

# सरलीकरण | Simplification

## Type-2

1.  $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots + \frac{1}{n \times (n+1)} = ?$

- (A)  $\frac{n}{n+1}$  (B)  $\frac{n}{n-1}$   
(C)  $\frac{n+1}{n}$  (D)  $\frac{n-1}{n}$

2.  $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots + \frac{1}{100 \times 101} = ?$

- (A)  $\frac{99}{100}$  (B)  $\frac{98}{99}$   
(C)  $\frac{100}{101}$  (D)  $\frac{97}{100}$

3.  $\frac{1}{10} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \dots + \frac{1}{90} = ?$

- (A)  $\frac{1}{3}$  (B)  $\frac{1}{4}$   
(C)  $\frac{79}{89}$  (D)  $\frac{81}{85}$

4.  $\frac{1}{3 \times 7} + \frac{1}{7 \times 11} + \frac{1}{11 \times 15} + \dots + \frac{1}{899 \times 903} = ?$

- (A)  $\frac{21}{509}$  (B)  $\frac{18}{403}$   
(C)  $\frac{25}{301}$  (D)  $\frac{29}{31}$

5. If  $x_1 = \frac{1}{1 \times 3}$ ,  $x_2 = \frac{1}{3 \times 5}$ ,  $x_3 = \frac{1}{5 \times 7}$ , then find  $x_1 + x_2 + x_3 + \dots + x_{100} = ?$

यदि  $x_1 = \frac{1}{1 \times 3}$ ,  $x_2 = \frac{1}{3 \times 5}$ ,  $x_3 = \frac{1}{5 \times 7}$  हो, तो ज्ञात कीजिये  $x_1 + x_2 + x_3 + \dots + x_{100} = ?$

- (A)  $\frac{101}{200}$  (B)  $\frac{100}{201}$   
(C)  $\frac{100}{200}$  (D) None of these

6.  $\frac{5}{2^2 \cdot 3^2} + \frac{7}{3^2 \cdot 4^2} + \frac{9}{4^2 \cdot 5^2} + \dots + \frac{39}{19^2 \cdot 20^2} = ?$

- (A)  $\frac{99}{400}$  (B)  $\frac{101}{400}$   
(C)  $\frac{99}{301}$  (D)  $\frac{75}{361}$

7.  $\frac{1}{2^2 - 1} + \frac{1}{4^2 - 1} + \frac{1}{6^2 - 1} + \dots + \frac{1}{20^2 - 1} = ?$

- (A)  $\frac{9}{19}$  (B)  $\frac{10}{19}$   
(C)  $\frac{10}{21}$  (D)  $\frac{11}{19}$

8.  $\frac{1}{7^2 - 3^2} + \frac{1}{13^2 - 3^2} + \frac{1}{19^2 - 3^2} + \dots + \frac{1}{31^2 - 3^2} = ?$

- (A)  $\frac{5}{31}$  (B)  $\frac{5}{136}$   
(C)  $\frac{15}{131}$  (D)  $\frac{15}{136}$

9.  $\frac{1}{1 \times 2 \times 3} + \frac{1}{2 \times 3 \times 4} + \frac{1}{3 \times 4 \times 5} + \dots + \frac{1}{n(n+1)(n+2)} = ?$

- (A)  $\frac{1}{4} - \frac{1}{2(n+1)(n+2)}$   
(B)  $\frac{1}{4} + \frac{1}{2(n+1)(n+2)}$   
(C)  $\frac{1}{4} + \frac{1}{2(n+1)(n-2)}$   
(D)  $\frac{1}{4} - \frac{1}{2(n+1)(n-2)}$

10.  $\frac{1}{1 \times 2 \times 3 \times 4} + \frac{1}{2 \times 3 \times 4 \times 5} + \dots +$

$$\frac{1}{n(n+1)(n+2)(n+3)} = ?$$

(A)  $\frac{1}{3} \left[ \frac{1}{1 \times 2 \times 3} - \frac{1}{(n+1)(n+2)(n+3)} \right]$

(B)  $\frac{1}{n} \left[ \frac{1}{1 \times 2 \times 3} - \frac{1}{(n+1)(n+2)(n+3)} \right]$

(C)  $\frac{1}{3} \left[ \frac{1}{1 \times 2 \times 3} - \frac{1}{(n+1)(n+2)(n-3)} \right]$

