्ब धि+' स रीरा'–' रा क्रिस्टा सरें ख बधि स्ख्ाा स रीरा ब,ा अग्रुकुपख्ए ख्ब टेज

7. Mathematical operation performed on each observation results in same effect on the average.

गव्दा गमिरि र अग्र ,ा तपर्तकस्ुदा ं क्कृदरा सरें ख अग्र ं ब्रु ा गपसी ा अस्मख्ब टेज

Related to numbers (र ने ब्ब ब स्तु अ

1. If there are 3 natural numbers and average of any two number when added with third number gives a, b, c. Then natural numbers.

दका 3 गख्छक्ह ं घदस्थ्टरसिेख्क ं ,ा स्ी घदस्सख्ी सिरें ख र दिका ख्ं ए ं घदरा बधि क्रीनस्थाख्ी ग्रख्ं घदस्थ क्रृबत्र्ह्ं a, b ग्रुस c टरख्ि

Sum of number) ं घदरा र दरसीचा = $\left(\frac{a+b+c}{2}\right) = k$

First number)अट्ा रु ध्पव्ा = 2a–k

Second number)स्ंलग्रु ध्पव्ा = 2b – k

Third number)ख्ं ए्रु ध्पव्ा = 2c – k

2. If the average of m numbers is x and out of these 'm' numbers the average of n numbers is y. (or vice versa) then the average of remaining numbers will be

दका m ा ं घदस्सख्ी सरें ख x ा टो सख्ेए्रु ा 'm' ा ं घदस्सख्ी बधिं f n ं घदस्सख्ी सरें ख y टेजा)दरा ं कि अग्र ख्व ा ख्ि त्रूरि ं घदस्सख्ी र सरें ख टरसिर

(i) Average of remaining numbers)त्रूरि ं घदस्सख्ी र सरें ख्व = $\frac{mx - ny}{m - n}$ (if m > n)

(ii) Average of remaining numbers)त्रूरि ं घदस्सख्ी र सरें ख्व

= $\frac{ny - mx}{n - m}$ (if n > m)

3. If from (n + 1) numbers, the average of first n numbers is 'F' and the average of last n numbers is 'L', and the first number is 'f' and the last number 'l' then

दका (n + 1) ा ं घदस्सख्ी ग्रि ख्ा n ा ं घदस्सख्ी सरें ख 'F' ा सख्ेए्रु स घख्ब्र्ा n ा ं घदस्सख्ी सरें ख 'L' ा ग्रुस अट्ा ,ा ं घदरा 'f' ा सख्ेए्रु स घख्ब्र्ा ं घदरा 'l' टरख्ि

$f - l = n(F - L)$

4. If the average of 'n' observations is 'x' and from these the average of 1st 'm' observation is 'y' and the average of last 'm' observations is 'z' then

दका 'n' ा सी ा रु ख्ी सरें ख 'x' ा टो सख्ेए्रु ब्धिं f अट्ा f 'm' सी ा रु ा सरें ख 'y' ा टो सख्ेए्रु स घख्ब्र्ा 'm' ा सी ा रु ख्ी सरें ख 'z' ा टो ख्ि

m^{th} observation = m(y + z) – nx

$(m + 1)^{th}$ observation = nx – m(y + z)

When data is misread (् भ्जबजये बन) बघो क्क े ब्श्च

1. If average of n numbers is m but later on it was found that a number 'a' was misread as 'b'. The correct average will be

दका n ं घदस्सख्ी सरें ख m टो ा क्रि ा भ्स्स बधिटा अट्रा तदरा क ा थ ं घदरा 'a' ा रीत ा ख्ा ं f 'b' ा अग्रात्क दरा तदरा स्ख्ं ट,ा सरें ख टरसिर

= $m + \frac{(a - b)}{n}$

2. If the average of n numbers is m but later on it was found that two numbers a and b misread as p and q.

दका n ं घदस्सख्ी सरें ख m टो ा क्रि ा भ्स्स बधिटा अट्रा तदरा क ा स्ी ं घदस्थ a ा सख्ेए्रु b ा रीत ा ख्ा ं f p ा सख्ेए्रु q ा अग्रात्क दरा तदरा ख्ज

The correct average) ं ट,ा सरें ख्व = $m + \frac{(a + b - p - q)}{n}$

Miscellaneous (कैकु अ

1. If the average of n students in a class is a, where average of passed students is x and average of failed students is y, then

दक्षाकं ,ा मसबंधिn कैं क्रमदर्शि रसरें खa टसिनटसृ्र, रकैं क्रमदर्शि रसरें ख x सरेख सु ैर्, रकैं क्रमदर्शि रसरें ख y टरिखि

Number of students passed/ृर्, रकैं क्रमदर्शि ,ं यदर

$$= \frac{n(a-y)}{(x-y)}$$

2. Bowling Average)तश्मिस ,ा सरें ख्या =

$$\frac{\text{Total runs given}}{\text{Total wickets taken}}$$

3. Batting Average)सरें ख र्ं । श्मिस ,चा =

$$\frac{\text{Total runs scored}}{\text{Total number of innings played}}$$

❖ Total runs = Bowling average × Wickets

ै। ।ए ा =ा तश्मिस ,ा सरें ख × के हि

Or

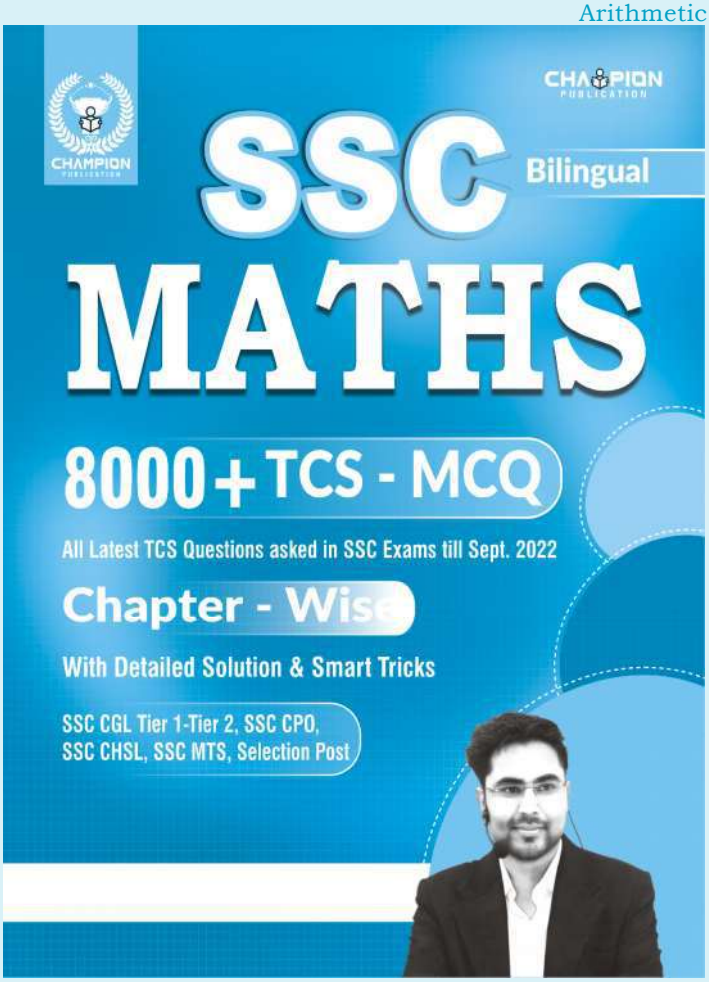
Bowling Average (1) Bowling Average (2)

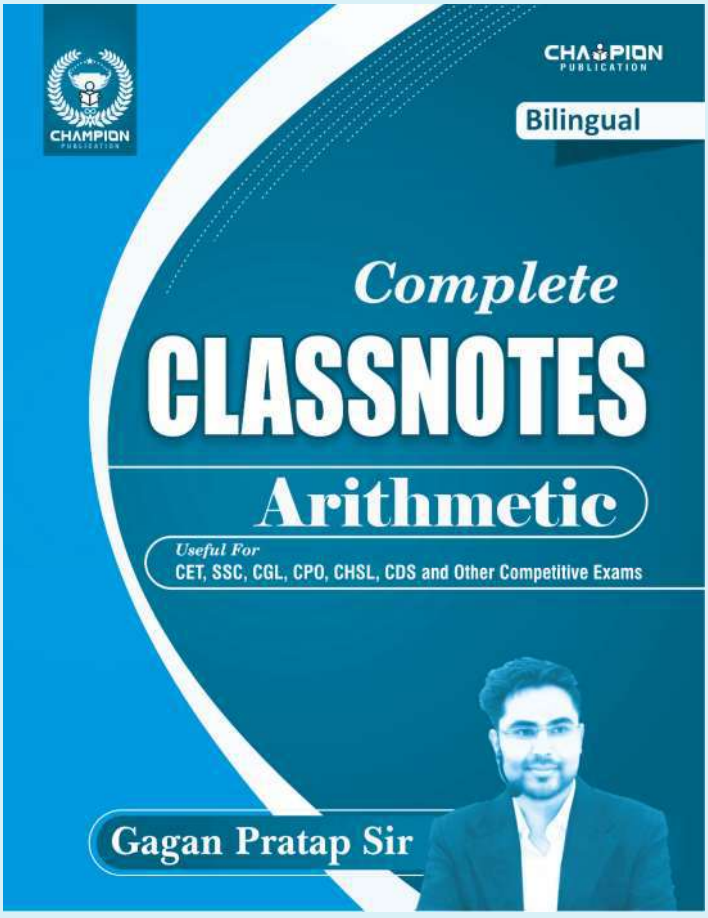
↖ ↗

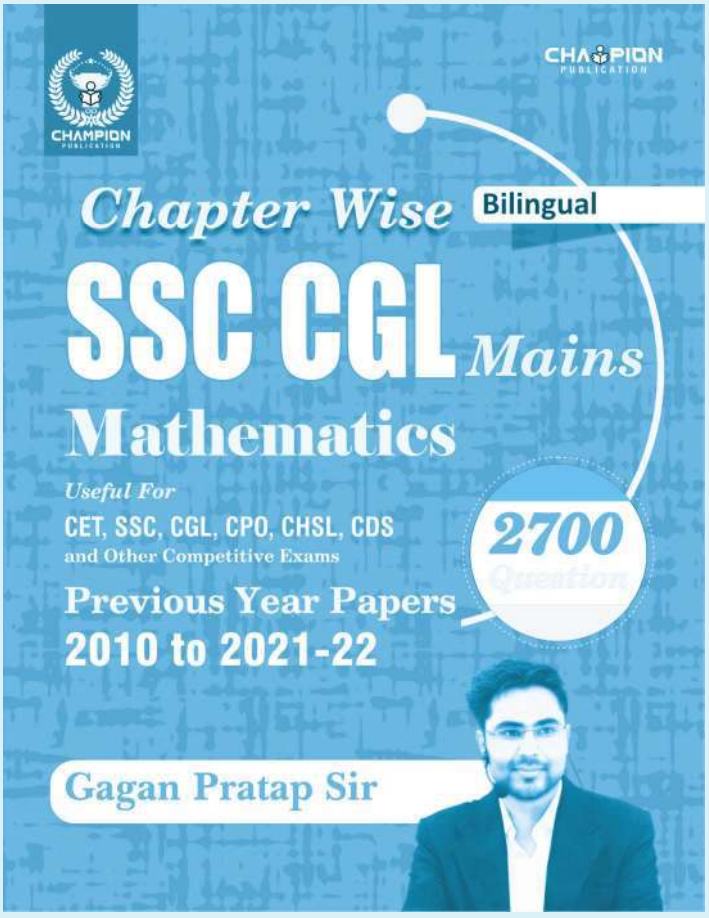
Total average

↙ ↘

Wicket (1) Wicket (2)







Time & Work (ा र्व बरे एक्ं ड्म

Total work = time × efficiency
ै।ा स्त =ां बदा ×ा स्मख्

Efficiency ∝ total work (When time is constant)
स्मख्∝ा ै।ा स्त)नभ ां बदाक्रूरए टरवि

- Efficiency/मख्बख ∝ wages/बनस्स् ∝ $\frac{1}{\text{Time}/)\ बदच}$
- For same time period wages ∝ efficiency
ं बुरा ां बदासी का कि थ बनस्स्ा ∝ा मख्बख
- For different time period wages ∝ work done
स। तध्स। ता ं बदरी का कि थी ड्डि।∝ाक दरा तदरा स्त

1. If M_1 men finish W_1 work in D_1 days, working T_1 time each day and M_2 men finish W_2 work in D_2 days, working T_2 time each day then
दका M_1 ऐ्ू र W_1 ा स्त री D_1 ाकु री्विधिएस्ा एवं टेष्वागवर्दा ाकु ा T_1 ं बदा स्त एवं टेष्वासरेए M_2 ा ऐ्ू र W_2 ा स्त री D_2 ाकु री्विधि बस्रु एवं टेष्वागवर्दा ाकु ा T_2 ा ं बदा स्त एवं टेष्वाख्

$$\frac{M_1D_1T_1}{W_1} = \frac{M_2D_2T_2}{W_2}$$

Ex. 5 men can finish a work in 10 days working 8 hours each day. How many men will be needed to finish the same work in 5 days working 4 hours each day?
5ा सस्ब,ा थ ा खा रीगक्छ्कु ा ?ा श्रा्थि खा ए ि10ाकु री्विधिएस्ा एं खटेष्वागक्छ्कु ा4ा श्रा्थि स्त ए ति ं ा स्त री5ाकु री्विधिएस्ा एु ि कि थक कु ड्डिकृएरवि ा ं सरे ्रद खा टसी,द्व
5 × 10 × 8 = m × 5 × 4
m = 20

2. If A completes a piece of work in 'x' days and B completes the same work in 'y' days, then.
दका Aाक ं ा स्त री 'x'ाकु री्विधिएस्ा एखटेाासरेए Bाते ं ा स्त री 'y'ाकु री्विधिएस्ा एखटेज

Work done by A in 1 day $= \frac{1}{x}$

Aा (रए 1 कु ा बक्कि दरा तदरा स्त $= \frac{1}{x}$

Work done by B in 1 day $= \frac{1}{y}$

Bा (रए 1ा कु ा बक्कि दरा तदरा स्तज $= \frac{1}{y}$

$\therefore$ Work done by A and B in 1 day $= \frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy}$

Aा सरेए Bा (रए 1ा कु ा बक्कि दरा तदरा स्त $= \frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy}$

$\therefore$Total time taken to complete the work by A and B both $= \left(\frac{xy}{x+y}\right)$

Aा सरेए Bा स्ुरि्वि(रए स्त रीअस्ा एु बिक्कि दरा तदरा ै।ा ं बद $= \left(\frac{xy}{x+y}\right)$

Ex. If A completes a piece of work in 20 days and B completes the same work in 30 days. If they work together then in how many days can they finish the total work?
दका Aा थ ा खा री20ाकु री्विधिएस्ा एखटेाासरेए Bाते ं ा खा री 30ाकु री्विधिएस्ा एखटेाादका ाि थ ं ा रा खा एवं टेष्वाख्वा ि ै। खा रि कु ड्डिकु री्विधिएस्ा एं खटेष्व

Work done by A and B in 1 day
Aा सरेए Bा (रए 1ा कु ा बक्कि दरा तदरा स्त

$= \frac{1}{20} + \frac{1}{30} = \frac{2+3}{60}$

Total time taken to complete the work by A and B together $= \frac{20 \times 30}{20+30} =$ **12 days**

3. If A can do a work in 'x' days. B can do the same work in 'y' days, C can do the same work in 'z' days then, total time taken by A, B and C to complete the work together
दका Aाक ं ा स्त री 'x'ाकु री्विधिएं खटेजB ं बुरा स्त री 'y'ाकु री्विधिएं खटेाC ते ं ा स्त री 'z'ाकु री्विधिएं ख टेाख्A, ा B सरेए C ा (रए का। ए स्त रीअस्ा एु बिधि ा तरा ै।

ं बदा $= \frac{1}{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}} = \frac{xyz}{xy+yz+zx}$

Ex. If A can do a work in 20 days, B can do the same work in 12 days. C can do the same work in 15 days then in how many days will taken by A, B, C to complete the work together?
दका Aाक ं ा स्त री20ाकु री्विधिएं खटेाख्Bाते ं ा स्त री 12ाकु री्विधिएं खटेजCते ं ा स्त री15ाकु री्विधिएं ख टेाख्A, B, C री्वा। एत ं ा स्त रीअस्ा एु बिक्कि कु ड्डि। ति्वि

Time $= \frac{1}{\frac{1}{20}+\frac{1}{12}+\frac{1}{15}}$

$= \frac{20 \times 12 \times 15}{20 \times 12 + 12 \times 15 + 15 \times 20} =$ **5 days**

4. If A can finish $\frac{m}{n}$ part of the work in D days.

Then, total time taken to finish the work by A

$= \frac{n}{m} \times D$ days

दका A क ं ,ा स्त र $\frac{m}{n}$ ा पस्ता D कु रिवधिअिस्ा ए ं खटेजाकम्ाठ

A ा (रण स्त रिअस्ा पु बिधि। तर ै।। ं बदा :ा $\frac{n}{m} \times D$ ा कु

Ex. Rahul can finish $\frac{4}{5}$th part of the work in 20 days.

In how many days he will complete the work?

एट।। खा र $\frac{4}{5}$ पस्ता ढ2ा कु रिवधिअिस्ा ए ं खटेजी टा स्त रि क ु कि ु रिवधिअिस्ा एिद्ध

∴ Time taken to complete the work

$\frac{5}{4} \times 20 =$ **25 days**

5. If A and B can do a work in 'x' days B and C can do the same work in 'y' days. C and A can do the same work in 'z' days. Then total time taken,

when A, B and C work together $= \frac{2}{\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)}$ OR

$\frac{2xyz}{xy+yz+zx}$ **days**

दका A सरेए B क ं ,ा खा रि'x'ा कु रिवधि ए ं खटेधाख B सरेए C त ं ,ा खा रि'y'ा कु रिवधि ए ं खटेधाख C सरेए A ं बुरा स्त रि 'z'ा कु रिवधि ए ं खटेधाख्ा A, B सरेए C थ ं ्ररत ं ा स्त रि क ु कि ु रिवधिअिस्ा ए। ि$= \frac{2}{\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)}$ ा : $\frac{2xyz}{xy+yz+zx}$ कु

Ex. If A and B can do a work in 20 days, B and C can do the same work in 10 days. C and A can do the same work in 60 days then in how many time will taken by A, B, C to complete the work together?

दका A सरेए B थ ा स्त रि20ा कु रिवधि ए ं खटेधाख B सरेए C त ं ,ा स्त रि10ा कु रिवधि ए ं खटेधाख C सरेए A ं बुरा स्त रि 60ा कु रिवधि ए ं खटेधाख A, B, C ा रिक्ा। ए स्तअस्ा पु बिधि क ु र ं बदा। ति द्ध

Time taken $= \frac{2}{\frac{1}{20}+\frac{1}{10}+\frac{1}{60}}$

$\frac{2\times20\times10\times60}{20\times10+10\times60+60\times20} =$ **12 days**

6. If A alone can do a certain work in 'x' days and A and B together can do the same work in 'y' days then B alone can do the same work in

दका A स रिक ं ,ा स्त रि'x'ा कु रिवधि ए ं खटेसरेए A सरेए B क्ा। ए त ं ,ा स्त रि'y'ा कु रिवधि ए ं खटेधाख B स रित ं , स्त रि क ु कि ु रिवधि ए ं खटेद्व

$\Rightarrow \left(\frac{xy}{x-y}\right)$ days

Ex. If A can do a work in 8 days and A + B together can do the same work in 5 days. Then B alone can do the work in how many days?

दका A क ं ,ा खा रि8ा कु रिवधि ए ं खटेसरेए A + B क्ा। ए त ं ,ा खा रि5ा कु रिवधि ए ं खटेधाख B स रित ं । स्त रि क ु कि ु रिवधि ए ं खटेद्व

Time taken by B alone

$= \frac{8\times5}{8-5} = \mathbf{\frac{40}{3}}$ **days**

7. If food is available for 'x' days for 'A' men at a certain place and after 'y' days 'B' men join, then the remaining food will serve total men for

दका थ ाकुकइ खरूसु ाअ्ा 'A' ा A ै ू रषि कि थ 'x' कु रषि कि थ पस्ुि त आ घ ाटेजा सरेए 'y' ा कु रषि भ्स 'B' ा A ै ू र त्स्क्ा। ा ट र खटेधाख ि्रूरि पस्ुि । ै।। ा A ै ू रषिक्रि क ु कि ु इ। ति्रज

Required time)सरे ्रद ं बदच

$= \frac{A(x-y)}{(A+B)}$ days

Ex. If food is available for 60 days for 35 men. After 15 days 10 new men join, then remaining food will serve total men for?

दका 35 ा स स्क्ा दरषि कि थ 60 ा कु रषि कि थ पस्ुि । त आ घ ाटेजा 15 कु रषि भ्स 10ु थ A ै ू र त्स्क्ा। ा ट र नि खटेधाख ि्रूरि पस्ुि । ै।। ा A ै ू रषि कि थ क ु कि ु इ। ति्रज

Required time $= \frac{35(60-15)}{35+10} =$ **35 days**

8. If A men or B boys or C women can do a certain work in 'x' days then A_1 men B_1 boys and C_1 women can do the same work in

दका A ै ू र दर B ा क्र त दिर C ा बक्ा। र क ं ,ा स्त रि'x'ा कु रिवधि ए ं खटेधाख A_1 ै ू र B_1 ा क्र त सरेए C_1 ा बक्ा। र ं बुरा स्त क ु कि ु रिवधि ए ं खटेद्व

Time taken $= \frac{x}{\frac{A_1}{A}+\frac{B_1}{B}+\frac{C_1}{C}}$

Ex. 16 men or 21 women or 18 children can complete a work in 93 days. In how many days 32 men, 35 women and 27 children working together complete the whole work?

