

# सरलीकरण | Simplification

## Type-9 (Surds & Indices)

1. If  $\sqrt[3]{0.014 \times 0.14x} = 0.014 \times 0.14\sqrt[3]{y}$  then  $\frac{x}{y}$  is equal to

यदि  $\sqrt[3]{0.014 \times 0.14x} = 0.014 \times 0.14\sqrt[3]{y}$  तो  $\frac{x}{y}$  का मान ज्ञात कीजिए?

- (A) 0.000196      (B) 0.00196  
 (C) 0.0196      (D) 0.196

2.  $\sqrt{x^{-1}y}, \sqrt{y^{-1}z}, \sqrt{z^{-1}x} = ?$

- (A) xyz      (B)  $\sqrt{xyz}$   
 (C)  $\frac{1}{xyz}$       (D) 1

3.  $\left(\frac{x^a}{x^b}\right)^{1/ab} \times \left(\frac{x^b}{x^c}\right)^{1/bc} \times \left(\frac{x^c}{x^a}\right)^{1/ca} = ?$

- (A) 1  
 (B)  $x^{1/abc}$   
 (C)  $x^{1/(ab+bc+ca)}$   
 (D) None of these

4. If  $5^{\sqrt{x}} + 12^{\sqrt{x}} = 13^{\sqrt{x}}$ , the value of x is—

यदि  $5^{\sqrt{x}} + 12^{\sqrt{x}} = 13^{\sqrt{x}}$  हो, तो x का मान होगा—

- (A)  $\frac{25}{4}$       (B) 4  
 (C) 9      (D) 16

5.  $\left[\left(x + \frac{1}{y}\right)^a \left(x - \frac{1}{y}\right)^b\right] \div \left[\left(y + \frac{1}{x}\right)^a \left(y - \frac{1}{x}\right)^b\right]$  is equal to—

$\left[\left(x + \frac{1}{y}\right)^a \left(x - \frac{1}{y}\right)^b\right] \div \left[\left(y + \frac{1}{x}\right)^a \left(y - \frac{1}{x}\right)^b\right]$  के बराबर हैं—

- (A)  $\left(\frac{x}{y}\right)^{a+b}$       (B)  $\left(\frac{x}{y}\right)^{a-b}$   
 (C)  $\left(\frac{y}{x}\right)^{a+b}$       (D)  $\left(\frac{y}{x}\right)^{a-b}$

6. The value of  $\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{c-b}+x^{a-b}}$   
 $+ \frac{1}{1+x^{a-c}+x^{b-c}}$  is —

$$\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{c-b}+x^{a-b}} + \frac{1}{1+x^{a-c}+x^{b-c}}$$

का मान है —

- (A) 1      (B) 2  
 (C) 3      (D) 4

7. The value of  $\frac{4^n \times 20^{m-1} \times 12^{m-n} \times 15^{m-m-2}}{16^m \times 5^{2m-n} \times 9^{m-1}}$  is —

$$\text{वर्णक } \frac{4^n \times 20^{m-1} \times 12^{m-n} \times 15^{m-n-2}}{16^m \times 5^{2m-n} \times 9^{m-1}}$$

का मान है —

- (A)  $\frac{1}{50}$       (B)  $\frac{1}{500}$       (C)  $\frac{1}{100}$       (D)  $\frac{1}{5}$

8. The value of

$$\frac{(0.3)^{1/3} \cdot \left(\frac{1}{27}\right)^{1/4} \cdot (9)^{1/6} \cdot (0.81)^{2/3}}{(0.9)^{2/3} \cdot (3)^{-1/2} \cdot \left(\frac{1}{3}\right)^2 \cdot (243)^{-1/4}}$$

is / का मान है —

- (A) 3      (B) 0.03  
 (C) 0.3      (D) 30

9. If the value of

$$\frac{52.5 \times 52.5 - 2493.75 + 47.5 \times 47.5}{525^3 + 475^3} = 10^{-k} \text{ Then}$$

find the value of k = ?

यदि  $\frac{52.5 \times 52.5 - 2493.75 + 47.5 \times 47.5}{525^3 + 475^3} = 10^{-k}$  फिर k का मान ज्ञात कीजिए?

