disha
Nurturing Ambitions

SSC MATHEMATICS

Guide

- Illustrations & Examples
- 25 Chapters with explanations
- 2 levels of Exercise
- 5 Practice Sets

Fully Solved with shortcut methods

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PRACTICE SETS

CHAPTER 1

FUNDAMENTALS

'BODMAS' RULE

A given series of calculations or operations is done in a specific order as each letter of BODMAS in order represent.

B → Brackets and order of operation of brackets is ( ), { }, [ ]
O → Of (Calculation is done the same as multiplication)
D → Division
M → Multiplication
A → Addition
S → Subtraction

So, first of all we solve the inner most brackets moving outwards. Then we perform 'of' which means multiplication, then division, addition and subtraction.

- Addition and subtraction can be done together or separately as required.
- Between any two brackets if there is not any sign of addition, subtraction and division it means we have to do multiplication
  $(20 \div 5)\ (7 + 3 \times 2) + 8 = 4\ (7 + 6) + 8$
  $= 4 \times 13 + 8 = 52 + 8 = 60$

BRACKETS

They are used for the grouping of things or entities. The various kind of brackets are:

(i) '–' is known as line (or bar) bracket or vinculum.
(ii) ( ) is known as parenthesis, common bracket or small bracket.
(iii) { } is known as curly bracket, brace or middle bracket.
(iv) [ ] is known as rectangular bracket or big bracket.

The order of eliminating brackets is:

(i) line bracket
(ii) small bracket (i.e., common bracket)
(iii) middle bracket (i.e., curly bracket)
(iv) big bracket (i.e., rectangular bracket)

Illustration 1: Find the value of

$$\left[5 - \left\{6 - (5 - \overline{4 - 3})\right\}\right] \text{ of } \frac{1 + \frac{1}{2}}{1 - \frac{1}{2}} \div \frac{\frac{1}{2} + \frac{1}{3}}{\frac{1}{2} - \frac{1}{3}}$$

Solution: $\left[5 - \left\{6 - (5 - \overline{4 - 3})\right\}\right] \text{ of } \dfrac{1 + \frac{1}{2}}{1 - \frac{1}{2}} \div \dfrac{\frac{1}{2} + \frac{1}{3}}{\frac{1}{2} - \frac{1}{3}}$

$$= [5 - \{6 - (5 - 1)\}] \text{ of } \frac{\frac{3}{2}}{\frac{1}{2}} \sqrt{} \frac{\frac{5}{6}}{\frac{1}{6}}$$

$$= \{5 - (6 - 4)\} \text{ of } \left(\frac{3}{2} \times \frac{2}{1}\right) \div \left(\frac{5}{6} \times \frac{6}{1}\right)$$

$$= (5 - 2) \text{ of } 3 \div 5$$

$$= 3 \text{ of } 3 \div 5 = 3 \times \frac{3}{5} = \frac{9}{5}$$

FACTORIAL

The product of n consecutive natural numbers (or positive integers) from 1 to n is called as the factorial 'n'. Factorial n is denoted by $n!$.

i.e., $n! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \ldots (n - 2)(n - 1)\,n$
$4! = 1 \times 2 \times 3 \times 4 = 4 \times 3 \times 2 \times 1$
$5! = 1 \times 2 \times 3 \times 4 \times 5 = 5 \times 4 \times 3 \times 2 \times 1$
$6! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 = 6 \times 5 \times 4 \times 3 \times 2 \times 1$

Note: $0! = 1$ and $1! = 1$

Properties

(i) $n!$ is always an even number if $n \geq 2$.
(ii) $n!$ always ends with zero if $n \geq 5$.

ROMAN NUMBERS

In this system there are basically seven symbols used to represent the whole Roman number system. The symbols and their respective values are given below.

I = 1, V = 5, X = 10, L = 50,
C = 100, D = 500 and M = 1000

In general, the symbols in the numeral system are read from left to right, starting with the symbol representing the largest value; the same symbol cannot occur continuously more than three times; the value of the numeral is the sum of the values of the symbols.

For example LX VII = 50 + 10 + 5 + 1 + 1 = 67.

An exception to the left to the right reading occurs when a symbol of smaller value is followed immediately by a symbol of greater value, then the smaller value is subtracted from the larger. For example.

CDXL VIII = (500 – 100) + (50 – 10) + 5 + 1 + 1 + 1 = 448.

Illustration 2: **The value of the numeral MCDLXIV is:**

(a) 1666 (b) 664 (c) 1464 (d) 656

Solution: MCDLXIV = 1000 + (500 – 100) + 50 + 10 + (5 – 1) = 1464

Hence (c) is the correct option.

Illustration 3: **Which of the following represents the numeral for 2949**

(a) MMMIXL (b) MMXMIX
(c) MMCMIL (d) MMCMXLIX

Solution: 2949 = 2000 + 900 + 40 + 9
= (1000 + 1000) + (1000 – 100) + (50 – 10) + (10 – 1)
= MMCMXLIX

Hence (d) is the correct option.

IMPORTANT CONVERSION

1 trillion = 10^{12} = 1000000000000
1 billion = 10^9 = 1000000000
1 million = 10^6 = 1000000
1 crore = 10^7 = 100 lakh
10 lakh = 10^6 = 1 million
1 lakh = 10^5 = 100000 = 100 thousand
1 thousand = 10^3 = 1000

ABSOLUTE VALUE OR MODULUS OF A NUMBER

Absolute value of a number is its numerical value irrespective of its sign.

If x be a real number N then | N | indicates the absolute value of N.

Thus | 6 | = 6, | – 6 | = 6, | 0 | = 0, | 1 | = 1, | 3.4 | = 3.4, | – 6.8 | = 6.8, etc.

| – 6 | = 6 can also be written as | – 6 | = – (– 6) = 6. Thus, if x is a negative number, then $|x| = -x$ and if x is non-negative number, then $|x| = x$

Hence $|x| = \begin{cases} x, \text{ if } x \geq 0 \\ -x, \text{ if } x < 0 \end{cases}$

PROPERTIES OF A MODULUS

(i) $|a| = |-a|$ (ii) $|ab| = |a||b|$

(iii) $\left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|}$ (iv) $|a+b| \leq |a| + |b|$

(The sign of equality holds only when the sign of a and b are same)

(v) If $|a| \leq k \Rightarrow -k \leq a \leq k$

(vi) If $|a-b| \leq k \Rightarrow -k \leq a-b \leq k \Rightarrow b-k \leq a \leq b+k$

Illustration 4: **Solution of the equation | x – 2 | = 5 is**

(a) 3, – 7 (b) – 3, 7
(c) 3, 6 (d) None of these

Solution: $|x-2| = 5 \Rightarrow x - 2 = 5$ or $x - 2 = -5$
$\Rightarrow$ $x = 7$ or $x = -3$

Hence (b) is the correct option.

Illustration 5: **The minimum value of the expression | 17x – 8 | – 9 is**

(a) 0 (b) – 9
(c) $\dfrac{8}{17}$ (d) none of these

Solution: The value of expression | 17x – 8 | – 9 is minimum only when | 17x – 8 | is minimum. But the minimum value of | k | is zero.

Hence minimum value of | 17x – 8 | – 9 = 0 – 9 = – 9

Hence (b) is the correct answer.

POWERS OR EXPONENTS

When a number is multiplied by itself, it gives the square of the number. i.e., $a \times a = a^2$ (Example $5 \times 5 = 5^2$)

If the same number is multiplied by itself twice we get the cube of the number i.e., $a \times a \times a = a^3$ (Example $4 \times 4 \times 4 = 4^3$)

In the same way $a \times a \times a \times a \times a = a^5$

and $a \times a \times a \times$ ... upto n times $= a^n$

There are five basic rules of powers which you should know:

If a and b are any two real numbers and m and n are positive integers, then

(i) $a^m \times a^n = a^{m+n}$ (Example: $5^3 \times 5^4 = 5^{3+4} = 5^7$)

(ii) $\dfrac{a^m}{a^n} = a^{m-n}$, if $m > n$ $\left(\text{Example: } \dfrac{6^5}{6^2} = 6^{5-2} = 6^3\right)$

$\dfrac{a^m}{a^n} = \dfrac{1}{a^{n-m}}$, if $m < n$ $\left(\text{Example: } \dfrac{4^3}{4^8} = \dfrac{1}{4^{8-3}} = \dfrac{1}{4^5}\right)$

and $\dfrac{a^m}{a^n} = a^0 = 1$, if $m = n$ $\left(\text{Example: } \dfrac{3^4}{3^4} = 3^{4-4} = 3^0 = 1\right)$

(iii) $(a^m)^n = a^{mn} = (a^n)^m$ (Example: $(6^2)^4 = 6^{2\times4} = 6^8 = (6^4)^2$

(iv) (a) $(ab)^n = a^n \cdot b^n$ (Example: $(6 \times 4)^3 = 6^3 \times 4^3$)

(b) $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$, $b \neq 0$ $\left(\text{Example: } \left(\dfrac{5}{3}\right)^4 = \dfrac{5^4}{3^4}\right)$

(v) $a^{-n} = \dfrac{1}{a^n}$ $\left(\text{Example: } 5^{-3} = \dfrac{1}{5^3}\right)$

(vi) For any real number a, $a^0 = 1$

Illustration 6: $\dfrac{5^{n+3} - 6 \times 5^{n+1}}{9 \times 5^n - 5^n \times 2^2} = ?$

Solution: $\dfrac{5^n \times 5^3 - 6 \times 5^n \times 5}{5^n (9 - 2^2)}$

$= \dfrac{5^n (5^3 - 6 \times 5)}{5^n \times 5}$

$= \dfrac{125 - 30}{5}$

$= \dfrac{95}{5} = 19$

Illustration 7: $\left\{\left(\sqrt[3]{(81)^2}\right)^{3/2}\right\}^{1/4} = ?$

Solution: $\left\{\left(\sqrt[3]{(81)^2}\right)^{3/2}\right\}^{1/4}$

$$= \left\{\left(81^2\right)^{1/3\times\frac{3}{2}}\right\}^{1/4}$$

$$= (81)^{2\times\frac{1}{3}\times\frac{3}{2}\times\frac{1}{4}} = \left(3^4\right)^{\frac{1}{4}} = 3$$

ALGEBRAIC IDENTITIES

Consider the equality $(x + 2)(x + 3) = x^2 + 5x + 6$

Let us evaluate both sides of this equality for some value of variable x say $x = 4$

LHS = $(x + 2)(x + 3) = (4 + 2)(4 + 3) = 6 \times 7 = 42$

RHS = $(4)^2 + 5 \times 4 + 6 = 16 + 20 + 6 = 42$

So for $x = 4$, LHS = RHS

Let us calculate LHS and RHS for $x = -3$

LHS = $(-3 + 2)(-3 + 3) = 0$

RHS = $(-3)2 + -(-3) + 6 = 9 - 15 + 6 = 0$

$\therefore$ for $x = -3$, LHS = RHS

If we take any value of variable x, we can find that LHS = RHS

Such an equality which is true for every value of the variable present in it is called an identity. Thus $(x + 2)(x + 3) = x^2 + 5x + 6$, is an identity.

Identities differ from equations in the following manners.

An equation is a statement of equality of two algebraic expression involving one or more variables and it is true for certain values of the variable.

For example:

$4x + 3 = x - 3$ ... (1)

$\Rightarrow \quad 3x = -6 \Rightarrow x = -2$

Thus equality (1) is true only for $x = -2$, no other value of x satisfy equation (1).

Standard Identities

(i) $(a + b)^2 = a^2 + 2ab + b^2$
(ii) $(a - b)^2 = a^2 - 2ab + b^2$
(iii) $a^2 - b^2 = (a + b)(a - b)$
(iv) $(x + a)(x + b) = x^2 + (a + b)x + ab$
(v) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Some More Identities

We have dealt with identities involving squares. Now we will see how to handle identities involving cubes.

(i) $(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

$\Rightarrow (a+b)^3 = a^3 + b^3 + 3ab(a+b)$

(ii) $(a-b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$

$\Rightarrow (a-b)^3 = a^3 - b^3 - 3ab(a-b)$

(iii) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

(iv) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

(v) $a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

If $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$

2. Multiplication of Two Numbers Using Formulae $(a - b)(a + b) = a^2 - b^2$

If the difference between two numbers x and y is a small even number, then the smaller is express as $(a - b)$ whereas larger is expressed as $(a + b)$, then the product of x and y is found out by the formulae $x \cdot y$ i.e., $(a - b)(a + b) = a^2 - b^2$

Here a should be such that a^2 is very easily calculated.

For example:

(i) $38 \times 42 = (40 - 2) \times (40 + 2) = (40)^2 - (2)^2 = 1600 - 4 = 1596$

(ii) $66 \times 74 = (70 - 4) \times (70 + 4) = (70)^2 - (4)^2 = 4900 - 16 = 4884$

(iii) $2094 \times 2106 = (2100 - 6) \times (2100 + 6) = (2100)^2 - (6)^2 = 4410000 - 36 = 4409964$

If the difference between the two numbers is not even, still this method is used by modify as

$47 \times 54 = 47 \times 53 + 47$
$= (50 - 3) \times (50 + 3) + 47$
$= (50)^2 - (3)^2 + 47$
$= 2500 - 9 + 47 = 2538$

SQUARES

When a number is multiplied by itself, then we get the square of the number.

For example, square of 5 = 5×5 (or 5^2) = 25

Square of 2 and 3 digits numbers and cube of 2 digits numbers are very useful in CAT and CAT like competitions.

For this it is advised to learn the square of 1 to 30 as given in the table:

Number	Square	Number	Square
1	1	16	256
2	4	17	289
3	9	18	324
4	16	19	361
5	25	20	400
6	36	21	441
7	49	22	484
8	64	23	529
9	81	24	576
10	100	25	625
11	121	26	676

Number	Square	Number	Square
12	144	27	729
13	169	28	784
14	196	29	841
15	225	30	900

SQUARE ROOTS

If $b = a \times a$ or a^2, then a is called square root of b and it is represented as $\sqrt{b} = a$ or $(b)^{1/2} = a$.

Since, $16 = 4 \times 4$ or 4^2, therefore $\sqrt{16} = 4$

And $25 = 5 \times 5$ or 5^2, therefore $\sqrt{25} = 5$

There are two methods for finding the square root of a number.

(i) Prime Factorisation Method

To find the square root by this method, we first factorise the given number into prime numbers as given below for the number 3136.

2	3136
2	1568
2	784
2	392
2	196
2	98
7	49
7	7
	1

$3136 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7$

Now pair the same prime factor like

$3136 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{7 \times 7}$

Now product of prime numbers staken one number from each pair of prime factors is the square root of the given number

$\therefore \sqrt{3136} = 2 \times 2 \times 2 \times 7 = 56$

If we write, $3136 = (2)^6 \times (7)^2$

Then square root of 3136 is the product of prime factors 2 and 7 with the powers half of the powers raised on 2 and 7 respectively.

i.e., $\sqrt{3136} = (2)^3 \times 7 = 56$

(ii) Division Method

In this method first of all pair the digits of the given number from right side. But there may be left a single digit at the left end of the number. Further process is shown below for the number 2304.

$\sqrt{2304} = 48$

	48
4	$\underline{23}\,\underline{04}$
4	16
88	704
8	704
	×××

Illustration 8: Find the square root of 15625.

Solution: $\sqrt{15625} = 125$

	125
1	$\underline{1}\,\underline{56}\,\underline{25}$
1	1
22	56
2	44
245	1225
5	1225
	××××

CUBES

When a number multiplies itself three times, we get the cube of the number.

Cube of $4 = 4 \times 4 \times 4 = 64$

Cubes of large numbers are rarely used. It is advised to you to learn the cube of the integers from 1 to 10.

Number	1	2	3	4	5	6	7	8	9	10
Cube	1	8	27	64	125	216	343	512	729	1000

Practice Exercise

Level - I

1. $287 \times 287 + 269 \times 269 - 2 \times 287 \times 269 = ?$
   (a) 534 (b) 446
   (c) 354 (d) 324
2. If $(64)^2 - (36)^2 = 20 \times x$, then $x = ?$
   (a) 70 (b) 120
   (c) 180 (d) 140
3. If $\sqrt{3} = 1.732$ and $\sqrt{2} = 1.414$, the value of $\frac{1}{\sqrt{3}+\sqrt{2}}$ is
   (a) 0.064 (b) 0.308
   (c) 0.318 (d) 2.146
4. $\sqrt{0.01 + \sqrt{0.0064}} = ?$
   (a) 0.3 (b) 0.03
   (c) $\sqrt{0.18}$ (d) None of these
5. $356 \times 936 - 356 \times 836 = ?$
   (a) 35600 (b) 34500
   (c) 9630 (d) 93600
6. The value of $\dfrac{\frac{1}{2} \div \frac{1}{2} \text{ of } \frac{1}{2}}{\frac{1}{2} + \frac{1}{2} \text{ of } \frac{1}{2}}$ is
   (a) 2/3 (b) 2
   (c) 4/3 (d) 3
7. The simplified value of

$$\frac{\left(1+\frac{1}{1+\frac{1}{100}}\right)\left(1+\frac{1}{1+\frac{1}{100}}\right)-\left(1-\frac{1}{1+\frac{1}{100}}\right)\left(1-\frac{1}{1+\frac{1}{100}}\right)}{\left(1+\frac{1}{1+\frac{1}{100}}\right)+\left(1-\frac{1}{1+\frac{1}{100}}\right)} \text{ is}$$

   (a) 100 (b) $\frac{200}{101}$
   (c) 200 (d) $\frac{202}{100}$
8. If $5^a = 3125$, then the value of $5^{(a-3)}$ is
   (a) 25 (b) 125
   (c) 625 (d) 1625
9. In a group of buffaloes and ducks, the number of legs are 24 more than twice the number of heads. What is the number of buffaloes in the group?
   (a) 6 (b) 8
   (c) 10 (d) 12
10. $\sqrt[3]{4\frac{12}{125}} = ?$
    (a) $1\frac{2}{5}$ (b) $1\frac{3}{5}$
    (c) $1\frac{4}{5}$ (d) $2\frac{2}{5}$
11. If $3^{4X-2} = 729$, then find the value of X.
    (a) 4 (b) 3
    (c) 2 (d) 5
12. What number must be added to the expression $16a^2 - 12a$ to make it a perfect square?
    (a) 9/4 (b) 11/2
    (c) 13/2 (d) 16
13. The value of $\left[\frac{1}{\sqrt{9}-\sqrt{8}}\right] - \left[\frac{1}{\sqrt{8}-\sqrt{7}}\right] + \left[\frac{1}{\sqrt{7}-\sqrt{6}}\right] - \left[\frac{1}{\sqrt{6}-\sqrt{5}}\right] + \left[\frac{1}{\sqrt{5}-\sqrt{4}}\right]$ is
    (a) 6 (b) 5
    (c) –7 (d) –6
14. Simplify: $5\sqrt[3]{250} + 7\sqrt[3]{16} - 14\sqrt[3]{54}$
    (a) $-2\sqrt[3]{2}$ (b) $-3\sqrt[3]{2}$
    (c) $3\sqrt[3]{2}$ (d) $2\sqrt[3]{3}$
15. The no. plate of a bus had peculiarity. The bus number was a perfect square. It was also a perfect square when the plate was turned upside down. The bus company had only five hundred buses numbered from 1 to 500. What was the number?
    (a) 169 (b) 36
    (c) 196 (d) Cannot say
16. If * means adding six times of second number into first number, then find the value of (1*2)*3.
    (a) 121 (b) 31
    (c) 93 (d) 91
17. If a and b are positive ingegers, such that $a^b = 125$, then $(a - b)^{a+b-4} = ?$
    (a) 16 (b) 25
    (c) 28 (d) 30
18. If $p \times q = p + q + \frac{p}{q}$, then value of $8 \times 2 = ?$
    (a) 2 (b) 10
    (c) 14 (d) 16

19. If $x*y = x^2 + y^2 - xy$, then value of 9*11 is
(a) 93 (b) 103
(c) 60.5 (d) 121

20. The least number by which we multiply to the 11760, so that we can get a perfect square number
(a) 2 (b) 3
(c) 5 (d) None of these

21. If $5\sqrt{5} \times 5^3 \div 5^{-3/2} = 5^{(a+2)}$, then value of a is
(a) 5 (b) 4
(c) 6 (d) 7

22. If difference between the $\frac{4}{5}$ of $\frac{3}{4}$ of a number and $\frac{2}{5}$ of $\frac{1}{6}$ of the same number is 648, then number is
(a) 1110 (b) 1215
(c) 1325 (d) 1440

23. If sum of two numbers is 42 and their product is 437, then find their difference.
(a) 3 (b) 4
(c) 5 (d) 7

24. 54.327 × 357.2 × 0.0057 is the same as:
(a) $5.4327 \times 3.572 \times 5.7$
(b) $5.4327 \times 3.572 \times 0.57$
(c) $54327 \times 3572 \times 0.0000057$
(d) None of these

25. Write the 44000 in Roman numerals
(a) XLI (b) XLVI
(c) XLIV (d) XLVIC

26. Write LXXIX in Hindu-Arabic numerals
(a) 70000 (b) 70009
(c) 7009 (d) 700009

27. If $\frac{a+b}{b+c} = \frac{c+d}{d+a}$, then
(a) a must equal c
(b) $a+b+c+d$ must equal zero
(c) either $a = c$ or $a+b+c+d = 0$, or both
(d) $a(b+c+d) = c(a+b+d)$

28. A number lies between 300 and 400. If the number is added to the number formed by reversing the digits, the sum is 888 and if the unit's digit and the ten's digit change places, the new number exceeds the original number by 9. Find the number.
(a) 339 (b) 341
(c) 378 (d) 345

29. x and y are 2 different digits. If the sum of the two digit numbers formed by using both the digits is a perfect square, then find $x + y$.
(a) 10 (b) 11
(c) 12 (d) 13

30. Arrange the following in the decending order;
$5^{1/4}, 4^{1/3}, 6^{1/5}$.
(a) $4^{1/3}, 5^{1/4}, 6^{1/5}$ (b) $5^{1/4}, 4^{1/3}, 6^{1/5}$
(c) $6^{1/5}, 4^{1/3}, 5^{1/4}$ (d) $5^{1/4}, 4^{1/3}, 6^{1/5}$

31. If $a + b + c = 13$, $a^2 + b^2 + c^2 = 69$, then find $ab + bc + ca$.
(a) –50 (b) 50
(c) 69 (d) 75

32. If $a - 8 = b$, then determine the value of $|a-b| - |b-a|$.
(a) 16 (b) 0
(c) 4 (d) 2

33. Find the possible integral value of x, if $x^2 + |x - 1| = 1$.
(a) 1 (b) –1
(c) 0 (d) 1 and 0

DIRECTIONS (Qs. 34-49) : ***What value should come in the place of question mark (?) in the following questions ?***

34. $3.6 + 36.6 + 3.66 + 0.36 + 3.0 = ?$ [*SBI Clerk-June-2012*]
(a) 44.22 (b) 77.22
(c) 74.22 (d) 47.22
(e) None of these

35. $23 \times 45 \div 15 = ?$ [*SBI Clerk-June-2012*]
(a) 69 (b) 65
(c) 63 (d) 71
(e) None of these

36. $4\frac{5}{6} + 7\frac{1}{2} - 5\frac{8}{11} = ?$ [*SBI Clerk-June-2012*]
(a) $2\frac{10}{33}$ (b) $6\frac{20}{33}$
(c) $2\frac{20}{33}$ (d) $6\frac{10}{33}$
(e) None of these

37. $\frac{210}{14} \times \frac{17}{15} \times ? = 4046$ [*SBI Clerk-June-2012*]
(a) 202 (b) 218
(c) 233 (d) 227
(e) None of these

38. 83% of 2350 = ? [*SBI Clerk-June-2012*]
(a) 1509.5 (b) 1950.5
(c) 1905.5 (d) 1590.5
(e) None of these

39. $\sqrt{1089} + 3 = (?)^2$ [*SBI Clerk-June-2012*]
(a) 5 (b) 6
(c) 3 (d) 8
(e) 4

40. $96 + 32 \times 5 - 31 = ?$ [*SBI Clerk-June-2012*]
(a) 223 (b) 225
(c) 229 (d) 221
(e) None of these

41. $? \div 36 = (7)^2 - 8$ [*SBI Clerk-June-2012*]
(a) 1426 (b) 1449
(c) 1463 (d) 1476
(e) None of these

42. $\sqrt{8281} = ?$ [*SBI Clerk- June-2012*]
(a) 89 (b) 97
(c) 93 (d) 91
(e) 83

43. $(63)^2 - (12)^2 = ?$ [*SBI Clerk-June-2012*]
(a) 3528 (b) 3852
(c) 3582 (d) 3825
(e) None of these

44. $1\frac{4}{5} + 3\frac{3}{5} = ? - 4\frac{3}{10}$ [*SBI Clerk-June-2012*]
(a) $9\frac{7}{10}$ (b) $7\frac{7}{10}$
(c) $9\frac{3}{10}$ (d) $7\frac{9}{10}$
(e) None of these

45. $17 \times 19 \times 4 \div ? = 161.5$ [*SBI Clerk-June-2012*]
(a) 8 (b) 6
(c) 7 (d) 9
(e) None of these

46. $1798 \div 31 \times ? = 348$ [*SBI Clerk-June-2012*]
(a) 3 (b) 6
(c) 4 (d) 5
(e) None of these

47. $(9.8 \times 2.3 + 4.46) \div 3 = (3)^?$ [*SBI Clerk-June-2012*]
(a) 3 (b) 9
(c) 5 (d) 2
(e) None of these

48. 43% of 600 + ?% of 300 = 399 [*SBI Clerk-June-2012*]
(a) 45 (b) 41
(c) 42 (d) 47
(e) None of these

49. The sum of three consecutive odd numbers is 1383. What is the largest number ? [*SBI Clerk-June-2012*]
(a) 463 (b) 49
(c) 457 (d) 461
(e) None of these

DIRECTIONS (Qs. 50-54) : ***What approximate value should come in place of the question mark (?) in the following questions? (NOTE: You are not expected to calculate the exact value)***

50. $1504 \times 5.865 - 24.091 = ?$ [*SBI Clerk-2012*]
(a) 7200 (b) 9500
(c) 6950 (d) 5480
(e) 8800