16 ा A ै ू र दर 21 ा बक्ा। थ र 18 ा भ्च्छ थ ि ा खा रि93 ा कु रिवधिअिस्ा ए ं खटेधा 32 ा A ै ू र स 35 ा बक्ा। थ र सरेए 27 ा भ्च्छ थ ि ं ्ररत ख ए अर्ा खा रि क ु कि ु रिवधिअिस्ा एिद्व

Time taken = $\frac{93}{\frac{32}{16}+\frac{35}{21}+\frac{27}{18}} = \frac{93}{\frac{31}{6}} = 18$ days

9. The comparison of rate of work done is called efficiency of doing work.

Efficiency (E) $\propto \frac{1}{\text{No. of days}}$

कथतथ स्त ,ास्ए ,ाखै ु र स्त ुए ि ,ास्मख्ा ट। ख्ाटेज

स्मख्ा (E) $\propto \frac{1}{\text{No. of days}}$

$E_1 : E_2 : E_3 = \frac{1}{D_1} : \frac{1}{D_2} : \frac{1}{D_3}$, $E = \frac{k}{D}$ or, ED = k or, $E_1D_1 = E_2D_2$

Ex. 3 persons can complete the work in 3 days, 4 days, 5 days respectively. Find ratio of their efficiency?

3ाऊकृप्रथ । स्त रकृिबत्रद्ध3ाकु ठ4ाकु ठ5ाकु ।बधिक्ष्ा एं खटेचाहु ,ास्मख्ा रसु ैअख. ख ,ादद्वि

Persons	→	A	B	C
Time	→	3	4	5

Efficiency → $\left(\frac{1}{3} : \frac{1}{4} : \frac{1}{5}\right) \times 60$ (LCM)

20 : 15 : 12

Ex. A can do a work in 6 days and B can do the same work in 5 days if they work together and the total salary is ₹550 then find the shear of both.

Aाक ं ,ा ख्ा रि6ाकु रकिबधि एं खटेाटेसरेए Bाते ,ा ख्ा रि5 कु रकिबधि एं खटेादकाीथि ं ।रर ख्ा एवटेाटेसरेए ै। ीहि ु ₹550ाटेठख्सिुरकिबधि रकृरूं र. ख एिि

$A = \frac{5}{(5+6)} \times 550 =$ **₹250**

$B = \frac{5}{(5+6)} \times 550 =$ **₹300**

11. If A can do a work in 'x' days and B is R% more efficient than A then 'B' alone will do the same work in $x \frac{100}{(100+R)}$ days

दकाAाक ं ,ा स्त रि'x'ाकु रकिबधि एं खटेाटेसरेए Bठा A ,ाखै ु र बधिR%ासक । ैत्रा ाटेख्ा'B'ास ि ितं ,ा स्त रि$x \frac{100}{(100+R)}$ कु रकिबधि एीरज

Ex. A can do a work in 10 days. B is 20% more efficient than A. Then B alone do the same work in?

Aाक ं ,ा ख्ा रि10ाकु रकिबधि एं खटेाजBठा A ,ाखै ु रबधि20% सकर । ैत्रा ाटेजाख्ा Bास ि किु किु रकिबधि बुर। स्त एीरद्ध

B alone will do $= 10 \times \frac{100}{100+20}$

$= 10 \times \frac{100}{120}$

$\Rightarrow \frac{25}{3}$ **days**

12. A can do a work in 'm' days and B can do the same work in 'n' days. If they work together and total wages is R, then.

Aाक ं ,ा ख्ा रि'm'ाकु रकिबधि एं खटेाटेसरेए Bाते ,ा ख्ा रि 'n'ाकु रकिबधि एं खटेाजादकाीथि ं ।रर ख्ा एवटेाटेसरेए ै। ीहि ु Rाटेठख्जि

Part of A द्वबA रकृरूं ख= $\frac{n}{(m+n)} \times R$

Part of B द्वबB रकृरूं र = $\frac{m}{(m+n)} \times R$

13. If A, B and C finish the work in m, n and p days respectively and they receive the total wages R, then the ratio of their wages is

दकाA, Bासरेए C। स्त रिm, nासरेए p कु रकिबधिक्ष्ा एवटेाजासरेए तथ्ाधि ै। ।बनस्ख्ा Rागख्ा टरख्ा।टेठख्ा ु ,ाबनस्ख्ा रसु ैअख्ा टे

$\frac{1}{m} : \frac{1}{n} : \frac{1}{p}$

14. If A working alone takes x days more than A & B and B working alone takes y days more than A & B. Then the number of days taken by A & B working together to finish a job $\Rightarrow \sqrt{xy}$ days.

दकाAास ि ि स्त एवटैथAासरेए Bां िxाकु ासक । ाख्टेासरेए Bास ि ि स्त एवटैथAासरेए Bां िyाकु ासक । ाख्टेाजाख्ा A सरेए B(रएथ ं ।रर स्त रिxाख्ा ु बिधि तु ीरा किु रकि ,ां घदर $\Rightarrow \sqrt{xy}$ ाकु ाटेज

Ex. If A working alone takes 4 days more than A and B and B working alone takes 9 days more than A and B working together. Then the number of days taken by A and B working together to finish a job?

दकाAास ि ि स्त एवटैथAासरेए Bां ि4ाकु ासक । ाख्टेासरेए Bास ि ि स्त एवटैथAासरेए B थ ं ।रर स्त ुए ि9ाकु सक । ाख्टेाजाख्ा Aासरेए B(रएका। एथ । स्त रिxाख्ा ुए बधिक्ष्ा किु किु ।। तख्टेाट्ठ

∴ Time taken by A and B together

Aासरेए B(रएथ ं ।रर कदरतदर ं बद

$= \sqrt{4 \times 9} =$ **6 days**

❖ Wages/बनस्ख्ा ∝ work done/कदरतदर स्त

❖ Work done/कदरतदर स्त ∝ Number of persons involved/ ंक्षबक खऊकृख्दरधि ,ां घदर

❖ Work done = Number of persons × efficiency × time period

कदरतदर स्त= ंक्षबक खऊकृख्दरधि ,ां घदर× मख्ख× ं बदासख्ाए

Pipe & Cistern (ि)बक्बधन द्श

Amount of water released or filled = Rate × time.
म क्रिंतिथ दरा पएु तथ अु ,ा ,ा बरख्स =ा स्प् ×ा ं बदज

1. Two taps 'A' and 'B' can fill a tank in 'x' hours and 'y' hours respectively. If both the taps are opened together, then how much time it will take to fill the tank?
स्रु ।।'A'ा सरेए'B'ा ४ हध,ा रकिृबत्रद्ध'x'ा श्रंथ्सिरेए'y'ा श्रंथ्बधिमिरए ं खटेचादका स्रुधि्। रधि रि४ ।ं ्रारलरी ाकदरानस्थ्ठखहध,ा रि पएु बिधि्का कुर ं बदा। तसिद्ध

$$\text{Required time} = \left(\frac{xy}{x+y}\right)\text{hrs}$$

2. Two taps 'A' and 'B' can empty a tank in 'x' hours and 'y' hours respectively. If both the taps are opened together, then time taken to empty the tank will be $\text{Required time} = \left(\frac{xy}{x+y}\right)\text{hrs}$

स्रु ।।'A'ा सरेए'B'ा ४ हेधा रकिृबत्रद्ध'x'ा श्रंथ्सिरेए'y'ा श्रंथ्बधिमिरा ,ा ए ं खटेजादका स्रुधि्। ।४ ।ं ्रारलरी निखटेचाखहध,ा रिलरा , पु बिधि्तर ं बदा $= \left(\frac{xy}{x+y}\right)$ ा श्रंथ्हि

3. If $x, y, z,$..... all taps are opened together then, the time required to fill/empty the tank will be:
दका $x, y, z,$.....ा ं पु ।।४ ।ं ्रारलरी निखटेचाखहध,ा रि पएु लिरा ,ा पु बिधि्तु ीिरा र ं बदा टसिरद्ध

$$\frac{1}{x} \pm \frac{1}{y} \pm \frac{1}{z} \pm .... = \frac{1}{T}$$

where T, is the required time.
नटसका ठसपू हा ं बदा टेज

Note: Positive result shows that the tank is filling and Negative result shows that the tank is getting empty.
ि चबसग ु रब ।अर्ंखा स्त्रसख्या टेक ाहध,ापएा एट,ाटेासरेए ऋंर्ंखा स्त्रसख्या टेक ाहध,ाल रा ,ाटर्ंए,ाटेज

4. If a pipe fills a tank in 'x' hours but it takes 't' more hours to fill it due to leakage in tank. If tank is filled completely, then in how many hours it will be empty? [due to leakage outlet]
दका४ ।अरुअक ं ,ाहध,ा रि'x'ा श्रंथ्बधिमिरएखटेा। कि ु ाहध,ाबधिमिं रि ि र्ंए रत ं पिएु बिधि't'ा श्रंथ्सिक ।। तर्खटेचादका हध,ाअस्ा खट्टापए नस्थ्ठखर्ी टाक कु श्रंथ्बधिमिरा ,ाटर्ंनस्थ्त,द्वा . वं्रं रा ।सस्रु हा हि ि र्ंए रा

$$\text{Required time} = \frac{x(x+t)}{t}$$

Ex. If a pipe fills a tank in 20 hours but it takes 4 hours more to fill it due to leakage in the tank. If the tank is filled completely, then in how many hours it will be empty?
दका४ ।अरुअक ं ,ाहध,ा रि20ा श्रंथ्बधिमिरएखटेा। कि ु ाहध,ाबधिमिं रि ि र्ंए रत ं पिएु बिधि्ा श्रंथ्सिक ।। तर्खटेचादका हध,ाअस्ा खट्टापए नस्थ्ठखर्ी टाक कु श्रंथ्बधिमिरा ,ाटर्ंनस्थ्त,द्वा

$$\text{Required time} = \frac{20 \times (20+4)}{4}$$
$$= 5 \times 24 = \textbf{120 hours}$$

5. A tap 'A' can fill a tank in 'x' hours and 'B' can empty the tank in 'y' hours. Then (a) time taken to fill the tank
४ ु ।।'A'ा४ ाहध,ा रि'x'ा श्रंथ्बधिमिरएं खटेासरेए'B'ाहध,ा रि 'y' श्रंथ्बधिमिरा ,ा ए ं खटेजाख्ा (a)ा हेधा रिपएु बिधि तु ीिरा र ं बद

when both are opened)नभा स्रुधिमिरा र्ी निखटेचा

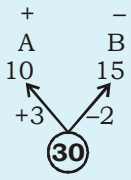
$$= \left(\frac{xy}{y-x}\right) : x < y$$

(b) time taken to empty the tank
हध,ा रिलरा ,ा पु बिधि्तर ं बद

when both are opened पनभा स्रुधिमिरा र्ी निखटेध

$$= \left(\frac{xy}{x-y}\right) : y < x$$

eg.→ (a) Tap A can fill the tank in 10 hours and tap B can empty it in 15 hours. Find the taken to fill the tank.

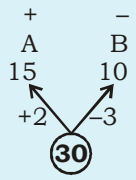


Time taken to fill the tank $= \frac{30}{3-2} = 30$ hours

Alternatively:-

Time taken to fill the tank $= \frac{10 \times 15}{15-10} = \frac{10 \times 15}{5}$
$= \textbf{30 hours}$

(b) Tap A can fill the tank in 15 hours and Tap B can empty it in 10 hours. Find time taken to empty the tank.



Time taken to empty the tank $= \frac{30}{1} = 30$ hours

Alternatively:-

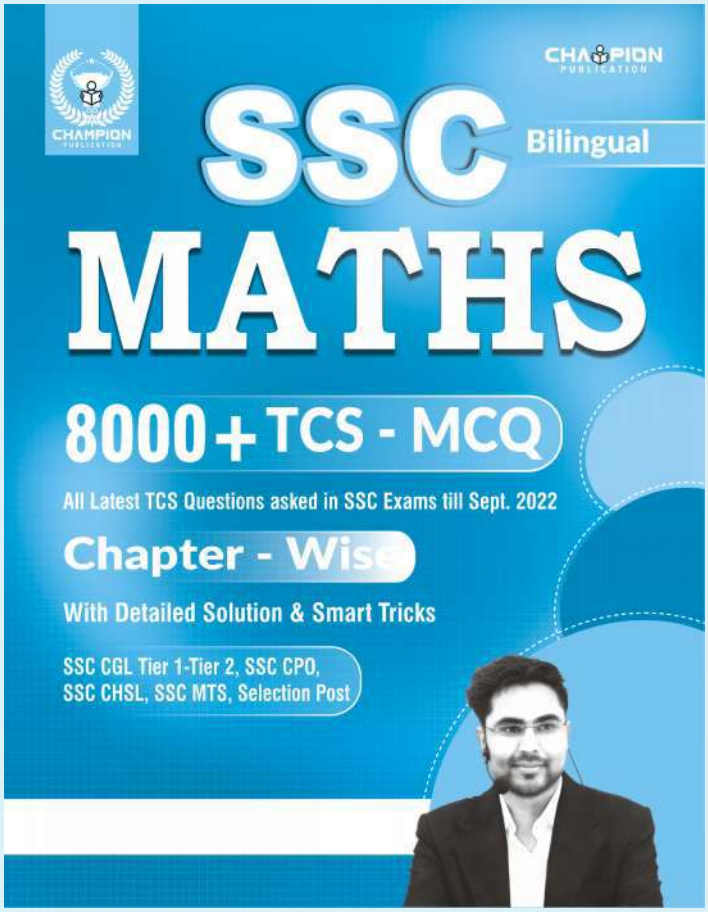
Time taken to empty the tank $= \frac{15 \times 10}{15 - 10} = \frac{150}{5}$

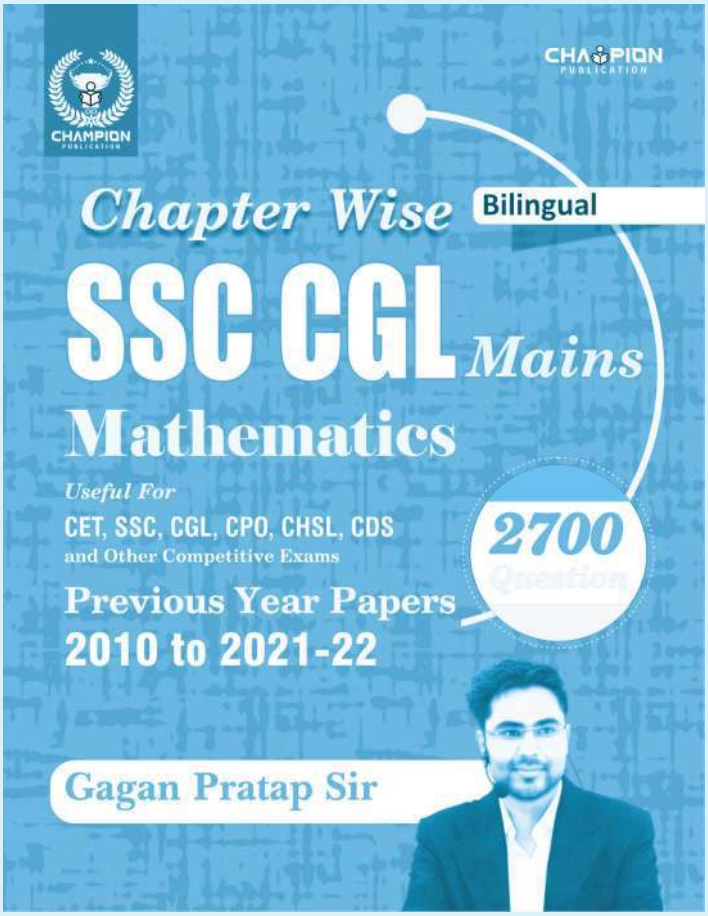
$=$ **30 hours**

6. Two taps A and B can fill a tank in x hours and y hours respectively. If both the pipes are opened together, then the time after which pipe B should be closed so that the tank is full in t hours

स्रु ।।A सरेए Bा थ ।हध ,। रिकृबत्रद्धxत श्राथ्यासिरेए yा श्राथ्यबिधिरए ं खि टेथ्वादका स्रु रिधिरए अधि रिथ ।ं त्र रा ल री ।कदरा न खा टेठ खि अ ए अ xत रि क छु ि बदा भ्स्रा भ्स्रा ए स्रु रि इ स्त्रथा ख्क ।हेधा tा श्राथ्यबिधिरए न स्थ ठ

Required time ए सरे त्रद ।ं बदा $= \left[y\left(1 - \frac{t}{x}\right) \right]$ hours





Time, Speed & Distance (ा र्व ध्वलक्षत्रे फ्रपे)अ

Time→ Time is a fundamental physical quantity which defines the sequence and duration of events and it is denoted by 'T'

ा र्व →ां बदा थ ाबेस्क ापसेक्ष ाबरअस टोनरश्रहु सरश्री सिुैकृब सरेए सी का रिअाप्सस्कूस्ख एढ़ा टोसरेए ष्ं ि'T'ा (रए कुय कृाख क दर नख्या टेज

- **Distance**→ The length of space between two points is considered the distance and it is denoted by 'D'

लम्बद→ स्रकिसैसरश्री भ्रि,इा रूसु ा ,ा। धरअ रिस्सा बुर ा नख्या टोसरेए ष्ं ि'D'ा (रए कुय कृाख क दर ा नख्या टेज

- **Speed**→ Speed is defined as the distance travelled in unit time and it is denoted by 'S'.

नै →ातकक्ष रिष्ं बदा बध्रिख्दा ,ातष स्सा यिअ बध्रिअाप्सस्कूस्ख क दर ातदर टोसरेए ष्ं ि'S'ा (रए स्त्रस्दर ा नख्या टेज

❖ **Distance = D, Speed = S, Time = T**

$\Rightarrow$ $D = S \times T, S = \frac{D}{T}, T = \frac{D}{S}$

$1km/hr = \frac{1km}{1Hr} = \frac{1000m}{3600sec} = \frac{5}{18} m/sec$

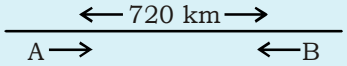
$1km/hr = \frac{5}{18} m/sec$

$1 m/sec = \frac{18}{5} km/hr.$

Case-I: If D = constant $\quad S \propto \frac{1}{T}, T \propto \frac{1}{S}$

$D = S_1T_1 \Rightarrow S_1T_1 = S_2T_2$

$\therefore \frac{S_1}{S_2} = \frac{T_2}{T_1}$



$S_1 = 60$ km/h $\quad S_2 = 90$ km/h

D = Constant

$A = \frac{720}{60} = 12Hr.$

$B = \frac{720}{90} = 8Hr.$

Speed →2 : 3

Time → 3 : 2

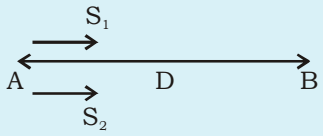
Case-II: If time constant D ∝ S

$D_1 = S_1 T$

$D_2 = S_2 T$

$\frac{D_1}{D_2} = \frac{S_1}{S_2}$

Proof:-



Diff. of time = Δt

Diff. of speed = $S_1 \sim S_2/S_2 \sim S_1 = \Delta S$

$\therefore \frac{D}{S_1} - \frac{D}{S_2} = \Delta t$

$\therefore D\left[\frac{S_2 - S_1}{S_1S_2}\right] = \Delta t$

$$\mathbf{D = \frac{S_1S_2}{S_1 \sim S_2} \times \Delta t}$$

Case-III: If speed constant D ∝ T

$D_1 = ST_1$

$D_2 = ST_2$

$$\mathbf{\frac{D_1}{D_2} = \frac{T_1}{T_2}}$$

❖ If A and B starts walking towards each other. After meeting each other. A covered his remaining distance in t_1 time and B covered his remaining distance in t_2 time. Then ratio of their speed is

दका Aा सरेए Bा थ ास्लर्ए ,ासरश्रि इ।ु ि। तर्खटेधासरअं ाबध्रिख्दा।ु ि ि भ्रस्त Aा सअु ,ा त्रूरि स्सा रि t_1 ां बदा बध्रिख्दा एख्ा टोसरेए Bा सअु ,ा त्रूरि स्सा रि t_2 ां बदा बध्रिख्दा एख्ा टेजा र्खि ु ,ातकक्ष र सुैअख्ा टे

$\Rightarrow \frac{A}{B} = \sqrt{\frac{t_2}{t_1}}$

If they meet after 't' hours then $t = \sqrt{t_1 \cdot t_2}$

दका ि't'ा श्रार्थ ि भ्रस्सा का। र्खटेधार्खि $t = \sqrt{t_1 \cdot t_2}$

Ex. A and B started their journey from X to Y and Y to X respectively. After crossing each other A and B completed the remaining part of their journey in 4h and 9h respectively. If the speed of B is 60 km/h then speed (in km/h) of A is:

Aा सरेए Bा कृबक्रद्ध Xां ि Yा सरेए Yां ि Xा ख ासअु ,ा दरअ क्रस्यै ा ,ज थ ास्लर्ए रिअाप्ु ि भ्रस्स Aा सरेए Bा सिअु ,ा दरअ र त्रूरि पस्त कृबक्रद्ध 4ा श्रार्थ सरेए 9ा श्रार्थ बध्रिख्दा क दर ा दका Bा ,ातकक्ष 60ा क ब,प्श्रार्थ टोर्खA ,ातकक्ष)क ब,प्श्रार्थ बध्रि टेद्ध

$\frac{S_A}{S_B} = \sqrt{\frac{9}{4}} \Rightarrow \frac{S_A}{60} = \frac{3}{2}$

$S_A = 90$ km/h

Relative speed (ो च्येबनै अ

1. When the two objects travel in same direction:-

नभा स्गी खैब्नक्ष्य ाट,ाक्रऽस बश्हि। ख्ा टेछ्क्ष

→30km/hr.

→70 km/hr.