- (A) 3      (B) 4  
 (C) 5      (D) 6



$$\left( k + \frac{1}{k} \right) \left( k^4 - \frac{1}{k^4} \right) \text{ is } k^{16} - \frac{1}{k^{16}}$$

$$\text{II. } \left(k^2 + \frac{1}{k^2}\right)\left(k - \frac{1}{k}\right)\left(k^4 + \frac{1}{k^4}\right) \text{ का मान } k^{16} - \frac{1}{k^{16}} \text{ है।}$$

- (A) Neither I nor II / ना ही ना ही  
(B) Only II / केवल II  
(C) Only I / केवल I  
(D) Both I and II / I तथा II दोनों

( )  
Simplify the following.

### निम्नलिखित को सरल :

- (A) - 281250      (B) 281350  
 (C) 271250      (D) - 281450

(C) 271230

$$\left( k - \frac{1}{k} \right) \left( k^2 + \frac{1}{k^2} \right) \left( k^4 + \frac{1}{k^4} \right)$$

$$\left( k^8 + \frac{1}{k^8} \right) \left( k^{16} + \frac{1}{k^{16}} \right) \left( k^{32} + \frac{1}{k^{32}} \right) ?$$

$$\left(k - \frac{1}{k}\right) \left(k^2 + \frac{1}{k^2}\right) \left(k^4 + \frac{1}{k^4}\right)$$

$$\left( k^8 + \frac{1}{k^8} \right) \left( k^{16} + \frac{1}{k^{16}} \right) \left( k^{32} + \frac{1}{k^{32}} \right) \text{ का मान क्या है?}$$

$$(A) \frac{k^{34} - \frac{1}{k^{34}}}{k + \frac{1}{k}} \quad (B) \frac{k^{32} - \frac{1}{k^{32}}}{k - \frac{1}{k}}$$

$$(C) \frac{k^{32} - \frac{1}{k^{32}}}{k + \frac{1}{k}} \quad (D) \frac{k^{32} + \frac{1}{k^{32}}}{k + \frac{1}{k}}$$

## SOLUTIONS

1. (B)  $\sqrt{0.014 \times 0.14x} = 0.014 \times 0.14\sqrt{y}$

Doing square of both sides

$$0.014 \times 0.14x = (0.014)^2 \times (0.14)^2 y$$

$$\Rightarrow \frac{x}{y} = 0.014 \times 0.14 = 0.00196$$

2. (D)  $\sqrt{x^{-1}y} \times \sqrt{y^{-1}z} \times \sqrt{z^{-1}x} = \sqrt{\frac{y}{x}} \times \sqrt{\frac{z}{y}} \times \sqrt{\frac{x}{z}} = 1$

3. (A)  $\left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}} \times \left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} \times \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} = x^{\frac{1}{a} + \frac{1}{b} - \frac{1}{b} + \frac{1}{c} - \frac{1}{c} + \frac{1}{a}} = x^0 = 1$

4. (B) ATQ  $5^{\sqrt{x}} + 12^{\sqrt{x}} = 13^{\sqrt{x}}$

As we know,  $5^2 + 12^2 = 13^2$

By comparison,

$$\sqrt{x} = 2$$

$$\Rightarrow x = 4$$

5. (A)  $\left[\left(x + \frac{1}{y}\right)^a \left(x - \frac{1}{y}\right)^b\right] \div \left[\left(y + \frac{1}{x}\right)^a \left(y - \frac{1}{x}\right)^b\right]$

$$= \frac{(xy+1)^a}{y^a} \times \frac{(xy-1)^b}{y^b} \times \frac{x^a}{(xy+1)^a} \times \frac{x^b}{(xy-1)^b}$$

$$= \left(\frac{x}{y}\right)^{a+b}$$

6. (A)  $\frac{1}{1 \times x^{b-a} + x^{c-a}} + \frac{1}{1 \times x^{c-b} + x^{a-b}} + \frac{1}{1 \times x^{a-c} + x^{b-c}}$

$$= \frac{x^a}{x^a + x^b + x^c} + \frac{x^b}{x^a + x^b + x^c} + \frac{x^c}{x^a + x^b + x^c}$$

$$= \frac{x^a + x^b + x^c}{x^a + x^b + x^c} = 1$$

7. (B)  $\frac{4^n \times 20^{m-1} \times 12^{m-n} \times 15^{m-n-2}}{16^m \times 5^{2m+n} \times 9^{m-1}}$

