A

$\sin\theta = \frac{L}{K} = \frac{P}{H}$

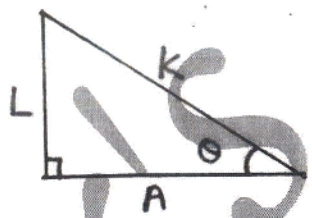
$\cos\theta = \frac{A}{K} = \frac{B}{H}$

$\tan\theta = \frac{L}{B} = \frac{P}{B}$

$\cot\theta = \frac{A}{L} = \frac{B}{P}$

$\sec\theta = \frac{K}{A} = \frac{H}{B}$

$\mathrm{cosec}\,\theta = \frac{K}{L} = \frac{H}{P}$



sin	cos	tan
L	A	L
K	K	A

sin	cos	tan
P	B	P
H	H	B
Cosec	sec	cot

$\sin\theta = \frac{1}{\mathrm{cosec}\,\theta}$

$\cos\theta = \frac{1}{\sec\theta}$

$\tan\theta = \frac{1}{\cot\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

B

1. $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \sin^2\theta = 1 - \cos^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

2. $1 + \tan^2\theta = \sec^2\theta$

$\Rightarrow 1 = \sec^2\theta - \tan^2\theta$

$1 = (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)$

$\frac{1}{(\sec\theta + \tan\theta)} = (\sec\theta - \tan\theta)$

3. $1 + \cot^2\theta = \mathrm{cosec}^2\theta$

$\Rightarrow 1 = \mathrm{cosec}^2\theta - \cot^2\theta$

$1 = (\mathrm{cosec}\,\theta - \cot\theta)(\mathrm{cosec}\,\theta + \cot\theta)$

$\frac{1}{(\mathrm{cosec}\,\theta + \cot\theta)} = (\mathrm{cosec}\,\theta - \cot\theta)$

$\frac{1}{(\mathrm{cosec}\,\theta - \cot\theta)} = (\mathrm{cosec}\,\theta + \cot\theta)$

Mother's

Note $\Rightarrow$

(1) $(a\sin\theta + b\cos\theta) = c$ ------- (1)

$(a\cos\theta - b\sin\theta) = x$ ------- (2)

$eq^n\ (1)^2 + (2)^2$

$$a^2\sin^2\theta + b^2\cos^2\theta + \cancel{2ab\sin\theta\cos\theta} = c^2$$
$$a^2\cos^2\theta + b^2\sin^2\theta - \cancel{2ab\cos\theta\sin\theta} = x^2$$

$$a^2(\sin^2\theta + \cos^2\theta) + b^2(\cos^2\theta + \sin^2\theta) = c^2 + x^2$$

$$a^2 + b^2 = c^2 + x^2$$

$(\therefore \sin^2\theta + \cos^2\theta = 1)$

$$a^2 + b^2 - c^2 = x^2$$

$$x = \pm\sqrt{a^2 + b^2 - c^2}$$

OR

$(a\sin\theta + b\cos\theta) = c$ ------- (1)

$(b\sin\theta - a\cos\theta) = x$ ------- (2)

$eq^n\ (1)^2 + (2)^2$

$$a^2\sin^2\theta + b^2\cos^2\theta + \cancel{2ab\sin\theta\cos\theta} = c^2$$
$$b^2\sin^2\theta + a^2\cos^2\theta - \cancel{2ab\sin\theta\cos\theta} = x^2$$

$$a^2(\sin^2\theta + \cos^2\theta) + b^2(\sin^2\theta + \cos^2\theta) = c^2 + x^2$$