(D)  $\frac{1}{3} \left[ \frac{1}{1 \times 2 \times 3} + \frac{1}{(n+1)(n+2)(n+3)} \right]$

11.  $\frac{1}{1 \times 3 \times 5 \times 7} + \frac{1}{3 \times 5 \times 7 \times 9} + \dots + 10 \text{ terms} = ?$

(A)  $\frac{134}{12075}$

(B)  $\frac{134}{12705}$

(C)  $\frac{134}{12750}$

(D) None of these

12.  $\frac{1}{1 \times 6 \times 11} + \frac{1}{6 \times 11 \times 16} + \dots + 10 \text{ terms} = ?$

(A)  $\frac{95}{5712}$

(B)  $\frac{97}{5712}$

(C)  $\frac{97}{5714}$

(D)  $\frac{95}{5713}$

13.  $S = \frac{1}{1 \times 3 \times 5} + \frac{1}{1 \times 4} + \frac{1}{3 \times 5 \times 7} + \frac{1}{4 \times 7} + \dots + 20 \text{ terms} = ?$

(A)  $\frac{6179}{15275}$

(B)  $\frac{6070}{14973}$

(C)  $\frac{7191}{15174}$

(D)  $\frac{5183}{16973}$

14. Which of the following statement(s) is/are true?  
निम्नलिखित में से कौन-सा/से कथन सत्य है/हैं?

I.  $\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \dots + \frac{1}{11 \times 13} = \frac{12}{13}$

II.  $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots + \frac{1}{12 \times 13} = \frac{12}{13}$

(A) Only I/ केवल I

(B) Only II/ केवल II

(C) Both I and II/ I और II दोनों

(D) Neither I nor II/ न तो I न ही II

15. If  $A = \frac{1}{1 \times 2} + \frac{1}{1 \times 4} + \frac{1}{2 \times 3} + \frac{1}{4 \times 7} + \frac{1}{3 \times 4} + \frac{1}{7 \times 10} + \dots$  upto 20 terms, then what is the value of A ?

यदि  $A = \frac{1}{1 \times 2} + \frac{1}{1 \times 4} + \frac{1}{2 \times 3} + \frac{1}{4 \times 7} + \frac{1}{3 \times 4} + \frac{1}{7 \times 10} + \dots$  20 पदों तक हो, तो A का मान क्या है?

(A)  $\frac{379}{308}$

(B)  $\frac{171}{140}$

(C)  $\frac{379}{310}$

(D)  $\frac{420}{341}$

**SOLUTIONS**

$$\begin{aligned}
 1. \quad (A) \quad & \frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots + \frac{1}{n \times (n+1)} \\
 &= \frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{n} - \frac{1}{(n+1)} \\
 &= 1 - \frac{1}{(n+1)} = \frac{n}{n+1}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad (C) \quad & \frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots + \frac{1}{100 \times 101} \\
 &= \frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{100} - \frac{1}{101} \\
 &= 1 - \frac{1}{101} = \frac{100}{101}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad (A) \quad & \frac{1}{10} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \dots + \frac{1}{90} \\
 &= \frac{1}{10} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \frac{1}{5 \times 6} + \dots + \frac{1}{9 \times 10} \\
 &= \frac{1}{10} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \dots + \frac{1}{9} - \frac{1}{10} = \frac{1}{3}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad (C) \quad & \frac{1}{3 \times 7} + \frac{1}{7 \times 11} + \frac{1}{11 \times 15} + \dots + \frac{1}{899 \times 903} \\
 &= \frac{1}{4} \left[ \frac{1}{3} - \frac{1}{7} + \frac{1}{7} - \frac{1}{11} + \frac{1}{11} - \frac{1}{15} + \dots + \frac{1}{899} - \frac{1}{903} \right] \\
 &= \frac{1}{4} \left[ \frac{1}{3} - \frac{1}{903} \right] = \frac{25}{301}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad (B) \quad & X_1 = \frac{1}{1 \times 3}, X_2 = \frac{1}{3 \times 5}, X_3 = \frac{1}{5 \times 7} \\
 \text{So, } & X_1 + X_2 + \dots + X_{100} \\
 &= \frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \dots + \frac{1}{199 \times 201} \\
 &= \frac{1}{2} \left[ \frac{1}{1} - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \dots + \frac{1}{199} - \frac{1}{201} \right]
 \end{aligned}$$