51. $16.928 + 24.7582 \div 5.015 = ?$ [*SBI Clerk-2012*]
(a) 20 (b) 24
(c) 22 (d) 26
(e) None of these

52. $\sqrt[3]{7.938} \times (6.120)^2 - 4.9256 = ?$ [*SBI Clerk-2012*]
(a) 70 (b) 55
(c) 30 (d) 25
(e) 90

53. $16.046 \div 2.8 \times 0.599 = ?$ [*SBI Clerk-2012*]
(a) 3.5 (b) 7.9
(c) 1.9 (d) 5.6
(e) 6.2

54. $\sqrt{963} + (4.895)^2 - 9.24 = ?$ [*SBI Clerk-2012*]
(a) 60 (b) 35
(c) 85 (d) 45
(e) 25

DIRECTIONS (Qs. 55-69) : ***What should come in place of the question mark (?) in the following questions ?***

55. $(12 \times 19) + (13 \times 8) = (15 \times 14) + ?$ [*SBI Clerk-2012*]
(a) 124 (b) 122
(c) 126 (d) 128
(e) None of these

56. $\sqrt{65 \times 12 - 50 + 54} = ?$ [*SBI Clerk-2012*]
(a) $\sqrt{28}$ (b) 28^2
(c) 28 (d) 784
(e) None of these

57. 15% of 524 – 2% of 985 + ? = 20% of 423 [*SBI Clerk-2012*]
(a) 25.9 (b) 27.7
(c) 25.7 (d) 24.9
(e) None of these

58. $151 \times 8 + (228 \div 19)^2 = ?$ [*SBI Clerk-2012*]
(a) 1360 (b) 1354
(c) 1368 (d) 1381
(e) None of these

59. $\sqrt{1521} + \sqrt{225} = ?$ [*SBI Clerk-2012*]
(a) 56 (b) 58
(c) 54 (d) 62
(e) None of these

60. $38.734 + 8.638 - 5.19 = ?$ [*SBI Clerk-2012*]
(a) 41.971 (b) 42.179
(c) 43.072 (d) 42.182
(e) None of these

61. $7^{8.9} \div (343)^{1.7} \times (49)^{4.8} = 7^?$ [*SBI Clerk-2012*]
(a) 13.4 (b) 12.8
(c) 11.4 (d) 9.6
(e) None of these

62. $\sqrt[5]{512} \div \sqrt[4]{16} + \sqrt{576} = ?$ [*SBI Clerk-2012*]
(a) 24 (b) 31
(c) 22 (d) 18
(e) None of these

63. $(42 \times 3.2) \div (16 \times 1.5) = ?$ [*SBI Clerk-2012*]
(a) 5.9 (b) 5.6
(c) 6.1 (d) 4.8
(e) None of these

64. $199 + 5^3 \div 4 \times 4^2 = ?$ [*SBI Clerk-2012*]
(a) 969 (b) 655
(c) 966 (d) 799
(e) None of these

65. $342 \div 6 \times 28 = 1099 + ?$ [*SBI Clerk-2012*]
(a) 478 (b) 502
(c) 486 (d) 504
(e) None of these

66. $\frac{9.8 \times 2.5 \times 7.6}{0.5} = ?$ [*SBI Clerk-2012*]
(a) 384.2 (b) 379.5
(c) 364.3 (d) 372.4
(e) None of these

67. $\frac{3}{5}$ of $\frac{2}{7}$ of $? = 426$ [*SBI Clerk-2012*]
(a) 2490 (b) 2565
(c) 2475 (d) 2485
(e) None of these

68. $3\frac{2}{5} + 1\frac{2}{9} = 4\frac{4}{5} - ?$ [*SBI Clerk-2012*]
(a) $\frac{8}{45}$ (b) $\frac{7}{47}$
(c) $\frac{7}{45}$ (d) $\frac{8}{51}$
(e) None of these

69. $\frac{13}{63} \div \frac{104}{14} \times \frac{52}{19} = ?$ [*SBI Clerk-2012*]
(a) $\frac{12}{173}$ (b) $\frac{13}{171}$
(c) $\frac{17}{171}$ (d) $\frac{18}{171}$
(e) None of these

DIRECTIONS (Qs. 70-84) : ***What should come in place of the question mark (?) in the following questions ?***

70. $\sqrt[3]{13824} \times \sqrt{?} = 864$ [*SBI Clerk-2014*]
(a) 1296 (b) 1156
(c) 1600 (d) 1024
(e) None of these

71. $(91)^2 + (41)^2 - \sqrt{?} = 9858$ [*SBI Clerk-2014*]
(a) 11236 (b) 10816
(c) 10404 (d) 9604
(e) None of these

72. $4900 \div 28 \times 444 \div 12 = ?$ [*SBI Clerk-2014*]
(a) 6575 (b) 6475
(c) 6455 (d) 6745
(e) None of these

73. 125% of 260 + ?% of 700 = 500 [*SBI Clerk-2014*]
(a) 32 (b) 56
(c) 23 (d) 46
(e) None of these

74. $3\frac{7}{11} + 7\frac{3}{11} \times 1\frac{1}{2} = ?$ [*SBI Clerk-2014*]
(a) $13\frac{10}{11}$ (b) $14\frac{6}{11}$
(c) $14\frac{9}{11}$ (d) $10\frac{17}{22}$
(e) None of these

75. $\frac{.23 - .023}{.0023 \div 23} = ?$ [*SBI Clerk-2014*]
(a) 0.207 (b) 207
(c) 2070 (d) 0.0207
(e) None of these

76. 1.05% of 2500 + 2.5% of 440 = ? [*SBI Clerk-2014*]
(a) 37.50 (b) 37.25
(c) 370.25 (d) 372.50
(e) None of these

77. $\frac{17 \times 4 + 4^2 \times 2}{90 \div 5 \times 12} = ?$ [*SBI Clerk-2014*]
(a) $\frac{25}{54}$ (b) $\frac{22}{57}$
(c) $\frac{11}{27}$ (d) $\frac{13}{27}$
(e) None of these

78. $17\frac{2}{5} \times 4\frac{5}{8} - ? = 46\frac{7}{8}$ [*SBI Clerk-2014*]
(a) $32\frac{3}{5}$ (b) $33\frac{3}{5}$
(c) $33\frac{2}{5}$ (d) $32\frac{2}{5}$
(e) None of these

79. 136% of 250 + ?% of 550 = 670 [*SBI Clerk-2014*]
(a) 64 (b) 55
(c) 56 (d) 65
(e) None of these

80. $3889 + 12.952 - ? = 3854.002$ [*SBI Clerk-2014*]
(a) 47.95 (b) 47.752
(c) 47.095 (d) 47.932
(e) None of these

81. $(5 \times 5 \times 5 \times 5 \times 5 \times 5)^4 \times (5 \times 5)^6 \div (5)^2 = (25)^?$ [*SBI Clerk-2014*]
(a) 10 (b) 17
(c) 19 (d) 12
(e) None of these

82. $\frac{28 \times 5 - 15 \times 6}{7^2 + \sqrt{256} + (13)^2} = ?$ [*SBI Clerk-2014*]
(a) $\frac{27}{115}$ (b) $\frac{22}{117}$
(c) $\frac{25}{117}$ (d) $\frac{22}{115}$
(e) None of these

83. $1.5 \times 0.025 + (?)^2 = 0.1$ [*SBI Clerk-2014*]
(a) 0.28 (b) 0.27
(c) 0.25 (d) 0.235
(e) None of these

84. $\dfrac{(3.537-0.948)^2+(3.537+0.948)^2}{(3.537)^2+(.948)^2} = ?$ [*SBI Clerk-2014*]
(a) 4.485 (b) 2.589
(c) 4 (d) 2
(e) None of these

DIRECTIONS (Qs. 85-89) : ***Find out the approximate value which should come in place of the question mark in the following questions. (You are not expected to find the exact value.)***

85. $\dfrac{(10008.99)^2}{10009.001} \times \sqrt{3589} \times 0.4987 = ?$ [*SBI Clerk-2014*]
(a) 3000 (b) 300000
(c) 3000000 (d) 5000
(e) 9000000

86. $196.1 \times 196.1 \times 196.1 \times 4.01 \times 4.001 \times 4.999 \times 4.999 = 196.1^3 \times 4 \times ?$ [*SBI Clerk-2014*]
(a) 100 (b) 16
(c) 10 (d) 64
(e) 32

87. $12.25 \times ? \times 21.6 = 3545.64$ [*SBI Clerk-2014*]
(a) 20 (b) 12
(c) 15 (d) 13
(e) None of these

88. ?% of $45.999 \times 16\%$ of $83.006 = 116.073$ [*SBI Clerk-2014*]
(a) 6 (b) 24
(c) 19 (d) 30
(e) 11

89. $[(1.3)^2 \times (4.2)^2] \div 2.7 = ?$ [*SBI Clerk-2014*]
(a) 7 (b) 21
(c) 18 (d) 11
(e) 16

DIRECTIONS (Qs. 90-94) : ***What should come in place of question mark (?) in the following number series ?***

90. 3 23 43 ? 83 103 [*SBI Clerk-2014*]
(a) 33 (b) 53
(c) 63 (d) 73
(e) None of these

91. 1 9 25 49 81 ? 169 [*SBI Clerk-2014*]
(a) 100 (b) 64
(c) 81 (d) 121
(e) None of these

92. 5 6 14 45 ? [*SBI Clerk-2014*]
(a) 183 (b) 185
(c) 138 (d) 139
(e) None of these

93. 7 8 18 57 ? [*SBI Clerk-2014*]
(a) 244 (b) 174
(c) 186 (d) 226
(e) None of these

94. 1, 8, 9, ?, 25, 216, 49 [*SBI Clerk-2014*]
(a) 60 (b) 64
(c) 70 (d) 75
(e) None of these

95. Last year my age was a perfect square number. Next year it will be a cubic number. What is my present age? [*SSC-Sub. Ins.-2012*]
(a) 25 years (b) 27 years
(c) 26 years (d) 24 years

96. What is the value of $(2.1)^2 \times \sqrt{0.0441}$? [*SSC-Sub. Ins.-2012*]
(a) 0.9261 (b) 92.61
(c) 92.51 (d) 0.9251

97. The value of $\sqrt[3]{1372} \times \sqrt[3]{1458}$ is [*SSC-Sub. Ins.-2012*]
(a) 116 (b) 126
(c) 106 (d) 136

98. Equal amounts of water were poured into two empty jars of different capacities, which made one jar $\frac{1}{4}$ full and the other jar $\frac{1}{3}$ full. If the water in the jar with lesser capacity is then poured into the jar with greater capacity, then the part of the larger jar filled with water is [*SSC-Sub. Ins.-2012*]
(a) $\frac{1}{2}$ (b) $\frac{7}{12}$
(c) $\frac{1}{4}$ (d) $\frac{1}{3}$

99. If $\dfrac{5x-3}{x}+\dfrac{5y-3}{y}+\dfrac{5z-3}{z}=0$, then the value of $\dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}$ is [*SSC-Sub. Ins.-2012*]
(a) 15 (b) 3
(c) 5 (d) 10

100. Minimum value of $x^2+\dfrac{1}{x^2+1}-3$ is [*SSC-Sub. Ins.-2012*]
(a) -3 (b) -2
(c) 0 (d) -1

101. If $a + b = 5$, $a^2 + b^2 = 13$, the value of $a - b$ (where $a > b$) is [*SSC-Sub. Ins.-2012*]
(a) 2 (b) -1
(c) 1 (d) -2

102. If $(3x - y) : (x + 5y) = 5 : 7$, then the value of $(x + y) : (x - y)$ is [*SSC-Sub. Ins.-2012*]
(a) 3 : 1 (b) 1 : 3
(c) 2 : 3 (d) 3 : 2

103. The value of $1+\cfrac{1}{1+\cfrac{2}{3+\cfrac{4}{5}}}$ is: [*SSC-Sub. Ins.-2013*]

(a) $\frac{12}{29}$ (b) $\frac{8}{19}$

(c) $\frac{48}{29}$ (d) $\frac{2}{19}$

104. The value of

$\sqrt{19.36}+\sqrt{0.1936}+\sqrt{0.001936}+\sqrt{0.00001936}$ is: [*SSC-Sub. Ins.-2013*]

(a) 4.8484 (b) 4.8694
(c) 4.8884 (d) 4.8234

105. If the square of the sum of two numbers is equal to 4 times of their product. then the ratio of these numbers is : [*SSC-Sub. Ins.-2013*]

(a) 2 : 1 (b) 1 : 3
(c) 1 : 1 (d) 1 : 2

106. If $a^2 + b^2 = 5ab$, then the value of $\left(\frac{a^2}{b^2}+\frac{b^2}{a^2}\right)$ is : [*SSC-Sub. Ins.-2013*]

(a) 32 (b) 16
(c) 23 (d) –23

107. If $x=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ and $y=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, then the value of x^3+y^3 is: [*SSC-Sub. Ins.-2013*]

(a) 950 (b) 730
(c) 650 (d) 970

108. The greatest among the following numbers $(3)^{\frac{1}{3}},(2)^{\frac{1}{2}},1,(6)^{\frac{1}{6}}$ is: [*SSC-Sub. Ins.-2013*]

(a) $(2)^{\frac{1}{2}}$ (b) 1

(c) $(6)^{\frac{1}{6}}$ (d) $(3)^{\frac{1}{3}}$

109. Evaluate $\frac{\sqrt{24}+\sqrt{6}}{\sqrt{24}-\sqrt{6}}$ [*SSC-Sub. Ins.-2014*]

(a) 2 (b) 3
(c) 4 (d) 5

110. The value of [*SSC-Sub. Ins.-2014*]

$3\div\left[(8-5)\div\left\{(4-2)\div\left(2+\frac{8}{13}\right)\right\}\right]$ is

(a) $\frac{15}{17}$ (b) $\frac{13}{17}$

(c) $\frac{15}{19}$ (d) $\frac{13}{19}$

111. If '+' means '÷', '×' means '–', '÷' means '×' and '–' means '+', what will be the value of the following expression?
$9 + 3 \div 4 - 8 \times 2 = ?$ [*SSC-Sub. Ins.-2014*]

(a) $6\frac{1}{4}$ (b) $6\frac{3}{4}$

(c) $-1\frac{3}{4}$ (d) 18

112. The next term of the sequence,

$\left(1+\frac{1}{2}\right),\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right),\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right),$ __ is [*SSC-Sub. Ins.-2014*]

(a) 3 (b) $\left(1+\frac{1}{5}\right)$

(c) 5 (d) $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{5}\right)$

113. If a = $\sqrt{6}+\sqrt{5}$, b = $\sqrt{6}-\sqrt{5}$, then $2a^2 - 5ab + 2b^2 =$ [*SSC-Sub. Ins.-2014*]

(a) 38 (b) 39
(c) 40 (d) 41

114. If $p=\frac{5}{18}$, then $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$ is equal to [*SSC-Sub. Ins.-2014*]

(a) $\frac{4}{27}$ (b) $\frac{5}{27}$

(c) $\frac{8}{27}$ (d) $\frac{10}{27}$

115. If $x+\frac{1}{x}=2$, then $x^{2013}+\frac{1}{x^{2014}}=?$ [*SSC-Sub. Ins.-2014*]

(a) 0 (b) 1
(c) –1 (d) 2

116. If a = 331, b = 336 and c = – 667, then the value of $a^3 + b^3 + c^3 - 3abc$ is [*SSC-Sub. Ins.-2014*]

(a) 1 (b) 6
(c) 3 (d) 0

117. The simplified value of

$\left(\sqrt{6}+\sqrt{10}-\sqrt{21}-\sqrt{35}\right)\left(\sqrt{6}-\sqrt{10}+\sqrt{21}-\sqrt{35}\right)$ is [*SSC-Sub. Ins.-2014*]

(a) 13 (b) 12
(c) 11 (d) 10

118. If $x = a - b, y = b - c, z = c - a$, then the numerical value of the algebraic expression $x^3 + y^3 + z^3 - 3xyz$ will be

(a) $a + b + c$ (b) 0
(c) $4(a + b + c)$ (d) $3abc$

119. The simplified value of $\frac{\sqrt{32}+\sqrt{48}}{\sqrt{8}+\sqrt{12}}$ is [*SSC-MT-2013*]

(a) 4 (b) 3
(c) 2 (d) 6

120. $\sqrt{\frac{9.5\times0.085}{0.0017\times0.19}}$ equals [*SSC-MT-2013*]

(a) 5 (b) 50
(c) 500 (d) 0.05

121. If $\frac{x}{b+c}=\frac{y}{c+a}=\frac{z}{a+b}$, then : [*SSC 10+2-2012*]

(a) $\frac{x-y}{b-a}=\frac{y-z}{c-b}=\frac{z-x}{a-c}$

(b) $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}$

(c) $\frac{x-y}{c}=\frac{y-z}{b}=\frac{z-x}{a}$

(d) none of the above is true

122. If $\frac{547.527}{0.0082}=x$, then the value $\frac{547527}{82}$ is :

[*SSC 10+2-2012*]

(a) 10x (b) 100x
(c) $\frac{x}{100}$ (d) $\frac{x}{10}$

123. If $\sqrt[3]{3^n}=27$, then the value of n is: [*SSC 10+2-2012*]

(a) 9 (b) 6
(c) 1 (d) 3

124. From 9.00 AM to 2.00 PM, the temperature rose at a constant rate from 21°C to 36°C. What was the temperature at noon ?

[*SSC 10+2-2012*]

(a) 27°C (b) 30°C
(c) 32°C (d) 28.5°C

125. If $\frac{3x+5}{5x-2}=\frac{2}{3}$, then the value of x is : [*SSC 10+2-2012*]

(a) 11 (b) 19
(c) 23 (d) 7

126. If the difference of two numbers is 3 and the difference of their squares is 39; then the larger number is :

[*SSC 10+2-2012*]

(a) 9 (b) 12
(c) 13 (d) 8

127. If $x=\sqrt{3}+\sqrt{2}$, then the value of $x^3-\frac{1}{x^3}$ is :

[*SSC 10+2-2012*]

(a) $14\sqrt{2}$ (b) $14\sqrt{3}$
(c) $22\sqrt{2}$ (d) $10\sqrt{2}$

128. If $a^2+b^2+c^2=2(a-b-c)-3$, then the value of $2a-3b+4c$ is [*SSC 10+2-2013*]

(a) 1 (b) 7
(c) 2 (d) 3

129. Let $a=\sqrt{6}-\sqrt{5}, b=\sqrt{5}-2, c=2-\sqrt{3}$.
Then point out the correct alternative among the four alternatives given below. [*SSC 10+2-2013*]

(a) $a<b<c$ (b) $b<a<c$
(c) $a<c<b$ (d) $b<c<a$;

130. If $a=\frac{b^2}{b-a}$ then the value of a^3+b^3 is [*SSC 10+2-2013*]

(a) 2 (b) 6 ab
(c) 0 (d) 1

131. If $xy+yz+zx=0$, then [*SSC 10+2-2013*]

$\left(\frac{1}{x^2-yz}+\frac{1}{y^2-zx}+\frac{1}{z^2-xy}\right)(x,y,z\neq0)$ is equal to

(a) 0 (b) 3
(c) 1 (d) $x+y+z$

132. The value of $\sqrt{40+\sqrt{9\sqrt{81}}}$ is [*SSC 10+2-2013*]

(a) 11 (b) $\sqrt{111}$
(c) 9 (d) 7

133. Which is greater $\sqrt[3]{2}$ or $\sqrt{3}$? [*SSC 10+2-2013*]

(A Equal (b) Cannot be compared
(c) $\sqrt[3]{2}$ (d) $\sqrt{3}$

134. If a + b + c = 9 (where a, b, c are real numbers), then the minimum value of $a^2+b^2+c^2$ is [*SSC 10+2-2013*]

(a) 81 (b) 100
(c) 9 (d) 27

135. Find the value of

$3+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{3}+3}+\frac{1}{\sqrt{3}-3}$. [*SSC 10+2-2013*]

(a) 6 (b) 3
(c) $\frac{3}{2(\sqrt{3}+3)}$ (d) $2\sqrt{3}$

136. 'a' divides 228 leaving a remainder 18. The biggest two-digit value of 'a' is [*SSC 10+2-2013*]

(a) 30 (b) 70
(c) 21 (d) 35

137. A teacher wants to arrange his students in an equal number of rows and columns. If there are 1369 students, the number of students in the last row are [*SSC 10+2-2014*]

(a) 37 (b) 33
(c) 63 (d) 47

138. Which one of the following is true ? [*SSC 10+2-2014*]

(a) $\sqrt{5}+\sqrt{3}>\sqrt{6}+\sqrt{2}$
(b) $\sqrt{5}+\sqrt{3}<\sqrt{6}+\sqrt{2}$
(c) $\sqrt{5}+\sqrt{3}=\sqrt{6}+\sqrt{2}$
(d) $(\sqrt{5}+\sqrt{3})(\sqrt{6}+\sqrt{2})=1$

DIRECTIONS (139-148) : ***What will come in place of the question mark (?) in the following questions?***

139. $(3325 \div 25) \times (152 \div 16) = ?$ [*IBPS Clerk-2012*]
(a) 1269.4 (b) 1264.9
(c) 1265.3 (d) 1263.5
(e) None of these

140. $\sqrt{3136} - \sqrt{1764} = \sqrt{?}$ [*IBPS Clerk-2012*]
(a) 14 (b) $(196)^2$
(c) – 14 (d) 144
(e) None of these

141. $5\frac{1}{5} + 2\frac{2}{15} + 3\frac{2}{3} = ?$ [*IBPS Clerk-2012*]
(a) 15 (b) 13
(c) $\frac{11}{15}$ (d) 12
(e) None of these

142. $-15 - 27 - 88 - 63 + 255 = ?$ [*IBPS Clerk-2012*]
(a) 55 (b) 74
(c) 62 (d) 59
(e) None of these

143. $(2525 \times 0.25 \div 5) \times 7 = ?$ [*IBPS Clerk-2012*]
(a) 889.43 (b) 883.75
(c) 886.45 (d) 881.75
(e) None of these

144. $\frac{14}{19} \times \frac{57}{70} \times \frac{20}{21} = ?$ [*IBPS Clerk-2012*]
(a) $\frac{2}{7}$ (b) $\frac{4}{7}$
(c) $\frac{2}{9}$ (d) $\frac{3}{7}$
(e) None of these

145. 32% of 500 + 162% of 50 = ? [*IBPS Clerk-2012*]
(a) 231 (b) 245
(c) 237 (d) 247
(e) None of these

146. $45316 + 52131 - 65229 = ? + 15151$ [*IBPS Clerk-2012*]
(a) 17063 (b) 17073
(c) 17076 (d) 17067
(e) None of these

147. $\sqrt{25 - 12 + 155 + 1} = ?$ [*IBPS Clerk-2012*]
(a) 13 (b) 14
(c) 17 (d) 16
(e) None of these

148. $\frac{184 \times 4}{23\% \text{ of } 400} = ?$ [*IBPS Clerk-2012*]
(a) 7 (b) 9
(c) 8 (d) 5
(e) None of these

149. What will come in place of both the question marks (?) in the following question ? [*IBPS Clerk-2012*]

$$\frac{(?)^{4/3}}{32} = \frac{128}{?^{5/3}}$$

(a) 16 (b) 12
(c) 18 (d) 14
(e) None of these

150. If the following fractions are arranged in a descending order (from left to right), which of them will be second from the right end ? [*IBPS Clerk-2012*]

$\frac{4}{9}, \frac{6}{13}, \frac{5}{11}, \frac{13}{16}, \frac{7}{12}$

(a) $\frac{6}{13}$ (b) $\frac{4}{9}$
(c) $\frac{13}{16}$ (d) $\frac{7}{12}$
(e) $\frac{5}{11}$

151. A factory produces 1515 items in 3 days. How many items will they produce in a week ? [*IBPS Clerk-2012*]
(a) 3530 (b) 3553
(c) 3533 (d) 3535
(e) None of these

152. What is the **least** number that can be added to 4800 to make it a perfect square ? [*IBPS Clerk-2012*]
(a) 110 (b) 81
(c) 25 (d) 36
(e) None of these

153. If $(11)^3$ is subtracted from $(46)^2$ what will be the remainder ? [*IBPS Clerk-2012*]
(a) 787 (b) 785
(c) 781 (d) 783
(e) None of these

154. The sum of the squares of two odd numbers is 11570. The square of the smaller number is 5329. What is the other number ? [*IBPS Clerk-2012*]
(a) 73 (b) 75
(c) 78 (d) 79
(e) None of these

155. The sum of three consecutive integers is 5685. Which of the following is the correct set of these numbers ? [*IBPS Clerk-2012*]
(a) 1893, 1894, 1895 (b) 1895, 1896, 1897
(c) 1899, 1900, 1901 (d) 1897, 1898, 1899
(e) None of these

156. The product of three consecutive odd numbers is 24273. Which is the smallest number? [*IBPS Clerk-2012*]
(a) 25 (b) 29
(c) 23 (d) 37
(e) 27

DIRECTIONS (Qs. 157-171) : *What will come in place of question mark (?) in the given question?*

157. $4\frac{1}{2}+\left(1\div 2\frac{8}{9}\right)-3\frac{1}{13}=?$ [IBPS Clerk-2013]

(a) $1\frac{9}{26}$ (b) $2\frac{7}{13}$

(c) $1\frac{11}{26}$ (d) $2\frac{4}{13}$

(e) $1\frac{10}{13}$

158. $\frac{6\times 136\div 8+132}{628\div 16-26.25}=?$ [IBPS Clerk-2013]

(a) 15 (b) 24
(c) 18 (d) 12
(e) 28

159. $\{(441)^{1/2}\times 207\times(343)^{1/3}\}\div\{(14)^2\times(529)^{1/2}\}$ [IBPS Clerk-2013]

(a) $6\frac{1}{2}$ (b) $5\frac{1}{2}$

(c) $5\frac{3}{4}$ (d) $6\frac{3}{4}$

(e) $6\frac{1}{4}$

160. $\left\{\sqrt{7744}\times(11)^2\right\}\div(2)^3=(?)^3$ [IBPS Clerk-2013]

(a) 7 (b) 9
(c) 11 (d) 13
(e) 17

161. $(4356)^{1/2}\div\frac{11}{4}=\sqrt{?}\times 6$ [IBPS Clerk-2013]