$$= \frac{2^{2n} \times 2^{2m-2} \times 5^{m-1} \times 2^{2m-2n} \times 3^{m-n} \times 3^{m-n-2} \times 5^{m+n-2}}{2^{4m} \times 5^{2m+n} \times 3^{2m-2}}$$

$$= 2^2 \times 3^0 \times 5^3 = \frac{1}{500}$$

8. (C)  $\frac{(0.3)^{\frac{1}{3}} \left(\frac{1}{27}\right)^{\frac{1}{4}} (9)^{\frac{1}{6}} (0.81)^{\frac{2}{3}}}{(0.9)^{\frac{2}{3}} (3)^{\frac{1}{2}} \left(\frac{1}{3}\right)^2 (243)^{\frac{1}{4}}}$

$$= \frac{3^{\frac{1}{3}} \times 3^{\frac{1}{3}} \times 3^{\frac{1}{3}} \times 10^{\frac{2}{3}} \times 3^{\frac{2}{3}} \times 3^{\frac{4}{3}}}{10^{\frac{1}{3}} \times 3^{\frac{4}{3}} \times 10^{\frac{5}{3}} \times 3^{\frac{3}{3}} \times 3^{\frac{2}{3}}}$$

$$= \frac{3^{\frac{10}{3}-\frac{7}{3}} \times 10^{\frac{2}{3}}}{3^{\frac{49}{3}-\frac{5}{3}}} = \frac{3}{10} = 0.3$$

9. (C)  $\Rightarrow \frac{10^{-2}}{(525+475)} = \frac{10^{-2}}{1000} = \frac{10^{-2}}{10^3} = 10^{-5}$

$$= k = 5$$

10. (A)  $2^x = 4^y = 8^z$

$$\Rightarrow 2^x = 2^{2y} = 2^{3z}$$

$$\Rightarrow x = 2y = 3z$$

Given, xyz = 288

$$\Rightarrow x \times \frac{x}{2} \times \frac{x}{3} = 288$$

$$\Rightarrow x = \sqrt[3]{6 \times 6 \times 2 \times 2 \times 2} = 12$$

$$\text{So, } y = 6, z = 4$$

$$\Rightarrow \frac{1}{2x} + \frac{1}{4y} + \frac{1}{6z} = \frac{1}{24} + \frac{1}{24} + \frac{1}{24} = \frac{1}{8}$$

11. (A)  $x^m = \sqrt[14]{x \times \sqrt{x^{3/2}}}$

$$x^{11} = \sqrt[14]{x \times x^{3/4}}$$

$$x^{11} = \sqrt[14]{x^{7/4}}$$

$$x^m = \frac{7}{14} \times \frac{1}{14} =$$

$$\Rightarrow x^m = \frac{1}{x^8}$$

$$\therefore m = \frac{1}{8}$$

12. (C) दिया है,  $2^m + 2^{1+m} = 24$

$$\Rightarrow 2^m + 2 \cdot 2^m = 24 \Rightarrow 2^m(1+2) = 24$$

$$\Rightarrow 2^m \cdot 3 = 24 \Rightarrow 2^m = 8 = 2^3$$

$$\therefore m = 3 \quad [\because a^m = a^n \Rightarrow m = n]$$

13. (D) दिया है  $16 \times 8^{n+2} = 2^m \Rightarrow (2)^4 \times 2^{3(n+2)} = 2^m$

$$\Rightarrow 2^{(4+3n+6)} = 2^m \Rightarrow 2^{(3n+10)} = 2^m$$

$$\Rightarrow 3n + 10 = m \quad [\because a^n = a^m \Rightarrow n = m]$$

$$\therefore m = 3n + 10$$

14. (D) दिया गया व्यंजक =  $\left[ \left( \sqrt{2} \right)^{\frac{x}{y}} \right]^{\frac{z}{w}} \left( \sqrt{2} \right)^2 = (2)^{\frac{2z}{w}} = 2$

जो कि एक वास्तविक संख्या है किन्तु परिमेय संख्या नहीं है।

15. (A) दिया है  $p^x = r^y \Rightarrow r = p^{x/y}$

$$\text{तथा } p^z = r^w \Rightarrow r = p^{z/w}$$

सभी (i) व (ii) से

$$p^{x/y} = p^{z/w} \Rightarrow \frac{x}{y} = \frac{z}{w}$$

$$\Rightarrow xw = yz$$

16. (A)  $7 \left[ \left( 64^{\frac{1}{3}} + 27^{\frac{1}{3}} \right)^3 \right]^{\frac{1}{4}} = 7 \left[ (4+3)^3 \right]^{\frac{1}{4}} = 7$