$(\therefore \sin^2\theta + \cos^2\theta = 1)$

$$a^2 + b^2 = c^2 + x^2$$

$$a^2 + b^2 - c^2 = x^2$$

$$x = \pm\sqrt{a^2 + b^2 - c^2}$$

(2). $\sin^2\theta - \cos^2\theta = a$

Multiply both side $(\sin^2\theta + \cos^2\theta)$

$$(\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta) = a(\sin^2\theta + \cos^2\theta)$$

$$\sin^4\theta - \cos^4\theta = a$$

$$\therefore a^4 - b^4 = (a^2-b^2)(a^2+b^2)$$

$$\sin^2\theta + \cos^2\theta = 1$$

උදා

$$\sin^4\theta - \cos^4\theta = \sin^2\theta - \cos^2\theta$$

$$\sec^4\theta - \tan^4\theta = \sec^2\theta + \tan^2\theta$$

$$\mathrm{cosec}^4\theta - \cot^4\theta = \mathrm{cosec}^2\theta + \cot^2\theta$$

C

	0°	30°	45°	60°	90°
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞
$\cot\theta$	∞	$\sqrt{3}$	1	$1/\sqrt{3}$	0
$\sec\theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	∞
$\mathrm{cosec}\theta$	∞	2	$\sqrt{2}$	$2/\sqrt{3}$	1