(a) 2 (b) 4
(c) 8 (d) 6
(e) 16

162. $\frac{3}{8}$ of $\{4624\div(564-428)\}=?$ [IBPS Clerk-2013]

(a) $13\frac{1}{4}$ (b) $14\frac{1}{2}$

(c) $11\frac{5}{6}$ (d) $12\frac{3}{4}$

(e) $12\frac{1}{8}$

163. $456\div 24\times 38-958+364=?$ [IBPS Clerk-2013]

(a) 112 (b) 154
(c) 128 (d) 136
(e) 118

164. $(43)^2+841=(?)^2+1465$ [IBPS Clerk-2013]

(a) 41 (b) 35
(c) 38 (d) 33
(e) 30

165. $3\frac{3}{8}\times 6\frac{5}{12}-2\frac{3}{16}\times 3\frac{1}{2}=?$ [IBPS Clerk-2013]

(a) 21 (b) 18
(c) 14 (d) 15
(e) 16

166. $(34.5\times 14\times 42)\div 2.8=?$ [IBPS Clerk-2013]

(a) 7150 (b) 7365
(c) 7245 (d) 7575
(e) 7335

167. $(216)^4\div(36)^4\times(6)^5=(6)^?$ [IBPS Clerk-2013]

(a) 13 (b) 11
(c) 7 (d) 9
(e) 10

168. $\frac{\sqrt{4356\times\sqrt{?}}}{\sqrt{6084}}=11$ [IBPS Clerk-2013]

(a) 144 (b) 196
(c) 169 (d) 81
(e) 121

169. $\left(3\frac{6}{17}\div 2\frac{7}{34}-1\frac{9}{25}\right)=(?)^2$ [IBPS Clerk-2013]

(a) $\frac{2}{5}$ (b) $\frac{1}{3}$

(c) $\frac{4}{5}$ (d) $\frac{1}{5}$

(e) $\frac{3}{5}$

170. $(1097.63+2197.36-2607.24)\div 3.5=?$ [IBPS Clerk-2013]

(a) 211.5 (b) 196.5
(c) 209.5 (d) 192.5
(e) 189.5

171. $\frac{1}{11}$ of $[(17424)^{1/2}\div(66)^2\times 3^3]=?^2$ [IBPS Clerk-2013]

(a) $\frac{1}{11}$ (b) $\frac{3}{11}$

(c) $\frac{2}{11}$ (d) $\frac{4}{11}$

(e) $\frac{5}{11}$

Level - II

1. Value of $999\frac{995}{999}\times 999 = ?$

(a) 990809 (b) 998996
(c) 153.6003 (d) 213.0003

2. $7892.35 \times 99.9 = ?$

(a) 753445.765 (b) 764455.765
(c) 788445.765 (d) None of these

3. How many $\frac{1}{12}$ in $18\frac{3}{4}$

(a) 522 (b) 252
(c) 225 (d) 253

4. The least possible positive number which should be added to 575 to make a perfect square number is

(a) 0 (b) 1
(c) 4 (d) None of these

5. If $a * b * c = \sqrt{\frac{(a+2)(b+3)}{(c+1)}}$, then the value of (6 * 15 * 3) is

(a) 6 (b) 3
(c) 4 (d) can't be determined

6. If $x = 3+\sqrt{8}$, then $\left(x^2+\frac{1}{x^2}\right) = ?$

(a) 34 (b) 24
(c) 38 (d) 36

7. If $x^a = y^b = z^c$ and $y^2 = zx$ then the value of $\frac{1}{a}+\frac{1}{c}$ is

(a) $\frac{b}{2}$ (b) $\frac{c}{2}$
(c) $\frac{2}{b}$ (d) $2a$

8. If $\frac{2x}{1+\frac{1}{1+\frac{x}{1-x}}} = 1$, then find the value of x.

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$
(c) 2 (d) $\frac{1}{2}$

9. Find the value of

$$\frac{1}{2\times 3}+\frac{1}{3\times 4}+\frac{1}{4\times 5}+\frac{1}{5\times 6}+.....+\frac{1}{9\times 10}$$

(a) $\frac{3}{2}$ (b) $\frac{2}{5}$
(c) $\frac{2}{3}$ (d) $\frac{3}{5}$

10. Find the square root of $7-2\sqrt{10}$.

(a) $\sqrt{5}+\sqrt{2}$ (b) $-\sqrt{5}-\sqrt{2}$
(c) $\pm(\sqrt{5}-\sqrt{2})$ (d) $\pm(\sqrt{5}+\sqrt{2})$

11. The product of two 2-digit numbers is 1938. If the product of their unit's digits is 28 and that of ten's digits is 15, find the larger number.

(a) 34 (b) 57
(c) 43 (d) 75

12. If $P + P! = P^3$, then the value of P is

(a) 4 (b) 6
(c) 0 (d) 5

13. For any real value of x the maximum value of $8x - 3x^2$ is

(a) $\frac{8}{3}$ (b) 4
(c) 5 (d) $\frac{16}{3}$

14. If x is a number satisfying the equation

$\sqrt[3]{x+9}-\sqrt[3]{x-9} = 3$, then x^2 is between

(a) 55 and 65 (b) 65 and 75
(c) 75 and 85 (d) 85 and 95

15. The value of $\left[35.7-\left(3+\frac{1}{3+\frac{1}{3}}\right)-\left(2+\frac{1}{2+\frac{1}{2}}\right)\right]$ is

(a) 30 (b) 34.8
(c) 36.6 (d) 41.4

16. Which one of the following sets of surds is in correct sequence of ascending order of their values?

(a) $\sqrt[4]{10},\sqrt[3]{6},\sqrt{3}$ (b) $\sqrt{3},\sqrt[4]{10},\sqrt[3]{6}$
(c) $\sqrt{3},\sqrt[3]{6},\sqrt[4]{10}$ (d) $\sqrt[4]{10},\sqrt{3},\sqrt[3]{6}$

17. The last three-digits of the multiplication 12345 × 54321 will be

(a) 865 (b) 745
(c) 845 (d) 945

18. The sum of the two numbers is 12 and their product is 35. What is the sum of the reciprocals of these numbers ?

(a) $\frac{12}{35}$ (b) $\frac{1}{35}$
(c) $\frac{35}{8}$ (d) $\frac{7}{32}$

19. Find the value of $\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right).....\left(1-\frac{1}{100}\right)$.

(a) $\frac{1}{5}$ (b) $\frac{1}{10}$

(c) $\frac{1}{50}$ (d) $\frac{2}{5}$

20. An employer pays ₹20 for each day a works, and forfeits ₹ 3 for each day he is idle. At the end of 60 days, a worker gets ₹280. For how many days did the worker remain idle?

(a) 28 (b) 40

(c) 52 (d) 60

21. Simplify : $\dfrac{1}{1+\dfrac{\frac{2}{3}}{1+\frac{2}{3}+\dfrac{\frac{8}{9}}{1-\frac{2}{3}}}}$

(a) $\frac{11}{13}$ (b) $\frac{13}{15}$

(c) $\frac{13}{11}$ (d) $\frac{15}{13}$

22. The value of $\sqrt{\dfrac{(0.03)^2+(0.21)^2+(0.065)^2}{(0.003)^2+(0.021)^2+(0.0065)^2}}$ is

(a) 0.1 (b) 10

(c) 10^2 (d) 10^3

23. If $\dfrac{x^2+y^2+z^2-64}{xy-yz-zx}=-2$ and $x + y = 3z$, then the value of z is

(a) 2 (b) 3

(c) 4 (d) None of these

24. If $\sqrt{24}$ = 4.899, the value of $\sqrt{\frac{8}{3}}$ is

(a) 0.544 (b) 1.333

(c) 1.633 (d) 2.666

25. If $(X+(1/X))=4$, then the value of X^4+1/X^4 is

(a) 124 (b) 64

(c) 194 (d) Can't be determined

26. If $\sqrt{15625}=125$, then the value of $\sqrt{15625}+\sqrt{156.25}+\sqrt{1.5625}$ is

(a) 1.3875 (b) 13.875

(c) 138.75 (d) 156.25

27. A hostel has provisions for 250 students for 35 days. After 5 days, a fresh batch of 25 students was admitted to the hostel. Again after 10 days, a batch of 25 students left the hostel. How long will the remaining provisions survive?

(a) 18 days (b) 19 days

(c) 20 days (d) 17 days

28. If $\dfrac{97}{19}=a+\dfrac{1}{b+\frac{1}{c}}$ where a, b and c are positive integers, then what is the sum of a, b and c?

(a) 16 (b) 20

(c) 9 (d) Cannot be determined

29. If $a > 1$, then arrange the following in ascending order.

I. $\sqrt[3]{\sqrt[4]{a^3}}$ II. $\sqrt[3]{\sqrt[5]{a^4}}$

III. $\sqrt{\sqrt[3]{a}}$ IV. $\sqrt{\sqrt[5]{a^3}}$

(a) I, II, III, IV (b) I, II, IV, III

(c) IV, I, III, II (d) III, I, II, IV

30. Which of the following is correct if $A=3^{3^{3}}$, $B=3^{33^{3}}$, $C=3^{3^{33}}$ and $D=3^{333}$?

(a) $A > B = C > D$ (b) $C > A > B > D$

(c) $A > C > D > B$ (d) $C > B > D > A$

31. Find the value of x in $\sqrt{x+2\sqrt{x+2\sqrt{x+2\sqrt{3x}}}}=x$.

(a) 1 (b) 3

(c) 6 (d) 12

32. Find two numbers such that their sum, their product and the differences of their squares are equal.

(a) $\left(\frac{3+\sqrt{3}}{2}\right)$ and $\left(\frac{1+\sqrt{2}}{2}\right)$ or $\left(\frac{3+\sqrt{2}}{2}\right)$ and $\left(\frac{1+\sqrt{2}}{2}\right)$

(b) $\left(\frac{3+\sqrt{7}}{2}\right)$ and $\left(\frac{1+\sqrt{7}}{2}\right)$ or $\left(\frac{3+\sqrt{6}}{2}\right)$ and $\left(\frac{1-\sqrt{6}}{2}\right)$

(c) $\left(\frac{3-\sqrt{5}}{2}\right)$ and $\left(\frac{1-\sqrt{5}}{2}\right)$ or $\left(\frac{3+\sqrt{5}}{2}\right)$ and $\left(\frac{1+\sqrt{5}}{2}\right)$

(d) None of these

DIRECTIONS (Qs. 33-37) : ***What will come in place of the question mark (?) in the following questions ?***

33. $\sqrt{11449}\times\sqrt{6241}-(54)^2=\sqrt{?}+(74)^2$ [*IBPS-PO-2011*]

(a) 384 (b) 3721

(c) 381 (d) 3638

(e) None of these

34. $\left[\left(3\sqrt{8}+\sqrt{8}\right)\times\left(8\sqrt{8}+7\sqrt{8}\right)\right]-98=?$ [*IBPS-PO-2011*]

(a) $2\sqrt{8}$ (b) $8\sqrt{8}$

(c) 382 (d) 386

(e) None of these

35. $3463\times295-18611=?+5883$ [*IBPS-PO-2011*]

(a) 997091 (b) 997071

(c) 997090 (d) 999070

(e) None of these

36. $\frac{28}{65} \times \frac{195}{308} \div \frac{39}{44} + \frac{5}{26} = ?$ [*IBPS-PO-2011*]

(a) $\frac{1}{3}$ (b) 0.75

(c) $1\frac{1}{2}$ (d) $\frac{1}{2}$

(e) None of these

37. $(23.1)^2 + (48.6)^2 - (39.8)^2 = ? + 1147.69$ [*IBPS-PO-2011*]

(a) $(13.6)^2$ (b) $\sqrt{12.8}$

(c) 163.84 (d) 12.8

(e) None of these

DIRECTIONS (Qs. 38-42) : ***What*** **approximate** ***value should come in place of the question mark (?) in the following questions?***

(**Note :** ***You are not expected to calculate the exact value.)***

38. $\sqrt[3]{4663} + 349 = ? \div 21.003$ [*IBPS-PO-2011*]

(a) 7600 (b) 7650

(c) 7860 (d) 7560

(e) 7680

39. 39.897% of 4331 + 58.779% of 5003 = ? [*IBPS-PO-2011*]

(a) 4300 (b) 4500

(c) 4700 (d) 4900

(e) 5100

40. $59.88 \div 12.21 \times 6.35 = ?$ [*IBPS-PO-2011*]

(a) 10 (b) 50

(c) 30 (d) 70

(e) 90

41. $43931.03 \div 2111.02 \times 401.04 = ?$ [*IBPS-PO-2011*]

(a) 8800 (b) 7600

(c) 7400 (d) 9000

(e) 8300

42. $\sqrt{6354} \times 34.993 = ?$ [*IBPS-PO-2011*]

(a) 3000 (b) 2800

(c) 2500 (d) 3300

(e) 2600

DIRECTIONS (Qs. 43–47) : ***In the following number series only one number is*** **wrong.** ***Find out the*** **wrong** ***number.***

43. 9050 5675 3478 2147 1418 1077 950 [*IBPS-PO-2011*]

(a) 3478 (b) 1418

(c) 5675 (d) 2147

(e) 1077

44. 7 12 40 222 1742 17390 208608 [*IBPS-PO-2011*]

(a) 7 (b) 12

(c) 40 (d) 1742

(e) 208608

45. 6 91 584 2935 11756 35277 70558 [*IBPS-PO-2011*]

(a) 91 (b) 70558

(c) 584 (d) 2935

(e) 35277

46. 1 4 25 256 3125 46656 823543 [*IBPS-PO-2011*]

(a) 3125 (b) 823543

(c) 46656 (d) 25

(e) 256

47. 8424 4212 2106 1051 526.5 263.25 131.625 [*IBPS-PO-2011*]

(a) 131.625 (b) 1051

(c) 4212 (d) 8424

(e) 263.25

48. Rubina could get equal number of ₹ 55, ₹ 85 and ₹ 105 tickets for a movie. She spents ₹ 2940 for all the tickets. How many of each did she buy? [*IBPS-PO-2011*]

(a) 12 (b) 14

(c) 16 (d) Cannot be determined

(e) None of these

49. Seema bought 20 pens, 8 packets of wax colours, 6 calculators and 7 pencil boxes. The price of one pen is ₹ 7, one packet of wax colour is ₹ 22, one calculator is ₹ 175 and one pencil box is ₹ 14 more than the combined price of one pen and one packet of wax colours. How much amount did Seema pay to the shopkeeper?[*IBPS-PO-2011*]

(a) ₹ 1,491 (b) ₹ 1,725

(c) ₹ 1,667 (d) ₹ 1,527

(e) None of these

DIRECTIONS (Qs. 50-54) : ***What will come in place of the question mark (?) in the following questions ?***

50. $4003 \times 77 - 21015 = ? \times 116$ [*IBPS-PO-2012*]

(a) 2477 (b) 2478

(c) 2467 (d) 2476

(e) None of these

51. $\left[\left(5\sqrt{7}+\sqrt{7}\right)+\left(4\sqrt{7}+8\sqrt{7}\right)\right]-(19)^2 = ?$ [*IBPS-PO-2012*]

(a) 143 (b) $72\sqrt{7}$

(c) 134 (d) $70\sqrt{7}$

(e) None of these

52. $(4444 \div 40) + (645 \div 25) + (3991 \div 26) = ?$ [*IBPS-PO-2012*]

(a) 280.4 (b) 290.4

(c) 295.4 (d) 285.4

(e) None of these

53. $\sqrt{33124} \times \sqrt{2601} - (83)^2 = (?)^2 + (37)^2$ [*IBPS-PO-2012*]

(a) 37 (b) 33

(c) 34 (d) 28

(e) None of these

54. $5\frac{17}{37}\times4\frac{51}{52}\times11\frac{1}{7}+2\frac{3}{4}=?$ [*IBPS-PO-2012*]
(a) 303.75 (b) 305.75
(c) $303\frac{3}{4}$ (d) $305\frac{1}{4}$
(e) None of these

DIRECTIONS (Qs. 55-61) : ***What approximate value should come in place of the question mark (?) in the following questions? (Note : You are not expected to calculate the exact value.)***

55. $8787\div343\times\sqrt{50}=?$ [*IBPS-PO-2012*]
(a) 250 (b) 140
(c) 180 (d) 100
(e) 280

56. $\sqrt[3]{54821}\times(303\div8)=(?)^2$ [*IBPS-PO-2012*]
(a) 48 (b) 38
(c) 28 (d) 18
(e) 58

57. $\frac{5}{8}$ of 4011.33 + $\frac{7}{10}$ of 3411.22 = ? [*IBPS-PO-2012*]
(a) 4810 (b) 4980
(c) 4890 (d) 4930
(e) 4850

58. 23% of 6783 + 57% of 8431 = ? [*IBPS-PO-2012*]
(a) 6460 (b) 6420
(c) 6320 (d) 6630
(e) 6360

59. $335.01\times244.99\div55=?$ [*IBPS-PO-2012*]
(a) 1490 (b) 1550
(c) 1420 (d) 1590
(e) 1400

60. Rachita enters a shop to buy ice-creams, cookies and pastries. She has to buy atleast 9 units of each. She buys more cookies than ice-creams and more pastries than cookies. She picks up a total of 32 items. How many cookies does she buy ? [*IBPS-PO-2012*]
(a) Either 12 or 13 (b) Either 11 or 12
(c) Either 10 or 11 (d) Either 9 or 11
(e) Either 9 or10

61. With a two digit prime number, if 18 is added, we get another prime number with digits reversed. How many such numbers are possible? [*SSC CGL-2012*]
(a) 2 (b) 3
(c) 0 (d) 1

62. If $x=\frac{4ab}{a+b}$, then the value of [*SSC CGL-2012*]
$\frac{x+2a}{x-2a}+\frac{x+2b}{x-2b}$ is
(a) a (b) b
(c) 0 (d) 2

63. If $x = 997$, $y = 998$, $z = 999$, then the value of $x^2 + y^2 + z^2 - xy - yz - zx$ will be [*SSC CGL-2012*]
(a) 3 (b) 9
(c) 16 (d) 4

64. If $a + b + c = 8$, then the value of [*SSC CGL-2012*]
$(a-4)^3 + (b-3)^3 + (c-1)^3 - 3(a-4)(b-3)(c-1)$ is
(a) 2 (b) 4
(c) 1 (d) 0

65. If $x=\sqrt{a}+\frac{1}{\sqrt{a}}$, $y=\sqrt{a}-\frac{1}{\sqrt{a}}$, then the value of $x^4+y^4-2x^2y^2$ is [*SSC CGL-2012*]
(a) 16 (b) 20
(c) 10 (d) 5

66. If $5a+\frac{1}{3a}=5$, then the value of $9a^2+\frac{1}{25a^2}$ is [*SSC CGL-2012*]
(a) $\frac{51}{5}$ (b) $\frac{29}{5}$
(c) $\frac{52}{5}$ (d) $\frac{39}{5}$

67. If $x=3+2\sqrt{2}$, then the value of $\sqrt{x}-\frac{1}{\sqrt{x}}$ is [*SSC CGL-2012*]
(a) $\pm2\sqrt{2}$ (b) ±2
(c) $\pm\sqrt{2}$ (d) $\pm\frac{1}{2}$

68. If $a+b+c=0$, the value of [*SSC CGL-2012*]
$\left(\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}\right)$ is
(a) 2 (b) 3
(c) 4 (d) 5

69. If *a, b, c* are real and $a^3 + b^3 + c^3 = 3abc$ and $a+b+c\neq0$, then the relation between a, b, c will be [*SSC CGL-2012*]
(a) $a+b=c$ (b) $a+c=b$
(c) $a=b=c$ (d) $b+c=a$

70. If $a = 2$, $b = 3$, then $(a^b + b^a)^{-1}$ is [*SSC CGL-2013*]
(a) $\frac{1}{31}$ (b) $\frac{1}{17}$
(c) $\frac{1}{21}$ (d) $\frac{1}{13}$

71. The smallest positive integer which when multiplied by 392, gives a perfect square is [*SSC CGL-2013*]
(a) 2 (b) 3
(c) 5 (d) 7

72. Divide 81 into three parts so that $\frac{1}{2}$ of 1st, $\frac{1}{3}$ of 2nd and $\frac{1}{4}$ of 3rd are equal. [*SSC CGL-2013*]
(a) 36, 27, 18 (b) 27, 18, 36
(c) 18, 27, 36 (d) 30, 27, 24

73. The expression $x^4 - 2x^2 + k$ will be a perfect square when the value of k is [*SSC CGL-2013*]
(a) 1 (b) 2
(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

74. If $3x - \frac{1}{4y} = 6$, then the value of $4x - \frac{1}{3y}$ is [*SSC CGL-2013*]
(a) 2 (b) 4
(c) 6 (d) 8

75. If $a + b + c = 0$, find the value of $\frac{a+b}{c} - \frac{2b}{c+a} + \frac{b+c}{a}$. [*SSC CGL-2013*]
(a) 0 (b) 1
(c) – 1 (d) 2

76. If $x + \frac{4}{x} = 4$, find the value of $x^3 + \frac{4}{x^3}$. [*SSC CGL-2013*]
(a) 8 (b) $8\frac{1}{2}$
(c) 16 (d) $16\frac{1}{2}$

77. If $x = 3 + 2\sqrt{2}$, then the value of $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)$ is [*SSC CGL-2013*]
(a) 1 (b) 2
(c) $2\sqrt{2}$ (d) $3\sqrt{3}$

78. If 'a' be a positive number, then the least value of $a + \frac{1}{a}$ is [*SSC CGL-2013*]
(a) 1 (b) 0
(c) 2 (d) $\frac{1}{2}$

79. Arrange the following in ascending order $3^{34}, 2^{51}, 7^{17}$, we get [*SSC CGL-2014*]
(a) $3^{34} > 2^{51} > 7^{17}$ (b) $7^{17} > 2^{51} > 3^{34}$
(c) $3^{34} > 7^{17} > 2^{51}$ (d) $2^{51} > 3^{34} > 7^{17}$

80. If $x = 2 + \sqrt{3}$, then $x^2 + \frac{1}{x^2}$ is equal to [*SSC CGL-2014*]
(a) 10 (b) 12
(c) –12 (d) 14

81. If a = 4.965, b = 2.343 and c = 2.622, then the value of $a^3 - b^3 - c^3 - 3abc$ is [*SSC CGL-2014*]
(a) –2 (b) –1
(c) 0 (d) 9.93^2

82. If $x + y + z = 0$, then the value of $\frac{x^2 + y^2 + z^2}{x^2 - yz}$ is [*SSC CGL-2014*]
(a) –1 (b) 0
(c) 1 (d) 2

83. In an examination, a boy was asked to multiply a given number by $\frac{7}{19}$. By mistake, he divided the given number by $\frac{7}{19}$ and got a result 624 more than the correct answer. The sum of digits of the given number is [*SSC CGL-2014*]
(a) 10 (b) 11
(c) 13 (d) 14

84. If $a^2 + b^2 + c^2 = 2a - 2b - 2$, then the value of $3a - 2b + c$ is [*SSC CGL-2014*]
(a) 0 (b) 3
(c) 5 (d) 2

85. If $a + b + c = 3$, $a^2 + b^2 + c^2 = 6$ and $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$, where a, b, c are all non-zero, then 'abc' is equal to [*SSC CGL-2014*]
(a) $\frac{2}{3}$ (b) $\frac{3}{2}$
(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

86. If $a^2 - 4a - 1 = 0$, $a \neq 0$, then the value of $a^2 + 3a + \frac{1}{a^2} - \frac{3}{a}$ is [*SSC CGL-2014*]
(a) 24 (b) 26
(c) 28 (d) 30

Hints & Solutions

Level-I

1. **(d)** Given Exp. = $a^2 + b^2 - 2ab$, where $a = 287$ and $b = 269 = (a-b)^2 = (287-269)^2 = (18)^2 = 324$.

2. **(d)** $20 \times x = (64+36)(64-36) = 100 \times 28$

$$\Rightarrow x = \frac{100 \times 28}{20} = 140.$$

3. **(c)** $\frac{1}{\sqrt{3}+\sqrt{2}} = \frac{1}{(\sqrt{3}+\sqrt{2})} \times \frac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})} = \left(\frac{\sqrt{3}-\sqrt{2}}{3-2}\right)$

$$= (\sqrt{3}-\sqrt{2}) = (1.732 - 1.414) = 0.318$$

4. **(a)** Given expression $= \sqrt{0.01+0.08} = \sqrt{0.09} = 0.3$

5. **(a)** $356 \times 936 - 356 \times 836 = 356 \times (936-836)$
$= 356 \times 100 = 35600$

6. **(a)** $\dfrac{\frac{1}{2} \div \frac{1}{2} \times \frac{1}{2}}{\frac{1}{2} + \frac{1}{2} \times \frac{1}{2}} = \dfrac{\frac{1}{2} \times 2 \times \frac{1}{2}}{\frac{3}{4}} = \frac{1}{2} \times \frac{4}{3} = \frac{2}{3}$

7. **(b)** Given exp.

$$= \frac{a^2-b^2}{a+b} = a - b = \left(1 + \frac{1}{1+\frac{1}{100}}\right) - \left(1 - \frac{1}{1+\frac{1}{100}}\right)$$

$$= 2 \times \frac{1}{(101/100)} = 2 \times \frac{100}{101} = \frac{200}{101}$$

8. **(a)** $5^a = 3125 \Rightarrow 5^a = 5^5 \Rightarrow a = 5$
$\Rightarrow 5^{(a-3)} = 5^{(5-3)} = 5^2 = 25$

9. **(d)** Let the number of buffaloes be x and the number of ducks be y.
Then, $4x + 2y = 2(x+y) + 24 \Leftrightarrow 2x = 24 \Leftrightarrow x = 12$.

10. **(b)** $\sqrt[3]{4\frac{12}{125}} = \sqrt[3]{\frac{512}{125}} = \left(\frac{8 \times 8 \times 8}{5 \times 5 \times 5}\right)^{1/3} = \frac{8}{5} = 1\frac{3}{5}$

11. **(c)** $729 = 9^3 = 3^6$, Now $4X - 2 = 6$ or $X = 2$.

12. **(a)** $16a^2 - 12a = (4a)^2 - 2(4a)(3/2)$
$\therefore$ The number is $(3/2)^2 = (9/4)$.