Relative speed)ं रुमि तक्छ्वा = 70 – 30 = 40km/hr.

∴ Distance between man & bus in one hr. will be 40 km.

Relative speed = diff of speeds

सस्ब,ासरेण्ं ा फ्रि,इा थ ाश्रथ्बश्हि2ाक ब,ास्ग्ा टसी,ज

ं रुमि तक्छ्व =ा तक्छ्व र सख्छए

2. When two objects travel in opposite direction:-

नभा स्गी खैब्नक्के अए,ख क्रऽस बश्हि। ख्ा टेछ्क्ष

→30 km/hr 70 km/hr←

Relative speed)ं रुमि तक्छ्व → 30+70 = 100 km/hr.

Relative speed (ो च्येबनै अ= sum of both speeds.

(लेग्मिब्नै बक्े बे यग्अ

When train passes (ऽ भ्बथ्मिब्नछप क्रथ्ख

1. When train passes a pole or stationary man

ऽ भ्बथ्मिबैन्ा क्ष्र सेबं बैटेहोपबरे ल्वक्ा ब्नछप क्रथ्ख

Distance covered will be equal to length of train

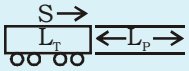
ख्ा ,ातफ स्ग्ा हुा ,ा। थ्रफ भ्एिभ्एट्सी,

$T = \frac{L}{S}$

T = crossing time)अए ए्ु ि रं बदच

2. When train passes a bridge/platform.

ऽ भ्बथ्मिबच्छेद्दुल)थ्चोल्वड्ा ब्नछप क्रथ्ख



distance covered will be equal to length of train + length of bridge/platform.

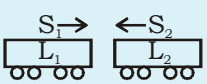
ख्ा ,ातफ स्ग्ा = हुा ,ा। थ्रफ +ाअै ण्ा हिअण्ब्ा ,ा। थ्रफ भ्एिभ्एट्सी,ज

$T = \frac{L_T + L_P}{S}$

T= Crossing time)अए ए्ु ि रं बदच

3. When a train pases another train in opposite direction

नभा थ ाहुा िकेअए,ख क्रऽस बश्हिस्लएाहुा ि रअए एड्ा टे

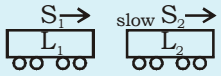


distance covered)ख्ा ,ातफ स्ग्चा = L_1+L_2

$T = \frac{L_1 + L_2}{S_1 + S_2} = \frac{\text{Distance Covered}}{\text{Relative Speed}}$

4. When a train passes another train in same direction.

नभा थ ाहुा िं ,ाक्रऽस बश्हिस्लएाहुा ि रअए एड्ा टेज



$T = \frac{L_1 + L_2}{S_1 - S_2} = \frac{\text{Distance Covered}}{\text{Relative Speed}}$

5. When a train passes a person sitting in another moving train

नभा थ ाहुा िस्लएाइ। ख्ा हुा िबश्हिेब्ऊक्रकृप्र रअए ए्नख्ा टे

$T = \frac{L_1}{\text{Relative Speed}} = \frac{L_1}{S_1 - S_2}$

Average speed (ो ख बनै अ

1. If a man travels different distances d_1, d_2, d_3, ... and so on in different time t_1, t_2, t_3 respectively then,

दक्ा थ ाऊक्रकृप्र स। तथ्स। ता स्ग्ा d_1, d_2, d_3, ...ाासरेण्ं ,ाख्ट स। तथ्स। ता ं बदा t_1, t_2, t_3 ाबश्कृबऽरुद्ध ख्ा एड्ा टेछ्क्ष ख्ि

Average speed प सरें ख तक्छ्व $= \frac{\text{total travelled distance}}{\text{total time taken in travelling distance}}$

$= \frac{d_1 + d_2 + d_3 + \ldots}{t_1 + t_2 + t_3 + \ldots}$

2. If a man travels different distances d_1, d_2, d_3, ... and so on with different speeds s_1, s_2, s_3, respectively then,

दक्ा थ ाऊक्रकृप्र स। तथ्स। ता स्ग्ा d_1, d_2, d_3, ...ाासरेण्ं ,ाख्ट स। तथ्स। ता तक्छ्व कृबऽरुद्ध s_1, s_2, s_3 ां दिऽऽस एड्ा टेछ्क्ष ख्ि

Average speed प सरें ख तक्छ्व $= \frac{(d_1 + d_2 + d_3 + ...)}{\frac{d_1}{s_1} + \frac{d_2}{s_2} + \frac{d_3}{s_3} + ...}$

Ex. A car travels 15 km, 20 km, 30 km and 12 km at speeds of 20 km/hr, 30 km/hr, 40 km/hr and 30 km/hr respectively. Find the average speed of the car in the total journey.

थ ा रयकृबऽरुद्ध15 क ब,ठ20ाक ब,ठ30ाक ब,ाासरेण्12ाक ब,ा , तक्छ्व ं ि20ा क ब,ण्प्राश्रऽस 30ा क ब,ण्प्राश्रऽस 40ा क ब,ण्प्राश्रऽस सरेण्ं 30 क ब,ण्प्राश्रऽस ,ाइरा ं ख्ा एड्ा टेजा ै। ादऽस्ब्ि रय ,ासरें ख तक्छ्व . ख एिि

Average speed $= \frac{77}{\frac{15}{20} + \frac{20}{30} + \frac{30}{40} + \frac{12}{30}} = \frac{77}{77} \times 30$

= **30 km/h**

3. If a distance is divided into n equal parts each travelled with different speeds, then,

दकाश ास्ता रीnा भएभए पस्तरीकिपस्ता खक दरा नख्या टेठगव्दि ु ि स। तध्स। ता तक्ख दिखस ,ठख्

Average speed प सरें ख तक्ह = $\frac{n}{\left(\frac{1}{s_1}+\frac{1}{s_2}+\frac{1}{s_3}+\frac{1}{s_4}\right)}$

where n number of equal parts s_1, s_2, s_3, ... s_n are speeds.

नटस्ताnा भएभए पस्तरीि ,ं घदर s_1, s_2, s_3, ... s_n ।तक्ख टेघ

4. If a bus travels from A to B with the speed of x km/h and returns from B to A with the speed of y km/h, then the average speed will be $\left(\frac{2xy}{x+y}\right)$

दकाश ।ं ।A ं ।B।ख ।x।क ब,प्पाक्षण ,ातक्ख दिखस एठाटोसरेए B ं ।A।ख ।y।क ब,प्पाक्षण ,ातक्ख ी।सं ।सख्ाटेठख्सिरें ख तक्ह टसी, $\left(\frac{2xy}{x+y}\right)$

Ex. A person travels from a station A to station B at a uniform speed of 60 km/h and returns to A at a uniform speed of 84 km/h. His average speed for the entire journey is:

श ।ऊकप्रस्हरि ।A ं स्हरि ।B।ख ।60।क ब,प्पाक्षण ,ाश ं बुर तक्ख दिखस एठाटोसरेए 84।क ब,प्पाक्षण ,ं बुर ।तक्ख ी।सं ।A अए। रेहख टेजा अस्ता दखस कि थत ं ,ासरें ख तक्ख टेद्ध

Average speed = $\frac{2\times 60\times 84}{84+60}$ = 70 km/h

Alternatively:-

Let D = 420 km (L.C.M of 84 and 60)

Average speed = $\frac{420+420}{\frac{420}{84}+\frac{420}{60}}=\frac{840}{5+7}$ = **70 km/h**

Speed increase/decrease (नै ब्वबबैकैं द्व- वस्श

1. If an object increases/decreases its speed from x km/hr to y km/hr. to cover a distance in t_2 hours in place of t_1 hours then (Here $(t_2 - t_1)$ will be given).

दका रीी खैवसअु ,।तक्ख रिx।क ब,प्पाक्षण ं भ्रिस्र एy।क ब,प्पाक्षण ए स्ह।टेजाक ं ,ास्ता रीt$_1$।श्रथ्ी क्रिसु ।अए t$_2$।श्रथ्बिभ्रिख्ा ए ु ि कि थ)दटस्ता$(t_2 - t_1)$।क दरा नस्थतस्चज

Distance = $\frac{xy}{(\text{Diff. of } x \text{ and } y)}\times$ (Change in time)

or,

Distance = $\left(\frac{\text{Product of Speeds}}{\text{Diff. in Speeds}}\right)\times$ (Change in time)

2. If an object travels certain distance with the speed of $\frac{A}{B}$ of its original speed and reaches its destination 't' hours before or after, then the time taken by object travelling at original speed is

दका रीी खैवसअु ,।बलातक्ख ,।$\frac{A}{B}$।तक्ख ि र ुकुकइ ख स्ता ख्द एठाटोसरेए सअु तिख्ख्ा 't'।श्रथ्ी अट। दिर भ्स बधि्रिटैइ ख्ाटेठख्ी खैवै (रए बलातक्ख दिखस ए ु बिभ्रिक दरा तदर ं बदा टे

Time = $\frac{A}{(\text{Diff. of A and B})}\times$ time (in hour)

3. If a man travels at the speed of s_1, he reaches his destination t_1 late while he reaches t_2 before when he travels at s_2 speed, then the distance between the two places is

दका श ।ऊकप्रस$_1$।ा ,।तक्ख दिखस एठाटेठख्ी टासअु तिख्ख्ाt_1 स्रि ं ि टैइ ख्ाटेनभ्कीाटाt_2।अट। ि टैइ ख्ाटेनभ्ी टाs_2।ा ,।तक्ह ं दिखस एठाटेठख्सिुरि रिसु रि ि भ्रइा ,ास्ता टे

Distance = $\frac{(s_1\times s_2)\times(t_1+t_2)}{s_2-s_1}$

Some important points (् एब्वथ्मक्ख्ष्ट ड्रैभ्तख्

1. Formula to calculate the no. of rounds.

Circular Distance = (circumference) × No of rounds,

ैखर एस्ता :।)अक्षक्च ×।इड एमि ,ं घदर

D = 2πr × n

2. If any one overtakes or follows another, then time taken to catch

सतए रिक ं ,। रिसरीि एहि।दरा अम्र एठाटोख्ति ं अि क्रुत्बिधि ं बदा। तख्ाटे

= $\frac{\text{distance between them}}{\text{Relative speed}}$

or, meeting time)क्ा।ु ि र ं बदच

= $\frac{(\text{Speed of 1st traveller})\times \text{time}}{(\text{Diff. of speeds})}$

Total travelled distance to catch the thief)इरि रि अ क्रुत्ि कि थख्दा ,।तष ै।ास्ताच

= $\frac{(\text{Product of speeds})\times \text{time}}{(\text{Diff. of speeds})}$

3. Formula to calculate the no. of poles,

वरश्भरि ,ं घदर अख्ा ए ु र

Distance प वरश्भरि ,ं घदर)स्ताच = (n – 1)x

where n = No. of poles.

x = distance between consecutive two poles.

x :।। तख्ाए स्र्ि वरश्भरि भ्रिइा ,ास्ताज

4. If a man covers $\frac{1}{x}$ part of Journey at u km/h, $\frac{1}{y}$ part at v km/h and $\frac{1}{z}$ part at w km/hr and so on, then his average speed for the whole journey will be

दका थ ाऊकृप्रसअु ,ादरश्य र $\frac{1}{x}$ ापस्ताuाक ब,प्थाश्य ,ातक्व ं हि $\frac{1}{y}$ पस्ताvाक ब,प्थाश्य ,ातक्व ं सिरेए $\frac{1}{z}$ ापस्ताwाक ब,प्थाश्य ,ातक्व ं ि ख्दा एआटेठख्वअिग्ा दरश्य कि थत ं ,ासरें खतक्व टसि,

$$\frac{1}{\frac{1}{xu}+\frac{1}{yv}+\frac{1}{zw}+....}$$

5. Let 'a' metre long train is travelling with the speed 'x' m/s and 'b' metre long train is travelling with the speed 'y' m/s in the opposite direction on parallel path. Then, time taken by the trains to cross each other

बुरा। ,क्थाक ा'a'ाब,हप्। ध,ाहुा'x' m/s ,ातक्व ं दिरश्य एएए, टोसरेए'b'ाब,हप्। ध,ाहुा ं बुरख्वअरअए के अए,खक्त्रसबधिy' m/s ,ातक्व ं दिरश्य एएए,ाटेजाक्मप्रहुाहि(रएए थ ास्ंलएि रिअएए पुि बधि्ा तु ीि रा र ं बद

$$=\left(\frac{a+b}{x+y}\right)\text{seconds}$$

6. If a train crosses a standing man/a pole in 't_1' sec time and crosses 'P' meter long platform in 't_2' sec time, then length of the train $=\frac{P\times t_1}{(t_2-t_1)}$

दका रिएि तक्त्राक ं ,ालक्त्रिकृप्रल्धाि री't_1'ा ि क्रा ं बदाबधिअए एआटोश्र स'P'ाब,हप्। श्भ्रि हिअक्त्रा री't_2'ा ि क्रा ं बदाबधिअए एआटेठख्वएि तक्त्रा ,ा। श्भ्रश्य $=\frac{P\times t_1}{(t_2-t_1)}$

7. If two trains of (same lengths) are coming from same direction and cross a man in t_1 and t_2 seconds, then time taken by both the trains to cross each other $=\frac{2\times\text{Product of time}}{\text{Diff. of time}}$

दका) बुरा। ध्रश्य ा ,स्रएि तक्त्रादस्थ ाट,ाक्त्रस ं सिरा एए,ाटोसरेए थ ासस्ब,ा रीt_1ासरेएt_2ा ं ि क्रा बधिअए एआटेठख्वस्रुरिअएि तक्त्रादस्थि (रएए थ ा स्ंलएि रिअएए पुि बधि्ा क दरा तदरा ं बद

$$=\frac{2\times\text{Product of time}}{\text{Diff. of time}}$$

8. If two trains of same length are coming from opposite directions and cross a man in t_1 seconds and t_2 seconds then time taken by both trains to cross each other $=\frac{2\times\text{Product of time}}{\text{Sum of time}}$

दका ं बुरा। ध्रश्य ,ास्रहुि्ा क्अए,खक्त्रस्सअरिअएि सिरा एए,ाटोसरेए थ सस्ब,ा रीt_1ा ं ि क्रा सरेएt_2ा ं ि क्रा बधिअए एआटेठख्वस्रुरिअएि हुि्ा (रएए थ ा स्ंलएि रिअएए पुि बधि्ा क दरा तदरा ं बदा $=\frac{2\times\text{Product of time}}{\text{Sum of time}}$

9. If a train of length l m passes a bridge/ platform of 'x' m in t_1 sec, then the time taken by the same train to cross another bridge/platform of length 'y' m is,

$$\text{Time taken}=\left(\frac{l+y}{l+x}\right)t_1$$

दका ाब,हप्। ध्रश्य ,थ ाहुा'x'ाब,हप्। ंअ ाहुा'x'ाब,हप्। ंअ ाहुा ं ि क्रा बधिअए एआटेठख्वि ,ाहुा(रएए। ध्रश्य'y'ाब,हप्। स्ंलएि ंअ ाहुा ं ि क्रा रिअएए पुि बधि्ा तु ीि रा र ं बदा टेठ $=\left(\frac{l+y}{l+x}\right)t_1$

10. From stations A and B, two trains start travelling towards each other at speeds a and b, respectively. When they meet each other, it was found that one train covers distance d more than that of another train. The distance between stations A and B is given as

स्हरिु रिएAा सरेए Bा ं हि स्रहुि्ा क्ि बत्रस्सद्धaा सरेए bा ,ातक्व ं थि ास्ंलएि ,ासरि दरश्य एुि रत्रैया एआटेठख्वन भीथि ध्स्ंलएि क्रिा। ख्वटेठख्वख्वि दटा अदरा तदरा क ा थ ाहुा स्ंलएा हुा ,ाखै ुि रबधि्ा सक ास्ख्रा ख्द एआटेजा स्हरिु रिएAा सरेए Bा ं थि ,इा ,ास्ख्रा ं ग रएए स,ा तश्य टे

$$\Rightarrow\left(\frac{a+b}{a-b}\right)\times d$$

11. Excluding stoppage, the average speed of a train is u and with stoppage its average speed is v. Then, the stoppage time per hour

ब्टएि। रि-क्रि्त एुथ ाहुा ,ासरें खतक्व uाटोसरेए ब्टएि। ि ्रर ं ,ासरें खतक्व vाटेजाक्मप्रहुाग क्व श्रथि= ु ि र ं बद

$$=\frac{\text{Diff. between their average speed}}{\text{Speed without stoppage}}$$

$$=\frac{u-v}{u}$$

With u > v and u, v ≠ 0

12. A train covers a distance between stations A and B in time t_1. If the speed is changed by S then the time taken to cover the same distance is t_2. Then the distance (D) between A and B is given by

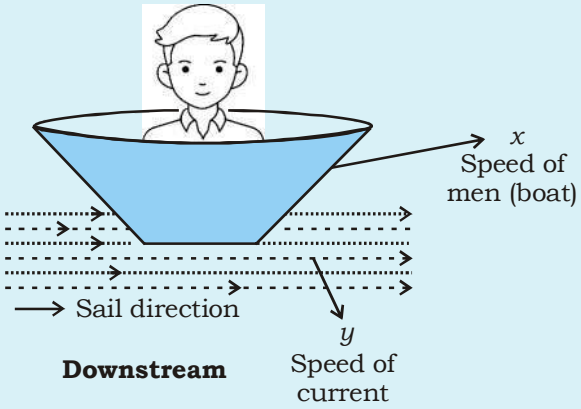
थ ाहुा स्हरिु रिएAा सरेए Bा थि ध्इा ,ास्ख्रा रीt_1ा ं बदाबधिख्दा एुटेजाद क ातक्व रSा ं थि स्। ाक्दरा नथ ाख्वि बुरा स्ख्रा ख्द पुि बधि्ा तु ि ीि रा र ं बदाt_2ाटेजाख्ा Aा सरेए Bा थि ध्इा ,ास्ख्रा (D)ा(रएए स,ा नख्ा टे

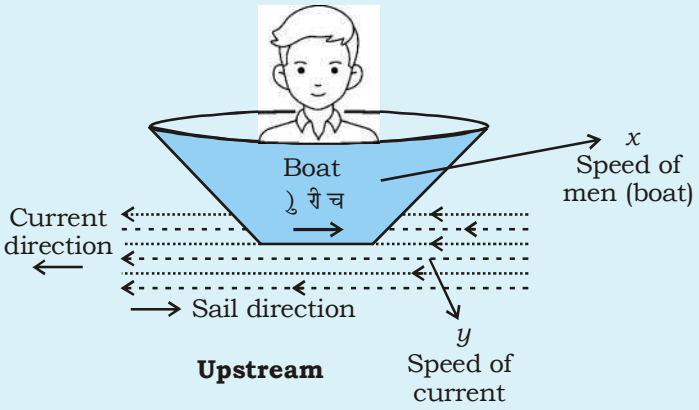
$$D=S\left(\frac{t_1t_2}{t_1-t_2}\right)\text{or}\left(\frac{S'}{t'}\right)t_1t_2$$

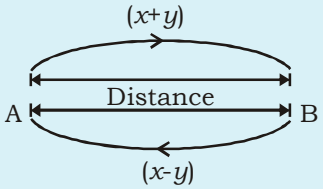
Where 't' : change in the time taken

नटक्त्रा t'ा द्धक थ त थ ं बदाबधिअक्ी खु

Boat & Stream (ि क्क्ब्ु पेअ







1. Speed of boat in still water = $x$km/hr

ञस्ख्न। ाबध्रि ैा ,ातक्ब =ा $x$km/hr

Speed of current/stream = $y$km/hr

ग्रण ,ातक्ब =ा $y$km/hr

❖ Speed of boat in same direction of stream = downstream = **D = *x* + *y***

ग्रण ,ा ट,ाक्रस बध्रि ैा ,ातक्ब =ा ग्रण सिुै ल = **D = *x* + *y***

❖ Speed of boat in opposite direction of stream = upstream = **U = *x* – *y***

ग्रण किअ्रख क्रस बध्रि ैा ,ातक्ब :ा ग्रण गिक्ह ला = **U = *x* – *y***

❖ $x > y$ $x = \frac{D+U}{2}$ = Speed of boat. ु ैा ,ातक्ब

$y = \frac{D-U}{2}$ = Speed of stream.)ग्रण ,ातक्ब

D > U

2. Let the speed of boat is x km/h and speed of stream is y km/h. To travel d_1 km downstream and d_2 km upstream, the time is 't' hours, then

बुरु ु ैा ,ातक्ब xाक ब,ण्प्रश्नग टोसरेण ग्रण ,ातक्ब yाक ब,ण्प्रश्नग टेज d_1ाक ब,ाग्रण सिुै लासरेण d_2ाक ब,ाग्रण गिक्ह लादरअ्ख ुए कि थ्ठं बदा 't'ा श्रण्टेठख्

$$\frac{d_1}{x+y}+\frac{d_2}{x-y}=t$$

Ex. The speed of a boat is 23 km/hr and the speed of the stream is 9 km/hr, it takes 't' time to cover a distance of 112 km downstream and 224 km upstream. Then find the value of t?

थ ु ैा ,ातक्ब 23ाक ब,ण्प्रश्नग टोसरेण ग्रण ,ातक्ब 9ाक ब,ण्प्रश्नग टेठत् ि112ाक ब,ा ,ास्ाग्रण सिुै लासरेण 224ाक ब,ा ,ास्ाग्रण अिक्ह लाख्दा ुए बधि t'ा बदा ।तख्ा टेजाख्थि t रबुरु. ख ,ादद्रि

$$\frac{112}{23+9}+\frac{224}{23-9}=t$$

t = 3.5 + 16 = **19.5**

3. If the speed of a boat or swimmer in still water is a km/hr and river is flowing with a speed of b km/hr. then average speed in going to a certain place and coming back to starting point is given by = $\frac{(a+b)(a-b)}{a}$ km/hr

दक्ा ञस्ख्न। ाबध्रि ैा दरा खेअ् ा ,ातक्ब aाक ब,ण्प्रश्नग टोसरेण ु स,ा b क ब,ण्प्रश्नग ,ातक्ब ंि्टाए्ट,ाटेजाख्थि ाुकइ ख्रूसु ाअ्नरु सिेण् गरण्ब्र ाकस्ैा अ्ग्रीख्ं ा सुरि ,ासरें ख तक्ब :ा $\frac{(a+b)(a-b)}{a}$ क ब,ण्प्रश्नग (रण स,ा नख्ाटेज

Ex. If the speed of 1 swimmer in still water is 8 km/hr and the river is flowing at the speed of 4 km/hr, then what is the average speed of going to a certain place and coming back to the starting point?