$\Rightarrow$

$$\sin 0^\circ = \cos 90^\circ$$

$$\sin 30^\circ = \cos 60^\circ$$

$$\sin 45^\circ = \cos 45^\circ$$

$$\sin 60^\circ = \cos 30^\circ$$

$$\sin 90^\circ = \cos 0^\circ$$

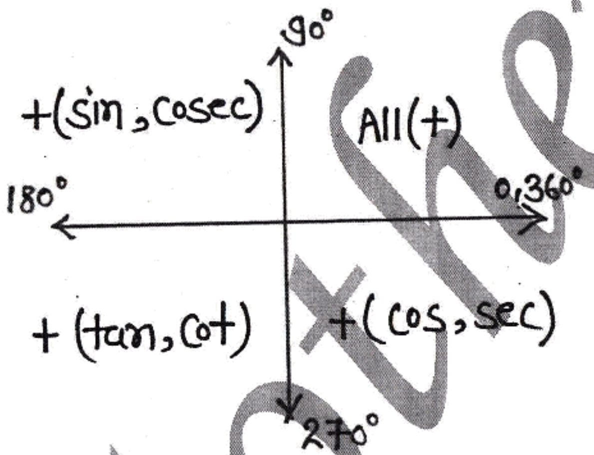
$\Rightarrow$

$\sin x = \cos y$

$\tan x = \cot y$ $\quad [\therefore x + y = 90°]$

$\sec x = \operatorname{cosec} y$

$\boxed{\text{D}}$



90°, 270° $\Rightarrow$

sin $\longleftrightarrow$ cos

tan $\longleftrightarrow$ cot

sec $\longleftrightarrow$ cosec

$\sin(90-\theta) = \cos\theta$

$\cos(90-\theta) = \sin\theta$

$\tan(90-\theta) = \cot\theta$

$\cot(90-\theta) = \tan\theta$

$\sec(90-\theta) = \operatorname{cosec}\theta$

$\operatorname{cosec}(90-\theta) = \sec\theta$

$\sin(90+\theta) = \cos\theta$

$\cos(90+\theta) = -\sin\theta$

$\tan(90+\theta) = -\cot\theta$

$\cot(90+\theta) = -\tan\theta$

$\sec(90+\theta) = -\operatorname{cosec}\theta$

$\operatorname{cosec}(90+\theta) = \sec\theta$

$\sin(180-\theta) = \sin\theta$

$\cos(180-\theta) = -\cos\theta$

$\tan(180-\theta) = -\tan\theta$

$\cot(180-\theta) = -\cot\theta$

$\sec(180-\theta) = -\sec\theta$

$\operatorname{cosec}(180-\theta) = \operatorname{cosec}\theta$

$\sin(180+\theta) = -\sin\theta$

$\cos(180+\theta) = -\cos\theta$

$\tan(180+\theta) = \tan\theta$

$\cot(180+\theta) = \cot\theta$

$\sec(180+\theta) = -\sec\theta$

$\operatorname{cosec}(180+\theta) = -\operatorname{cosec}\theta$

$\sin(270-\theta) = -\cos\theta$

$\cos(270-\theta) = -\sin\theta$

$\tan(270-\theta) = \cot\theta$

$\cot(270-\theta) = \tan\theta$

$\sec(270-\theta) = -\operatorname{cosec}\theta$

$\operatorname{cosec}(270-\theta) = -\sec\theta$

$\sin(270+\theta) = -\cos\theta$

$\cos(270+\theta) = \sin\theta$

$\tan(270+\theta) = -\cot\theta$

$\cot(270+\theta) = -\tan\theta$

$\sec(270+\theta) = \operatorname{cosec}\theta$

$\operatorname{cosec}(270+\theta) = -\sec\theta$

$\Rightarrow \sin(-\theta) = -\sin\theta$

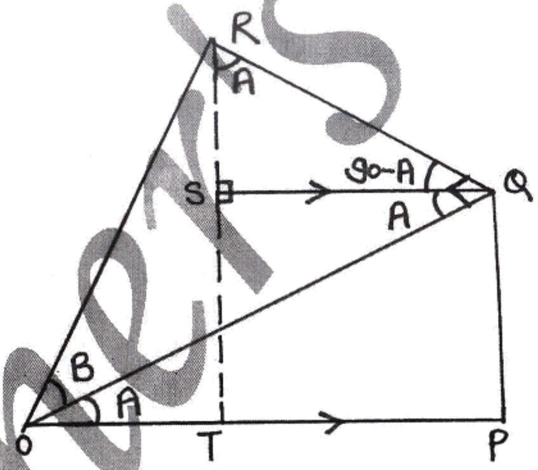
$\therefore [\sin(360-\theta) = -\sin\theta]$

$\cos(-\theta) = \cos\theta$

$\therefore [\cos(360-\theta) = \cos\theta]$

E

$\sin(A+B) = \sin A \cos B + \cos A \sin B$



$\sin(A+B) = \dfrac{RT}{OR}$

$= \dfrac{SR+ST}{OR}$

$= \dfrac{SR}{OR} + \dfrac{ST}{OR}$

$= \dfrac{SR}{OR} + \dfrac{PQ}{OR}$

$= \dfrac{SR}{RQ} \times \dfrac{RQ}{OR} + \dfrac{PQ}{OQ} + \dfrac{OQ}{OR}$

$\boxed{\sin(A+B) = \cos A \times \sin B + \sin A \times \cos B}$

1. $\sin(A+B) = \sin A \cos B + \cos A \sin B$
2. $\sin(A-B) = \sin A \cos B - \cos A \sin B$
3. $\cos(A+B) = \cos A \cos B - \sin A \sin B$
4. $\cos(A-B) = \cos A \cos B + \sin A \sin B$
5. $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$
6. $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$
7. $\cot(A+B) = \dfrac{\cot A \cot B - 1}{\cot B + \cot A}$
8. $\cot(A-B) = \dfrac{\cot A \cot B + 1}{\cot B - \cot A}$

Proof ⇒ $\therefore \tan(A+B) = \dfrac{\sin(A+B)}{\cos(A+B)}$

$\dfrac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}$

divide by $\cos A \cos B$

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

$\therefore \cos 15^\circ \Rightarrow \cos(45^\circ - 30^\circ) \Rightarrow$

$$\cos 45^\circ \cos 30^\circ + \sin 45 \sin 30$$

$$\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$\cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

9. $2\sin A \cos B = \sin(A+B) + \sin(A-B)$
10. $2\cos A \sin B = \sin(A+B) - \sin(A-B)$
11. $2\cos A \cos B = \cos(A+B) + \cos(A-B)$
12. $-2\sin A \sin B = \cos(A+B) - \cos(A-B)$

$\therefore$ Put $A+B = C$

$$\begin{array}{l} A-B = D \\ \hline A = \frac{C+D}{2} \\ B = \frac{C-D}{2} \end{array}$$

13. $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$
14. $\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$
15. $\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$
16. $\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$

Special formula $\Rightarrow$

1. $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$
   $= \cos^2 B - \cos^2 A$