$$= \frac{1}{2} \left[ 1 - \frac{1}{201} \right] = \frac{100}{201}$$

$$\begin{aligned}
 6. \quad (A) \quad & \frac{5}{2^2 \cdot 3^2} + \frac{7}{3^2 \cdot 4^2} + \frac{9}{4^2 \cdot 5^2} + \dots + \frac{39}{19^2 \cdot 20^2} \\
 &= \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{4^2} - \frac{1}{5^2} + \dots + \frac{1}{19^2} - \frac{1}{20^2} \\
 &= \frac{1}{4} - \frac{1}{400} = \frac{99}{400}
 \end{aligned}$$

$$\begin{aligned}
 7. \quad (C) \quad & \frac{1}{2^2 - 1} + \frac{1}{4^2 - 1} + \dots + \frac{1}{20^2 - 1} \\
 &= \frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \dots + \frac{1}{19 \times 21} \\
 &= \frac{1}{2} \left[ \frac{1}{1} - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \dots + \frac{1}{19} - \frac{1}{21} \right] \\
 &= \frac{1}{2} \left[ 1 - \frac{1}{21} \right] = \frac{10}{21}
 \end{aligned}$$

$$\begin{aligned}
 8. \quad (B) \quad & \frac{1}{7^2 - 3^2} + \frac{1}{13^2 - 3^2} + \frac{1}{19^2 - 3^2} + \frac{1}{25^2 - 3^2} + \frac{1}{31^2 - 3^2} \\
 &= \frac{1}{4 \times 10} + \frac{1}{10 \times 16} + \frac{1}{16 \times 22} + \frac{1}{22 \times 28} + \frac{1}{28 \times 34} \\
 &= \frac{1}{6} \left[ \frac{1}{4} - \frac{1}{10} + \frac{1}{10} - \frac{1}{16} + \frac{1}{16} - \frac{1}{22} + \frac{1}{22} - \frac{1}{28} + \frac{1}{28} - \frac{1}{34} \right] \\
 &= \frac{1}{6} \left[ \frac{1}{4} - \frac{1}{34} \right] = \frac{15}{6 \times 68} = \frac{5}{136}
 \end{aligned}$$

$$\begin{aligned}
 9. \quad (A) \quad & \frac{1}{1 \times 2 \times 3} + \frac{1}{2 \times 3 \times 4} + \dots + \frac{1}{n(n+1)(n+2)} \\
 &= \frac{1}{(3-1)} \left[ \frac{1}{1 \times 2} - \frac{1}{2 \times 3} + \frac{1}{2 \times 3} - \frac{1}{3 \times 4} + \dots \right. \\
 & \quad \left. + \frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)} \right] \\
 &= \frac{1}{2} \left[ \frac{1}{2} - \frac{1}{(n+1)(n+2)} \right] = \frac{1}{4} - \frac{1}{2(n+1)(n+2)}
 \end{aligned}$$

$$\begin{aligned}
 10. (A) \quad & \frac{1}{1 \times 2 \times 3 \times 4} + \frac{1}{2 \times 3 \times 4 \times 5} + \\
 & \dots + \frac{1}{n(n+1)(n+2)(n+3)} \\
 & = \frac{1}{(4-1)} \left[ \frac{1}{1 \times 2 \times 3} - \frac{1}{2 \times 3 \times 4} + \frac{1}{2 \times 3 \times 4} - \frac{1}{3 \times 4 \times 5} + \dots + \right] \\
 & \left[ \frac{1}{n(n+1)(n+2)} - \frac{1}{(n+1)(n+2)(n+3)} \right] \\
 & = \frac{1}{3} \left[ \frac{1}{1 \times 2 \times 3} - \frac{1}{(n+1)(n+1)(n+3)} \right]
 \end{aligned}$$

$$\begin{aligned}
 11. (A) \quad & \frac{1}{1 \times 3 \times 5 \times 7} + \frac{1}{3 \times 5 \times 7 \times 9} + \dots \text{ 10 terms} \\
 \text{10th term} & = \frac{1}{19 \times 21 \times 23 \times 25}
 \end{aligned}$$