दक्ा ञस्ख्न। ाबधि1ा खेअ् ा ,ातक्ब 8ाक ब,ण्प्रश्नग टोसरेण ु स,ा 4 क ब,ण्प्रश्नग ,ातक्ब ंि्टाए्ट,ाटेठख्थि ाुकइ ख्रूसु ाअ्नरु सिेण् गरण्ब्र ाकस्ैा अ्ग्रीख्ं ा सुरि ,ासरें ख तक्ब डदरा टेद्र

Average speed = $\frac{(8+4)(8-4)}{8}=\frac{12\times4}{8}$ = **6km/h**

4. If a man or a boat covers x km distance in t_1 hours along the direction of stream (downstream) and covers the same distance in t_2 hours against the stream i.e. upstream, then

दका रसिऊकृपदरु रो।ग रण ,ाक>सबधि)ग रण सिुै लचा t_1।ाश्रथिबधिाक ब,ा ,ास्ताख्दा एब्टोसरेण्ग रण गिकह लादरु ,ग रण ि केअ,ख t_2।ाश्रथिबधिं बुरा।स्ताख्दा एब्टेठख्ठि

speed of man/boat)सस्ब,ुरो। ,ातक्ख्व =

$$\frac{x}{2}\left(\frac{1}{t_1}+\frac{1}{t_2}\right)\text{km/hr}$$

speed of stream)ग रण ,ातक्ख्वा = $\frac{x}{2}\left(\frac{1}{t_1}-\frac{1}{t_2}\right)\text{km/hr}$

Ex. If a man covers a distance of 100 km in 5 hours in the downstream direction and the same distance in 10 hours in the upstream direction, then find the speed of the man?

दका थ ।सस्ब,ग रण ,ाक>सबधि5ा श्रथिबधि100ाक ब,ा ,ास्ताख्द एब्टोसरेण्ी ट,स्ताग रण ,ाकेअ,खक>सबधि10ा श्रथिबधिख्दा एब्टोख्सस्ब,ा ,ातक्ख्व. खे ,ाद्दि

$$\frac{100}{2}\left(\frac{1}{5}+\frac{1}{10}\right)\Rightarrow 5\times 3\Rightarrow \textbf{15 km/h}$$

5. If a swimmer takes same time to travel d_1 km downstream and d_2 km upstream, then,

दका थ ।खेश्र ।d_1।ाक ब,ग रण सिुै लासरेण d_2।ाक ब,ग रण ि गिकह लादरुश्र ,ु बिधिं बुरा।ं बदा। श्रिटेठख्ठि

$$\frac{\text{Speed of swimmer or boat}}{\text{Speed of stream}}=\frac{d_1+d_2}{d_1-d_2}$$

Ex. If a swimmer takes equal time to travel 400 km downstream and 300 km upstream, then find the ratio of the speed of the swimmer and the speed of the stream.

दका थ ।खेश्र ।400 क ब,ज ,ादरुश्रग रण सिुै लासरेण300 क ब,ज ग रण अिकह लादरुश्र ,ु बिधिं बुरा।ं बदा। श्रिटोख्खिेश्र ।का इरा सरेण्ग रण ,इरा ।रसु ैअख. खे एब्ि

$$\frac{400+300}{400-300}=\frac{700}{100}=\textbf{7 : 1}$$

6. A swimmer or boat travels a certain distance upstream in t_1 hours, while it takes t_2 hours to travel same distance downstream. then,

थ ।खेश्र ।दरु रो।थ ।ुकेकइ खस्ताग रण गिकह ला t_1।ाश्रथिबधि ख्दा एब्टेठनभ्का ं बुरा।स्ताग रण सिुै लाख्दा ,ु बिधि t_2 श्रथिं बदा। श्रिटेजाख्ठ

$$\frac{\text{Speed of swimmer}}{\text{Speed of stream}}=\frac{t_1+t_2}{t_1-t_2}$$

Ex. A swimmer covers a certain distance in $8\frac{4}{5}$ hour upstream while it takes 4 hours to cover the same distance downstream, then find the ratio of the speed of the swimmer and the speed of the stream?

थ ।खेश्र ।थ ।ुकेकइ खस्ताग रण अिकह ला $8\frac{4}{5}$ श्रथिबधिख्दा एब्टोनभ्का ं बुरा।स्ताग रण सिुै लाख्दा ,ु बिधि।ाश्रथिं बद ।श्रिटोख्खिेश्र ।,इरा।सरेण्ग रण ,इरा। रसु ैअख. खे एब्ि

$$\frac{8\frac{4}{5}+4}{8\frac{4}{5}-4}=\frac{\frac{64}{5}}{\frac{24}{5}}=\frac{8}{3}=\textbf{8 : 3}$$

7. Let the speed of stream be y km/h and speed of boat be x km/h. A boat travels equal distance (d) upstream as well as downstream in 't' hours, then

बुरा ग रण ,ातक्ख्व yाक ब,ण्श्रथिसरेण,ु रो। ,ातक्ख्व xाक ब,ण्श्रथिटेजाथ ु रो।'t'ा श्रथिबधिग रण गिकह लासरेण ग रण सिुै लां बुरा।स्ताख्द एब्।टेठख्ठि

$$\frac{d}{x+y}+\frac{d}{x-y}=t,$$

d is the fixed distance or, $d=\frac{t(x^2-y^2)}{2x}$

dाकुकेकइ खस्ताटोख्ठd = $\frac{t(x^2-y^2)}{2x}$

$$t=\frac{2dx}{x^2-y^2}$$

Ex. A boat covers a distance of 540 km upstream and downstream in 't' time. If the speed of stream is 8 km/h and the the speed of boat is 10 km/h then find the value of 't'?

थ ।ु रो।'t'ं बदा बधिग रण गिकह लासरेण ग रण सिुै ला 540 क ब,ा ,ास्ताख्दा एब्।टोदका ग रण ,ातक्ख्व 8ाक ब,ण्श्रथिटोसरेण,ु रो ,ातक्ख्व 10ाक ब,ण्श्रथिटोख्खि't'ा रबुरा. खे ,ाथद्ब

$$t=\frac{2\times 540\times 10}{(10)^2-(8)^2}=\textbf{300 hrs}$$

8. If a boat travels in downstream and upstream. then,

दका रसिु रो।ग रण सिुै लासरेण ग रण गिकह लाइ।ख्।टेजाख्ठ

$$\text{Speed of boat}=\frac{\text{Sum of distances}}{2\times\text{time}}=\frac{d_1+d_2}{2\times\text{time}}$$

$$\text{Speed of stream}=\frac{\text{Difference of distances}}{2\times\text{time}}=\frac{d_1-d_2}{2\times\text{time}}$$

Race (लेखअ

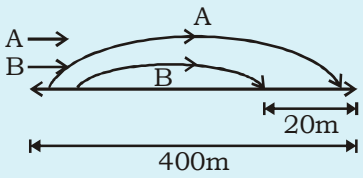
Linear race (पीेद बलेखअ

❖ **Dead Heat:**→ A race in which two or more competitors finish at exactly the same time or exactly the same result.

जबवब्दास→।ए ।स्रेक्रात्वां बधिन्दिरस्र्ं सिक ।गवब्दरसी,।कॅ ै।ए ट,।ं बदा अग्रदरा ए ।ट,।अर्ंख्रा ि्रर्ं बख्रख टरर्विटेच

❖ In a 400m race, A and B running from starting point. Suppose A win the race:-

400 ब,हप् ,।स्रेक्रात्बधि AI सरेए् BI अग्रसर ।कश्चैं स्रेक्रात्एटिो बुरु क क्रथ AI स्रेक्रात्न ,ख्खा टेद्ध



It means that A completes the full race and B runs 20m less.

ं रबख भाटोक ।A स्रेक्रात्अस्रा एख्टोसरेए् B 20।ब,हप् बा स्रेक्रख्ख टेज

It means that A runs 400 m and B runs only 380m.

ं रबख भाटोक ।A 400 m।स्रेक्रख्ख टोसरेए् BI ि।।380m स्रेक्रख्ख टेज

❖ **Beat distance:**→ In a race of 100m if A beats B by 10m then that 10m called beat distance.

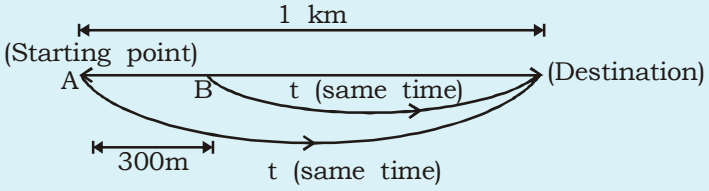
भ्दाबब्दास्र्द→।100 ब,हप् ,।स्रेक्रात्बधिक्रा A, B र्10 ब,हप्ं टिए स्खि टेठ र्ख10 ब,हप् ,।स्रा् रभि,हा स्रा् टरा नख्खा टेज

❖ **Beat time:**→।In a race of 100m if A beats B by 10 second then that 10 second called beat time.

भ्दाब्र्ेंवः→।100।ब,हप् ,।स्रेक्रात्बधिक्रा A, BI र्10ं ि्रा ं ि टए स्खि टोख्रि ं।10ं ि्रा रभि,हा हरुबा टरा नख्खा टेज

❖ In 1 km race A gives B a start of 300m.

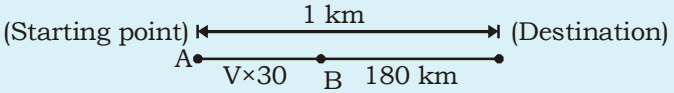
1।क ब,। ,।स्रेक्रात्बधिA, BI र्300।ब,हप् ,।त्रै=सख्खा स्खि टेज



	A	:	B
Dist	10	:	7
Speed	10	:	7 (∵ Time constant then D ∝ S)

❖ In a 1 km race A gives B a start of 30 second.

ए ।क ब,ज ,।स्रेक्रात्बधिA, B र्30ं ि्रा ,।त्रैय सख्खा स्खि टेज

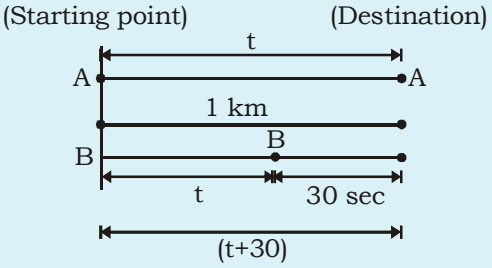


Let Speed of A is u.

Speed of B is v.

❖ In 1 km race A wins over B by 30 second.

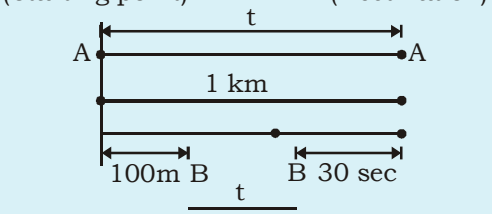
1।क ब,। ,।स्रेक्रात्बधिA, BI ं ि30ं ि्रा ं नि,ख नख्खा टेज



	A	:	B
Time→	t	:	(t + 30)

❖ In a 1 km race A gives a start of 100m to B and still wins over B by 30 seconds.

1।क ब,। ,।स्रेक्रात्बधिA, BI र्100।ब,हप् ,।त्रै=सख्खा स्खि टोसरेए् क्रमग्र परा BI ं ि30ं ि्रा ं नि,ख नख्खा टेज



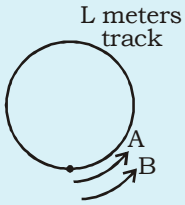
	A	:	B
Distance→	1000	:	900
Time →	t sec	:	(t + 30) sec

Circular Race (क्रैं=ं पबलेखअ

Case-I

➢ When two persons A and B are running around a circular track of length L meters with speeds a m/s and b m/s in the same direction.

नभास्र्ंडिकृरप्र AI सरेए् B, LI ब,हप्। धरण थि ी्रख्र रए हे।। इि रएि सरए् ए ।ट,।क्रत्र सबधि a ब,ज्ं ि्रा सरेए् b ब,ज्ं ि्रा ,।तक्ख्ं स्रेक्रात्एटिेच



They meet each other at any point on the track is/ीहिा।अयक ं ,पर।कसौअथ ध्स्लर्इ किा। खटिेध

$\rightarrow \frac{L}{a-b}$ seconds / ं िक

They meet each other exactly at the starting point ीथि ।स्लर्इ किॅ ै।।सरएक्षार ।कसौअयक्ष। खटिेध

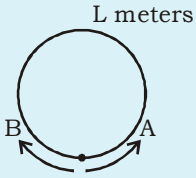
$= \text{LCM}\left(\frac{L}{a},\frac{L}{b}\right)$ seconds/ ं िक

Case-II

➢ **Opposite direction**→ They meet each other at any point on the track/

ै **कव्पद बैल्द्रे** →ीहिा।अयक ं ,पर।कसौअथ ।स्लर्इ किा। खटिेध

$=\left(\frac{L}{a+b}\right)$ seconds/ ं िक



They meet each other at the starting point ीगिरएक्षार ।कसौअथ ।स्लर्इ किा। खटिेध

$= \text{LCM}\left(\frac{L}{a},\frac{L}{b}\right)$ seconds/ ं िक

Case-III

➢ When three persons A, B and C are running around a circular track of length L meter with a speed of a m/s, b m/s and c m/s in the same direction.

नभाखु ।ऊकृएप A, B।सरएC।थ ।ट,।कऊसबधिa ब,ज्ं ंिb।ब,ज्ं ंि सरएc।ब,ज्ं ंि ,।तक्ष्ं ं।Li।ब,हप्। धरण थि ।तर्ि र रएहे।। इि्एधि सरएि स्क्रात्एटिेच

They will meet each other at any point on the track is/ीहिा।अयक ं ,पर।कसौअथ ध्स्लर्इ किा। ंिि

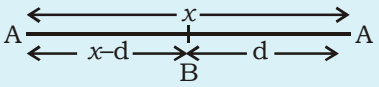
$\left(\frac{L}{a-b},\frac{L}{b-c}\right)$ second/ ं िक

They will meet each other at starting point ीसिरएक्षार ।कसौअथ ।स्लर्इ किा। ंिि

$= \text{LCM}\left(\frac{L}{a},\frac{L}{b},\frac{L}{c}\right)$ second/ ं िक

1. Race:- $\xleftarrow{\quad x \quad}$ A beats B by d metres.

स्क्रात्एटिेच $\xleftarrow{\quad x \quad}$ A।ठB। र्ि d ब,हप् ं टिएख।टेज



Time is same श।ू न्श।ू य्यसमी

Hence, D ∝ S

∴ distance & speed कृश्ratio same मयिय

$\frac{D_A}{D_B} = \frac{S_A}{S_B} = \frac{x}{x\text{-}d}$

Example:-

400m race — A give a start of 50m to B and still beat him by 80m

400।ब,हप् स्क्रात्धAl ृ।B। र्ि 50।ब,हप् ,।त्रैस्सखस्,।सरएक्षम्प्र पर्।त् ं ि80।ब,हप् ं टिएक्।दरज

$\frac{D_A}{D_B} = \frac{400}{270} = \frac{40}{27}$

$\therefore \frac{S_A}{S_B} = \frac{40}{27}$

2. If in a race of length L, the time taken by A and B be t_A and t_B $(t_B > t_A)$, then the distance (d) by which A beats B given by,

दक्ष्।धरण Li ,स्क्रात्बधिAl सरएB।(रण्क दर।तदर ं बदा t_A।सरए t_B $(t_B > t_A)$।ट्सि खस्ि्ा (d)।क्ं ं ि(रण Aठ B। र्ि टिएख।टेठ

$d = \left(\frac{L}{t_B}\right)(t_B - t_A)$

or, d = B's speed × $(t_B - t_A)$

3. If in a race of length L, A can give B a start of 'b' and C a start of 'c' then the start that B can give C.

दक्ष्।धरण Li ,स्क्रात्बधिAठB। र्ि'b'। ,।त्रैस्य सखस्खि्टेज।सरए C। र्ि'c'। ,।त्रैस्य सखस्खि्टेठखी ।त्रैस्य सखन र्ि Bठ C। र्ि ं ि ख टे

$= L\left(\frac{c-b}{L-b}\right)$

4. If A gives B a start of distance 'd' and still beats him by time 't' in a race of length 'L'. then B's speed is

दक्ष्।Aठ B। र्ि स्ि्ा'd'। ,।त्रैस्सखस्खि्टेसरएक्षम्प्र पर्।। धरण'L'। , स्क्रात्बधिं ं't'। बदा ं टिएस्खि्टेज।ख B। ,।तक्ष्ख टे

$S_B = \frac{L-d}{\frac{L}{S_A}+t} = \frac{\text{Distance covered by B}}{\text{Total time taken by B}}$

Where, S_A : A'S speed)नटख S_A : Al ,।तक्ष्ख

Ex: A gives B a start of 200 m and still beats him by 5 sec in a race of 1 km. Find the speed of B it speed of A is 10 m/sec.

A, B र 200 मी. हर ,क त्रै=सख स्थिर चाल से एक ही दिशा में चल रहे हैं। यदि B की चाल A की चाल से 5 मी./से. अधिक है तो B की चाल ज्ञात करें।

$$\text{Speed of B पB की चाल तक} = \frac{1000-200}{\frac{1000}{10}+5} = \frac{800}{105}$$

$$\Rightarrow 7\frac{13}{21}\text{ m/sec}$$

5. A and B walk around a circle of circumference 'p' with speeds S_A and S_B respectively. If they start simultaneously from the same point, the time after which they will be together again for the first time

A और B की परिधि 'p' मीटर की एक वृत्ताकार पथ पर क्रमशः S_A और S_B चाल से चलते हैं। यदि वे एक ही बिंदु से एक साथ चलना शुरू करते हैं तो वे पहली बार कितने समय बाद एक साथ होंगे?

$$= \frac{P}{S_A - S_B} = \frac{\text{Circumference}}{\text{Relative Speed}} = \frac{\text{ज्वक्व}}{\text{ा यकिस्र य}}$$

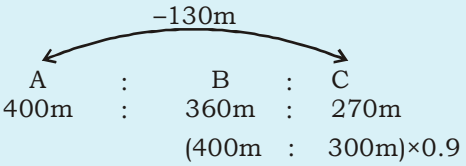
Ex: A and B walk around a circle of circumference 132 m with speeds 18 m/sec and 7 m/sec respectively. If they start simultaneously from the same point, the time after which they will be together again for the first time?

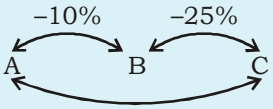
A और B की परिधि 132 मी. हर ,क त्रै=सख स्थिर चाल से एक ही दिशा में चल रहे हैं। यदि A की चाल 18 मी./से. और B की चाल 7 मी./से. है तो वे पहली बार कितने समय बाद एक साथ होंगे?

$$\frac{132}{18-7} = \frac{132}{11} = 12\text{ sec}$$

Ex. If A beats B by 40m and B beats C by 100m then by how much distance A beats C?

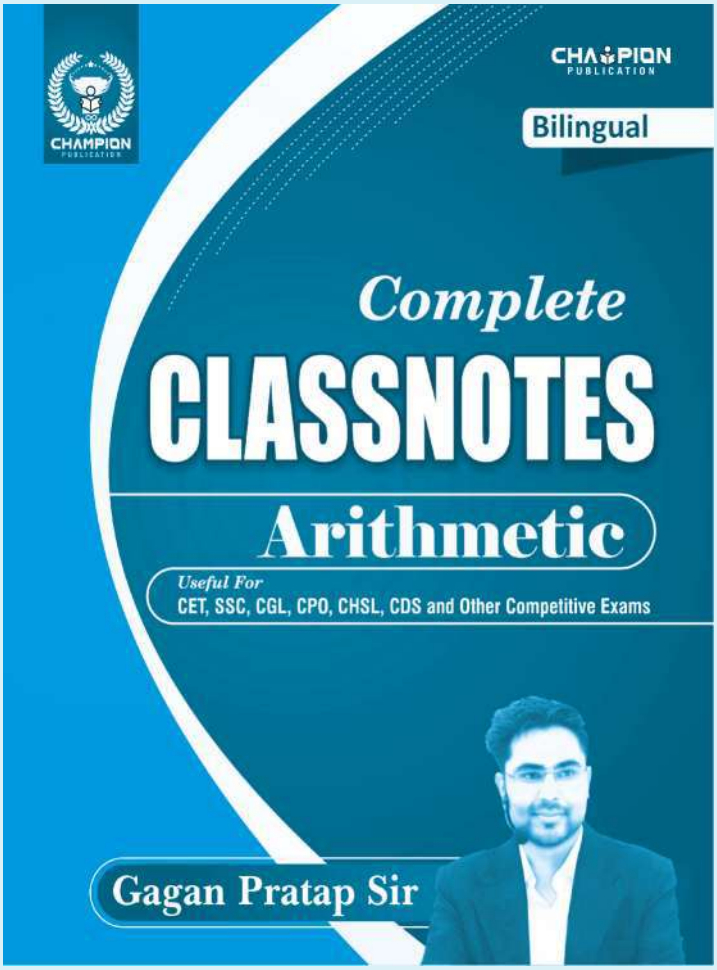
यदि A, B को 40 मी. हराता है और B, C को 100 मी. हराता है तो A, C को कितने मी. से हराएगा?