2. $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$
   $= \cos^2 B - \sin^2 A$

Proof $\Rightarrow$ $\sin(A+B) = \sin A\cos B + \cos A \sin B$ ----- (1)

$\sin(A-B) = \sin A \cos B - \cos A \sin B$ ----- (2)

(2) $\cos\theta\cos(60-\theta)\cos(60+\theta) = \frac{1}{4}\cos 3\theta$

(3) $\tan\theta\tan(60-\theta)\tan(60+\theta) = \tan 3\theta$

(4) $\sin 2A = \sin(A+A)$

$= \sin A\cos A + \cos A\sin A$

$= 2\sin A\cos A$

$\sin 2A = 2\sin A\cos A$

multiply & divide by $\cos A$

$$\sin 2A = \frac{2\sin A \times \cos^2 A}{\cos A}$$

$$\sin 2A = \frac{2\tan A}{\sec^2 A}$$

$$= \frac{2\tan A}{1+\tan^2 A}$$

(5) $\cos 2A = \cos(A+A)$

$= \cos A\cos A - \sin A\sin A$

$= \left[\cos^2 A - (\sin^2 A) \times \frac{\cos^2 A}{\cos^2 A}\right]$

$= \cos^2 A(1-\tan^2 A)$

$$\cos 2A = \frac{1-\tan^2 A}{\sec^2 A}$$

$$\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$$

$\cos 2A = 1-2\sin^2 A$

$\cos 2A = 2\cos^2 A - 1$

$\cos 2A = \cos^4 A - \sin^4 A$

(6) $$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

Multiply eq^n ① & ②

$\sin(A+B)\sin(A-B) = \sin^2 A\cos^2 B - \cos^2 A\sin^2 B$

$= (1-\cos^2 A)(1-\sin^2 B) - \cos^2 A\sin^2 B$

$= 1-\cos^2 A - \sin^2 B + \cancel{\cos^2 A\sin^2 B} - \cancel{\cos^2 A\sin^2 B}$