13. **(b)** By rationalization we have

$$\left[\frac{1}{\sqrt{9}-\sqrt{8}}\right] = \left[\frac{1}{\sqrt{9}-\sqrt{8}}\right] \times \frac{\sqrt{9}+\sqrt{8}}{\sqrt{9}+\sqrt{8}} = \frac{\sqrt{9}+\sqrt{8}}{9-8}$$

$$= \sqrt{9}+\sqrt{8}$$

Similarly, $\left[\frac{1}{\sqrt{8}-\sqrt{7}}\right] = \sqrt{8}+\sqrt{7}$ and $\frac{1}{\sqrt{7}-\sqrt{6}} = \sqrt{7}+\sqrt{6}$

and so on. The given expression
$= (\sqrt{9}+\sqrt{8}) - (\sqrt{8}+\sqrt{7}) + (\sqrt{7}+\sqrt{6}) - (\sqrt{6}+\sqrt{5}) + (\sqrt{5}+\sqrt{4})$
$= \sqrt{9}+\sqrt{4} = 3 + 2 = 5$.

14. **(b)** $5\sqrt[3]{250} + 7\sqrt[3]{16} - 14\sqrt[3]{54}$

$$= 5\sqrt[3]{125 \times 2} + 7\sqrt[3]{8 \times 2} - 14\sqrt[3]{27 \times 2}$$

$$= 5 \times 5\sqrt[3]{2} + 7 \times 2\sqrt[3]{2} - 14 \times 3 \times \sqrt[3]{2}$$

$$= (25 + 14 - 42)\sqrt[3]{2} = -3\sqrt[3]{2}$$

15. **(a)** Work from the choices: only 169 when reversed becomes 961 and both numbers are squares.

16. **(b)** $1*2 = 1 + 2 \times 6 = 13$
$13*3 = 13 + 3 \times 6 = 31$

17. **(a)** $a^b = 125 \Rightarrow a^b = 5^3$
$\therefore a = 5, b = 3$
$(a-b)^{a+b-4} = (5-3)^{5+3-4} = 2^4 = 16$

18. **(c)** $p \times q = p + q + \frac{p}{q} \Rightarrow 8 \times 2 = 8 + 2 + \frac{8}{2} = 14$

19. **(b)** $x*y = x^2 + y^2 = xy$
$9*11 = 9^2 + 11^2 - 9 \times 11$
$= 81 + 121 - 99 = 103$

20. **(d)** Since the factors of 11760 are $\underline{2 \times 2} \times \underline{2 \times 2} \times 3 \times 5 \times \underline{7 \times 7}$ so we need to multiply it with 3×5 because all the factos are paired but 3 and 5 are unpaired, hence (d) is the correct choice.

21. **(b)** $5\sqrt{5} \times 5^3 \div 5^{-3/2} = 5^{(a+2)}$

$$5^1 \times 5^{\frac{1}{2}} \times 5^3 \times 5^{3/2} = 5^{a+2}$$

$$5^{1+\frac{1}{2}+3+\frac{3}{2}} = 5^{a+2},\ 5^{\frac{12}{2}} = 5^{a+2},\ a + 2 = 6$$

$\therefore a = 4$

22. **(b)** Let number be x

$$x \times \frac{4}{5} \times \frac{3}{4} - x \times \frac{2}{5} \times \frac{1}{6} = 648$$

$$\frac{3x}{5} - \frac{x}{15} = 648$$

$$\frac{9x - x}{15} = 648$$

$$8x = 648 \times 15 \Rightarrow x = \frac{648 \times 15}{8} = 81 \times 15 = 1215$$

23. **(b)** If sum of two is even, their difference is always even, So (b) is right answer.

24. **(a)** Number of decimal places in the given expression = 8
Number of decimal places in (a) = 8
Number of decimal places in (b) = 9
Number of decimal places in (c) = 7.
Clearly, the expression in (a) is the same as the given expression.

25. **(c)** **26.** **(b)**

27. **(c)** $\frac{a+b}{b+c}=\frac{c+d}{d+a}$

$\Rightarrow ad+a^2+bd+ab=bc+c^2+bd+cd$

$\Rightarrow (a^2-c^2)+(ad-cd)+(ab-bc)=0$

$\Rightarrow (a-c)(a+c+d+b)=0$

$\Rightarrow a=c$ or $a+b+c+d=0$ or both

28. **(d)** Sum is 888 $\Rightarrow$ unit's digit should add up to 8. This is possible only for 4th option as "3" + "5" = "8".

29. **(b)** The numbers that can be formed are xy and yx. Hence $(10x + y) + (10y + x) = 11(x + y)$. If this is a perfect square then $x + y = 11$.

30. **(a)** Comparing $4^{1/3}$ and $5^{1/4}$
$(4^{1/3})^{12}$ and $(5^{1/4})^{12}$ *i.e.,* 4^4 and 5^3
$=256>125$
$\therefore \quad 4^{1/3}>5^{1/4}$
Similarly, comparing $5^{1/4}$ and $6^{1/5}$
$(5^{1/4})^{20}$ and $(6^{1/5})^{20}$ *i.e.,* 5^5 and 6^4 $=3125>1296$
$\therefore \quad 5^{1/4}>6^{1/5}$

31. **(b)** $(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$
$\Rightarrow \quad 2(ab+bc+ca)=(a+b+c)^2-(a^2+b^2+c^2)$
$=169-69=100$
$ab+bc+ca=50$

32. **(b)** $|a-b|=|8|=8 \Rightarrow |b-a|=|-8|=8$

$\Rightarrow |a-b|-|b-a|=8-8=0$

33. **(d)** At a value of $x = 0$ we can see that the expression $x^2 + |x - 1| = 1 \rightarrow 0 + 1 = 1$. Hence, $x = 0$ satisfies the given expression. Also at $x = 1$, we get $1 + 0 = 1$.

34. **(d)** $3.6+36.6+3.66+0.36+3.0=47.22$

35. **(a)** $23\times 45\div 15=69$

36. **(b)** $4\frac{5}{6}+7\frac{1}{2}-5\frac{8}{11}=\frac{29}{6}+\frac{15}{2}-\frac{63}{11}$

$=\frac{319+495-378}{66}=\frac{436}{66}=\frac{218}{33}=6\frac{20}{33}$

37. **(e)** $\frac{210}{14}\times\frac{17}{15}\times ? = 4046$

$?=\frac{4046\times 15\times 14}{210\times 17}=238$

38. **(b)** 83% of 2350 = ?

$?=\frac{83\times 2350}{100}=1950.50$

39. **(b)** $\sqrt{1089}+3=(?)^2$

$33+3=(6)^2$

40. **(b)** $96+32\times 5-31=225$

41. **(d)** $?\div 36=(7)^2-8$

$\frac{?}{36}=49-8$

$\therefore \quad ?=36\times 41$
$=1476$

42. **(d)** $\sqrt{8281}=?=91$

43. **(d)** $(63)^2-(12)^2=3825$

44. **(e)** $\frac{9}{4}+\frac{18}{5}=?-\frac{43}{10}$

$?=\frac{9}{4}+\frac{18}{5}+\frac{43}{10}$

$=\frac{45+72+86}{20}=\frac{203}{20}=10\frac{3}{20}$

45. **(a)** $17\times 19\times 4\div ?=161.5$

$\frac{1}{?}=\frac{1615}{10\times 17\times 19\times 4}$

$?=8$

46. **(b)** $1798\div 31\times ?=348$

$?=\frac{348\times 31}{1798}=6$

47. **(d)** $(9.8\times 2.3+4.46)\div 3=3^?$
$27\div 3=3^? \quad 3^2=3^?$
$\therefore \quad ?=2$

48. **(d)** 43% of 600 + ?% of 300 = 399
$43\times 6+3x=399$
$3x=141$
$x=47$

49. **(e)** $x+(x+1)+(x+2)=1383$
$\Rightarrow 3x+3=1383$
$\Rightarrow 3x=1380$

$\Rightarrow x=\frac{1380}{3}=460$

Largest number $=x+2=462$

50. **(e)** $1504\times 5.865-24.091=?$
$\therefore \quad ?=8796.869\approx 8800$

51. **(c)** $16.928+(24.7582\div 5.015)=?$
$16.928+(4.93)=?$
$\therefore \quad ?=21.86\approx 22$

52. **(a)** $?=\sqrt[3]{7.938}\times(6.120)^2-4.9256$
$=(2\times 37.4)-4.9256$
$=74.8-4.9256$
$\approx 70 \quad \therefore\ ?\approx 70$

53. **(a)** $16.046\div 2.8\times 0.599=?$
$(5.73)\times 0.599=?$
$\therefore \quad ?=3.43\approx 3.5$

54. **(d)** $\sqrt{963}+(4.895)^2-9.24=?$
$31+23.9-9.24=?$
$54.91-9.24=?$
$\therefore \quad ?=45.6\approx 45$

55. **(b)** $(12\times 19)+(13\times 8)=(15\times 14)+?$
$228+104=210+?$
$\therefore \quad ?=122$

56. **(c)** $\sqrt{65\times 12-50+54}=?$
$\Rightarrow \sqrt{780-50+54}=?$
$\therefore \quad ?=28$

57. **(c)** $15\%\text{ of }524-2\%\text{ of }985+?=20\%\text{ of }423$

$$\frac{15\times524}{100}-\frac{2\times985}{100}+?=\frac{20\times423}{100}$$

$78.6-19.7-84.6=-?$

$\therefore\quad ?=25.7.$

58. **(e)** $151\times8+(228\div19)^2=?$

$1208+(12)^2=?$

$1208+144=?$

$\therefore\quad ?=1352$

59. **(c)** $\sqrt{1521}+\sqrt{225}=?$

$39+15=?$

$\therefore\quad ?=54$

60. **(d)** $38.734+8.638-5.19=?$

$47.372-5.19=?$

$\therefore\quad ?=42.182$

61. **(a)** $7^{8.9}\div(343)^{1.7}\times(49)^{4.8}=7^?$

$$\frac{7^{8.9}}{7^{3\times1.7}}\times7^{2\times4.8}=7^?,$$

$$\frac{7^{8.9}}{7^{5.1}}\times7^{9.6}=7^?$$

$7^{8.9-5.1+9.6}=7^?,$

$\therefore\quad ?=13.4$

62. **(e)** $\sqrt[3]{512}\div\sqrt[4]{16}+\sqrt{576}=?$

$(8\div2)+24=?$

$4+24=?$

$\therefore\quad ?=28$

63. **(b)** $(42\times3.2)\div(16\times1.5)=?$

$134.4\div24=?$

$\therefore\quad ?=5.6$

64. **(e)** $(199+((5^3\div4)\times4^2))=?$

$(199+(31.25\times4^2)=?$

$(199+500)=?$

$\therefore\quad ?=699$

65. **(e)** $(342\div6)\times28=1099+?$

$57\times28=1099+?$

$1596-1099=?$

$\therefore\quad ?=497$

66. **(d)** $\frac{9.8\times2.5\times7.6}{0.5}=?$

$$=\frac{186.2}{0.5}=?$$

$\therefore\quad ?=372.4$

67. **(d)** $\frac{3}{5}\text{ of }\frac{2}{7}\text{ of }?=426$

$$?=\frac{5}{3}\text{ of }\frac{7}{2}\text{ of }426$$

$$?=\frac{5}{3}\text{ of }1491$$

$\therefore\quad ?=2485$

68. **(a)** $3\frac{2}{5}+1\frac{2}{9}=4\frac{4}{5}-?$

$$\frac{17}{5}+\frac{11}{9}=\frac{24}{5}-?$$

$$\frac{17}{5}+\frac{11}{9}-\frac{24}{5}=-?$$

$$\frac{153+55-216}{45}=-?$$

$$\therefore\quad ?=\frac{8}{45}$$

69. **(b)** $\left(\frac{13}{63}\div\frac{104}{14}\right)\times\frac{52}{19}=?$

$$\left(\frac{13}{63}\times\frac{14}{104}\right)\times\frac{52}{19}=?$$

$\therefore\quad ?=13/171$

70. **(a)** $\sqrt[3]{13824}\times\sqrt{?}=864$

$\sqrt[3]{24\times24\times24}\times\sqrt{?}=864$

$\Rightarrow 24\times\sqrt{?}=864$

$$\Rightarrow\sqrt{?}=\frac{864}{24}$$

$\therefore ?=36\times36=1296$

71. **(b)** $(91)^2+(41)^2-\sqrt{?}=9858$

$\Rightarrow 8281+1681-\sqrt{?}=9858$

$\Rightarrow\sqrt{?}=9962-9858=104$

$\therefore ?=104\times104=10816$

72. **(b)** $?=4900\div28\times444\div12$

$\Rightarrow ?=175\times37$

$\Rightarrow ?=6475$

73. **(e)** $125\%\text{ of }260+?\%\text{ of }700=500$

$\Rightarrow\ ?\%\text{ of }700=500-125\%\text{ of }260$

$\Rightarrow\ ?\%\text{ of }700=175$

$$\therefore\quad ?=\frac{175\times100}{700}=25$$

74. **(b)** $3\frac{7}{11}+7\frac{3}{11}\times1\frac{1}{2}=\frac{40}{11}+\frac{80}{11}\times\frac{3}{2}=\frac{160}{11}=14\frac{6}{11}$

75. **(c)** Given Expression $=\frac{0.207}{\frac{0.0023}{23}}=\frac{0.207}{0.0001}=\frac{0.2070}{0.0001}=2070.$

76. **(b)** $?=1.05\%\text{ of }2500+2.5\%\text{ of }440$

$$\Rightarrow ?=\frac{1.05}{100}\times2500+\frac{2.5}{100}\times440$$

$$\Rightarrow ?=\frac{2625}{100}+\frac{1100}{100}$$

$$\Rightarrow ?=\frac{3725}{100}=37.25$$

77. **(a)** $? = \dfrac{17 \times 4 + 4^2 \times 2}{90 \div 5 \times 12}$

$\Rightarrow ? = \dfrac{68 + 16 \times 2}{18 \times 12}$

$\Rightarrow ? = \dfrac{68 + 32}{216}$

$\Rightarrow ? = \dfrac{100}{216} = \dfrac{25}{54}$

78. **(b)** $\dfrac{87}{5} \times \dfrac{37}{8} - ? = \dfrac{375}{8}$

$\Rightarrow ? = \dfrac{3219}{40} - \dfrac{375}{8}$

$= \dfrac{3219 - 1875}{40} = \dfrac{1344}{40}$

$= \dfrac{168}{5} = 33\dfrac{3}{5}$

79. **(e)** $\dfrac{250 \times 136}{100} + \dfrac{550 \times ?}{100} = 670$

$\Rightarrow 340 + 5.5 \times ? = 670$

$\Rightarrow 5.5 \times ? = 670 - 340 = 330$

$\Rightarrow ? = \dfrac{330}{5.5} = 60$

80. **(a)** $3889 + 12.952 - ? = 3854.002$

or $? = 3889 + 12.952 - 3854.002 = 47.95$

81. **(b)** $(25)^? = (5 \times 5 \times 5 \times 5 \times 5 \times 5)^4 \times (5 \times 5)^6 \div (5)^2$

$= (25 \times 25 \times 25)^4 \times (25)^6 \div (25)^1$

$= (25^3)^4 \times (25)^6 \div 25^1 = (25)^{12} \times (25)^6 \div (25)^1$

$= (25)^{12+6-1} = (25)^{17}$

$\therefore \quad ? = 17$

82. **(c)** $? = \dfrac{28 \times 5 - 15 \times 6}{7^2 + \sqrt{256} + (13)^2}$

$\Rightarrow ? = \dfrac{140 - 90}{49 + 16 + 169}$

$\Rightarrow ? = \dfrac{50}{234} = \dfrac{25}{117}$

83. **(c)** $1.5 \times 0.025 + (?)^2 = 0.1 \Rightarrow (?)^2 = 0.1 - 1.5 \times 0.025$

$\Rightarrow (?)^2 = 0.1 - 0.0375 \Rightarrow ? = \sqrt{.0625} = 0.25$

84. **(d)** Given Expression $= \dfrac{(a-b)^2 + (a+b)^2}{(a^2 + b^2)} = \dfrac{2(a^2 + b^2)}{(a^2 + b^2)} = 2$

85. **(b)** $? = \dfrac{(10008.99)^2}{10009.001} \times \sqrt{3589} \times 0.4987$

$= (10009) \times \sqrt{3600} \times 0.50$

$= 10009 \times 60 \times 0.50 \approx 300000$

86. **(a)** $196.1 \times 196.1 \times 196.1 \times 4.01 \times 4.01 \times 4.001 \times 4.999 \times 4.999$

$= (196.1)^3 \times 4 \times ?$

or $4 \times ? = 4.01 \times 4.001 \times 4.999 \times 4.999$ or $? = 4 \times 5 \times 5 = 100$

87. **(d)** $\because 12.25 \times ? \times 21.6 = 3545.64$

$\because ? = \dfrac{3545.64}{264.6} = 13.4 \approx 13$

88. **(c)** Let x be there in place of question mark so, x% of $45.999 \times 16\%$ of $83.006 = 116.073$.

We get, $\dfrac{x}{100} \times 46 \times \dfrac{16}{100} \times 83 = 116$

$x \times 0.46 \times 13.28 = 116$

or $x \times 6.11 = 116$

$\Rightarrow x = 18.98 \approx 19.$

89. **(d)** $? = \dfrac{1.69 \times 17.64}{2.7} = 11.04 \approx 11$

90. **(c)** 3 23 43 [63] 83 103

+20 +20 +20 +20 +20

91. **(d)** 1 9 25 49 81 [121] 169

↑ ↑ ↑ ↑ ↑ ↑ ↑

1^2 3^2 5^2 7^2 9^2 11^2 13^2

92. **(e)** Pattern of the series would be as follows

$5 \times 1 + 1 = 6$

$6 \times 2 + 2 = 14$

$14 \times 3 + 3 = 45$

$\therefore \quad 45 \times 4 + 4 = 184$

93. **(e)** The pattern of the number series is:

$7 \times 1 + 1 = 8$

$8 \times 2 + 2 = 18$

$18 \times 3 + 3 = 57$

$57 \times 4 + 4 = \boxed{232}$

94. **(b)** Can you see that the pattern is

$1^2, 2^3, 3^2, 4^3, 5^2, 6^3, 7^2$

95. **(c)** By going options, 26 years is the present age. Present age be 26, then last year age was 25 which represents a perfect square and next year age would be 27 which represents a cubic number.

96. **(a)** Expression is $(2.1)^2 \times \sqrt{0.0441} = 4.41 \times 0.21 = 0.9261$

97. **(b)** $\sqrt[3]{1372} \times \sqrt[3]{1458}$

$= 7\sqrt[3]{4} \times 9\sqrt[3]{2} = 63 \times \sqrt[3]{4 \times 2} = 63 \times 2 = 126$

98. **(a)** Amounts of water in two jars are equal; the jar with the greater capacity is $\dfrac{1}{4}$ full, and the Jar with lesser capacity is $\dfrac{1}{3}$ full.

$\therefore$ When the water in smaller jar is poured into the larger Jar, the addition of an equal amount of water will double the amount in the larger jar, which will then be $2\times\frac{1}{4}=\frac{1}{2}$ full.

99. (c) $\frac{5x-3}{x}+\frac{5y-3}{y}+\frac{5z-3}{z}=0$

$\frac{5x}{x}-\frac{3}{x}+\frac{5y}{y}-\frac{3}{y}+\frac{5z}{z}-\frac{3}{z}=0$

$5-\frac{3}{x}+5-\frac{3}{y}+5-\frac{3}{z}=0$

$-3\left[\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right]+15=0$

$-3\left[\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right]=-15$

$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{-15}{-3}=5$

100. (b) $x^2+\frac{1}{x^2+1}-3$

is minimum when $x=0$

$0+\frac{1}{0+1}-3=-2$

101. (c) $a+b=5$

Squaring on both sides

$(a+b)^2=(5)^2$

$a^2+b^2+2ab=25$

$13+2ab=25$

$2ab=25-13=12$ ... (1)

Again, $a^2+b^2=13$

Subtracting (– 2ab) from both sides

$a^2+b^2-2ab=13-2ab$

$(a-b)^2=13-12$ from equation (1)

$(a-b)^2=1$

TRICK $\Rightarrow a=3$

$b=2\ (a>b)$

$a-b=1$

102. (a) $\frac{3x-y}{x+5y}=\frac{5}{7}\Rightarrow 21x-7y=5x+25y$

$\Rightarrow 16x=32y$

$\Rightarrow x=2y$ or $\frac{x}{y}=\frac{2}{1}$, ... (1)

Now, to calculate value of $\frac{x+y}{x-y}$, divide numerator & denominator by y.

$\Rightarrow \frac{\frac{x}{y}+1}{\frac{x}{y}-1}$

Putting value of $\frac{x}{y}$ from equation (1)

$\frac{\frac{2}{1}+1}{\frac{2}{1}-1}=\frac{3}{1}$ or 3:1

103. (c) $1+\frac{1}{1+\frac{2}{\frac{15+4}{5}}}=1+\frac{1}{1+\frac{2\times5}{19}}$

$=1+\frac{1}{\frac{19+10}{19}}=1+\frac{19}{29}=\frac{29+19}{29}=\frac{48}{29}$

104. (c) $\sqrt{19.36}+\sqrt{0.1936}+\sqrt{0.001936}+\sqrt{0.00001936}$

$=4.4+0.44+0.044+0.0044=4.8884$

105. (c) Let the number be x and y.

According to question,

$(x+y)^2=4xy$

$\Rightarrow x^2+y^2+2xy-4xy=0$

$\Rightarrow (x-y)^2=0$

$\Rightarrow x=y$

106. (c) $a^2+b^2=5ab$

$\Rightarrow \frac{a^2+b^2}{ab}=5$

$\Rightarrow \frac{a}{b}+\frac{b}{a}=5$

On squaring both sides.

$\therefore \left(\frac{a}{b}+\frac{b}{a}\right)^2=25$

$\Rightarrow \frac{a^2}{b^2}+\frac{b^2}{a^2}+2=25$

$\Rightarrow \frac{a^2}{b^2}+\frac{b^2}{a^2}=25-2=23$

107. (d) $x=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}=\frac{(\sqrt{3}-\sqrt{2})(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$

$=\frac{(\sqrt{3}-\sqrt{2})^2}{3-2}=3+2-2\sqrt{3}.\ \sqrt{2}=5-2\sqrt{6}$

$\therefore y=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}=5+2\sqrt{6}$

$\therefore x+y=5-2\sqrt{6}+5+2\sqrt{6}=10$

$xy=(5-2\sqrt{6})\cdot(5+2\sqrt{6})$

$=25-24=1$

$\therefore x^3+y^3=(x+y)^3-3xy\,(x+y)$

$=(10)^3-3(10)=1000-30=970$

108. (d) LCM of 3, 2 and 6 = 6

$\therefore (3)^{\frac{1}{3}} = (3^2)^{\frac{1}{6}} = (9)^{\frac{1}{6}}$

$2^{\frac{1}{2}} = (2^3)^{\frac{1}{6}} = (8)^{\frac{1}{6}}$

$(1)^{\frac{1}{6}} = 1(6)^{\frac{1}{6}} = (6)^{\frac{1}{6}}$

109. (b) $\dfrac{\sqrt{24}+\sqrt{6}}{\sqrt{24}-\sqrt{6}} = \dfrac{2\sqrt{6}+\sqrt{6}}{2\sqrt{6}-\sqrt{6}} = \dfrac{3\sqrt{6}}{\sqrt{6}} = 3$

110. (b) $3 \div \left[3 \div \left\{2 \div \dfrac{34}{13}\right\}\right]$

$3 \div \left[3 \div 2 \times \dfrac{13}{34}\right]$

$3 \div \left[3 \times \dfrac{34}{2 \times 13}\right]$

$\dfrac{3 \times 2 \times 13}{3 \times 34} = \dfrac{13}{17}$

111. (d) $9 + 3 \div 4 - 8 \times 2 = ?$

Applying rules

$9 \div 3 \times 4 + 8 - 2 = ?$

$3 \times 4 + 8 - 2 = ?$

$20 - 2 = ?$

$? = 18$

112. (a) Next term will be

$\left(1+\dfrac{1}{2}\right)\left(1+\dfrac{1}{3}\right)\left(1+\dfrac{1}{4}\right)\left(1+\dfrac{1}{5}\right)$

$= \dfrac{3}{2} \times \dfrac{4}{3} \times \dfrac{5}{4} \times \dfrac{6}{5} = 3$

113. (b) $2a^2 - 5ab + 2b^2$

$2(a^2 - 2ab + b^2) - ab$

$2(a - b)^2 - ab$

$2\left[\sqrt{6}+\sqrt{5}-\sqrt{6}+\sqrt{5}\right]^2 - \left(\sqrt{6}+\sqrt{5}\right)\left(\sqrt{6}-\sqrt{5}\right)$

$2 \times 4 \times 5 - 1 = 39$

114. (c) $27P^3 - \dfrac{1}{216} - \dfrac{9}{2}P^2 + \dfrac{1}{4}P$

$= (3P)^3 - \left(\dfrac{1}{6}\right)^3 - 3.(3P)^2.\dfrac{1}{6} + 3.3P.\left(\dfrac{1}{6}\right)^2$

$= \left(3P - \dfrac{1}{6}\right)^3$

$= \left(3 \times \dfrac{5}{18} - \dfrac{1}{6}\right)^3 = \dfrac{8}{27}$

115. (d) $x + \dfrac{1}{x} = 2$

$x^2 - 2x + 1 = 0; (x - 1)^2 = 0; x = 1$

$x^{2013} + \dfrac{1}{x^{2014}} = 1 + 1 = 2$

116. (d) Here, $a + b + c = 0$

$\therefore a^3 + b^3 + c^3 - 3abc = 0$

117. (d)

$\left[\left(\sqrt{6}-\sqrt{35}\right)+\left(\sqrt{10}-\sqrt{21}\right)\right]\left[\left(\sqrt{6}-\sqrt{35}\right)-\left(\sqrt{10}-\sqrt{21}\right)\right]$

$= \left(\sqrt{6}-\sqrt{35}\right)^2 - \left(\sqrt{10}-\sqrt{21}\right)^2$

$= 6 + 35 - 2\sqrt{6}.\sqrt{35} - 10 - 21 + 2\sqrt{10}.\sqrt{21}$

$= 10 - 2\sqrt{210} + 2\sqrt{210}$

$= 10$

118. (b) $x + y + z = a - b + b - c + c - a = 0$

$\therefore x^3 + y^3 + z^3 - 3xyz = 0$

119. (c) $\dfrac{\sqrt{32}+\sqrt{48}}{\sqrt{8}+\sqrt{12}} = \dfrac{\sqrt{2\times2\times2\times2\times2}+\sqrt{2\times2\times2\times2\times3}}{\sqrt{2\times2\times2}+\sqrt{2\times2\times3}}$

$\Rightarrow \dfrac{4\sqrt{2}+4\sqrt{3}}{2\sqrt{2}+2\sqrt{3}} = \dfrac{2(2\sqrt{2}+2\sqrt{3})}{(2\sqrt{2}+2\sqrt{3})} = 2$

120. (b) $\sqrt{\dfrac{9.5 \times 0.085}{0.0017 \times 0.19}} = \sqrt{\dfrac{95}{10} \times \dfrac{85}{1000} \times \dfrac{10000}{17} \times \dfrac{100}{19}}$

$\Rightarrow \sqrt{5 \times 5 \times 100} = 50$

121. (a) $\dfrac{x}{b+c} = \dfrac{y}{c+a} = \dfrac{z}{a+b} = k(\text{say})$

So, $x = k(b + c)$

$\Rightarrow x - y = k(b + c) - k(c + a)$

$= k(b - a)$

$y = k(c + a)$

$\Rightarrow y - z = k(c + a) - k(a + b)$

$= k(c - b)$

$z = k(a + b) \Rightarrow z - x = k(a + b) - k(b + c) = k(a - c)$

So,check option (a)

$\dfrac{x-y}{b-a} = \dfrac{y-z}{c-b} = \dfrac{z-x}{a-c}$

$\dfrac{k(b-a)}{b-a} = \dfrac{k(c-b)}{c-b} = \dfrac{k(a-c)}{a-c}$

$k = k = k$

option (a) is true.