$$\begin{aligned}
 \text{So, Sum} & = \frac{1}{(7-1)} \left[ \frac{1}{1 \times 3 \times 5} - \frac{1}{21 \times 23 \times 25} \right] \\
 & = \frac{1}{6} \left[ \frac{804}{21 \times 23 \times 25} \right] = \frac{134}{12075}
 \end{aligned}$$

$$\begin{aligned}
 12. (A) \quad & \frac{1}{1 \times 6 \times 11} + \frac{1}{6 \times 11 \times 16} + \dots \text{ 10 terms} \\
 \text{10th term} & = \frac{1}{46 \times 51 \times 56}
 \end{aligned}$$

$$\begin{aligned}
 \text{So, Sum} & = \frac{1}{11} \left[ \frac{1}{1 \times 6} - \frac{1}{51 \times 56} \right] \\
 & = \frac{1}{10} \left[ \frac{476-1}{51 \times 56} \right] = \frac{95}{5712}
 \end{aligned}$$

$$\begin{aligned}
 13. (B) \quad S & = \frac{1}{1 \times 3 \times 5} + \frac{1}{1 \times 4} + \frac{1}{3 \times 5 \times 7} + \frac{1}{4 \times 7} + \dots \\
 & \text{20 terms}
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow S & = \frac{1}{1 \times 4} + \frac{1}{4 \times 7} + \dots + \frac{1}{28 \times 31} + \frac{1}{1 \times 3 \times 5} \\
 & + \frac{1}{3 \times 5 \times 7} + \dots + \frac{1}{19 \times 21 \times 23}
 \end{aligned}$$

$$\Rightarrow S = \frac{1}{(4-1)} \left[ \frac{1}{1} - \frac{1}{31} \right] + \frac{1}{(5-1)} \left[ \frac{1}{1 \times 3} - \frac{1}{21 \times 23} \right]$$

$$S = \frac{10}{31} + \frac{40}{483} = \frac{4830+1240}{14973} = \frac{6070}{14973}$$

$$14. (B) \quad (i) \quad \frac{2}{2} \left[ \frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \dots \right]$$

$$\frac{1}{2} \left[ 1 - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \frac{1}{5} - \dots + \frac{1}{12} - \frac{1}{13} \right]$$

$$= \frac{1}{2} \left[ 1 - \frac{1}{13} \right]$$

$$= \frac{1}{2} \left[ \frac{12}{13} \right]$$

$$= \frac{6}{13} \text{ but in question given } \frac{12}{13} \text{ not true.}$$

Alternative

$$= \frac{1}{\text{Difference}} \left[ \frac{1}{\text{First}} - \frac{1}{\text{last}} \right]$$

$$= \frac{1}{2} \left[ \frac{1}{1} - \frac{1}{13} \right] = \frac{6}{13}$$

$$(ii) \quad \frac{1}{\text{Difference}} \left[ \frac{1}{\text{First}} - \frac{1}{\text{last}} \right]$$

$$\frac{1}{2-1} \left[ \frac{1}{1} - \frac{1}{13} \right] = \frac{1}{1} \left[ \frac{12}{13} \right] = \frac{12}{13} \text{ is true}$$

$$\begin{aligned}
 15. (D) \quad A & = \frac{1}{1 \times 2} + \frac{1}{1 \times 4} + \frac{1}{2 \times 3} + \dots + \frac{1}{7 \times 10} \dots \\
 & \text{upto 20 terms}
 \end{aligned}$$

$$A = \left[ \frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \dots + \frac{1}{10 \times 11} \right]$$

$$+ \left[ \frac{1}{1 \times 4} + \frac{1}{4 \times 7} + \dots + \frac{1}{27 \times 31} \right]$$

$$A = \left[ 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \dots + \frac{1}{10} - \frac{1}{11} \right]$$

$$+ \frac{1}{3} \left[ 1 - \frac{1}{4} + \frac{1}{4} - \frac{1}{7} + \dots + \frac{1}{27} - \frac{1}{31} \right]$$

$$= 1 - \frac{1}{11} + \frac{1}{3} \left[ 1 - \frac{1}{31} \right] = \frac{10}{11} + \frac{30}{3 \times 31} = \frac{420}{341}$$