$= \sin^2 A - \sin^2 B$

and $[(1-\cos^2 A) - 1 + \cos^2 B]$

$\cos^2 B - \cos^2 A$

17. $2\sin x\cos y = \sin(x+y) + \sin(x-y)$

18. $2\cos x\sin y = \sin(x+y) - \sin(x-y)$

19. $2\cos x\cos y = \cos(x+y) + \cos(x-y)$

20. $-2\sin x\sin y = \cos(x+y) - \cos(x-y)$

Special formula ⇒

1. $\sin\theta\sin(60-\theta)\sin(60+\theta) = \frac{1}{4}\sin 3\theta$

Proof ⇒

$\frac{\sin\theta}{2}\, 2\sin(60-\theta)\sin(60+\theta) \Rightarrow$

$-\frac{\sin\theta}{2}[\cos 120° - \cos 2\theta]$

$= \frac{\sin\theta}{2}[\cos 2\theta - \cos 120°]$

$= \frac{\sin\theta}{2}[\cos 2\theta + \frac{1}{2}]$

$= \frac{\sin\theta}{2}[1-2\sin^2\theta + \frac{1}{2}]$

$= \frac{\sin\theta}{2}[\frac{3}{2} - 2\sin^2\theta]$

$= \frac{1}{4}[3\sin\theta - 4\sin^3\theta]$

$= \frac{1}{4}\sin 3\theta$

(iv) $A+B=45$

$$\tan(A+B)=\tan 45$$

$$\frac{\tan A+\tan B}{1-\tan A\tan B}=1$$

$$\tan A+\tan B=1-\tan A\tan B$$

$$\tan A+\tan B+\tan A\tan B+1=1+1$$

$$\tan A(1+\tan B)+(\tan B+1)=2$$

$$(1+\tan B)(1+\tan A)=2$$

$(1-\cot A)(1-\cot B)=2$

$\boxed{F}$ **Degree & Radian** ⇒

$$1^\circ \rightarrow 60'$$

$$1' \rightarrow 60''$$

$$1^\circ \rightarrow 3600''$$

⇒ $\pi^c = 180^\circ$

$$1^\circ = \frac{\pi^c}{180^\circ} \qquad \therefore c = \text{Radian}$$

$$1^c = \frac{180}{\pi} \qquad \pi = \frac{22}{7}$$

Note ⇒ $1^c = x^\circ y' z''$

Now $\pi^c \rightarrow 180^\circ$

$$\frac{22}{7}^c \rightarrow 180^\circ$$

$$22^c \rightarrow 180\times 7$$

$$1^c \rightarrow \frac{180\times 7}{22}$$

$$1^c \rightarrow 57^\circ 16' 22''$$

(7) (i) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

(ii) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

(iii) $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

(8) (i) $A + B + C = 180^\circ$

$A + B = 180^\circ - C$

$\tan(A+B) = \tan(180 - C)$

$\dfrac{\tan A + \tan B}{1 - \tan A \tan B} = -\tan C$

$\tan A + \tan B = -\tan C + \tan A \tan B \tan C$

$\tan A + \tan B + \tan C = \tan A \tan B \tan C$ ------(1)

(ii) from eq^n (1)

$\tan A + \tan B + \tan C = \tan A \tan B \tan C$

$\dfrac{1}{\cot A} + \dfrac{1}{\cot B} + \dfrac{1}{\cot C} = \dfrac{1}{\cot A \cot B \cot C}$

$\cot A \cot B + \cot B \cot C + \cot C \cot A = 1$

(iii) $A + B + C = 90^\circ$

$A + B = 90^\circ - C$

$\tan(A+B) = \tan(90^\circ - C)$

$\dfrac{\tan A + \tan B}{1 - \tan A \tan B} = \dfrac{1}{\tan C}$ $\quad \tan(90^\circ - C) = \cot C$

$\tan A \tan C + \tan B \tan C = 1 - \tan A \tan B$

$\tan A \tan C + \tan B \tan C + \tan A \tan B = 1$ ------(1)

Now from eq^n (1)

$\tan A \tan C + \tan B \tan C + \tan A \tan B = 1$

divide by $\tan A \tan B \tan C$

$\dfrac{1}{\tan B} + \dfrac{1}{\tan A} + \dfrac{1}{\tan C} = 1$

$\cot C + \cot A + \cot B = \cot A \cot B \cot C$

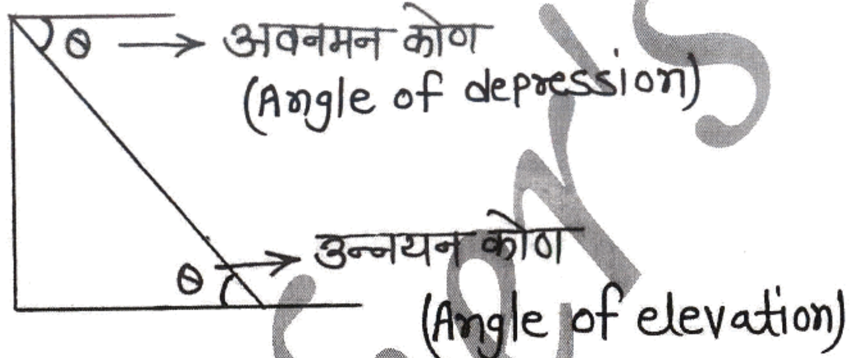
Note ⇒

(1) एक Particular(विशेष) Time में

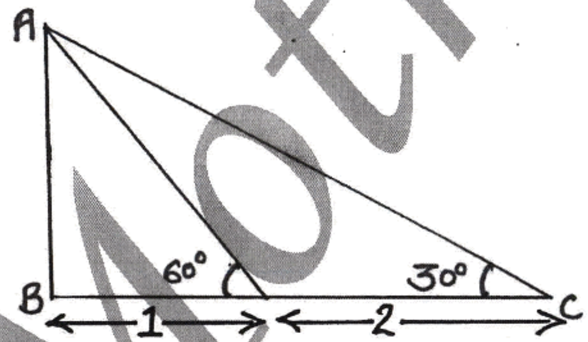
angle ⇒ $\left[\text{Hour} \times 30 - \text{minute} \times \frac{1}{2}\right]$