122. (d) $\dfrac{547.527}{0.0082} = x \Rightarrow \dfrac{547527}{1000} \times \dfrac{10000}{82} = x$

$\Rightarrow \dfrac{547527}{82} = \dfrac{x \times 1000}{10000} \Rightarrow \dfrac{x}{10}$

123. (a) $\left[3^n\right]^{\frac{1}{3}} = 27$

$\Rightarrow \quad 3^{\frac{n}{3}} = 3^3$

Comparing, $\frac{n}{3} = 3$

x = 9

124. (b) Time difference between 9.00 A.M & 2.00 P.M = 5 hours
Temperature difference between 21°C & 36°C
= 36 – 21 = 15°C
Now, Time difference between 9.00 A.M & 12.00 Noon = 3 hrs.

In 5 hours $\xrightarrow[\text{difference}]{\text{temperature}} 15°C$

So, In 3 hours $\xrightarrow[\text{difference}]{\text{temperature}} \left(\frac{15}{5} \times 3\right) = 9°C$

So, temperature at noon = 21 + 9 = 30°C

125. (b) $\frac{3x+5}{5x-2} = \frac{2}{3}$

$\Rightarrow \quad 9x + 15 = 10x - 4$

$\Rightarrow \quad 15 + 4 = 10x - 9x$

$\Rightarrow \quad x = 19$

126. (d) Let the numbers are x, y.

$x - y = 3$ ...(1)

$x^2 - y^2 = 39$

$\Rightarrow \quad (x - y)(x + y) = 39$

$\Rightarrow \quad x + y = 13$ ...(2)

Adding eqn (1) and (2)

$x + y + x - y = 16$

$\Rightarrow \quad x = 8$

$\therefore \quad y = 3$

Hence, 8 is the larger number.

127. (c) $x = \sqrt{3} + \sqrt{2}$

$\frac{1}{x} = \frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{3}-\sqrt{2}}{3-2} = \sqrt{3} - \sqrt{2}$

Now, $x^3 - \frac{1}{x^3} = \left(\sqrt{3}+\sqrt{2}\right)^3 - \left(\sqrt{3}-\sqrt{2}\right)^3$

$= (a + b)^3 - (a - b)^3$ [Let $\sqrt{3}$ = a and $\sqrt{2}$ = b]

$= a^3 + b^3 + 3a^2b + 3b^2a - (a^3 - b^3 - 3a^2b + 3b^2a)$

$= a^3 + b^3 + 3a^2b + 3b^2a - a^3 + b^3 + 3a^2b - 3b^2a$

$= 2b^3 + 6a^2b = 2\left(\sqrt{2}\right)^3 + 6\left(\sqrt{3}\right)^2\left(\sqrt{2}\right)$

$= 4\sqrt{2} + 18\sqrt{2} = 22\sqrt{2}$

128. (a) $a^2 + b^2 + c^2 = 2(a - b - c) - 3$

$\Rightarrow \quad a^2 + b^2 + c^2 - 2(a - b - c) + 3 = 0$

$\Rightarrow \quad a^2 + b^2 + c^2 - 2a + 2b + 2c + 3 = 0$

$\Rightarrow \quad (a^2 + 1 - 2a) + (b^2 + 1 + 2b) + (c^2 + 1 + 2c) = 0$

$\Rightarrow \quad (a-1)^2 + (b+1)^2 + (c+1)^2 = 0$

This is possible when $(a-1)^2 = 0, (b+1)^2 = 0$ and $(c+1)^2 = 0.$

$\Rightarrow \quad a = 1, b = -1, c = -1$

Thus, $2a - 3b + 4c = 2(1) - 3(-1) + 4(-1)$
$= 2 + 3 - 4 = 1.$

129. (a) $\sqrt{6} = 2.44, \sqrt{5} = 2.23, \sqrt{3} = 1.73$

$a = \sqrt{6} - \sqrt{5} = 0.21$

$b = \sqrt{5} - 2 = 0.23$

$c = 2 - \sqrt{3} = 0.27$

130. (c) Given a = $\frac{b^2}{b-a}$ or $ab - a^2 = b^2$ or $ab = b^2 + a^2$

We know, $a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$

$\therefore$ (a + b) (ab – ab) $\Rightarrow$ 0 (using given)

131. (a) $\frac{1}{x^2 + xy + xz} + \frac{1}{y^2 + xy + zy} + \frac{1}{z^2 + yz + zx}$

$\frac{1}{x(x+y+z)} + \frac{1}{y(x+y+z)} + \frac{1}{z(x+y+z)}$

$\frac{xy + yz + zx}{xyz(x+y+z)} = 0 \quad (\because xy + yz + zx = 0)$

132. (d) $\sqrt{40 + \sqrt{9 \times 9}} = \sqrt{49} = 7$

133. (d) $\sqrt[3]{2} = 2^{1/3}$ or $2^{\frac{1}{3} \times \frac{2}{2}} = 2^{\frac{2}{6}} = \sqrt[6]{4}$

$\sqrt{3} = 3^{\frac{1}{2}}$ or $3^{\frac{1}{2} \times \frac{3}{3}} = 3^{\frac{3}{6}} = \sqrt[6]{27}$

$\sqrt{3} > \sqrt{2}$

134. (d) $a^2 + b^2 + c^2 = (a + b + c)^2 - 2(ab + bc + ca)$

$= 9^2 - 2(ab + bc + ca)$

$a^2 + b^2 + c^2$ will be minimum if ab + bc + ca is maximum.

$ab + bc + ca$ is maximum when $a = 3, b = 3,$ and $c = 3.$

$[\because a + b + c = 9]$

$\therefore$ minimum value of $a^2 + b^2 + c^2$

$= 81 - 2(3 \times 3 + 3 \times 3 + 3 \times 3)$

$= 81 - 54 = 27$

135. (b) $3+\frac{1}{\sqrt{3}}+\frac{1}{3+\sqrt{3}}+\frac{1}{\sqrt{3}-3}$

$\Rightarrow 3+\frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}+\frac{1}{3+\sqrt{3}}\times\frac{3-\sqrt{3}}{3-\sqrt{3}}+\frac{1}{\sqrt{3}-3}\times\frac{\sqrt{3}+3}{\sqrt{3}+3}$

$\Rightarrow \frac{3}{1}+\frac{\sqrt{3}}{3}+\frac{3-\sqrt{3}}{6}+\frac{\sqrt{3}+3}{-6}$

$\Rightarrow \frac{18+2\sqrt{3}+3-\sqrt{3}-\sqrt{3}-3}{6}$

$\Rightarrow \frac{18+2\sqrt{3}-2\sqrt{3}}{6} \quad \Rightarrow \quad 3$

136. (b) 228 – 18 = 210 is exactly divisible biggest two digit no. 70

137. (a) If they are equal number of rows and columns then,

$\sqrt{1369}=37$

138. (a) $\sqrt{5}+\sqrt{3}>\sqrt{6}+\sqrt{2}$

Squaring both sides

$5+3+2\sqrt{15}>6+2+2\sqrt{12}$

$\sqrt{15}>\sqrt{12}$ which is true

139. (d) Given expression implies $? = \frac{3325}{25}\times\frac{152}{16}$

$=133\times 9.5=1263.5$

140. (e) $\sqrt{3136}-\sqrt{1764}=\sqrt{?}$

$\Rightarrow 56-42=\sqrt{?}$

$\Rightarrow \sqrt{?}=14$

On squaring both the side

$\therefore\ ?=14\times 14=196$

141. (e) $?=5+\frac{1}{5}+2+\frac{2}{15}+3+\frac{2}{3}$

$=10+\frac{1}{5}+\frac{2}{15}+\frac{2}{3}$

$=10+\frac{3+2+10}{15}=10+\frac{15}{15}$

$=10+1=11$

142. (c) $?=-15-27-88-63+255$

$=-193+255=62$

143. (b) Given expression can be written as

$?=\frac{2525\times 0.25\times 7}{5}=883.75$

144. (b) $?=\frac{14}{19}\times\frac{57}{70}\times\frac{20}{21}=\frac{2}{1}\times\frac{3}{10}\times\frac{20}{21}=\frac{2}{1}\times\frac{1}{1}\times\frac{2}{7}=\frac{4}{7}$

145. (e) $?=\frac{500\times 32}{100}+\frac{50\times 162}{100}$

$=160+81=241$

146. (d) $45316+52131-65229$

$=?+15151$

$\Rightarrow 32218=?+15151$

$\therefore\ ?=32218-15151=17067$

147. (a) $?=\sqrt{25-12+155+1}$

$=\sqrt{169}=13$

148. (c) $?=\frac{184\times 4}{\frac{400\times 23}{100}}=\frac{184\times 4}{4\times 23}=8$

149. (a) $?^{4/3}\times ?^{5/3}=32\times 128$

$\Rightarrow ?^3=2^5\times 2^7=2^{12}$

$\therefore\ ?=(2^{12})^{1/3}=2^4=16$

150. (e) Given fractions can be written in decimal forms as

$\frac{4}{9}=0.44;\quad \frac{6}{13}=0.46;\quad \frac{5}{11}=0.45;\quad \frac{13}{16}=0.8125$

$\frac{7}{12}=0.583$

$\therefore$ Clearly,

$\frac{13}{16}>\frac{7}{12}>\frac{6}{13}>\frac{5}{11}>\frac{4}{9}$

151. (d) Number of items produced in 3 days = 1515

Number of items produced in 1 day = $\frac{1515}{3}$

Required number of items

$=\frac{1515\times 7}{3}=3535$

152. (e) $4800<4900$

$\sqrt{4900}=70$

$\therefore$ Required least number

$=4900-4800=100$

153. (b) Required remainder

$=(46)^2-(11)^3$

$=2116-1331=785$

154. (d) $(\text{Larger number})^2=11570-5329=6241$

$\therefore$ Larger number $=\sqrt{6241}=79$

155. (e) Smallest number $=\frac{5685-3}{3}=1894$

156. (e) $27\times 29\times 31=24273$

157. (e) $4\frac{1}{2}+\left(1\div 2\frac{8}{9}\right)-3\frac{1}{13}=?$

$4+\frac{1}{2}+1\times\frac{9}{26}-\left(3+\frac{1}{13}\right)$

$4+\frac{1}{2}+\frac{9}{26}-3-\frac{1}{13}$

$1+\frac{1}{2}-\frac{1}{13}+\frac{9}{26}=\frac{26+13-2+9}{26}=1\frac{10}{13}$

158. (c) $\dfrac{6\times 136\div 8+132}{628\div 16-26.25}$

$=\dfrac{6\times 136\times \frac{1}{8}+132}{628\times \frac{1}{16}-26.25}$

$=\dfrac{102+132}{39.25-26.25}=\dfrac{234}{13}=18$

159. (d) $\{(441)^{1/2}\times 207\times (343)^{1/3}\}\div \{(14)^2\times (529)^{1/2}\}$

$\{(21^2)^{1/2}\times 207\times (7^3)^{1/3}\}\div \{(14)^2\times (23^2)^{1/2}\}$

$(21\times 207\times 7)\div ((14)^2\times 23)$

$\dfrac{21\times 207\times 7}{14\times 14\times 23}=6\dfrac{3}{4}$

160. (c) $\left\{\sqrt{7744}\times (11)^2\right\}\div (2)^3=(?)^3$

$\left\{88\times (11)^2\right\}\div (2)^3$

$88\times (11)^2\times \dfrac{1}{8}=(11)^3$

$?=11$

161. (e) $(4356)^{½}\div \dfrac{11}{4}=\sqrt{?}\times 6$

$(66^2)^{½}\times \dfrac{4}{11}$

$66\times \dfrac{4}{11}=4\times 6=\sqrt{16}\times 6$

$?=16$

162. (d) $\dfrac{3}{8}\text{ of }\{4624\div (564-428)\}=?$

$\dfrac{3}{8}\times \left\{4624\times \dfrac{1}{136}\right\}$

$\dfrac{3}{8}\times 34=12\dfrac{3}{4}$

163. (c) $456\div 24\times 38-958+364=?$

$=456\times \dfrac{1}{24}\times 38-958+364$

$=722-958+364$

$=128$

164. (b) $(43)^2+841=(?)^2+1465$

$1849+841=(?)^2+1465$

$1225=(?)^2$

$?=35$

165. (c) $3\dfrac{3}{8}\times 6\dfrac{5}{12}-2\dfrac{3}{16}\times 3\dfrac{1}{2}$

$\dfrac{22}{8}\times \dfrac{77}{12}-\dfrac{35}{16}\times \dfrac{7}{2}$

$\dfrac{2079}{96}-\dfrac{245}{32}=\dfrac{2079-7365}{96}=14$

166. (c) $(34.5\times 14\times 42)\div 2.8$

$34.5\times 14\times 42\times \dfrac{1}{2.8}$

$=7245$

167. (d) $(216)^4\div (36)^4\times (6)^5=(6)^?$

$(6^3)^4\div (6^2)^4\times (6)^5$

$(6^3)^4\times \dfrac{1}{6^8}\times (6)^5$

$6^{12+5-8}=6^9$

$?=9$

168. (c) $\dfrac{\sqrt{4356}\times \sqrt{?}}{\sqrt{6084}}=11$

$\dfrac{\sqrt{66\times 66}\times \sqrt{?}}{\sqrt{78\times 78}}=11$

$\dfrac{66\times \sqrt{?}}{78}=11$

$\sqrt{?}=\dfrac{11\times 78}{66}$

$\sqrt{?}=13$

$?=169$

169. (a) $\left(3\dfrac{6}{17}\div 2\dfrac{7}{34}-1\dfrac{9}{25}\right)=(?)^2$

$\dfrac{57}{17}\times \dfrac{34}{75}-\dfrac{34}{25}$

$\dfrac{19\times 2}{25}-\dfrac{34}{25}=\dfrac{4}{25}=\left(\dfrac{2}{5}\right)^2$

$?=\dfrac{2}{5}$

170. (b) $(1097.63+2197.36-2607.24)\div 3.5$

$(3294.99-2607.24)\times \dfrac{1}{3.5}$

$687.75\times \dfrac{1}{3.5}=196.5$

171. (b) $\dfrac{1}{11}\times \left[(17424)^{½}\times \dfrac{1}{(66)^2}\times 3^3\right]$

$\dfrac{1}{11}\times \left[(132^2)^{½}\times \dfrac{1}{(66)^2}\times 3^3\right]$

$\dfrac{1}{11}\times \dfrac{132}{(66)^2}\times 3^3=\dfrac{\cancel{2}\times \cancel{27}^{\,9}}{11\times \underset{\underset{11}{\cancel{33}}}{\cancel{66}}}=\left(\dfrac{3}{11}\right)^2$

$?=11$

Level-II

1. (b) $999\frac{995}{999}\times 999 = \left(999+\frac{995}{999}\right)\times 999$

$= 999 \times 999 + \frac{995}{999}\times 999 = 999^2 + 995$

$= 998001 + 995 = 998996$

2. (c) 7892.35×99.9

$= \frac{789235\times 999}{1000} = \frac{789235\times(1000-1)}{1000}$

$= \frac{789235000-789235}{1000} = 788445.765$

3. (c) Total number of $\frac{1}{12} = \frac{18\frac{3}{4}}{\frac{1}{12}}$

$= \frac{75}{4}\times\frac{12}{1} = 225$

4. (b) This problem can't be solved by factorisation because we need not factor. So we have to solve it by division method as follows

	23.9
2	575
2	4
43	175
3	129
469	4600
9	4221

(If the number is not a perfect square then by putting decimal we can increase the zeros in pairs for further calculation.)

The result obtained is ≈ 23.9.

So by adding some number we can make it the perfect square of 24. Now since we know that $(24)^2 = 576$. So we need to add 1 (∵ 576 – 575 = 1)

Thus (b) is the correct option.

Alternatively : Using options we can solve this problem as if we consider optin (a) then 575 itself be a perfect square but its not a perfect square. Again if we add 1 (i.e., using option (b)) we get the number 576 and then check it, we find that 576 is a perfect square. Hence (b) is correct.

Alternatively : Since we know that $(20)^2 = 400$ and $(25)^2 = 625$. It means the value of perfect square must lies in the range of 400 and 625. So we can try it manually and get that $(23)^2 = 529$ and $(24)^2 = 576$. So simply we need to add 1 to make a perfect square number.

5. (a) $6*15*3 = \sqrt{\frac{(6+2)(15+3)}{(3+1)}} = \sqrt{\frac{8\times 18}{4}} = 6$

6. (a) $x+\frac{1}{x} = 3+\sqrt{8}+\frac{1}{3+\sqrt{8}}$

$= \frac{(3+\sqrt{8})^2+1}{(3+\sqrt{8})} = \frac{9+8+6\sqrt{8}+1}{(3+\sqrt{8})}$

$= \frac{18+6\sqrt{8}}{(3+\sqrt{8})} = \frac{6(3+\sqrt{8})}{(3+\sqrt{8})} = 6,\quad \left(x+\frac{1}{x}\right)^2 = 6^2$

$x^2+\frac{1}{x^2}+2.x.\frac{1}{x} = 36,\ x^2+\frac{1}{x^2} = 36 - 2 = 34$

7. (c) If $x^a = y^b = z^c$ and $y^2 = zx$

Let $x^a = y^b = z^c = k$

$\Rightarrow\ x = k^{1/a},\ y = k^{1/b},\ z = k^{1/c}$

Now, ∵ $y^2 = zx$

∴ $(k^{1/b})^2 = (k^{1/c}).(k^{1/a})$

$\Rightarrow\ k^{2/b} = k^{\frac{1}{c}+\frac{1}{a}}$

$\Rightarrow\ \frac{1}{a}+\frac{1}{c} = \frac{2}{b}$

Hence (c) is the correct option.

8. (a) We have : $\frac{2x}{1+\frac{1}{\frac{(1-x)+x}{1-x}}} = 1 \Leftrightarrow \frac{2x}{1+\frac{1}{[1/(1-x)]}} = 1$

$\Leftrightarrow\ \frac{2x}{1+(1-x)} = 1$

$\Leftrightarrow\ 2x = 2 - x \Leftrightarrow 3x = 2 \Leftrightarrow x = \frac{2}{3}$

9. (b) Given expression

$= \left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{6}\right)+ \ldots\ldots + \left(\frac{1}{9}-\frac{1}{10}\right)$

$= \left(\frac{1}{2}-\frac{1}{10}\right) = \frac{4}{10} = \frac{2}{5}$

10. (c) $7-2\sqrt{10} = 5+2-2\sqrt{5\times 2}$

$\Rightarrow\ 7-2\sqrt{10} = (\sqrt{5})^2+(\sqrt{2})^2-2\sqrt{5}.\sqrt{2}$

$\Rightarrow\ 7-2\sqrt{10} = (\sqrt{5}-\sqrt{2})^2$

Thus the $\sqrt{7-2\sqrt{10}} = \pm(\sqrt{5}-\sqrt{2})$

11. (b) We have,

Product of unit's digits = 28

$\Rightarrow$ Product of units digits = 4 × 7

[$\because$ Unit's digits are one digit numbers]

$\Rightarrow$ Unit's digits are 4 and 7.

$\Rightarrow$ Product of ten's digits = 15

$\Rightarrow$ Product of ten's digits = 3 × 5

[Ten's digit are one digit numbers]

$\Rightarrow$ Ten's digits are 3 and 5.

Thus, the two numbers either 34 and 57 or 37 and 54.

Now, 34 × 57 = 34 × (50 + 7) [$\because$ 57 = 50 + 7]

= 34 × 50 + 34 × 7 [$\because$ $a \times (b + c) = a \times b + a \times c$]

= 1700 + 238 = 1938

and, 37 × 54 = 37 × (50 + 4) [$\because$ 54 = 50 + 4]

[$\because$ $a \times (b + c) = a \times b + a \times c$]

= 1850 + 148 = 1998

12. (d) Consider $P = 5$, then

$$5 + 5! = 5^3$$
$$5 + 120 = 125$$
$$125 = 125$$

Thus (d) is correct option.

13. (d) Let $Z = 8x - 3x^2$

$$\Rightarrow Z = -3\left[x^2 - \frac{8}{3}x\right]$$

$$\Rightarrow Z = -3\left[x^2 - 2 \times x \times \frac{4}{3} + \left(\frac{4}{3}\right)^2 - \left(\frac{4}{3}\right)^2\right]$$

$$\Rightarrow Z = -3\left(x - \frac{4}{3}\right)^2 + 3 \times \left(\frac{4}{3}\right)^2$$

So the maximum value occurs when $x = \frac{4}{3}$

Maximum value $= -3 \times 0 + 3 \times \frac{16}{9} = \frac{16}{3}$

14. (c) $\sqrt[3]{x+9} - \sqrt[3]{x-9} = 3,$

$$\Rightarrow (x+9) - (x-9) - 3\sqrt[3]{x+9}.\sqrt[3]{x-9} = 3,$$

$$\left(\sqrt[3]{x+9} - \sqrt[3]{x-9}\right) = 3^3$$

$$\left(\because (a-b)^3 = a^3 - b^3 - 3ab(a-b)\right)$$

$$\Rightarrow 18 - 3\sqrt[3]{x^2 - 81} \times 3$$

$$\Rightarrow \frac{18 - 27}{9} = \sqrt[3]{x^2 - 81}$$

$$\Rightarrow -1 = x^2 - 81 \Rightarrow x^2 = 80$$

15. (a) Given expression

$$= 35.7 - \left(3 + \frac{1}{\frac{10}{3}}\right) - \left(2 + \frac{1}{\frac{5}{2}}\right)$$

$$= 35.7 - \left(3 + \frac{3}{10}\right) - \left(2 + \frac{2}{5}\right)$$

$$= 35.7 - \frac{33}{10} - \frac{12}{5} = 35.7 - \left(\frac{33}{10} + \frac{12}{5}\right)$$

$$= 35.7 - \frac{57}{10} = 35.7 - 5.7 = 30.$$

16. (b) $\sqrt[4]{10} = (10)^{1/4} = (10)^{3/12} = (1000)^{1/12}$

$\sqrt[3]{6} = (6)^{1/3} = (6)^{4/12} = (1296)^{1/12}$

$\sqrt{3} = (3)^{1/2} = (3)^{6/12} = (729)^{1/12}$

$\therefore \sqrt{3} < \sqrt[4]{10} < \sqrt[3]{6}$ is the correct order and hence (b) is correct.

17. (b) The unit's digit will be 1 × 5 = 5 (no carry over). The tens digit will be (4*1 + 5*2) = 4 (carry over 1). The hundreds digit will be (3*1 + 4*2 + 5*1) = 6 + 1 (carried over) = 7. Hence, answer is 745.

18. (a) Let the numbers be a and b. Then, $a + b = 12$ and $ab = 35$.

$$\therefore \quad \frac{a+b}{ab} = \frac{12}{35} \qquad \Rightarrow \left(\frac{1}{b} + \frac{1}{a}\right) = \frac{12}{35}$$

$\therefore$ Sum of reciprocals of given numbers $= \frac{12}{35}$

19. (c) Given expression

$$= \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times ..... \times \frac{99}{100} = \frac{2}{100} = \frac{1}{50}$$

20. (b) Suppose the worker remained idle for x days. Then, he worked for $(60 - x)$ days.

$\therefore$ $20(60 - x) - 3x = 280 \Leftrightarrow 1200 - 23x = 280 \Leftrightarrow 23x = 920 \Leftrightarrow x = 40.$

So, the worker remained idle for 40 days.