17. (B)  $9^x = \sqrt[11]{243}$

$$\Rightarrow 9^{11x} = \sqrt{243}$$

$$\Rightarrow 9^{11x} = \sqrt[9]{3}$$

$$\Rightarrow x = \frac{5}{22}$$

18. (C)  $\frac{P}{\sqrt{540}} = \frac{\sqrt{240}}{P}$

$$\Rightarrow P = \sqrt[6]{10}$$

19. (A)  $\sqrt[3]{3125} + 4\sqrt[3]{25} + 3\sqrt[3]{675}$

$$= 5\sqrt[3]{25} + 4\sqrt[3]{25} + 3 \times 3\sqrt[3]{25}$$

$$= 18\sqrt[3]{25}$$

20. (A)  $\frac{(17)^3 + (7)^3}{(17^2 + 7^2 - k)} = 24$

$$a+b = 17+7 = 24$$

$$\therefore \frac{a^3 + b^3}{a^2 + b^2 - ab} = (a+b)$$

$$k = ab = 17 \times 7 = 119.$$

21. (C)  $\frac{4 \left[ (17)^3 - (7)^3 \right]}{(17^2 + 7^2 + P)} = 40$

$$\frac{\left[ (17)^3 - (7)^3 \right]}{17^2 + 7^2 + P} = 10$$

$$\therefore a^3 - b^3 = (a-b)(a^2 + b^2 + ab) \{ \text{identity} \}$$

$$\frac{[17-7][17^2 + 7^2 + 7 \times 17]}{17^2 + 7^2 + P} = 10$$

$$17^2 + 7^2 + P = 17^2 + 7^2 + 7 \times 17$$

$$P = 17 \times 7 = 119.$$

22. (A)  $\frac{428 \times 428 \times 428 + 348 \times 348 \times 348}{428 \times 428 - 428 \times 348 + 348 \times 348}$

$$\text{Identity, } a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$\frac{(428+348)(428 \times 428 - 428 \times 348 + 348 \times 348)}{428 \times 428 - 428 \times 348 + 348 \times 348}$$

$$= 428 + 348 = 776.$$

23. (C) Statement - 1

$$1. \quad 199 + 195 + 191 + \dots + 43.$$

$$\ell = a + (n-1)d$$

$$199 = 43 + (h-1) \times 4$$

$$h = 40$$

$$S_n = \frac{n}{2} (a + \ell)$$

$$= \frac{40}{2} [43 + 199]$$

$$= 20 \times 242 = 4840$$

Statement - 1 is correct

Statement - 2 is wrong.

Only statement - 1 is correct.

24. (B)  $97 \times 103$

$$\begin{aligned} & (100 - 3) \times (100 + 3) \\ \Rightarrow & (a - b)(a + b) = a^2 - b^2 \\ = & (100)^2 - (3)^2 \\ = & 9991. \end{aligned}$$

25. (A)  $\left(k - \frac{1}{k}\right)\left(k^2 + \frac{1}{k^2}\right)\left(k^4 + \frac{1}{k^4}\right)\left(k^8 + \frac{1}{k^8}\right)$

$$\left(k^{16} + \frac{1}{k^{16}}\right)\left(k^{32} + \frac{1}{k^{32}}\right) \times \frac{\left(k + \frac{1}{k}\right)}{\left(k + \frac{1}{k}\right)}$$

$$\frac{\left(k^2 - \frac{1}{k^2}\right)\left(k^2 + \frac{1}{k^2}\right)\left(k^4 + \frac{1}{k^4}\right)\left(k^8 + \frac{1}{k^8}\right)\left(k^{16} + \frac{1}{k^{16}}\right)\left(k^{32} + \frac{1}{k^{32}}\right)}{k + \frac{1}{k}}$$

similarly,

$$a^2 - b^2 = (a - b)(a + b)$$

$$= \frac{k^{64} - \frac{1}{k^{64}}}{k + \frac{1}{k}}.$$

Mother's