(2) Radius के equal arc द्वारा center पर बनाया गया कोण 1^c होता हैं।

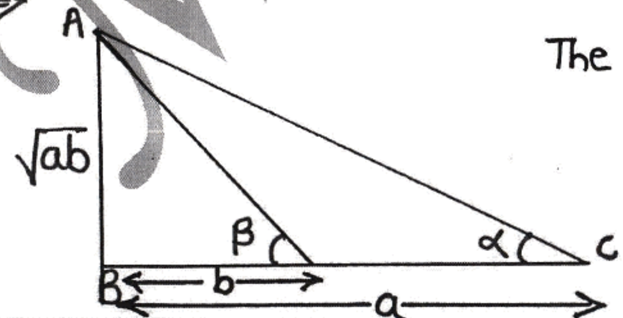
G Height & Distance (ऊँचाई और दूरी) ⇒



⇒



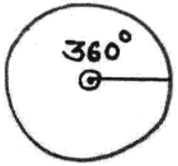
⇒



The Height $AB = \sqrt{ab}$

$[\because \alpha + \beta = 90°]$

⇒



circumference (परिधि) = 2π

$2\pi r \longrightarrow \text{Angle} = 2\pi^c$

$1 \longrightarrow \text{Angle} = \dfrac{2\pi^c}{2\pi r}$

$r \longrightarrow \text{Angle} = \dfrac{2\pi^c}{2\pi r} \times r = 1^c$

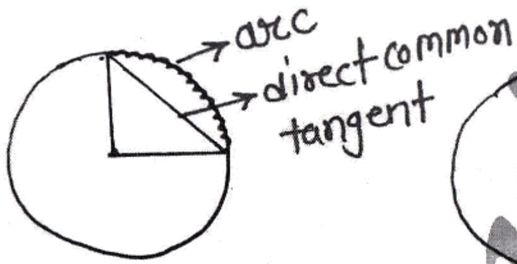
if $l = r$

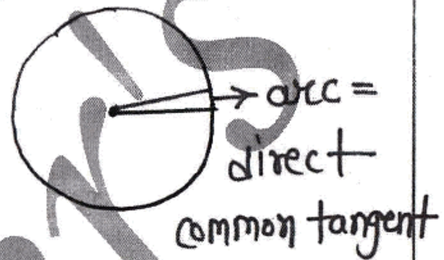
$l \longrightarrow \dfrac{2\pi}{2\pi r} \times l$

$\text{Angle} = \dfrac{l^c}{r} = \dfrac{\text{चाप}}{\text{त्रिज्या}} = \dfrac{\text{Arc}}{\text{Radius}}$

⇒ अगर कोण बहुत छोटा हो तो चाप और कोण के सामने की भुजा की लम्बाई समान होगी।

if angle is less than 1° then opposite side of angle is equal to length of arc.





⇒ घंटे की सुई 1 घंटे में 30° का कोण बनाती है।

Hand of Hour makes angle in 1 Hour is 30°.

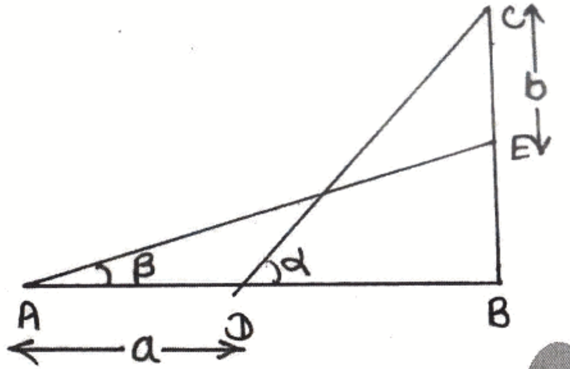
⇒ मिनट की सुई 1 मिनट में 6° का कोण बनाती है।

Hand of minute makes angle in 1 minute is 6°.