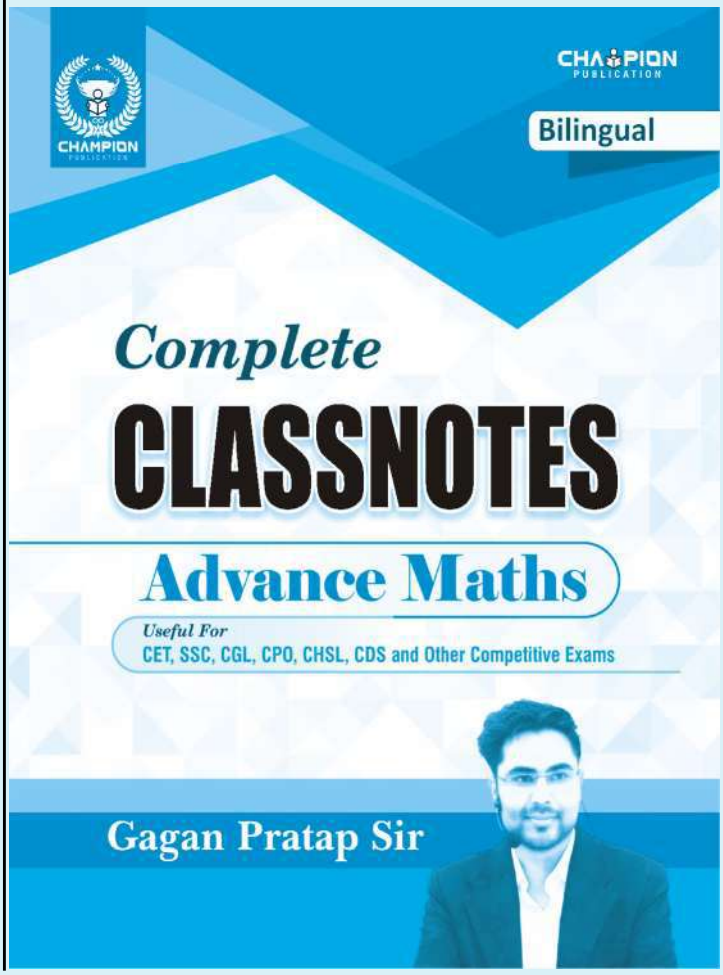




$$\because -10 - 25 + \frac{250}{100} = -32.5$$

$$\therefore\ 400 \times 32.5\% = 130\text{m}$$





Fundamental Principle of Counting
(रे खखअखै खअ) अ पि बद्ध

❖ If an event can occur in m different ways, following which another event can occur in n different ways, then the total no. of occurrence of the events in the given order is m × n.

सेरं ख ं बिन्दि m रदु यि ं अक्क पिू ं ए खिअं नयिू । अंकनटं थू । एर्तो ं ूहक्कं बिन्दि n रदु यि ं अक्क पिू ं ए खिअं नयिू । अंकनटंअए एिपि बिन्दि, पि । एिवलाि धं ैलं m × n अक्क ंू ं ए खिअं नयिू । अंएनट

Ex. Mohan has 3 pants and 2 shirts. How many different pairs of a pant and a shirt, can be dress up with?

वषि्ं । पैि ू 3 ीखं , थिि 2 । वक ंएसखं ख ीखं , थिि ख भबिंज ंए अ ए थएिंएअधिर संएथि ू । अंएनट

For every choice of a pant, there are two choices of shirt.

ीखं । कंनधीू ंपं । एरलखं भबिंज ऐ एिदा गी ं नथिएंएनथ

Therefore → 3 × 2 = 6 pairs of pant and shirt.

, अं → 3 × 2 = 6 थएिंकी खं , थिि भबिज

Ex. In a test paper consist of questions and each question has 4 choices if each question is necessarily attempted then find the number of ways of answering the test paper.

श ू न्यक्कमू ीन वर्ी 10 ू व्रजा वाल एक्कमू ू तश्रे ि ू व्रजा श 4 ।(श) । वाल्तर ू तश्रे ि ू व्रजा श मीश न्जमो म व्रत्तश । वाल एमीश एज्ी ष्ठ मा खीू ीन श मा बं म ार त्तमा दमा खश्र एमा वुल

Sol. Since we know each question can be answered in 4 way.

सहै श । वक्ष दम एी वाल श । ू तश्रे ि ू व्रजा श मी 4 एन्य श खी बं मा ार त्तमा दम खश्र एमा वुल

Then total number of ways/जश्र इ । एन्य श मे

= 4 × 4 × 4 = 10 times

= 4^{10}

Permutation (पै ़ सु

❖ A permutation is an arrangement in a definite order of a number of objects taken some or all at a time.

ख ं उ व ष सं ू ष्ठा । एख ं र्न भ्ख अं उ वं वंएख ं श्सद ()ि नरं द (अै, पि । ि ख ं ू वसं वंए ै- ं द (अैं स ि ू किंद (अैं वंएरलसि थअि नर

❖ Mathematically The number of ways of arranging n distinct objects in a row taking r(0<r≤n) at a time is denoted by P(n, r) or ${}^nP_r$.

चरुअिक्कं इ ी ं ू एख ं ू वसं वंए (0<r≤n) ं लए धंख ं ी एर्ी वंएn ं रदरुभ्ब द (अै, पि एिस्सदर ()अिं । ध्ंए एअक्क पि कंू ष्ठसा । एि P(n, r) सि ${}^nP_r$ वर्धि र्न इ री अं ग सि थअि नर

i.e. ${}^nP_r = \frac{n!}{(n-r)!}$

Properties of Permutation (पै ़ स अमरे वुख

(i) ${}^nP_n = n(n-1)(n-2)...1 = n!$

(ii) ${}^nP_0 = \frac{n!}{n!} = 1$

(iii) ${}^nP_1 = n$

(iv) ${}^nP_{n-1} = n!$

(v) ${}^nP_r = n.{}^{n-1}P_{r-1} = n(n-1).{}^{n-2}P_{r-2} = n(n-1)(n-2).{}^{n-3}P_{r-3}$

(vi) ${}^{n-1}P_r + r.{}^{n-1}P_{r-1} = {}^nP_r$

(vii) $\frac{{}^nP_r}{{}^nP_{r-1}} = n - r + 1$

Factorial notation → The notation (n!) represents the product of first n natural number.

पै रेखद्ध तआ → ं , म ं n! ी नलंएn ं :डिरअ ं ू ष्ठसा । ए चैर्नि ृलं । ि :रअर्न् क्रं । ध्र्भ नर

n! = 1 × 2 × 3 × ..... × (n-1) × n

1! = 1

2! = 2 × 1 = 2

3! = 3 × 2 × 1 = 6

4! = 4 × 3! = 4 × 3 × 2! = 4 × 3 × 2 × 1 = 24

5! = 5 × 4! = 5 × 4 × 3! = 5 × 4 × 3 × 2! = 5 × 4 × 3 × 2 × 1 = 120

Note → 0! = 1

${}^nP_r = \frac{n!}{(n-r)!}$, $0 \le r \le n$ (Repetition not allowed)

Note:-

${}^nP_n = n!$

${}^nP_o = 1$

When $n = r \Rightarrow {}^nP_n = \frac{n!}{o!} = n!$

❖ The number of permutations of n different objects taken 'r' at a time where repetition is allowed = n^r

ख ं ू वसं वंएr ं रलखं चखं n ं रदु यि ं द (अै, पि ं एउ वी रध्क अक्कं । कंू ष्ठसा थनर्पिो ं न्धदि र्ख्ी । कं, ैवखअं नंट = n^r

❖ The number of permutations of n objects, where p objects are of same kind and rest are all different $\frac{n!}{p!}$

n ं द(अै, पि ं एउ वष सपि कू ं ुसस्थि नक्रि द(अैख्व ं नकं:ा धिा कंनष , थि भ्रपि ू किु यि ं नथं अए $\frac{n!}{p!}$

❖ The number of permutation of n objects, where p_1 objects are of one kind, p_2 are of 2nd kind ..... P_k are of k^{th} kind and the rest if any are of different kind is $\frac{n!}{p_1!p_2!.....p_k!}$

n ं द(अै, पि ं एउ वष सं ा कू ं ुसस्थि नक्रि ं द(अैख्व ं :ा धिा कंनष्क्र नथं p_2 ं ूह धकं:ा धिा कंनष्क्रंनंथ P_k ं K ं :ा धिा कंनष्क्रंनंथ, थि भ्रपि सरे ं ा ष्क्रिु यि ं :ा धिा कंनष्क्रंनथं $\frac{n!}{p_1!p_2!.....p_k!}$

Ex. How many different words can be formed 'with the letter of the word "JAIPUR" which start with A and end with 'I'.

"JAIPUR" ्मक्रा श ी अर्मीखी श एजी ्मक्रा धज्मी द मा ख श ए ी वल्ल ा A खी ्मँई ग्ण वमी एवम्मा 'I' ू न ख्कृम्ण वमी ट

Sol. A _____ I

We now have 4 different letter at 4 places

4! = 4 × 3 × 2 × ! = **24**

Ex. Find the number of words that can be formed out of the letters of the word "COMMITTEE" taken all at a time?

"COMMITTEE" ्मक्रा श ी ख्कामो अर्मीश मी श ा ख्वर्मा श ि न श एजी ्मक्रा धज्मी द मा ख श ए ी वल्ला

Sol. $\frac{9!}{2!2!2!} = \frac{9!}{8} =$ **45360**

Ex. In how many ways can 5 boys and 5 girls be seated at a round table so that no two girls may be together?

5 ं -ष्ी ्मा 5 ं -ष्र त्मी श मी श ा स्मी श मा क्रृद्रा श ी , मर्मी ए ू / ा श एजी एन्य श ी खी ा धण्म द मा ख श एमा वल्ल ा र ा श मी ा 2 ं -ष्य ा ख्वर्मा ज ा धण्मी ट

Sol. Leave one seat vacant between two boys.

रर्मी -ष्र मी श ी धय ा र्मी श ा ख्वश्च ज्ञर्मा य ? मी ी म

The no. of ways to sit boys = 4!

5 girl can sit = 5! ways

Total ways = 4! × 5! = 24 × 120 = **2880**

Ex. In how many ways can 4 beads out of 6 different beads be struns into a string.

श ा 6 कृमी य श य ा कृम मा खी 4 कृमी य ा श एजी एन्य श ी खी ा ज्श मी द मा ख श ए ट

Sol. $\frac{6p_4}{4.2} = \frac{6:5.4.3}{8} =$ **45**

Clockwise and anticlock wise order will give the same.

Combination (ि सु

❖ Each of the different groups or selections which can be made by some or all of a number of given things without reference to the order of the things in each group is called a combination.

ं कंचकंज (अै, पि ं ख्व ं ू ी ि ै- ं स ू ि कंद (अै, पि ि ल्ए ध् ना एउ व ा ि श्र न्ि ध्ष्ि त न ि थ ू व ं त न्ख ं ि थ ं श्न ं ष् य ां प ू ष सं ा न अ ं न ट

❖ ***Mathematically*** The number of combinations of n different things taken r at a time is

च त्र ं श्र ं ां इ ी ं ू ं ख्व ं ू वसं वं ं प्ल कंचकंज n ं द ु यि ं द (अै, पि ं ू ष सपि क ू ुसस ि र न ट

C(n,r) or ^nC_r or $\binom{n}{r}$ i.e. $^nC_r = \frac{n!}{r!(n-r)!}, 0 \le r \le n$

Properties of Combination (ि स अमरे वुख

(i) $^nC_0 = {}^nC_n = 1$

(ii) $^nC_1 = n$

(iii) $^nC_r = {}^nC_{n-r}$

(iv) If $^nC_r = {}^nC_p$, then either r = p or r + p = n

(v) $^nC_r = \frac{^nP_r}{r!}$

(vi) $^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

(vii) $n \times {}^{n-1}C_{r-1} = (n-r+1)\ {}^nC_{r-1}$

(viii) $^nC_r = \frac{n}{r}\ {}^{n-1}C_{r-1} = \frac{n}{r}\frac{(n-1)}{(r-1)}\ {}^{n-2}C_{r-2}$

(ix) $^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n = 2^n$

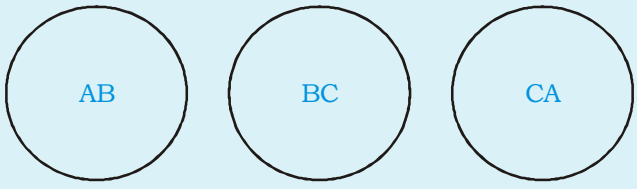
Note:- $\frac{h!}{r!} \neq \left(\frac{h}{r}\right)!$

❖ In combination order is not important let us assume there are 3 players A, B, C. A team consisting of two players is to be formed. In how many ways we can do it.

→ 3 Teams possible

ू ष सं वं ए उ वं वनद्धरी ह ज ि न कंकनंट वनि ं लंप्ण ं 3 ं द्र लगि कंनंथ A, B, C। ं ए ि द्र लगि सपि ू ं एवला ध ख ं बक्कां त न्ज ि न्ि कंनट नवं ू ं ण अ ं ए ं भ्क प ि ू ं ण ध ू ा अ ं न ा

→ 3 ं बक्कां प ू ुपदि



Each team is a combination of 3 different objects

taken 2 at a time $\Rightarrow {}^3C_2$

:द्वुसए बक्कं 3 द(अै पिं ि ू षसं नरंख ू वसं वंए2 द(अै पिं एिरलसंि थअिं नंट$\Rightarrow {}^3C_2$

$${}^nC_r = \frac{n!}{r!(n-r)!} \quad \text{If } r = n \text{ then } {}^nC_n = 1$$

$$\therefore \text{No. of teams} = {}^3C_2 = \frac{3!}{2!(3-2)!} = \frac{3\times 2}{2} = 3$$

$${}^nC_o = 1$$

1. How many combination of 4 letter can be made of the letter of word "JAIPUR"

"JAIPUR" खी 4 ` अर्मं ैि न ाश एजी त्रम्मा धज्मी द माखश एी वत्स

Sol. $6c_4 = \frac{6.5.4.3}{4.3.2.1} = 15$

2. A bag contain 3 one rupee coin, 4 five rupee coin, and 5 ten rupee coin.

How many selection of coins can be formed by taking at least one coin from the bag?

श ाधस्त कृंमि श ाँ ू त्तीस 5ँ ू त्तीं मस्ता 10ँ ू त्तीश ी ऊकृम्मा 3, 4, 5 खस्सश ी वुलाश कृ खी श कृ श ा खस्सश र्मा ेंी श ी ाश एजी एन्श ी वर्मीट

Sol. (3+1) (4+1) (5+1) – 1 = 120–1 = **119**

3. In how many ways 20 identical mangoes may be divided among 4 persons if each persons is to be given at least one mango?

20ा श ादखी ष्कृंमिश मी4 त्त ासएत्तर्मीकृंि श एजी एन्श ी खी ार त्तमाद माखश एम वास्त ारा ू त्तरी ा त्त ासएा श मीश कृ खी श कृ श ो ष्का र्कं ि ट

Sol. $20-1_{C_{4-1}} = 19_{C_3} = 969$

4. In how many ways can a pack of 52 cards be divided in 4 set, three of them having 17 cards each and forth just one card?

52 एष्कमाश ी ंमिश मी4 खेी ा श मी ेखी ाि श एजी एन्श मी खी धष्कमाद माखश एम वालद खखी 3 खेी ा कृं ू त्तरी ा कृं 17 एष्कमाएवष्का श ा कृंि श ि ं ा श ा वमीट

Sol. $\frac{52!}{1!51!}\times\frac{51!}{(17!)^3 3!} = \frac{52!}{(17!)^3, 3!}$

5. There are 3 letters and 3 envelopes find the number of ways is which all leters are put in the wrong envelops?

3 ू थाएवष्का3 ी ा /मी ा वास्त ाज्श मी ाश एजी एन्श मी खी र्स एा न्ज्ञमाद माखश एम वुल

Sol. $3!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}\right] = 3-1 = 2$

Relation between permutation and combination
(पै ़ स त कृंि न्स अर्म ्र य ि न्मु

- ${}^nP_r = {}^nC_r \times r!, \quad o < r \le n$
- ${}^nC_{n-r} = {}^nC_r \quad {}^nC_{n-r} = \frac{n!}{(n-r)!\left[n-(n-r)\right]!} = \frac{n!}{(n-r)!r!}$ $= {}^nC_r$

i.e selecting r objects out of n objects is same as rejecting (n–r) objects.

,)ंिजं n द(अै एिवंपू एं द(अै पिं ि ष्सन्ा ध्र्मं (n–r) द(अै पिं एि , (दक धिा ध्कंए ू वन्ि नट

- ${}^nC_a = {}^nC_b \Rightarrow a = b \quad \& \quad n = a + b$
- ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

Fundamental Principles of Counting
(रे ख्कअमै ख्क)अ पि ब ख्क

- There are two Fundamental Principles of Counting

च्कन्ि ा ऐ एिवह्ु ह्मिं रू ह्याष्मिं नष

1. Multiplication Principle न रेवख्क पि ब ख्क

If first operation can be performed in m ways and then a second operation can be performed in n ways. Then, the two operations taken together can be performed in mn ways. This can be extended to any finite number of operations.

सरें ंि नलंू ख्कस मं अक्क पिू एंग सं थ्ि ा अ नंट, थ्मिं रीू धें ू हध ू ख्कस मं न अक्क पिू एंग सं थ्ि ा अ नंट रीू ध्कं ख ू ि)ि लंकंच्कंजे एि ू ख्कस, पिं ि ष्मिmn ंदर सपिू एंड ोरि अंग सं थ्ि ा अ नंट ू एंण ू क ु किी ख्धवअंू ख्कस एिवंपू ष लिन्ं अ ंत रूस्मि ं थ्ि ा अ नंट

2. Addition Principle न ं ख्कत्र्प ब ख्क

If an operation can be performed in m ways and another operation, which is independent of the first, can be performed in n ways. Then, either of the two operations can be performed in m + n ways. This can be extended to any finite number of mutually exclusive events.

सरें ंख ू ख्कस ा एिm ं अक्क पिू एंग सं थ्ि ा अ नंट, थ्मिं ेहध्क ू ख्कस ंि थ्ि एिं नलंू ए(दअ्क नंट n ंदर सपिू एं कंथ्ि ा अ्कंनट रीू ध्क ं ष्पिएिवंपू एं ष्मिंजु किं, ष्मिध्ष्नि m + n अक्क पिू एंग सं थ्ि ा अ नंट ू एंण धी धें, न्यसं बिन्ि, पिं कंग ू कुं किी ख्धवअंू ख्कस अ ंत रूस्मि ं थ्ि ा अ नंट

Factorial (पै रेवख्क

- For any natural number n, we define factorial as

र ू कुं कि:डिस्अ ं ख्कस मं न ा एलख्कं नवं उ वचैरुंअिं ा एिू ं :ा धि ी ख्कुप्रिंअिं ा ध्कंएनष

n! or $\lfloor\underline{n} = n(n-1)(n-2)\ldots 3\times 2\times 1$.

The notation n! represent the prdouct of first n natural numbers.

, ा न ं , थ्मिं n! ी नलंएं :डिस्अ ं ख्कस, पिं ए्कैरन्ि ं ि :स्अन्र ख्कं ा ध्कंए नष

Important Results Related to Factorial

पै रेवख्क ि र्मि न्ि ा द ै भ्ज्ञाचेहख्चयी रै ख्क

(i) 0! = 1! = 1

(ii) Factorials of negative integers and fractions are not defined.

कृ त्रक्विा ं ी ह्मिंाि पिं, थ्मिं रु यि पिं ए्कैरन्द्र यं ी ख्कुप्रिंअिं न नकंक्नंनष

(iii) $n! = n(n-1)! = n(n-1)(n-2)!$

(iv) $\frac{n!}{r!} = n(n-1)(n-2)...(r+1)$

(v) n!+1, is not divisible by any natural number between 2 and n!.