21. (b) Given exp. $= \dfrac{1}{1 + \dfrac{\frac{2}{3}}{\frac{5}{3} + \frac{\frac{8}{9}}{(1/3)}}} = \dfrac{1}{1 + \dfrac{2/3}{\frac{5}{3} + \frac{8}{9} \times 3}}$

$$= \frac{1}{1 + \frac{2/3}{(13/3)}} = \frac{1}{1 + \frac{2}{13}} = \frac{13}{15}$$

22. **(b)** Given exp. $= \sqrt{\dfrac{(0.03)^2 + (0.21)^2 + (0.065)^2}{\left(\dfrac{0.03}{10}\right)^2 + \left(\dfrac{0.21}{10}\right)^2 + \left(\dfrac{0.065}{10}\right)^2}}$

$= \sqrt{\dfrac{100\left[(0.03)^2 + (0.21)^2 + (0.065)^2\right]}{(0.03)^2 + (0.21)^2 + (0.065)^2}}$

$= \sqrt{100} = 10.$

23. **(c)** Given : $x^2 + y^2 + z^2 - 64 = -2(xy - yz - zx)$ ...(i)

Now, $[x + y + (-z)]^2 = x^2 + y^2 + z^2 + 2(xy - yz - zx)$

$\Rightarrow$ $(3z - z)^2 = x^2 + y^2 + z^2 + 2(xy - yz - zx)$

$\Rightarrow$ $-2(xy - yz - zx) = (x^2 + y^2 + z^2) - (2z)^2$ ...(ii)

From (i) and (ii), we get:

$(2z)^2 = 64 \Leftrightarrow 4z^2 = 64 \Leftrightarrow z^2 = 16 \Leftrightarrow z = 4.$

24. **(c)** $\sqrt{\dfrac{8}{3}} = \sqrt{\dfrac{8\times 3}{3\times 3}} = \dfrac{\sqrt{24}}{3} = \dfrac{4.899}{3} = 1.633.$

25. **(c)** $\left[X + \dfrac{1}{X}\right]^2 = X^2 + \dfrac{1}{X^2} + 2 = 16$ or $X^2 + \dfrac{1}{X^2} = 14$

Now, $X^4 + \dfrac{1}{X^4} + 2 = 196$ or $X^4 + \dfrac{1}{X^4} = 194.$

26. **(c)** Given expression $= \sqrt{15625} + \sqrt{\dfrac{15625}{100}} + \sqrt{\dfrac{15625}{10000}}$

$= \left(125 + \dfrac{125}{10} + \dfrac{125}{100}\right) = (125 + 12.5 + 1.25) = 138.75$

27. **(b)** Provisions for one student $= 250 \times 35 = 8750$

250 students used provisions for 5 days.

Total provisions used by 250 students in 5 days

$= 250 \times 5 = 1250$

Remaining provision $= 8750 - 1250 = 7500$

After 5 days total number of student $= 250 + 25 = 275$

Total provisions used by 275 student in 10 days

$275 \times 10 = 2750$

Now remaining $= 7500 - 2750 = 4750$

After 15 days no. of student $= 275 - 25 = 250$

$4750 = 250 \times$ no. of extra dayus

No. of extra days $= \dfrac{4750}{250} = 19$ days

28. **(a)** $\dfrac{97}{19} = 5 + \dfrac{2}{19}$. Also, $\dfrac{19}{2}$ can be written as $9 + \dfrac{1}{2}$. So the values of a, b and c are 5, 9 and 2 respectively. Hence, the sum of a, b and c is 16.

29. **(d)** $\text{I} = \sqrt[3]{\sqrt[4]{a^3}} = \left((a^3)^{1/4}\right)^{1/3} = a^{1/4}$

$\text{II} = \sqrt[3]{\sqrt[5]{a^4}} = \left((a^4)^{1/5}\right)^{1/3} = a^{4/15}$

$\text{III} = \sqrt{\sqrt[3]{a}} = \left(a^{1/3}\right)^{1/2} = a^{1/6}$

$\text{IV} = \sqrt{\sqrt[5]{a^3}} = \left((a^3)^{1/5}\right)^{1/2} = a^{3/10}$

Now again, to compare these numbers, we need to bring the indices to a common denominator.

$\therefore$ I = a1/4 = a15/60. II = a4/15 = a16/60.

III = a1/6 = a10/60 . IV = a3/10 = a18/60.

$\therefore$ The ascending order is III, I, II, IV.

30. **(b)** $A^{3^{3^{3^3}}} = 3^{3^{27}}$

and $C^{3^{33}} = 3^{3^{33}}$

Hence $C > A$.

Hence either (b) and (d) option is correct.

Now $A = 3^{3^{3^3}} = 3^{3^{27}}$

and $D = 3^{333}$

Hence $A > D$ (Since $3^{27} > 333$)

Thus the correct relation is $C > A > B > D$.

Hence, option (b) is correct.

31. **(b)** If we try to put x as 12, we get the square root of $3x$ as 6. Then the next point at which we need to remove the square root sign would be 12 + 2(6) = 24 whose square root would be an irrational number. This leaves us with only 1 possible value (x = 3). Checking for this value of x we can see that the expression is satisfied as LHS = RHS.

32. **(d)** Solve this question through options. Also realize that $a \times b = a + b$ only occurs for the situation $2 \times 2 = 2 + 2$.

Hence, clearly the answer has to be none of these.

33. **(b)** $\sqrt{11449} \times \sqrt{6241} - (54)^2 = \sqrt{?} + (74)^2$

$\Rightarrow$ $\sqrt{?} = 107 \times 79 - 2916 - 5476$

$= 8453 - 2916 - 5476 = 61$

$\therefore$ $? = (61)^2 = 3721$

34. **(c)** $? = \left[\left(3\sqrt{8} + \sqrt{8}\right) \times \left(8\sqrt{8} + 7\sqrt{8}\right)\right] - 98$

$= \left(4\sqrt{8} \times 15\sqrt{8}\right) - 98 = (60 \times 8) - 98$

$= 480 - 98 = 382$

35. **(a)** $? + 5883 = 3463 \times 295 - 18611$

$\therefore$ $? = 1021585 - 18611 - 5883 = 997091$

36. **(d)** $? = \dfrac{28}{65} \times \dfrac{195}{308} \div \dfrac{39}{44} + \dfrac{5}{26} = \dfrac{28}{65} \times \dfrac{195}{308} \times \dfrac{44}{39} + \dfrac{5}{26}$

$= \dfrac{4}{13} + \dfrac{5}{26} = \dfrac{8+5}{26} = \dfrac{13}{26} = \dfrac{1}{2}$

37. **(c)** $? + 1147.69 = (23.1)^2 + (48.6)^2 - (39.8)^2$

$\therefore$ $? = 533.61 + 2361.96 - 1584.04 - 1147.69 = 163.84$

38. **(e)** $? \div 21.003 = \sqrt[3]{4663} + 349$

$\Rightarrow ? \div 21 = 17 + 349 = 366$

$\therefore$ $? = 366 \times 21 = 7686 \approx 7680$

39. (c) $? = 4331\times\frac{39.897}{100}+5003\times\frac{58.779}{100}$

$= 4330\times\frac{40}{100}+5000\times\frac{59}{100}$

$=1732+2950=4682\approx 4700$

40. (c) $?=59.88\div 12.21\times 6.35$

$\approx 60\div 12\times 6 = 60\times\frac{1}{12}\times 6 = 30$

41. (e) $?=43931.03\div 2111.02\times 401.04$

$\approx 43930\div 2110\times 400$

$\approx 43930\times\frac{1}{2110}\times 400\approx 8300$

42. (b) $?=\sqrt{6354}\times 34.993\approx 80\times 35 = 2800$

43. (e) The given number series is based on the following pattern:

9050 → 5675 → 3478 → 2147 → 1418 → [1077] (1075) → 950

$-(15)^3$, $-(13)^3$, $-(11)^3$, $-(9)^3$, $-(7)^3$, $-(5)^3$

Hence, the number 1077 is wrong and it should be replaced by 1075.

44. (d) The given number series is based on the following pattern :

7 → 12 → 40 → 222 → [1742] (1744) → 17390 → 208608

$\times 2-(2\times 1)$, $\times 4-(4\times 2)$, $\times 6-(6\times 3)$, $\times 8-(8\times 4)$, $\times 10-(10\times 5)$, $\times 12-(12\times 6)$

Hence, the number 1742 is wrong and it should be replaced by 1744.

45. (c) The given number series is based on the following pattern:

6 → 91 → [584] (582) → 2935 → 11756 → 35277 → 70558

$\times 7+(7)^2$, $\times 6+(6)^2$, $\times 5+(5)^2$, $\times 4+(4)^2$, $\times 3+(3)^2$, $\times 2+(2)^2$

Hence, the number 584 is wrong and it should be replaced by 582.

46. (d) The given number series is based on the following pattern.

1, 4, [25] (27), 256, 3125, 46656, 823543

$(1)^1$, $(2)^2$, $(3)^3$, $(4)^4$, $(5)^5$, $(6)^6$, $(7)^7$

Hence, the number 25 is wrong and it should be replaced by 27.

47. (b) The given number series is based on the following pattern :

8424 → 4212 → 2106 → [1051] (1053) → 526.5 → 263.25 → 131.625

$\times\frac{1}{2}$, $\times\frac{1}{2}$, $\times\frac{1}{2}$, $\times\frac{1}{2}$, $\times\frac{1}{2}$, $\times\frac{1}{2}$

Hence, the number 1051 is wrong and it should be replaced by 1053.

48. (a) Value of one ticket of each kind = 55 + 85 + 105 = ₹ 245

∴ Required number of ticket of each kind

$=\frac{2940}{245}=12$

49. (c) Cost of one pencil box = 7 + 22 + 14 = ₹ 43

∴ Required amount

$=(20\times 7)+(8\times 22)+(6\times 175)+(7\times 43)$

$=140+176+1050+301=$ ₹ 1667

50. (d) $4003\times 77-21015=?\times 116$

$\Rightarrow 308231-21015=?\times 116 \Rightarrow 287216=?\times 116$

$\Rightarrow ?=\frac{287216}{116}=2476$

51. (a) $\left[\left(5\sqrt{7}+\sqrt{7}\right)\times\left(4\sqrt{7}+8\sqrt{7}\right)\right]-(19)^2=?$

$\Rightarrow (6\sqrt{7}\times 12\sqrt{7})-(361)=?$

$\Rightarrow 72\times\sqrt{7}\times\sqrt{7}-361=?$

$\therefore ?=504-361=143$

52. (b) $(4444\div 40)+(645\div 25)+(3991\div 26)=?$

$\Rightarrow ?=(111.1)+(25.8)+(153.5)\Rightarrow ?=290.4$

53. (e) $\sqrt{33124}\times\sqrt{2601}-(83)^2=(?)^2+(37)^2$

$\Rightarrow (?)^2=\sqrt{33124}\times\sqrt{2601}-(83)^2-(37)^2$

$\Rightarrow (?)^2=182\times 51-6889-1369$

$\Rightarrow (?)^2=9282-6889-1369$

$\Rightarrow (?)^2=1024$

$\therefore ?=\sqrt{1024}=32$

54. (b) $5\frac{17}{37}\times 4\frac{51}{52}\times 11\frac{1}{7}+2\frac{3}{4}=?$

$\Rightarrow\left(\frac{202}{37}\times\frac{259}{52}\times\frac{78}{7}\right)+\left(\frac{11}{4}\right)=?$

$\Rightarrow 303+\frac{11}{4}=?$

$\therefore ?=\frac{1223}{4}=305.75$

55. (c) $8787\div 343\times\sqrt{50}=?$

$\Rightarrow 25\times 7=?$

$\therefore\ ?=175\approx 180$

56. (b) $\sqrt[3]{54821} \times (303 \div 8) = (?)^2$

$\Rightarrow \; 38 \times 37.5 = (?)^2$

$? = \sqrt{38 \times 38}$

$? = 38$

57. (c) $\frac{5}{8}$ of $4011.33 + \frac{7}{10}$ of $3411.22 = ?$

$\Rightarrow \; \frac{5}{8} \times 4010 + \frac{7}{10} \times 3410 = ? \;\Rightarrow\; ? = 2506 + 2387$

$\Rightarrow \; ? = 4893 \approx 4890$

58. (e) 23% of $6783 + 57\%$ of $8431 = ?$

$\Rightarrow ? = 1560 + 4805$

$\therefore ? = 6365 \approx 6360$

59. (a) $335.01 \times 244.99 \div 55$

$\Rightarrow ? = \frac{335 \times 245}{55}$

$\therefore ? = 1492 \approx 1490$

60. (c) By options

(a) Either 12 or 13

then ice-creams should not be given atleast 9. This can be rejected.

(b) Either 11 or 12

Ice-cream should be atleast 9. By this combination ice cream gets less than 9.

(c) Either 10 or 11

By giving cookies 10 or 11, we get all the possible condition fulfilled.

(d) and (e), the ice-cream distribution can be more than cookies which violates our condition.

$\therefore$ option (c) is the write answer.

61. (a) Let the number be $10x + y$.

According to condition

$10x + y + 18 = 10y + x$

$y - x = 2$

So those numbers are 02, 13, 24, 35, 46, 57, 68, 79, 80

But 13 and 79 are prime numbers.

62. (c) Given, $x = \frac{4ab}{a+b}$

$\Rightarrow \; \frac{x}{2a} = \frac{2b}{a+b}$

Applying componendo and dividendo, we get

$$\frac{x+2a}{x-2a} = \frac{2b+a+b}{2b-a-b} = \frac{a+3b}{b-a} \quad \ldots(i)$$

Also, $\frac{x}{2b} = \frac{2a}{a+b}$

Applying componendo and dividendo, we get

$$\frac{x+2b}{x-2b} = \frac{2a+a+b}{2a-a-b} = \frac{3a+b}{a-b} \quad \ldots(ii)$$

Add (i) & (ii),

$$\frac{x+2a}{x-2a} + \frac{x+2b}{x-2b} = \frac{a+3b}{b-a} + \frac{3a+b}{a-b}$$

$$= \frac{1}{b-a}[a+3b-3a-b] = \frac{2(b-a)}{(b-a)} = 2$$

63. (a) $x^2 + y^2 + z^2 - xy - yz - zx$

$$= \frac{2}{2}(x^2 + y^2 + z^2 - xy - yz - zx)$$

$$= \frac{1}{2}(2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx)$$

$$= \frac{1}{2}(x^2 + y^2 - 2xy + y^2 + z^2 - 2yz + x^2 + z^2 - 2zx)$$

$$= \frac{1}{2}[(x-y)^2 + (y-z)^2 + (z-x)^2]$$

$$= \frac{1}{2}[(997-998)^2 + (998-999)^2 + (999-997)^2]$$

$$= \frac{1}{2}[1^2 + 1^2 + 2^2] = \frac{1}{2} \times 6 = 3$$

64. (d) We have $x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx)$

Here $x = a - 4, y = b - 3, z = c - 1$

So, given expression is $(x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= (a - 4 + b - 3 + c - 1)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= (a + b + c - 8)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= (8 - 8)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= 0$

65. (a) $x^4 + y^4 - 2x^2y^2$

$\Rightarrow (x^2 - y^2)^2 \Rightarrow [(x+y)(x-y)]^2$

$$\Rightarrow \left[\left(\sqrt{a} + \frac{1}{\sqrt{a}} + \sqrt{a} - \frac{1}{\sqrt{a}}\right)\left(\sqrt{a} + \frac{1}{\sqrt{a}} - \sqrt{a} + \frac{1}{\sqrt{a}}\right)\right]^2$$

$$\Rightarrow \left(2\sqrt{a} \times \frac{2}{\sqrt{a}}\right)^2 \Rightarrow 16$$

66. (d) $5a + \frac{1}{3a} = 5$

Multiply by $\frac{3}{5}$ on both sides

$$\frac{3}{5}\left(5a + \frac{1}{3a}\right) = 5 \times \frac{3}{5}$$

$$3a + \frac{1}{5a} = 3$$

Squaring on both sides

$$9a^2 + \frac{1}{25a^2} + 2 \times 3a \times \frac{1}{5a} = 9$$

$$\Rightarrow 9a^2 + \frac{1}{25a^2} = 9 - \frac{6}{5} = \frac{39}{5}$$

67. (b) $x = 3 + 2\sqrt{2}$

$$\frac{1}{x} = \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$$

$$\frac{1}{x} = \frac{3-2\sqrt{2}}{9-8} = 3-2\sqrt{2}$$

$$\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 = x + \frac{1}{x} - 2$$

$$\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 = 3+2\sqrt{2}+3-2\sqrt{2}-2 = 4$$

$$\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) = \sqrt{4} = \pm 2$$

68. (b) If $a + b + c = 0$,
then $a^3 + b^3 + c^3 = 3abc$
Dividing both sides by abc

$$\frac{a^3}{abc} + \frac{b^3}{abc} + \frac{c^3}{abc} = \frac{3abc}{abc}$$

$$\frac{a^2}{bc} + \frac{b^2}{ac} + \frac{c^2}{ab} = 3$$

69. (c)

70. (b) $(a^b + b^a)^{-1} = (2^3 + 3^2)^{-1} = (8+9)^{-1} = (17)^{-1} = \frac{1}{17}$

71. (a) $392 \times 2 = 784 \Rightarrow (27)^2$
Hence, 2 can be multiplied by 392 which gives perfect square.

72. (c) Let lst, 2nd and 3rd part represented by x, y, z

Let $\frac{1}{2}x = \frac{1}{3}y = \frac{1}{4}z = k$

$\therefore \quad x = 2k, y = 3k, = 4k$
According to question
$x + y + z = 81$
$\Rightarrow \quad 2k + 3k + 4k = 81 \Rightarrow 9k = 81 \Rightarrow k = 9$
Hence, parts are 18, 27, 36.

73. (a) $x^4 - 2x^2 + k$
$(x^2)^2 - 2x^2 + k \Rightarrow (x^2)^2 - 2.1.\, x^2 + k$
For above expression to make a perfect square, the k value is equal to 1.

74. (d) $3x - \frac{1}{4y} = 6 \quad 3x = 6 + \frac{1}{4y}$

Taking 3 common on both sides

$$x = \frac{6}{3} + \frac{1}{4.3y} \Rightarrow x = 2 + \frac{1}{12y}$$

Multiplying equation by 4 on both sides

$$4x = 8 + \frac{1}{3y} \Rightarrow 4x - \frac{1}{3y} = 8$$

75. (a) $a + b + c = 0$
i.e. $a = -(b + c); b = -(c + a); c = -(a + b)$

Now, $\frac{a+b}{c} - \frac{2b}{c+a} + \frac{b+c}{a}$

$$\Rightarrow \frac{a+b}{-(a+b)} - \frac{2[-(c+a)]}{c+a} + \frac{b+c}{-(b+c)}$$

$\Rightarrow -1 + 2 - 1 = 0$

76. (b) $x + \frac{4}{x} = 4$
$x^2 + 4 = 4x \Rightarrow x^2 - 4x + 4 = 0 \Rightarrow (x-2)^2 = 0$
$x = 2$

$$x^3 + \frac{4}{x^3} = (2)^3 + \frac{4}{(2)^3} \Rightarrow 8 + \frac{4}{8} \Rightarrow 8 + \frac{1}{2} \Rightarrow 8\frac{1}{2}$$

77. (b) $x = 3 + 2\sqrt{2}$

$x = 2 + 1 + 2\sqrt{2}$

$x = (\sqrt{2})^2 + (1)^2 + 2.1.\sqrt{2}$

$x = (\sqrt{2} + 1)^2$

$\sqrt{x} = (\sqrt{2} + 1) \qquad$ ...(1)

$$\frac{1}{\sqrt{x}} = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{2-1} = \sqrt{2}-1$$

Now, $\sqrt{x} - \frac{1}{\sqrt{x}} = \sqrt{2} + 1 - (\sqrt{2} - 1) = \sqrt{2} + 1 - \sqrt{2} + 1$

$$\sqrt{x} - \frac{1}{\sqrt{x}} = 2$$

78. (c) The least value of $a + \frac{1}{a}$ is 2 where $a = 1$.

79. (a) $3^{34} = (3^2)^{17} = 9^{17}$
$2^{51} = (2^3)^{17} = 8^{17}$
Clearly, $7^{17} < 8^{17} < 9^{17}$
or $7^{17} < 2^{51} < 3^{34}$

80. (d) $x = 2 + \sqrt{3}$

$$\frac{1}{x} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = 2-\sqrt{3}$$

$$x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$$

$$= \left(2+\sqrt{3}+2-\sqrt{3}\right)^2 - 2$$

$= 16 - 2 = 14$

81. (c) $a = 4.965 \approx 5, b = 2.343 \approx 2$
$c = 2.622$
$a - b = c$

taking cube both sides

$a^3 - b^3 - 3a^2b + 3ab^2 = c^3$

$a^3 - b^3 - c^3 - 3ab(a - b) = 0$

$a^3 - b^3 - c^3 - 3abc = 0$

82. **(d)** $x + y + z = 0$

$y + z = -x$

$y^2 + z^2 + 2yz = x^2$

$\Rightarrow y^2 + z^2 = x^2 - 2yz$ ...(1)

$$\frac{x^2 + y^2 + z^2}{x^2 - yz} = \frac{x^2 - 2yz + x^2}{x^2 - yz} = \frac{2(x^2 - yz)}{x^2 - yz}$$

$= 2$

83. **(d)** Let the number be x

$$\frac{x}{\frac{7}{19}} - \frac{7}{19} \times x = 624$$

$$x\left(\frac{19}{7} - \frac{7}{19}\right) = 624$$

$$x\left(\frac{361 - 49}{7 \times 19}\right) = 624$$

$$x = \frac{624 \times 133}{312}$$

$x = 266$

Sum of digits $(2 + 6 + 6) = 14$

84. **(c)** $a^2 + b^2 + c^2 = 2a - 2b - 2$

$(a^2 - 2a + 1) + (b^2 + 2b + 1) + c^2 = 0$

$(a - 1)^2 + (b + 1)^2 + c^2 = 0$

This equation is possible if

$a - 1 = 0, b + 1 = 0$ and $c = 0$

$a = 1, b = -1, c = 0$

$3a - 2b + c = 3 \times 1 - 2 \times (-1) + 0$

$= 3 + 2 = 5$

85. **(b)** $a + b + c = 3$

Squaring both sides

$a^2 + b^2 + c^2 + 2(ab + bc + ac) = 9$

$6 + 2(ab + bc + ca) = 9$

$ab + bc + ca = \frac{3}{2}$ ...(1)

given $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$

$\Rightarrow ab + bc + ac = abc = \frac{3}{2}$ [from (1)]

86. **(d)** $a^2 - 4a - 1 = 0$

$a^2 - 4a = 1$

$a(a - 4) = 1$

$a - 4 = \frac{1}{a}$

$a - \frac{1}{a} = 4$ ...(1)

We have $a^2 + 3a + \frac{1}{a^2} - \frac{3}{a}$

$$\left(a^2 + \frac{1}{a^2}\right) + 3\left(a - \frac{1}{a}\right)$$

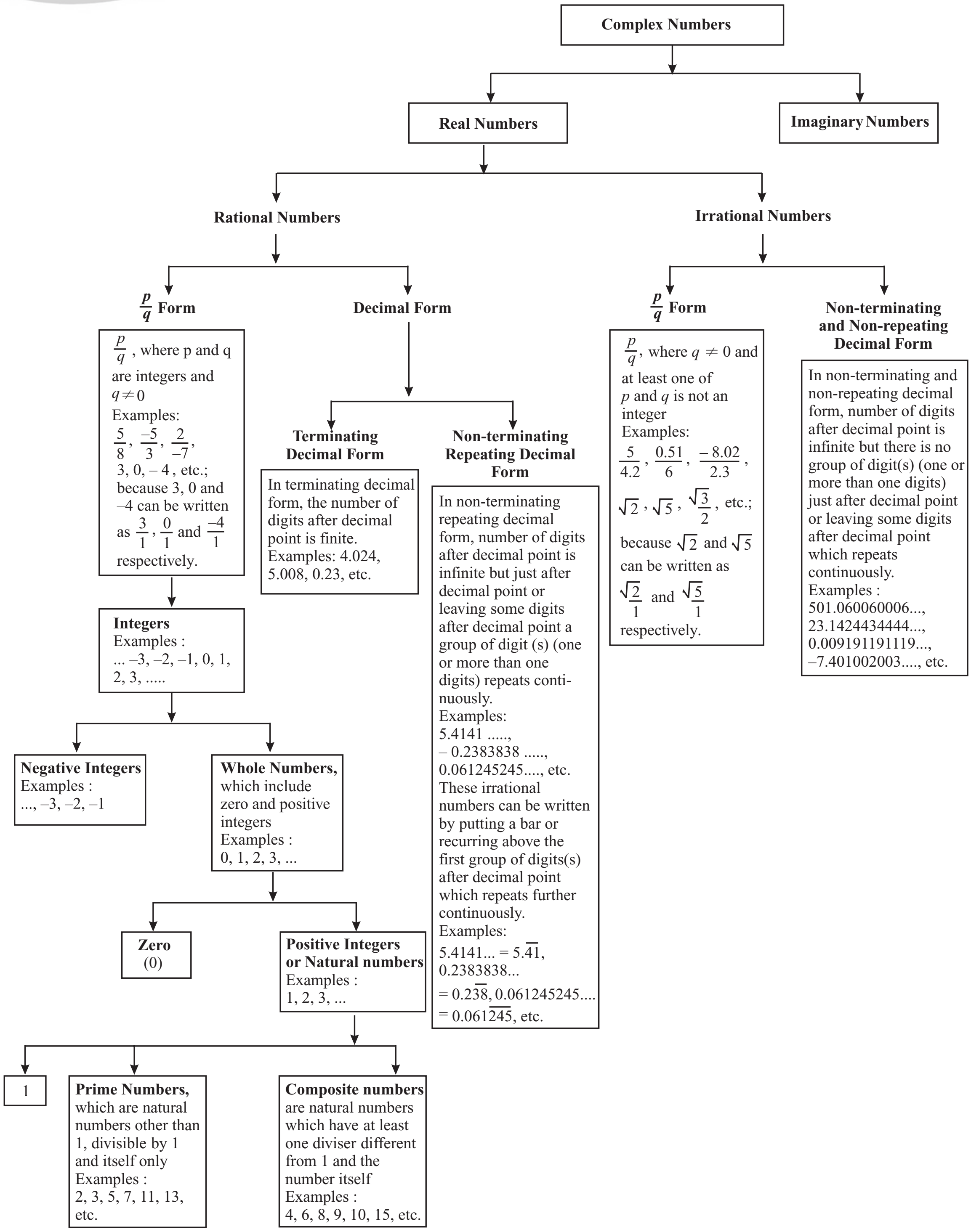
$$\left(a - \frac{1}{a}\right)^2 + 3\left(a - \frac{1}{a}\right) + 2$$

$4^2 + 3 \times 4 + 2 = 30$

CHAPTER 2

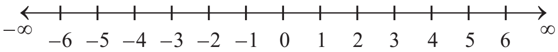
NUMBER SYSTEM

Chart: Classification of Numbers



CONCEPT OF NUMBER LINE (OR NUMBER LINE)

A number line is a straight line from negative infinitive $(-, \infty)$ in left hand side to positive infinitive $(+, \infty)$ in right hand side as given:



Each point on the number line represents a unique real number and each real number is denoted by a unique point on the number line.