⇒ घंटे की सुई 1 मिनट में $\frac{1}{2}°$ का कोण बनाती है।

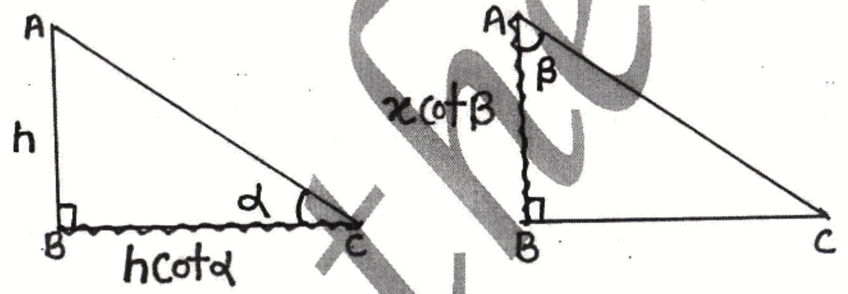
Hand of Hour makes angle in 1 minute is $\frac{1}{2}°$.

⇛



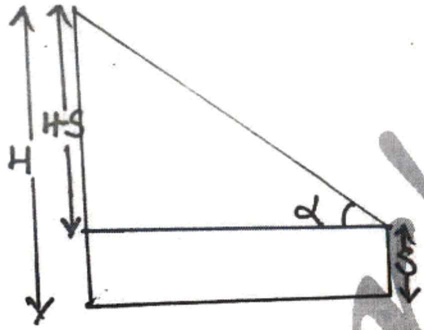
$a = b\tan\left(\frac{\alpha+\beta}{2}\right)$

⇛



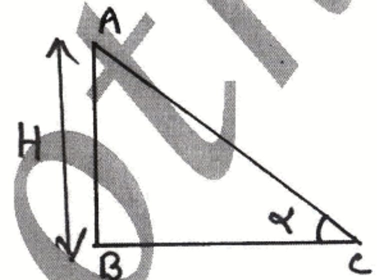
⇛ यदि प्रक्षेप की लम्बाई 5cm है और मीनार की ऊँचाई H सेमी हो तो प्रेक्षक की आँख से मीनार की चोटी का उन्नयन कोण α हो।

if Height of the observer 5cm & the height of the tower is Hcm & the elevation angle from the eye of the observer to the peak of tower is α°.



⇛ अगर प्रेक्षक की लम्बाई नहीं दी हो।

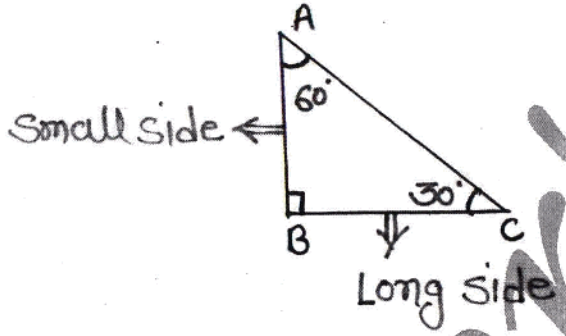
if height of observer are not given.



⇛ किसी भी प्रतिबिम्बीय सतह से वस्तु जितनी ऊँचाई पर होता हैं प्रतिबिम्ब उतना ही नीचे बनता है।

The distance b/w object & Reflecting surface is same as distance b/w image & Reflecting surface.

⇒



⇒ कोण 30° के पास की भुजा बडी होती हैं व कोण 60° के पास वाली भुजा छोटी होती है।

side adjacent to angle 30° is always greater than side adjacent to angle 60°

⇒ 90° के सामने की भुजा हमेशा 30° के सामने वाली भुजा की दोगुनी होगी।

side which is opposite to angle 90° is always double then the side which is opposite to angle 30°

Note ⇒

(i) बडी भुजा बनाने के लिए $\sqrt{3}$ से गुणा।

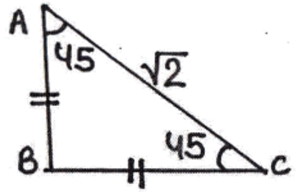
Multiply by $\sqrt{3}$ for find out greater side.