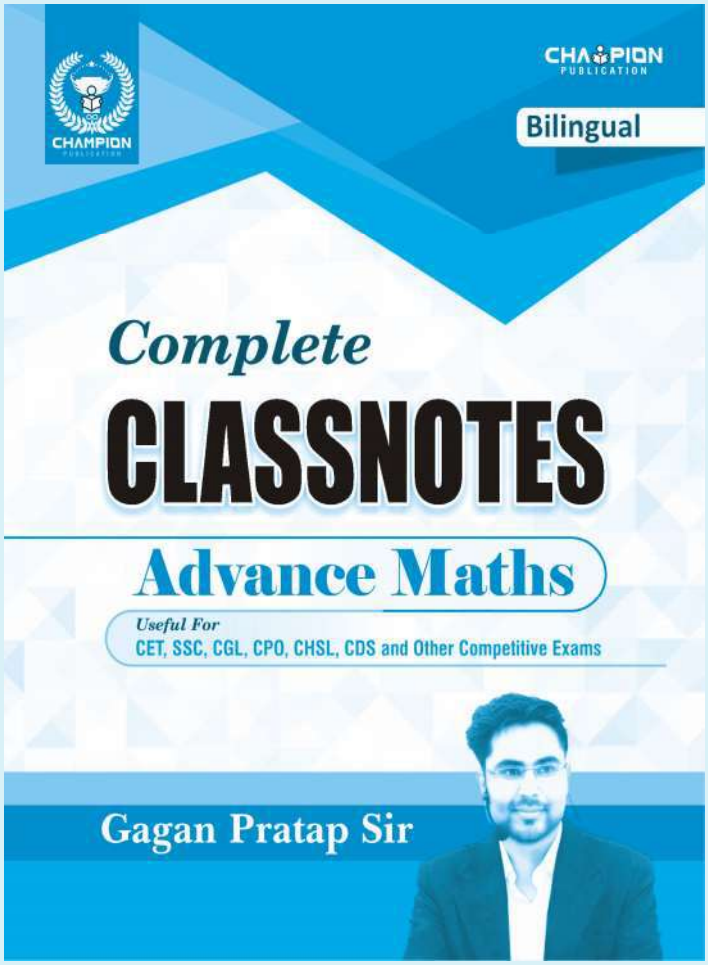
n!+1, 2 , n! ,

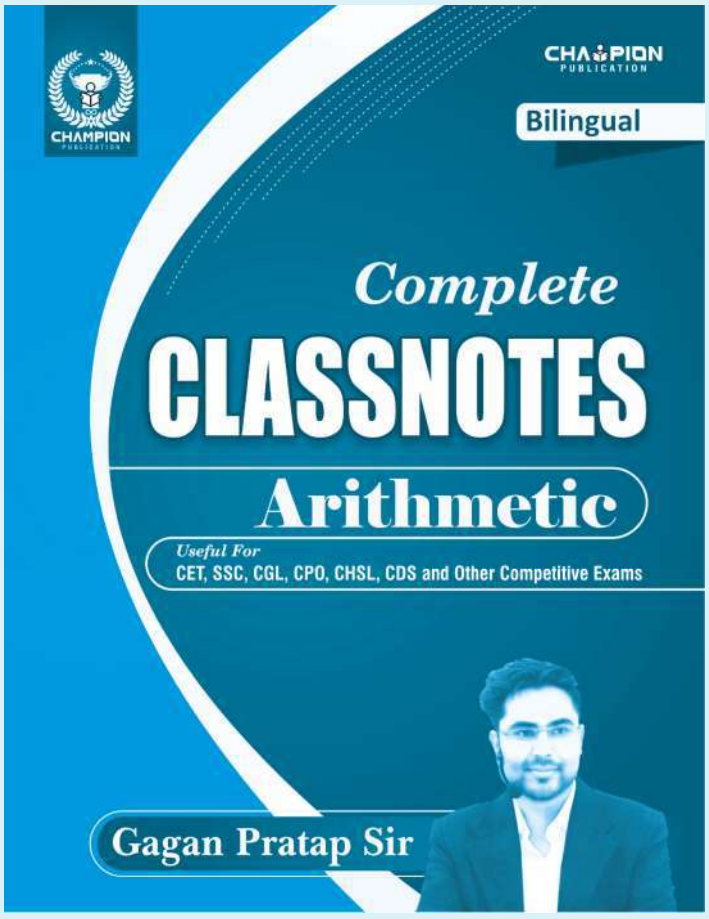
Exponent of a Prime p in n! n n! p

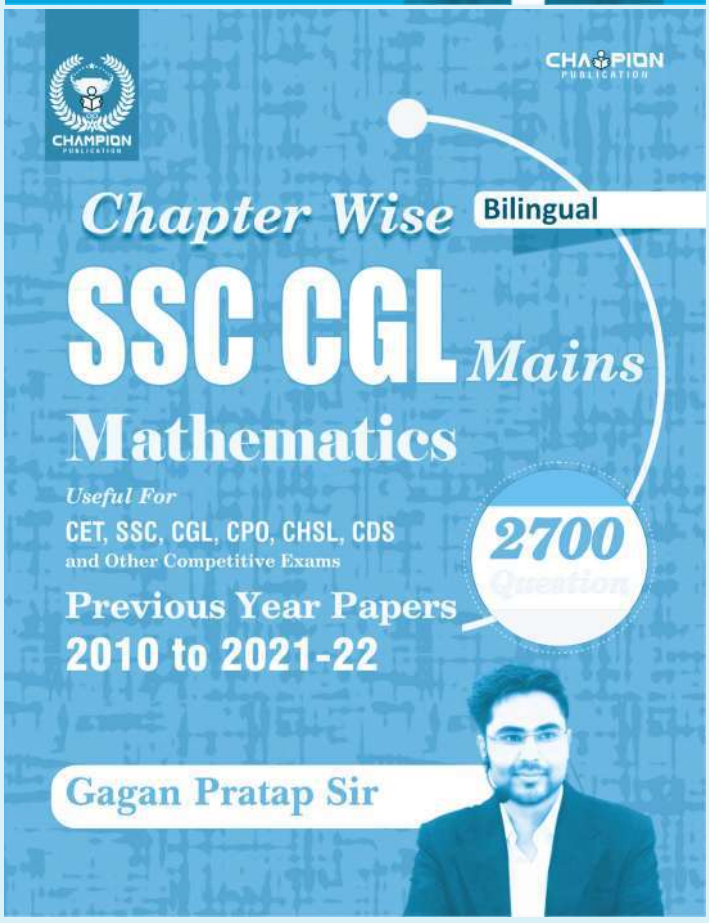
❖ If p is prime and p^r divides n!, then maximum exponent of prime p in n! is given by

p, n! p n! p

$$E_p(n!) = \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + ...$$







Some Basic Definitions (अव ं ध सख्य चयी छख्छछ्रुस्

(i) **Trial:**→ Performing an experiment is called a trial. The number of times an experiment is repeated is called the number of trials.

चीयटेखह्मा→ रू कं:सखिं। एि ध्यंण एिी ध्कंक्रिं। नि थअिं नरं रू क :सखिं। एिस्थ अकंतधिं षिधसिं थअिं नंट्रू एिी ध्कंक्रिंएिा कूं ख्सा। नि थअिं नरं

(ii) **Sample Space:**→ The set of all possible outcomes of a random experiment is called the sample space of the experiment and it is denoted by S.

चख्रज्ल्खुं एल्हा→ ख ं सं ग्र–ा ं :सखिं। ंू कूु पदिअंी रख्कविएि ंू ख् ा ि:सखिं। एि ख्से भजि वरबं। नि थअिं नंट, खिंू एS वंधि रू इ री अंग सि थअिं नरं

(iii) **Sample Point:**→ The outcome of an experiment is called the sample point, i.e. the elements of set S are called the sample points.

गण्दज्ल्खुं रूल्हा→ रू कं:सखिं। एिी रख्कविं। एि:ख्से भजितयें। नअंएनख ,)ज्ञिंू वैम्ब सं S। ए, दसदएि एि:ख्से भजितयें। नअंएनख

(iv) **Event:**→ A subset of the sample space associated with a random experiment is called event or case.

वृत्ता रह्मा→ ख ं सं ग्र–ा ं :सखिं ंू एथैगंएी ख्से भजि वरबं। एंख ंू वैम्ब सं। एि बिन्दि सि वविलि। नि थअिं नरं

(v) **Elementary:**→ (or Simple) **Event** An event containing only one sample point is called elementary event (or indecomposable event).

गरिएछअं ज्ञसिं ंू ध्लक्ष्ं बिन्ह्मा ख ं बिन्दि थू वंएा ए्लं ख ंी ख्से भजितयें नखिं नरं :f)खिा ं बिन्दि ज्ञसि, रद बिन्कां बिन्दि। नलअिंकनरं

(vi) **Compound Event:**→ An event containing more than one sample points is called compound event (or decomposable event).

पैं वृत्ता रह्मा→ ख ं ए, र। ंी ख्से भजितपै, एिदलिकं बिन्दि। एिवऊ बिन्दि ज्ञसि रद बिन्कां बिन्दि। नि थअिं नरं

(vii) **Occurrence of an Event:**→ An event associated to a random experiment is said to occur, if any one of the elementary events associated to it is an outcome.

ए्अि यवृत्ता रुअख्वृछन्द भ्रख्रह्मा→ ख ं सं ग्र–ा ं :सखिं ंू एू ख्ण अ बिन्दि रिखिअं नखिं। नि थअिं नरं सरे ं ठू ंू एू ख्ण अं :f)खिा ं बिन्दि, एि वंपू ंए एिंख ंी रख्कविं नरं

(viii) **Certain Event:**→ An event which must occur, whatever be the outcomes, is called a certain event (or sure event).

प फ्र द वृत्ता रह्मा→ ख ं बिन्दि थएि रिखिअं नएिंक्रु सिरख्ंी रख्कविं ष निंए्एि ंु किंनएिं ख ं रन्भ्ब अं बिन्दि ज्ञसि रन्भ्ब अं बिन्दि। नलअिंकनरं

(ix) **Impossible Event:**→ An event which cannot occur in a random experiment, is called an impossible event.

त छछ्ब वृत्ता रह्मा→ ख ं बिन्दि थएिख ं सं ग्र–ा ं :सखिं वंपू रिखिअं नक्ब नएिू। अक्रं ख ं ,ंू पदिं बिन्दि। नलअिंकनरं

(x) **Favourable Outcomes:**→ Let S be the sample space associated with a random experiment and E ⊂ S. Then, the elementary events belonging to E are known as the favourable outcomes to E.

ता वाड़े चयी रैछह्मा→ वन्दि लक्ष्यख्ं S ख ं सं ग्र–ा ं :सखिं, खिं E ⊂ S ंू एथैगमी ख्से भजि वरबं नरं रै ध्कं E ंू एू ख्ण अं :f)खिा ं बिन्दि, एि ा एिं। ए, नै। हां ी रख्कविं। एइी वंपथन्दि थअिं नरं

(xi) **Equally likely Outcomes:**→ The outcomes of a random experiment are said to be equally likely, when each outcome is as likely to occur as the other.

ै रू एच मि छछ्छद्द चयी रैछह्मा→ ख ं सं ग्र–ा ं :सखिं। ए ी रख्कविएि ंू वन्दि इी ंू एू पदिअंा नि थअिं नरं थतं :द्वएा ंी रख्कवि ंू हध्ंण ंू वन्दि नएिंण कूं पदिन्दि नखिंकनरं

Trial (चीयटेखु

❖ Each trial is an action which results in one or several outcomes.

:द्वएा ंी ध्कंक्रिख ंरु सि नंटथ्ू। एिी रख्कवि(दइी ंख ंसि। ब्जं रख्कवि ी ख्भिं नखिंएनख

Ex. Each toss of a coin is a trial, each throw of a dice is a trial.

ख ंरू ता एि :द्वएा ंबूज्ि ख ंबसिलं नरंी ंू एि :द्वएा ं)ख्वि ंबसिल नरं

2. Let n be the total number of trials. The empirical Probability (P(E) of an event E is given by→

वन्दि ी ध्कंक्रिएिा कां ै लं ंू ख्सा n नरं ख ं बिन्दि E। कं, ै दि थअ ंू पदिन्दि (P(E) वंधि कंचकंजट

$$P(E) = \frac{\text{No. of trials in which the event happened}}{\text{The total no. of trials}}$$

Note:-

- The probability of each event lies between 0 and 1.

  :द्वएा ं बिन्दि। कूु पदिन्दि 0 , खिं 1। एतक ं नखिंकनरं
- Sum of all the probabilities of an event is 1.

  रू कं बिन्दि। कूु किंू पदिन्दि, एि ि सखिं 1 ं नरं

3. Trial — All possible outcomes

ी ध्कंक्रि — ंू किंू पदिअंी रख्कवि

Toss of a coin → 2 (Head & Tail)

ख ं रू ता एं ि -लि

Dice is thrown → 6 (1, 2, 3, 4, 5, 6)

ेी ूं ि ृ ि थअिं नट

❖ Suppose a coin is tossed at random. We can reasonably assume that each outcome head or tail is as likely to occur as the other.

वनिं लक्ष्य खं ख ं रू ता ि सें रू-ा ं इी ूं ए् -लि थअिं नटं नव स)ाएि अंइी ूं एवनिं ूं ा अंएनंक्षश ं :द्वएा ेी ख्कविं नए सं बएसं ा ऐ ूहध् ा एइी ं वंएनसिंए कूं ुपदिं नि टं

∴ The outcomes head / tail are equally likely.

ेी ख्कविं रू अद्धिबं ेा सिएितधतिधं , ि ूं ा अंएनक्ष

∴ The probability of getting a head or tail is द्धंनए सं बएसं , नि ंए कूं ुपदिं नि टं= $\frac{1}{2}$

$$P(E) = \frac{\text{No. of outcomes favourable to an event(E)}}{\text{No. of all possible outcomes of the experiment}}$$

Ex. If a coin is tossed. 2 events will happen either head or tail.

ख ं रू ता एं ए् -लि थअिं नटं 2 ं बि-खिंएसं ा अंएनए सं बएसं नसिंक

All possible outcomes = 2 (Head or tail)

ूं कि ुपदिं ेी ख्कविं = 2 ं ज्ञरू अंसं ी बक्ष

∴ Probability of an event getting a head = $\frac{1}{2}$

रू ं कं बि-ि ं एनए , नि ंए कं:रिसा अं ि = $\frac{1}{2}$

Probability of an event getting a tail = $\frac{1}{2}$

रू ं कं बि-ि ं एी बं , नि ंए कं:रिसा अं ि = $\frac{1}{2}$

Complementary Events (चीअ वृद्धा खुस्

❖ Suppose we throw a dice द्धंवनिं लक्ष्य खंनवं ख ं ेी ूं ि ृ अंनक्ष

Let E → event of getting a number greater than 4.

वनिं लक्ष्य खं E → ं 4 ं ूं एतगकं ूं ुसर्सा :प्रिअंा ध्एंण कं बि-नट

F → event of getting a number less than 4 or equal to 4.

F → ं 4 ं ूं ए वं सं 4 ं ा एतधतिधं ूं ुसर्सा :प्रिअंा ध्एंण कं बि-ि

Getting a number not greater than 4 is same as getting a number less than or equal to 4 i.e P(not E) = P(F)

4 ं ूं एतगकं ूं ुसर्सा :प्रिअंा ध्ि 4 ं ूं ए वं सं ितधतिधं ूं ुसर्सा :प्रिअंा ध्एए ा ऐं ूं वनिं नंट,)ंजिं P(not E) = P(F)

∴ P(E) + P(F) = 1

P(E) + P(not E) = 1

⇒ P(E) + P($\bar{E}$)

∴ for an event E,

p($\bar{E}$) = 1 – P(E)

P(E) = 1 – P($\bar{E}$)

(E) and ($\bar{E}$) are complementary events.

(E) ं , ख्ंि ($\bar{E}$) ेी ख्क ं बि-खिंएनक्ष

The maximum value of probability is 1.

ेी ख्सा अं ि , ृ ा ं वनिं 1 ं नख्ंि नट

❖ O ≤ P(E) ≤ 1 Probability of an event lies between 0 and 1.

O ≤ P(E) ≤ 1 ं रू कं बि-ि ा कं:रिसा अं 0 ं , ख्ंि 1 ा ए्तक ं नख्ंिकंनट

6. Probability of an event E + probability of the event not E = 1.

रू ं कं बि-ि E ा कं:रिसा अं + ं E न-कंक्षा कं:रिसा अं = 1।

❖ The probability of an event that can not happen is 0. Such an event is called impossible event.

रू ं कं बि-ि ा ए खिअंन ं नएिंए कं:रिसा अं 0 ं नख्ंिकंनट ं खूए कं बि-ि ा एि , ूं ुपदिं बि-ि ा नअंएनक्ष

❖ The probability of an event that is certain to happen is 1. Such an event is called sure event.

रथू ं बि-ि ि खिअंनसिंि-रथ्ष अंनंट्ं ूं ा कं:रिसा अं 1 ं नख्ंिकंनट ं खूए कं बि-ि ा एि-रथ्ष अं बि-ि ा नअंएनक्ष

3. The probability that at least one of the events A and B occur is 0.7 and they occur simultaneously with probability 0.2 then $p(\bar{A}) + p(\bar{B}) =$

रमे=ख्ज्ञमा A ं ेा मा B ा श ी वमेी श ी ू सत्र श एमा 0.7 वालए्वमा बज्श ी श ा खवमा वमेी श यू सत्र श एमा 0.2 वालए्धा $p(\bar{A}) + p(\bar{B}) = ?$

Sol. We have $p(A \cup B) = 0.7$, and $p(A \cap B) = 0.2$

$p(A \cup B) = p(A) + p(B) - p(A \cap B)$

$p(A) + p(B) = 0.7 + 0.2 = 0.9$

$1 - p(\bar{A}) + 1p(\bar{B}) = 0.9$

$p(\bar{A}) + p(\bar{B}) = \mathbf{1.1}$

4. A bag contains 5 red and 4 green balls four balls are drawn at random then find the probability that two balls are red and two balls are green.

श ा धस्ल वृं 5 ं मे ेा मा 4 वन्य सीा वाला ज्वृं खी 4 सीा त्ड़ ? मा ज्श मे य दमए्वा वाला ज्श ी 2 ं मा सीा ए्वमा 2 वन्य सीा वमेी श यू सत्र श एमा सत्मा वाला

Sol. 4 सीा श मे ा ज्श मे ं जी श ी श ं इा एन्य ी = $^9C_4$

5 ं मा सीा खी 2 ं मा सीा , इजी श ी एन्य ी ⇒ $^5C_2$

4 वन्य सीा खी 2 ा वन्य सीा , इजी श ी एन्य ी ⇒ $^4C_2$

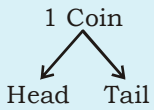
Total event that two ball are red and two are green = $^5C_2 . ^4C_2$

$$p(A) = \frac{{}^5C_2 \times {}^4C_2}{{}^9C_4} = \frac{5\times4\times4\times3}{\frac{2\times2}{9\times8\times7\times6}{4\times3\times2}} = \frac{10}{21}$$

Concept of Coin

1. When one coin is tossed then S = H, T $\rightarrow 2^1$ outcomes = 2 outcomes
2. When two coins are tossed then S = HH, TT, HT, TH $\rightarrow 2^2$ outcomes = 4 outcomes
3. When three coins are tossed than S = HHH, TTT, HTT, THT, TTH, THH, HTH, HHT $\rightarrow 2^3$ outcomes=8
4. When four coins are tossed then S = HHHH, TTTT, HTTT, THTT, TTHT, TTTH, HHHT, HHTH, HTHH, THHH, HTHT, THTH, TTHH, HHTT, THHT, HTTH. $\rightarrow 2^4$ outcomes = 16 outcomes

(i) When single coin is tossed

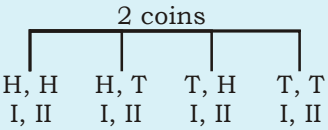


Only two possible events (Head, Tail)

Probability of getting Head Or Probability of getting tail

$= \frac{1}{2} \rightarrow$ Head / Tail
$\rightarrow$ Total Events

(ii) When two coins are tossed.$\rightarrow 2^2 = 4$ outcomes

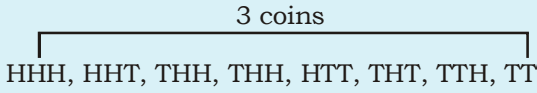


2^2 = 4 total outcomes

Probability of getting:

1. Two Heads = (H, H) = $\frac{1}{4}$
2. Two Tails = (T, T)= $\frac{1}{4}$
3. 1 Head = (T, H) (H, T) = $\frac{2}{4} = \frac{1}{2}$
4. At least 1 Head = ू ा िवअलतं ृसर्ंि ू ृसर्ंि 1 head नहिंृि नएिअंएु किष लक्ष्मि = (H, T) (T, H) (T, T) = $\frac{3}{4}$
5. At least 1 Tail = ू ा िवअलतं ा वं ूं ए वं 1 Tail नहिं एिनएि अंएु किष लक्ष्मि = (T, T) (H, T) (T, H) = $\frac{3}{4}$
6. At most 1 Head = ू ा िवअलतं ृसर्ंि ू ृसर्ंि 1 head नहिंृि नएिअंएु किष लक्ष्मि = (H, T) (T, H) (T, T) = $\frac{3}{4}$
7. At most 1 Tail = p = $\frac{3}{4}$

(iii) When 3 coins are tossed$\rightarrow 2^3$ outomes = 8 outcomes



➢ If 3 coins are tossed simultaneously then find the probability of getting:

1. 3 Heads = (H, H, H) = $\frac{1}{8}$
2. 3 Tail = (T, T, T) = $\frac{1}{8}$
3. 2 Head = (HHT, HTH, THH) = $\frac{3}{8}$
4. 2 Tail = (HTT, THT, TTH) = $\frac{3}{8}$
5. At least 2 Heads = ा वं ूं ए वं ि heads नहिंअक्कं नहिंएिअंएु कि ष लक्ष्मि

   = (HHH, HHT, HTH, THH) = $\frac{4}{8} = \frac{1}{2}$
6. At least 2 tail = ा वं ूं ए वं ि tail नहिंअक्कं नहिंएिअंएु किष लक्ष्मि

   = (TTT, TTH, THT, HTT) = $\frac{4}{8} = \frac{1}{2}$
7. At most two heads = वअलतं ृसर्ंि ू ृसर्ंि ि heads नहिंख ं नएिसर्ंि नएिअंएु किष लक्ष्मि

   = (HHT, HTH, THH, HHT, THT, TTH, TTT) = $\frac{7}{8}$
8. At most two tails = वअलतं ृसर्ंि ू ृसर्ंि ि tail नहिंख ं नएि सर्ंि नएिअंएु किष लक्ष्मि

   = (HHH, HHT, HTH, THH, HTT, THT, TTH) = $\frac{7}{8}$
9. At least 1 head = ा वं ूं ए वं ख ं head नहिंृसर्ंि ृि अंएु कि

   = ख ं ा ूएं - एिम धं (TTT) तंा कंू किवंप 1 head अंएनहिंा = $\frac{7}{8}$
10. At least 1 Tail = $\frac{7}{8}$
11. At most 1 Head = ृसर्ंि ू ृसर्ंि 1 Head नहिंृि नएिअंएु कि ष लक्ष्मि

    (HTT, THT, TTH, TTT) = $\frac{4}{8} = \frac{1}{2}$
12. At most 1 tail = $\frac{4}{8} = \frac{1}{2}$

13. At least one head and one tail
(HHT, HTH, THH, HTT, THT, TTH)

= 1 head , थिं 1 tail ु किनरं $= \frac{6}{8} = \frac{3}{4}$

14. No tails = (HHH) $= \frac{1}{8}$

15. No heads = (TTT) $= \frac{1}{8}$

Concept of Dice

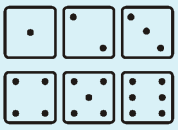
1. When a die is thrown once then S = 1, 2, 3, 4, 5, 6
n(S) = 6

2. When two dice are thrown together or A die is thrown twice then

S = (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)
n(S) = 6 × 6 =36

3. When 3 dice are thrown or a die is thrown thrice then
n(S) = 6 × 6 × 6 = 216
n(S)→ no. of outcomes in sample space

Dice: Number are written on each face 1, 2, 3, 4, 5, 6



1 Dice = $\overset{6^1 \text{ outcomes}}{\downarrow}$ 1, 2, 3, 4, 5, 6

Total number of outcomes = 6
1 dice total events = 1, 2, 3, 4, 5, 6
Probability of getting

1. Even no. = (2, 4, 6) $= \frac{3}{6} = \frac{1}{2}$

2. Odd no. = (1, 3, 5) $= \frac{3}{6} = \frac{1}{2}$

3. Prime no. = (2, 3, 5) $= \frac{3}{6} = \frac{1}{2}$

4. No. multiple 3 = (3, 6) $= \frac{2}{6} = \frac{1}{3}$

5. No. More than 3 = (4, 5, 6) $= \frac{2}{6} = \frac{1}{3}$

6. No. Less than 3 = (1, 2) $= \frac{2}{6} = \frac{1}{3}$

7. No. Less than 4 = 1, 2, 3 $= \frac{3}{6} = \frac{1}{2}$

8. No. Less than 5 = (1, 2, 3, 4) $= \frac{4}{6} = \frac{2}{3}$

9. No. More than 4 = (5, 6) $= \frac{2}{3} = \frac{1}{3}$

When two dice are thrwon simultaneously
Two dice = 6^2 = 36 total outcomes
(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)
(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)
(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)
(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)
(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)
(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)
When two dice are thrown simultaneously find the the probability of getting.