Symbols of some special sets are:

$\boldsymbol{N}$: the set of all natural numbers
$\boldsymbol{Z}$: the set of all integers
$\boldsymbol{Q}$: the set of all rational numbers
$\boldsymbol{R}$: the set of all real numbers
$\boldsymbol{Z}^+$: the set of positive integers
$\boldsymbol{Q}^+$: the set of positive rational numbers, and
$\boldsymbol{R}^+$: the set of positive real numbers

The symbols for the special sets given above will be referred to throughout the text.

Even Integers

An integer divisible by 2 is called an even integer. Thus, ..., – 6, – 4, – 2, 0, 2, 4, 6, 8, 10, 12,...., etc. are all even integers. $2n$ always represents an even number, where n is an integer.

For example, by putting $n = 5$ and 8 in $2n$, we get even integer $2n$ as 10 and 16 respectively.

Odd Integers

An integer not divisible by 2 is called an odd integer.

Thus, ..., –5, –3, –1, 1, 3, 5, 7, 9, 11, 13, 15,..., etc. are all odd integers.

$(2n - 1)$ or $(2n + 1)$ always represents an odd number, where n is an integer.

For example by putting $n = 0$, 1 and 5 in $(2n - 1)$, we get odd integer $(2n - 1)$ as – 1, 1 and 9 respectively.

Properties of Positive and Negative Numbers

If n is a natural number then

$$(\text{A positive number})^{\text{natural number}} = \text{A positive number}$$
$$(\text{A negative number})^{\text{even positive number}} = \text{A positive number}$$
$$(\text{A negative number})^{\text{odd positive number}} = \text{A negative number}$$

CONVERSION OF RATIONAL NUMBER OF THE FORM NON-TERMINATING RECURRING DECIMAL INTO THE RATIONAL NUMBER OF THE FORM $\frac{p}{q}$

First write the non-terminating repeating decimal number in recurring form i.e., write

64.20132132132..... as $64.20\overline{132}$

Then using formula given below we find the required $\frac{p}{q}$ form of the given number.

Rational number in the form $\frac{p}{q}$

$$= \frac{\begin{bmatrix}\text{Complete number neglecting} \\ \text{the decimal and bar over} \\ \text{repeating digit(s)}\end{bmatrix} - \begin{bmatrix}\text{Non-recurring part of} \\ \text{the number neglecting} \\ \text{the decimal}\end{bmatrix}}{m \text{ times 9 followed by } n \text{ times 0}}$$

where m = number of recurring digits in decimal part
and n = number of non-recurring digits in decimals part

Thus, $\frac{p}{q}$ form of $64.20\overline{132} = \frac{6420132 - 6420}{99900}$

$$= \frac{6413712}{99900} = \frac{534476}{8325}$$

In short; $0.\overline{a} = \frac{a}{9}$, $0.\overline{ab} = \frac{ab}{99}$, $0.\overline{abc} = \frac{abc}{999}$, etc. and

$$0.a\overline{b} = \frac{ab - a}{90},\ 0.a\overline{bc} = \frac{abc - a}{990},\ 0.ab\overline{c} = \frac{abc - ab}{900},$$

$$0.ab\overline{cd} = \frac{abcd - ab}{9900},\ ab \cdot cd\overline{e} = \frac{abcde - abc}{990}, \text{etc.}$$

Illustration 1: Convert $2.\overline{46102}$ in the $\frac{p}{q}$ form of rational number.

Solution: Required $\frac{p}{q}$ form $= \frac{246102 - 2}{99999} = \frac{246100}{99999}$

Illustration 2: Convert $0.167\overline{3206}$ in the $\frac{p}{q}$ form of rational number.

Solution: Required $\frac{p}{q}$ form $= \frac{1673206 - 167}{9999000} = \frac{1673039}{9999000}$

Illustration 3: Convert 31.026415555 ... into $\frac{p}{q}$ form of rational number.

Solution: First write 31.026415555... as $31.02641\overline{5}$

Now required $\frac{p}{q}$ form $= \frac{31026415 - 3102641}{900000} = \frac{27923774}{900000}$

$$= \frac{13961887}{450000}.$$

DIVISION

```
4) 275 (68
   24
    35
    32
     3
```

Here 4 is the divisor, 275 is the dividend, 68 is the quotient and 3 is the remainder. Remainder is always less than divisor.

```
Thus, Divisor) Dividend (Quotient
               abc
               Remainder
```

Thus,

Dividend = Divisor × Quotient + Remainder

For example, $275 = 4 \times 68 + 3$

When quotient is a whole number and remainder is zero, then dividend is divisible by divisor.

TESTS OF DIVISIBILITY

I. Divisibility by 2:

A number is divisible by 2 if its unit digit is any of 0, 2, 4, 6, 8.

Ex. 58694 is divisible by 2, while 86945 is not divisible by 2.

II. Divisible by 3:

A number is divisible by 3 only when the sum of its digits is divisible by 3.

Ex. (i) Sum of digits of the number 695421 = 27, which is divisible by 3.

∴ 695421 is divisible by 3.

(ii) Sum of digits of the number 948653 = 35, which is not divisible by 3.

∴ 948653 is not divisible by 3.

III. Divisible by 4:

A number is divisible by 4 if the number formed by its last two digits i.e. ten's and unit's digit of the given number is divisible by 4.

Ex. (i) 6879376 is divisible by 4, since 76 is divisible by 4.

(ii) 496138 is not divisible by 4, since 38 is not divisible by 4.

IV. Divisible by 5:

A number is divisible by 5 only when its unit digit is 0 or 5.

Ex. Each of the numbers 76895 and 68790 is divisible by 5.

V. Divisible by 6:

A number is divisible by 6 if it is simultaneously divisible by both 2 and 3.

Ex. 90 is divisible by 6 because it is divisible by both 2 and 3 simultaneously.

VI. Divisible by 7:

A number is divisible by 7 if and only if the difference of the number of its thousands and the remaining part of the given number is divisible by 7 respectively.

Ex. 473312 is divisible by 7, because the difference between 473 and 312 is 161, which is divisible by 7.

VII. Divisible by 8:

A number is divisible by 8 if the number formed by its last three digits i.e. hundred's, ten's and unit's digit of the given number is divisible by 8.

Ex. (i) In the number 16789352, the number formed by last 3 digits, namely 352 is divisible by 8.

∴ 16789352 is divisible by 8.

(ii) In the number 576484, the number formed by last 3 digits, namely 484 is not divisible by 8.

∴ 576484 is not divisible by 8.

VIII. Divisible by 9:

A number is divisible by 9 only when the sum of its digits is divisible by 9.

Ex. (i) Sum of digits of the number 246591 = 27, which is divisible by 9.

∴ 246591 is divisible by 9.

(ii) Sum of digits of the number 734519 = 29, which is not divisible by 9.

∴ 734519 is not divisible by 9.

IX. Divisible by 10:

A number is divisible by 10 only when its unit digit is 0.

Ex. (i) 7849320 is divisible by 10, since its unit digit is 0.

(ii) 678405 is not divisible by 10, since its unit digit is not 0.

X. Divisible by 11:

A number is divisible by 11 if the difference between the sum of its digits at odd places from right and the sum of its digits at even places also from right is either 0 or a number divisible by 11.

Ex. (i) Consider the number 29435417.

(Sum of its digits at odd places from right) – (Sum of its digits at even places from right)

(7 + 4 + 3 + 9) – (1 + 5 + 4 + 2) = (23 – 12) = 11, which is divisible by 11.

∴ 29435417 is divisible by 11.

(ii) Consider the number 57463822.

(Sum of its digits at odd places) – (Sum of its digits at even places)

= (2 + 8 + 6 + 7) – (2 + 3 + 4 + 5) = (23 – 14) = 9, which is neither 0 nor divisible by 11.

∴ 57463822 is not divisible by 11.

XI. Divisible by 12:

A number is divisible by 12, if it is simultaneously divisible by both 3 and 4.

Illustration 4: Find the least value of * for which 7* 5462 is divisible by 9.

Solution: Let the required value be x. Then,

$(7 + x + 5 + 4 + 6 + 2) = (24 + x)$ should be divisible by 9.

$\Rightarrow \quad x = 3$

Illustration 5: Find the least value of * for which 4832*18 is divisible by 11.

Solution: Let the digit in place of * be x.

(Sum of digits at odd places from right) – (Sum of digits at even places from right)

$= (8 + x + 3 + 4) – (1 + 2 + 8 = (4 + x)$,

which should be divisible by 11.

$\therefore \quad x = 7.$

PRIME NUMBERS

A number other than 1 is called a prime number if it is divisible by only 1 and itself.

All prime numbers less than 100 are:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

Note that 2 is the smallest prime number. 2 is the only even prime number.

Smallest odd prime number is 3.

Twin Primes: A pair of prime numbers are said to be twin prime when they differ by 2. For example 3 and 5 are twin primes.

Co-primes or Relative primes: A pair of numbers are said to be co-primes or relative primes to each other if they do not have any common factor other than 1. For example 13 and 21.

Some Properties which Help in Finding Two Co-prime Numbers

(i) Two consecutive natural numbers are always co-prime.

Ex. 8 and 9 are co-prime.

Also 12 and 13 are co-prime.

(ii) Two consecutive odd integers are always co-prime.

Ex. 7, 9; 15, 17; 21, 23; etc.

(iii) Two prime numbers are always co-prime.
Ex. 19 and 23 are co-prime.
Also 29 and 41 are co-prime.

(iv) A prime number and a composite number such that the composite number is not a multiple of the prime number are always co-prime.
Ex. 7 and 15 are co-prime.

(v) Square of two co-prime numbers are always co-prime numbers.

Some Properties which Help in Finding Three Co-prime Numbers

3 numbers are co-prime to each other means all the possible pair of numbers out of these three numbers are co-prime. For example from three numbers 7, 8, 13 three pairs (7, 8), (7, 13) and (8, 13) are formed and each of these pair is a pair of co-prime. Hence, 7, 8, 13 are three co-prime numbers.

Following are some properties helping in finding three co-prime numbers:

(i) Three consecutive odd integers are always co-prime.
Ex. 9, 11, 13 are co-prime.

(ii) Three consecutive natural numbers with first one being odd are always co-primes.
Ex. 7, 8, 9 are co-prime.

(iii) Two consecutive natural numbers along with the next odd numbers are always co-primes.
Ex. 12, 13, 15 are co-prime. Also 17, 18, 19 are co-prime.

(iv) Three prime numbers are always co-prime.
Ex. 3, 11, 13 are co-prime.

To Test Whether a Given Number is Prime Number or Not

In CAT and CAT like competitions you are required to check whether a given number maximum upto 400 is prime number or not.

If you want to test whether any number is a prime number or not, take an integer equal to the square root of the given number but if square root is not an integer then take an integer just larger than the approximate square root of that number. Let it be 'x'. Test the divisibility of the given number by every prime number less than 'x'. If the given number is not divisible by any prime number less than, then the given number is prime number; otherwise it is a composite number.

Square root of 361 is 19. Prime numbers less than 19 are clearly 2, 3, 5, 7, 11, 13 and 17. Since, 361 is not divisible by any of the numbers 2, 3, 5, 7, 11, 13 and 17. Hence, 361 is a prime number.

It is advisable to learn the squared numbers of all integers from 1 to 20, which are very useful to find whether a given number is a prime or not.

$1^2 = 1$
$2^2 = 4$
$3^2 = 9$
$4^2 = 16$
$5^2 = 25$
$6^2 = 36$
$7^2 = 49$
$8^2 = 64$
$9^2 = 81$
$10^2 = 100$
$11^2 = 121$
$12^2 = 144$
$13^2 = 169$
$14^2 = 196$
$15^2 = 225$
$16^2 = 256$
$17^2 = 289$
$18^2 = 324$
$19^2 = 361$
$20^2 = 400$

From the table it is clear that if any number, say 271 lies between 256 and 289, then its square root lies between 16 and 17, because $16^2 = 256$ and $17^2 = 289$. Thus square root of the given number is not an integer. So, we take 17 as an integer just greater than the square root of the given number. Now all the prime numbers less than 17 are 2, 3, 5, 7, 11 and 13. Since 271 is not divisible by any of the numbers 2, 3, 5, 7, 11 and 13. Hence 361 is a prime number.

Illustration 6: Is 171 is a prime number ?

Solution: Square root of 171 lies between 13 and 14, because $13^2 = 169$ and $14^2 = 196$. Therefore, the integer just greater than the square root of 171 is 14.

Now prime numbers less than 14 are 2, 3, 5, 7, 11 and 13.

Since 171 is divisible by 3, therefore 171 is not a prime number.

Illustration 7: Is 167 is a prime number ?

Solution: Square root of 167 lies between 12 and 13, because $12^2 = 144$ and $13^2 = 169$. Therefore the integer just greater than the square root of 167 is 13.

Now prime numbers less than 13 are 2, 3, 5, 7 and 11.

Since 167 is not divisible by any of the prime numbers 2, 3, 5, 7 and 11; therefore 167 is a prime number.

GENERAL OR EXPANDED FORM OF 2 AND 3 DIGITS NUMBERS

(i) In a two digits number AB, A is the digit of tenth place and B is the digit of unit place, therefore AB is written using place value in expanded form as
$AB = 10A + B$
Ex. $35 = 10 \times 3 + 5$

(ii) In a three digits number ABC, A is the digit of hundred place, B is the digit of tenth place and C is the digit of unit place, therefore ABC is written using place value in expanded form as
$ABC = 100A + 10B + C$
Ex. $247 = 100 \times 2 + 10 \times 4 + 7$
These expanded forms are used in forming equations related to 2 and 3 digits numbers.

Illustration 8: In a two digit prime number, if 18 is added, we get another prime number with reversed digits. How many such numbers are possible ?

Solution: Let a two-digit number be pq.

$\therefore \quad 10p + q + 18 = 10q + p$

$\Rightarrow \quad -9p + 9q = 18 \;\Rightarrow\; q - p = 2$

Satisfying this condition and also the condition of being a prime number (pq and qp both), there are 2 numbers 13 and 79.

FACTORISATION

It is a process of representing a given number as a product of two or more prime numbers.

Here each prime number which is present in the product is called a factor of the given number.

For example, 12 is expressed in the factorised form in terms of its prime factors as $12 = 2^2 \times 3$.

Illustration 9: If $N = 2^3 \times 3^7$, then

(a) What is the smallest number that you need to multiply with N in order to make it a perfect square ?

(b) What is the smallest number that you need to divide by N in order to make it a perfect square ?

Solution:

(a) Any perfect square number in its factorised form has prime factors with even powers. So in order to make $2^3 \times 3^7$ a perfect square, the smallest number that we need to multiply it with would be 2×3 i.e. 6. The resulting perfect square will be $2^4 \times 3^8$.

(b) Similarly, in order to arrive at a perfect square by dividing the smallest number, we need to divide the number by 2×3 i.e., 6. The resulting perfect square will be $2^2 \times 3^6$.

NUMBER OF WAYS OF EXPRESSING A COMPOSITE NUMBER AS A PRODUCT OF TWO FACTORS

(i) Number of ways of expressing a composite number N which is not a perfect square as a product of two factors

$$= \frac{1}{2} \times (\text{Number of prime factors of the } N)$$

(ii) Number of ways of expressing a perfect square number M as a product of two factors $= \frac{1}{2}$ [(Number of prime factors of M + 1]

Illustration 10: Find the number of ways of expressing 180 as a product of two factors.

Solution: $180 = 2^2 \times 3^2 \times 5^1$

Number of factors = (2 + 1) (2 + 1) (1 + 1) = 18

Since 180 is not a perfect square, hence there are total $\frac{18}{2} = 9$ ways in which 180 can be expressed as a product of two factors.

Illustration 11: Find the number of ways expressing 36 as a product of two factors.

Solution: $36 = 2^2 \times 3^2$

Number of factors = (2 + 1) (2 + 1) = 9

Since 36 is a perfect square, hence the number of ways of expressing 36 as a product of two factors

$= \frac{9+1}{2} = 5$, as 36 = 1 × 36, 2 × 18, 3 × 12, 4 × 9 and 6 × 6.

SUM OF FACTORS (OR DIVISORS) OF A COMPOSITE NUMBER

Let N be a composite number in such a way that $N = (x)^a (y)^b (z)^c$ ... where x, y, z... are prime numbers. Then, the sum of factors (or divisors) of $N = \frac{x^{a+1}-1}{x-1} \times \frac{y^{b+1}-1}{y-1} \times \frac{z^{c+1}-1}{z-1} \ldots$

Illustration 12: What is the sum of the divisors of 60 ?

Solution: $60 = 2^2 \times 3 \times 5$

$\Rightarrow$ Sum of the divisors $= \frac{2^3-1}{2-1} \times \frac{3^2-1}{3-1} \times \frac{5^2-1}{5-1} = 168$.

SUM OF UNIT DIGITS

For given n different digits $a_1, a_2, a_3, \ldots, a_n$; the sum of the digits at unit place of all different numbers formed is $(a_1 + a_2 + a_3 + \ldots + a_n)(n-1)!$ *i.e.*, (Sum of the digits) $(n-1)!$

Illustration 13: Find the sum of unit digits of all different numbers formed from digits 4, 6, 7 and 9.

Solution: Required sum $= (4 + 6 + 7 + 9) - (4 - 1)!$
$= 26 - 3! = 26 - 6 = 20.$

THE LAST DIGIT FROM LEFT (i.e., UNIT DIGIT) OF ANY POWER OF A NUMBER

The last digits (from left) of the powers of any number follow a cyclic pattern i.e., they repeat after certain number of steps. If we find out after how many steps the last digit of the powers of a number repeat, then we can find out the last digit of any power of any number.

Let us look at the powers of 2:

Last digit of 2^1 is 2 . Last digit of 2^6 is 4 .
Last digit of 2^2 is 4 . Last digit of 2^7 is 8 .
Last digit of 2^3 is 8 . Last digit of 2^8 is 6 .
Last digit of 2^4 is 6 . Last digit of 2^9 is 2 .
Last digit of 2^5 is 2 .

Since last digit of 2^5 is the same as the last digit of 2^1, then onwards the last digit will start repeating, i.e., digits of $2^5, 2^6, 2^7, 2^8$ will be the same as those of $2^1, 2^2, 2^3, 2^4$. Then the last digit of 2^9 is again the same as the last digit of 2^1 and so on. Thus, we see that when power of 2 increases, the last digits repeat after every 4 steps.

In above pattern, we can see that whenever the power of 2 is a multiple of 4, the last digit of that number will be the same as the last digit of 2^4.

Suppose we want to find out the last digit of 2^{66}, we should look at a multiple of 4 which is just less than or equal to the power 66 of 2. Since 64 is a multiple of 4, the last digit of 2^{64} will be the same as the last digit of 2^4.

Then the last digits of $2^{65}, 2^{66}$ will be the same as the last digits of $2^1, 2^2$ respectively. Hence the last digit of 2^{66} is the same as the last digit of 2^2 i.e., 4.

Similarly, we can find out the last digit of 3^{75} by writing down the pattern of the powers of 3.

Last digit of 3^1 is 3. Last digit of 3^4 is 1.
Last digit of 3^2 is 9. Last digit of 3^5 is 3.
Last digit of 3^3 is 7. Last digit of $3^6 = 9$
Last digit of $3^7 = 7$
Last digit of $3^8 = 1$
Last digit of $3^9 = 3$

The last digit repeats after 4 steps (like in the case of powers of 2).

Whenever the powers of 3 is a multiple of 4, the last digit of that number will be the same as the last digit of 3^4.

To find the last digit of 3^{75}, we look for a multiple of 4 which is just less than or equal to the power 75 of 3. Since, 72 is multiple of 4, the last digit of 3^{72} will be the same as that of 3^4. Hence the last digit of 3^{75} will be the same as the last digit of 3^3 i.e., 7.

Last Digit (i.e., Unit Digit) of a Product

Last digit of the product $a \times b \times c$ ... is the last digit of the product of last digits of a, b, c, ...

Illustration 14: Find the last digit of $2^{416} \times 4^{430}$.

Solution: Writing down the powers of 2 and 4 to check the pattern of the last digits, we have

We have seen that whenever the power of 2 is a multiple of 4, the last digit of that number will be the same as the last digit of 2^4.

Now, Last digit of $4^1 = 4$.

Last digit of $4^2 = 6$.

Last digit of $4^3 = 4$.

Last digit of $4^4 = 6$.

Thus last digit of any power of 4 is 4 for an odd power and 6 for an even power. The last digit of 2^{416} will be the same as 2^4 because 416 is a multiple of 4. So the last digit of 2^{416} is 6.

Last digit of 4^{430} is 6, since the power of 4 is even.

Hence the last digit of $2^{416} \times 4^{430}$ will be equal to the last digit of $6 \times 6 = 6$.

CONCEPT OF REMAINDERS

(I) Suppose the numbers $N_1, N_2, N_3, \ldots$ give quotients $Q_1, Q_2, Q_3, \ldots$ and remainder $R_1, R_2, R_3, \ldots$ when divided by a common divisor D.

Let S be the sum of $N_1, N_2, N_3, ..$

Therefore, $S = N_1 + N_2 + N_3 + \ldots$

$= (D \times Q_1 + R_1) + (D \times Q_2 + R_2) + (D \times Q_3 + R_3) + \ldots$

$= D \times K + (R_1 + R_2 + R_3 \ldots)$, ... (1)

where K is some number

Hence the remainder when S is divided by D is the remainder when $(R_1 + R_2 + R_3 \ldots\ldots)$ is divided by D.

(II) Suppose the numbers, $N_1, N_2, N_3, \ldots$ give quotients $Q_1, Q_2, Q_3, \ldots$ and remainders $R_1, R_2, R_3, \ldots$ respectively, when divided by a common divisor D.

Therefore $N_1 = D \times Q_1 + R_1, N_2 = D \times Q_2 + R_2,$

$N_3 = D \times Q_3 \times R_3 \ldots$ and so on.

Let P be the product of $N_1, N_2, N_3, \ldots$

Therefore,

$P = N_1 N_2 N_3 \ldots$

$= (D \times Q_1 + R_1)(D \times Q_2 + R_2)(D \times Q_3 + R_3)\ldots$

$= D \times K + (R_1 R_2 R_3 \ldots)$, ... (2)

where K is some number

In the above equation, since only the product $(R_1 R_2 R_3 \ldots)$ is free of D, therefore the remainder when P is divided by D is the remainder when the product $(R_1 R_2 R_3 \ldots)$ is divided by D.

Illustration 15: What is the remainder when the product 1991 × 1992 × 2000 is divided by 7 ?

Solution: The remainder when 1991, 1992 and 2000 are divided by 7 are 3, 4 and 5 respectively.

Hence the final remainder is the remainder when the product $3 \times 4 \times 5 = 60$ is divided by 7. Therefore, remainder = 4.

Illustration 16: What is the remainder when 2^{2010} is divided by 7 ?

Solution: 2^{2010} is a product $(2 \times 2 \times 2 \ldots (2010 \text{ times}))$. Since, 2 is a number less than 7, we try to convert the product into product of numbers higher than 7. Notice that $8 = 2 \times 2 \times 2$. Therefore, we convert the product in the following manner

$$2^{2010} = 8^{670} = 8 \times 8 \times 8 \ldots (670 \text{ times.})$$

The remainder when 8 is divided by 7 is 1. Hence the remainder when 8^{670} is divided by 7 is the remainder obtained when the product $1 \times 1 \times 1 \ldots$ (670 times) is divided by 7. Therefore, remainder = 1.

Illustration 17: What is the remainder when 25^{24} is divided by 9 ?

Solution: Again $25^{24} = (18 + 7)^{24} = (18 + 7)(18 + 7) \ldots$ 24 times $= 18K + 7^{24}$.

Hence, remainder when 25^{24} is divided by 9 is the remainder when 7^{24} is divided by 9.

Now, $7^{24} = 7^3 \times 7^3 \times 7^3 \ldots (8 \text{ times}) = 343 \times 343 \times 343 \ldots (8 \text{ times})$

Now when 343 is divided by 9 the remainder is 1

So, the remainder when dividing $(343)^8$ by 9 means remainder when dividing $(1)^8$ by 9. So the required remainder is 1.

NUMBER OF ZEROES IN AN EXPRESSION LIKE *a* × *b* × *c* × ..., WHERE *a, b, c*,... ARE NATURAL NUMBERS

Consider an expression $8 \times 15 \times 20 \times 30 \times 40$.

The expression can be written in the standard form as :

$8 \times 15 \times 20 \times 30 \times 40$

$= (2^3) \times (3 \times 5) \times (2^2 \times 5) \times (2 \times 3 \times 5) \times (2^3 \times 5)$

$= 2^9 \times 3^2 \times 5^4$, in which base of each factor is a prime number.

A zero is formed by the product of 2 and 5 i.e. 2×5. Hence number of zeroes is equal to the number of pair(s) of 2's and 5's formed.

In the above standard form of the product there are 9 twos and 4 fives. Hence number of pairs of 2 and 5 i.e. (2×5) is 4. Hence, there will be 4 zeroes at the end of the final product.

In the same above way, we can find the number of zeroes at the end of any product given in the form of an expression like $a \times b \times c \times \ldots$, where *a, b, c*,... are natural numbers.

If there is no pair of 2 and 5 i.e. 2×5, then there is no zero at the end of the product. For example, consider the expression $9 \times 21 \times 39 \times 49$.

The given expression in standard form,

$9 \times 21 \times 39 \times 49 = (3^2) \times (3 \times 7) \times (3 \times 13) \times (7^2)$

$= 3^4 \times 7^3 \times 13$

There is no pair of 2 and 5 in the standard form of expression given as product, therefore there will be no zero at the end of the final product.

Illustration 18: Find the number of zeroes in the product $1^1 \times 2^2 \times 3^3 \times 4^4 \times 5^5 \times 6^6 \times \ldots\ldots\ldots\ldots \times 49^{49}$

Solution: Clearly the fives will be less than the twos. Hence, we need to count only the fives.