(ii) छोटी भुजा बनाने के लिए $\sqrt{3}$ से भाग।

Divide by $\sqrt{3}$ for Find out smaller side.

⇒ अगर कोई कोण 45° हो तो दो भुजा बराबर होती है तथा कर्ण प्रत्येक भुजा का $\sqrt{2}$ होता हैं

if there is a angle of 45° then two sides of a triangle are equal & hypotenius is $\sqrt{2}$ times of each side.



[H] Maxima & Minima ⇒

$\sin\theta = [-1, 1]$ $\qquad \sin^2\theta = [0, 1]$

$\cos\theta = [-1, 1]$ $\qquad \cos^2\theta = [0, 1]$

$\tan\theta = (-\infty, \infty)$ $\qquad \tan^2\theta = (0, \infty)$

$\cot\theta = (-\infty, \infty)$ $\qquad \cot^2\theta = (0, \infty)$

$\sec\theta = R - (-1, 1)$ $\qquad \sec^2\theta = R^+ - (0, 1)$ या $1 - \infty$

$\text{cosec}\,\theta = R - (-1, 1)$ $\qquad \text{cosec}^2\theta = R^+ - (0, 1)$ या $1 - \infty$

⇒ $a\sin\theta + b\cos\theta$

Max. ⇒ $\sqrt{a^2+b^2}$

Min. ⇒ $-\sqrt{a^2+b^2}$

Proof ⇒ $f(x) = a\sin\theta + b\cos\theta$

Now $\quad a = r\cos x$

$a^2 = r^2\cos^2 x$ ------- (1)

$b = r\sin x$

$b^2 = r^2\sin^2 x$ ------ (2)

add eq^n (1) & (2)

$a^2 + b^2 = r^2[1]$

$r^2 = \sqrt{a^2+b^2}$

Put $f(\theta)$ value of a & b

$f(\theta) = r\cos x\sin\theta + r\sin x\cos\theta$

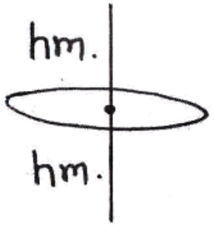
$f(\theta) = r\sin(\theta + x)$

$-1 \le \sin(\theta + x) \le 1$

for function

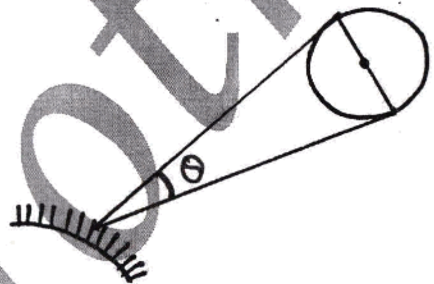
$-r \le r\sin(\theta + x) \le r$

$-\sqrt{a^2+b^2} \le f(\theta) \le \sqrt{a^2+b^2}$



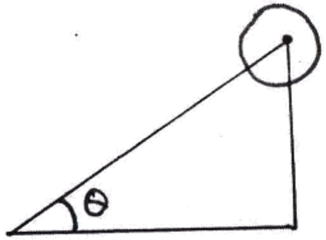
⇛ गुब्बारे से व्यक्ति की आँख पर बना कोण θ हो तो

The angle made by diametrically opposite points of ballon on observer's eye is θ.



⇛ गुब्बारे से समतल पर θ कोण बने तो

if the angle made by center of ballon on a flat surface is θ.



⇛ किसी भी समान ऊँचाई के टावर से एक ही बिन्दु पर अलग - अलग कोण बने तो।

if two towers of same height are standing opposite to each other than the point in b/w them from where their angles of elevation are 30° & 60° respectivelly are at a distance of 3:1 from their bases.