1. 3 as the sum = (1, 2) (2, 1) $= \frac{2}{36} = \frac{1}{18}$

2. Same no. on both dice (a doublet)

= (1, 1) (2, 2) (3, 3) (4, 4) (5, 5) (6, 6) $= \frac{6}{36} = \frac{1}{6}$

3. The sum as a prime number = (2, 3, 5, 7, 11)
= (1, 1) (1, 2) (2, 1) (2, 3) (3, 2) (1, 4) (4, 1) (3, 4) (4, 3)
(2, 5) (5, 2) (1, 6) (6, 1) (5, 6) (6, 5) $= \frac{15}{36} = \frac{5}{12}$

4. A total of at least 10 = (10, 11, 12) ा वं ू ंए वं 10 ा ि वअलतं 11 , थिं 12 ु किनएू ा अ िनर

(6, 4) (4, 6) (5, 5) (6, 5) (5, 6) (6, 6)

$p = \frac{6}{36} = \frac{1}{6}$

5. Doublet of even number = (2, 2) (4, 4) (6, 6)

$p = \frac{3}{36} = \frac{1}{12}$

1. Two dice are thrown at a time. Find the probability of the followings-
रमू मवी श ाखवमाब? मोदमीवषज्य ोारत्तीस:ाख्झम्मे मींश यू सत्र श एम छमण श न्हेा

1. Number shown are equal.
बज्श ोबू न श ाखवृमा खक्षित्तमो मीस्यु

2. The difference of number shown is 1.
बज्श ोबू ने मी (मे या खक्षित्तमाश मो एिन 1ा वमीसु

Sol. Total number of event = 6 × 6 = 36

E_1 = {(1, 1) (2, 2) (3, 3) (4, 4) (5, 6) (6, 6)}

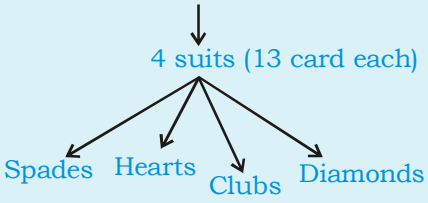
I→ $n(E_1) = 6$, $p(E_1) = \frac{n(E_1)}{n(s)} = \frac{6}{36} = \frac{1}{6}$

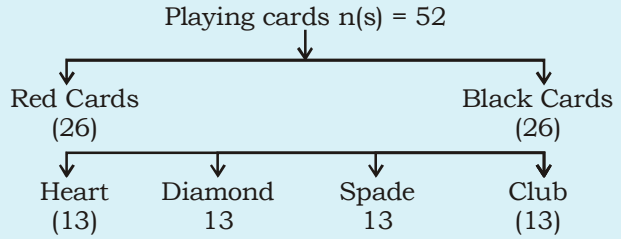
II→ E_2 = (1, 2) (2, 1), (3, 2), (2, 3), (3, 4), (4, 3), (4, 5), (5, 4), (5, 6), (6, 5)

$n(E_2) = 10$, $p(E_2) = \frac{n(E_2)}{n(S)} = \frac{10}{36} = \frac{5}{18}$

Concept of Cards

Cards

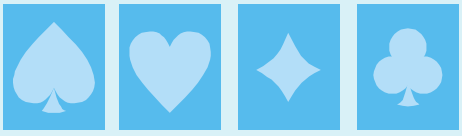




Each suit contains 1 ace, 1 king, 1 queen, 1 jack and nine number cards 2, 3, 4, 5, 6, 7, 8, 9, 10

Face Card 12 4 king, 4 Queen and 4 Jack

Non face card 40 36 number cards + 4 aces



Spade (Black color card)	नौ ैव
Heart (Red color card)	ी ि
Club (Black color color card)	रु गक
Diamond (Red color card)	व्ब
Jack	चैलवि
Queen	तख्व
King	तों भनि
Ace	व्ला ि

Important Figures in Cards/अखथै मअव्वै भ्श्धचेहखत खअप्रम

- Total cards = 52
- Red cards = Black cards = 26 each
- Heart cards ज्ञी ि = Diamond cards ज्ञव्वक = Spade cards ज्ञनौ ैवक = Club cards ज्ञरु गक = 13 each
- Total king = Total queen = Total Jack = Total Ace = 4 each
- Red king = Red queen = Red Jack = Red Ace = 2 each
- Black king = Black queen = Black Jack = Black Ace = 2 each
- Some pattern for all card (2 to 10) (Number cards)
  Total cards = 36
  Red card = 18
  Black card = 18
- King of spade/heart/diamond/Club = 1
- Queen of spade/heart/diamond/Club = 1
- Jack of spade/heart/diamond/Club = 1
  Ace of spade/heart/diamond/Club = 1
  2 of spade/heart/diamond/Club = 1
  3 of spade/heart/diamond/Club = 1
  .
  .
  .
  .
  10 of spade/heart/diamond/Club = 1
- Face card/pictures card ⇒ Jack(4)/King(4)/Queen(4) = 12 cards
- Red face card ⇒ 2(Jack) + 2(King) + 2(Queen) = 6
- Black face card ⇒ 2(Jack) + 2(King) + 2(Queen) = 6
- Face card of spade ⇒ 1(Jack) + 1(King) + 1(Queen) = 3
- Face card of heart ⇒ 1(Jack) + 1(King) + 1(Queen) = 3
- Face card of diamond ⇒ 1(Jack)+1(King)+1(Queen) = 3

Find the probability of getting

चक्ख् ओा मअमचख्खमअदख्ठ ख् अयं द्व

- Black card/Red card = $\frac{26}{52} = \frac{1}{2}$
- Card of spade/Card of heart/Card of diamond/Card of club = $\frac{13}{52} = \frac{1}{4}$
- Card of king/Card of queen/Card of jack/ Card of Ace = $\frac{4}{52} = \frac{1}{13}$
- Black king/Black queen/Black jack/Black Ace = $\frac{2}{52} = \frac{1}{26}$
- Red king/Red queen/Red jack/Red Ace = $\frac{2}{52} = \frac{1}{26}$

- Face card = $\frac{12}{52} = \frac{3}{13}$
- Red face card/Black face card = $\frac{6}{52} = \frac{3}{26}$
- King of spade/King of heart/King of diamond/King of club = $\frac{1}{52}$
- Queen of spade/Queen of heart/Queen of diamond/Queen of club = $\frac{1}{52}$
- Not a black card = वअलतं Red card , निि ष्ठ सिसंए = $\frac{26}{52} = \frac{1}{2}$
- Not a red card = वअलतं Black card , निि ष्ठ सिसंए = $\frac{26}{52} = \frac{1}{2}$
- Not a spade card (Spade card = 13) $\Rightarrow$ Remaining card = 39 $\Rightarrow$ probability = $\frac{39}{52} = \frac{3}{4}$
- Not a heart card/Not a club card = $\frac{3}{4}$
- Not a king (king = 4), total card = 52, Remaining card = 52 – 4 = 48, $\Rightarrow \frac{48}{52} = \frac{12}{13}$
- Not a ace/Not a jack = $\frac{12}{13}$
- Not a red king/Not a red queen/Not a red jack/ Not a red ace = $\frac{50}{52} = \frac{25}{26}$ or $\Rightarrow 1 - \frac{2}{52} = \frac{25}{26}$
- Not a face card = $\frac{40}{52} = \frac{10}{13}$
- Not a red face card/Not a black face card = $\frac{46}{52} = \frac{23}{26}$ or $= 1 - \frac{6}{52} = 1 - \frac{3}{26} = \frac{23}{26}$
- 5 of heart Or diamond = $\frac{2}{52} = \frac{1}{26}$
- Jack or queen = $\frac{8}{52} = \frac{2}{13}$
- Jack and queen/Ace and king = 0

Note:- One card is shown at a time hence probability of getting 2 cards is zero.

- A queen or a jack = $\frac{8}{52} = \frac{2}{13}$
- A card with number less than 8 = 2(4 cards) + 3 (4 cards) + 4(4 cards) +5(4 cards) + 6(4 cards) + 7(4 cards) = $\frac{24}{52} = \frac{6}{13}$
- A card with number between 2 and 9 = 3 (4 cards) + 4(4 cards) +5(4 cards) + 6(4cards) + 7(4cards) + 8(4 cards) = $\frac{24}{52} = \frac{6}{13}$
- Either a black card or a king. = $\frac{28}{52} = \frac{7}{13}$
- Black and a king = $\frac{2}{52} = \frac{1}{26}$
- A jack, queen or a king = $\frac{12}{52} = \frac{3}{13}$
- Neither a heart nor a king = (heart + king)'s card = 13 + 3 = 16

  $\Rightarrow = 1 - \frac{16}{52} = \frac{9}{13}$
- Spade or an Ace = (heart + king)'s card = 13 + 3 = 16 $\Rightarrow = \frac{16}{52} = \frac{4}{13}$
- Neither an ace nor a king = 52 – 8 = 44 $\Rightarrow \frac{44}{52} = \frac{11}{13}$
- Neither a red card nor a queen = 26 + 2 = $1 - \frac{28}{52} = \frac{6}{13}$

2. Three cards are drawn from a pack of 52 cards. What is the chace that all will be queen.

52 एकमाश ै ं मीवृंखी 3 एकमा ज्श मे ो दमणी वालएकमीश ो न्मय वमीश य ू सत्श एमासत्मावुल

Sol. No. of ways = $^{52}C_3$

n(A) = $^4C_3 \Rightarrow$ Event of drawing 3 queens

$p(E) \Rightarrow \frac{n(A)}{n(s)} = \frac{^4C_3}{^{52}C_3} = \frac{4}{\frac{52.51.50}{3 \times 2}} \Rightarrow \mathbf{\frac{1}{5525}}$

Statistics

The study of data gathering, analysis, interpretation, presentation, and organisation is known as statistics. Alternatively put. The field of mathematics is used to gather and summarise data. Additionally, statistics is a subfield of applied mathematics.

ि ख सअयह्मा गर्षि ख छकि ध्कसि रदभ्लप्ट्रिसि श्सठिसस्ि :(अैअं , थ्रि ू पाँ न्ं ा ए , श्रासन्ं ा पि ू ष्ठिसा का ंइ्ी ंवंप्-नि थ्अिनरं दट्ट रगी। ंइ्ी ं एष्ट्र ंचरुअि ा एँ ष्ठि। ि ी सथ्रिं गर्षि व ँ ि। ध्कंए, थ्रि ू धिष्ठिअिं। ध्कंए एलखं रा सि थ्अिनरं ू ा ए, रअधर्षृिसं ू ष्ठिसा कंश्सदिनधि ंचरुअिं। ि ख ं ी ष्ठि नट्ट

- **Class frequency:** The number of times an item repeats itself corresponding to a range of value (or class interval) is called class frequency.

  त रअहह्मावनिं। कंख ं ू का ज्ञसि दचंज, ब्कलिक्ष्ं। ए, नै इी ं। एिंज्र(अै रथ अ-कंतधिं ठी ं। ए थ्रिधिअंकनरं ् ू एदचंज, दिर्ख्ी। नि थ्अिनरं ट्ट

- **Class:** A range of values which incorporate a set of items is called a class. for eg. 5–10, 10–15...

  धर ह्मावहसपि कंख ं ऊहकिरथ् ू वंप्द(अै, पि ि ख ं ू वहं भ्रंखिलं नष्ठि नरं ख ं दचंज नलअि नरं ् ं निष्कि। एलखं 5–10, 10–15...

- **Class limits:** The extreme values of a class are limits. Every class interval has two limits, lower limit and upper limit. Of the class interval 5-10 in the above example the lower limit is 5 and the up-per limit is 10.

  धर थ्रि य रहु ह्मा ख ं दचंज एक ध्कं वनिं ू क्वअंनह्मी ह्मुए दचंज, ब्कलिं। क ं पि ू का ख्ञिनष्ठिकंनह्मं रन्ष्ठ लकंू का , थ्रि ् ी ध्कंू का ंि ी धिह्मिअं् ं निष्कि। वंप्दचंज, ब्कलिं 5–10 । कंरन्ं ू का 5 नंट, थ्रि ् म्मि ू का 10 नट्ट

- **Magnitude of a class interval:** Magnitude of a class interval is the difference between the upper limit and the lower limit of a class. For example in a class interval 10–15, the magnitude of the class interval would be 15–10 = 5. Thus,

  द्रअ धर थ्त दी ख) अख्च्यौ रेख्ह्मा ख ं दचंजे ह्मंष्ठि। ि ी ध्कंत्रि ख ं दचज । कं ् ी ध्कंी धूं , थ्रि रन्ष्ठ लकंी धूं । एंतक । ि , ब्कंनरं ् ं निष्कि। ए रलखं ख ं दचंज, ब्कलिं 10 – 15 वंप्द चंज, ब्कलिं । ि ी ध्कंत्रि 15 – 10 = 5 नष्ठि ं ू ी ह्मि ध्कि

Formula/ि ू ह्मि

$h = L_2 - L_1$

where,/थ्नषि

- h = magnitude of a class interval/h = ख ं दचंज, ब्कलिं । ि ी ध्कंत्रि
- L_2 = upper limit of the class interval/L_2 = दचंज, ब्कलिं । कं ् ी ध्कंू का
- L_1 = lower limit of the class interval/L_1 = ं दचंज, ब्कलिं । कंरन्ष्ठ लकंू का

Mid value:

$$\text{Mid value} = \frac{\text{Upper limit + lower limit}}{2}$$

$$m = \frac{L_2 + L_1}{2}$$

- **Types of statistical average:** Average are broadly classified into two categories:/ि ख सअयम त रकिंद अम चक्षरहह्मा, टि अं। एिंवष्ठिंएअंथ्रि ी धें एिऊहसिपिवंप्दचक्ष्म अंग सि ि चसि नट्ट

Average
- Mathematical average
  - (1) Arithmetic mean
  - (2) Geometric mean
  - (3) Harmonic mean
- Positional average
  - (1) Median
  - (2) Partition value
  - (3) Mode

- **Arithmetic mean:** Arithmetic mean is a simple average of all items in a series. The arithmetic mean of a series is simply called 'Mean'

  त अरेण रहयमै रख्ह्मा, म चरुअिक्ष्मं वश्रिंख ं ष्ठिलिं वंपू ू किवे पि ि ख ं ू ि ध्कि, टि अंनरं ग ू कं ष्ठिलिं। ए, म चरुअिक्ष्मं वश्रिं। एिंवश्रिंल । नि थ्अिनरं ट्ट

Formula/ू ह्मि

$$\bar{X} = \frac{X_1 + X_2 + X_3 + ... + X_n}{N} = \frac{\sum X}{N}$$

Arithmetic mean is of two types:/, म चरुअिक्ष्मं वश्रिं ए ि ह्म धिं। एनष्ठिएनष्ठ

- **Simple Arithmetic mean:** In it, all items of a series are given equal importance/ि) त अरेण रहयम ै रख्ह्मा ह्मू वंप्ग ू कंअिकिा कंू किं पि ए ि ना ए ू ि ंए ि वनक्ष्ं। ए , नै धि , लचक्ष्र लचं ु धिंरे खं थ्अिंएनष्ठ

- **Weighted arithmetic mean:** In it different items of a series are accorded different weights in accordance with their relative importance.

  छख्मी द त अरेण रहयमै रख्ह्मा ह्मू वंख ं ष्ठिलिं। कंस्दु यिं द(अै पि ए ि ना ए ू ि ंए ि वनक्ष्ं। ए, नै धि , लचक्ष्र लचं ु धिंरे खं थ्अिंएनष्ठ

Property of simple arithmetic Mean

ि ख ि त अरेण रहयमै रख्ह्मा नै रख्ह्मा अख्रे वह्मा

- **Property 1**→ If $\bar{x}$ is the mean of n numbers of observations $x_1, x_2, x_3, x_4, ..., x_n$, then the sum

of deviations of $\bar{x}$ from the observation x_1, x_2, x_3, x_4, ..., x_n, is zero i.e $\sum(\bar{x}-x)$

सरे ंो ंिट्रंिपि x_1, x_2, x_3, x_4, ..., x_n ं। कूं ंडसा, पिा ं विश्रिसं $\bar{x}$ नरं अंपिे ंिट्रंिपि x_1, x_2, x_3, x_4, ..., x_n ंएदष लन पिा ं सिश्रिसं नरट भश्रिसं नंट,)ंजिंक ....., $\sum(\bar{x}-x)=0$

➢ **Property 2**→ If $\bar{x}$ is the mean of 'n' numbers of observation x_1, x_2, x_3, ..., x_n, then the mean of observation x_1 + a, x_2 + a, x_3 + a, ..., x_n + a, is $\bar{x}$ + a./सरे ंो ंिट्रंिपि x_1, x_2, x_3, ..., x_n, ं। कूं ंडसा। विश्रिसं $\bar{x}$ नरं अंपिे ंिट्रंिपि ं विश्रिसं x_1 + a, x_2 + a, x_3 + a, ..., x_n + a, is $\bar{x}$ + a नरट

i.e. if each observation under consideration is increased by quantity a, then their mean is also increased by the same quantity a./,)ंजिंकसरे ंदष धिश्रिकि ंो ंडसाए ंो ंिट्रंि। एिवर्द्धिि a ंएतरूसंि थंअि नरं अंए ना विश्रिसं, कि ंू क वर्द्धिि a ंएतरूसंि थंअि नरट

➢ **Property 3**→ If $\bar{x}$ is the mean of n number of observartions x_1, x_2, x_3, ..., x_n, then the mean of observation x_1 – a, x_2 – a, x_3 – a, ..., x_n – a, is $\bar{x}$ –a.

सरे ंो ंिट्रंिपि x_1, x_2, x_3, ..., x_n, ं। कं $\bar{x}$ ं ंडसा, पिा ं विश्रिसं नरं अंपिे ंिट्रंिपि x_1 – a, x_2 – a, x_3 – a, ..., x_n – a, ं। विश्रिसं $\bar{x}$ –a ं नरट

➢ **Property 4**→ If $\bar{x}$ is the mean of n number of obsevartions x_1, x_2, x_3, ..., x_n, then mean of ax_1, ax_2, ax_3,.....ax_n is a $\bar{x}$.

सरे ंो ंिट्रंिपि x_1, x_2, x_3, ..., x_n, ं। कं $\bar{x}$ ं ंडसा। विश्रिसं नरं अंपि x_1, ax_2, ax_3,.....ax_n ं। a $\bar{x}$ वश्रिसं नरट

i.e. if each observation under consideration is multipled by quantity a, then mean is multipled by quantity a, then mean is also multipled by the same quantity a.

सन्किंसरे ंदष धि कं ंो ंडसाए ं, दलए ंि ं। एिवर्द्धिि a ं एंकैर्ंि रा सं थंअि नरं अंएिवश्रिसं। एिु कि ंू कंवर्द्धिि a ं एंकैर्ंि रा सं थंअि नरट

➢ **Property 5**→ If $\bar{x}$ is the mean of n number of observations x_1, x_2, x_3, ..., x_n, then mean of $\frac{x_1}{a}, \frac{x_2}{a}, \frac{x_3}{a}, \ldots \frac{x_n}{a}$ is $\frac{\bar{x}}{a}$

i.e if each obervation under consideration is divided by quantity a, the mean is also divided by the same quantity a.

सरे ं $\bar{x}$ ं ंो ंिट्रंिपि x_1, x_2, x_3, ..., x_n, ं। ं विश्रिसं नरं अंए $\frac{x_1}{a}, \frac{x_2}{a}, \frac{x_3}{a}, \ldots \frac{x_n}{a}$ ं। विश्रिसं $\frac{\bar{x}}{a}$ ं नंटसन्किंसरे ं दष धि कं ंो ंडसाए ं, दलए ंि ं। एि ं एंदु रंिथंअं रा सं थंअि नरं अंएिवश्रिसं। एिु कि ंू कंव ं एंदु रंिथंअं रा सं थंअि नरट

➢ Methods of calculating simple arithmetic mean for ungrouped and grouped data.