Now, $5^5 \times 10^{10} \times 15^{15} \times 20^{20} \times 25^{25} \times 30^{30} \times 35^{35} \times 40^{40} \times 45^{45}$

$= (5)^5 \times (5 \times 2)^{10} \times (5 \times 3)^{15} \times (5 \times 4)^{20} \times (5 \times 5)^{25} \times (5 \times 6)^{30} \times (5 \times 7)^{35} \times (5 \times 8)^{40} \times (5 \times 9)^{45}$

It gives us $5 + 10 + 15 + 20 + 25 \times 25 + 30 + 35 + 40 + 45$ fives *i.e.*, 825 fives

Thus the product has 825 zeroes.

BASE SYSTEM

The number system in which we carry out all calculation is decimal (base 10) system. It is called decimal system because there are 10 digits 0 to 9.

There are other number systems also depending on the number of digits contained in the base system. Some of the most common systems are Binary system, Octal system, and Hexadecimal system. A number system containing two digits 0 and 1 is called binary (base 2) system. Number system containing eight digits 0, 1, 2, 3, ..., 7 is called Octal (base 8) system.

Hexadecimal (base 16) system has 16 digits 0, 1, 2, 3, .., 9, *A*, *B, C, D, E, F*; where *A* has a value 10, *B* has a value 11 and so on.

> Let a number *abcde* be written in base *p*, where *a, b, c, d* and *e* are single digits less than *p*. The value of the number abcde in base 10 = $e \times p^0 + d \times p^1 + c \times p^2 + b \times p^3 + a \times p^4$
>
> For example, The number 7368 can be written as
>
> $8 + 6 \times 10 + 3 \times (10)^2 + 7 \times (10)^3 = 7368$ in decimal (base 10) number system.
>
> The number 7368 in base 9 is written in decimal number system as
>
> $8 \times 9^0 + 6 \times 9 + 3 \times 9^2 + 7 \times 9^3 = 5408$

There are mainly two types of operations associated with conversion of bases: First conversion from any base to base ten and second conversion from base 10 to any base.

(i) Conversion From Any Base to Base Ten

The number $(pqrstu)_a$ (i.e., the number *pqrstu* on base *a*) is converted to base 10 by finding the value of the number.

$(pqrstu)_a = u + ta + sa^2 + ra^3 + qa^4 + pa^5$.

Here subscript '*a*' in $(pqrst)_a$ denotes the base of the number *pqrstu*.

Illustration 19: Convert $(21344)_5$ to base 10.

Solution:

$(21344)_5 = 4 \times 5^0 + 4 \times 5^1 + 3 \times 5^2 + 1 \times 5^3 + 2 \times 5^4$
$= 4 + 4 \times 5 + 3 \times 25 + 1 \times 125 + 2 \times 625 = 1474.$

(ii) Conversion From Base 10 to Any Base

A number written in base 10 can be converted to any base '*a*' by first dividing the number by '*a*' and then successively dividing the quotients by '*a*'. The remainders, written in reverse order, give the equivalent number in base '*a*'.

For example the number 238 in base 3 is found as

$$\begin{array}{r|rl} 3 & 238 & \\ \hline & 79 & 1 \\ & 26 & 1 \\ & 8 & 2 \\ & 2 & 2 \end{array} \quad \leftarrow \text{Remainders}$$

The remainders in reverse order is 22211.

Hence, 22211 is the required number in base 3.

> **Note:** Value of a single digit number to all bases are the same. For example,
>
> $5_4 = 5_7 = 5_8 = 5_{10}$

Addition, Subtraction and Multiplication in the Same Bases

Illustration 20: Add the numbers $(4235)_7$ and $(2354)_7$.

Solution: The numbers are written as

$$\begin{array}{r} 4\ 2\ 3\ 5 \\ 2\ 3\ 5\ 4 \\ \hline \\ \hline \end{array}$$

The addition of 5 and 4 (first digit from right of both numbers) is 9 which being more than 7 would be written as $9 = 7 \times 1 + 2$. Here 1 is the quotient and 2 is the remainder when 9 is divided by 7. The remainder 2 is placed at the first place from right of the answer and the quotient 1 gets carried over to the second place from the right.

At the second place from the right $3 + 5 + 1$ (carry) $= 9 = 7 \times 1 + 2$

$$\begin{array}{cccc} & & +1 & +1 \\ 4 & 2 & 3 & 5 \\ 2 & 3 & 5 & 4 \\ \hline 6 & 6 & 2 & 2 \\ \hline \end{array}$$

The remainder 2 is placed at the second place from right of the answer and the quotient 1 carry over to the third place from right.

In the same way, we can find the other digits of the answer.

Illustration 21: $(52)_7 + 46_8 = (?)_{10}$

(a) $(75)_{10}$ **(b) $(50)_{10}$**
(c) $(39)_{39}$ **(d) $(28)_{10}$**

Solution: (a) $(52)_7 = (5 \times 7^1 + 2 \times 7^0)_{10} = (37)_{10}$

Also, $(46)_8 = (4 \times 8^1 + 6 \times 8^0)_{10} = (38)_{10}$

Sum $= (37)_{10} + (38)_{10} = (75)_{10}$

Illustration 22: $(11)_2 + (22)_3 + (33)_4 + (44)_5 + (55)_6 + (66)_7 + (77)_8 + (88)_9 = (?)_{10}$

(a) 396 **(b) 276**
(c) 250 **(d) 342**

Solution: (b) $(11)_2 = (1 \times 2^1 + 1 \times 2^0)_{10} = (3)_{10}$

$(22)_3 = (2 \times 3^1 + 2 \times 3^0)_{10} = (8)_{10}$

$(33)_4 = (3 \times 4^1 + 3 \times 4^0)_{10} = (15)_{10}$

$(44)_5 = (4 \times 5^1 + 4 \times 5^0)_{10} = (24)_{10}$

$(55)_6 = (5 \times 6^1 + 5 \times 6^0)_{10} = (35)_{10}$

$(66)_7 = (6 \times 7^1 + 6 \times 7^0)_{10} = (48)_{10}$

$(77)_8 = (7 \times 8^1 + 7 \times 8^0)_{10} = (63)_{10}$

$(88)_9 = (8 \times 9^1 + 8 \times 9^0)_{10} = (80)_{10}$

Sum $= (3)_{10} + (8)_{10} + (15)_{10} + (24)_{10} + (35)_{10} + (48)_{10} + (63)_{10} + (80)_{10}$
$= (276)_{10}$

Illustration 23: Subtract $(247)_8$ from $(345)_8$.

Solution:

(i) 5 is less than 7. So borrow 1 from the previous digit 4. Since, we are working in octal system, so 5 become $5 + 8 = 13$. Subtract 7 from 13, you will get 6.

$$\begin{array}{r} 3\ 4\ 5 \\ -2\ 4\ 7 \\ \hline 6 \\ \hline \end{array}$$

(ii) Since, we have borrowed 1, the 4 in the first row has now become 3, which is less than the digit (4), just below it in the second row, So borrow 1 from 3 of first row. So, the 4 in first row is now becomes 3 + 8 = 11. Subtracting 4 of second row from 11, we get 7. Hence,

$$\begin{array}{r} 3\ 4\ 5 \\ -2\ 4\ 7 \\ \hline 0\ 7\ 6 \\ \hline \end{array}$$

FACTORS AND MULTIPLES

If one number '*a*' completely divides a second number '*b*' then 1st number '*a*' is said to be a factor of the 2nd number '*b*'. For example 3 completely divides 15, so 3 is a factor of 15; while 4 does not divide 15 completely, so 4 is not a factor of 15.

Factors of 30 are 1, 2, 3, 5, 6, 10, 15 and 30

Factors of 40 are 1, 2, 4, 5, 8, 10, 20 and 40.

If a number '*a*' is exactly divisible by *a* number '*b*' then the 1st number '*a*' is said to be a multiple of 2nd number '*b*'. For example, 35 is exactly divisible by 7, so 35 is a multiple of 7. Multiple of a number '*b*' can be written down as '*nb*' where *n* is a natural number. So multiples of 5 are 5, 10, 15, 20, 25, ...

HIGHEST COMMON FACTOR (HCF) OR GREATEST COMMON DIVISOR (GCD)

The highest (i.e. largest) number that divides two or more given numbers is called the highest common factor (HCF) of those numbers.

Consider two numbers 12 and 15.

Factors of 12 are 1, 2, 3, 4, 6, 12.

Factors of 30 are 1, 2, 3, 5, 6, 10, 15, 30.

We have some common factors out of these factors of 12 and 30, which are 1, 2, 3, 6. Out of these common factors, 6 is the highest common factor. So, 6 is called the Highest Common Factor (HCF) of 12 and 30.

Methods to Find The HCF or GCD

There are two methods to find HCF of the given numbers

(i) Prime Factorization Method

When a number is written as the product of prime numbers, then it is called the prime factorization of that number. For example, $72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$. Here, $2 \times 2 \times 2 \times 3 \times 3$ or $2^3 \times 3^2$ is called prime factorization of 72.

To find the HCF of given numbers by this methods, we perform the prime factorization of all the numbers and then check for the common prime factors. For every prime factor common to all the numbers, we choose the least index of that prime factor among the given numbers. The HCF is the product of all such prime factors with their respective least indices.

Illustration 24: Find the HCF of $36x^3y^2$ and $24x^4y$.

Solution $36x^3y^2 = 2^2.3^2.x^3.y^2$, $24x^4y = 2^3.3.x^4.y$. The least index of 2, 3, *x* and *y* in the numbers are 2, 1, 3 and 1 respectively. Hence the HCF $= 2^2.3.x^3.y = 12x^3y$.

Illustration 25: The numbers 400, 536 and 646; when divided by a number N, give the remainders of 22, 23 and 25 respectively. Find the greatest such number *N*.

Solution: *N* will be the HCF of (400 – 22), (536 – 23) and (646 – 25). Hence, *N* will be the HCF of 378, 513 and 621. Hence, *N* = 27.

Illustration 26: The HCF of two numbers is 12 and their product is 31104. How many such numbers are possible.

Solution: Let the numbers be 12*x* and 12*y*, where *x* and *y* are co-prime to each other.

Therefore, $12x \times 12y = 31104 \rightarrow xy = 216$.

Now we need to find co-prime pairs whose product is 216.

$216 = 2^3 \times 3^3$. Therefore, the co-prime pairs will be (1, 216) and (8, 27). Therefore, (12, 12 × 216) and (8 × 12, 27 × 12) are two possible numbers.

(ii) Division Method

To find the HCF of two numbers by division method, we divide the larger number by the smaller number. Then we divide the smaller number by the first remainder, then first remainder by the second remainder.. and so on, till the remainder becomes 0. The last divisor is the required HCF.

Illustration 27: Find the HCF of 288 and 1080 by the division method.

Solution:

```
288 | 1080 | 3
       864
      ----
       216 | 288 | 1
             216
             ---
              72 | 216 | 3
                   216
                   ---
                     0
```

The last divisor 72 is the HCF of 288 and 1080.

Shortcut for Finding HCF or GCD

To find the HCF of any number of given numbers, first find the difference between two nearest given numbers. Then find all factors (or divisors) of this difference. Highest factor which divides all the given numbers is the HCF.

Illustration 28: Find the HCF of 12, 20 and 32.

Solution: Difference of nearest two numbers 12 and 20 = 20 – 12 = 8

All factors (or divisor) of 8 are 1, 2, 4 and 8.

1, 2 and 4 divides each of the three given numbers 12, 20 and 32. Out of 1, 2 and 4; 4 is the highest number. Hence, HCF = 4.

LEAST COMMON MULTIPLE (LCM)

The least common multiple (LCM) of two or more numbers is the lowest number which is divisible by all the given numbers.

Consider two numbers 12 and 15.

Multiples of 12 are 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132,...

While the multiples of 15 are 15, 30, 45, 60, 75, 90, 105, 120, 135, 150,.....

Out of these series of multiples, we have some common multiples like 60, 120, 180, ..., etc. Out of these common multiples, 60 is the lowest, so 60 is called the Lowest Common Multiple (LCM) of 12 and 15.

Methods to Find The LCM

There are two methods to find the LCM.

(i) Prime Factorization Method

After performing the prime factorization of all the given numbers, we find the highest index of all the prime numbers among the given numbers. The LCM is the product of all these prime numbers with their respective highest indices because LCM must be divisible by all of the given numbers.

Illustration 29: Find the LCM of 72, 288 and 1080.

Solution: $72 = 2^3 \times 3^2$

$288 = 2^5 \times 3^2$

$1080 = 2^3 \times 3^3 \times 5$

Hence, $\text{LCM} = 2^5 \times 3^3 \times 5^1 = 4320$

(ii) Division Method

To find the LCM of 5, 72, 196 and 240, we use the division method in the following way:

Check whether any prime number that divides at least two of all the given numbers. If there is no such prime number, then the product of all these numbers is the required LCM, otherwise find the smallest prime number that divides at least two of the given numbers. Here, we see that smallest prime number that divides at least two given numbers is 2.

Divide those numbers out of the given numbers by 2 which are divisible by 2 and write the quotient below it. The given number(s) that are not divisible by 2 write as it is below it and repeat this step till you do not find at least two numbers that are not divisible by any prime number.

2	5,	72,	196,	240
2	5,	36,	98,	120
2	5,	18,	49,	60
3	5,	9,	49,	30
5	5,	3,	49,	10
	1,	3,	49,	2

After that find the product of all divisors and the quotient left at the end of the division. This product is the required LCM.

Hence, LCM of the given numbers = product of all divisors and the quotient left at the end.

$= 2 \times 2 \times 2 \times 3 \times 5 \times 3 \times 49 \times 2 = 35280$

Illustration 30: On a traffic signal, traffic light changes its colour after every 24, 30 and 36 seconds in green, red and orange light. How many times in an hour only green and red light will change simultaneously.

Solution: LCM. of 24 and 30 = 120

So in 1 hr both green and red light will change simultaneously 3600/120 times = 30 times

LCM of 24, 30 and 36 is 360

Hence in 1 hr all three lights will change simultaneously 3600/360 times = 10 times

So in 1 hr only red and green lights will change 30 – 10 = 20 times simultaneously.

Shortcut For Finding LCM

Using idea of co-prime, you can find the LCM by the following shortcut method:

LCM of 9, 10, 15 and 36 can be written directly as 9 × 10 × 2.

The logical thinking that behind it is as follows:

Step 1: If you can see a set of 2 or more co-prime numbers in the set of numbers of which you are finding the LCM, write them down by multiply them.

In the above situation, since we see that 9 and 10 are co-prime to each other, we start off writing the LCM by writing 9 × 10 as the first step.

Step 2: For each of the other numbers, consider what prime factor(s) of it is/are not present in the LCM (if factorised into primes) taken in step 1. In case you see some prime factors of each of the other given numbers separately are not present in the LCM (if factorised into primes) taken in step 1, such prime factors will be multiplied in the LCM taken in step 1.

Prime factorisation of 9 × 10 = 3 × 3 × 2 × 5

Prime factorisation of 15 = 3 × 5

Prime factorisation of 36 = 2 × 2 × 3 × 3

Here we see that both prime factors of 15 are present in the prime factorisation of 9 × 10 but one prime factor 2 of 36 is not present in the LCM taken in step 1. So to find the LCM of 9, 10, 15 and 36; we multiply the LCM taken in step 1 by 2.

Thus required LCM = 9 × 10 × 2 = 180

Rule For Finding HCF and LCM of Fractions

(I) HCF of two or more fractions

$$= \frac{\text{HCF of numerator of all fractions}}{\text{LCM of denominator of all fractions}}$$

(II) LCM of two or more fractions

$$= \frac{\text{LCM of numerator of all fractions}}{\text{HCF of denominator of all fractions}}$$

Illustration 31: Find the HCF and LCM of $\frac{4}{5}, \frac{6}{11}, \frac{3}{5}$.

Solution: $\text{HCF} = \frac{\text{HCF of } 4, 6, 3}{\text{LCM of } 5, 11, 5} = \frac{1}{55}$

$$\text{LCM} = \frac{\text{LCM of } 4, 6, 3}{\text{HCF of } 5, 11, 5} = \frac{12}{1} = 12$$

For any two numbers, HCF × LCM = product of the two numbers

This formula is applicable only for two numbers.

For example, HCF of 288 and 1080 is 72 and LCM of these two numbers is 4320.

We can see that 72 × 4320 = 311040 = 288 × 1080.

GREATEST INTEGRAL VALUE

If x be a real number, then $[x]$ indicates greatest integer equal or less than x.

If the given number is an integer, then the greatest integer gives the number itself, otherwise it gives the first integer towards the left of the number x on the number line.

For example $[4] = 4$, $[3.4] = 3$, $[6.8] = 6$, $[-2.3] = -3$, $[-5.6] = -6$ and so on.

Note that -3 is less than -2.3 and -6 is less than -5.6, etc.

Illustration 32: What is the value of

$$\left[\sqrt{1}\right]+\left[\sqrt{2}\right]+\left[\sqrt{3}\right]+ \ldots +\left[\sqrt{49}\right]+\left[\sqrt{50}\right]$$

where $[x]$ denotes greatest integer function?

Solution: $1^2 = 1$, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$, $5^2 = 25$, $6^2 = 36$, $7^2 = 49$, $8^2 = 64$

Therefore, from $\left[\sqrt{1}\right]$ to $\left[\sqrt{3}\right]$, the value will be 1, from $\left[\sqrt{4}\right]$ to $\left[\sqrt{8}\right]$ the value will be 2, from $\left[\sqrt{9}\right]$ to $\left[\sqrt{15}\right]$ the value will be 3 and so on.

Therefore, the total value

$= 1 \times 3 + 2 \times 5 + 3 \times 7 + 4 \times 9 + 5 \times 11 + 6 \times 13 + 7 \times 2$

$= 3 + 10 + 21 + 36 + 55 + 78 + 14 = 217.$

Illustration 33: What is the value of x for which $x[x] = 32$?

Solution: If the value of x is 5, $x[x] = 25$, and if the value of x is 6, then $x[x] = 36$

Therefore, the value of x lies between 5 and 6.

If x lies between 5 and 6, then $[x] = 5$.

$$\Rightarrow \quad x = \frac{28}{[x]} = \frac{32}{5} = 6.4 .$$

Practice Exercise

Level - I

1. The greatest number which will divide 116, 221, 356 leaving the same remainder in each case is
(a) 15 (b) 5
(c) 10 (d) 20

2. What number has to be added to 345670 in order to make it divisible by 6?
(a) 2 (b) 4
(c) 5 (d) 6

3. The least number which when divided by 35 leaves a remainder 25, when divided by 45 leaves the remainder 35 and when divided by 55 leaves 45 is
(a) 3465 (b) 3645
(c) 3655 (d) 3455

4. If n is any even number, then $n(n^2 + 20)$ is always divisible by
(a) 15 (b) 20
(c) 24 (d) 32

5. When 2^{256} is divided by 17 the remainder would be
(a) 1 (b) 16
(c) 14 (d) None of these

6. The last digit of 2137^{753} is
(a) 9 (b) 7
(c) 3 (d) 1

7. Find the least square number which is divisible by 3, 5, 6, and 9.
(a) 900 (b) 90
(c) 8100 (d) 81

8. In order that the number 1 y 3 y 6 be divisible by 11, the digit y should be
(a) 1 (b) 2
(c) 5 (d) 6

9. If n is an even natural number, then the largest natural number by which $n(n + 1)(n + 2)$ is divisible is
(a) 6 (b) 8
(c) 12 (d) 24

10. Which number should be added to 459045 to make it exactly divisible by 27 ?
(a) 3 (b) 9
(c) 0 (d) None of these

11. Find the last digit of the sum $19^{81} + 4^{9k}$, $K \in N$.
(a) 4 (b) 9
(c) 3 (d) Cannot be determined

12. The sum of prime numbers that are greater than 60, but less than 70 is
(a) 128 (b) 191
(c) 197 (d) 260

13. The number 311311311311311311311 is
(a) divisible by 3 but not by 11
(b) divisible by 11 but not by 3
(c) divisible by both 3 and 11
(d) neither divisible by 3 nor by 11

14. A difference between two numbers is 1365, when larger number is divided by the smaller one, the quotient is 6 and the remainder is 15. What is the smaller number?
(a) 240 (b) 360
(c) 270 (d) 295

15. If the number 517 * 324 is completely divisible by 3, then the smallest whole number in place of * will be:
(a) 0 (b) 1
(c) 2 (d) None of these

16. If the product 4864 × 9 P 2 is divisible by 12, the value of P is
(a) 2 (b) 5
(c) 6 (d) None of these

17. The largest 4-digit number exactly divisible by 88 is
(a) 9944 (b) 9768
(c) 9988 (d) 8888

18. $(x^n - a^n)$ is completely divisible by $(x + a)$, when
(a) n is any natural number
(b) n is an even natural number
(c) n is an odd natural number
(d) n is prime

19. When $0.\overline{47}$ is converted into a fraction the result is
(a) $\frac{46}{90}$ (b) $\frac{46}{99}$
(c) $\frac{47}{90}$ (d) $\frac{47}{99}$

20. Which of the following statements are true:
(i) The rational number $\frac{29}{23}$ lies to the left of zero on the number line.
(ii) The rational number $\frac{-12}{-17}$ lies to the right of zero on the number line.
(iii) The ratinal numbers $\frac{-12}{5}$ and $\frac{-7}{17}$ are on the opposite side of zero on the number line.
(v) The rational numbers $\frac{-21}{5}$ and $\frac{7}{-31}$ are on the opposite side of zero on the number line.
(a) Only (i) (b) (i) & (ii)
(c) Only (iii) (d) (i), (ii) & (iv)

21. I have a certain number of beads which lie between 600 and 900. If 2 beads are taken away the remainder can be equally divided among 3, 4, 5, 6, 7 or 12 boys. The number of beads I have

(a) 729 (b) 842
(c) 576 (d) 961

22. Find the digit at the unit's place of
$(377)^{59} \times (793)^{87} \times (578)^{129} \times (99)^{99}$

(a) 1 (b) 2
(c) 7 (d) 9

23. Four different electronic devices make a beep after every 30 minutes, 1 hour, $1\frac{1}{2}$ hour and 1 hour 45 minutes respectively. All the devices beeped together at 12 noon. They will again beep together at:

(a) 12 midnight (b) 3 a.m.
(c) 6 a.m. (d) 9 a.m.

24. If N is the sum of first 13,986 prime numbers, then N is always divisible by

(a) 6 (b) 4
(c) 8 (d) None of these

25. If two numbers when divided by a certain divisor give remainder 35 and 30 respectively and when their sum is divided by the same divisor, the remainder is 20, then the divisor is

(a) 40 (b) 45
(c) 50 (d) 55

26. Find the least number which when divided by 12, leaves a remainder 7, when divided by 15, leaves a remainder 10 and when divided by 16, leaves a remainder 11

(a) 115 (b) 235
(c) 247 (d) 475

27. How many even integers n, where $100 \le n \le 200$, are divisible neither by seven nor by nine ?

(a) 40 (b) 37
(c) 39 (d) 38

28. A number is *interesting* if on adding the sum of the digits of the number and the product of the digits of the number, the result is equal to the number. What fraction of numbers between 10 and 100 (both 10 and 100 included) is *interesting?*

(a) 0.1 (b) 0.11
(c) 0.16 (d) 0.22

29. In a cricket match, Team A scored 232 runs without losing a wicket. The score consisted to byes, wides and runs scored by two opening batsmen : Ram and Shyam. The runs scored by the two batsman are 26 times wides. There are 8 more byes than wides. If the ratio of the runs scored by Ram and Shyam is 6 : 7, then the runs scored by Ram is

(a) 88 (b) 96
(c) 102 (d) 112

30. If $x + y + z = 1$ and x, y, z are positive real numbers, then the least value of $\left(\frac{1}{x}-1\right)\left(\frac{1}{y}-1\right)\left(\frac{1}{z}-1\right)$ is

(a) 4 (b) 8
(c) 16 (d) None of these

31. The last digit of $3^{3^{4n}} + 1$, is

(a) 0 (b) 4
(c) 8 (d) 2

32. The last digit in $(25\_)^{32}$ and $(25\_)^{33}$ both is 6. The missing digit is :

(a) 4 (b) 8
(c) 6 (d) 5

33. Which digits should come in place of * and $ if the number 62684*$ is divisible by both 8 and 5?

(a) 4, 0 (b) 0, 4
(c) 0, 0 (d) 4, 4

34. At a college football game, 4/5 of the seats in the lower deck of the stadium were sold. If 1/4 of all the seating in the stadium is located in the lower deck, and if 2/3 of all the seats in the stadium were sold, then what fraction of the unsold seats in the stadium was in the lower deck ?

(a) 3/20 (b) 1/6
(c) 1/5 (d) 1/3

35. The integers 1, 2, ..., 40 are written on a blackboard. The following operation is then repeated 39 times; In each repetition, any two numbers, say a and b, currently on the blackboard are erased and a new number $a + b - 1$ is written. What will be the number left on the board at the end?

(a) 820 (b) 821
(c) 781 (d) 819

36. If $653xy$ is divisible by 80 then the value of $x + y$ is

(a) 2 (b) 3
(c) 4 (d) 6

37. How many numbers are there between 200 and 800 which are divisible by both 5 and 7?

(a) 35 (b) 16
(c) 17 (d) can't be determined

38. How many numbers are there in the set $S = \{200, 201, 202, ..., 800\}$ which are divisible by neither of 5 or 7?

(a) 411 (b) 412
(c) 410 (d) None of these

39. When a number divided by 9235, we get the quotient 888 and the remainder 222, such a least possible number is

(a) 820090 (b) 8200920
(c) 8200680 (d) None of these

40. A number which when divided by 32 leaves a remainder of 29. If this number is divided by 8 the remainder will be

(a) 0 (b) 1
(c) 5 (d) 3

41. $(0.\overline{1})^2\left[1-9(0.\overline{16})^2\right] = ?$

(a) $-\frac{1}{162}$ (b) $\frac{1}{108}$
(c) $\frac{7696}{106}$ (d) $\frac{833}{88209}$

42. A six digit number which is consisting of only one digits either 1, 2, 3, 4, 5, 6, 7, 8 or 9, *e.g.,* 111111, 222222... etc. This number is always divisible by :

(a) 7 (b) 11
(c) 13 (d) All of these