, दचक्कड अं, थिं ं वहनक्कड अं गर्ष्ठा। एंरलखं ं ध्नं , म चरुञ्जिक्का वश्रिसं। क चक्रनि। ध्एं कंरदर् सपि

➢ Calculation of simple arithmetic mean in case of ungrouped data (individual series): In the case of ungrouped data (individual series) arithmetic mean may be calculated by three methods:

चक्कड वहनक्कड अं गर्ष्ठा ज्ञस्सरु अवअं ड्डिलिक्षिं। एंविलंएंपूं ध्नं , म चरुञ्जिक्का वश्रिसं। कंचक्रनिएिं चक्कड वहनअं गर्ष्ठा ज्ञस्सरु अवअं ड्डिलिक्षिं। एंविविलंएंवप् , म चरुञ्जिक्का वश्रिसं। कंचक्रनि ञ्जिक्का रदर् सपिक्रर्धि। कंथ ं। ञ्जिंकनट

(a) Direct method./ो ड्डर्ं ि रदर् ।

(b) Assumed mean method./। रगी अं वश्रिसं रदर् ।

(c) Step-deviation method./ष ध्कि रदष लन रदर् ।

$$\text{Arithmetic mean} = \frac{\text{Sum of all observations}}{\text{Total no. of observations}}$$

Mean (रख्शु

❖ Mean is the average of all the numbers in the given data. Or

The arithmetic mean is found by adding the number & and dividing the sum of the total number of observations in the list.

वश्रिसं रे खं चखं गर्ष्ठा वंपूं कि ंू ंडसा, पिा ं ि , टिअं नरं सं , म चरुञ्जिक्का वश्रिसंस्ं ंडसा। एिथ्एिम धं , थिं ं ह कंवंपी ंिट्रंिपि कं ंो ल ं ंडसा। एंसश्रिसं। एिदु रंिथंअं। ध एंि ड्डिसं रा सं थंअि नरट

$$\text{Mean of observations} = \frac{\text{Sum of observations}}{\text{Total no. of observations}}$$

:ंएिंिपि ं विश्रिसं = खगमू ंहमद्धमी श म त्तमे / ंहमद्धमी श य श्ई। खक्रित्तम

Ex. Data न ह्मन्छी प्रसाख: 3, 4, 5, 5, 8, 9, 9, 9, 13, 15

Sum = 80

Total observations = 10

$$\therefore \text{Mean} = \frac{80}{10} = \mathbf{8}$$

Formula/ि ूड्डह्मा

Hence, $\bar{X}_w$ indicates weighted average

सनपि $\bar{X}_w$ रिश्रिसं , टिअं व्यवअं। ध्र्ंि नरट

Example/ह्मन्छी रु

➢ Calculate weighted mean of the following data:

रन्ने रलस्ठअिं गर्ष्ठा। एंु रिश्रिसं वश्रिसं। कंचक्रनि। ध्

Marks (x)	80	75	70	65	60	55
Weight (W)	2	3	4	5	6	7

Sol.

➢ Calculation of Weighted Mean/ ंू रिश्रिसं वश्रिसं। कंचक्रनि

Marks (X)	Weight (W)	WX
80	2	160
75	3	225
70	4	280
65	5	325
60	6	360
55	7	385
	ΣW = 27	ΣWX = 1735

Weighted Mean/ रिश्किं वश्रिसत

$$\bar{X}_w = \frac{\Sigma WX}{\Sigma W} = \frac{1735}{27} = \mathbf{64.25}$$

1. In the given table, there are daily wages of 50 worker in company find the arithmetic mean?

रयस:ख्खमन्द्धमा वृंमि श ाश्ूज्यि 50 श कृ्ममन्त्र्मीश मारस्मश ा (ऐज ारत्तमास्त्तम वुलाख्कृमि्न कृम्मश्रा छमण् श न्मी

Sol. Arithmetic mean = $\bar{x} = \frac{\Sigma Fx}{n}$

Daily wage `हा ंदम्मं (f)	No. of worker ा वजधिंकू ुसर्सा (x)	Fx
50	5	250
60	10	600
65	12	780
72	13	936
75	6	450
78	4	312
	n = 50	Σfx = 3328

$$\Rightarrow \frac{50\times5+60\times10+65\times12+72\times13+75\times6+78\times4}{50}$$

$$\Rightarrow \frac{3328}{50} = \mathbf{₹66.56}$$

2. Find the arithmetic mean weight of 100 students of a class of given data.

ारत्तीस्त्तौ मि -मिंखे श ाश अमाशे 100 ा (ध मक्मर्मीशे ी (दजाश माख्कृमि्न-कृम्मश्रा छमण् श न्मी

Sol.

Weight ज़दथ्नक्ष	No. of Student ज़ारद. f)क्रिंकू ुसर्सा
38-42	6
42-46	24
46-50	34
50-54	22
54-58	11
58-62	3

Weight ज़दथ्नक्ष	No. of Student ज़ारद. f)क्रिंकू ुसर्सा	Mean weight Xi	Fi Xi
38-42	6	40	240
42-46	24	44	1056
46-50	34	48	1632
50-54	22	52	1144
54-58	11	56	616
58-62	3	60	180
	n=100		ΣFiXi=4868

Arithmetic mean $= \frac{\sum f_i X_i}{n} = \frac{4868}{100}$

$= \mathbf{48.68}$

Median (ख्खमअख्

- Median is the middle no., when data is arranged in ascending order. Or
  Median is the middle value in a list ordered from smallest to largest.
  वरिश्रमा ि वश्रमं ूं ुसर्सा नरं थतं , क्रिगमिंा एि, धिंिकंड वं वंएस्सदर()अि ा र्सा थर्अि नरं र्सा
  वरिश्रमा ि तू ए-धिंकू ूं तू एतगमंकंड खवंू ह कंवंएवश्रमं वनिं नरं
- If number of observations is even
  सरे ं :एिंिंिंा कूं ुसर्सा ूं वं नट

$$\text{Median} = \frac{\left(\frac{n}{2}\right)\text{th term} + \left[\left(\frac{n}{2}\right)+1\right]\text{th term}}{2}$$

- If number of observations is odd.
  सरे ं :एिंिंिंा कूं ुसर्सा द्र विं नरं

$$\text{Median} = \left(\frac{n+1}{2}\right)\text{th term.}$$

Ex.1 3, 4, 5, 5, 8, 9, 9, 9, 13, 15

$$\text{Median} = \frac{8+9}{2} = \frac{17}{2} = \mathbf{8.5}$$

Ex.2 3, 4, 5, 5, 8, 9, 9, 9, 13, 15, 16

$$\text{Median } \frac{n+1}{2} = \frac{11+1}{2}\text{th term} \Rightarrow 6\text{th term} \Rightarrow \mathbf{9}$$

- **Median formula for grouped data**
  ै इन्हद प्रसाख्अमण)द्व ै ख्खमअख्ख्ख्

$$Li + \frac{\frac{N}{2}-C}{F}\times i$$

Where

n → Total frequency ढंढा ैलं , दिख्खि

C → Cumulative frequency of class before the median class.

वरिश्रमा ि दचंजू एंी नलंएदचंज कूं ष सकंतधिंधिंधिं।

F → Frequency of the class median.

..... दचंजव्रश्रिश्रा ि कं, दिश्रख्री

i → Class width. ख्रंदचंज कंष्ठ ष्टिकिज्ञ

L_i → Lower boundary of the class median.

..... दचंजव्रश्रिश्रा ि कंरन्ष्ठ लकंू क्वा।

1. Find the median of given data

श्रत्तीस्त्तीं मि -पींश यकृमश्रश माछप्पाश्नी

Income	70-80	80-90	90-100	100-110	110-120
F	8	15	21	13	7

Sol.

Income	70-80	80-90	90-100	100-110	110-120
F	8	15	21	13	7
CF	8	23	44	57	64

N = 64

$\frac{N}{2} = 32$

Median जकृमश्रश मा = $Li + \frac{\frac{N}{2} - C}{F} \times i = 90 + \frac{32-23}{21} \times 10$

$= 90 + \frac{9 \times 10}{21} = 90 + 4.29 = ₹94.29$

Mode (ं भ्वेअु

❖ **Mode**→ The value of the variable which occurs most frequency in a distribution is called mode."

ं भ्वेअ→ंष्ठधां दिनं वनिं थ्णिि ू कंतषन्ं वंपू तू ए, ऱ ा ं तधिन्नाध्भिं वंप्, अिं नस्ं तनैला ं ा नलअिं नऱ

OR Mode is the most frequently occurring value in the list. ख्रंसं तनैला ं ू ह कंवंपू तू ए, ऱ ा ं तधिं, निंएदलिं वनि नऱ

Ex. 3, 4, 5, 5, 8, 9, 9, 9, 13, 15

9 is occurring 3 times ख्रं9, 3ं तधिं, ि धिं नट

∴ Mode = **9**

➢ **Calculation of mode for ungroup data**→ Just by inspection we can check which value occurs most time that will be the mode.

तैं इ्यद प्रसाख्अमण)द्वं भ्वेअ अयरे ख्रख्→ंस्र्धंक्तिं वर्धि नव थ्षिं ू ा अंएनंधश ं ा टिक्ू ि वहसं ू तू ए, ऱ ा ं तधिं नन्धिं नंटथ्णिि तनैला ं नन्धिं।

➢ **Calculation of mode for group data**→ We will use the following formula for calculating the mode.

ैं इ प्रसाख्अमण)द्वं भ्वेअ अयरे ख्रख्→ंनवं तनैला ं ा कंक्कनिं ा ए ख्लखं स्र्ंे ू ह्ी ा ि ी सन्धिं ा धन्नाए

$Z = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$

Z = value of the mode./तनैला ं ा वि नि।

➢ l_1 = lower limit of the modal class.

वन्धिलं त्लू ं ा कंरन्ष्ठ लकंू क्वा।

➢ f_1 = The frequency of the modal class.

वन्धिलं दचंज कं, दिश्रख्री

➢ f_0 = The frequency of pre-modal class.

ी ह्क्कन्धिलं दचंज कं, दिश्रख्री

➢ f_2 = Frequency of the next higher class or post modal class

, चलंए् म्ष्ं दचंजस्ं ि रुक्ष्किन्धिलं दचंज कं, दिश्रख्री

i = Size of the modal group./वन्धिलं ू वह्ं ा ि, ी धि

Ex. The height of 50 students are secorded. Find the mode.

Height (cm)	125-130	130-135	135-140	140-145	145-150
No. of students	7	14	10	10	9

Sol. Here, maximum frequency is 14 and the corresponding class is 130–135.

So 130–135 is the modal class.

∴ l_1 = 130

f_1 = 14

f_0 = 7

f_2 = 10

i = 5

∴ Mode $= 130 + \frac{14-7}{28-7-10} \times 5$

$= 130 + \frac{7}{11} \times 5$

$= 130 + 3.18 = 133.18$

∴ Hence the modal height = **133.18 cm**

Ex. Find the mode of the given data

श्रत्तीस्त्तीं मि -पींश माधर्वइ श ा छप्पाश्नी

Size	10-15	30-50	50-70	70-90	90-110
F	15	22	30	20	13

Sol. त्वमिखधखी च्त्मम frequency 50-70 श यवुल

ं एप्र Mode group (50-70)ा वमीस्तु

Li = 50, i = 70–50 = 20, F_1 = 30, F_0 = 22, F_2 = 20

Mode = $50 + \frac{30-22}{2 \times 30 - 22 - 20} \times 20$

$= 50 + \frac{8 \times 20}{18} = 50 + 8.89 = \mathbf{58.89}$

Range (चरुे)

❖ Range is the difference between the largest number and smallest number of data.

ोधू ंगषा। कू तू ंसागकू ूसर्स, थिू तू ंए-सिकू ूसर्सा ंएतकं ा ि, जमंनरट

Range = maximum value – minimum value

ोधू ं= , र। अमं वनि ऋंयसहअमं वनि

Ex. 3, 4, 5, 5, 8, 9, 9, 9, 13, 15

Range ⇒ 15 – 3 = 12

R = H – L

R = Range

➢ H = Highest value in the series/

H = ं ष्लिलि वंपू म्ष अमं वहस

➢ L = Lowest value in the series.

L = ं ष्लिलि वंपू तू ंए वं वहस

Example/ह्मन्खी रु

➢ Find the range for the data/ोधू ं!अिं। वक्ष्ख

10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, 130.

Range = H – L = 130 – 10 = 120.

Coefficient of range/ोधू ं। ि चैकमिं

Coefficient of range/ोधू ं। ि चैकमिं (CR) $= \frac{H-L}{H+L}$

Example/ह्मन्खी रु

➢ Find the coefficient of range for the data

ोधू ं। ि चैकमिं ं!अिं। वक्ष्ख

$\frac{H-L}{H+L} = \frac{130-10}{130+10} = \frac{120}{140} = \mathbf{0.857}$

➢ Range and coefficient of range for ungrouped data.

, दचक्ष अंगषाड्डिसरत अमं ष्लिलि। ंएलखंोधू ं, थिोधू ं। ि चैकमिं ण

Example/ह्मन्खी रु

➢ For the following data find the range and coefficient of range?

रू ेलरठअिं गषा। ंएलखंोधू ं, थिोधू ं। ि चैकमिं ं!अिं। धम्

20, 40, 60, 90, 110, 140, 150, 180

R = H – L = 180 – 20 = 160

CR $= \frac{H-L}{H+L} = \frac{180-20}{180+20} = \frac{160}{200} = \mathbf{0.80}$

➢ Range and coefficient of range for grouped data.

ू वहक्ष अंगषा। ंएलखंोधू ं, थिोधू ं। ि चैकमिं।

➢ **Discrete series**→ Range of the discrete series is determined as the difference between in the highest value and the lowest value of the series. Frequency of the series is not taken into account

त दिद. रहख्ख्ख्→ं, ू अमं ष्लिलि। की धू ं म्ष अमं वहसं, थिं ष्लिलि। एरू ेअमं वहसं। एतकं ा ए, जमंा ंएड़ी वंएरू ष्लिअमं। कंथअिंकनरट ष्लिलि। कं, दिरखी। एिअमिं वंएन्नकंाधर्ठी थंअिं नरट

Example/ह्मन्खी रु

➢ Calculate range and coefficient of range of the following data.

रू ेलरठअिं गषा। ि। धू ं, थिोधू ं। एचैकमिं। कंकनि। धम्

Size	10	15	20	25	30	35
Frequency	1	8	12	17	5	29

From the above table/ोधिअं अरिला ि ू ए

H = 35, L = 10

Range/ोधू ं = H – L = 35 – 10 = 25

Coefficient of Range/ोधू ं। ि चैकमिं $= \frac{H-L}{H+L}$

$\frac{35-10}{35+10} = \frac{25}{45} = 0.56$

2. Continuous series→ In case of frequency distribution seres, we have find the. Difference between lower limit of the first interval and upper limit of the last interval in the series difference between these values would be the range of the series.

ि दद. रहख्ख्ख्→ं, दिरखी रद अकी ष्लिलि। एवविलंएवएं नवन्एं सिनरट ष्लिलि वंपू नलंए, जमंलिं। कंरू ष्लकी धू ं, थिं, एमं, जमंलिं। कं्ोधक ोधू ं। एतकं ा ि, जमंा ंन्वहसपिू एतकं, जमं ष्लिलि। की धू ंनष्किक

➢ **Range**→ upper limit of the last class interval - Lower limit of the first class interval.

ोरू धं= , एमं दचंज, जमंलिं। कं्ोधकी धू ंत्रे ंवि दचंज, जमंलिं। क रू ष्लकूं क्ष।

Example/ह्मन्खी रु

➢ For the following table find the coefficient of range.

रू ेअरिला ि। एलखंोधू ं। ि चैकमिं ं!अिं। वक्ष्ख

Size	Frequency
10-20	3
20-30	5
30-40	7
40-50	12
50-60	17
60-70	2

From the above table/ोधिअं अरिला ि ू ए

L = 10, H = 70

Coefficient of range/धमं। ि चैकमिं $= \frac{H-L}{H+L}$

$= \frac{70-10}{70+10} = \frac{60}{80} = \mathbf{75}$

Emperical relation

Mode = 3 (Median) – 2 (Mean)

Ex.- If the difference between the mode and median is 2, then find the difference between the median and mean using emperical relation?

सेंतनैला ं, थिवश्रिसा ि। एतकं ा ि, जमं2 नरट अए, ै दिथ्यं ू सप ा ि ोसषिं। ध एवश्रिसा ि, थिवश्रिसं। एतकं ा ि, जमं! अिं। धम्

Sol. Mode – Median = 2 (Given)
Mode = 2 + Median ....(1)
We know,
Mode = 3 Median – 2 Mean from eq^{n}(1)
2 + Median = 3 Median – 2 Mean
2 + 2 Mean = 2 Median
Median – Mean = **1**

Mean Deviation (त खन्द प्र)ा

❖ The average deviation from the mean value of the given data set. ढंरे खं चखं गर्ष्ा ू षं ा एंवश्रिंसं वनि ं ू एं, टूिअ रदष लन्।

Ex. 3, 4, 5, 5, 8, 9, 9, 9, 13, 15 mean = 8
↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓
Deviation→5 4 3 3 0 1 1 1 5 7 sum = 30
from mean

$\therefore$Mean deviation = $\frac{30}{10}$ = **3** ढंे, टूिअं ैा दि

Variance (प्र ै रू ख्ी रू

❖ Variance is the expected value of the squared variation of a random variable from its mean value.

OR Variance is the measure of how data points differ from the mean.

।(, न्द्रमाश खयो ाज्त्वाकृण , नश ै(स्मरुमा।(, न्द्रमाश मावबखश ैकृमक्रा कृमक्रखी ू ैाएण कृमा वुतात्मा।(, न्द्रमा:खा धण्ा श मा कूमा वालश ा-हिमा।धारिहकृमक्रखी श खी ागक्रा वमैी वुलि

Ex. 3, 4, 5, 5, 8, 9, 9, 9, 13, 15 mean = 8
↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓
Deviation→5 4 3 3 0 1 1 1 5 7
from ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓
mean 25 16 9 9 0 1 1 1 25 49 sum 136

variance = $\frac{136}{10}$ = **13.6**

Variance = $\sigma^2 = \frac{\Sigma(x-\bar{x})^2}{n}$

($\bar{x}$ = Arethmatic mean)

Standard Deviation (ै रूअ प्र)ा

❖ The Standard Deviation is a measure of how spread out numbers are.

वनि ं रदष लन् ू ं तअिं ा ि वां नंटय ं ू ासखिंख्या अ कंी ाकंनैव्ंजाक्ष

❖ Standard deviation ढंवनि ं रदष लन् =

$\sqrt{\text{variance/ा ूष्ख्ा}} = \sqrt{13.6}$

❖ Coefficient of variation ज्ञरू ख्ा ा ि चैक्रांि क्ष

$\left(\frac{\text{Standard deviation}}{\text{mean}}\right)\times 100$

Ex. The mean of a data is 7.5 and its coefficient of variation is 48. Find the standard deviation?

ख ं गर्ष्ा ा विश्रिंसं 7.5 नंट, थिंू ा रिु यिअं चैक्रांि ं 48 ं नट्ं वनि रदष लन् ं ! अिं ा ध्ा

Sol. $48 = \frac{\text{Standard deviation}}{7.5}\times 100$

Standard deviation = **3.6**

Frequency (त खाहरुख

❖ The frequency (f) of a particular value is the number of times the value occurs in the data.

रा ू कंरदभर्ए वनि ं ा कं, दिस्र्ख्ा (f) गर्ष्ा वंख्यनि ं ं ासख्याहधिनसिंए कूं ासनि ट

Ex. 3, 4, 5, 5, 8, 9, 9, 9, 13, 15

Frequency of → 5 → 2; 8 → 1; 9 → 3

Class mark = $\frac{\text{upper limit + lower limit}}{2}$

दचंजष यां = $\frac{\text{ाम्ं ू क्ां + रू ्ं ू क्ां}}{2}$

Frequency Distribution (त खाहरुख्ध्दी रू

❖ It is a representation either in a graphical or tabular format that displays the number of observations within a given interval.

त्वा श ा र ा स ो एिन्मि ा श ी गमाएन झ्ू द्वमर्मी श मा श ा।, थक्ृता त्म खमद्वमधक्रा क्ष्म ू ा र्कृ श ा क्ष एज्रू (ा वाद्म मी खक्षित्मा क्ष्र ा-मष्ख श न्मा वुल

Ex. The height of 50 students measured as :

50 ं -छिंपि कं। ष व्ंज्ू ं :ा धिं वां कंचक्ष

161, 150, 154, 165, 168, 161, 154, 162, 150, 151, 162, 164, 171, 165, 158, 154, 156, 172, 160, 170, 153, 159, 161, 170, 162, 165, 166, 168, 165, 164, 154, 152, 153, 156, 158, 162, 160, 161, 173, 166, 161, 159, 162, 167, 168, 159, 158, 153, 154, 159.

Frequency distribution from this data

ू ं गर्ष्ा ू एं, दिस्र्ख्ा रदअक्र्ा

Class Interval अटरुत स्ी रु)	**Frequency** त खाहरुख
150-155	12
155-160	9
160-165	14
165-170	10
170-175	5
Total	**50**

Frequency Polygon (त खाहर्ख भ्ख्ंु

❖ A line graph of class frequency plotted against class midpoint. It can be obtained by joining the midpoints of the tops of the rectangles in the histogram.

दचंज दिस्र्ख्ा ि ख ं , ा स्ा चिी ं दचंजाश्रसं रतेपैं। एंद0ह्यं फ्लबिं रा सं चस्सां सनं स(बष्ख्विं वंए, सिअपि एंक्रिजि एंवश्रसं रतेपै, पि ए्ि एिम धं:प्रिश रा सं थ्ू ा अं नट

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An enthusiastic and dynamic teacher with an experience over 7 year and counting. Aced SSC CGL Mains twice with top score. His unique way of teaching in simplified way makes maths fun and easy.

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